Topic: Time Series Analysis

Concepts: Detrending, Stationarity, Baseline Correction

In this assignment, you will correct ground motion acceleration data. The ground motion recorded during an earthquake contains many issues:

(a) Baseline drift (slowly varying trends due to instrument bias)
(b) Transient offsets before and after shaking
(c) High-frequency noise

If this ground motion is directly used for velocity and displacement computation by using integration, it leads to imperfections and errors in these quantities. The ground motion by itself is expected to be a stationary signal.

In this assignment, you will use the time series modeling concepts to correct earthquake accelerogram. You are provided with a file “*ground_motion.npz*” containing *t* and *y* arrays. The t array contains the time in the units of seconds and the y array contains the acceleration in the units of g.

Task 1:

Load *t* and *y* from *ground_motion.npz*. Plot acceleration versus time. Comment on visible trends or offsets. Hint: The signal should oscillate around zero for a stationary accelerogram.

Task 2:

Now you will fit a **linear model** $y(t) = y_0 + rt + ou_k(t) + \epsilon(t)$ to the acceleration data. For this recall that we can write this as $Y = AX + \epsilon$, where Y, A and ϵ are:

$$Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_{k-1} \\ Y_k \\ \vdots \\ Y_m \end{bmatrix} \quad , \quad A = \begin{bmatrix} 1 & t_1 & 0 \\ \vdots & \vdots & \vdots \\ 1 & t_{k-1} & 0 \\ 1 & t_k & 1 \\ \vdots & \vdots & \vdots \\ 1 & t_m & 1 \end{bmatrix} \quad , \quad X = \begin{bmatrix} y_0 \\ r \\ o \end{bmatrix}$$

where the first column containing ones is for intercept, second column containing times is for rate and the third column containing the Heaviside step function is for offset.

Your task now is to assemble the A matrix. As a side note, Y is already in this format.

Task 3:

In practice if we can get multiple ground motions, we would use Best Linear Unbiased Estimator. Unfortunately, we might not have it. This is what we assume in this case as well. So, we will just use Least-Squares fit. For this, you can use lstsq() function from scipy.linalg package. Using this function estimate $\hat{X} = \begin{bmatrix} \widehat{y_0} \\ \hat{r} \\ \hat{o} \end{bmatrix}$

Task 4:

Now you will compute the clean ground motion. To do this, estimate the trend and offset in the original ground motion and remove it from the original ground motion, that is

$$y_{clean}(t) = y(t) - \left(\widehat{y_0} + \hat{r}t + \hat{o}u_k(t)\right)$$

Plot the original and clean ground motion and write down your observations. Check if your clean ground motion has near zero mean.

Task 5:

Plot the frequency response of clean ground motion. If you had to classify this as a type of noise, what type of noise would you classify?