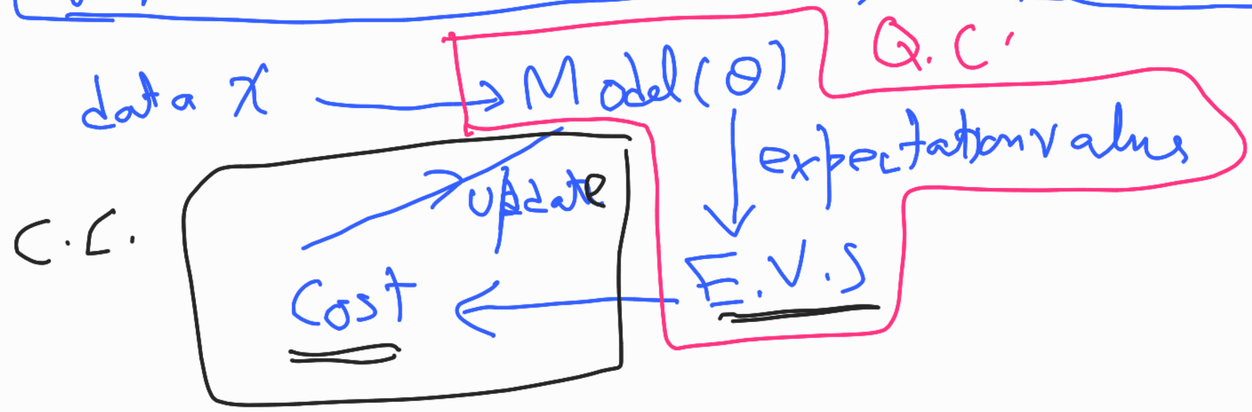
QML Workshop

QML = Learning from data + Q.C. + classical data

$\mathcal{X} \xrightarrow{f(x)} \mathcal{Y}$ $f(x) \approx \boxed{f_\theta(x)}$

$\boxed{\text{Variational Circuits}}$, Kernel methods

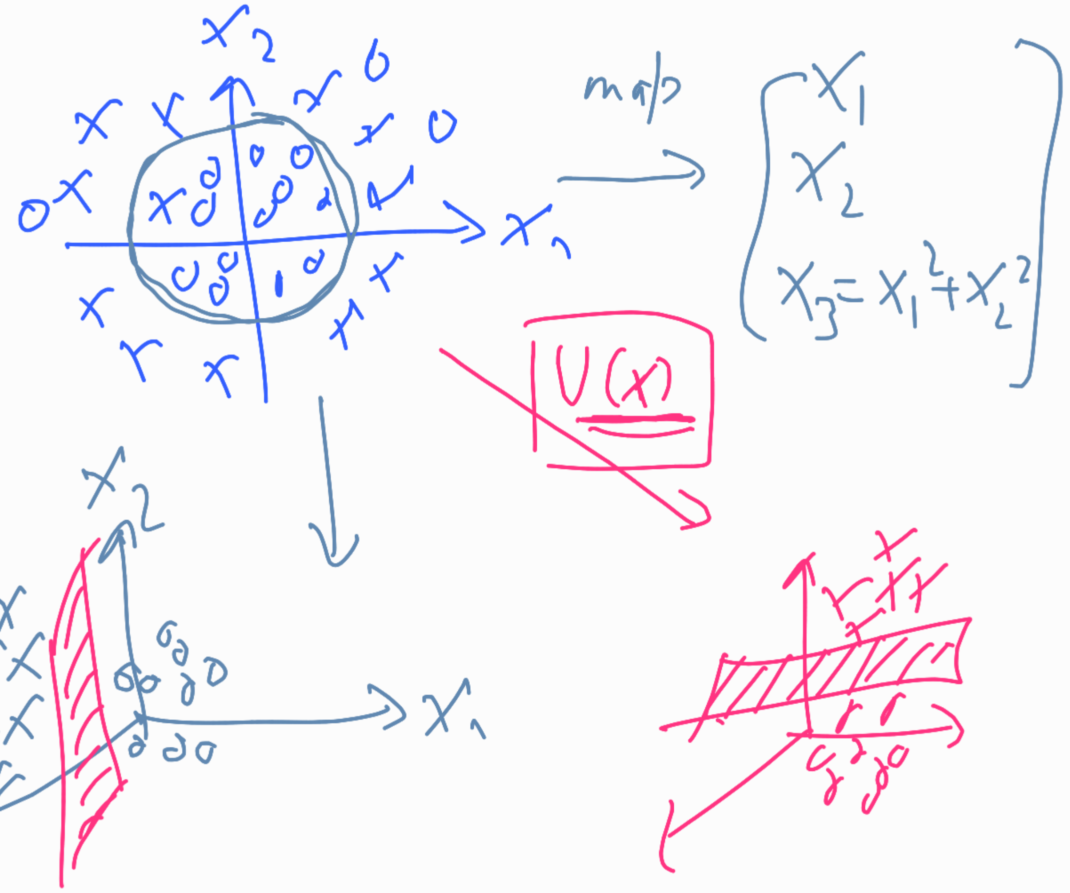


Hybrid classical-Quantum

Kernel Methods: Q.K.

Suppose:

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$



Today: $\rightarrow$ Kernels + QVC
$\rightarrow$ Example & Toy classification

2 $\rightarrow$ Data encoding / Feature maps

3 $\rightarrow$ VQC

4 $\rightarrow$ Quantum SVM

QML Model:
1. Deterministic
2. Probabalistic

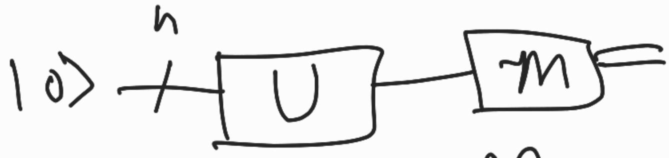
Deterministic Models

$\mathcal{X} \times \mathcal{Y}$
dom Range

$\mathcal{X} : (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_M) \xmapsto{f(x)} (y_1, y_2, \ldots, y_M)$

$\boxed{f_\theta(x)}$

$$|\psi(x, \theta)\rangle = U(\bar{x}, \bar{\theta})|0\rangle^{\otimes n}$$

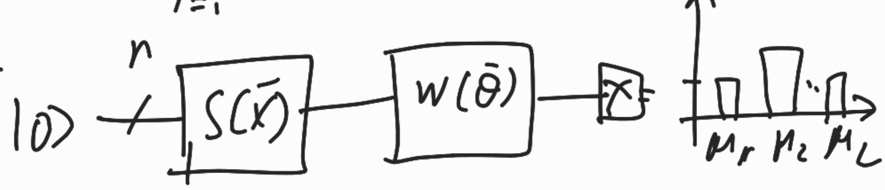


observable M

$$f_\theta(x) = \langle \psi(x,\theta) | M | \psi(x,\theta) \rangle$$

$$M = \sum_{i=1}^{L \le 2^n} \mu_i |\mu_i\rangle\langle \mu_i|$$

An example:



$$\boxed{M = \sum_i \mu_i f(\mu_i)}$$

$$\underline{C(x,\theta)} = F(f_\theta(x))$$

$$\boxed{f_{\tilde{\theta}}(x)}$$

$$\underline{U(x,\theta)} = \overbrace{\underbrace{W(\bar{\theta})}_{\text{ansatz}}\ \underbrace{S(\bar{x})}_{\text{feature map}}}$$

$$M = \sum_i y_i |y_i\rangle\langle y_i|$$

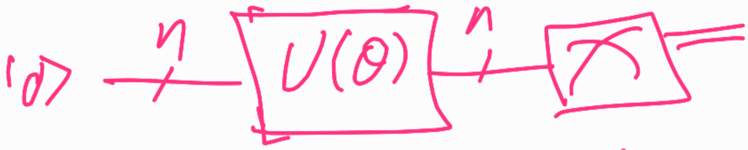
Probabalistic Models:

$$p_\theta(y_i|\bar{x}) = |\langle y_i|\psi(\bar{x},\underline{\theta})\rangle|^2$$

$x_1 \to y_1$

$x_2 \to y_2$

Supervised ML $\longrightarrow$ $(\bar{x}_1, \bar{x}_2, \dots \bar{x}_M)$

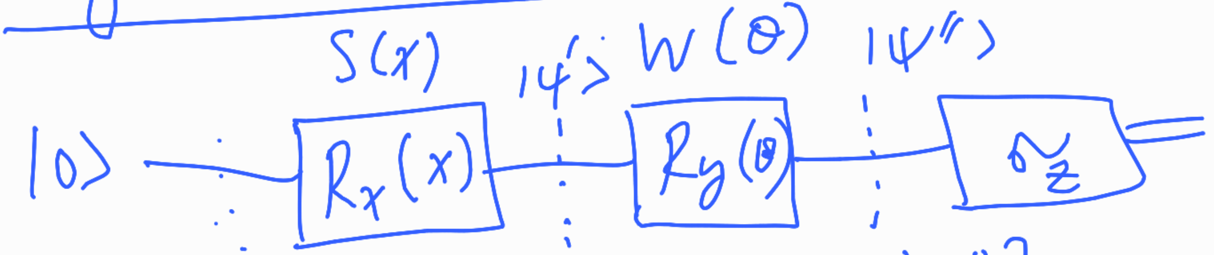


$$p_\theta(x_i) = |\langle x_i|\psi(\theta)\rangle|^2$$

Example: $\mathcal{X}: \{x_1, x_2, \dots, x_m\}$

$y: \{1, 1, \dots, -1, -1, \dots 1\} \in [-1, 1]$

Toy Variational Quantum Circuit (VQC)



$$M = \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$R_x(x) = \begin{bmatrix} \cos\frac{x}{2} & -i\sin\frac{x}{2} \\ -i\sin\frac{x}{2} & \cos\frac{x}{2} \end{bmatrix} \longrightarrow e^{-i\frac{x}{2}\sigma_x}, \quad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\psi'\rangle = \begin{bmatrix} \cos\frac{x}{2} \\ -i\sin\frac{x}{2} \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} = e^{-i\frac{\theta}{2}\sigma_y}$$

$$|\psi''\rangle = R_y|\psi'\rangle = \begin{bmatrix} \cos\frac{x}{2}\cos\frac{\theta}{2} + i\sin\frac{x}{2}\sin\frac{\theta}{2} \\ \cos\frac{x}{2}\sin\frac{\theta}{2} - i\sin\frac{x}{2}\cos\frac{\theta}{2} \end{bmatrix}$$

$$\boxed{|\psi(x,\theta)\rangle =}$$

$$f_\theta(x) = \langle\psi|\sigma_z|\psi\rangle$$
$$= |\langle 0|\psi\rangle|^2 - |\langle 1|\psi\rangle|^2$$

$$f_\theta(x) = \cos^2\frac{x}{2}\cos^2\frac{\theta}{2} + \sin^2\frac{x}{2}\sin^2\frac{\theta}{2} - \left(\cos^2\frac{x}{2}\sin^2\frac{\theta}{2} + \sin^2\frac{x}{2}\cos^2\frac{\theta}{2}\right)$$

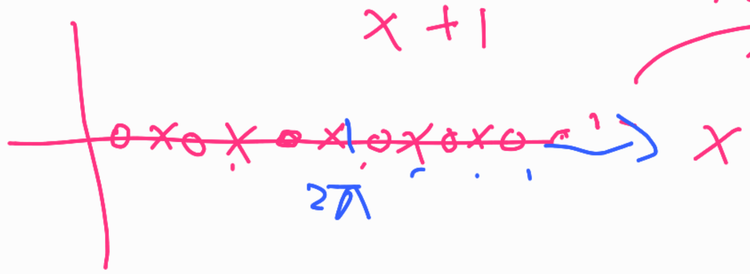
$$= \cos^2\frac{x}{2}\left(\cos\theta\right) - \sin^2\frac{x}{2}\left(\cos\theta\right)$$

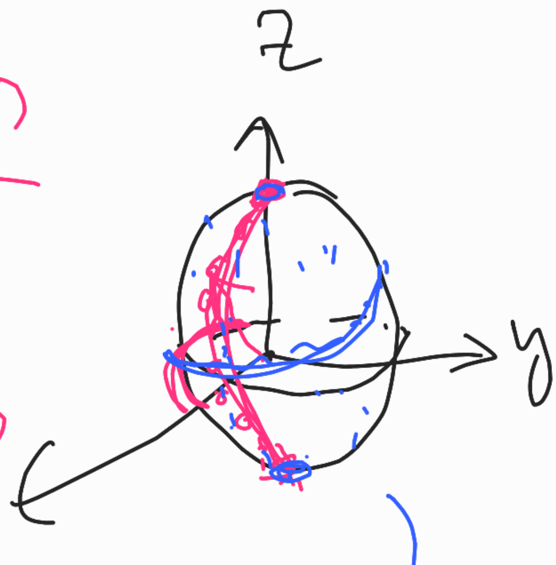
$$\boxed{f_\theta(x) = \cos x \cos\theta}$$

$0 \; -1$

$x \; +1$

$\xrightarrow{R_x(x)}$



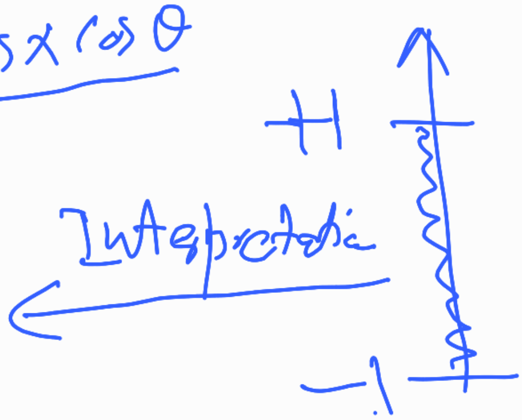


$$|\psi'\rangle = \cos\frac{x}{2}|0\rangle - i\sin\frac{x}{2}|1\rangle$$

$$= \cos\frac{x}{2}|0\rangle + e^{-i\frac{\pi}{2}}\sin\frac{x}{2}|1\rangle$$

$f_\theta = \cos x \cos\theta$

f_θ



$f_\theta < 0 \longrightarrow y_p = -1$

$f_\theta > 0 \longrightarrow y_p = +1$

Lesson:

$\longrightarrow x \in (0, 2\pi)$

$\longrightarrow U(x, \theta)$

$\longrightarrow |\psi(x,\theta)\rangle$ should be in larger part of H

$$f_\theta(x) = \cos\theta \cos x$$

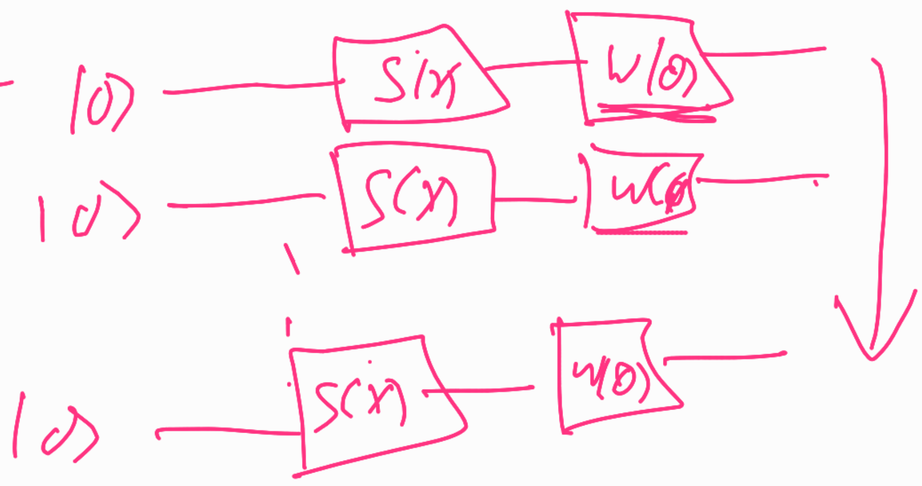
$$= \frac{1}{2} e^{-ix} \cos\theta + \frac{1}{2} e^{ix} \cos\theta$$

$$\boxed{f_\theta(x) = \sum_{n=-1}^{1} c_i e^{inx}}$$

$$f(x) = \sum_{n=-\infty}^{\infty} f_n e^{inx}$$

Repetition:

$M = \sigma_z \otimes \sigma_z \otimes \dots \sigma_z$



$$f_\theta(x) = (\cos\theta\cos x)^n$$

$$= \cos^n\theta\,\underbrace{\cos^n x}$$

$${}_nC_0\cos nx + {}_nC_1\cos(n-1)x + \dots$$

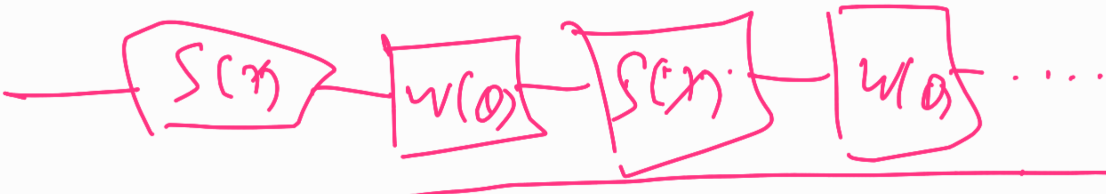
$$\cos^n x = \left(\frac{e^{ix}+e^{-ix}}{2}\right)^n$$

$$= \frac{1}{2^n}\left\{ e^{inx},\ e^{i(n-1)x} \dots e^{-(n-1)x}\ e^{-inx} \dots \right\}$$

$$\boxed{f_\theta(x) = \sum_{j=-n}^{n} c_j e^{ijx}}$$

$=$

n



$$\boxed{f_\theta(x) = \sum_{j=-n}^{n} c_j e^{ijx}}$$

expressivity of QML

$$U \sim e^{-i\theta H}$$