Graph Embedding and Extensions: A General Framework for Dimensionality Reduction

Graph Embedding

Definition

The definition of **Graph Embedding** in this paper is different from original definition.

Let $G = \{X, W\}$ be undirected weighted graph with vertex set X and similarity matrix W.

We want to find a low-dimensional representation of the Graph G which can maintain graph relation.

Hence, we define:

- $y = f(x)$ which map vertex to $\mathbb{R}^m$
- A diagonal matrix D. $D_{ii} = \sum_{i \neq j} W_{ij}$
- $L = D - W$
- $B = I$ or $B = D^p - B^p$ for scalar preserving, where W^p denotes penalty graph.

and a programming target:

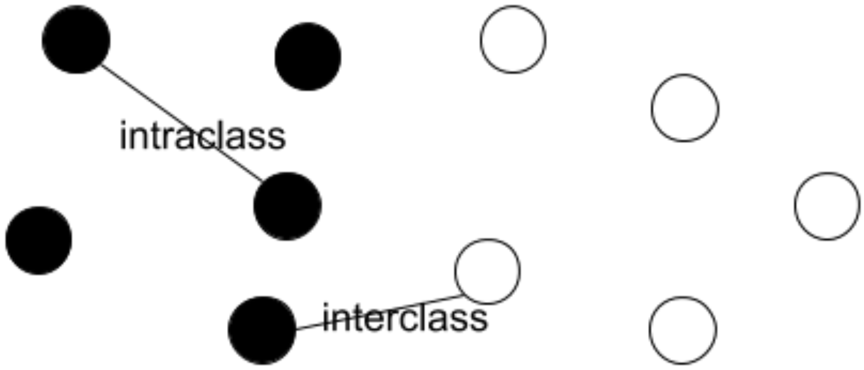
$$y^* = \arg \min_{y^T By = d} \sum_{i \neq j} |y_i - y_j|^2 W_{ij} = \arg \min_{y^T By = d} y^T Ly$$

Dimensionality Reduction

Previous works on dimensionality reduction can be represented as graph embedding framework[Table crop from paper]:

Algorithm	W&B Definition
PCA	$W_{ij} = \frac{1}{N}$, $B = I$
LDA	$W_{ij} = \frac{\delta_{c_i, c_j}}{n_{c_i}}$, $B = I - \frac{1}{Nee^T}$
LPP	$W_{ij} = \exp\left\{\frac{-\|x_i - x_j\|^2}{t}\right\}, \text{if } i \in N_k(j) \text{ or } j \in N_k(i)$, $B = D$

Marginal Fisher Analysis



Steps

Step1: PCA

Step2: Intraclass compatness and interclass separability

- Intraclass: $W_{ij} = 1$ if (i, j) is nearest neighbor pair in same class
- Interclass: $W_{ij}^p = 1$ if (i, j) is nearest neighbor pair in different class

Step3: Marginal Fisher Criterion

$w^* = \arg\min_w \frac{w^T X(D-W)X^T w}{w^T X(D^p-W^p)X^T w}$

Step4: Resolve PCA

Result

The result is greater than previous work at face recognition.