From Classical Mechanics to Modern Physics

Muhammad Faryad

Associate Professor of Physics,

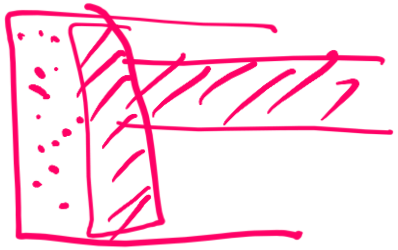
Lahore University of Management Sciences

Topics:

1. Classical vs Quantum Systems
2. Postulates of Classical Mechanics
3. Postulates of Quantum Mechanics

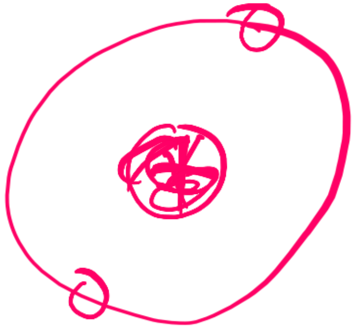
Classical World

- 
Falling
h, t=?
- 

- 
- Bridge, building 
- gases 
- Engine
- Macroscopic
- 

Quantum World

- Microscopic



- white dwarf
- neutron star
- Low T

E_2
E_2
E_1
E_0

light

gases

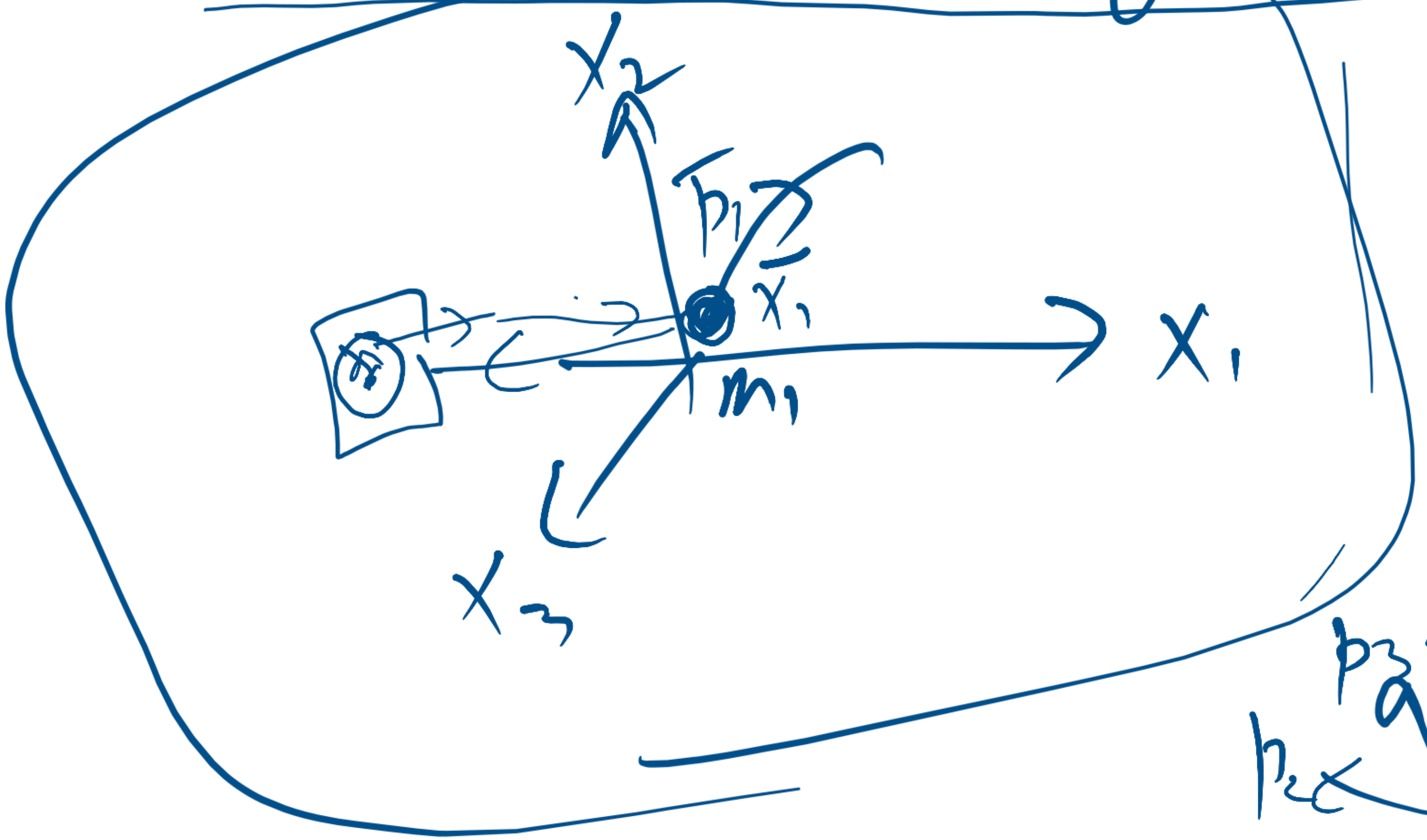
low pressure

metal

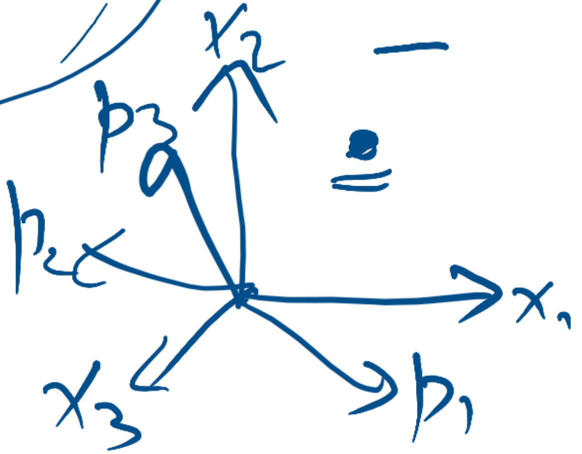
- Quantum Mechanics

Classical Mechanics Postulates

① State of C.S. given by $\bar{x}$, $\bar{p}$, $\quad \bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\bar{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$



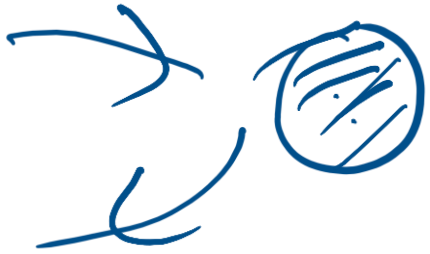
- real
- directly observable
- continuous
-



6-dim phase space

② measurements can give us $\bar{p}$, $\bar{x}$, without changing $\bar{x}$, $\bar{p}$

- copy state



③ Evolution of state / change in state

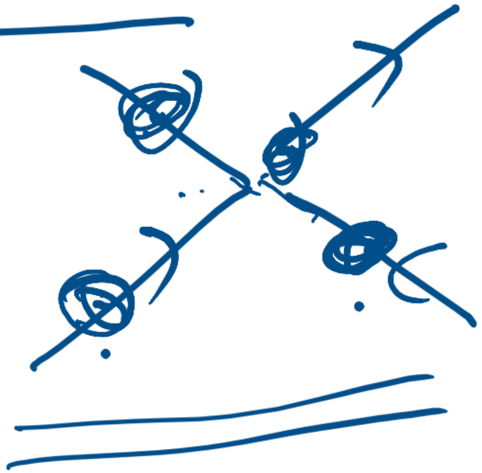
→ Newton's law

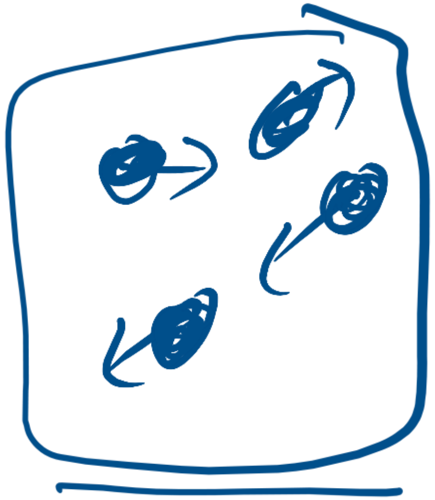
$$\boxed{\bar{p}(t) = \int F \, dt}$$

$$\boxed{\frac{d\bar{p}}{dt} = \bar{F}}$$

4th Composite system:

m_1 , $\frac{d\bar{p}_1}{dt} = \bar{F}_1$





Composite $\equiv$ sum of its parts

T = ?
K.E = .
P = :

$$\bar{X} = \begin{pmatrix} x_{1x} \\ x_{1y} \\ x_{1z} \\ x_{2x} \\ x_{2y} \\ x_{2z} \\ \vdots \end{pmatrix}, \quad \bar{p} = \begin{pmatrix} p_{1x} \\ p_{1y} \\ \vdots \\ \vdots \\ \vdots \end{pmatrix}$$

Quantum Mechanics Postulates

- (1) State of Q.S. by state vector

$|\psi\rangle$ = psi ket = $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ 4x1

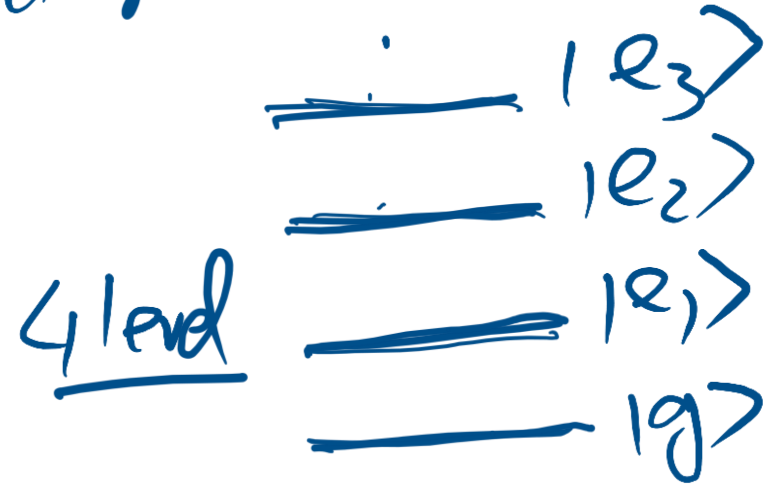
— complex numbers

$a = 1-3i$

$b = \sqrt{2} + 0.7i$

$\vdots$

$\boxed{i = \sqrt{-1}}$



— normalized

$$\boxed{|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1}$$

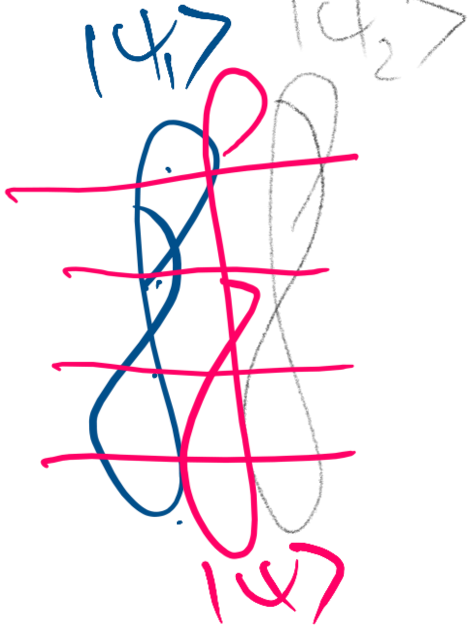
$$|a|^2 = aa^*$$

$$= (1-3i)(1+3i)$$

$$= 1 - \cancel{3i} + \cancel{3i} - 9i^2 = 1 + 9 = 10$$

— ① $|\psi_1\rangle = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$, $|\psi_2\rangle = \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix}$

$\boxed{|\psi\rangle = \alpha|\psi_1\rangle + \beta|\psi_2\rangle}$ Also a state

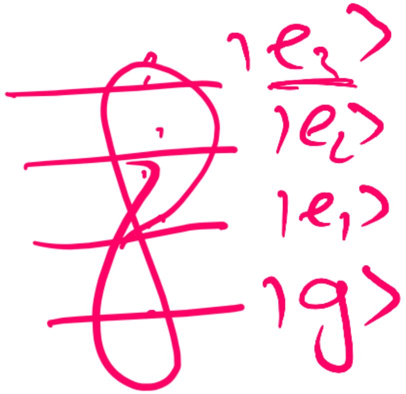


— superposition principle $|\alpha|^2 + |\beta|^2 = 1$

$|a|^2$ = prob of measuring to be $|g\rangle$

$|b|^2$ = prob . . . . . $|e_1\rangle$

$|c|^2$ = ... $|e_2\rangle$



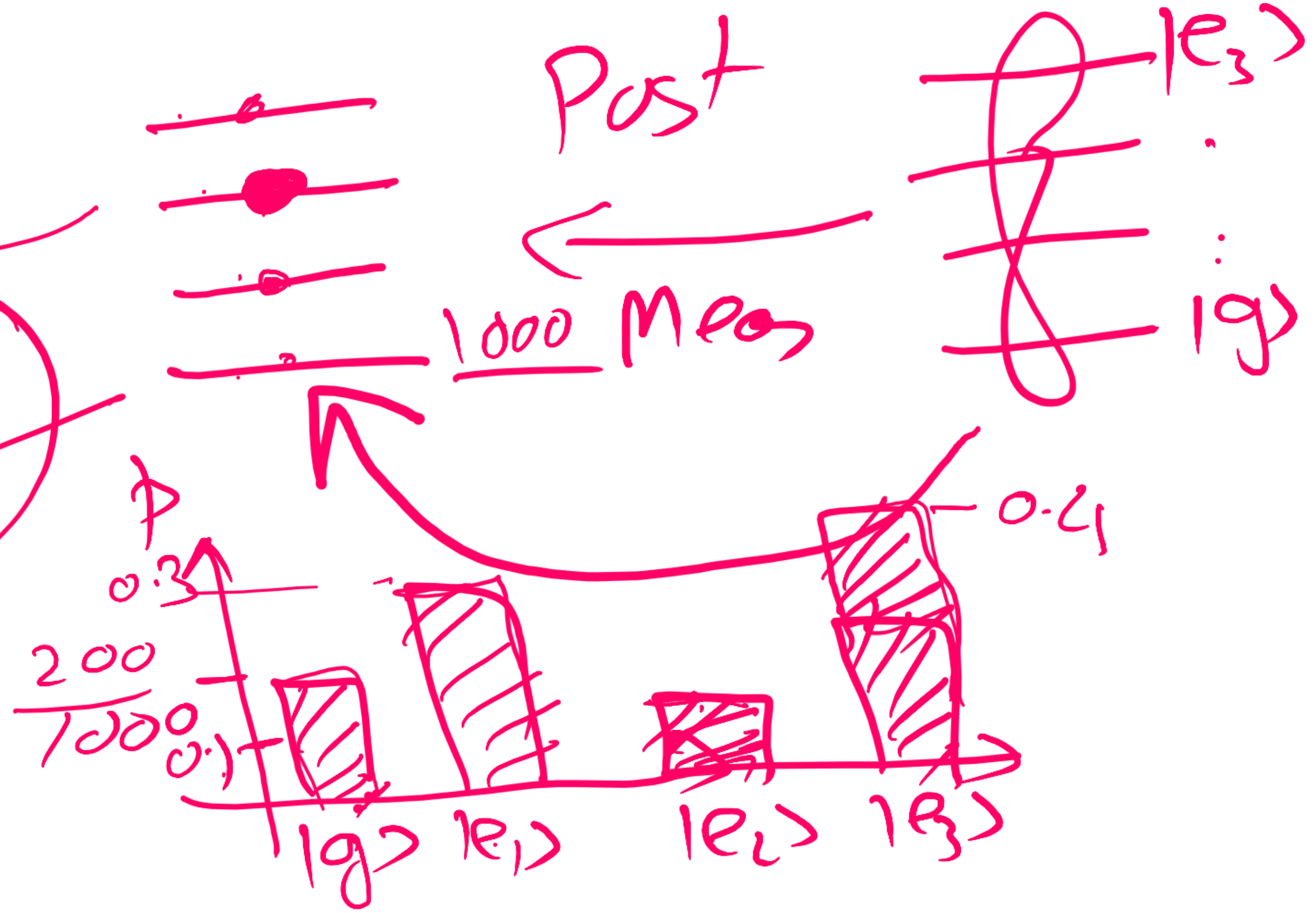
2nd measurements can disturb the system

$$|\psi\rangle = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = a|g\rangle + b|e_1\rangle + c|e_2\rangle + d|e_3\rangle$$

$$|c|^2$$

$$|\psi\rangle_{|e_2\rangle} = \frac{c|e_2\rangle}{|c|}$$

$$=$$



3rd Evolution of state

$$U = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

$$(U^{\dagger})^{*} = \begin{bmatrix} u_{11}^{*} & u_{21}^{*} \\ u_{12}^{*} & u_{22}^{*} \end{bmatrix}$$

$$U^{\dagger} = (U^{T})^{*}$$

Unitary matrix

$$\begin{pmatrix} a' \\ b' \\ c' \\ d' \end{pmatrix} = \begin{bmatrix} \quad U_{4\times 4} \quad \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$$

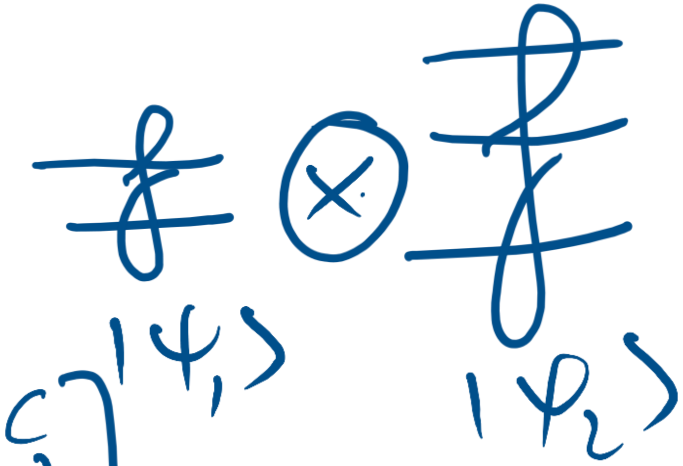
$$|a'|^2 + |b'|^2 + |c'|^2 + |d'|^2 = 1$$

$$U^{-1} = (U^{T})^{*} = U^{\dagger}$$

$$\Rightarrow \boxed{U^{\dagger} U = I}$$

4 Composite Quantum system

$$|\psi_1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}, \quad |\psi_2\rangle = \begin{bmatrix} c \\ d \\ f \end{bmatrix}$$



$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle = \begin{bmatrix} a \\ b \end{bmatrix} \otimes \begin{bmatrix} c \\ d \\ f \end{bmatrix}$$

$$\boxed{|\psi\rangle = |\psi_1\rangle |\psi_2\rangle}$$

$$|\psi\rangle = \begin{bmatrix} ac \\ ad \\ af \\ bc \\ bd \\ bf \end{bmatrix}$$

– Composite systems:

– superposition principle applies to C.S.

$$|\psi_1\rangle = |a_1\rangle \otimes |b_1\rangle$$

$$|\psi_2\rangle = |a_2\rangle \otimes |b_2\rangle$$

$$\boxed{|\psi\rangle = \alpha|\psi_1\rangle + \beta|\psi_2\rangle}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$$\boxed{|\psi\rangle = \alpha|a_1\rangle|b_1\rangle + \beta|a_2\rangle|b_2\rangle}$$

Summary

- Classical world is described by position and momentum and measurements do not disturb it
- Quantum world is described by normalized state vectors and measurements can project state to eigen states
- Composite systems in classical world are just sums of its parts
- Quantum composite systems are more than just sums of its parts
- Classical systems live in phase space
- Quantum systems live in Hilbert space
- Quantum state evolution is unitary