EXERCISE 2(G)

Given

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix},$$

we can produce $\mathbf{M} \to \mathbf{I}_3$ via:

1. Divide row 3 by 3.
2. Add multiples of -2 from row 2 to row 1.
3. Swap rows 2 and 3[1].

This effect can be produced by the following elementary matrices.

The swap matrix can be created from emulation with I_3:

$$\mathbf{E}_{swap} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

similarly, the division matrix would be

$$\mathbf{E}_{div} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix},$$

and lastly, the additive matrix would be

$$\mathbf{E}_{add} = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Altogether, the following multiplication should hold true:

$$\mathbf{E}_{div}\mathbf{E}_{add}\mathbf{E}_{swap}\mathbf{M} = \mathbf{I}_3$$

[1]This was originally wrong and placed first. You need to swap last because matrix multiplication is right-associative, so the rightmost operation occurs first, not last. In other words, a "correct" way to approach this would be to do the entire process in reverse when multiplying.

EXERCISE 2(H)

Given

$$\begin{aligned} x + 2y &= 4 \\ 3z &= 6 \\ y &= 8, \end{aligned}$$

we can produce the following coefficient matrix $\mathbf{A}$ and constant vector $\mathbf{B}$:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 4 \\ 6 \\ 8 \end{pmatrix},$$

and given that $\mathbf{A} = \mathbf{M}$, from exercise 2(g) the solutions can be calculated by performing the RREF calculations on $\mathbf{B}$ to produce solution vector $\mathbf{S}$:

$$\mathbf{E}_{div}\mathbf{E}_{add}\mathbf{E}_{swap}\mathbf{B} = \mathbf{S},$$

yielding

$$\mathbf{S} = \begin{pmatrix} -12 \\ 8 \\ 2 \end{pmatrix},$$

showing that $x = -12, y = 8, z = 2$.

EXERCISE 2(I)

This isn't solvable because the best RREF form is

$$[\mathbf{M}|\mathbf{B}] = \begin{pmatrix} 1 & 0 & -6 & 4 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 6 \end{pmatrix}$$

or, in other words, equations (2) and (3) contradict each other.

EXERCISE 2(J)

The initial state of the system is

$$\begin{pmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & 3 & 6 \\ 0 & 2 & 6 & 12 \end{pmatrix},$$

and the RREF of this system becomes

$$\begin{pmatrix} 1 & 0 & -6 & -8 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Again, this system is not solvable as equations 2 and 3 are equivalent, resulting in this system not containing enough information to be adequately solved, but the reduced echelon form of the coefficient matrix is the same as in exercise 2(i).