Advance Numerical Technique Laboratory

Lab 7

Q.1 Solve a partial differential equation using Alternate Direction Implicit (ADI Scheme)

du/dt = k * (d2u/dx2 + d2u/dy2)

u(0,x,y) = 0 0 <= x,y <= 1

On the boundary : u(t,x,y) = exp(0.2*pi*x)* sin(0.2*pi*y)

dx = dy = 1/4, r = 1/6

Solution :-

Here, please note that the variable u is twice the size of t/dt. The values at the even indices are actual and the values at the odd indices are the values of t + ½*dt.

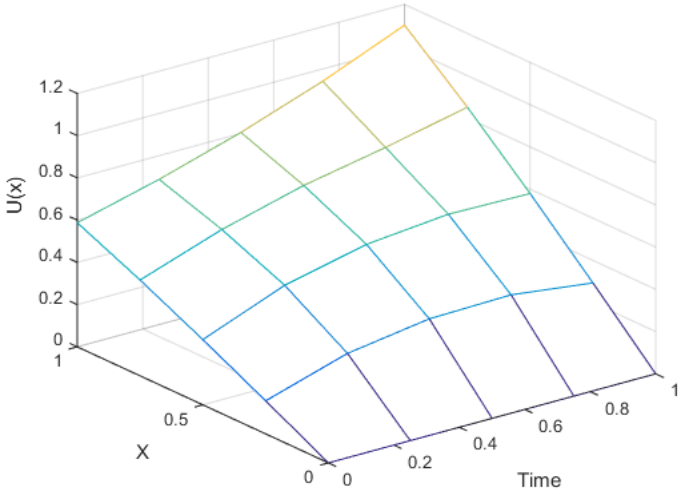

Figure 1. U(x,y) at t=to

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u(:,:,1) =

     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0

u(:,:,2) =

     0    0.1564    0.3090    0.4540    0.5878
     0         0         0         0    0.6878
     0         0         0         0    0.8047
     0         0         0         0    0.9416
     0    0.2932    0.5792    0.8510    1.1018

u(:,:,3) =

     0    0.1564    0.3090    0.4540    0.5878
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         0    0.0130    0.0254    0.0342    0.6878
         0         0         0         0    0.8047
         0    0.0244    0.0477    0.0642    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,4) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.0227    0.0436    0.1120    0.6878
         0    0.0065    0.0125    0.0832    0.8047
         0    0.0440    0.0843    0.1844    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,5) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.0326    0.0658    0.1310    0.6878
         0    0.0112    0.0252    0.0884    0.8047
         0    0.0616    0.1226    0.2204    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,6) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.0416    0.0838    0.1938    0.6878
         0    0.0229    0.0503    0.1726    0.8047
         0    0.0797    0.1571    0.3135    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,7) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.0499    0.1037    0.2044    0.6878
         0    0.0298    0.0679    0.1767    0.8047
         0    0.0929    0.1876    0.3329    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,8) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.0596    0.1227    0.2584    0.6878
         0    0.0450    0.0990    0.2551    0.8047
         0    0.1106    0.2196    0.4078    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,9) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.0670    0.1404    0.2637    0.6878
         0    0.0526    0.1172    0.2564    0.8047
         0    0.1205    0.2437    0.4168    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,10) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.0778    0.1603    0.3116    0.6878
         0    0.0700    0.1504    0.3268    0.8047
         0    0.1381    0.2734    0.4789    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,11) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.0843    0.1757    0.3131    0.6878
         0    0.0774    0.1672    0.3252    0.8047
         0    0.1454    0.2921    0.4813    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,12) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.0963    0.1962    0.3561    0.6878
         0    0.0961    0.2003    0.3877    0.8047
         0    0.1629    0.3196    0.5339    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,13) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1018    0.2091    0.3549    0.6878
         0    0.1025    0.2147    0.3835    0.8047
         0    0.1680    0.3340    0.5318    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,14) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1148    0.2298    0.3938    0.6878
         0    0.1220    0.2467    0.4388    0.8047
         0    0.1855    0.3594    0.5772    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,15) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1191    0.2402    0.3903    0.6878
         0    0.1270    0.2582    0.4326    0.8047
         0    0.1886    0.3700    0.5721    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,16) =
         0    0.1564    0.3090    0.4540    0.5878
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         0    0.1330    0.2606    0.4256    0.6878
         0    0.1469    0.2885    0.4814    0.8047
         0    0.2061    0.3936    0.6119    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,17) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1358    0.2685    0.4204    0.6878
         0    0.1504    0.2974    0.4736    0.8047
         0    0.2073    0.4011    0.6046    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,18) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1503    0.2884    0.4526    0.6878
         0    0.1702    0.3257    0.5170    0.8047
         0    0.2246    0.4230    0.6400    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,19) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1517    0.2940    0.4460    0.6878
         0    0.1721    0.3320    0.5080    0.8047
         0    0.2241    0.4279    0.6312    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,20) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1665    0.3133    0.4755    0.6878
         0    0.1918    0.3584    0.5467    0.8047
         0    0.2413    0.4482    0.6630    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,21) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1664    0.3167    0.4677    0.6878
         0    0.1920    0.3625    0.5368    0.8047
         0    0.2392    0.4509    0.6530    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,22) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1815    0.3352    0.4948    0.6878
         0    0.2114    0.3870    0.5716    0.8047
         0    0.2562    0.4699    0.6819    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,23) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1800    0.3367    0.4861    0.6878
         0    0.2100    0.3890    0.5609    0.8047
         0    0.2527    0.4706    0.6710    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,24) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1951    0.3545    0.5111    0.6878
         0    0.2290    0.4117    0.5924    0.8047
         0    0.2694    0.4885    0.6976    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,25) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.1922    0.3543    0.5016    0.6878
         0    0.2262    0.4120    0.5812    0.8047
         0    0.2646    0.4876    0.6860    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,26) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2074    0.3713    0.5250    0.6878
         0    0.2448    0.4332    0.6099    0.8047
         0    0.2812    0.5044    0.7106    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,27) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2032    0.3695    0.5149    0.6878
         0    0.2406    0.4319    0.5983    0.8047
         0    0.2751    0.5022    0.6986    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,28) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2184    0.3859    0.5367    0.6878
         0    0.2588    0.4517    0.6246    0.8047
         0    0.2915    0.5181    0.7215    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,29) =
         0    0.1564    0.3090    0.4540    0.5878
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         0    0.2130    0.3828    0.5260    0.6878
         0    0.2533    0.4491    0.6127    0.8047
         0    0.2844    0.5147    0.7091    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,30) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2281    0.3985    0.5466    0.6878
         0    0.2711    0.4676    0.6370    0.8047
         0    0.3005    0.5298    0.7307    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,31) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2216    0.3942    0.5355    0.6878
         0    0.2645    0.4638    0.6249    0.8047
         0    0.2925    0.5254    0.7179    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,32) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2367    0.4094    0.5550    0.6878
         0    0.2819    0.4813    0.6475    0.8047
         0    0.3084    0.5398    0.7384    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,33) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2292    0.4041    0.5436    0.6878
         0    0.2743    0.4765    0.6352    0.8047
         0    0.2996    0.5346    0.7254    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,34) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2442    0.4188    0.5621    0.6878
         0    0.2914    0.4930    0.6563    0.8047
         0    0.3153    0.5484    0.7449    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,35) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2359    0.4126    0.5504    0.6878
         0    0.2829    0.4874    0.6439    0.8047
         0    0.3057    0.5424    0.7317    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,36) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2507    0.4268    0.5681    0.6878
         0    0.2996    0.5031    0.6638    0.8047
         0    0.3213    0.5557    0.7503    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,37) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2417    0.4199    0.5562    0.6878
         0    0.2903    0.4968    0.6512    0.8047
         0    0.3111    0.5491    0.7370    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,38) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2564    0.4338    0.5732    0.6878
         0    0.3067    0.5117    0.6702    0.8047
         0    0.3265    0.5620    0.7550    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,39) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2467    0.4262    0.5611    0.6878
         0    0.2968    0.5048    0.6575    0.8047
         0    0.3157    0.5549    0.7415    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,40) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2614    0.4397    0.5776    0.6878
         0    0.3129    0.5191    0.6755    0.8047
         0    0.3310    0.5674    0.7589    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,41) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2510    0.4315    0.5653    0.6878
         0    0.3024    0.5117    0.6628    0.8047
         0    0.3197    0.5598    0.7453    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,42) =
         0    0.1564    0.3090    0.4540    0.5878
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         0    0.2656    0.4448    0.5813    0.6878
         0    0.3182    0.5254    0.6801    0.8047
         0    0.3348    0.5720    0.7622    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,43) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2548    0.4361    0.5689    0.6878
         0    0.3072    0.5175    0.6673    0.8047
         0    0.3231    0.5640    0.7485    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,44) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2693    0.4491    0.5844    0.6878
         0    0.3229    0.5308    0.6840    0.8047
         0    0.3382    0.5759    0.7650    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,45) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2580    0.4401    0.5719    0.6878
         0    0.3113    0.5226    0.6711    0.8047
         0    0.3261    0.5676    0.7513    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,46) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2725    0.4528    0.5871    0.6878
         0    0.3268    0.5355    0.6873    0.8047
         0    0.3411    0.5793    0.7674    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,47) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2608    0.4434    0.5745    0.6878
         0    0.3149    0.5269    0.6744    0.8047
         0    0.3287    0.5707    0.7536    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,48) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2752    0.4560    0.5893    0.6878
         0    0.3302    0.5394    0.6901    0.8047
         0    0.3435    0.5821    0.7695    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,49) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2632    0.4463    0.5767    0.6878
         0    0.3180    0.5305    0.6771    0.8047
         0    0.3309    0.5733    0.7556    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,50) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2776    0.4587    0.5913    0.6878
         0    0.3332    0.5428    0.6924    0.8047
         0    0.3457    0.5846    0.7712    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,51) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2653    0.4488    0.5785    0.6878
         0    0.3206    0.5337    0.6795    0.8047
         0    0.3328    0.5756    0.7573    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,52) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2796    0.4610    0.5929    0.6878
         0    0.3357    0.5457    0.6945    0.8047
         0    0.3475    0.5867    0.7727    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,53) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2671    0.4509    0.5801    0.6878
         0    0.3229    0.5363    0.6815    0.8047
         0    0.3344    0.5775    0.7587    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,54) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2813    0.4630    0.5943    0.6878
         0    0.3379    0.5481    0.6962    0.8047
         0    0.3490    0.5885    0.7739    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,55) =
         0    0.1564    0.3090    0.4540    0.5878
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         0    0.2686    0.4527    0.5815    0.6878
         0    0.3248    0.5386    0.6832    0.8047
         0    0.3358    0.5791    0.7600    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,56) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2828    0.4647    0.5955    0.6878
         0    0.3397    0.5502    0.6977    0.8047
         0    0.3504    0.5900    0.7750    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,57) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2699    0.4542    0.5826    0.6878
         0    0.3265    0.5406    0.6847    0.8047
         0    0.3370    0.5805    0.7610    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,58) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2841    0.4662    0.5965    0.6878
         0    0.3413    0.5520    0.6989    0.8047
         0    0.3515    0.5913    0.7759    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,59) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2710    0.4555    0.5836    0.6878
         0    0.3279    0.5423    0.6859    0.8047
         0    0.3380    0.5817    0.7619    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,60) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2852    0.4674    0.5974    0.6878
         0    0.3427    0.5536    0.7000    0.8047
         0    0.3525    0.5924    0.7767    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,61) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2720    0.4567    0.5844    0.6878
         0    0.3291    0.5437    0.6870    0.8047
         0    0.3388    0.5827    0.7627    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,62) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2861    0.4685    0.5981    0.6878
         0    0.3438    0.5549    0.7009    0.8047
         0    0.3534    0.5933    0.7773    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,63) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2728    0.4576    0.5851    0.6878
         0    0.3302    0.5449    0.6879    0.8047
         0    0.3396    0.5836    0.7633    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,64) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2869    0.4694    0.5988    0.6878
         0    0.3448    0.5560    0.7017    0.8047
         0    0.3541    0.5942    0.7779    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,65) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2735    0.4584    0.5858    0.6878
         0    0.3311    0.5460    0.6886    0.8047
         0    0.3402    0.5844    0.7638    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,66) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2876    0.4701    0.5993    0.6878
         0    0.3457    0.5570    0.7023    0.8047
         0    0.3547    0.5949    0.7784    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,67) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2741    0.4591    0.5863    0.6878
         0    0.3318    0.5468    0.6893    0.8047
         0    0.3408    0.5850    0.7643    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,68) =
         0    0.1564    0.3090    0.4540    0.5878
```

```
         0    0.2882    0.4708    0.5998    0.6878
         0    0.3464    0.5578    0.7029    0.8047
         0    0.3552    0.5954    0.7788    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,69) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2746    0.4597    0.5867    0.6878
         0    0.3325    0.5476    0.6899    0.8047
         0    0.3412    0.5855    0.7647    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,70) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2887    0.4714    0.6002    0.6878
         0    0.3470    0.5585    0.7034    0.8047
         0    0.3557    0.5959    0.7791    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,71) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2750    0.4602    0.5871    0.6878
         0    0.3331    0.5482    0.6903    0.8047
         0    0.3416    0.5860    0.7651    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,72) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2891    0.4718    0.6005    0.6878
         0    0.3475    0.5591    0.7038    0.8047
         0    0.3561    0.5964    0.7794    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,73) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2754    0.4607    0.5874    0.6878
         0    0.3335    0.5488    0.6907    0.8047
         0    0.3420    0.5864    0.7654    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,74) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2895    0.4722    0.6008    0.6878
         0    0.3480    0.5596    0.7041    0.8047
         0    0.3564    0.5967    0.7797    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,75) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2757    0.4610    0.5877    0.6878
         0    0.3339    0.5493    0.6911    0.8047
         0    0.3423    0.5867    0.7656    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,76) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2898    0.4726    0.6010    0.6878
         0    0.3484    0.5600    0.7044    0.8047
         0    0.3567    0.5971    0.7799    0.9416
         0    0.2932    0.5792    0.8510    1.1018
u(:,:,77) =
         0    0.1564    0.3090    0.4540    0.5878
         0    0.2760    0.4614    0.5879    0.6878
         0    0.3343    0.5497    0.6914    0.8047
         0    0.3425    0.5870    0.7658    0.9416
         0    0.2932    0.5792    0.8510    1.1018
```