Uniform (a,b):

$$R_X = [a,b]$$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & , \text{if } x \in R_X \\ 0 & , \text{if } x \notin R_X \end{cases}$$

Mean:

$$\langle X \rangle = \int_{-\infty}^{+\infty} x\, f_X(x)\, dx = \int_a^b x \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a} \int_a^b x\, dx = \frac{1}{b-a} \left[\frac{1}{2} x^2\right]_a^b$$

$$= \frac{1}{2} \cdot \frac{1}{b-a} \left(b^2 - a^2\right)$$

$$= \frac{1}{2} \frac{\cancel{(b-a)}\,(b+a)}{\cancel{(b-a)}} = \frac{1}{2}(a+b)$$

Variance:

$$\langle X^2 \rangle_c = \langle (X - \langle X \rangle)^2 \rangle = \langle (X - \tfrac{1}{2}(a+b))^2 \rangle$$

$$= \int_{-\infty}^{+\infty} (x - \tfrac{1}{2}(a+b))^2 f_X(x)\, dx$$

$$= \frac{1}{b-a} \left(\int_a^b x^2 dx - \int_a^b x(a+b)\, dx + \frac{1}{4} \int_a^b (a+b)^2 dx \right)$$

$$= \frac{1}{b-a} \left(\left[\frac{1}{3} x^3\right]_a^b - (a+b) \left[\frac{1}{2} x^2\right]_a^b + \frac{1}{4}(a+b)^2 \left[x\right]_a^b \right)$$

$$= \frac{1}{b-a} \left(\frac{1}{3} b^3 - \frac{1}{3} a^3 - \frac{1}{2} ab^2 - \frac{1}{2} b^3 + \frac{1}{2} a^3 + \frac{1}{2} a^2 b + \frac{1}{4}(a^2 + 2ab + b^2)(b-a) \right)$$

$$= \frac{1}{b-a} \left(\frac{1}{3} b^3 - \frac{1}{3} a^3 - \frac{1}{2} ab^2 - \frac{1}{2} b^3 + \frac{1}{2} a^3 + \frac{1}{2} a^2 b - \frac{1}{4} a^2 b + \frac{1}{4} ab^2 + \frac{1}{4} b^3 \text{ [illegible] } - \frac{1}{4} ab^2 \right)$$

$$= \frac{1}{b-a} \left(\frac{1}{12} b^3 - \frac{1}{12} a^3 - \frac{3}{12} ab^2 + \frac{3}{12} a^2 b \right) = \frac{1}{12} \frac{1}{b-a} \left(b^3 - a^3 - 3ab^2 + 3a^2 b \right)$$

$$\langle X^2 \rangle_c = \frac{1}{12}\frac{1}{b-a}(b-a)^3 = \frac{1}{12}(b-a)^2$$

skewness

$$\langle X^3 \rangle_s = \frac{\langle X^3 \rangle_c}{\langle X^2 \rangle_c^{3/2}}$$

$$\cancel{\langle X^3 \rangle_c = \int_{-\infty}^{+\infty} x f_x(x)\,dx = \int_a^b x^3 \frac{1}{b-a}\,dx}$$

$$\neq$$

$$\langle X^3 \rangle_c = \langle (X - \langle X \rangle)^3 \rangle$$

$$= \int_a^b \left(x - \frac{1}{2}(a+b)\right)^3 \cdot \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a}\left(\int_a^b x^3 - \frac{3}{2}(a+b)x^2 + \frac{3}{4}(a+b)^2 x - \frac{1}{8}(a+b)^3\,dx\right)$$

$$= \frac{1}{b-a}\left(\left[\frac{1}{4}x^4\right]_a^b - \frac{3}{2}(a+b)\left[\frac{1}{3}x^3\right]_a^b + \left[\frac{3}{8}(a+b)^2 x^2\right]_a^b - \frac{1}{8}(a+b)^3 [x]_a^b\right)$$

$$= \frac{1}{b-a}\left(\frac{1}{4}(b^4-a^4) - \frac{1}{2}(a+b)b^3 + \frac{1}{2}(a+b)a^3 + \frac{3}{8}(a+b)^2(b^2-a^2) - \frac{1}{8}(a+b)^3(b-a)\right)$$

$$= \frac{1}{64}\frac{1}{b-a}\frac{1}{64}(a+b-2x)^4$$

Gauß (μ, σ)

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

Mean

$$\langle X \rangle = \int_{-\infty}^{+\infty} x\, f_X(x)\, dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} x \cdot \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} [(x-\mu)+\mu] \cdot \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} (x-\mu) \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx + \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mu \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} (-\sigma^2) \int_{-\infty}^{+\infty} \frac{(-x+\mu)}{\sigma^2} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx$$

$$+ \cancel{\frac{1}{\sigma\sqrt{2\pi}}}\, \mu \underbrace{\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx}_{1}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} (-\sigma^2) \left[\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)\right]_{-\infty}^{+\infty} + \mu$$

$$= \underline{\underline{\mu}}$$

Variance

$$\langle X^2 \rangle_2 = \langle (X-\mu)^2 \rangle$$

$$= \int_{-\infty}^{+\infty} (x-\mu)^2 f_X(x)\, d_x$$

$$= \int_{-\infty}^{+\infty} (x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx$$

subs $t = \dfrac{x-\mu}{\sigma\sqrt{2}}$

$$= \int_{-\infty}^{+\infty} \sigma^2 2t^2 \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-t^2\right) \sigma\sqrt{2}\, dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{+\infty} t^2 \cdot \exp(-t^2)\, dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \left[-\frac{1}{2} t \cdot \exp(-t^2)\right]_{-\infty}^{+\infty} \leftarrow \text{is } 0 \text{ (L'Hôpitals rule)}$$

$$- \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{+\infty} -\frac{1}{2} \exp(-t^2)\, dt$$

$$= 0 + \frac{2\sigma^2}{\sqrt{\pi}} \frac{1}{2} \int_{-\infty}^{+\infty} \exp(-t^2)\, dt$$

$$= \frac{\sigma^2}{\sqrt{\pi}} \sqrt{\pi} = \underline{\underline{\sigma^2}}$$

	Gauß (μ, σ)	Uniform (a,b)
mean	μ	$\frac{1}{2}(a+b)$
variance	σ^2	$\frac{1}{12}(b-a)^2$