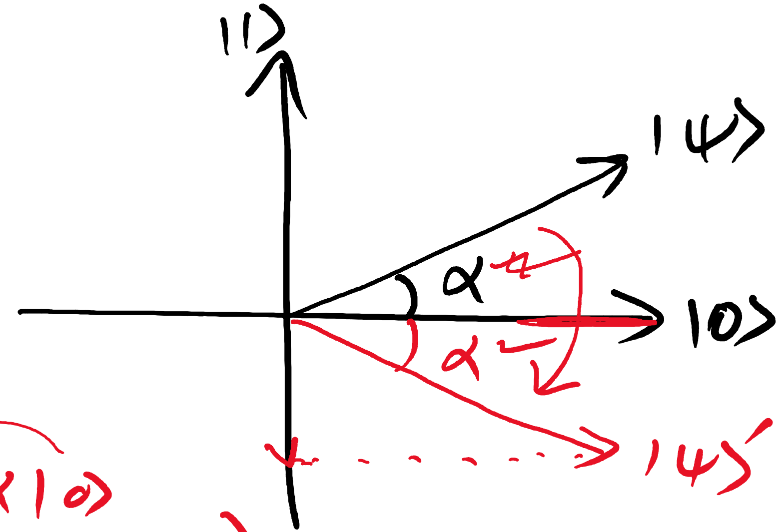
Reflection operators:

$$|\psi\rangle = \cos\alpha|0\rangle + \sin\alpha|1\rangle$$



$$A = 2|0\rangle\langle 0| - I$$

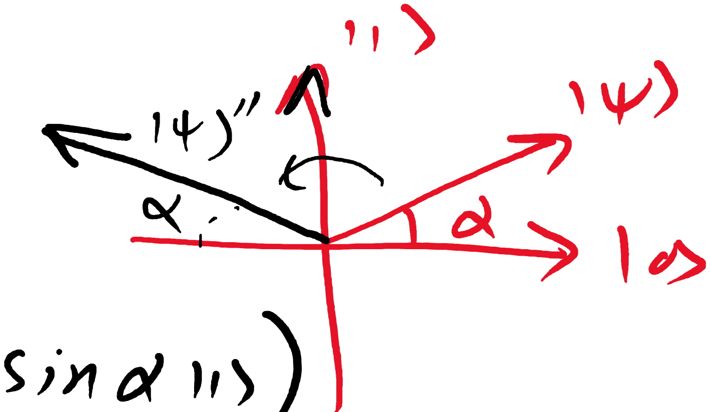
$$|\psi'\rangle = A|\psi\rangle = \left(2|0\rangle\langle 0| - I\right)\left(\cos\alpha|0\rangle + \sin\alpha|1\rangle\right)$$

$$= 2|0\rangle\langle 0|0\rangle\cos\alpha + 2|0\rangle\langle 0|1\rangle\sin\alpha - (\cos\alpha|0\rangle - \sin\alpha|1\rangle)$$

$$= 2\cos\alpha|0\rangle - \cos\alpha|0\rangle - \sin\alpha|1\rangle$$

$$\boxed{|\psi'\rangle = \cos\alpha|0\rangle - \sin\alpha|1\rangle}$$

$$\boxed{B = 2\,|1\rangle\langle 1| - I}$$



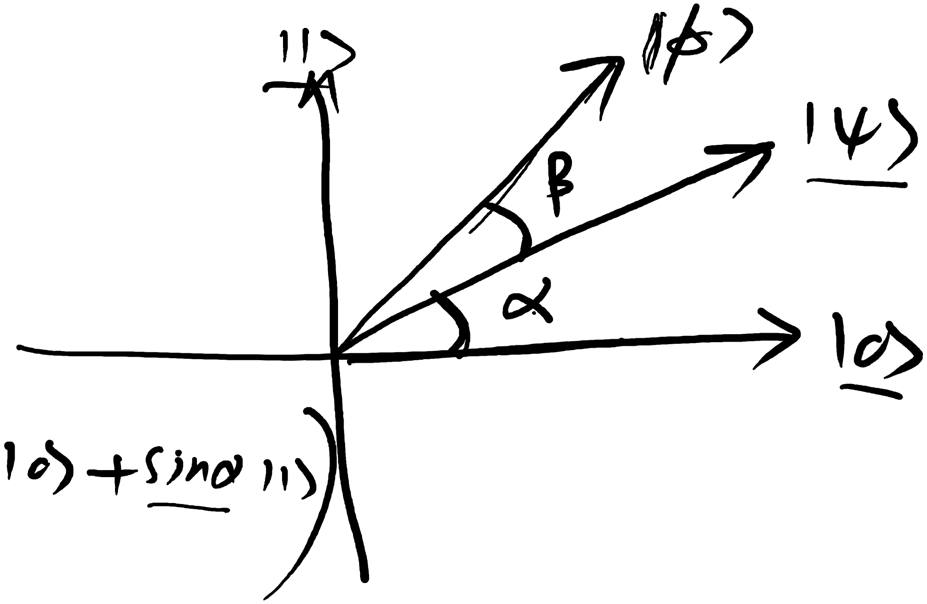
$$|\psi\rangle'' = B|\psi\rangle$$

$$= \left(2\,|1\rangle\langle 1| - I\right)\left(\cos\alpha\,|0\rangle + \sin\alpha\,|1\rangle\right)$$

$$= 2\,|1\rangle\langle 1|0\rangle\cos\alpha + 2\,|1\rangle\langle 1|1\rangle\sin\alpha - \cos\alpha\,|0\rangle - \sin\alpha\,|1\rangle$$

$$= 2\sin\alpha\,|1\rangle - \cos\alpha\,|0\rangle - \sin\alpha\,|1\rangle$$

$$= -\cos\alpha\,|0\rangle + \sin\alpha\,|1\rangle$$



$$C = 2|\phi\rangle\langle\phi| - I$$

$$|\psi\rangle''' = C|\psi\rangle$$

$$= \left(2|\phi\rangle\langle\phi| - I\right)\left(\cos\alpha|0\rangle + \sin\alpha|1\rangle\right)$$

$$= 2|\phi\rangle\langle\phi|0\rangle\cos\alpha + 2|\phi\rangle\langle\phi|1\rangle\sin\alpha$$

$$- \cos\alpha|0\rangle - \sin\alpha|1\rangle$$

$$\langle\phi|0\rangle = \cos(\alpha+\beta), \quad \langle\phi|1\rangle = \sin(\alpha+\beta)$$

$$|\psi\rangle'' = 2\cos\alpha\cos(\alpha+\beta)|\phi\rangle + \sin\alpha\sin(\alpha+\beta)|\phi\rangle$$

$$- \cos\alpha|0\rangle - \sin\alpha|1\rangle$$

$$|\psi\rangle'' = 2\cos\alpha\cos(\alpha+\beta)\,|\phi\rangle + 2\sin\alpha\sin(\alpha+\beta)\,|\phi\rangle$$

$$- \cos\alpha\,|0\rangle - \sin\alpha\,|1\rangle$$

$$|\psi\rangle''' = 2\Big(\cos\alpha\cos(\alpha+\beta) + \sin\alpha\sin(\alpha+\beta)\Big)|\phi\rangle$$

$$- \cos\alpha\,|0\rangle - \sin\alpha\,|1\rangle$$

$$= 2\cos\big(\alpha-(\alpha+\beta)\big)\,|\phi\rangle - \cos\alpha\,|0\rangle - \sin\alpha\,|1\rangle$$

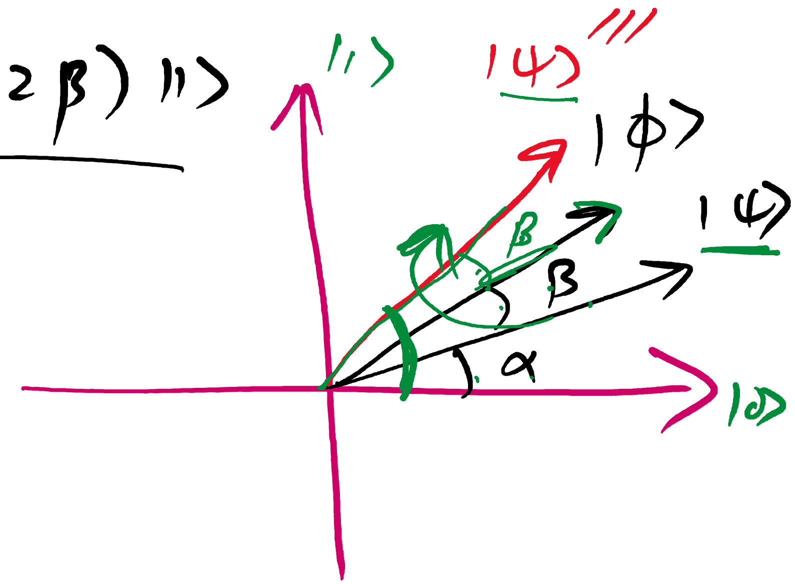
$$= 2\cos(-\beta)\Big[\cos(\alpha+\beta)\,|0\rangle + \sin(\alpha+\beta)\,|1\rangle\Big]$$

$$- \cos\alpha\,|0\rangle - \sin\alpha\,|1\rangle$$

$$= \Big[2\cos\beta\cos(\alpha+\beta) - \cos\alpha\Big]|0\rangle + \Big[2\cos\beta\sin(\alpha+\beta) - \sin\alpha\Big]|1\rangle$$

$$= \Big[\cos(\beta+\alpha+\beta) + \cos(\beta-\alpha-\beta) - \cos\alpha\Big]|0\rangle + \Big(\sin(\beta+\alpha+\beta) + \sin\alpha - \sin\alpha\Big)|1\rangle$$

$$|\psi'''\rangle = \cos(\alpha+2\beta)|0\rangle + \sin(\alpha+2\beta)|1\rangle$$



$$\alpha+2\beta = \alpha+\beta+\beta$$

$$\boxed{C = 2|\phi\rangle\langle\phi| - I}$$

Reflection operators: reflect around $|\phi\rangle$

$$C|\psi\rangle$$

- $A = 2|0\rangle\langle 0| - I$

$$= 2|0\rangle\langle 0| - \left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- $A = 2|00\rangle\langle 00| - I$

$$= \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$