University of Basel

Mutex Based Potential Heuristics

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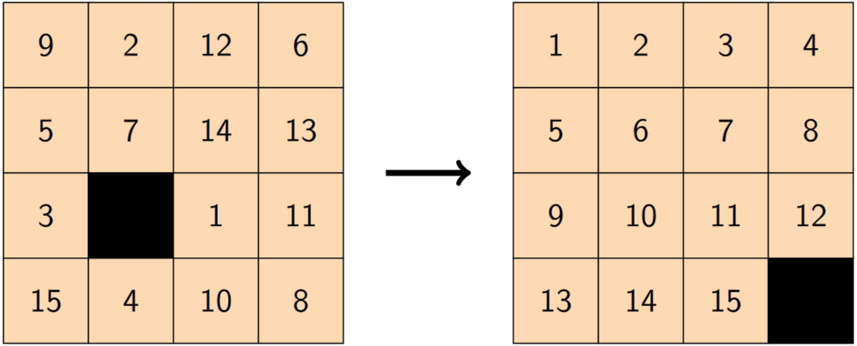
Classical Planning

- States $s \in \mathcal{S}$
- Facts $f = \langle V, v \rangle$, $f \in \mathcal{F}$
- Operators $o \in \mathcal{O}$
  - $\mathrm{cost}(o)$
  - $\mathrm{pre}(o) \subset \mathcal{F}$
  - $\mathrm{eff}(o) \subset \mathcal{F}$
- Path π

15-Puzzle



[1]

Initial State Goal State

[1] Image from Lecture *Introduction to Artificial Intelligence* (FS 2020)

Potential Heuristics

Definition (General Heuristic)

$$h : \mathcal{R} \rightarrow \mathbb{R} \cup \{\infty\}$$

Definition (Potential Heuristic)

$$h^{\mathrm{P}}(s) = \sum_{f \in s} P(f)$$

Linear Program

- Optimization Functions
  - Linear combination of potentials
  - Different optimization functions yield different heuristics
- Constraints
  - Inequalities
  - Assure admissibility

Mutexes and Disambiguations

Variables: A, B, C

Domain: $\mathrm{dom}(A) = \{1,2,3\}$
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Mutex Set: $\{ \{\langle A,1\rangle, \langle B,1\rangle\}, \{\langle B,2\rangle, \langle C,3\rangle\}, \{\langle B,3\rangle, \langle C,3\rangle\} \}$

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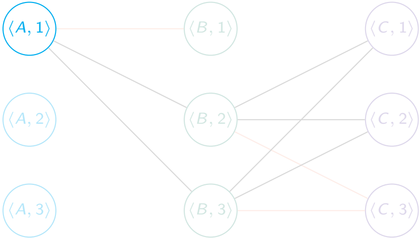

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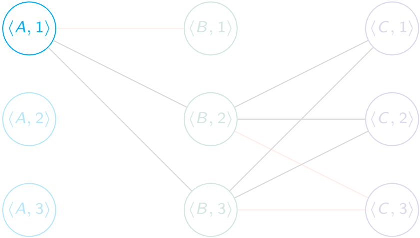

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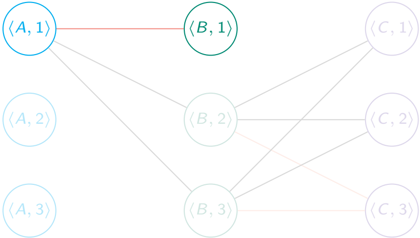

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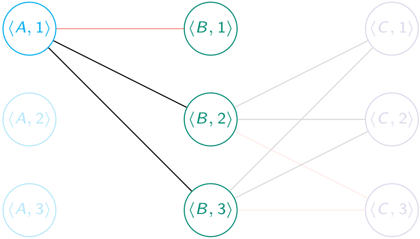

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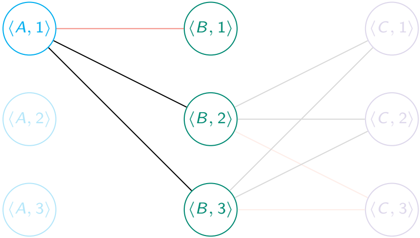

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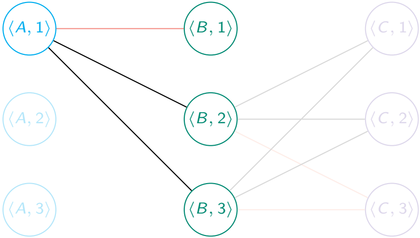

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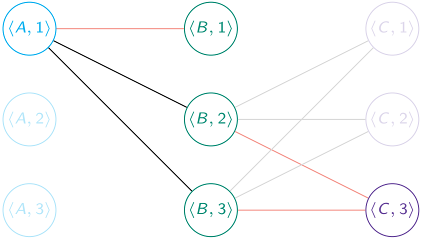

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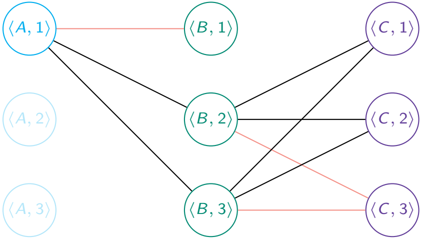
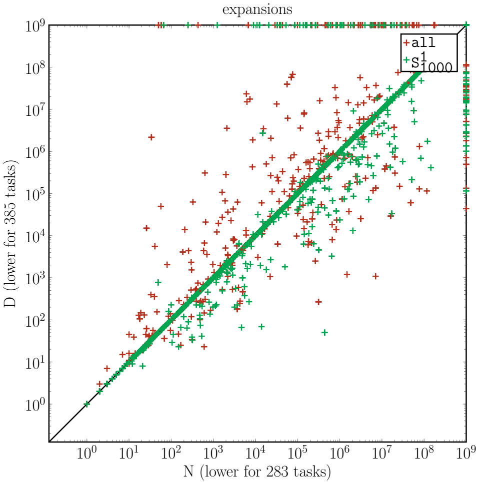


Strengthen LP Constraints

Definition (Constraint for Goal-Awareness)

$$\sum_{f \in G} \mathrm{P}(f) + \sum_{V \in \mathcal{V} \setminus \mathrm{vars}(G)} \max_{f \in \mathcal{F}_V} \mathrm{P}(f) \leq 0$$

Strengthen LP Constraints: Results



Mutex Based Optimization Functions

Definition (Mutex Based Optimization Function)

$$\mathrm{opt}^{k}_{\mathcal{M}} = \sum_{f=\langle V,v\rangle \in \mathcal{F}} \frac{\mathcal{C}^{k}_{f}(\mathcal{M})}{\sum_{f' \in \mathcal{F}_V} \mathcal{C}^{k}_{f'}(\mathcal{M})} \mathrm{P}(f)$$

Definition (Mutex Based Ensemble Optimization Function)

$$\mathrm{opt}^{t,k}_{\mathcal{M}} = \sum_{f=\langle V,v\rangle \in \mathcal{F}} \frac{\mathcal{K}^{k}_{f}(\mathcal{M}, t)}{\sum_{f' \in \mathcal{F}_V} \mathcal{K}^{k}_{f}(\mathcal{M}, t)} \mathrm{P}(f)$$

Mutex Based Optimization Functions: Results

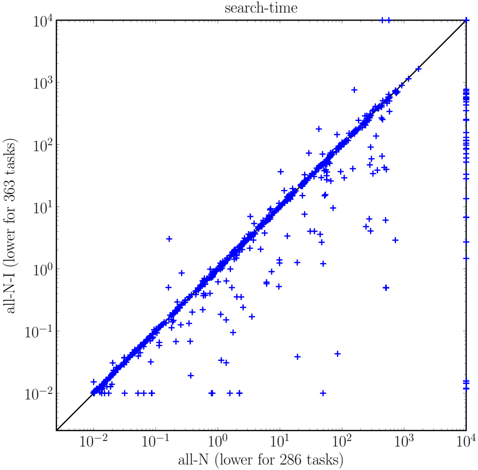
	`all-N`	$\mathtt{M}_1$`-D`	$\mathtt{J}_1^{10}$`-D`
Coverage	929	900	922
Expansions	10244	8297	6197
Total Time	0.29	0.59	1.02
Search Time	0.23	0.20	0.89

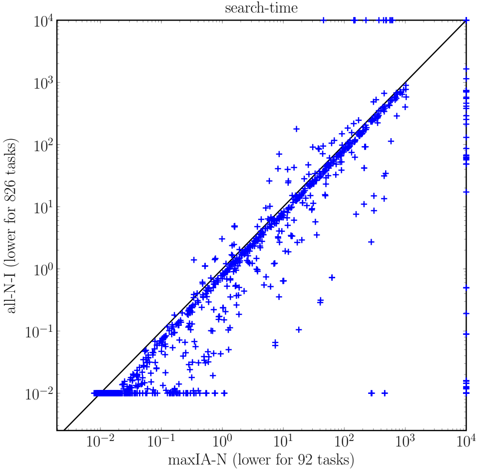
Additional Constraints

Definition (Additional Constraint)

$$\sum_{f \in s} \mathrm{P}(f) = h^{\mathrm{P}}(s)$$

Additional Constraints: Results





Conclusion

- Too computationally expensive
- Additional constraints good

Questions?

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Max(all, init)

	all-N	all-N-I	all-N-I-WP	max-N	max-N-WP	max-N-I
Coverage	929	**965**	**965**	948	957	958
Expansions	53184	49875	40519	38914	36556	**35804**
Total Time	1.12	**0.98**	1.00	1.13	1.13	1.10
Search Time	0.90	**0.75**	0.77	0.85	1.11	0.83
Out of Memory	870	837	**835**	854	843	843
Out of Time	11	**8**	10	**8**	10	9

Standard Potential Heuristics

	lmc	all-N	init-N	max-N	div-N	S_1^{100}-N	S_{1000}^{1}-N
Coverage	958	929	891	948	**963**	945	961
Expansions	**1287**	10244	22415	8270	6904	7181	9238
Total Time	0.57	**0.29**	0.54	0.33	0.74	0.94	0.33
Search Time	0.52	0.23	0.43	0.24	0.74	0.94	**0.22**
Out of Memory	**0**	870	911	854	623	170	844
Out of Time	852	11	8	8	224	695	**5**

Strengthened Constraints

	all-D	init-D	max-D	div-D	S_1^{100}-D	S_{1000}^{1}-D
Coverage	879	881	932	837	853	**952**
Expansions	12101	18964	7863	**5269**	5503	7697
Total Time	0.68	0.90	0.84	4.18	3.29	**0.64**
Search Time	0.27	0.39	0.24	**0.19**	0.31	**0.19**
Out of Memory	824	824	770	560	**273**	726
Out of Time	75	**73**	76	381	652	100

Strengthened Optimization Functions

	$\mathtt{M}_1$-D	$\mathtt{M}_2$-D	$\mathtt{K}_1^{10}$-D	$\mathtt{K}_2^{10}$-D	$\mathtt{L}_1^{10}$-D	$\mathtt{L}_2^{10}$-D	$\mathtt{J}_1^{10}$-D	$\mathtt{J}_2^{10}$-D
Coverage	900	859	911	831	921	840	**922**	845
Expansions	8297	8240	6790	6847	6126	6273	6197	**6039**
Total Time	**0.59**	1.23	0.89	4.23	0.99	3.93	1.02	3.20
Search Time	**0.20**	**0.20**	0.86	4.08	0.86	3.78	0.89	3.04
Out of Memory	802	726	714	608	691	589	677	**586**
Out of Time	**77**	203	155	351	169	364	193	364

Additional Constraint on the Initial State

	all-N-I	div-N-I	S^1_{1000}-N-I	M_1-D-I	M_2-D-I
Coverage	**965**	956	963	950	906
Expansions	8532	7741	9040	6585	**6561**
Total Time	**0.27**	0.70	0.33	0.60	1.21
Search Time	0.21	0.70	0.21	**0.17**	**0.17**
Out of Memory	837	716	843	729	**705**
Out of Time	8	127	**4**	115	183

Additional Constraints on Random States

	all-N-R	init-N-R	div-N-R	M_2-D-R
Coverage	**930**	898	905	863
Expansions	11672	16182	11306	**9190**
Total Time	**0.35**	0.44	0.76	1.51
Search Time	**0.34**	0.44	0.76	1.37
Out of Memory	858	899	798	**728**
Out of Time	16	**11**	105	205

Mutex Based Optimization Functions

Definition (Mutex Based Optimization Function)

$$\mathcal{C}_f^k(\mathcal{M}) = \sum_{p \in \mathcal{P}_k^{\{f\}}} \prod_{V \in \mathcal{V}} |F_V \setminus \mathcal{M}_p|$$

Definition (Mutex Based Ensemble Optimization Function)

$$\mathcal{K}_f^k(\mathcal{M}, t) = \sum_{p \in \mathcal{P}_{|t|+k}^{t \cup \{f\}}} \prod_{V \in \mathcal{V}} |\mathcal{F} \setminus \mathcal{M}_p|$$

LP Constraints

Theorem

Let $\Pi = \langle \mathcal{V}, \mathcal{O}, I, G \rangle$ denote a planning task, $\mathtt{P}$ a potential function, and for every operator $o \in \mathcal{O}$, let $\mathrm{pre}^*(o) = \{\langle V, \mathrm{pre}(o)[V] \rangle | V \in \mathrm{vars}(\mathrm{pre}(o)) \cap \mathrm{vars}(\mathrm{eff}(o))\}$ and $\mathrm{vars}^*(o) = \mathrm{vars}(\mathrm{eff}(o)) \setminus \mathrm{vars}(\mathrm{pre}(o))$. If

$$\sum_{f \in G} \mathtt{P}(f) + \sum_{V \in \mathcal{V} \setminus \mathrm{vars}(G)} \max_{f \in \mathcal{F}_V} \mathtt{P}(f) \leq 0 \tag{1}$$

and for every operator $o \in \mathcal{O}$ it holds that

$$\sum_{f \in \mathrm{pre}^*(o)} \mathtt{P}(f) + \sum_{V \in \mathrm{vars}^*(o)} \max_{f \in \mathcal{F}_V} \mathtt{P}(f) - \sum_{f \in \mathrm{eff}(o)} \mathtt{P}(f) \leq c(o) \tag{2}$$

then the potential heuristic for $\mathtt{P}$ is admissible.