Example 1:

$G = \{1, -1\}$.

$\cdot, +$

$(G, \cdot)$ is a group.?

(i) <u>closure Property</u>

$1 \times 1 = 1 \in \{1, -1\}$.

$1 \times -1 = -1 \in \{1, -1\}$

$-1 \times 1 = -1 \in \{1, -1\}$.

$-1 \times -1 = 1 \in \{1, -1\}$.

It is closed. w.r.t. '$\cdot$'

(ii) <u>Associative prop.</u> $1 \times 1 \times -1$

$(1 \times 1) \times -1 = 1 \times (1 \times -1)$

$1 \times -1 = 1 \times -1$

$-1 = -1$ Hold.

(iii) <u>Multiplicative Identity</u>

1 is Multiplicative Identity

$1 \times 1 = 1. \cancel{\in \{1, -1\}}$

$1 \times -1 = -1 \cancel{\in \{1, -1\}}$

$2 \times \frac{1}{2}$

(iv). Multiplicative Inverse.

The M.I of 1 is $\frac{1}{1} = 1$. $(1 \times \frac{1}{1})$

and M.I of -1 is $\frac{+1}{-1} = +1$

$(-1 \times \frac{1}{-1})$

Example 2.

$(Z, +)$ $(Q, +)$ $(R, +)$ $(C, +)$

are groups.

Example 3:

$G = \{1, \omega, \omega^2\}$ $\quad \underline{3}$

$\cdot$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	$\omega^3 = 1$
ω^2	ω^2	1	$\omega^4 = \omega$

$\omega^4 = \omega^3 \cdot \omega$

$\omega^4 = 1 \cdot \omega$

(i) **closure property:**

$1 \times 1 = 1$

$1 \times \omega = \omega \in \{1, \omega, \omega^2\}$.

$1 \times \omega^2 = \omega^2$ $\quad \omega \times \omega^2 = \omega^3 = 1 \in \{1, \omega, \omega^2\}$

(ii) **Associative prop:**

$(1 \times \omega) \times \omega^2 = 1 \times (\omega \times \omega^2)$

$\omega \times \omega^2 = 1 \times \omega^3$

$\omega^3 = \omega^3$

$1 = 1$

(iii) **Identity Element:** / Multiplicative Identity

1 is an Identity element.

$\therefore 1 \times 1 = 1$

$1 \times \omega = \omega$

$1 \times \omega^2 = \omega^2$

(iv) Multiplicative Inverse $\quad 2 \times \frac{1}{2} = 1$

The M. inverse of ω is ω^2. $\quad 2 + (-2) = 0$

The M. I of ω^2 is ω $\quad - \times - = 1$

The M. I. of 1 is 1..

$(C, +)$

Let $z = a + ib \in \underline{C}$

i) <u>Closure prop.</u>

Let $a + bi \in C$ and $c + di \in C$

$(a+bi) + (c+di) \in C.$

$= (a+c) + (b+d)i \in C.$

(ii) Associative prop

$a+bi, c+di, e+fi \in C.$

$(a+bi + c+di) + e+fi = a+bi + (c+di + e+fi)$

$[(a+c) + (b+d)i] + (e+fi)$

$= a+c+e + (b+d+f)i$

$= a + bi + [(c+e) + (d+f)i]$

$= (a+c+e) + (b+d+f)i$

$L.H.S = R.H.S.$

(iii) $0 + 0i \in C$. M.I.

$(a+ib) + (0+0i) = a+ib.$

$(0+0i) + (a+ib) = a+ib.$

(iv) $-a-bi \in C$. M.Inverse

$(a+bi) + (-a-bi) = 0+0i$

$(-a-bi) + (a+bi) = 0+0i$

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