University of Basel

Mutex Based Potential Heuristics

Bachelor thesis

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1 Introduction

My work is brilliant, as can be shown with this very nice example.

2 Background

The goal of this chapter is to define and explain the terminology in this thesis. For visualization the 8-Tiles problem is used as an example. This is a classical planning problem, in which 8 tiles are arranged in a 3x3-Grid. One spot remains empty, the goal is to bring the tiles in a specific order by sliding them around.

2.1 Planning Tasks

In order to solve a classical planning problem with heuristic search it is represented as a **planning task**. Fišer et al. use the finite domain representation (**FDR**) where Π is specified by a tuple $\Pi = \langle \mathcal{V}, \mathcal{O}, I, G \rangle$.

$\mathcal{V}$ is a set finite set of **variables**, each of the variables $V \in \mathcal{V}$ has a finite set of **domains** $\mathrm{dom}(V)$. For 8-Tiles, the variables could be defined as the 9 fields in the grid (1 to 9), and their domains hold the values of all tiles and the blank space (1 to 8 and 0 for the blank tile). A **fact** $f = \langle V, v \rangle$ consists of a variable $V \in \mathcal{V}$ and one of its values $v \in \mathrm{dom}(V)$. The fact for tile number 5 being in the first position would be $\langle 1, 5 \rangle$. $\mathcal{F}_V$ is the set of all possible facts of variable $V \in \mathcal{V}$ while $\mathcal{F}$ is the set of all facts of this problem.

A **partial state** p of size t contains t facts of t different variables, i.e., it is the variable assignment over the variables $\mathrm{vars}(p) \subseteq \mathcal{V}$ with $|\mathrm{vars}(p)| = t$. $p[V]$ is the value assigned to V in p. In other words, $p = \{\langle V, p[v] \rangle | V \in \mathrm{vars}(p)\}$. A **state** s is not partial, if all variables are assigned, i.e., $\mathrm{vars}(s) = \mathcal{V}$. It **extends** the partial state $p \subseteq s$, if $s[v] = p[v]$ for all $v \in \mathrm{vars}(p)$. The partial state $p = \{1 \mapsto 0, 2 \mapsto 1\}$ represents the state where the first grid in the field of the 8-Tiles puzzle is the blank space while tile number one lies in the first field.

I is the **initial state**, in 8-Tiles this is some specific random order of the tiles. G is a partial state representing the **goal**. s is a **goal state**, if it is an extension of G. In 8-Tiles it is one specific order of the tiles e.g. sorted by number: $s = \{1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 3, 4 \mapsto 4, 5 \mapsto 5, 6 \mapsto 6, 7 \mapsto 7, 8 \mapsto 8, 9 \mapsto 0\}$.

$\mathcal{O}$ is a finite set of **operators**. Each $o \in \mathcal{O}$ has a precondition $\mathrm{pre}(o)$ and an effect $\mathrm{eff}(o)$ which are both partial states over $\mathcal{V}$ and a cost $\mathrm{c}(o) \in \mathbb{R}_0^+$.

o is **applicable** in state s iff $\mathrm{pre}(o) \subseteq s$, the **resulting state** is $o[\![s]\!]$. $o[\![s]\!][v] = \mathrm{eff}(o)[v]$

holds for all $v \in \text{eff}(o)$ in resulting state $o[\![s]\!]$, while $o[\![s]\!][v] = s[v]$ for all $v \notin \text{eff}(o)$. In 8-Tiles the operators are to encode the movement of one tile to the blank space. The precondition assures that the tile is next to the blank space, the effect swaps the values of the corresponding two variables, while all other tiles remain at the same position.
In order to reach the goal multiple operators need to be applied in a specific order. Such a sequence of operators $\pi = \langle o_1, \ldots, o_n \rangle$ is called a plan, $\pi[\![s]\!] = s_n$. π is a **s-plan**, if π is applicable in s and $\pi[\![s]\!]$ is a an extension of G. If it has minimal cost among all s-plans it is called **optimal**.
The set $\mathcal{R}$ is defined as the set of all **reachable** states. A state s is reachable, if a plan π is applicable in I such that $\pi[\![I]\!] = s$. An operator o is reachable, if it is applicable in a reachable state. A state is a **dead-end state** if it does not extend the goal state, and no s-plan exists.

2.2 Heuristics

A **heuristic** $h : \mathcal{R} \rightarrow \mathbb{R} \cup \{\infty\}$ estimates the cost of the optimal plan for a state s. For 8-Tiles it would estimate how many tiles need to be slid to reach the goal state. The **optimal heuristic** $h^*(s)$ maps each state s to its actual optimal cost, or to ∞ if it is a dead-end state. We aim to approach this heuristic.
A heuristic is **admissible**, if it never overestimates the optimal heuristic, i.e., $h(s) \leq h^*(s)$. It is **goal-aware** iff $h(s) \leq 0$ for all reachable goal states, i.e., it recognizes a goal sate as such. Further, it is **consistent** iff $h(s) \leq h(o[\![s]\!]) + c(o)$. This means, the difference between the heuristic values of two successive states is not greater than the cost of the operator applied to the first one to obtain the second one A heuristic which is goal aware and consistent is also admissible.
This thesis uses heuristics in the forward heuristic search where unreachable states are never expanded. Therefore they are defined over $\mathcal{R}$ instead of over all states and the above defined rules hold for reachable states only.
One class of heuristics are potential heuristics which assign a potential to each possible fact of the planning task.

Definition 1. *Let Π denote a planning task with facts $\mathcal{F}$. A **potential function** is a function $\mathtt{P} : \mathcal{F} \mapsto \mathbb{R}$. A **potential heursistic** for $\mathtt{P}$ maps each state $s \in \mathcal{R}$ to the sum of potentials of facts in s, i.e., $h^{\mathtt{P}}(s) = \sum_{f \in s} P(f)$.*

Potential heuristics are goal aware, consistent and admissible [1]. The potentials themselves are obtained through optimization which will be further analyzed in chapter 3.
One further approach are **ensemble heuristics**. **TODO: Lektuere?**

2.3 Mutexes and Disambiguations

Mutex means, that two or more things mutually exclude each other.

Definition 2. *Let Π denote a planning task with facts $\mathcal{F}$. A set of facts $\mathcal{M} \subseteq \mathcal{F}$ is a **mutex** if $\mathcal{M} \nsubseteq s$ for every reachable state $s \in \mathcal{R}$*

Facts are a mutex if they never appear together in any reachable state. If a partial state p in 8-Tiles holds $p[3] = 1$, then tile one may not be in any other spot of the grid, i.e., the fact $\langle 3, 1\rangle$ is mutex with all other facts $\langle v, 1\rangle$ with $v \in \mathcal{V} \setminus \{3\}$.
A mutex-set contains all mutexes which can directly be inferred from the FDR-representation.

Definition 3. *Let* Π *denote a planning task with variables* $\mathcal{V}$ *and facts* $\mathcal{F}$*. A set of sets of facts* $\mathcal{M} \subseteq 2^{\mathcal{F}}$ *is called a* ***mutex-set*** *if the following hold: (a) every* $M \in \mathcal{M}$ *is a mutex; and (b) for every* $M \in \mathcal{M}$ *and every* $f \in \mathcal{F}$ *it holds that* $M \cup \{f\} \in \mathcal{M}$*; and (c) for every variable* $V \in \mathcal{V}$ *and every pair of facts* $f, f' \in \mathcal{F}_V$*,* $f \neq f'$*, it holds that* $\{f, f'\} \in \mathcal{M}$*.*

We can say that $s \in \mathcal{M}$ if s contains a subset of facts which are a mutex.
Mutexes can be used to derive disambiguations.

Definition 4. *Let* Π *denote a planning task with facts* $\mathcal{F}$ *and variables* $\mathcal{V}$*, let* $V \in \mathcal{V}$ *denote a variable, and let* p *denote a partial state. A set of facts* $F \subseteq \mathcal{F}_V$ *is called a* ***disambiguation*** *of* V *for* p *if for every reachable state* $s \in \mathcal{R}$ *such that* $p \subseteq s$ *it holds that* $F \cap s \neq \emptyset$ *(i.e.,* $\langle V, s[V]\rangle \in F$*).*

The disambiguation of a variable V for a partial state p is the set of facts $F \in \mathcal{F}_V$ which occur in all reachable extended states of p. This means, that each fact of V which is not in F is a mutex with p. If F contains exactly one fact then p can be safely extended with that fact, as it is the only non-dead-end extension of the state. If F is the empty set every extended state of p is a dead-end. This knowledge can be used to prune operators o for which $p \subseteq \text{pre}(o)$ and unreachable states $s \subseteq p$. If the goal state G is one of this states, the problem is unsolvable.
If a partial state s of the 8-Tiles problem holds $p[3] = 1$ and $p[2] = 1$, then it is a dead-end, as these facts are a mutex. If $p = \{1 \mapsto 1, 2 \mapsto 2, 3 \mapsto 3, 4 \mapsto 4, 5 \mapsto 5, 6 \mapsto 6, 7 \mapsto 7, 8 \mapsto 8\}$ then p is not a dead-end and 9 can safely be assigned with 0, as it is the only fact in $\mathcal{F}_9$ which does not form a mutex with any of the already assigned facts.
The set $\mathcal{M}_p = \{f | f \in \mathcal{F}, p \cup \{f\} \in \mathcal{M}\}$ is the set of facts which are mutex with p. All facts of a variable $f \in \mathcal{F}_V$ not contained in $\mathcal{M}_p$ build the disambiguation F of V for p.

3

Strengthening Potential Heuristics

Fišer et al. proposed to improve potential heuristics with mutexes and disambiguations This chapter contains the changes of the transformation of the planning task into TNF and the adaption of the optimization functions. It shows how the equations which were implemented were derived.

3.1 Potential Heuristics

When Pommerening et al. first introduced potential heuristics, they showed that two inequalities are sufficient to proof admissibility.

Theorem 5. *Let* $\Pi = \langle \mathcal{V}, \mathcal{O}, I, G \rangle$ *denote a planning task,* $\mathtt{P}$ *a potential function, and for every operator* $o \in \mathcal{O}$*, let* $\mathrm{pre}^*(o) = \{\langle V, \mathrm{pre}(o)[V] \rangle | V \in \mathrm{vars}(\mathrm{pre}(o)) \cap \mathrm{vars}(\mathrm{eff}(o))\}$ *and* $\mathrm{vars}^*(o) = \mathrm{vars}(\mathrm{eff}(o)) \setminus \mathrm{vars}(\mathrm{pre}(o))$*. If*

$$\sum_{f \in G} \mathtt{P}(f) + \sum_{V \in \mathcal{V} \setminus \mathrm{vars}(G)} \max_{f \in \mathcal{F}_V} \mathtt{P}(f) \leq 0 \tag{1}$$

and for every operator $o \in \mathcal{O}$ *it holds that*

$$\sum_{f \in \mathrm{pre}^*(o)} \mathtt{P}(f) + \sum_{V \in \mathrm{vars}^*(o)} \max_{f \in \mathcal{F}_V} \mathtt{P}(f) - \sum_{f \in \mathrm{eff}(o)} \mathtt{P}(f) \leq c(o) \tag{2}$$

then the potential heuristic for $\mathtt{P}$ *is admissible.*

Eq. (1) of the theorem by Fišer et al. assures goal-awareness of the potential heuristic. As all variables are assigned in the goal state, the potential of one fact per variable has to be summed up. For the variables $v \in \mathrm{vars}(G)$ we can simply use the potentials of their respective facts. Meanwhile we assume the worst case for the other variables, by using the maximal potential over their facts, as we do not know what fact they are assigned.
Eq.(2) assures consistency. Recall the general consistency equation $h(s) \leq h(o[\![s]\!]) + \mathrm{c}(o)$. It can be rewritten as $h(s) - h(o[\![s]\!]) \leq \mathrm{c}(o)$. As the facts which do not occur in the effect are the same in both s and $o[\![s]\!]$ we cam leave them aside. For s we know what facts of the variables of the preconditions are assigned and sum the potentials of those which are also in effect. For the variables which are in effect but not in the precondition we proceed similar

to (1), as we do not know their values. The potentials of the facts in the effect can be used without modification for $o[\![s]\!]$.
The Advantage of this equations is that they are not state-dependent, even though they do not tell us what exactly the potentials should be. However, they can be used as the constraints for a linear program (**LP**), the solution of which is a potential function that forms an admissible potential heuristic. More about this in 3.1.2.

3.1.1 Generalize with Mutexes

some general writng about how mutexes can be used for narrowing down domains

Using this [above], 5 can be generalized by the following theorem.

Theorem 6. *Let $\Pi = \langle \mathcal{V}, \mathcal{O}, I, G \rangle$ denote a planning task with facts $\mathcal{F}$, and let $\mathtt{P}$ denote a potential function, and*

(i) for every variable $V \in \mathcal{V}$, let $G_V \subseteq \mathcal{F}_V$ denote a disambiguation of V for G s.t. $|G_V| \geq 1$, and

(ii) for every operator $o \in \mathcal{O}$ and every variable $V \in \mathrm{vars}(\mathrm{eff}(o))$, let $E_V^o \subseteq \mathcal{F}_V$ denote a disambiguation of V for $\mathrm{pre}(o)$ s.t. $|E_V^o| \geq 1$.

If

$$\sum_{V \in \mathcal{V}} \max_{f \in G_V} \mathtt{P}(f) \leq 0$$

and for every operator $o \in \mathcal{O}$ it holds that

$$\sum_{V \in \mathrm{vars}(\mathrm{eff}(o))} \max_{f \in E_V^o} \mathtt{P}(f) - \sum_{f \in \mathrm{eff}(o)} \mathtt{P}(f) \leq \mathrm{c}(o)$$

then the potential heuristic $\mathtt{P}$ is admissible.

Fišer et al. proof the theorem by showing that equations (6) and (6) are generalizations of equations (1) and (2) respectively.
G_V is the disambiguation of $V \in \mathcal{V}$ for G, i.e., it holds all facts of $\mathcal{F}_V$ which are not a mutex with any of tha facts $f \in G$. If it is empty for any of the variables, then the problem is unsolvable. E_V^o is the disambiguation of $V \in \mathrm{vars}(\mathrm{eff}(o))$ for $\mathrm{pre}(o)$, $o \in \mathcal{O}$. In words, for all variables which are affected by the application of o, remove facts which are a mutex with any of the facts in the precondition. o is not applicable in any (partial) state, if E_V^o is empty for any $V \in \mathrm{vars}(\mathrm{eff}(o))$.
anything else for this subsection? something kind of concluding is missing

3.1.2 Transition Normal Form

What is TNF?
does some of this belong to the background?

3.2 Optimization

4 Implementation

I wrote some code.
Embedded in Fast-Downward. One class -¿ evtl. uml der Klasse? All major methods, what, why (map instead of vector etc.)

5 Experimantal Evaluation

Some things that I tested my code with.

6 Conclusion

In conclusion, I did well.

Bibliography

[1] Daniel Fišer, Rostislav Horčík, and Antonín Komenda. Strengthening potential heuristics with mutexes and disambiguations. In *Proceedings of the International Conference on Automated Planning and Scheduling*, volume 30, pages 124–133, 2020.

[2] Florian Pommerening, Malte Helmert, Gabriele Röger, and Jendrik Seipp. From non-negative to general operator cost partitioning. 2015.

A

Appendix

Declaration on Scientific Integrity
Erklärung zur wissenschaftlichen Redlichkeit

includes Declaration on Plagiarism and Fraud
beinhaltet Erklärung zu Plagiat und Betrug

Author — Autor
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Title of work — Titel der Arbeit
Mutex Based Potential Heuristics

Type of work — Typ der Arbeit
Bachelor thesis

Declaration — Erklärung
I hereby declare that this submission is my own work and that I have fully acknowledged the assistance received in completing this work and that it contains no material that has not been formally acknowledged. I have mentioned all source materials used and have cited these in accordance with recognised scientific rules.

Hiermit erkläre ich, dass mir bei der Abfassung dieser Arbeit nur die darin angegebene Hilfe zuteil wurde und dass ich sie nur mit den in der Arbeit angegebenen Hilfsmitteln verfasst habe. Ich habe sämtliche verwendeten Quellen erwähnt und gemäss anerkannten wissenschaftlichen Regeln zitiert.

Basel, 06. 11. 2020

Signature — Unterschrift