Advance Numerical Technique Laboratory

Lab 1

Q.1 Solve this boundary value problem using shooting method and classical runge kutta method.

$y'y'' + 1 + (y')^2 = 0$

$y(0) = 1$

$y(1) = 2$

$a_0 = 0.5$

$a_1 = 1$

$h = 0.2$

Solution :-

Rearranging differential equation, we get,

$$y'' = \frac{-1 - (y')^2}{y'}$$

Iteration 1

$For\ a_0 = 0.5$

x/y/y'	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	1.0	1.0770	1.1135	1.1135	1.0770	1.0000
y'	0.5	0.2785	0.0898	-0.0898	-0.2786	-0.5001

$For\ a_1 = 1.0$

x/y/y'	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	1.0	1.1661	1.2805	1.3563	1.3998	1.4139
y'	1.0000	0.6858	0.4683	0.2947	0.1427	-0.0002

Using secant method,

$a_{new} = 1.7079$

Iteration 2

$For\ a_0 = 1.0$

x/y/y'	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	1.0	1.1661	1.2805	1.3563	1.3998	1.4139
y'	1.0000	0.6858	0.4683	0.2947	0.1427	-0.0002

$For\ a_1 = 1.7079$

x/y/y'	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	1.0000	1.2812	1.4844	1.6388	1.7572	1.8466
y'	1.7079	1.1751	0.8794	0.6745	0.5152	0.3820

Using secant method,

$a_{new} = 1.9589$

Iteration 3

$For\ a_0 = 1.9589$

x/y/y'	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	1.0000	1.2812	1.4844	1.6388	1.7572	1.8466
y'	1.7079	1.1751	0.8794	0.6745	0.5152	0.3820

$For\ a_1 = 2.0039$

x/y/y'	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	1.0000	1.3261	1.5613	1.7428	1.8859	1.9990
y'	2.0039	1.3568	1.0240	0.8026	0.6356	0.4996

Using secant method,

$a_{new} = 2.0058$

Iteration 4

$For\ a_0 = 2.0039$

x/y/y'	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	1.0000	1.3261	1.5613	1.7428	1.8859	1.9990
y'	2.0039	1.3568	1.0240	0.8026	0.6356	0.4996

$For\ a_1 = 2.0058$

x/y/y'	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	1.0000	1.3263	1.5618	1.7434	1.8867	2.0000
y'	2.0058	1.3579	1.0249	0.8034	0.6364	0.5003

Using secant method,

$a_{new} = 2.0058$

Here the last two values of a are nearly same, which means the value of a converges. Below is the graph of the differential equation solved numerically.

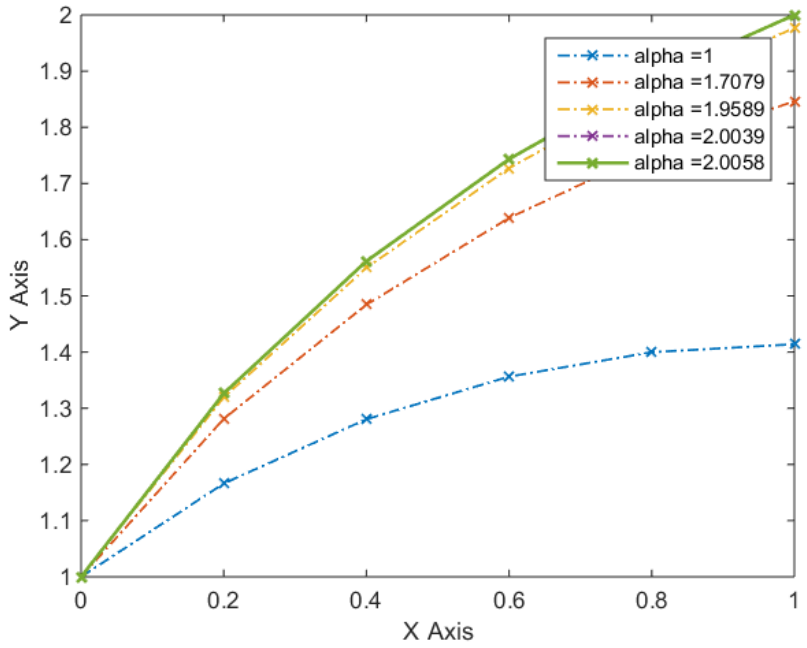

Fig. 1 Graph of the y values calculated

Q.2 Solve this boundary value problem using shooting method and classical runge kutta method.

$y'' - y = 0$

$y(0) = 0$

$y(1) = 1$

$a_0 = 0.3$

$a_1 = 0.4$

$h = 0.1$

Solution :-

Iteration 1

$For\ a_0 = 0.3$

	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	0	0.0852	0.1713	0.2591	0.3495	0.4434	0.5417	0.6455	0.7557	0.8735	1
y'	0.8509	0.8552	0.868	0.8895	0.9199	0.9595	1.0087	1.068	1.138	1.2194	1.313

$For\ a_1 = 0.4$

	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	0	0.0401	0.0805	0.1218	0.1643	0.2084	0.2547	0.3034	0.3552	0.4106	0.4701
y'	0.4	0.402	0.408	0.4181	0.4324	0.4511	0.4742	0.5021	0.535	0.5732	0.6172

Using secant method,

$a_{new} = -0.85092$

Iteration 2

$For\ a_0 = 0.4$

	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	0	0.0401	0.0805	0.1218	0.1643	0.2084	0.2547	0.3034	0.3552	0.4106	0.4701
y'	0.4	0.402	0.408	0.4181	0.4324	0.4511	0.4742	0.5021	0.535	0.5732	0.6172

$For\ a_1 = -0.85092$

	x = 0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
y	0	-0.0852	-0.1713	-0.2591	-0.3495	-0.4434	-0.5417	-0.6455	-0.7557	-0.8735	-1.0000
y'	-0.8509	-0.8552	-0.868	-0.8895	-0.9199	-0.9595	-1.0087	-1.068	-1.138	-1.2194	-1.3130

Using secant method,

$a_{new} = -0.8509$

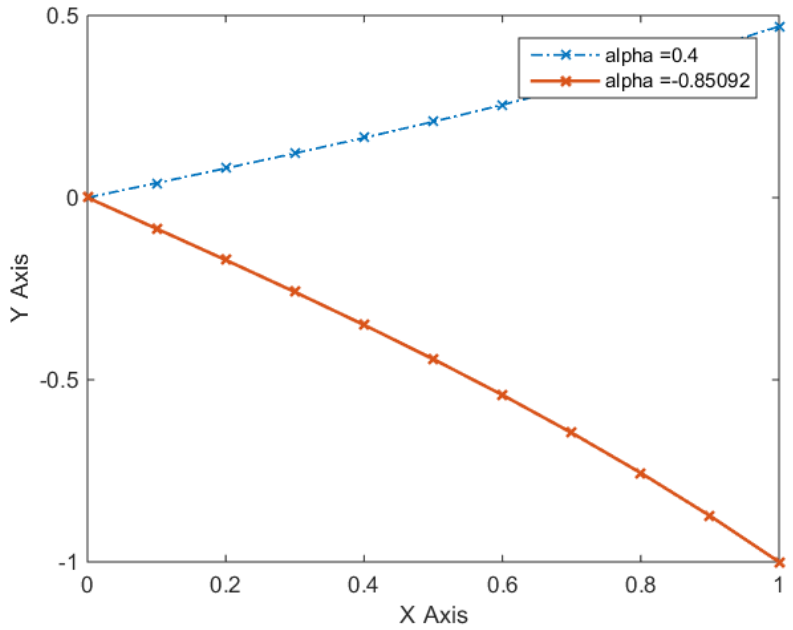

Fig. 2 Graph of the y values calculated