UNIVERSITY OF THE PUNJAB

Roll No. ....

Sixth Semester - 2017
Examination: B.S. 4 Years Programme

PAPER: Linear Algebra (MA)　　　　　　　TIME ALLOWED: 30 mins.
Course Code: IT-312　　　　　　　　　　　MAX. MARKS: 10

Attempt this Paper on this Question Sheet only.

SECTION-I

Q. 1　　　　　　　　　MCQs (1x10 = 10 Marks)

(i) The set $S = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ of vectors in R^2 is ---------

a) Linearly Independent　b) Linearly dependent　c) Basis of R^2　d) None of these

(ii) A system $Ax = b$ of m linear equations in n unknowns in n variables has a solution if and only if---------

(a) $rank\ of\ A = rank\ of\ A_b$　　　　(b) $rank\ of\ A \neq rank\ of\ A_b$

(c) $rank\ of\ A = m$　　　　　　　　　(d) None of these

(iii) If A is a matrix of order 3×3 and $\det(A) = 3$, then the value of $\det(2A)$ is --------

(a) 24　　　　　(b) 27　　　　　(c) 54　　　　　(d) None of these

(iv) A unit vector orthogonal to both $(1, 1, 2)$ and $(0, 1, 3)$ in R^3 is ---------

(a) $\left(\frac{1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}} \right)$　(b) $\left(\frac{-1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}} \right)$　(c) $\left(\frac{2}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{-1}{\sqrt{11}} \right)$　(d) $\left(\frac{-1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}} \right)$

(v) The set $W = \left\{ (x, y, z) \in R^3 : x + y + z = c \right\}$ is a subspace of R^3 if ---------

(a) $c > 0$　　　　(b) $c < 0$　　　　(c) $c = 0$　　　　(d) None of these

(vi) The property $\forall\ a, b \in R$ then $a + b = b + a$ is called ---------

(a) Associative property　　　(b) Transitive property

(c) Closure property　　　　　(d) None of these

(vii) A linear transformation $T : U \to V$ is one-to-one if and only if ---------

(a) $N(T) = \{0\}$　　(b) $N(T) \neq \{0\}$　　(c) $N(T) = \{1\}$　　(d) $N(T) = \{-1\}$

(viii) The transformation $T : R^3 \to R^2$ be defined by $T(x_1, x_2, x_3) = (x_1 + 1, x_2 + x_3)$　is

(a) Linear　　　　(b) Not Linear　　　(c) Rational　　(d) None of these

(ix) The dimension of $KerT$ is called ---------

(a) Rank　　　　(b) Nullity　　　　(c) basis　　　(d)} none of these

(x) The characteristic polynomial of the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$ is---------

(a) $p(\lambda) = (3 - \lambda)^2$　　　(b) $p(\lambda) = (3 - \lambda)(4 - \lambda)$

(c) $p(\lambda) = \lambda^2$　　　　　　(d) None of these

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Sixth Semester - 2017
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PAPER: Linear Algebra (MA)   TIME ALLOWED: 2 hrs. & 30 mins.
Course Code: IT-312   MAX. MARKS: 50

Attempt this Paper on Separate Answer Sheet provided.

SECTION-II

Q. 2   **SHORT QUESTIONS**   **(4x5 = 20 Marks)**

(i)   Solve the system of linear equations

$$6x - 6y + 6z = 6$$
$$2x - 4y - 6z = 12$$
$$10x - 5y + 5z = 30$$

(ii)   Prove that

$$\begin{vmatrix} \dfrac{a^2 + b^2}{c} & c & c \\ a & \dfrac{b^2 + c^2}{c} & a \\ b & b & \dfrac{c^2 + a^2}{c} \end{vmatrix} = 4abc$$

(iii) Show that the vectors $(1,2)$, and $(3,5)$ span the vector space $\mathbb{R}^2$

(iv)   Define $T : R^3 \to R^3$ by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3, x_2 + x_3)$. Find the basis and dimension of $N(T)$.

(v)   Find inverse of the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 4 & 1 \\ 1 & 3 & 0 \end{bmatrix}$

SECTION-III

LONG QUESTIONS   **(6x5 = 30 Marks)**

Q.3   Solve the system of linear equations by Gaussian elimination method

$$x + y + 2z = 9, \qquad 2x + 4y - 3z = 1, \qquad 3x + 6y - 5z = 0$$

Q.4   Find the characteristic polynomial, eigen Values and eigen vectors of the matrix $\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$

Q.5   Show that the matrix $\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$ is nilpotent.

Q.6   Determine whether or not the given set of vectors is a basis of R^3
$$v_1 = (1,2,-1), v_2 = (0,3,1), v_3 = (1,-5,3).$$

Q.7   Write $v = (1,-2,5)$ as a linear combination of the vectors
$$v_1 = (1,1,1), v_2 = (1,2,3), v_3 = (2,-1,1)$$