Stable Tree Labelling for Accelerating Distance Queries on Dynamic Road Networks

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Problem Definition

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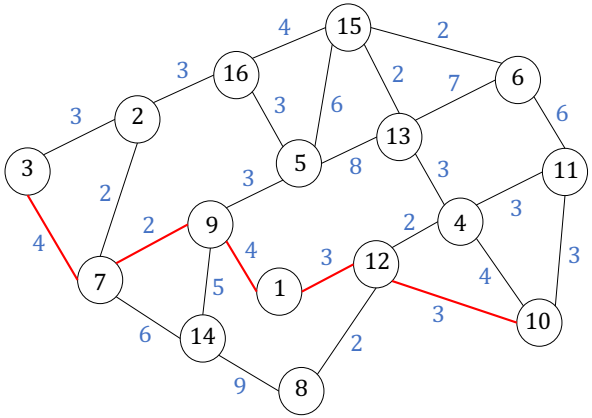
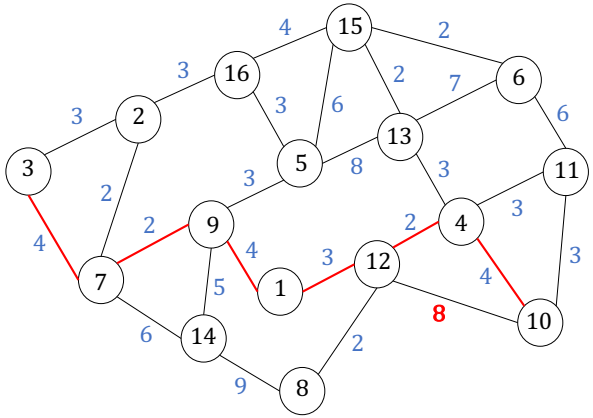


Before update



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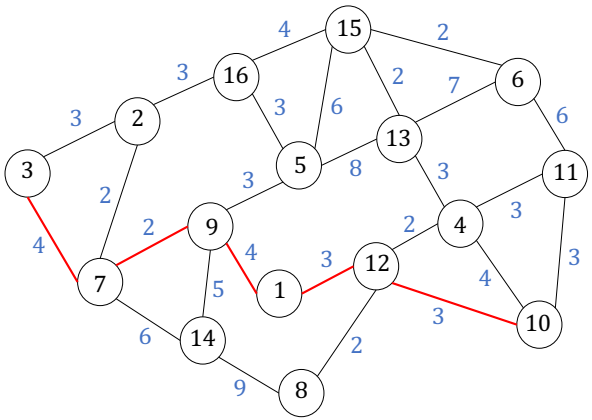

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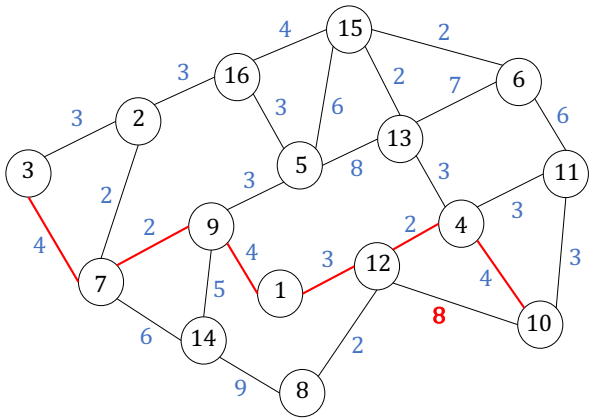

After update

– A shortest path between 3 and 10:

Before update: $\langle 3, 7, 9, 1, 12, 10 \rangle$

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– The distance between 3 and 10:

Before update: $d_G(3, 10) = 16$

After update: $d_G(3, 10) = 19$

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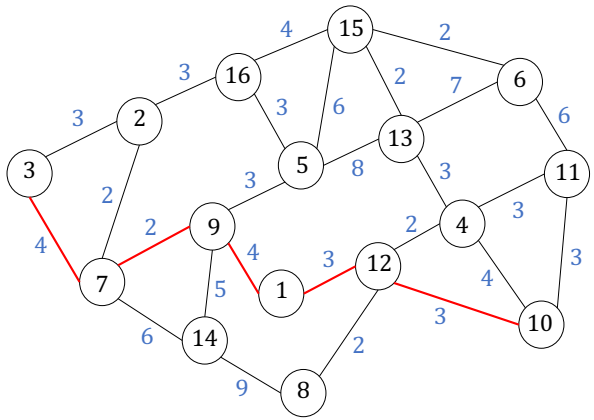

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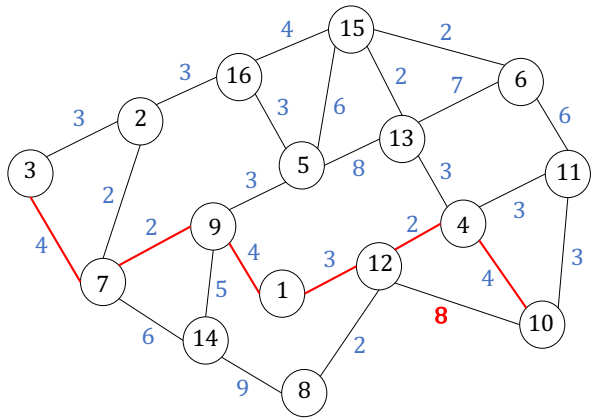

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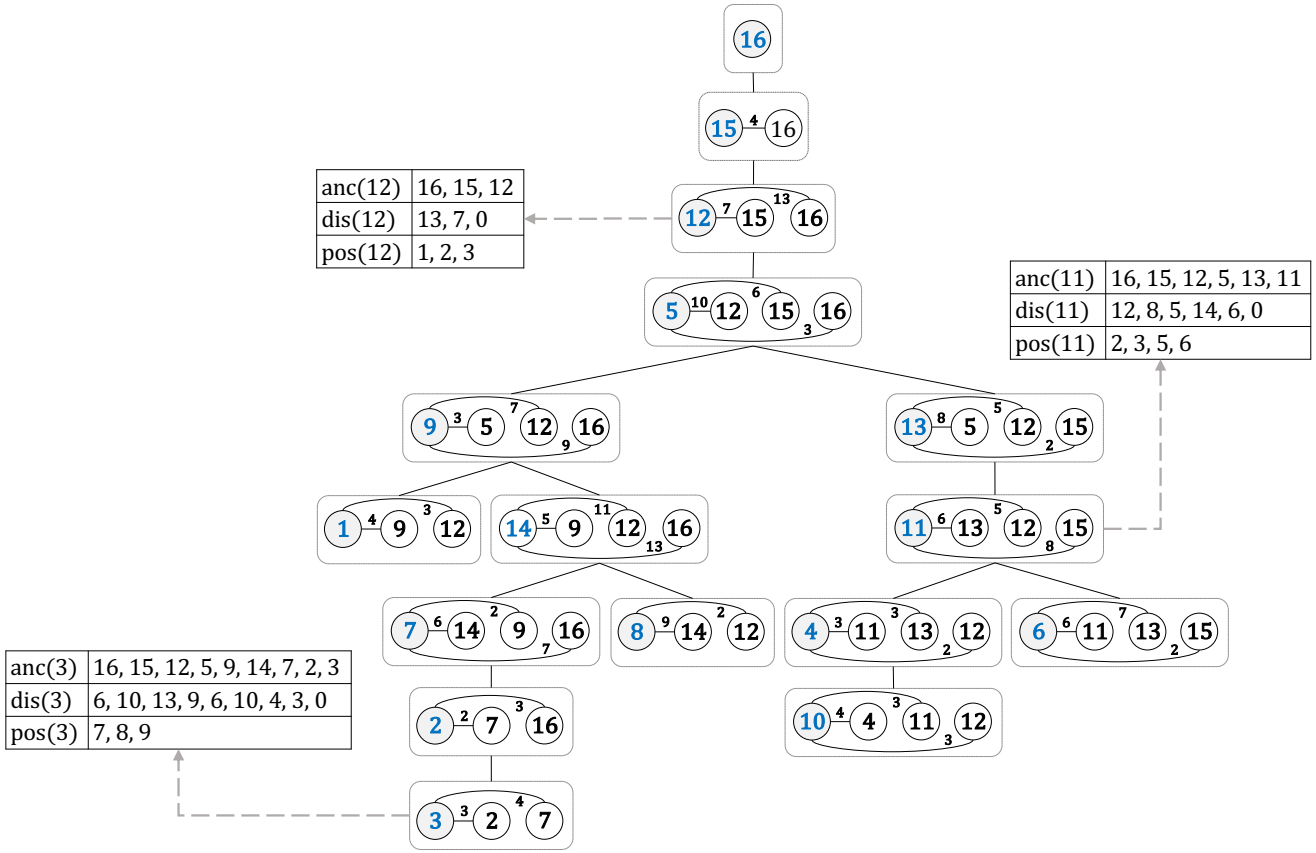
Applications: GPS navigation, route planning, traffic monitoring, etc.

Incremental Hierarchical 2-Hop (IncH2H)



H2H index

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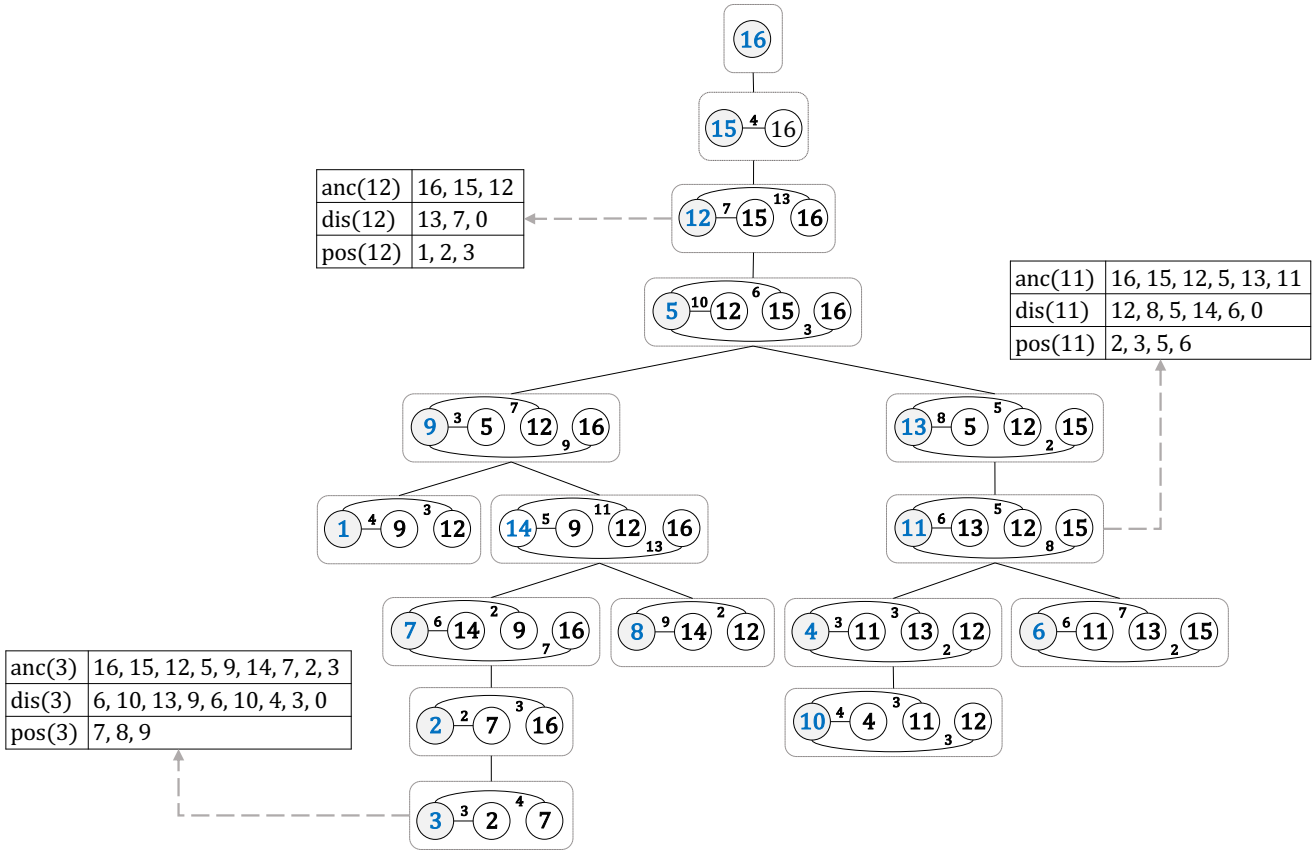
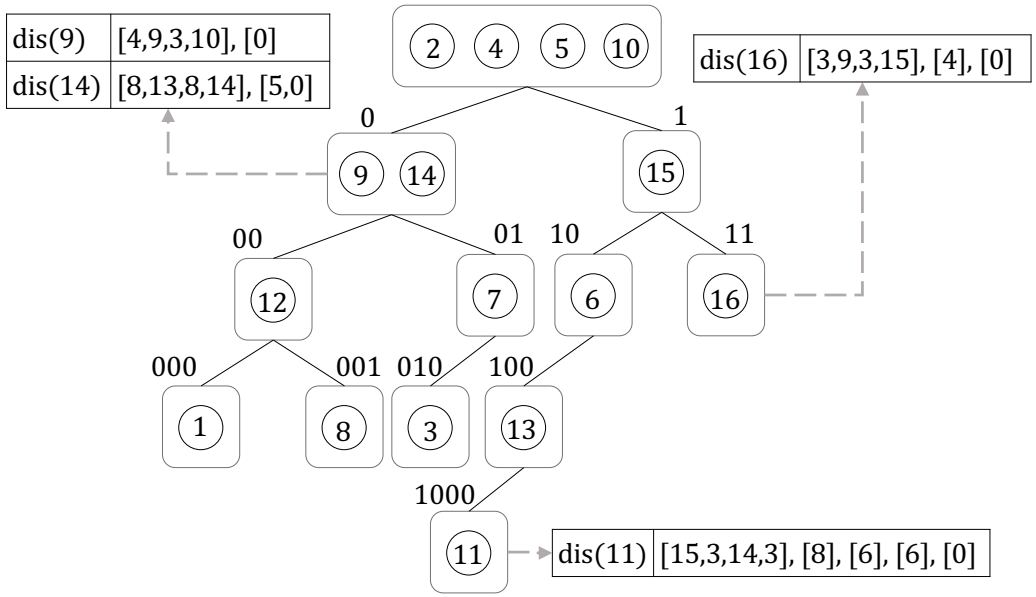


H2H index

- **Limitations:**
  - Huge index to maintain due to very large tree width and height
  - Additional computational cost to compute LCA(s,t)

Hierarchical Cut 2-Hop Labelling (HC2L)



HC2L index.

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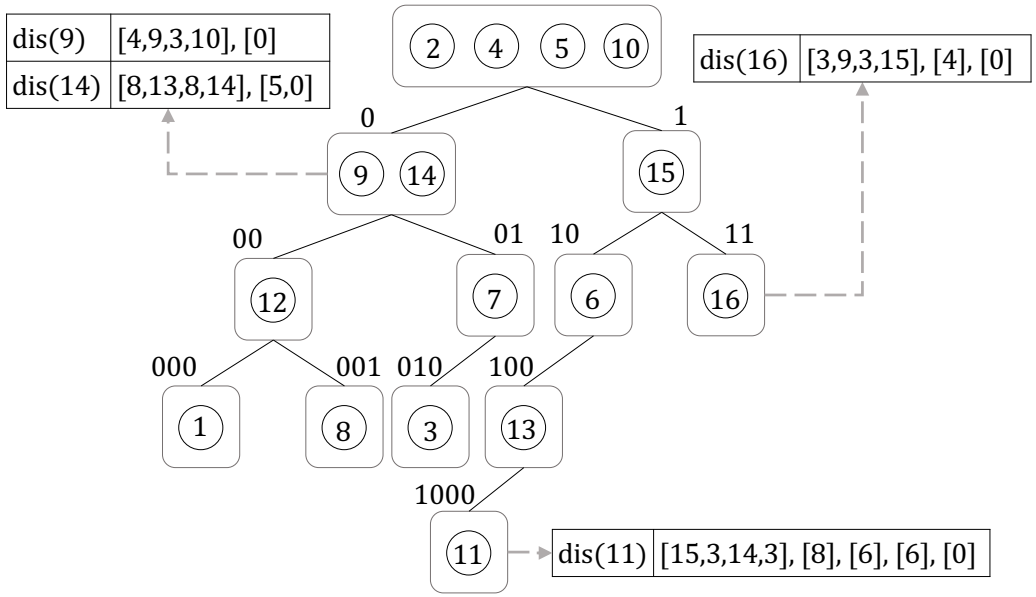

HC2L index.

- **Limitations:**
  - The distance-preserving property hinders efficient maintenance of the HC2L index.

Our Solution - Stable Tree Labelling

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  - Every path between any two vertices contains at least one of their common ancestors.
  - 2-hop labelling is constructed over Stable Tree Hierarchy and labels only store distances within subgraphs, not in the entire graph.

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(a) Stable Tree Hierarchy T

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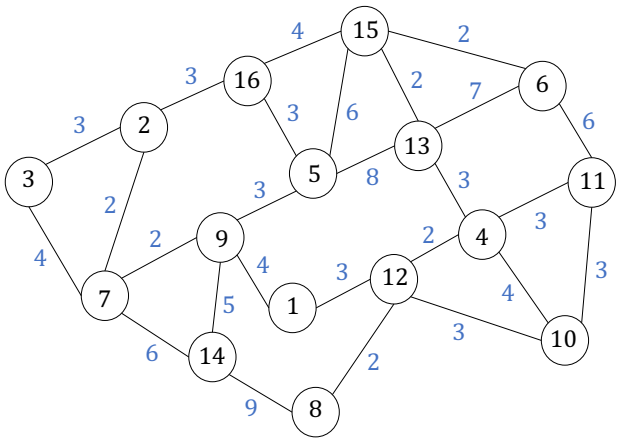
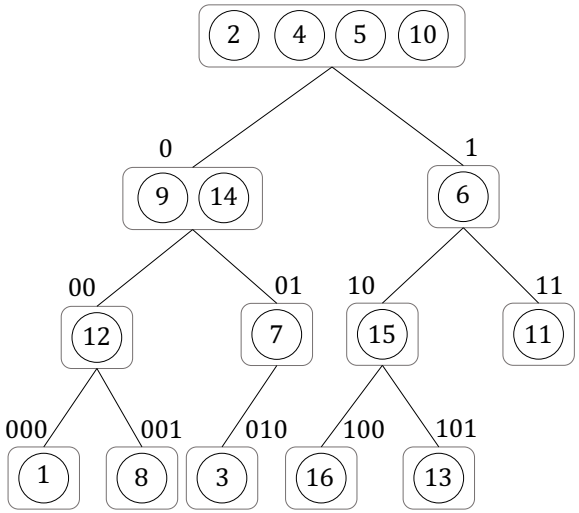


G



T

Label	Distance Entries
L(1)	[8,5,7,6], [4,14], [3], [0]
L(2)	[0]
L(3)	[3,15,9,16], [6,10], [4], [0]
L(4)	[12,0]
L(5)	[6,11,0]
L(6)	[9,7,8,9], [0]
L(7)	[2,11,5,12], [2,6], [0]
L(8)	[13,4,12,5], [9,9], [2], [0]
L(9)	[4,9,3,10], [0]
L(10)	[14,4,13,0]
L(11)	[15,3,14,3], [6], [0]
L(12)	[11,2,10,3], [7,11], [0]
L(13)	[9,3,8,13], [4], [2], [0]
L(14)	[8,13,8,14], [5,0]
L(15)	[7,5,6,11], [2], [0]
L(16)	[3,9,3,15], [6], [4], [0]

L

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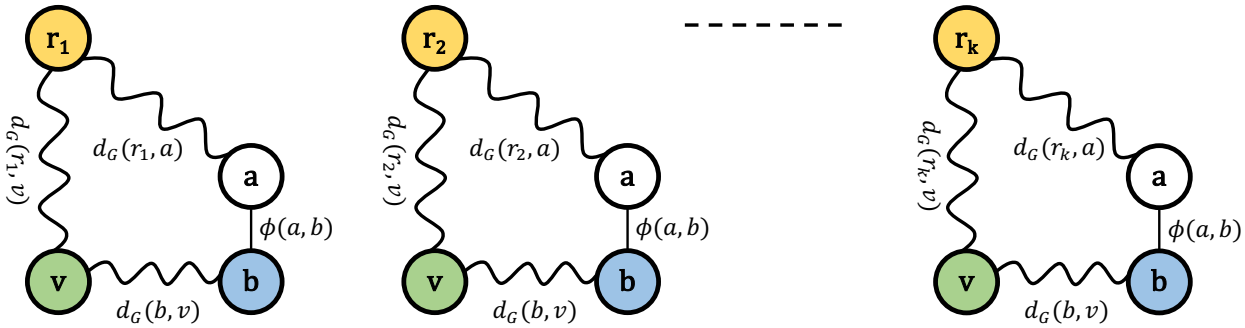
    $\hookrightarrow$ Develop *Label Search algorithms* – search from a very small set of ancestors to maintain 2-hop labelling.

    $\hookrightarrow$ Develop *Pareto Search algorithms* which improve *Label Search algorithms* – explore the interaction between search spaces of different ancestors.

Label Search Algorithms

- For each ancestor, identify affected labels to update using triangle inequality



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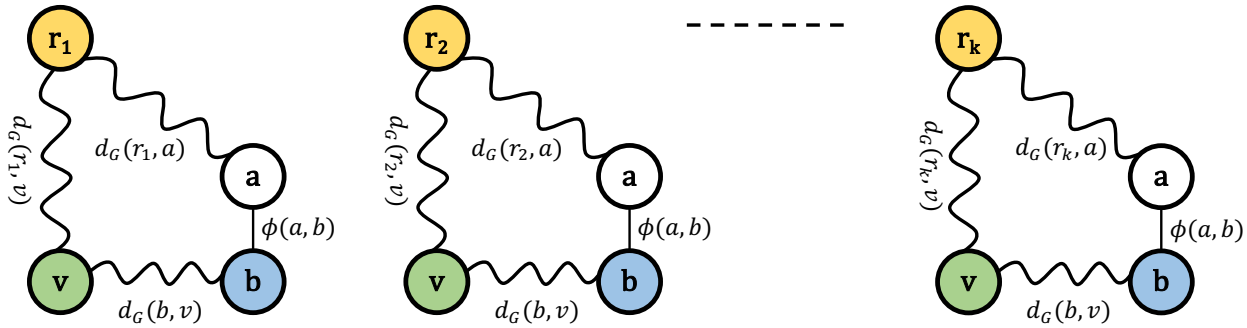
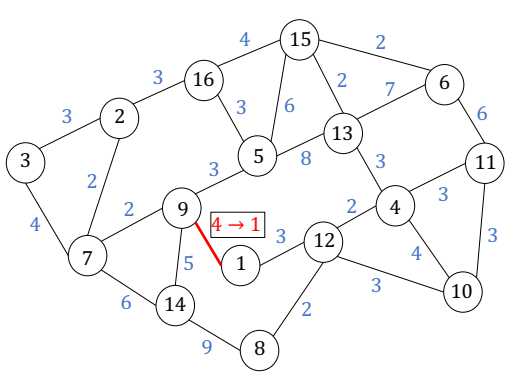
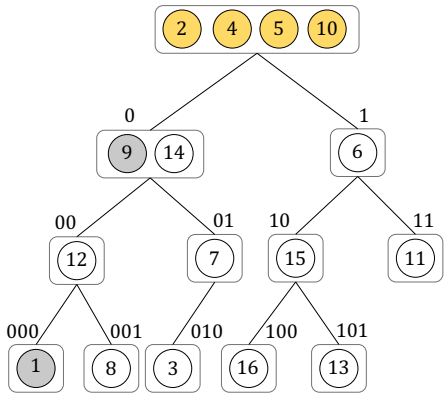
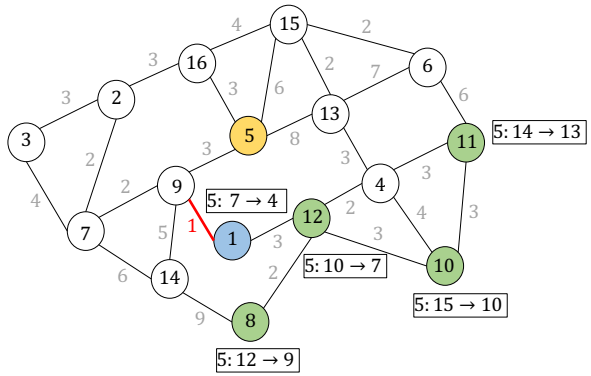


- Label search algorithm for handling weight decrease (STL-L$^-$)







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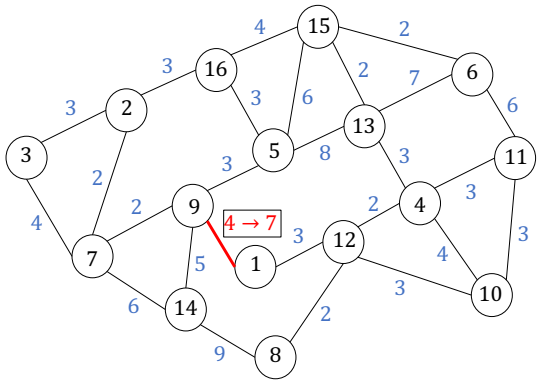

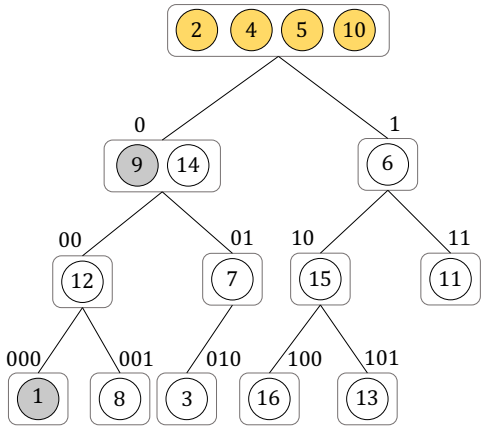

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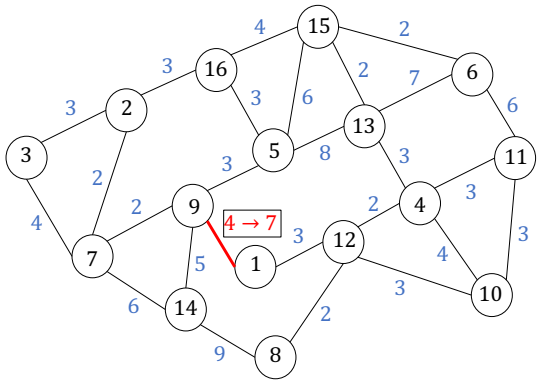

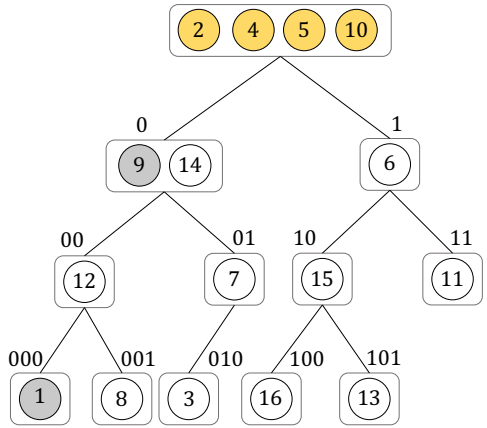

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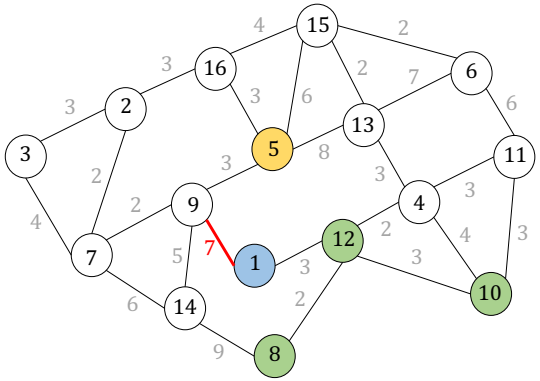

Find Affected Labels

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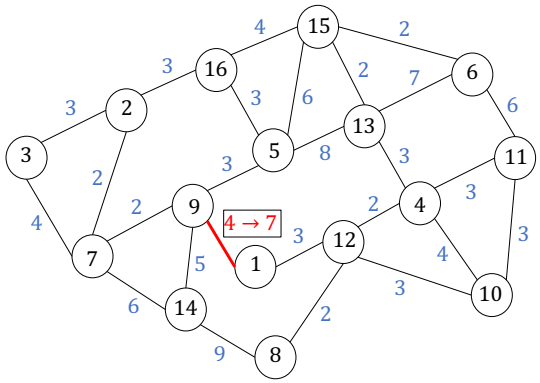

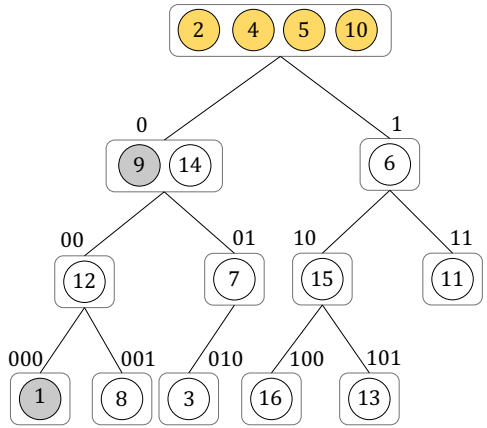

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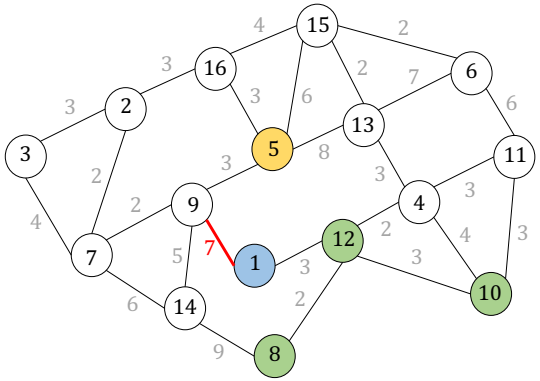

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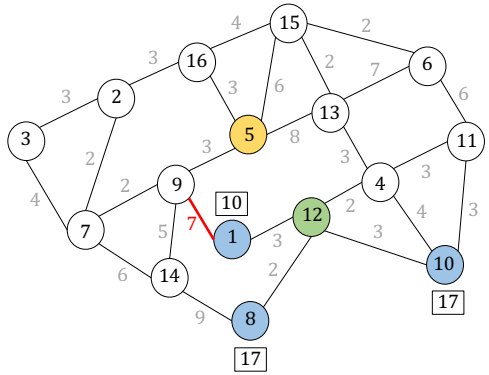

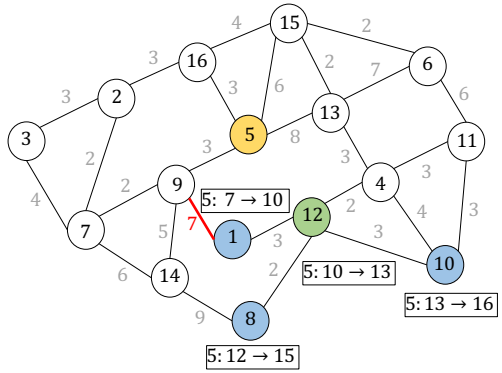

Repair Affected Labels

Pareto Search Algorithms

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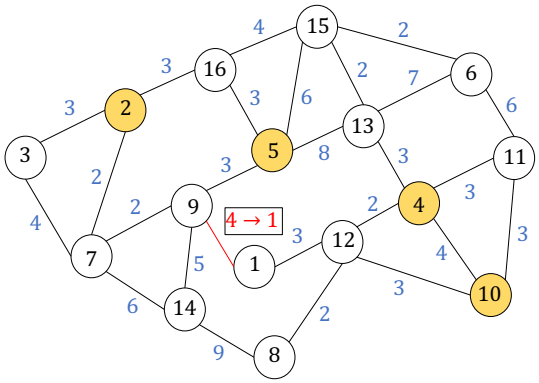
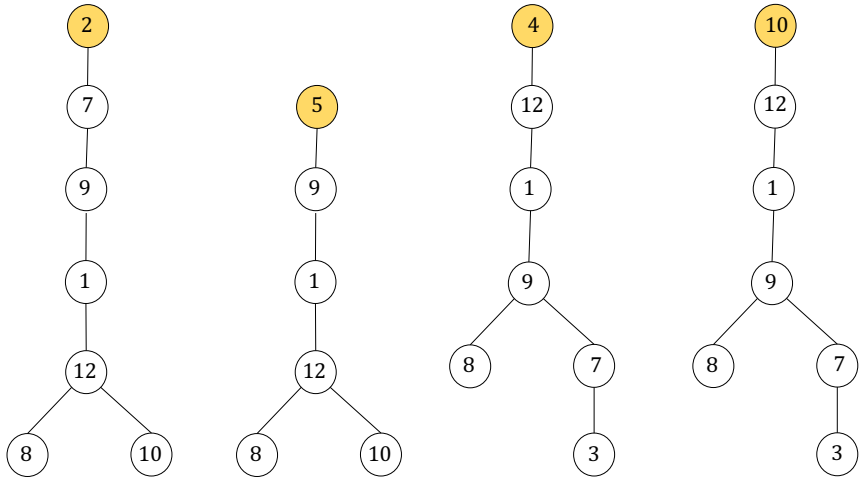
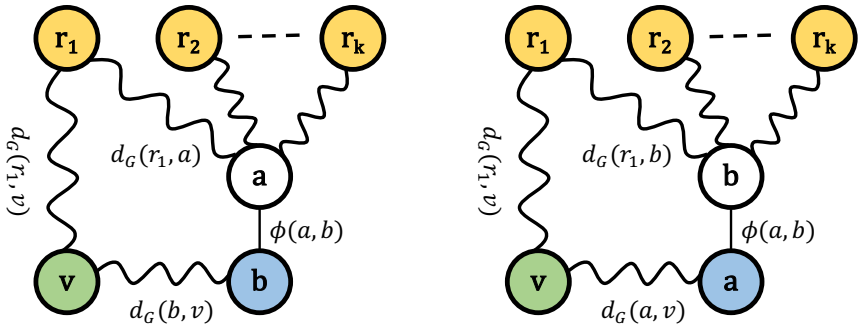




Pareto Search Algorithms

- For each update (a, b), combine searches from multiple ancestors into only two searches



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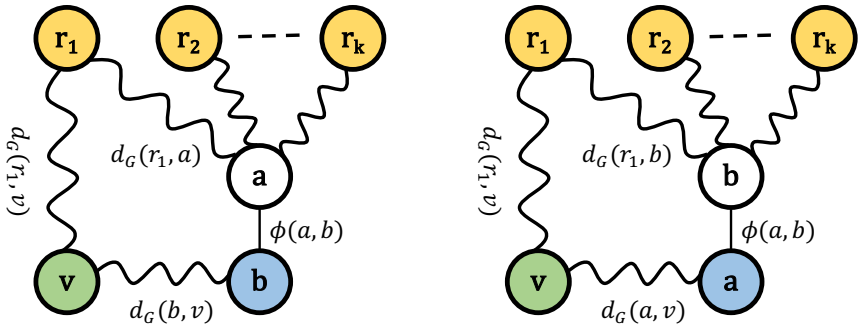

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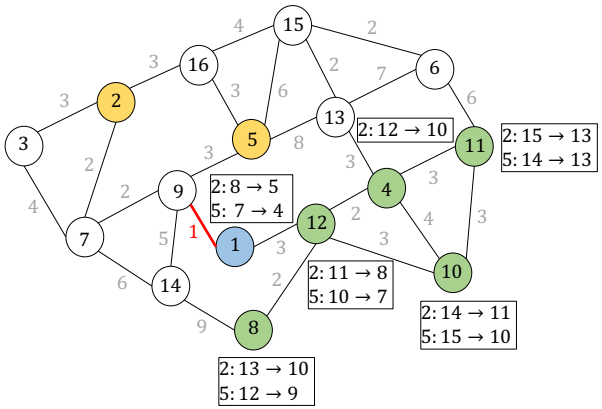

Edge Weight Decrease

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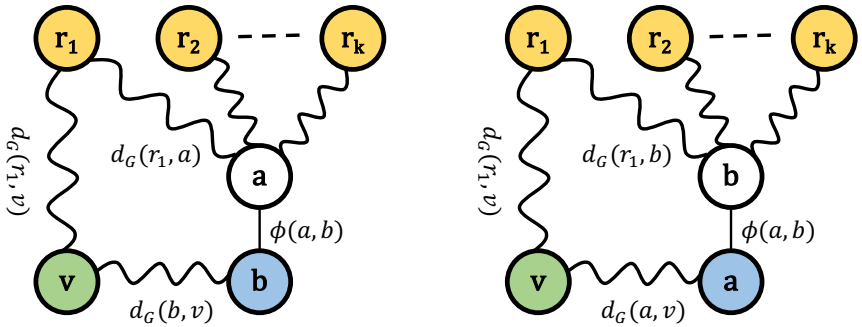

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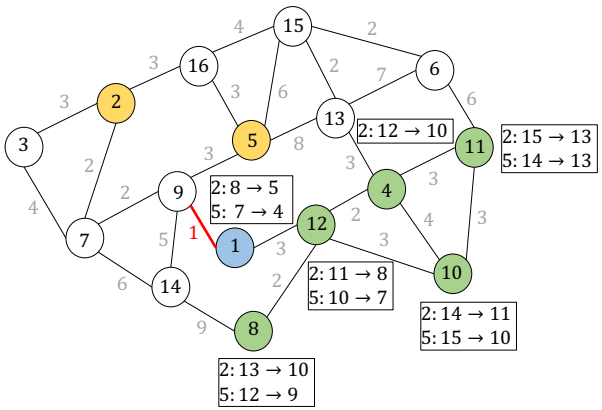

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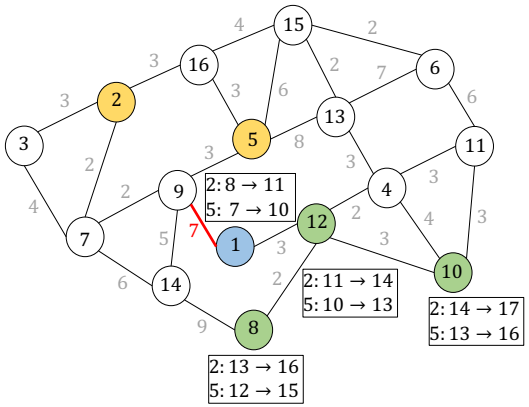

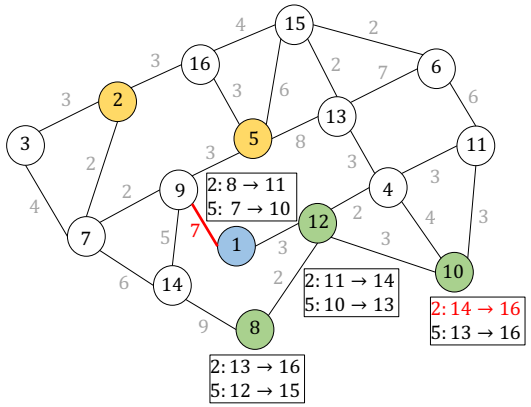

Edge Weight Increase

Empirical Evaluation

Network	Update Time - Decrease [ms]				Update Time - Increase [ms]			
	STL-P$^-$	STL-L$^-$	IncH2H$^-$	DTDHL$^-$	STL-P$^+$	STL-L$^+$	IncH2H$^+$	DTDHL$^+$
NY	0.845	1.978	2.006	11.40	1.712	3.561	2.900	13.87
BAY	0.917	1.788	1.769	8.899	1.695	3.233	2.498	14.53
COL	1.898	3.882	3.306	12.74	3.456	6.977	4.613	34.35
FLA	2.303	5.209	3.585	32.45	4.109	9.554	4.981	34.22
CAL	4.975	16.67	13.89	99.24	10.11	31.04	20.20	106.4
E	7.996	39.21	29.33	261.5	17.48	73.76	43.57	273.1
W	12.26	52.71	47.76	604.9	25.14	100.2	68.99	1,292
CTR	27.23	164.4	213.1	2,329	54.03	314.5	309.7	5,347
USA	32.67	216.4	239.8	–	82.78	412.9	356.3	–
EUR	13.68	68.25	66.97	–	61.57	131.4	96.63	–

- *5-7 times faster in terms of update time on large road networks compared to SOTA IncH2H*
- *several orders of magnitude faster in terms of update time compared to DTDHL*

Empirical Evaluation

Network	Query Time [μs]				Labelling Size				Construction Time [s]			
	STL	HC2L	IncH2H	DTDHL	STL	HC2L	IncH2H	DTDHL	STL	HC2L	IncH2H	DTDHL
NY	0.287	0.264	0.913	0.852	129 MB	172 MB	850 MB	391 MB	2	3	4	9
BAY	0.299	0.258	0.841	0.785	104 MB	134 MB	814 MB	377 MB	2	3	3	5
COL	0.349	0.318	1.018	0.988	175 MB	238 MB	1.37 GB	587 MB	4	6	5	7
FLA	0.396	0.349	1.019	0.958	423 MB	561 MB	2.43 GB	1.30 GB	11	16	11	17
CAL	0.490	0.484	1.333	1.380	1.03 GB	1.48 GB	8.21 GB	3.91 GB	28	44	30	48
E	0.630	0.550	1.683	1.585	2.92 GB	4.22 GB	20.7 GB	9.68 GB	75	129	74	111
W	0.664	0.601	1.702	1.819	4.82 GB	7.01 GB	36.3 GB	20.6 GB	120	249	126	194
CTR	0.812	0.702	2.483	2.658	19.7 GB	30.2 GB	178 GB	80.3 GB	540	1,140	858	766
USA	0.834	0.734	3.428	–	35.6 GB	53.6 GB	308 GB	–	852	1,721	1,081	–
EUR	1.185	0. 879	3.888	–	36.4 GB	51.2 GB	322 GB	–	1,236	2,354	1,254	–

Compared to IncH2H

- *about 3 times faster in terms of query time*
- *consuming about an order of magnitude less space*

Compared to DTDHL

- *significantly faster in terms of query time*
- *consuming 20%-30% of space for labelling*

Concluding Remarks

- Stable tree labeling (STL)

  serve as the foundation for designing efficient dynamic algorithms.

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  Label Search is ancestor-centric while Pareto Search is update-centric.

- Our solutions are scalable on large and dynamic road networks

  The power lies in "stable tree hierarchy".

Questions

Thank You