
Math primer

Exercise T1.1: Learning paradigms

(tutorial)

- (a) Describe the difference between *supervised*, *unsupervised*, and *reinforcement learning*.
- (b) Which of the above learning techniques would be most appropriate in the following cases and what would be the corresponding *observations*, *labels* and/or *rewards*?
- To teach a dog to catch a ball
 - To read hand written addresses from letters
 - To identify groups of users with the same taste of music

Exercise H1.1: Distributions and expected values

(homework, 3 points)

Let X be a random variable and $p : \mathbb{R} \rightarrow \mathbb{R}$ with:

$$p(x) = \begin{cases} c \cdot \sin(x), & x \in [0, \pi] \\ 0, & \text{elsewhere} \end{cases}$$

- (a) Determine the parameter value $c \in \mathbb{R}$ so that $p(x)$ is a probability density.
- (b) For such c , determine the expected value $\langle X \rangle_p$
- (c) Determine the variance $\langle X^2 \rangle_p - \langle X \rangle_p^2$.

Exercise H1.2: Marginal densities

(homework, 2 points)

Assume the probability density function of a two-dimensional random vector $\mathbf{z} = (x, y)^\top$

$$p_{\mathbf{z}}(\mathbf{z}) = p_{x,y}(x, y) = \begin{cases} \frac{3}{7}(2-x)(x+y), & x \in [0, 2], y \in [0, 1] \\ 0, & \text{elsewhere} \end{cases}$$

- (a) Write down the marginal densities $p_x(x)$ and $p_y(y)$ of the variables x and y .
- (b) Determine if the two variables are independent.

Exercise H1.3: Taylor expansion

(homework, 1 point)

For the function $\sqrt{1+x}$, write down the Taylor series expansion around $x_0 = 0$ up to 3rd order.

Exercise H1.4: Determinant of a matrix

(homework, 1 point)

Consider the 3×3 matrix

$$A = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{pmatrix}$$

Calculate the determinant and the trace of A (directly, not via eigenvalues).

Exercise H1.5: Critical points**(homework, 2 points)**

Consider the two functions

$$f(x, y) := c + x^2 + y^2$$

$$g(x, y) := c + x^2 - y^2,$$

where c is a constant.

- (a) Show that $\mathbf{a} = (0, 0)$ is a critical point of both functions.
- (b) Check for f and for g whether \mathbf{a} is a minimum, maximum, or no extremum by calculating the Hessian matrix. Make use of the fact that a matrix is positive (negative) definite if and only if all its eigenvalues are positive (negative).

Exercise H1.6: Bayes rule**(homework, 1 point)**

Assume it is known that 1% of the population suffer from a certain disease. A company has developed a test for diagnosing the disease, which comes up either positive (“+”, disease found) or negative (“-”, disease not found). People suffering from the disease (D) are diagnosed positive with probability 0.95, and healthy people (\bar{D}) are diagnosed negative with probability 0.999.

Apply Bayes’ rule to find

- (a) the probabilities that a person for which the test yielded a positive result is indeed suffering from the disease $P(D|+)$, respectively is healthy $P(\bar{D}|+)$.
- (b) the probabilities that a person for which the test yielded a negative result is indeed healthy $P(\bar{D}|-)$, respectively is suffering from the disease $P(D|-)$.

Total 10 points.