CHAPTER # 7

PROBABILITY THEORY

7.1 INTRODUCTION

The concept of probability plays an important role in all problems of science, business and every day life, which in any way involve an element of uncertainty.

It is mainly concerned with drawing inferences from uncertain experiments or situations. For example, when we make the statements that it is likely to rain today, Pakistan will win the Hockey World Cup, A train will be late or will be in time etc. We face such type of problems in our daily life. All these problems are uncertain in nature. A similar type of uncertainty occurs when we toss a coin, throw a dice or draw a card from well-shuffled pack of cards etc. The uncertainty in all these cases is measured in terms of probability.

Two French mathematicians Blaise Pascal and De Fermat in connection with gambling problems found the theory of probability. Later on Jacob Bernoulli, Abraham De Moivire and Laplace developed it. The modern probability theory was developed during the twenties and thirties of this century.

Probability is widely used in Economics, Psychology, Meteorology, Astronomy, Sociology, Physics and Insurance where risk and uncertainty are involved.

Now a day, probability theory is best understood through the application of set theory, therefore basic concepts, notations, and operations of set theory that are relevant to probability are given below.

7.2 SET

A set is a well-defined collection of distinct objects. The objects of the set are called elements. A set is denoted by capital letters A, B, C etc. Sets are always written between pair of brackets.

e.g. $A = \{1, 2, 3, 4, 5\}$

B = {Karachi, Lahore, Peshawar, Quetta}

Finite Set

A set is finite if it contains limited number of elements. *e.g.* The number of students in a college, the books in a library etc.

Infinite Set

A set is called infinite if it contains infinite number of elements *e.g.* the number of stars on the sky, the drops of water in a lake. etc.

Null or Empty Set

A set, which has no element, is called Null set or empty set. It is denoted by ϕ or { }.

e.g. A = {A male student admitted in girl's college}

Singleton Set

A set, which has only one element, is called singleton set.

e.g. $A = \{2\}$, $B = \{\text{Lahore}\}$

Subset

If every element of set A is also an element of set B, then set A is called subset of set B. It is denoted by $A \subset B$.

e.g. $A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4, 5\}$

Then $A \subset B$

Universal Set

A set, which contains all elements of all the sets under consideration, is called universal set. It is denoted by U or S.

e.g. $A = \{1, 3, 5\}$

$B = \{2, 4\}$

$\cup = \{1, 2, 3, 4, 5\}$

7.3 VENN DIAGRAM

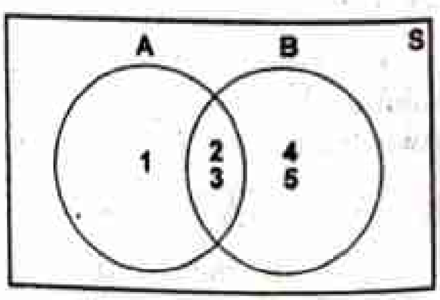
Venn diagram is a pictorial (Diagrammatic) way of representing sets and their relationship. The universal set "U" or "S" is shown by a rectangle. Inside the rectangle, circles show other sets.

e.g.

Let, $U = \{1, 2, 3, 4, 5\}$

$A = \{1, 2, 3\}$

$B = \{2, 3, 4, 5\}$



7.4 OPERATIONS ON SETS

The basic operations are union, intersection, difference and complement.

(i) Union of Sets

The union of two sets A and B is the set of all elements which belongs to either A or B or both A and B. It is denoted by $A \cup B$.

e.g. $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

Then $A \cup B = \{1, 2, 3, 4, 5\}$

(ii) Intersection of Sets

The intersection of two sets A and B is the set of those elements which are common in A and B.

It is denoted by $A \cap B$.

e.g. $A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

(iii) Difference of Sets

The difference of two sets A and B is denoted by "A–B" is the set of all elements which belong to A, but not to B.

e.g. $A = \{1, 2, 3, 4, 5\}$

$B = \{2, 5\}$

$A - B = \{1, 3, 4\}$

(iv) Complement of a Set

The complement of a set A denoted by $\bar{A}$ is a particular difference "S – A" *i.e.* the set of all those elements which belongs to S, but not to A.

e.g. $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3\}$

$$\bar{A} = S - A = \{1, 2, 3, 4, 5, 6\} - \{1, 2, 3\}$$
$$= \{4, 5, 6\}$$

7.5 PRODUCT SET

The product sets A and B denoted by $A \times B$ is a set of all ordered pairs (x, y), where x belongs to A and y belongs to B.

e.g. $A = \{1, 2, 3\}$, $B = \{5, 6\}$

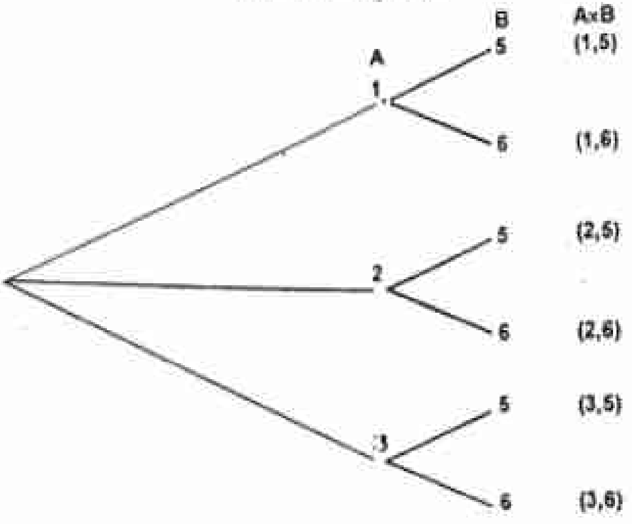
Then $A \times B = \{(1, 5), (1, 6), (2, 5), 2, 6), 3, 5), 3, 6)\}$

Similarly

$B \times A = \{(5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$

The product set $A \times B$ may be easily formed by means of "Tree Diagram"

Tree Diagram



Similarly Tree diagram for $B \times A$ my be formed.

Ordered Pair

An ordered pair consists of two elements say x and y, in which one of them (say x) is designated as first element and the other as a second element.

An ordered pair is denoted by (x, y)

7.6 TECHNIQUES OF COUNTING

Factorial

The numbers 1, 2, 3, ..., n are called natural numbers. The product of first "n" natural numbers is called "n" factorial. It is denoted by n!.

Mathematically,

$n! = n(n - 1)(n - 2), \ldots\ldots \times 3 \times 2 \times 1$

e.g. $4! = 4 \times 3 \times 2 \times 1 = 24$

Permutation

An orderly arrangement of distinct objects is called permutation. The number of permutations of "r" objects selected from "n" distinct objects is given by

$$^{n}P_r = \frac{n!}{(n - r)!}$$

e.g. $^{4}P_2 = \dfrac{4!}{(4 - 2)!}$

$$= \frac{4!}{2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1} = 12$$

Combination

A non-orderly arrangement of objects is called combination.

The umber of combinations of "r" objects selected from "n" objects is given by

$$^nC_r = \binom{n}{r} = \frac{n!}{r!(n - r)!}$$

e.g. $\binom{6}{2} = \frac{6!}{2!(6 - 2)!}$

$$= \frac{6!}{2!(4!)} = \frac{6 \times 5 \times 4!}{2 \times 1 \times 4!} = 15$$

7.7 EXPERIMENT

An experiment is a well-defined action of obtaining an observation.

Trial

Single performance of an experiment is called trial.

Out Come

The result obtained from an experiment is called outcome.

Random Experiment

An experiment which gives different results even though it is repeated a large number of times under similar conditions is called random experiment. The tossing of a fair coin and throwing a die etc. are the examples of random experiments.

7.8 SAMPLE SPACE

The set of all possible outcomes of a random experiment is called a sample space. *e.g.* when we toss a coin the possible outcomes are head and tail. It is denoted by "S".

$S = \{H, T\}$

If we throw a balance die, then the sample space will be

$S = \{1, 2, 3, 4, 5, 6\}$

Sampie Point

Each element of the sample space is called sample point. For example, when we toss a fair coin head and tail are two sample points.

Event

Any subset of a sample space is called an event.

Simple Event

An event that contains only one sample point is called a simple event *e.g.* the occurrence of 3, when a die is thrown.

i.e. Let $A = \{3\}$

Compound Event

An event that contains more than one sample point is called compound event. *e.g.* occurrence of even numbers when a die is thrown.

i.e. {2, 4, 6}

Sure Event

An event that is certain to occur is called certain event or sure event. The sample space S is a sure event.

Impossible Event

An event, which can never occur, is called an impossible event. The empty set *i.e.* ϕ is an impossible event.

Equally Likely Events

Two events A and B are said to be equally likely, if they have same chances of occurrence. *e.g.* when a coin is tossed, it has to possible out comes *i.e.* head and tail. Both these faces are equally likely.

Mutually Exclusive Events

Two events A and B are mutually exclusive if and only if they can not both occur at the same time. For example, when a coin is tossed, we get either a head or a tail, but not both. Head and tail are mutually exclusive events.

Collectively Exhaustive Events

Two mutually exclusive events are called collectively exhaustive if their union constitutes the entire sample space. For example, when we toss a coin, the two events *i.e.* head and tail are collectively exhaustive.

7.9 BASIC INFORMATIONS ABOUT PLAYING CARDS

Total number of cards = 52

They are arranged in four suits. Each suit contains 13 cards. The four suits are diamond, heart, club and spade.

Diamond and Heart are red cards while club and space are black.

The face values of 13 cards in each suit is Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King.

Jack, Queen and King are called picture cards.

Mathematically,

Total cards = 52

Diamond cards = 13

Heart cards = 13

Spade cards = 13

Club cards = 13

Red cards = Diamonds + Hearts = 13 + 13 = 26

Black cards = Spades + Clubs = 13 + 13 = 26

13+13=26

Total number of kings = 4

Total number of queens = 4

Total number of jacks = 4

Picture cards = 12 (three from each suit)

EXAMPLE 7.1

Construct the sample space for the following random experiments.

(i) When a fair coin is tossed.

(ii) When two coins are tossed.

(iii) When a die is rolled.

(iv) When pair of dice is thrown

SOLUTION

Coin N=2 Die N=6

SOLUTION

(i) Total number of outcomes = $2^1 = 2$

The two possible outcomes are head and tail therefore,

S = {H, T}

(ii) Total number of outcomes when two coins are tossed

$= 2^2 = 4$

Possible outcomes are as under

	Coin I	
Coin II	**H**	**T**
H	HH	HT
T	TH	TT

Therefore

S = {(HH), (HT), (TH), (TT)}

(iii) When a dire is rolled.

Total no. of samples / outcomes = $6^1 = 6$

The sample space will be

S = {1, 2, 3, 4, 5, 6}

(iv) When a pair of dice is rolled / thrown.

Total number of outcomes $= 6^2 = 36$

The possible outcomes are shown below.

Ist die	IInd die					
	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Sample space of the experiment is

$$S = \left\{ \begin{array}{l} (1,1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) \\ (2,1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\ (3,1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \\ (4,1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \\ (5,1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \\ (6,1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \end{array} \right\}$$

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

7.10 DEFINITIONS OF PROBABILITY

There are two ways to define probability namely objective approach and subjective approach.

(a) Objective Approach

In term of objective approach there are three different definitions.

1. Classical or A Priori Definition of Probability

This definition may be stated as follows if a random experiment can produce 'n' mutually exclusive and equally likely outcomes and 'm' of which are favorable to the occurrence of event A, then the probability of event A denoted by P(A) is defined as the ratio $\frac{m}{n}$.

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

i.e. $$P(A) = \frac{m}{n}$$

This definition was given by P.S. Laplace. The classical definition has following shortcomings.

(a) This definition is not applicable, when the outcomes are not equally likely.

(b) This definition becomes violate, when the number of possible outcomes may be infinite.

2. **Relative Frequency or Posteriori Definition of Probability**

If a random experiment is repeated a large number of times say n under uniform conditions and if an event A occurs m times, then the probability of A is defined as the limit of relative frequency $\frac{m}{n}$ as n tends to infinity.

i.e. $$P(A) = \lim_{n \to \infty} \frac{m}{n}$$

This definition has two defects.

(i) It is not applicable if the total numbers of ways are limited.

(ii) This definition becomes violate if the conditions are not same.

3. **The Axiomatic Definition of Probability**

Let S be the sample space with the sample points $E_1, E_2, \ldots, E_i, \ldots, E_n$. Each sample point is assigned a real number like $P(E_1), P(E_2), P(E_i), \ldots P(E_n)$, which are called the probabilities of E_i, that must satisfy the following axioms.

(i) $0 \leq P(Ei) \leq 1$

(ii) $P(S) = 1$, for the sure event.

(iii) If A and B are mutually exclusive events.

Then $P(A \cup B) = P(A) + P(B)$

(a) **Probability of an Event**

Let an event belongs to a sample space which contains a finite and equally likely sample points, then probability of an event A is given by

$$P(A) = \frac{\text{Number of sample points in A}}{\text{Number of smaple point in S}}$$

i.e. $$P(A) = \frac{n(A)}{n(S)}$$

(b) **Subjective or Personalistic Probability**

In this approach probability is based on the degree of confidence or belief that a reasonable person has in the occurrence of an event A. This probability is subjective in nature, because the probability vary from person to person for the same situation.

EXAMPLE 7.2

A fair coin is tossed. What is the probability of getting a

(i) Head

(ii) Tail

SOLUTION

Total number of outcomes $= 2^1 = 2$

The possible outcomes are head and tail.

Therefore the sample space is

$S = \{H, T\},\ n(s) = 2$

(i) Let A denote the event that head appears,

$A = \{H\},\ n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

(ii) Let B denote the event that tails appears.

$B = \{T\},\ n(B) = 1$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{2}$$

EXAMPLE 7.3

A die is rolled. What is the probability that it shows.

(i) a six

(ii) even number

(iii) odd number

SOLUTION

Total number of possible outcomes, when a die is rolled $= 6^1 = 6$

Sample space will be

$S = \{1, 2, 3, 4, 5, 6\}\ ,\ n(S) = 6$

(i) Let A denotes the event that 6 appears.

$A = \{6\},\ n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

(ii) Let B denotes the event that a die shows an even numbers.

$B = \{2, 4, 6\}, \; n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(iii) Let C denotes the event that die shows an odd numbers.

$C = \{1, 3, 5\} \; , \; n(C) = 3$

$$P(C) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

EXAMPLE 7.4

What is the probability of getting exactly two heads when 3 coins are tossed?

SOLUTION

Total number of possible outcomes $= 2^3 = 8$

The sample space of the experiment is

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$

Let A denote the event that exactly two heads appears,

$A = \{HHT, HTH, THH\} \; , \; n(A) = 3$

Therefore

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

EXAMPLE 7.5

From a well-shuffled pack of 52 cards, a card is drawn at random. What is the probability that it is

(i) Black card
(ii) An ace
(iii) A card of spade
(iv) A jack of club
(v) A pictured card

SOLUTION

The total number of possible outcomes $= \binom{52}{1} = 52$

$n(S) = 52$

(i) Let A represents the event that the card is black.

Then $n(A) = 26$

Therefore

$$P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

(ii) Let B represents the event that the card drawn is an ace $n(B) = 4$.

(There are four aces in a pack of cards)

Therefore

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(iii) Let $C = \{\text{A card is spade}\}$, $n(C) = 13$

$$P(C) = \frac{n(C)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(iv) Let $D = \{\text{A jack of club}\}$

then $n(D) = 1$ (There is only one jack of club)

Therefore

$$P(D) = \frac{n(D)}{n(S)} = \frac{1}{52}$$

(v) Let $E = \{\text{A pictured card}\}$, $n(E) = 12$

$$P(E) = \frac{n(E)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

EXAMPLE 7.6

(a) If two six faced balance dice are thrown. Find the probability that

(i) The same number appears on both the dice.

(ii) The sum of two numbers is greater than 9.

(b) Two fair dice are thrown. Find the chance that sum of numbers on two dice is at least seven.

(c) What is the probability of throwing a sum 7 with two dice.

SOLUTION

Total number of outcomes when two dice are thrown $= 6^2 = 36$

The sample space for the experiment is

$$S = \left\{ \begin{array}{l} (1,1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) \\ (2,1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\ (3,1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \\ (4,1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \\ (5,1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \\ (6,1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \end{array} \right\}$$

$n(S) = 36$

(a) **(i)**

Let A = {The same number appears on both the dice}

$A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

Then $n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) Let B = {The sum of two number is greater than 9}

$B = \{(4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)\}$

Then $n(B) = 6$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(b) Let C = {Sum of numbers on two dice is at least seven}

$$C = \left\{ \begin{array}{l} (1, 6), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 3), (4, 4) \\ (4, 5), (4, 6), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1) \\ (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \end{array} \right\}$$

$n(C) = 21$

Therefore

$$P(C) = \frac{n(C)}{n(S)} = \frac{21}{36} = \frac{7}{12}$$

(c) Let D = {Sum is 7 with two dice}

$D = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

Then $n(D) = 6$

$$P(D) = \frac{n(D)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

EXAMPLE 7.7

A digit is selected at random from the first ten natural numbers. Find the probability that the selected digit is

(i) Greater than 6

(ii) Multiple of 3

(iii) An even number

SOLUTION

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $n(S) = 10$

(i) Let A be the event that the selected digit is greater than 6,

then $A = \{7, 8, 9, 10\}$

Then $n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{10} = \frac{2}{5}$$

(ii) Let B be the event that the selected digit is multiple of 3.

$B = \{3, 6, 9\}$, $n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{10}$$

(iii) Let C be the event that the selected digit is an even number.

Then $C = \{2, 4, 6, 8, 10\}$, $n(C) = 5$

$$P(C) = \frac{n(C)}{n(S)} = \frac{5}{10} = \frac{1}{2}$$

EXAMPLE 7.8

Two cards are drawn from a well-shuffled pack of 52 playing cards. Find the probability that

(i) Both are King

(ii) One is king and other is queen

(iii) One is red and other is black

(iv) Both cards are of the same colour.

* One card is queen of diamond

* Both are of different colour

SOLUTION

Total no. of possible outcomes $= \binom{N}{n} = \binom{52}{2} = 1326$

$n(S) = 1326$

(i) Let A denotes the event that both cards are king

Let E = Both are of different colour

$n(A) = \binom{4}{2}\binom{48}{0}$, $n(A) = 6$

$E = \binom{26}{1}\binom{26}{1} = 676$

$P(A) = \frac{n(A)}{n(S)} = \frac{6}{1326} = \frac{1}{221}$

(ii) Let B denotes the event that one card is king and other is queen.

$n(B) = \binom{4}{1}\binom{4}{1}\binom{44}{0}$

$= 4 \times 4 \times 1 = 16$

$P(B) = \frac{n(B)}{n(S)} = \frac{16}{1326} = \frac{8}{663}$

(iii) Let C be the event that one card is red and other is black.

$n(C) = \binom{26}{1}\binom{26}{1}$

$= 26 \times 26 = 676$

$P(C) = \frac{n(C)}{n(S)} = \frac{676}{1326}$

(iv) Let D denotes the event that both cards are of the same colour (*i.e.* either both are black or both are red)

$n(D) = \binom{26}{2} + \binom{26}{2}$

$= 325 + 325 = 650$

$P(D) = \frac{n(D)}{n(S)} = \frac{650}{1326} = \frac{25}{51}$

EXAMPLE 7.9

A bag contains 5 red and 7 black balls. What is the probability of drawing at random.

(i) One red ball

(ii) One black ball

SOLUTION

Total balls in the bag $= 5 + 7 = 12$

Total no. of outcomes when one ball is selected from 12 balls $= \binom{12}{1} = 12$

(i) Let $A = \{\text{Drawing red ball}\}$

$$n(A) = 5$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{12}$$

(ii) Let $B = \{\text{Drawing black ball}\}$

$$n(B) = 7$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{12}$$

EXAMPLE 7.10

A bag contains 5 white and 7 black balls. If 3 balls are drawn at random from the bag. What is the probability that

(i) All are black

(ii) Two are white

(iii) All balls are of the same colour

SOLUTION

Whit balls	Black balls	=	Total
5	7	=	5 + 7 = 12

Total number of possible outcomes $= \binom{12}{3} = 220$

$$n(S) = 220$$

(i) Let $A = \{\text{All balls are black}\}$

$$n(A) = \binom{7}{3}\binom{5}{0}$$

$$= 35 \times 1 = 35$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{35}{220} = \frac{7}{44}$$

(ii) Let $B = \{\text{Two balls are white}\}$

$$n(B) = \binom{5}{2} \times \binom{7}{1}$$

$$= 10 \times 7 = 70$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{70}{220} = \frac{7}{22}$$

(iii) Let C = {All balls are of the same colour}

$$n(C) = \binom{5}{3} + \binom{7}{3}$$

$$= 10 + 35 = 45$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{45}{220} = \frac{9}{44}$$

7.11 IMPORTANT POINTS THAT ARE COMMONLY USED IN THE PROBABILITY QUESTIONS

Either, Or, At Least

While solving the different questions on probability we face the word Either, Or, At least used in the question. For example, A or B, either A or B, At least one of them. In these situations we use the addition law of probability for mutually and not mutually exclusive events.

A and B, Both

If A and B or both appear in the question, we use multiplication law of probabilities.

Law of Complementation

If $\bar{A}$ is the complement of an event A relative to the sample space S, then

$$P(\bar{A}) = 1 - P(A)$$

7.12 ADDITION LAW FOR NOT MUTUALLY EXCLUSIVE EVENTS

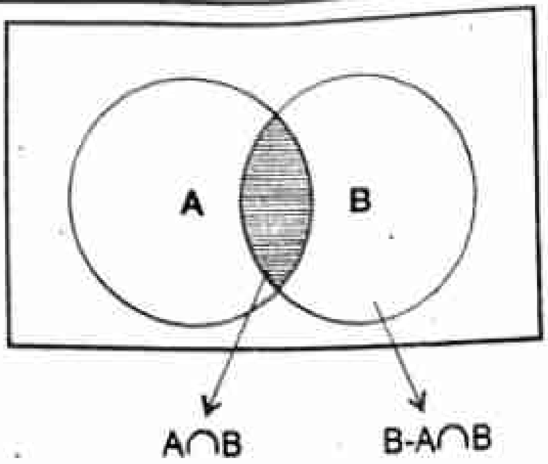
If A and B are any two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This law states that If A and B are two not mutually exclusive events, then the probability that at least one of them occurs is given by sum of separate probabilities of events A and B minus the probability of the joint event $A \cap B$.

Proof

From the Venn diagram we know that, the set $A \cup B$ may be written as the union of the two mutually exclusive event A and $(B - A \cup B)$.



That is

$A \cup B = A \cup [B - A \cap B]$

$P(A \cup B) = P[A \cup (B - A \cap B)]$

$P(A \cup B) = P(A) + P(B - A \cap B)$ .... (i)

Again Event B may be written as the union of two mutually exclusive event $A \cap B$ and $(B - A \cap B)$.

i.e. $B = (A \cap B) \cup (B - A \cap B)$

$P(B) = P(A \cap B) + P(B - A \cap B)$

$P(B) - P(A \cap B) - P(B - A \cap B) = 0$

$P(B - A \cap B) = P(B) - P(A \cap B)$

Putting the value of $P(B - A \cap B)$ in equation (i) we have

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

EXAMPLE 7.11

Two coins are tossed. Find the probability that both faces are head or at least one head.

SOLUTION

Total no. of possible outcomes $= 2^2 = 4$

The sample space for the experiment is

$S = \{HH, HT, TH, TT\}$, $n(S) = 4$

Let $A = \{\text{Both faces are head}\}$

$B = \{\text{At least one head}\}$

We have to calculate $P(A \cup B)$

$A = \{HH\}$, $n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

$B = \{TH, HT, HH\}$, $n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$$

Events A and B are not mutually exclusive, therefore

$A \cap B = \{HH\}$, $n(A \cap B) = 1$

$$P(A \cap B = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$$

We know that from addition law

$$P(A \cap B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{4} + \frac{3}{4} - \frac{1}{4} = \frac{3}{4}$$

EXAMPLE 7.12

One card is selected at random from a deck of 52 playing cards. What is the probability that the card is Heart or picture card or both?

SOLUTION

Total number of possible outcomes $= \binom{52}{1} = 52$

$n(S) = 52$

Let $A = \{\text{Card is heart}\}$

$B = \{\text{Pictured card}\}$

$A \cap B = \{\text{Card selected is both heart and pictured}\}$

Then we need $P(A \cup B)$

$n(A) = 13$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52}$$

$n(B) = 12$ (3 cards in each suit)

i.e. $3 \times 4 = 12$

$$P(B) = \frac{n(B)}{n(S)} = \frac{12}{52}$$

$n(A \cap B) = 3$, (Because 3 of hearts are also picture cards)

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{52}$$

Therefore

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$$

$$= \frac{13 + 12 - 3}{52} = \frac{22}{52} = \frac{11}{26}$$

EXAMPLE 7.13

An integer is chosen at random from the first 50 positive integers. What is the probability that the integer chosen is divisible by 4 or by 6?

SOLUTION

The sample space for this experiment is

$S = \{1, 2, 3, ..., 50\}$, $n(S) = 50$

Let A = {Integer is divisible by 4}

B = {Integer chosen is divisible by 6}

$A \cap B$ = {Int. chosen is divisible by both 4 & 6 *i.e.* 12}

Then we need $P(A \cup B)$

$A = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\}$

$$n(A) = 12 \;,\; P(A) = \frac{12}{50}$$

$B = \{6, 12, 18, 24, 30, 36, 42, 48\}$

$$n(B) = 8 \;,\; P(B) = \frac{8}{50}$$

$A \cap B = \{12, 24, 36, 48\}$

$n(A \cap B) = 4$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{4}{50}$$

Therefore

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{12}{50} + \frac{8}{50} - \frac{4}{50} = \frac{16}{50} = \frac{8}{28}$$

EXAMPLE 7.14

Two students A and B can solve 50% and 80% problems respectively from the exercise. What is the probability that either A or B can solve a problem chosen at random from that exercise.

SOLUTION

P(A can solve the problem) = 50%

$$P(A) = \frac{50}{100} = 0.5$$

P(B can solve the problem) = 80%

$$P(A) = \frac{80}{100} = 0.8$$

P(Both A and B can solve the problem)

$$P(A \cap B) = P(A) \,.\, P(B)$$

$$= 0.5 \times 0.8 = 0.4$$

Therefore

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.5 + 0.8 - 0.4$$

$$= 13 - 0.4 = 0.90$$

7.12.1 Addition Law for Mutually Exclusive Events

If two events A and B are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B)$$

Proof

As we know that from addition law for not mutually exclusive events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \text{..... (i)}$$

In this case events A and B are mutually exclusive, therefore

$$A \cap B = \phi \;,\; P(A \cap B) = 0$$

Putting the value of $P(A \cap B)$ in equation (i), we have

$$P(A \cup B) = P(A) + P(B)$$

EXAMPLE 7.15

Two six faced balance dice are thrown. Find the probability that the sum of two numbers is 8 or 10.

SOLUTION

Total number of outcomes $= 6^2 = 36$

The sample space for the experiment is

$$S = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

$n(S) = 36$

Let A be the event that the sum is 8 and B be the event that the sum is 10.

We need to find $P(A \cup B)$

Then the two events are

$A = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}, \ n(A) = 5$

$B = \{(4, 6), (5, 5), (6, 4)\}, \ n(B) = 3$

The events A and B are mutually exclusive, *i.e.* sum of 8 and 10 cannot both occur at the same time.

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

Therefore

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{5}{36} + \frac{3}{36} = \frac{8}{36} = \frac{2}{9}$$

EXAMPLE 7.16

A card is selected at random from a well-shuffled pack of 52 playing cards. Find the probability that the card is king or queen.

SOLUTION

There are 52 possible outcomes

$n(S) = 52$

Let $A = \{\text{A card is king}\}$

$B = \{\text{A card is queen}\}$

Then we need $P(A \cup B)$

$n(A) = 4$, (as there are 4 kings)

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$n(B) = 4$, (as there are 4 queens)

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

The two events A and B are mutually exclusive, because king and queen can not both occur together.

Therefore

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

EXAMPLE 7.17

For any two events A and B, it is known that $P(A) = \frac{2}{3}$

$P(A \cup B) = \frac{7}{12}$ and $P(A \cap B) = \frac{5}{12}$, Find P(B).

SOLUTION

We know that form Addition law of probabilities

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Putting the values, we have

$$\frac{7}{12} = \frac{2}{3} + P(B) - \frac{5}{12}$$

$$P(B) = \frac{7}{12} - \frac{2}{3} + \frac{5}{12}$$

$$= \frac{7-8+5}{12} = \frac{4}{12} = \frac{1}{3}$$

7.13 CONDITIONAL PROBABILITY

If A and B are two events in a sample space S, then conditional probability of event A, given that event B has occurred, denoted by P(A/B) is defined as

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \; P(B) \neq 0$$

Similarly

$$P(BA) = \frac{P(A \cap B)}{P(A)}, \; P(A) \neq 0$$

EXAMPLE 7.18

A pair of dice is rolled. What is the conditional probability that the sum of two dice will be 8, given that the two dice had the same outcomes.

SOLUTION

The sample space consists of 36 sample points.

i.e. $n(S) = 36$ (See example 5.15)

Let A = {Sum is 8}

B = {the two dice had the same outcome}

Then we require P(A/B)

Therefore

$A = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

$B = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$n(A) = 5$

$(A \cap B) = \{(4, 4)\}$

$n(A \cap B) = 1$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{36}$$

$$p(B) = \frac{n(B)}{n(S)} = \frac{6}{36}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1}{36} \times \frac{36}{6} = \frac{1}{6}$$

7.14 INDEPENDENT AND DEPENDENT EVENTS

Two events A and B are said to be independent if the occurrence of one does not effect the other.

Mathematically,

Two events A and B are independent iff

(i) $P(A/B) = P(A)$

$P(B/A) = P(B)$

(ii) $P(A \cap B) = P(A) \cdot P(B)$

Otherwise they are dependent.

7.15 MULTIPLICATION LAW

If A and B are any two events

Then $P(A \cap B) = P(A) \cdot P(B / A)$

$= P(B) \cdot P(A / B)$

Proof

As we know that from conditional probability

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Multiplying both sides by P(A), we have

$$P(A) \cdot P(B/A) = \frac{P(A \cap B)}{P(A)} \cdot P(A)$$

$$P(A) \cdot P(B/A) = P(A \cap B)$$

Or $P(A \cap B) = P(A) \cdot P(B/A)$

Similarly

$P(A \cap B) = P(B) \cdot P(A/B)$ can also be proved.

EXAMPLE 7.19

A box contains 8 items, 3 of which are defective and 5 are good. Two items are selected at random. What is the probability that the first is good and second is defective.

SOLUTION

Let A be the event that the first item selected is good and B be the event that second item is defective.

Then we need to calculate the probability of $A \cap B$ which is given by the formula

$$P(A \cap B) = P(A) \,.\, P(B/A)$$

$$n(A) = 5$$

$$P(A) = \frac{5}{8}$$

One good item is selected. Now there remains 7 items of which 4 are good and 3 are defective. Therefore the probability of selecting a defective after a good has been selected.

$$P(B/A) = \frac{3}{7}$$

$$P(A \cap B) = P(A) \,.\, P(B/A)$$

$$= \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$$

7.15.1 Multiplication Law for Independents Events

If A and B are two independent events

Then $P(A \cap B) = P(A) \,.\, P(B)$

EXAMPLE 7.20

Two dice are thrown twice. Find the probability of getting a total of 6 on the first throw and a total of 10 on the second.

SOLUTION

The sample space of this experiment consists of 36 sample points. *i.e.* $n(S) = 36$. (See example 5.15)

Let A be the event that getting a total of 6 on first throw and B be the event that getting a total of 10 on the second throw.

Then $A = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$

$$n(A) = 5$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

$$B = \{(4, 6), (5, 5), (6, 4)\}$$

$$n(B) = 3$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{36}$$

The event A and B are independent, therefore

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{5}{36} \times \frac{3}{36} = \frac{5}{432}$$

EXAMPLE 7.21

The probability that A will be alive after 10 years is $\frac{5}{7}$ and for B it is $\frac{7}{9}$. Find the probability

(i) Both of them will die

(ii) A will be alive and B dead

(iii) B will be alive and A dead

(iv) Both of them will be alive

(v) At least one will be alive in 10 years to come.

SOLUTION

$$P(A \text{ alive}) = \frac{5}{7} \quad \Rightarrow P(A) = \frac{5}{7}$$

$$P(A \text{ dead}) = 1 - P(A) \quad \Rightarrow P(\bar{A}) = 1 - \frac{5}{7} = \frac{2}{7}$$

$$P(B \text{ alive}) = P(B) = \frac{7}{9}$$

$$P(B \text{ dead}) = 1 - P(B) \Rightarrow P(B) = 1 - \frac{7}{9} = \frac{2}{9}$$

(i) We need the probability that both of them will die.

i.e. $P(\bar{A} \cap \bar{B})$ since events A and B are independent.

Therefore

$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \, . \, (\bar{B})$

$= \frac{2}{7} \times \frac{2}{9} = \frac{4}{63}$

(ii) We need the probability that A will be alive and B dead.

i.e. $P(A \cap \bar{B})$

Therefore

$P(A \cap \bar{B}) = P(A) \, . \, (\bar{B})$

$= \frac{5}{7} \times \frac{2}{9} = \frac{10}{63}$

(iii) We need the probability that A will be dead and B alive.

i.e. $P(\bar{A} \cap B)$

$P(\bar{A} \cap B) = P(\bar{A}) \, . \, P(B)$

$= \frac{2}{7} \times \frac{7}{9} = \frac{2}{9}$

(iv) We require the probability that both of them will be alive.

i.e. $P(A \cap B)$

Therefore

$P(A \cap B) = P(A) \, . \, P(B)$ (Events A and B are independent)

$= \frac{5}{7} \times \frac{7}{9} = \frac{5}{9}$

(v) We require the probability that at least one will be alive.

i.e. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{7} + \frac{7}{9} - \frac{5}{9}$

$= \frac{45 + 49 - 35}{63} = \frac{59}{63}$

EXAMPLE 7.22

Two cards are drawn at random from a well shuffled pack of ordinary cards. Find the probability that the first card is a king and the second cards is queen, if the first cards is

(i) replaced (ii) not replaced

SOLUTION

Let A be the event that the first card is king and B be the event that second card is queen.

(i) In case of replacement, event A and B are independent,

Therefore

$$P(A \cap B) = P(A) \, . \, P(B)$$

$$= \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$$

(ii) If the first card is not replaced, then A and B are dependent events.

Therefore

$$P(A \cap B) = P(A) \, . \, P(B/A)$$

= P (First card is King) P (Second card is queen, given that first card is king)

$$P(A \cap B) = P(A) \, . \, P(B / A)$$

$$= \frac{4}{52} \times \frac{4}{51} = \frac{4}{663}$$

SUMMARY

The summary of probability is given below:

NAME	FORMULA
Probability	$= \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$ $P(A) = \frac{n(A)}{n(S)}$ $P(B) = \frac{n(B)}{n(S)}$
Addition law for mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
Addition law for not mutually exclusive events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Law of complementation	$P(\overline{A}) = 1 - P(A)$
Conditional Probability	$P(A/B) = \frac{P(A \cap B)}{P(B)}$
Independent law	$P(A/B) = P(A)$ $P(B/A) = P(B)$
Multiplication law for dependent events	$P(A \cap B) = P(A) \cdot P(B/A)$
Multiplication law for Independent events	$P(A \cap B) = P(A) \cdot P(B)$

EXERCISES

7.1 **Define the following with examples.**

(i)	Set	**(ii)**	Null set
(iii)	Sub set	**(iv)**	Intersection of two sets
(v)	Universal set	**(vi)**	Product of sets

7.2 Given

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3\}, B = \{2, 4, 6, 8\}$

$C = \{4, 5, 6, 7, 8\}$, D $\{3, 4\}$

Find

(i) $A \cap B$ **(ii)** $B \cup D$

(iii) $A \cap C$ **(iv)** $A \times D$

7.3 **(a)** Define complement of the set

(b) Let $U = \{1, 2, 3, ..., 10\}$

$A = \{1, 3, 5, 7, 9\}$

$B = \{2, 4, 6, 8, 10\}$

Find **(i)** $\bar{A}$ **(ii)** $\bar{B}$

(iii) $\bar{A} \cup \bar{B}$ **(iv)** $\bar{A} \cap \bar{B}$

7.4 **(a)** Explain the followings.

(i) Permutation

(ii) Combination

(iii) Factorial

(b) Solve the following

(i) ${}^{9}P_{2}$

(ii) ${}^{20}P_{3}$

(iii) $\binom{10}{2}$

(iv) $\binom{52}{5}$

7.5 Define the following terms with examples.

(i) Random experiment

(ii) Sample space

(iii) Simple and compound event

(iv) Mutually exclusive events

(v) Equally likely events

7.6 (a) Define classical or priori definition of probability.

(b) Find the probability of each of the following outcomes.

(i) A tail appears, when a fair coin is tossed.

(ii) Two head appears when two coins are tossed.

(iii) An ace comes up, when a fair die is rolled.

(iv) An odd number comes up, when a true die is rolled.

(c) Two coins are tossed. What is the probability of getting?

(i) One head (ii) Two heads

(iii) At least one head (iv) At most two heads

7.7 (a) If three coins are tossed? Construct the sample space and find the probability of 3 heads.

(b) Three coins are tossed. What is the probability of getting?

(i) Two heads

(ii) At least one head

(iii) At most one head

7.8 (a) Two fair dice are rolled. Find the probability that

(i) Sum of two dice will be even

(ii) The two dice had the same outcomes.

(b) Two fair dice are rolled. Find the probability.

(i) The sum is odd

(ii) The sum is at least 9

(iii) Sum is exactly seven

(iv) A double six

(c) Two dice are rolled. Find the probability of getting sum is less than 7 is equal to sum is greater than 7.

(d) Two dice are rolled once. Find the probability that

(i) Sum is greater than 9. (ii) Sum is at most 5.

(iii) Their product is even

7.9 (a) a bag contains 3 white, 5 yellow and 2 red balls. What is the probability of drawing at random.

(i) One red ball

(ii) One yellow ball.

(b) A bag contains 3 white and 4 black balls. Two balls are drawn at random find the probability that

(i) One is white and other is black

(ii) Both are white

(iii) At least one is white

7.10 (a) A bag contains 3 black, 4 white and 5 red balls. If 3 balls are drawn at random from the bag. Find the probability that

(i) One black, one white and one red ball

(ii) Two red and one black ball

(iii) All balls are black

(b) A box contains 3 red, 4 white and 5 black balls. Three balls are drawn from the box. Find the probability that they may be

(i) all of different colours

(ii) all are of the same colours

7.11 From a well shuffled pack of 52 playing cards, a card is drawn at random. What is the probability that it is

(i) Red card (ii) Kind of Diamond

(iii) Card is Queen (iv) Card is Heart

7.12 (a) Two cards are drawn at random from a well shuffled pack of playing cards. Find the probability that

(i) Both are red. 4

(ii) One is heart and other is diamond

(iii) Both are of the same colour

(b) A digit is selected at random from first 15 natural numbers. Find the probability that the selected digit is

(i) An odd number

(ii) An even number

(iii) Less than 5

(iv) Complete square

(v) Divisible by 5

7.18 (a) State and prove addition law of probabilities for not mutually exclusive events.

(b) A fair die is rolled. What is the probability that the outcome is either an odd number or a prime number?

HINT

Odd Number

A number which is not divisible by 2

i.e. 1, 3, 5, ...

Prime Number

A number which is divisible by 1 and itself only

i.e. 2, 3, 5, ...

7.14 (a) An integer is chosen at random from the first positive 100 integers. Find the probability that the chosen integer is divisible by 8 or 12.

(b) A class contains 10 men and 20 women out of which half men and half women have brown eyes. Find the probability that the person chosen is a women or has brown eyes.

7.15 (a) A card is drawn from a well shuffled pack of 52 playing cards. What is the probability that card is either king or diamond?

(b) The probability that a boy will pass the examination is 0.70 and that for girl is 0.60. What is the probability that at least one of them will pass the examination?

.6 (a) Define addition law of probability for mutually exclusive events.

(b) A card is drawn from a well shuffled pack of 52 playing cards. Find the probability that the card is either king or queen.

7.17 (a) A ball is drawn at random from a bag containing 5 red, 6 white, 4 blue and 3 orange balls. Find the probability that the ball is either red or blue.

(b) Two balance dice are rolled. Find the probability of getting a total of either 7 or 9.

7.18 (a) What do you meant by conditional probability?

(b) A fair die is rolled. Find the conditional probability that the face is odd given that the face is less than 4.

7.19 (a) A man tosses two fair dice. What is the conditional probability that the sum of two top sides is 7, given that

(i) the sum is odd

(ii) the sum is greater than 6.

(iii) the two dice had the same outcome.

(b) Three fair coins are tossed. What is the conditional probability that 2 head appears given that at least one head.

7.20 (a) State and prove multiplication law of probabilities.

(b) Define independent and dependent events.

(c) Two cards are drawn from a well-shuffled pack of 52 playing cards. Find the probability that they are both aces if the first card is

(i) replaced (ii) not replaced

7.21 (a) Find $P(A \cap B) = ?$

Given that $P(A) = 0.25$, $P(B) = 0.60$

Assuming that A and B are independent events.

(b) given $P(A) = 0.60$

$P(B) = 0.40$, $P(A \cap B) = 0.24$

Find $P(A/B)$, $P(B/A)$ and $P(A \cup B)$

7.22 Two friends A and B appear in an interview for two posts in an office. The probability that A will be selected is $\frac{1}{3}$ and the probability that B will be selected is $\frac{1}{4}$. What is the probability that

(i) Both will be selected (ii) Only A will be selected

(iii) Only B will be selected (iv) At least one will be selected

(v) Neither will be selected.

7.23 Two coins are tossed. Show that the probability of getting at least one head is ¾.

7.24 A fair dice is rolled twice. Find the probability that sum of numbers appearing is

(i) at most five

(ii) at least 10

7.25 An unbiased coin is tossed twice. Find the probability that

(i) one tail appears

(ii) at least 1one head

7.26 A card is drawn from well shuffled pack of 52 cards. What is the prob that it is

(i) Black card (ii) A face card

7.27 EACH QUESTION HAS FOUR POSSIBLE ANSWERS. SELECT THE CORRECT ONE AND ENCIRCLE IT.

(i) The probability of an event always lies between

(a) −1 and 0 (b) 0 and 1

(c) −1 and +1 (d) 1 and 100

(ii) A letter is chosen as random from the word "SAMPLE", the probability of vowel is

(a) $\frac{3}{6}$ (b) $\frac{2}{3}$

(c) $\frac{1}{3}$ (d) $\frac{1}{6}$

(iii) The probability of an impossible event is always equal to

(a) 1 (b) 0

(c) −1 (d) none of these

(iv) Two coins are tossed. The probability of getting 2 heads is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$.

(c) $\frac{3}{4}$ (d) 1

(v) If A and B are independent events then $P(A \cap B)$ is

(a) $P(A) + P(B)$ (b) $P(A) . P(B/A)$

(c) $P(A) . P(B)$ (d) $P(A). P(A/B)$

(vi) The probability of drawing a king from pack of 52 cards is

(a) $\frac{1}{52}$ (b) $\frac{1}{13}$

(c) $\frac{2}{13}$ (d) $\frac{2}{52}$

(vii) A tail appears when a fair coin is tossed

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) 1

(viii) An ace comes up, when a die is rolled

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{3}{6}$

(ix) **When two coins are tossed, the possible outcomes are**

(a) 1 (b) 2

(c) 4 (d) 6

(x) **Two events A and B are called mutually exclusive if**

(a) $A \cup B = \phi$ (b) $A \cap B = \phi$

(c) $A \cap B = S$ (d) $A \cup B = S$

(xi) **An event that contains at least two sample points is called**

(a) Simple event (b) Compound event

(c) Multiple event (d) Equally likely event

(xii) **If two dice are rolled the possible outcomes are**

(a) 6 (b) 12

(c) 18 (d) 36

(xiii) **If A and B are mutually exclusive events, then**

(a) $P(A \cup B) = P(A) + P(B)$

(b) $P(A \cup B) = P(A) . P(B)$

(c) $P(A \cup B) = P(A) . P(B/A)$

(d) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(xiv) **The probability of drawing a picture card from a pack of 52 playing cards is**

(a) $\frac{1}{52}$ (b) $\frac{1}{32}$

(c) $\frac{1}{2}$ (d) $\frac{12}{52}$

(xv) **The probability of getting a red card from a pack of cards is**

(a) $\frac{1}{52}$ (b) $\frac{4}{52}$

(c) $\frac{26}{52}$ (d) $\frac{3}{52}$

(xvi) **If two events A and B are not independent, then**

(a) $P(A \cap B) = P(A) . P(B)$ (b) $P(A \cap B) = P(A) . P(B/A)$

(c) $P(A \cap B) = P(A) + P(B/A)$ (d) $P(A \cap B) = P(A) + P(A/B)$

(xvii) **A digit is selected from first ten natural numbers, the probability of even number is**

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

(xviii) In case of objective approach, the definitions of probability are

(a) 2 (b) 3

(c) 4 (d) 5

(xix) In conditional probability P(A/B) is equal to

(a) $\frac{P(A \cup B)}{P(B)}$ (b) $\frac{P(A \cap B)}{P(B)}$

(c) $\frac{P(A \cap B)}{P(A)}$ (d) $\frac{P(A \cup B)}{P(A)}$

(xx) $\binom{n}{r}$ is equal to

(a) $\frac{n!}{(n-r)!}$ (b) $\frac{n!}{n!(n-r)!}$

(c) $\frac{n!}{r!(n-r)!}$ (d) $\frac{n!}{(n+r)!}$

(xxi) 0! is equal to

(a) 0 (b) 1

(c) 2 (d) 3

(xxii) If $P(A \cap B) = 0.25$, $P(A) = 0.75$ then P(B/A) is:

(a) 1 (b) 0

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(xxiii) An orderly arrangement of objects is called:

(a) Combination (b) Permutation

(c) Sample space (d) Factorial

(xxiv) If A and B are mutually exclusive events then $P(A \cap B)$ is:

(a) 1 (b) 0.5

(c) 0 (d) None of these

ANSWERS 7.27

(i)	(b)	(ii)	(c)	(iii)	(b)	(iv)	(a)	(v)	(c)
(vi)	(b)	(vii)	(a)	(viii)	(c)	(ix)	(c)	(x)	(b)
(xi)	(b)	(xii)	(d)	(xiii)	(a)	(xiv)	(d)	(xv)	(c)
(xvi)	(b)	(xvii)	(d)	(xviii)	(b)	(xix)	(b)	(xx)	(c)
(xxi)	(b)	(xxii)	(c)	(xxiii)	(b)	(xxiv)	(c)		

SHORT QUESTION ANSWERS

1. **Define set**

Ans. A set is a well-defined collection of distinct objects. The objects of the set are called elements.

2. **What is the difference between permutation and combination?**

Ans. An orderly arrangement of distinct objects is called permutation while non-orderly arrangement of objects is called combination.

3. **Define outcome and sample space.**

Ans. **Outcome**

The result obtained from an experiment is called outcome.

Sample Space

The set of all possible outcomes of a random experiment is called sample space *e.g.* when we toss a coin the possible outcomes are head and tail, therefore

$S = \{H, T\}$

4. **What do you meant by Random experiment?**

Ans. An experiment which give different results even though it is repeated a large number of times under similar conditions is called random experiment *e.g.* tossing a coin, rolling a die etc.

5. **What is the difference between simple event and compound event?**

Ans. **Simple Event**

An event that contains only one sample point is called simple event *e.g.* occurrence of 3, when a die is rolled.

i.e. $A = \{3\}$

Compound Event

An event that contains more than one sample points is called compound event. *e.g.* occurrence of even numbers when a die is rolled.

i.e. $A = \{2, 4, 6\}$

6. **Define Sure Event.**

Ans. An event that is certain to occur is called sure event. For example, sample space is a sure event.

7. **Define impossible event.**

Ans. An event which can never occur is called an impossible event. For example, empty set is an impossible event.

8. **Define equally likely events.**

Ans. Two events A and B are said to be equally likely, if they have same chances of occurrence. For example, when we toss a coin, head and tail are called equally likely events.

9. **What is meant by Mutually Exclusive events?**

Ans. The events A and B are mutually exclusive if and only if they cannot both occur at the same time for example when a coin is tossed, we get either head or tail, but not both. Head and tail are mutually exclusive events.

10. **How we define probability in terms of objective approach?**

Ans. In terms of objective approach there are three definitions of probability namely,

(a) Classical or A priori definition

(b) Relative frequency or A posteriori definition

(c) The axiomatic definition of probability

11. **Write down the classical definition of probability.**

Ans. If a random experiment can produce 'n' mutually exclusive and equally likely outcomes and m of which are favourable to the occurrence of event A, then the probability of event A is given by

$$P(A) = \frac{m}{n} = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}}$$

12. **Give the relative frequency definition of probability.**

Ans. If a random experiment is repeated a large number of times say n under uniform conditions and if an event A occurs m times, then the probability of A is defined as the limit of relative frequency $\frac{m}{n}$ as n tends to infinity

$$P(A) = \lim_{n \to \infty} \frac{m}{n}$$

13. **What is the axiomatic definition of probability?**

Ans. Let S be the sample space with sample points $E_1, E_2, \ldots\ldots, E_i, \ldots\ldots E_n$. Each sample point is assigned a real number like $P(E_1) \ldots\ldots P(E_2) \ldots\ldots P(E_i) \ldots\ldots P(E_n)$, which are called probabilities of E_i, that must satisfy the following axioms.

(a) $0 \leq P(E_i) \leq 1$

(b) $P(S) = 1$

(c) If A and B are mutually exclusive events,

then, $P(A \cup B) = P(A) + P(B)$

14. Define subjective approach of probability.

Ans. In this approach probability is based on the degree of confidence or belief that a reasonable person has in the occurrence of event A.

15. Define Probability of an event.

Ans. $P(A) = \dfrac{\text{Number of sample points in A}}{\text{Number of sample points in S}}$

i.e. $P(A) = \dfrac{n(A)}{n(S)}$

16. Define collectively exhaustive events.

Ans. Two mutually exclusive events are said to be collectively exhaustive events if their union constitute the entire sample space.

17. What is conditional probability?

Ans. If A and B are two events in a sample space S then conditional probability of event A, given that event B has occurred, denoted by P(A/B) is defined as

$$P(A/B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

18. What is the difference between independent and dependent events?

Ans. Two events A and B are said to be independent if the occurrence of one does not effect the other.

Mathematically,

Two events A and B are independent iff

(a) $P(A/B) = P(A)$

(b) $P(B/A) = P(B)$

(c) $P(A \cap B) = P(A) \cdot P(B)$

Otherwise events are dependent.

19. Define addition law for not mutually exclusive events.

Ans. If A and B are two not mutually exclusive events, then the probability that at least one of them occurs is given by sum of separate probabilities of event A and B minus the probability of the joint event $A \cap B$.

i.e. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

20. Define addition law for mutually exclusive events.

Ans. If A and B are mutually exclusive events.

then $P(A \cup B) = P(A) + P(B)$