UNIVERSITY OF THE PUNJAB

Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Linear Algebra (MA) **TIME ALLOWED: 2 Hrs. & 45 Mints.**

Course Code: IT-312 Part – II **MAX. MARKS: 50**

Attempt this Paper on Separate Answer Sheet provided.

Q. 2 **SHORT QUESTIONS** **(4x5 = 20 Marks)**

(i) Determine the values of a for which the system of linear equations has no solution, exactly one solution and infinitely many solutions.

$x+y+7z=-7$

$2x+3y+17z=-16$

$x+2y+(a^2+1)z=3a.$

(ii) Prove that $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{c} & a \\ b & b & \frac{c^2+a^2}{c} \end{vmatrix} = 4abc$

(iii) Show that the vectors $(1,-2)$, and $(3,-5)$ span the vector space $\mathbb{R}^2$

(iv) Define $T: R^3 \to R^3$ by $T(x_1,x_2,x_3)=(-x_3,x_1,x_1+x_3)$. Find $N(T)$. Is T one-to-one?

(v) Show that the matrix $\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ is involutery

SECTION-III

LONG QUESTIONS **(6x5 = 30 Marks)**

Q.3 If A and B are 3×3 matrices such that $\det(A^2B^3)=108$ and $\det(A^3B^2)=72$ then find $\det(2A)$ and $\det(B^{-1})$.

Q.4 Find the real orthogonal matrix P for which $P^{-1}AP$ is orthogonal, where $A=\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Q.5 If possible, find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 0 \\ -2 & -2 & 2 \end{bmatrix}$

Q.6 Show that any finite dimensional vector space contains a basis.

Q.7 Determine whether or not the given set of vectors is a basis of $\mathbb{R}^3$

$\{(1,2,-1),(0,3,1),(1,-5,3)\}$

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Sixth Semester - 2018

Examination: B.S. 4 Years Programme

Roll No. ....

PAPER: Linear Algebra (MA) **TIME ALLOWED: 15 Mints.**

Course Code: IT-312 Part – I (Compulsory) **MAX. MARKS: 10**

Attempt this Paper on this Question Sheet only.

Please encircle the correct option. Each MCQ carries 1 Mark. This Paper will be collected back after expiry of time limit mentioned above.

Q. 1 MCQs (1x10 = 10 Marks)

(i) The set $S=\{(1,2),(2,3),(0,0)\}$ of vectors in R^2 is ----------

(a) linearly independent (b) linearly dependent (c) basis of R^2 (d) None of these

(ii) The basis of trivial subspace $A=\{0\}$ is ----------

(a) $\{0\}$ (b) $\{1\}$ (c) $\{\ \}$ (d) None of these

(iii) If A is a matrix of order 3×3 and $\det(A)=-2$, then the value of $\det(3A)$ is --------

(a) -24 (b) -6 (c) -27 (d) -54

(iv) A system of m homogeneous linear equations $Ax=0$ in n variables has a non-trivial solution if and only if the rank of A is ----------

(a) equal to n (b) less than n (c) greater to n (d) None of these

(v) The subspace of R^3 spanned by the vector (a, b, c) is ----------

(a) $x=t, y=bt, z=ct$ (b) $x=-at, y=-bt, z=-ct$

(c) $x=at, y=bt, z=ct$ (d) None of these

(vi) The property $\forall$ a, b$\in$R then $a+b\in R$ is called

(a) Associative property (b) Transitive property

(c) Closure property (d) None of these

(vii) A linear transformation $T:U\to V$ is one-to-one if and only if ----------

(a) $N(T)=\{0\}$ (b) $N(T)\neq\{0\}$ (c) $N(T)=\{1\}$ (d) $N(T)=\{-1\}$

(viii) Let R^3 be the vector space of all ordered triples of real numbers. Then the transformation $T:R^3\to R^3$ defined by $T(x,y,z)=(x,y,0)$ is

a) Linear b) Not Linear c) Rational d) None of these

(ix) The dimension of $KerT$ is called ....

(a) Rank (b)Nullity (c) basis (d)} none of these

(x) The characteristic polynomial of the matrix $\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}$ is...............

(a) $p(\lambda)=(1-\lambda)^2$ (b) $p(\lambda)=(2-\lambda)(3-\lambda)$

(c) $p(\lambda)=\lambda^2$ (d) None of these