More multi qubit gates

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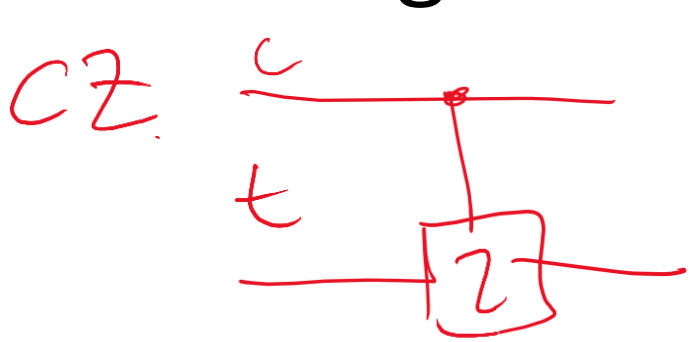
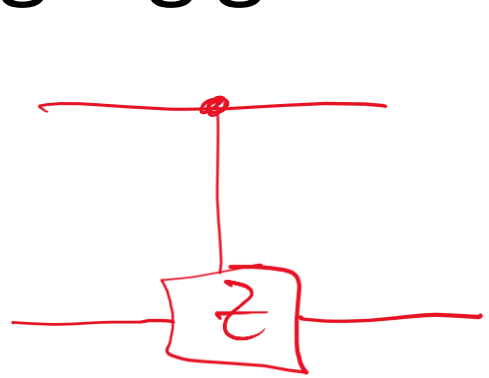
Topics:

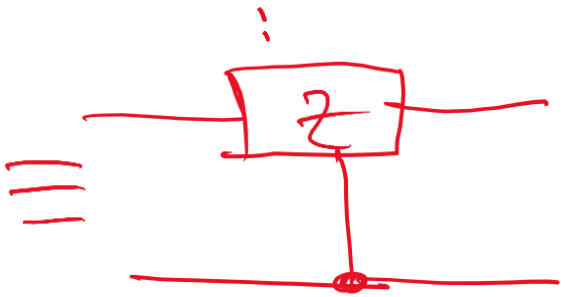
1. Control Z, Swap gates
2. Toffoli gate
3. Multi qubit gates

Control Z gate is another entangling gate



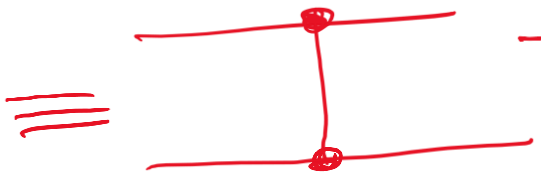
ct	ct
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|10\rangle$
$\|11\rangle$	$-\|11\rangle$





$Z|0\rangle = |0\rangle$

$Z|1\rangle = -|1\rangle$



Z gate

$$\underline{CZ} = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11| = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$CZ|1\rangle|+\rangle = CZ\,|1\rangle\,\frac{(|0\rangle + |1\rangle)}{\sqrt{2}} = \frac{|1\rangle|0\rangle - |1\rangle|1\rangle}{\sqrt{2}}$$

$$= CZ\,\frac{|1\rangle|0\rangle + |1\rangle|1\rangle}{\sqrt{2}} = |1\rangle|-\rangle$$

$CZ\,|+\rangle|1\rangle$

$= CZ\,\dfrac{|0\rangle|1\rangle + |1\rangle|1\rangle}{\sqrt{2}}$

$= \dfrac{|0\rangle|1\rangle - |1\rangle|1\rangle}{\sqrt{2}}$

$= |-\rangle|1\rangle$

$CZ\,|+\rangle|+\rangle$

$= CZ\,\dfrac{1}{\sqrt{2}}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$

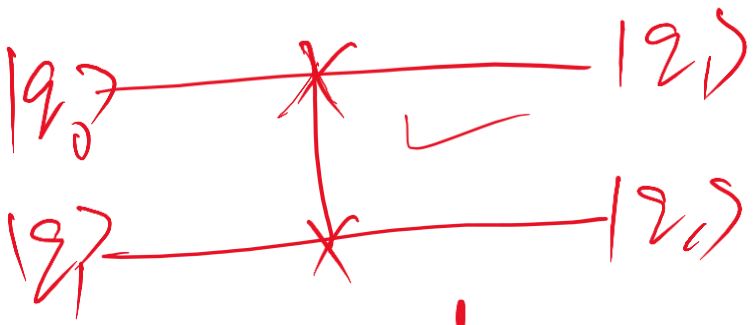
$= CZ\,\dfrac{|0\rangle|0\rangle + |01\rangle + |10\rangle + |11\rangle}{2}$

$= \dfrac{|00\rangle + |01\rangle + |10\rangle - |11\rangle}{2}$

$\neq |\psi\rangle|\phi\rangle$

Entangled state!

Swap gate is not an entangling gate



SWAP gate

0 0	0 0		
0 1	1 0		
1 0	0 1		
1 1	1 1		

$$SWAP = |00\rangle\langle 00| + |10\rangle\langle 01| + |01\rangle\langle 10| + |11\rangle\langle 11|$$

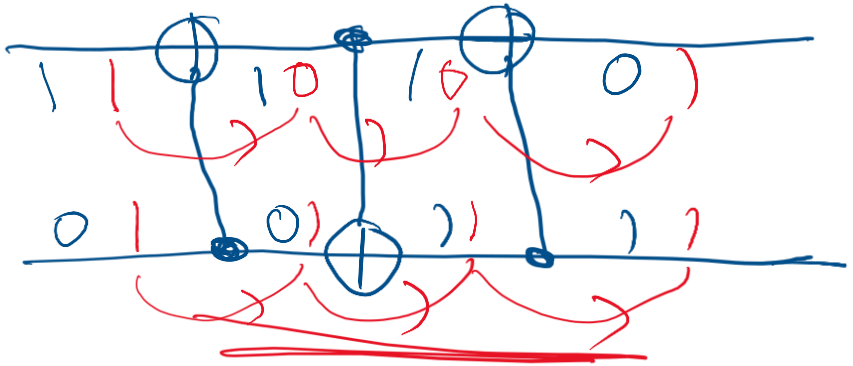
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$SWAP\,|+\rangle|-\rangle$$

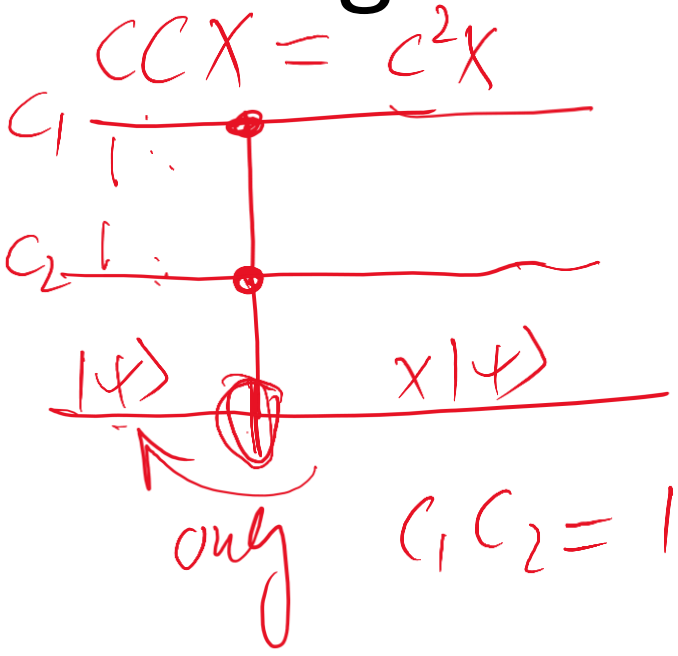
$$= SWAP\,\frac{|0\rangle + |1\rangle}{\sqrt{2}}\,\frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

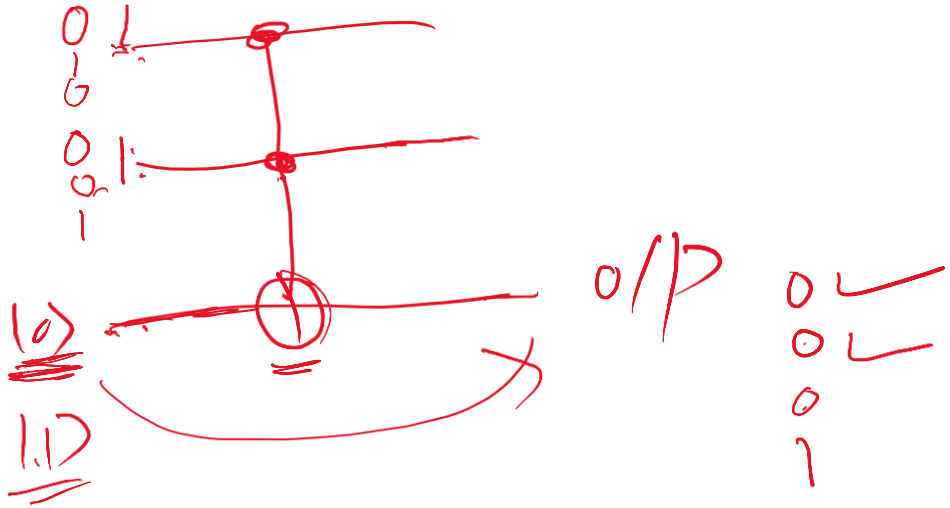
$$= SWAP\,\frac{|0\rangle|0\rangle + |1\rangle|0\rangle - |0\rangle|1\rangle - |1\rangle|1\rangle}{2}$$

$$= \frac{|0\rangle + |01\rangle - |10\rangle - |11\rangle}{2} = |-\rangle|+\rangle$$



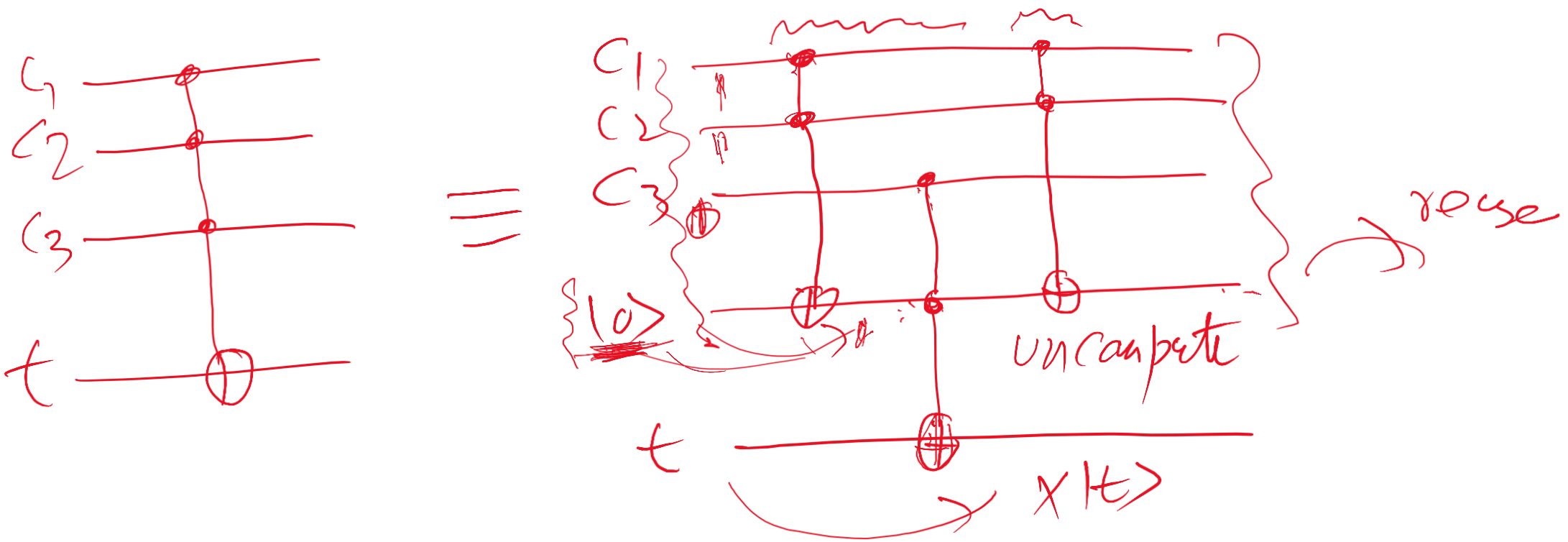
Toffoli gate can produce all classical logic



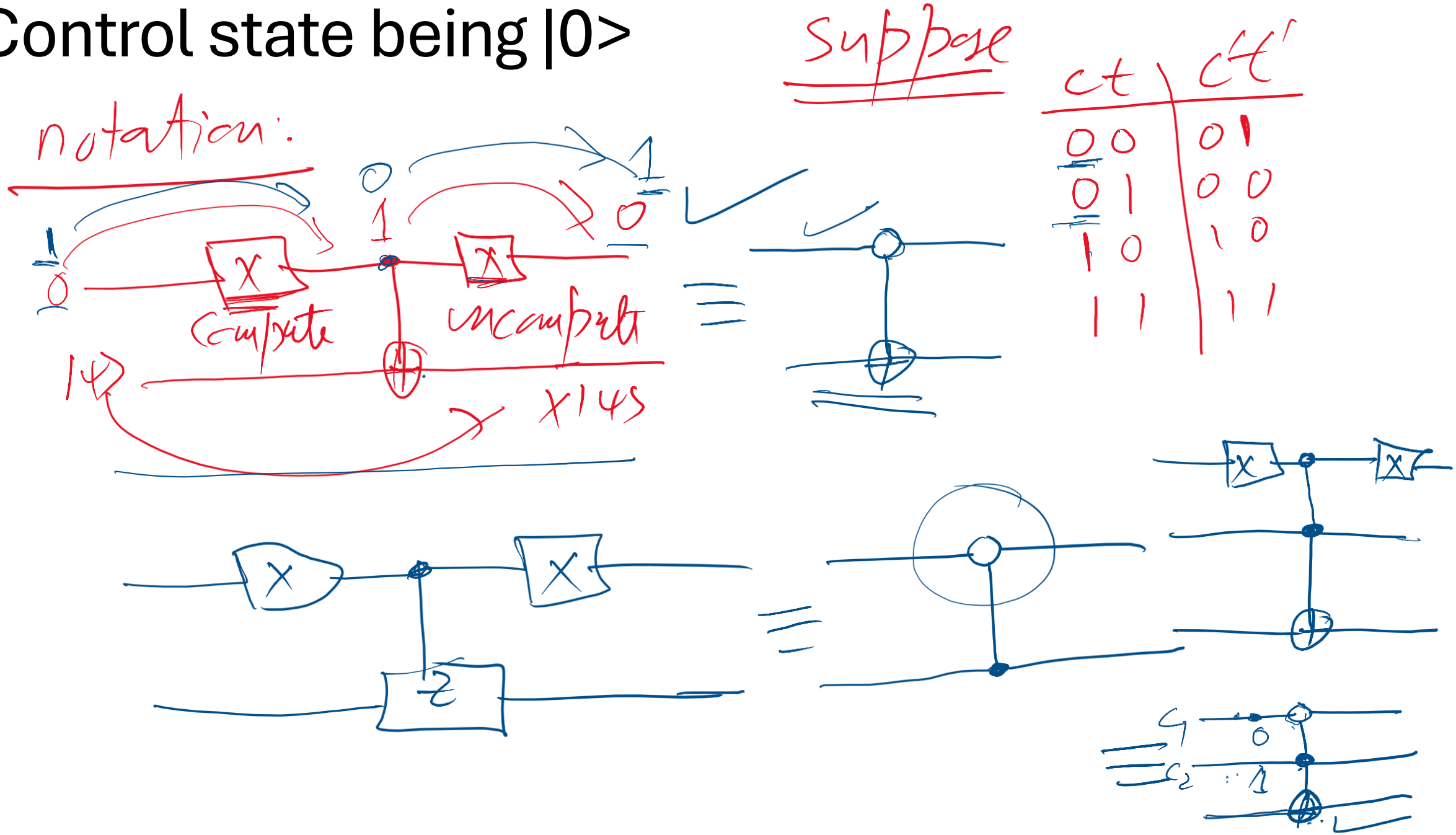


Toffoli gate can simulate all classical binary logical circuit

we can construct logic gates ⇐ Control x gates using angle Toffoli gate!


c1
c2
c3
t
≡
|0⟩
uncompute
reuse
t
X|t⟩

Control state being |0>



Summary

- Control Z is also an entangling gates
- Swap gate swaps the qubit states
- A large variety of logical structures available with Toffoli