Experiment #04
RC and LRC Circuits as Filters - Frequency Domain Measurements

Sarah Burke

1 Objective

In this experiment you will measure the frequency-dependent response of the RC and LRC circuits when you apply a sine wave to them. As discussed in the tutorial, the transient response, and the frequency dependent response to sine waves are closely related. One is measured in the time domain (transients), and today you will work in the frequency domain. The experiment will involve measuring the output amplitude of the circuit as you vary the frequency of the input signal. Aside from learning about the relationship between the time-domain and frequency domain, these two circuits are the most basic examples of electrical filters (devices that attenuate some signals, while allowing others to pass), and you will make use of these in the design of the radio at the end of term.

2 Learning Goals

After finishing the lab you will:

- know the relationship between time-domain and frequency-domain response of RC and LRC circuits and relate this to filtering properties
- learn how to take amplitude measurements on the oscilloscope
- develop decision-making skills around efficient sampling of a parameter space (in this case frequency) to capture key characteristics of a function
- learn how to fit to two important models - variations of the Lorentzian

3 The RC Circuit in the Frequency Domain

3.1 Background

You will rebuild the RC circuit that you used in Lab # 02, shown in Fig. 3.1. Make sure you use the same capacitor and 1 kΩ resistor that you used in that lab.

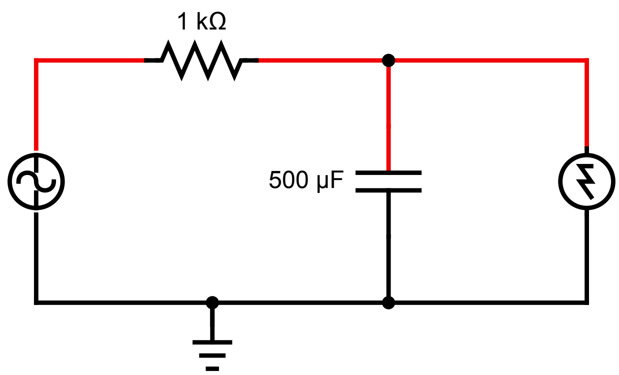

Figure 1: Circuit diagram for the series RC circuit with the scope connected to measure the voltage signal across the capacitor. To see the input signal from the function generator, use. BNC T to connect it to both your circuit and the other oscilloscope channel. Remember to note which channel is which.

The only difference in this set-up compared to Experiment 02 is that you will apply a sinusoidal signal from the function generator instead of a square wave, and vary its frequency to examine the response. On the output side, the voltage across the capacitor, you will measure the amplitude of the output signal, which will vary with frequency. This variation in amplitude is how the circuit operates as a filter; the low frequency response is different from the high frequency response.

If the sinusoidal signal from the generator is :

$$V_{in}(t) = V_{in}^0 \cos(\omega t) \tag{1}$$

where V_{in}^0 is the amplitude of the input signal, and ω is the angular frequency with $\omega = 2\pi f$. The output voltage signal measured across the capacitor is expected to follow:

$$V_c(f,t) = V_c(f) \cos\left[2\pi f t + \phi(f)\right]. \tag{2}$$

Note that the frequency is unchanged, but the amplitude and phase of the output voltage can be different from the input signal. Depending on the frequency, you might see something like Fig. 2, with both input (CH1, yellow) and output (CH2, cyan) showing sinusoidal time-dependence, but with different phase and amplitude from the input signal.

For this experiment, we will concentrate on the amplitude.

$$V_c(f) = V_{in}^0 / \sqrt{1 + (2\pi f RC)^2} \tag{3}$$

Note that this frequency-domain result contains the time constant $\tau = RC$. So this equation can be written with the time constant τ as a parameter in the model.

For this experiment, we will concentrate on the amplitude response as a function of frequency, which follows:

$$V_c(f) = V_{in}^0 / \sqrt{1 + (2\pi f \tau)^2} \quad (4)$$

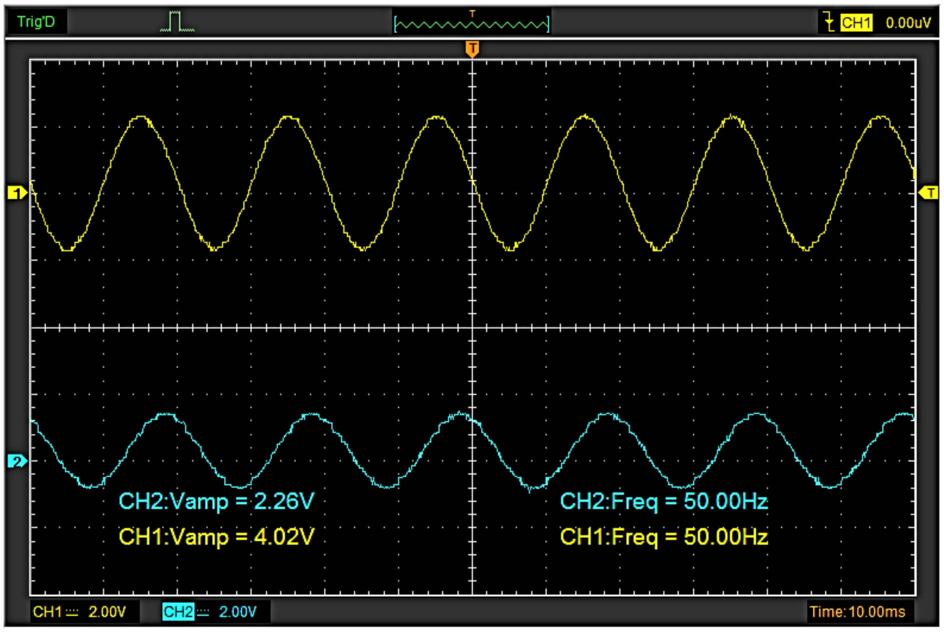

Figure 2: Output voltage signal across the capacitor CH2 (cyan) in response to an oscillating input signal CH1 (yellow) applied to a series RC circuit. Note the frequency is the same but amplitude of the output signal is reduced and there is a phase shift relative to the input signal.

3.2 Experiment

Before connecting up your circuit, check your components and compare to the values you measured in Experiment 02, and again make a note of their values. If they are not the same, discuss with a TA or instructor. Then build the RC series circuit in the same way you did in Experiment 02, but with the function generator set to output a sine wave. Use the plots you made in the Prelab to help you decide what frequency to start with.

As before, you will find it most convenient to use a BNC Tee to connect the function generator to both your circuit and one of the oscilloscope channels. Connect the output voltage to the other channel of the oscilloscope.

Tip 1

Use the function generator output to trigger the oscilloscope as this will not change amplitude as you take your measurements.

Try varying the frequency and make some qualitative comments about what happens to the amplitude and phase of the output signal. For more detailed measurements, you can use the cursors, or the internal measurement tools of the MicSig oscilloscope to measure amplitudes. You might find use of the cursors to be more reliable, since the internal tools sometimes measure high frequency noise that you are not even seeing in the measurement range you care about. If using the measurement tools, don't use them blindly and cross check with some cursor measurements.

Checkpoint 1

When you have a signal for the RC response on the oscilloscope and have made a rough measurement strategy, call a TA over to show your signal and discuss your plan.

Some important tips for making your measurements:

- cover a very wide frequency range
- data does not need to be evenly spaced in frequency
- data does not need to be taken in any sequential frequency order
- concentrate your data in ranges where something interesting is happening
- take care in estimating the uncertainty for each measurement; the uncertainty may be different at different frequencies
- you might be dealing with instrumental uncertainty as well as noise; reminder that the digital resolution of the oscilloscope depends on the vertical sensitivity and time base, so this will change as you rescale to fit your signal.

You will use the spreadsheet utility that you tried out in the pre-lab, to record your data and do initial checks by plotting it. Plot it early and make decisions about how much more data you need to capture the behaviour as a function of frequency.

Checkpoint 2

When you have taken around 10 data points, or when you feel you have sufficient data to capture the response curve, call a TA over to discuss your data.

Finish collecting any data you need to constrain the shape of the curve.

4 The LRC circuit in the Frequency Domain

4.1 Background

Next you will build the LRC series circuit shown in Figure 4.1 like you used in Experiment 03 to measure the transient oscillatory behaviour.

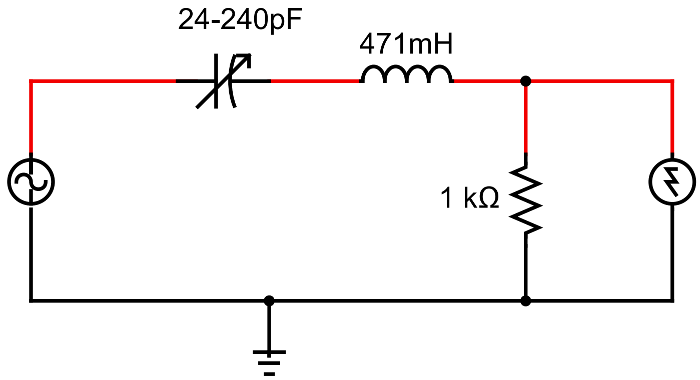

Figure 3: Circuit diagram for a series LRC circuit. The properties of the circuit can be measured by making a sudden change/decrease to an otherwise constant input voltage V_{in}^0. Alternatively one can drive the circuit with a sinusoidal input voltage $V_{in}(t)$. The voltage drop across the resistor can be used to directly monitor the time dependence of the current in the circuit.

In this case, the amplitude has the following behaviour as a function of input frequency:

$$V_r^0(\omega) = \frac{V_{in}^0}{\sqrt{1 + \left(\omega \frac{L}{R} - \frac{1}{\omega CR}\right)^2}} = \frac{V_{in}^0}{\sqrt{1 + \left(\frac{1}{\gamma\omega}\right)^2 (\omega^2 - \omega_o^2)^2}} \tag{5}$$

where $\omega_o = 1/\sqrt{LC}$ is the resonant angular frequency and $\gamma = R/2L$ determines the width of the resonance. One can rewrite this as a function of frequency $f = \omega/2\pi$:

$$V_r^0(f) = \frac{V_{in}^0}{\sqrt{1 + \left(\frac{2\pi}{\gamma f}\right)^2 (f^2 - f_o^2)^2}} \tag{6}$$

where $f_o = 1/(2\pi\sqrt{LC})$ is the resonant frequency.

4.2 Experiment

Rebuild the circuit that you used in Experiment #03, being careful to use the same three circuit components and try your best to set the capacitor to one of the values used in Lab 03 (this is tricky and you might not get it quite right, but do your best). Record all of your component values including your best attempt to match the capacitance.

Just as you did in the previous part of the lab, you will measure the frequency dependence of the amplitude at the output (voltage across the resistor) when driving with a sinusoidal input from the function generator. Once you get a signal, start by exploring the frequency response a little to help you plan out the data you will take.

Checkpoint 3

Once you have a signal and have qualitatively explored the frequency response, call a TA over to show your signal and discuss your plan to acquire the data you need.

Remember to keep plotting as as you take data so you can make decisions about where to focus your effort. You may also want to do some calculations of where you think the data will be most interesting. Your data does not need to be equally spaced! If nothing is changing between measurements, rethink your strategy.

5 Analysis of RC and LRC response

You will use the fitting code in this week's Jupyter notebook template to fit the amplitude response functions derived in Equations 4 and 6 to the data you have acquired. If you have time, start this in the lab as you may find you need/want more data!

Before you leave the lab!

Make sure:

- Your names, date and experiment number are recorded (TA check)
- Your component measurements (with uncertainty!) are all recorded
- You have notes recorded throughout, including what signals were on what channels of the oscilloscope, how your amplitude measurements were taken, how you determined uncertainties, etc (TA check)
- The data you have acquired for both the RC and LRC frequency responses shows the characteristic curve you expect (compare to pre-lab) (TA check)
- If time: begin fitting to confirm you have taken sufficient data in the most important regions
- Submit an HTML export of your notebook to Canvas (even if preliminary)
- Suggest backing up the .csv files created by the data_entry package

