CHAPTER # 8

RANDOM VARIABLES

8.1 RANDOM VARIABLES

An experiment is a well define action of obtaining an observation. Experiments in which the outcomes vary from trial to trial are called random experiment. The individual outcomes of a random experiment are said to be sample points. The set of all possible outcomes of a random experiment is called sample space.

Generally, we are not interested associated with each sample point, but only some number associated with these points. For example, when a coin is tossed two times, there are four possible outcomes, the sample space of the experiment is

$S = \{HH, HT, TH, TT\}$

If we are interested in the number of head appears, then a numerical value of 0, 1, 2 will be assigned to each sample point. Such a numerical quantity whose value is determined by the outcome of a random experiment, is called a random variable. In other words we can say A random variable is a function which assigns a numerical value to each simple event in a sample space.

A random variable is also called a chance variable, a stochastic variable and is abbreviated as r.v. The random variables are usually denoted by capital letters such as X, Y. A random variable can be either discrete or continuous.

8.2 DISCRETE RANDOM VARIABLE AND ITS PROBABILITY DISTRIBUTION

A random variable x is said to be discrete if it can assume only a limited number of values, which can be listed. For example, the number of heads obtained in coin tossing experiments, the number of fatal accidents etc.

Let X be a discrete r.v. taking value $x_1, x_2, \ldots, x_n$. Then the function, denoted by P(x) and defined by

$P(x) = f(x) = P(X = x_i)$

For $i = 1, 2, \ldots, n$ is called the probability function of the r.v.x.

A probability distribution must satisfy the following two properties.

(i) $f(x_i) \geq 0$

(ii) $\Sigma f(x_i) = 1$

EXAMPLE 8.1

Find the probability distribution for the number of heads, when 3 coins are tossed. Also draw probability Histogram and line graph.

SOLUTION

Total number of outcomes $= 2^3 = 8$

S = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

Let X be the r.v. that denotes the number of heads. Then x can take values 0, 1, 2 and 3 and their probabilities are

$$f(0) = P(X = 0) = P[\{TTT\}] = \frac{1}{8}$$

$$f(1) = P(X = 1) = P[\{HTT, THT, TTH\}] = \frac{3}{8}$$

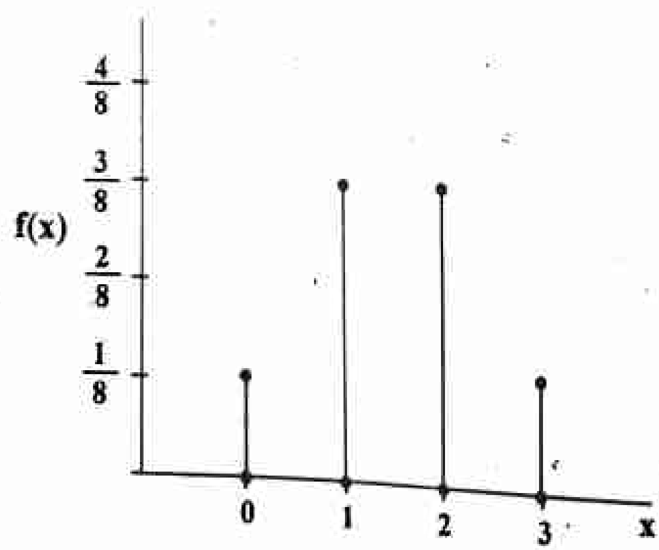
$$f(2) = P(X = 2) = P[\{HHT, HTH, THH\}] = \frac{3}{8}$$

$$f(3) = P(X = 3) = P[\{HHH\}] = \frac{1}{8}$$

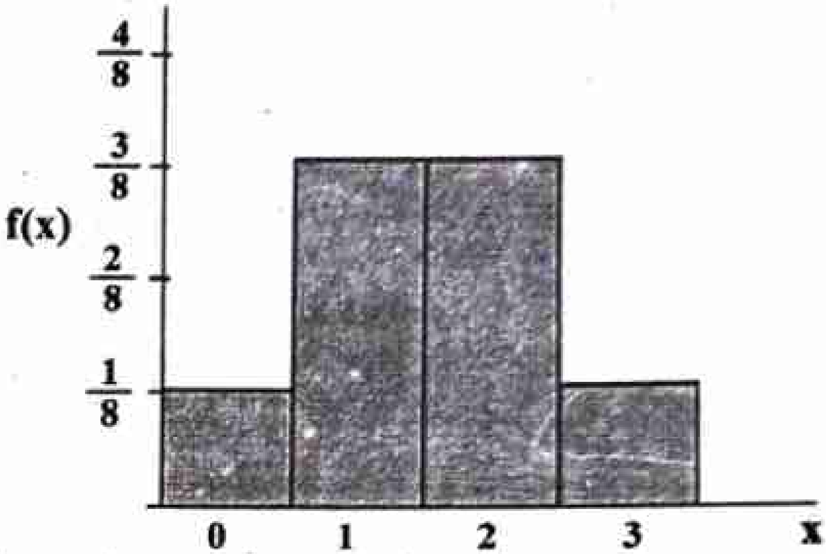
Probability distribution of No. of heads.

No. of Heads (x_i)	0	1	2	3
Probability $f(x_i)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(i) **Line Graph of Probability Distribution**



(ii) Histogram of Probability Distribution



EXAMPLE 8.2

Find the probability distribution of the sum of the dots when two fair dice are rolled.

SOLUTION

Total number of outcomes = $6^2 = 36$

The sample space of this experiment is

$$S = \left\{\begin{array}{l}(1,1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) \\ (2,1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\ (3,1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \\ (4,1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \\ (5,1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \\ (6,1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\end{array}\right\}$$

Let X be the random variable that denotes the sum of dots. Then x can take values 2, 3, 4, ..., 12.

$$f(2) = P(X = 2) = P[\{(1, 1)\}] = \frac{1}{36}$$

$$f(3) = P(X = 3) = P[\{(1, 2), (2, 1)\}] = \frac{2}{36}$$

$$f(4) = P(X = 4) = P[\{(1, 3), (2, 2), (3, 1)\}] = \frac{3}{36}$$

$$f(5) = P(X = 5) = P[\{(1, 4), (2, 3), (3, 2), (4, 1)\}] = \frac{4}{36}$$

$$f(6) = P(X = 6) = P[\{(1,5), (2,4), (3,3), (4,2), (5,1)\}] = \frac{5}{36}$$

$$f(7) = P(X = 7) = P[\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}] = \frac{6}{36}$$

$$f(8) = P(X = 8) = P[\{(2,6), (3,5), (4,4), (5,3), (6,2)\}] = \frac{5}{36}$$

$$f(9) = P(X = 9) = P[\{(3, 6), (4, 5), (5, 4), (6, 3)\}] = \frac{4}{36}$$

$$f(10) = P(X = 10) = P[\{(4, 6), (5, 5), (6, 4)\}] = \frac{3}{36}$$

$$f(11) = P(X = 11) = P[\{(5, 6), (6, 5)\}] = \frac{2}{36}$$

$$f(12) = P(X = 12) = P[\{6, 6\}] = \frac{1}{36}$$

Probability distribution of the r.v. X

x_i	2	3	4	5	6	7
$f(x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$
x_i	8	9	10	11	12	
$f(x_i)$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	

EXAMPLE 8.3

Three balls are drawn from a bag containing 5 white, 3 black balls. If x denotes the number of white balls drawn. Find the probability distribution of x.

SOLUTION

$$\text{Total No. of outcomes} = \binom{8}{3} = 56$$

Let X be a.r.v that denotes the no. of white balls, then x can take values 0, 1, 2, 3.

$$f(0) = P(X = 0) = \frac{\binom{5}{0}\binom{3}{3}}{\binom{8}{3}} = \frac{1 \times 1}{56} = \frac{1}{56}$$

$$f(1) = P(X = 1) = \frac{\binom{5}{1}\binom{3}{2}}{\binom{8}{3}} = \frac{5 \times 3}{56} = \frac{15}{56}$$

$$f(2) = P(X = 2) = \frac{\binom{5}{2}\binom{3}{1}}{\binom{8}{3}} = \frac{10 \times 3}{56} = \frac{30}{56}$$

$$f(3) = P(X = 3) = \frac{\binom{5}{3}\binom{3}{0}}{\binom{8}{3}} = \frac{10 \times 1}{56} = \frac{10}{56}$$

Probability distribution of r.v.X

x	0	1	2	3
f(x)	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$

8.3 CONTINUOUS RANDOM VARIABLE AND ITS PROBABILITY DENSITY FUNCTION

A random variable X is said to be continuous if it can assume any value within a given range. For example, weight of a person, the amount of rainfall, the temperature at a certain place etc.

The function f(x) is called probability density function abbreviated as p.d.f or simply density function of the r.v.x.

A function f(x) which satisfy the following conditions is called probability density function.

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

EXAMPLE 8.4

A continuous random variable x can assume values between 0 and 2 have a density function.

$f(x) = A(2 - x)(2 + x)$

Find (i) The value of A

(ii) $P\left(x = \frac{1}{2}\right)$

(iii) $P(x \leq 1)$

(iv) $P(x \geq 2)$

(v) $P(1 \leq x \leq 2)$

SOLUTION

(i) To find the value of A, we must have

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{0}^{2} A(2 - x)(2 + x)\, dx$$

$$A\int_{0}^{2} (4 - x^2)\, dx = 1$$

$$A\left|\frac{4x^1}{1} - \frac{x^3}{3}\right|_0^2 = 1$$

$$A\left[\left\{4(2) - \frac{(2)^3}{3}\right\} - \left\{4(0) - \frac{(0)^3}{3}\right\}\right] = 1$$

$$A\left[\left(8 - \frac{8}{3}\right) - (0)\right] = 1$$

$$A\left[\frac{24 - 8}{3}\right] = 1$$

$$16A = 3$$

$$A = \frac{3}{16}$$

After substituting the value of A, function f(x) becomes

$$f(x) = \frac{3}{16}(2 - x)(2 + x)\ ,\ 0 \le x \le 2$$

(ii) $P\left(X = \frac{1}{2}\right) = 0$, (as probability at a particular value is 0)

(iii) $P(x \le 1) = \frac{3}{16}\int_0^1 (4 - x^2)\,dx$

$$= \frac{3}{16}\left[4x - \frac{x^3}{3}\right]_0^1$$

$$= \frac{3}{16}\left[4(1) - \frac{(1)^3}{3}\right] = \frac{3}{16}\left(4 - \frac{1}{3}\right)$$

$$= \frac{3}{16}\left[\frac{12 - 1}{3}\right]$$

$$= \frac{3}{16} \times \frac{11}{3} = \frac{11}{16}$$

(iv) $P(x \ge 2) = 0$, (as outside the range 0 to 2)

(v) $P(1 \le x \le 2) = \frac{3}{16}\int_1^2 (4 - x^2)\,dx$

$$= \frac{3}{16}\left[4x - \frac{x^3}{3}\right]_0^1$$

$$= \frac{3}{16}\left[\left\{4(2) - \frac{(2)^3}{3}\right\} - \left\{4(1) - \frac{(1)^3}{3}\right\}\right]$$

$$= \frac{3}{16}\left[\left\{8 - \frac{8}{3}\right\} - \left\{4 - \frac{1}{3}\right\}\right]$$

$$= \frac{3}{16}\left[\left\{\frac{24 - 8}{3}\right\} - \left\{\frac{12 - 1}{3}\right\}\right]$$

$$= \frac{3}{16}\left[\frac{16}{3} - \frac{11}{3}\right]$$

$$= \frac{3}{16}\left(\frac{16 - 11}{3}\right)$$

$$= \frac{3}{16} \times \frac{5}{3} = \frac{5}{16}$$

EXAMPLE 8.5

A continuous random variable x with P.d.f. is given below.

$f(x) = K(x + 30)\,,\; 2 \leq x \leq 8$

Where K is constant, find

(i) Value of K

(ii) $P(3 \leq x \leq 6)$

(iii) $P(x \leq 6)$

(iv) $x \geq 4)$

SOLUTION

(i) To find the value of K, we must have

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{2}^{8} K(x + 30) = 1$$

$$K \left| \frac{x^2}{2} + 30x \right|_{2}^{8} = 1$$

$$K\left[\left\{\frac{(8)^2}{2} + 30(8)\right\} - \left\{\frac{(2)^2}{2} + 30(2)\right\}\right] = 1$$

$$K\left[\left\{\frac{64}{2} + 240\right\} - \left\{\frac{4}{2} + 60\right\}\right] = 1$$

$$K\,[\{32 + 240\} - \{2 + 60\}] = 1$$

$$K\,[272 - 62] = 1$$

$$210\,K = 1$$

$$K = \frac{1}{210}$$

$$f(x) = \frac{1}{210}(x + 30) \qquad 2 \le x \le 8$$

(ii) $$P(3 \le x \le 6 = \int_3^6 \frac{1}{210}(x + 30)\,dx$$

$$= \frac{1}{210}\int_3^6 (x + 30)\,dx$$

$$= \frac{1}{210}\left| \frac{x^2}{2} + 30x \right|_3^6$$

$$= \frac{1}{210}\left[\left\{\frac{(6)^2}{2} + 30(6)\right\} - \left\{\frac{(3)^2}{2} + 30(3)\right\}\right]$$

$$= \frac{1}{210}\left[(18 + 180) - \left(\frac{9}{2} + 90\right)\right]$$

$$= \frac{1}{210}\left[198 - \left(\frac{9 + 180}{2}\right)\right]$$

$$= \frac{1}{210}\left[198 - \frac{189}{2}\right]$$

$$= \frac{1}{210}\left[\frac{396 - 189}{2}\right]$$

$$= \frac{1}{210}\left[\frac{207}{2}\right] = \frac{207}{420}$$

(iii) $$P(x \le 6) = \int_2^6 \frac{1}{210}(x + 30)\,dx$$

$$= \frac{1}{210}\int_2^6 (x + 30)\,dx$$

$$= \frac{1}{210}\left| \frac{x^2}{2} + 30x \right|_2^6$$

$$= \frac{1}{210}\left[\left\{\frac{(6)^2}{2} + 30(6)\right\} - \left\{\frac{(2)^2}{2} + 30(2)\right\}\right]$$

$$= \frac{1}{210}\ [(18 + 180) - (2 + 60)]$$

$$= \frac{1}{210}\ [198 - 62]$$

$$= \frac{1}{210}\ (136) = \frac{136}{210} = \frac{68}{105}$$

(iv) $$P(x \geq 4) = \frac{1}{210}\int_{4}^{8} (x + 30)\, dx$$

$$= \frac{1}{210}\left|\frac{x^2}{2} + 30x\right|_{4}^{8}$$

$$= \frac{1}{210}\left[\left\{\frac{(8)^2}{2} + 30(8)\right\} - \left\{\frac{(4)^2}{2} + 30(4)\right\}\right]$$

$$= \frac{1}{210}\ [(32 + 240) - (8 + 120)]$$

$$= \frac{1}{210}\ [272 - 128]$$

$$= \frac{1}{210}\ (144) = \frac{144}{210} = \frac{72}{105}$$

8.4 MATHEMATICAL EXPECTATION OF A RANDOM VARIABLE

Let X be a discrete random variable taking values $x_1, x_2, \ldots x_n$ with corresponding probabilities $f(x_1), f(x_2), \ldots, f(x_n)$. Then the expectation of a random variable X is denoted by E(X) and defined as

$$E(X) = x_1 f(x_1) + x_2 f(x_2) + \ldots, + x_n f(x_n)$$

$$E(X) = \Sigma x_i f(x_i) \qquad \text{(For discrete variable)}$$

$$E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx \qquad \text{(For continuous variable)}$$

8.4.1 Properties of Expected Values

(i) If a is constant, then $E(a) = a$

(ii) $E(ax) = a\,E(x)$

(iii) $E(ax + c) = a\,E(x) + c$

(iv) $E(x \pm y) = E(x) \pm E(y)$

(v) $E(x\,y) = E(x)\,.\,E(y)$ (If x and y are independent)

EXAMPLE 8.6

What is the mathematical expectation of the number of heads when two coins are tossed.

SOLUTION

Total number of outcomes $= 2^2 = 4$

$S = \{HH, HT, TH, TT\}$, $n(s) = 4$

Let X be the r.v. that denotes the no. of heads, then x can take values 0, 1 and 2.

$$f(0) = P(X = 0) = P[\{TT\}] = \frac{1}{4}$$

$$f(1) = P(X = 1) = P[\{HT, TH\}] = \frac{2}{4}$$

$$f(2) = P(X = 2) = P[\{HH\}] = \frac{1}{4}$$

X	f(x)	xf(x)
0	$\frac{1}{4}$	0
1	$\frac{2}{4}$	$\frac{2}{4}$
2	$\frac{1}{4}$	$\frac{2}{4}$
Σ	1	$\frac{4}{4} = 1$

$$E(x) = \Sigma x\, f(x) = \frac{4}{4} = 1$$

EXAMPLE 8.7

If $f(x) = \dfrac{6 - |7 - x|}{36}$ for $x = 2, 3, \ldots, 12$, then find the mean and variance of the random variable x.

SOLUTION

Calculation of mean and variance

x	$f(x) = \frac{6 - \lvert 7 - x \rvert}{36}$	$x\,f(x)$	$x^2 f(x)$
2	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$
3	$\frac{2}{36}$	$\frac{6}{36}$	$\frac{18}{36}$
4	$\frac{3}{36}$	$\frac{12}{36}$	$\frac{48}{36}$
5	$\frac{4}{36}$	$\frac{20}{36}$	$\frac{100}{36}$
6	$\frac{5}{36}$	$\frac{30}{36}$	$\frac{180}{36}$
7	$\frac{6}{36}$	$\frac{42}{36}$	$\frac{294}{36}$
8	$\frac{5}{36}$	$\frac{40}{36}$	$\frac{320}{36}$
9	$\frac{4}{36}$	$\frac{36}{36}$	$\frac{324}{36}$
10	$\frac{3}{36}$	$\frac{30}{36}$	$\frac{300}{36}$
11	$\frac{2}{36}$	$\frac{22}{36}$	$\frac{242}{36}$
12	$\frac{1}{36}$	$\frac{12}{36}$	$\frac{144}{36}$
Σ	1	$\frac{252}{36} = 7$	$\frac{1974}{36}$

$$\text{Mean} = E(x) = \Sigma x\, f(x) = 7$$

$$\begin{aligned}
\text{Var}(x) &= E(x^2) - [E(x)]^2 \\
&= \Sigma x^2 f(x) - [E(x)]^2 \\
&= \frac{1974}{36} - 49 \\
&= \frac{1974 - 1764}{36} = \frac{210}{36} = 5.83
\end{aligned}$$

EXAMPLE 8.8

Construct a probability distribution based on the following frequency distribution.

Outcome	102	105	108	111	114	117
Frequency	10	20	45	15	20	15

Also compute the expected value of the outcome.

SOLUTION

Outcome (x)	Frequency	f(x)	x f(x)
102	10	$\frac{10}{125} = 0.08$	8.16
105	20	$\frac{20}{125} = 0.16$	16.80
108	45	$\frac{45}{125} = 0.36$	38.88
111	15	$\frac{15}{125} = 0.12$	13.32
114	20	$\frac{20}{125} = 0.16$	18.24
117	15	$\frac{15}{125} = 0.12$	14.04
Σ	125		109.44

Expected outcome $= E(x) = \Sigma x\, f(x) = 109.44$

EXAMPLE 8.9

Given the following discrete probability distribution.

x	0	1	2	3	4	5
f(x)	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

Compute mean, variance, S.D and C.V.

SOLUTION

x	f(x)	x f(x)	x^2 f(x)
0	$\frac{6}{36}$	0	0
1	$\frac{10}{36}$	$\frac{10}{36}$	$\frac{10}{36}$
2	$\frac{8}{36}$	$\frac{16}{36}$	$\frac{32}{36}$
3	$\frac{6}{36}$	$\frac{18}{36}$	$\frac{54}{36}$
4	$\frac{4}{36}$	$\frac{16}{36}$	$\frac{64}{36}$
5	$\frac{2}{36}$	$\frac{10}{36}$	$\frac{50}{360}$
Σ	1	$\mathbf{\frac{70}{36}}$	$\mathbf{\frac{210}{36}}$

$$\text{Mean} = E(x) = \Sigma x\, f(x) = \frac{70}{36} = 1.94$$

$$E(x^2) = \Sigma x^2 f(x) = \frac{210}{36} = 5.83$$

$$\text{Var}(x) = E(x^2) - [E(x)]^2$$

$$= 5.83 - (1.94)^2 = 2.07$$

$$\text{S.D.} = \sqrt{\text{var}(x)} = 1.44$$

$$\text{C.V.} = \frac{S}{\bar{x}} \times 100$$

$$= \frac{1.44}{1.94} \times 100 = 74.16\%$$

EXAMPLE 8.10

A variable x has P.d.f.

$f(x) = 20x^3(1 - x) \qquad 0 < x < 1$

Find mean and variance.

SOLUTION

$$\text{Mean} = E(x) \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= 20 \int_{0}^{1} x \cdot x^3 (1 - x)\, dx$$

$$= 20 \int_{0}^{1} (x^4 - x^5)\, dx$$

$$= 20 \left| \frac{x^5}{5} - \frac{x^6}{6} \right|_{0}^{1}$$

$$= 20\left[\frac{1}{5} - \frac{1}{6}\right] = \frac{20}{30} = \frac{2}{3}$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2\, f(x)\, dx$$

$$= 20 \int_{0}^{1} x^2 \cdot x^3 (1 - x)\, dx$$

$$= 20 \int_{0}^{1} (x^5 - x^6)\, dx$$

$$= 20 \left| \frac{x^6}{6} - \frac{x^7}{7} \right|_{0}^{1}$$

$$= 20\left[\frac{1}{6} - \frac{1}{7}\right]$$

$$= \frac{20}{42} = \frac{10}{21}$$

$$\text{Var}(x) = E(x^2) - [E(x)]^2$$

$$= \frac{10}{21} - \left(\frac{2}{3}\right)^2 \quad \Rightarrow \quad = \frac{10}{21} - \frac{4}{9} = \frac{2}{63}$$

EXAMPLE 8.11

If it rains, an umbrella saleman can earn Rs. 200 per day. If it is fair he can lose Rs. 50 per day. What is his expectation if the probability of rain is 0.4?

SOLUTION

Let x denote the number of rupees the saleman earns. Then x is a r.v. with possible values 200 and –50, with corresponding probabilities are 0.4 and 0.6 respectively.

$$E(x) = (200 \times 0.4) + (-50 \times 0.6)$$

$$= 80 - 30 = 50$$

$$= \text{Rs. } 50 \text{ per day}$$

EXAMPLE 8.12

A and B throw with one die for a prize of Rs. 55, which is to be won by the player who first throws 1. If A has the first throw, what are their expectations?

SOLUTION

The probability of throwing a 1 with one die is $\frac{1}{6}$ and that of not throwing a 1 is $\frac{5}{6}$.

As A has the first throw, therefore he can win in the first, third, fifth, ... throws.

$$P(A) = \frac{1}{6} + \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \ldots\ldots$$

$$a = \frac{1}{6}$$

$$r = \frac{\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)}{\left(\frac{1}{6}\right)} = \left(\frac{5}{6}\right)^2$$

$$S_\infty = \frac{a}{1 - r}$$

$$= \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{\frac{1}{6}}{\frac{11}{36}}$$

$$= \frac{1}{6} \times \frac{36}{11} = \frac{6}{11}$$

The probability that B wins = 1 – P(A)

$$= 1 - \frac{6}{11} = \frac{5}{11}$$

Hence A's expectation $= \frac{6}{11} \times 55 =$ Rs. 30

B's expectation $= \frac{5}{11} \times 55 =$ Rs. 25

EXAMPLE 8.13

A random variable "x" has the following probability distribution.

x_i	0	1	2	3	4	5
$f(x_i)$	0.10	0.25	0.30	0.20	0.10	0.05

Find (i) E(x), E(x^2) and var (x)

(ii) Construct probability distribution y = 3x + 2 then find E(y), E(y^2) and var (y).

(iii) What is the relation between E(x) and E(y) and var (x) and var (y)

SOLUTION

(i)

x_i	$f(x_i)$	$x_i \cdot f(x_i)$	$x_i^2 f(x_i)$
0	0.10	0	0
1	0.25	0.25	0.25
2	0.30	0.60	1.20
3	0.20	0.60	1.80
4	0.10	0.40	1.60
5	0.05	0.25	1.25
Σ	1.00	2.10	6.10

$E(x) = \Sigma x\, f(x) = 2.10$

$E(x^2) = \Sigma x^2 f(x) = 6.10$

$Var(x) = E(x^2) - [E(x)]^2$

$= 6.10 - (2.10)^2 = 1.69$

(ii)

$y = 3x + 2$	$f(y)$	$y\,f(y)$	$y^2 f(y)$
2	0.10	0.20	0.40
5	0.25	1.25	6.25
8	0.30	2.40	19.20
11	0.20	2.20	24.20
14	0.10	1.40	19.60
17	0.05	0.85	14.45
Σ	**1.10**	**8.30**	**84.10**

$$E(y) = \Sigma y\,f(y) = 8.30$$

$$E(y^2) = \Sigma\, y^2 f(y) = 84.10$$

$$\begin{aligned} var(y) &= E(y^2) - [E(y)]^2 \\ &= 84.10 - (8.30)^2 \\ &= 15.21 \end{aligned}$$

(iii) $y = 3x + 2$

$$\begin{aligned} E(y) &= 3E(x) + 2 \\ &= 3(2.10) + 2 \\ &= 6.30 + 2.0 = 8.30 \end{aligned}$$

$$y = 3x + 2$$

$$\begin{aligned} var\,(y) &= 9\,var\,(x) \\ &= 9(1.69) = 15.21 \end{aligned}$$

SUMMARY

APPLICATION	FORMULA
Probability function	$f(x) = P(x = x_i)$ Where $i = 1, 2, 3 \ldots\ldots n$
Properties of probability distribution	(i) $f(x) \geq 0$ (ii) $\Sigma f(x) = 1$
Probability density function	A function is said to be Pdf if it satisfy the following conditions (i) $f(x) \geq 0$ (ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$
Mathematical Expectation i.e. (Mean) Variance	$E(X) = \Sigma x f(x)$ Variance $= E(x^2) - [E(x)]^2$ Where, $E(x) = \Sigma x f(x)$ $E(x^2) = \Sigma x^2 f(x)$

EXERCISES

8.1 (a) What do you meant by Random Variable?

(b) A die is rolled. Let x denote the number of dots. Write down the possible outcomes and the values assigned to random variable. Find its probability distribution.

8.2 (a) Define discrete random variable and its probability distribution. Write down two basic properties of probability distribution.

(b) From an urn containing 4 red and 6 white marbles. A man draws three marbles at random with out replacement. Find the probability distribution of number of red marbles.

8.3 A committed size 5 is to be selected at random from 3 women and 5 men. Find the probability distribution of number of women on the committee.

8.4 Given the discrete probability distribution

$$f(x) = \binom{4}{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{4-x} \quad ; \; x = 0, 1, 2, 3, 4$$

(i) Find probability distribution.

(ii) Calculate mean and variance.

8.5. Three coins are tossed. Find the mean and variance of number of heads.

8.6 Two unbiased dice are rolled. Find the probability distribution of sum of dots. Also calculate its mean and variance.

8.7 Two dice are rolled. Find the expected value of the absolute difference of the number on two dice.

8.8 Construct a probability distribution based on the following frequency distribution.

Outcome	2	4	6	8	10	12	14
Frequency	24	22	16	12	7	3	1

Compute Mean and variance of the distribution.

8.9 A coin is tossed 4 times. If x denote the number of heads? What is the probability distribution of x. Find the mean and variance of number of heads.

8.10 Find the probability distribution of number of girls in families with three children assuming equal probabilities for boys and girls.

8.11 (a) Define expectation. Write down its properties.

(b) Let x be a random variable with probability distribution.

x	-1	0	1	2	3
f(x)	0.125	0.50	0.20	0.05	0.125

(i) Find E(x) and var(x).

(ii) Find the P.d of r.v. $y = 2x + 1$, using this P.d of y, determine E(y) and var(y)

(iii) How E(x) and E(y) and var(x) and var(y) related?

8.12 A and B throw with one die for a prize of Rs. 66, which is to be won by the player who first throws 3. If A has the first throw, what are their respective expectations?

8.13 A, B and C cut a pack of card successively in the order mentioned. If the person who cuts a Heart first, receives Rs. 175? What are their expectations?

8.14 A man draws 2 balls from a bag containing 3 white and 5 black balls. If he receives Rs. 70.00 for every white ball, he draws and Rs. 7.00 for every black ball. Find his expectations.

8.15 A fair die is rolled. If x is the random variable, the spots on the top, then obtain the probability distribution of x and hence obtained the expected value of the random variable x.

8.16 (a) Define continuous random variable. What do you mean by probability density function?

(b) A continuous random variable has a P.d.f given below.

$$f(x) = \frac{2(1 + x)}{27} \quad ; \quad 2 \le x \le 5$$

(i) Show that $P(2 \le x \le 5) = 1$

(ii) Find $P(x < 4)$

(iii) Find $P(3 \le x \le 4)$.

8.17 Given the following function

$f(x) = A(4x - 2x^2) \qquad 0 \leq x \leq 2$

$= 0 \quad$ Otherwise

Calculate the value of A. So as f(x) may be probability density function. Also compute E(x) and Var (x).

8.18 A variable x has P.d.f.

$f(x) = \frac{3}{8}(x - 2)^2 \, , \; 0 \leq x \leq 2$

$= 0 \quad$ Otherwise

Find expected value of x and its standard deviation.