Dissertation in Interdisciplinary Mathematics

Learning parameters of discrete determinantal point processes

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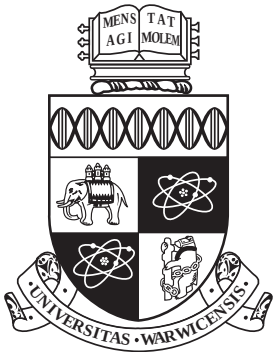
June 12, 2018



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Acknowledgements

Determinantal point processes are random subsets that exhibit a diversifying behaviour in the sense that the randomly selected points tend to be not similar in some way. This repellent structure first arrose in theortical physics and pure mathematics, but they have recently been used to model a variety of many real world scenarios in a machine learning setup. We aim to give an overview over the main ideas of this approach which is easily accessible even without prior knowledge in the area of machine learning and sometimes omit technical calculations in order to keep the focus on the concepts.

Abstract

Determinantal point processes are random subsets that exhibit a diversifying behaviour in the sense that the randomly selected points tend to be not similar in some way. This repellent structure first arrose in theortical physics and pure mathematics, but they have recently been used to model a variety of many real world scenarios in a machine learning setup. We aim to give an overview over the main ideas of this approach which is easily accessible even without prior knowledge in the area of machine learning and sometimes omit technical calculations in order to keep the focus on the concepts.

Contents

I Introduction and motivating examples **1**
I.1 Motivation . . . 1
I.2 Previous work . . . 1
I.3 Aim and outline of the dissertation . . . 1

II Determinantal points processes: Basic notions and properties **2**
II.1 Historical remarks . . . 2
II.2 Definitions and properties . . . 2
II.3 L-ensembles . . . 2
II.4 Related structures: conditional DPPs, structured DPPs and k-DPPs . . . 2
II.5 The magic properties of DPPs . . . 2
II.6 The mode problem . . . 2

III Learning setups **3**
III.1 What does learning mean? . . . 3
III.2 Motivation for learning DPPs . . . 3
III.3 Direct reconstruction of the kernel . . . 3
III.4 A learning approach to estimating the kernel . . . 3
III.5 Learning the quality functions . . . 3
III.6 Estimating the mixture coefficients of k-DPPs . . . 3

IV Toy examples and experiments **4**
IV.1 Minimal example? . . . 4
IV.2 Points on the line . . . 4
IV.3 Points in the square . . . 4
IV.4 Toy example for quality learning . . . 4

V Summary and conclusion **5**

A Generated code **6**

Bibliography **7**

Chapter I

Introduction and motivating examples

I.1 Motivation

I.2 Previous work

I.3 Aim and outline of the dissertation

Chapter II

Determinantal points processes: Basic notions and properties

II.1 Historical remarks

II.2 Definitions and properties

II.3 L-ensembles

II.4 Related structures: conditional DPPs, structured DPPs and k-DPPs

II.5 The magic properties of DPPs

II.6 The mode problem

Chapter III

Learning setups

III.1 What does learning mean?

III.2 Motivation for learning DPPs

III.3 Direct reconstruction of the kernel

III.4 A learning approach to estimating the kernel

III.5 Learning the quality functions

III.6 Estimating the mixture coefficients of k-DPPs

Chapter IV

Toy examples and experiments

IV.1 Minimal example?

IV.2 Points on the line

IV.3 Points in the square

IV.4 Toy example for quality learning

Chapter V

Summary and conclusion

Chapter A

Generated code

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