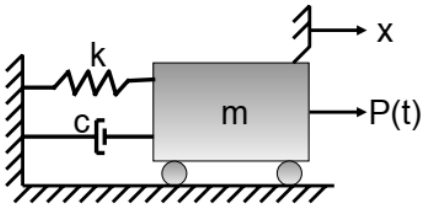
Topic: Signal Processing

Concepts: Fourier Series

In this assignment, you will see how to use Fourier Series to determine the response of SDOF system. In dynamics, the SDOF system looks as follows:



This system is typically used to model response of a system with a force varying as a function of time. The system therefore also has a response varying with time. SDOF system has the following equation of motion:

$$m\ddot{x} + c\dot{x} + kx = P(t)$$

Which describes how the response of the mass varies due to varying forcing. In this equation, m is the mass, k is the stiffness, $P(t)$ is applied force as a function of time and c is the viscous damping coefficient which represents the energy dissipation in the system. For the sake of this assignment, we will consider $m = 1, k = 45, c = 0$. This system has a natural frequency of $\omega_n = \sqrt{\frac{k}{m}}$. The applied force, $P(t)$ in this assignment is a periodic force.

Note that the response of this SDOF under purely sinusoidal forcing $P(t) = P_0 \sin(\omega t)$ will be $x(t) = \frac{P_0}{k-m\omega^2}\sin(\omega t)$.

Task 1:

In your GitHub folder, you will find a file called "*forcing1.npz*". This file consists of an array applied forcing, $P(t)$ and another array of time vector. You will find this loaded into your python program by using the command

npzfile = np.load('forcing1.npz')

Then the arrays are unpacked using the commands:

tspan = npzfile['tspan'] *% contains the time*

force1 = npzfile['force1'] *% contains the force*

Plot the force $P(t)$ as a function of time. Is this a signal? What is the amplitude of this signal? Find out the period T_0 of the signal.

Task 2:

Now you will extract out the signal for just one T_0. For this you would need to find the indices in $tspan$ where $tspan \leq T_0$. You can use the find this using $np.where()$. Use these indices to extract out signal for 0 to T_0.

Task 3:

Let us use Fourier Series to express $P(t) = a_0 + \sum_j a_j \cos(j\omega_0 t) + \sum_j b_j \sin(j\omega_0 t)$, where $\omega_0 = 2\pi/T_0$. We saw in the class that

$$a_0 = \frac{1}{T_0}\int_0^{T_0} P(t)dt$$

$$a_j = \frac{2}{T_0}\int_0^{T_0} P(t)\cos(j\omega_0 t)\,dt$$

$$b_j = \frac{2}{T_0}\int_0^{T_0} P(t)\sin(j\omega_0 t)\,dt$$

You will now determine these Fourier Coefficients for the forcing function. Note that each of the three expressions shown earlier are just area under the curve represented by the function in the integral. In this exercise, we will discretize our time as a multiple of Δts, i.e. (time = Δt, $2\Delta t$, ..., $n\Delta$t). Consequently, we will also discretize our forcing function and sinusoidals (i.e. $P(t) = P(i\Delta t)$, $\cos(j\omega_0 t) = \cos(j\omega_0 i\Delta t)$, $\sin(j\omega_0 t) = \sin(j\omega_0 i\Delta t)$. Then, we will now find the area under the curve numerically as:

$$a_0 = \frac{1}{T_0}\sum_i P(i\Delta t)\Delta t$$

$$a_j = \frac{2}{T_0}\sum_i P(i\Delta t)\cos(j\omega_0 i\Delta t)\,\Delta t$$

$$b_j = \frac{2}{T_0}\sum_i P(i\Delta t)\sin(j\omega_0 i\Delta t)\,\Delta t$$

This is perfectly reasonable to do if our Δt is small, which it is for our case. Run a for loop for first 15 js. Then plot a_0, a_j and b_j.

Task 4:

Find the response of the system using:

$$x(t) = \sum_j \frac{a_j}{k - m(j\omega_0)^2}\cos(j\omega_0 t) + \frac{b_j}{k - m(j\omega_0)^2}\sin(j\omega_0 t)$$

Plot the response as a function of time.

Task 5:

Now load “*forcing2.npz*”. Plot $P_0 \; vs \; t$. Comment on the magnitude of this forcing vs forcing in Task 1. Repeat all the tasks with this forcing and find the response. Comment on the response in this case. Note that in this case, it is a little hard to tell the period visually. You can use the period as $1/0.99$ seconds.