Algorithm on Face Recognition

Based on the paper by Hongun Li, we are going to use a relatively novel approach for Face Recognition.

The main algorithm is as follows:

Input:

First, we have the training data $X = \{x_1, x_2, \ldots, x_q\}$ and the testing data $Y = \{y_1, y_2, \ldots, y_p\}$. The size of training and testing data is $S_1 \times S_2$. The parameters are λ, ρ. A matrix of normalized training samples $D = [D_1, D_2, \ldots, D_k] \in R^{m\times n}$ for K classes, and a test sample $y \in R^m$. As you can see, the dimension m of the training sample is the same as the dimension of each D_i, and the dimension n denotes the number of images on each class.

Algorithm:

- Make the Dynamic Rank estimation and get the optimal rank r.
- For each class i do
  - Obtain the low-rank part M_i of the training data D_i.
    In order to do this, first we need to conduct a SVD of the matrix $M_i = U\Sigma V^T$, then, having obtained the matrix Σ, where its diagonal are the ordered singular values, take just the first r singular values and set the remaining to zero, this will give a new matrix Σ'.
    Finally $M_i = U\Sigma' V^T$.
  - Given M_i, solve the following optimization problem

$$\min_{Z_i^*, L_i^*, E_i^*} ||Z_i||_* + \lambda||M_i - M_i Z_i||_F^2$$

$$s.t. \qquad M_i = M_i Z_i + L_i M_i + E_i, \qquad Z_i = V_{M_i} W_{Z_i} V_{M_i}^T,$$

$$L_i = U_{M_i}\left(I - W_{Z_i}\right) U_{M_i}^T, \qquad \operatorname{rank}(Z_i) = r$$

    As you can see, the variables we are looking are W_{Z_i} and E_i, in order to find the best Z_i and L_i. The other components can be found by conducting a skinny SVD of $M_i = U_{M_i}\Sigma_{M_i}V_{M_i}^T$ to find U_{M_i} and V_{M_i}. The meaning of skinny is that Σ_{M_i} is a square matrix of size $\operatorname{rank}(M_i)$.
    This is a kind of optimization problem called convex optimization problem (you can read more about it in the following link), where $||Z_i||_* + \lambda||M_i - M_i Z_i||_F^2$ is called the "objective function", there are no inequality constraints and there are four equality constraint functions.

There are different approaches to solve this kind of problems, and there is a Python library created called CVXOPT.

- Having got the optimal L_i, calculate the discriminative information of face image by $L_i M_i$.
- For the identification, calculate the residual associated with the ith class as

$$e_i(y) = ||L_i y - L_i M_i||_2^2$$

Note that this will be a function of y, so that the identification is given by

$$\text{identify}(y) = \text{argmin}_i(e_i(y))$$

i.e. the class i where the function e_i is minimized