

Partial Differential Equations
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Lecture notes T_EXed by Thomas Eingartner

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1 Introduction

A differential equation is an equation of a function and its derivatives.

Example 1.1

$$f : \mathbb{R} \rightarrow \mathbb{R}$$

$$\begin{cases} f(t) = af(t) \text{ for all } t \geq 0, a \in \mathbb{R} \\ f(0) = a_0 \end{cases}$$

Solution: $f(t) = a_0 e^{at}$ for all $t \geq 0$. This is a linear ODE (Ordinary differential equation).

Example 1.2 (Non-Linear ODE)

$$f : \mathbb{R} \rightarrow \mathbb{R}$$

$$\begin{cases} f'(t) = 1 + f^2(t) \\ f(0) = 1 \end{cases}$$

A PDE (Partial Differential Equation) is an equation of a function of 2 or more variables and its derivatives. Recall $\Omega \subseteq \mathbb{R}^d$, $f : \Omega \rightarrow \{\mathbb{R}, \mathbb{C}\}$ open,

$$\partial_{x_1} f(x) = \lim_{h \rightarrow 0} \frac{f(x + he_1) - f(x)}{h}, e_1 = (0, 0, 1, \dots, 0, 0) \in \mathbb{R}^d$$

$$D^\alpha f(x) = \partial_{x_1}^{\alpha_1} \cdots \partial_{x_d}^{\alpha_d} f(x), |\alpha| = \sum_{i=1}^d |\alpha_i|$$

$$Df = \nabla f = \text{gradient of } f = (\partial_{x_1} \dots \partial_{x_d})$$

$$\Delta f = \partial_{x_1}^2 + \cdots + \partial_{x_d}^2 f$$

$$D^k f = (D^\alpha f)_{|\alpha|=k}$$

$$D^2 f = (\partial_{x_i} \partial_{x_j} f)_{1 \leq i, j \leq d}$$

Goals: For *solving a PDE* we want to

- Find an explicit solution! In many cases, it is impossible
- Prove a *well-posed theory* (existence of solutions, uniqueness of solutions, continuous dependence of solutions on the data)