Topic: Uncertainty and Estimation
Concepts: random variables, probability basics, distributions

Note: *Start each problem with the starter code shared in the repository.*
https://docs.scipy.org/doc/scipy/tutorial/stats.html

https://docs.scipy.org/doc/scipy/reference/stats.html

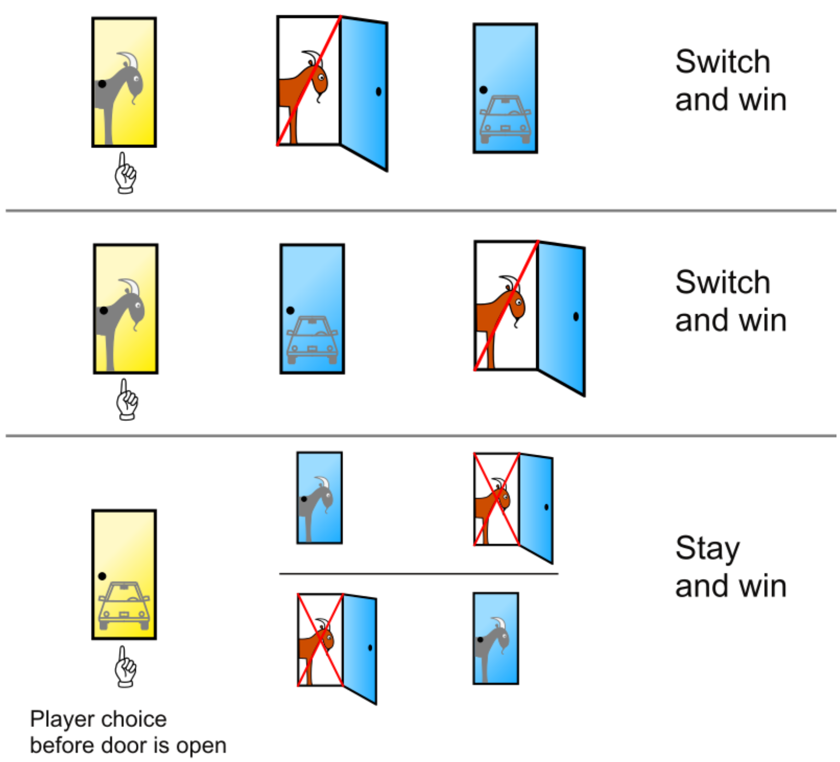
Problem-1 (Monty Hall problem):

You are on a game show with three closed doors. Behind one door is a car (prize); behind the other two are goats. You pick one door. The host (who knows where the car is) opens a different door to reveal a goat, and then offers you the option to switch your choice to the remaining unopened door. **Should you switch?**

Simulate and show how the probability of winning evolves if you always switch.

Discussion:
An overwhelming majority of people assume that each door has an equal probability of 1/3 and conclude that switching does not matter (i.e., they assume that the probability of winning remains unchanged at 1/3 regardless of the decision to switch).

In reality, the switching is advantageous (the probability of winning increases to 2/3 upon switching). Here is an explanation (source: wikipedia).



Objective:

Plot the evolution of the probability of winning when the player **always switches**, and use the simulation to illustrate convergence (law of large numbers), variability (confidence intervals), and the difference between empirical and analytic probability.

Step-by-step tasks:

Task 1. Implement a single trial with strategy of switching

Task 2. Build a vectorized simulator and compute running/cumulative probability

Task 3. Compare with the analytical result (2/3, when switching; 1/3, when not switching)

Task 4. Plot confidence interval

Task 5. Compare the switch versus non-switch decisions

For above tasks, now move on to the provided startup python file named **assm5_q1_student.py** and work on each task. Look out for #WRITE_YOUR_CODE sections; understand the provided program and complete the leftover lines.

Task 6. (*optional*; challenging) Generalize Monty Hall problem to N total doors and the host now opens k doors. This set up looks like:

- N doors: 1 car, $N-1$ goats.
- You, the player, pick one door.
- Host opens k doors from the remaining doors, but never the car.
- You may switch to one of the remaining unopened doors at random.

Simulate the switching strategy in this case for arbitrary N and k. Can you derive the winning probability in the generalized Monty Hall problem for the switch strategy? It is given as:

$$P(\text{win if switched}) = \frac{N-1}{N} \cdot \frac{1}{N-k-1}$$

Problem-2 (Modelling uncertain concrete strength):

Refer to the jupyter notebook `assm5_q2_student.ipynb` on GitHub repository. You will learn:

- reading data file,
- getting summary statistics of data,
- exposure to method of moments,
- develop empirical CDF (ECDF), and
- compare ECDF with fitted CDF.