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Global Credit Products - Case Study Report

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1 Convertible Bond Pricing

This section describes the solution to the first part of the case study on pricing of a convertible bond. Please refer to the attached OTPP_Convertible_Bond.xlsm file for the computation in Excel VBA. The specific details will be explain in Section 1.3.

1.1 Description of a Convertible Bond

A *convertible bond* is a derivative security that depends on the underlying stock price. Specifically, the convertible bond behaves as a regular bond while also providing the holder a right to convert the bond into stock at a prespecified ratio. For this problem, our convertible bond will continue to pay a semiannual coupon until maturity without exercise of the conversion (and default); when exercised, each bond will convert into a number (*conversion ratio*) of stock.

Since exercise is allowed at anytime, the security behaves like an American option: it will always take the maximum of the holding value and the exercise value (also known as *parity price*). Due to this complexity, there does not exist a closed-form solution for the price, and we must rely on a numerical approximation instead.

It is also common for the convertible bond to be callable or puttable; a *callable bond* allows the issuer of the bond to buy back the bonds before the maturity date; similarly a *puttable bond* allows the holder of the bond to demand early repayment of bond. In this problem, we will not deal with the complexity of such instruments.

1.2 The Binomial Tree Approach to Pricing

A *binomial tree* (also known as *binomial lattice*) is a discretized model of a continuous stochastic process, such that at any stage, the process can only evolve to two potential values in the next step. As the size of the time steps is taken to zero, the discretized process will converge to the continuous process (under some technical conditions). In practice, the convergence is more than sufficient for modeling simple derivative prices.

In this problem, we allow a third potential value in the event of a default, where the stock price will become zero, and the bond price will be equal to the *recovery value.* The underlying principle remains the same for find the price of the convertible bond P_{cb}

$$P_{\text{cb}}(t, S(t)) = \max \left\{ P_{\text{exercise}}(t, S(t)), P_{\text{hold}}(t, S(t)) + P_{\text{coupon}}(t) \right\}$$

where $P_{\text{coupon}}(t)$ is the coupon payment, and $P_{\text{exercise}}(t, S(t))$ is the value of the conversion at conversion rate C_r

$$P_{\text{exercise}}(t, S(t)) = C_r S(t)$$

$P_{\text{hold}}(t, S(t))$ can be computed as the discounted (conditional) expectation of the price at the next time step

$$\begin{aligned} P_{\text{hold}}(t, S(t)) &= e^{-r\Delta t}\mathbb{E}\Big[P_{\text{cb}}(t+\Delta t, S(t+\Delta t))\Big| S(t)\Big] \\ &= e^{-r\Delta t}\Big[p_{\text{up}}P_{\text{cb}}(t+\Delta t, S_{\text{up}}(t+\Delta t)) + p_{\text{down}}P_{\text{cb}}(t+\Delta t, S_{\text{down}}(t+\Delta t)) + p_{\text{default}}P_{\text{recovery}}] \end{aligned}$$

Here $p_{\text{up}}, p_{\text{down}}, p_{\text{default}}$ are (risk-neutral) probabilities of each of the branches in the binomial tree, r is the risk-free rate, and $S_{\text{up}}, S_{\text{down}}$ correspond to the up and down branches of the binomial tree of the stock.

Observe that $P_{\text{cb}}(t, S(t))$ is can be found recursively from the values of $P_{\text{cb}}(t+\Delta t, S(t+\Delta t))$. Therefore, by determining the values of $P_{cb}(T)$ where T is the maturity time. Since the holding value of convertible bond at maturity is just the par value plus the final coupon, we have

$$P_{\text{cb}}(T) = \max\Big\{C_r S(T), P_{\text{par}} + P_{\text{coupon}}(T)\Big\}$$

1.3 VBA Implementation

1.4 Computing Delta