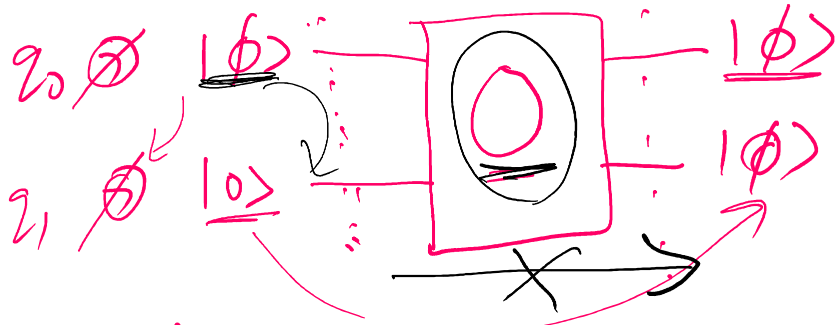
- No cloning Theorem



$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|0\rangle \otimes |\phi\rangle \xrightarrow{O} |\phi\rangle \otimes |\phi\rangle$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \xrightarrow{O} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \otimes \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\begin{bmatrix} \alpha \\ \beta \\ 0 \\ 0 \end{bmatrix} \xrightarrow{O} \begin{bmatrix} \alpha^2 \\ \alpha\beta \\ \alpha\beta \\ \beta^2 \end{bmatrix}$$

$$\begin{pmatrix} \alpha^2 \\ \alpha\beta \\ \alpha\beta \\ \beta^2 \end{pmatrix} = O_{4\times 4} \begin{pmatrix} \alpha \\ \beta \\ 0 \\ 0 \end{pmatrix}$$

O has to non-linear for copying to happen

→ In QM, only U

$$O \neq U$$

In Q. Comp: no-cloning theorem

State transfer algorithm:

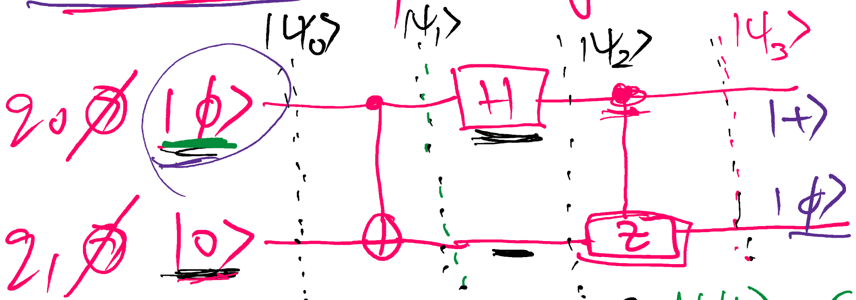
$H|0\rangle = |+\rangle$

$H|1\rangle = |-\rangle$

$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$

$Z|0\rangle = |0\rangle$

$Z|1\rangle = -|1\rangle$



⓪ Initial state:

$|\psi_0\rangle = |0\rangle \otimes |\phi\rangle$

$= |0\rangle \otimes [\alpha|0\rangle + \beta|1\rangle]$

$|\psi_0\rangle = \alpha \underset{t}{|0\rangle}\underset{c}{|0\rangle} + \beta \underset{t}{|0\rangle}\underset{c}{|1\rangle}$

① $|\psi_1\rangle = CX|\psi_0\rangle$

$|\psi_1\rangle = \alpha|0\rangle|0\rangle + \beta|1\rangle|1\rangle$

② After Hadamard:

$|\psi_2\rangle = I \otimes H|\psi_1\rangle$

$\boxed{|\psi_2\rangle = \alpha|0\rangle|+\rangle + \beta|1\rangle|-\rangle}$

$$|\psi_2\rangle = \alpha|0\rangle|+\rangle + \beta|1\rangle|-\rangle = \alpha|0\rangle\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) + \beta|1\rangle\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$$

$$= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle|0\rangle + \alpha|0\rangle|1\rangle + \beta|1\rangle|0\rangle - \beta|1\rangle|1\rangle\right]$$

t c t c

$$|\psi_3\rangle = CZ\,|\psi_2\rangle$$

$$= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle|0\rangle + \alpha|0\rangle|1\rangle + \beta|1\rangle|0\rangle + \beta|1\rangle|1\rangle\right]$$

$$= \frac{1}{\sqrt{2}}\left[\alpha|0\rangle\otimes(|0\rangle+|1\rangle) + \beta|1\rangle\otimes(|0\rangle+|1\rangle)\right]$$

$$|\psi_3\rangle = (\alpha|0\rangle + \beta|1\rangle)\otimes\frac{|0\rangle+|1\rangle}{\sqrt{2}} = |\phi\rangle\otimes|+\rangle$$

$$|0\rangle\otimes|\phi\rangle \xrightarrow{\text{Algorithm}} |\phi\rangle\otimes|+\rangle$$