Experiment #03
LRC Transients

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1 Objective

In this experiment you will measure the transient response of an LRC circuit after a sudden change in the applied voltage.

The LRC circuit is important for several reasons. A key one is that it is a canonical example of a damped harmonic oscillator; mathematically it behaves like many other oscillators such as a mass on a spring in classical mechanics, and many oscillator examples in quantum mechanics. The data you collect will illustrate the transient behaviour of a harmonic oscillator, which exhibits damped oscillations. This will give you an opportunity to further practice your electronics skills on a slightly more complicated circuit, and you will be fitting your data to a slightly more complicated model and explore how component parameters influence the transient oscillatory behaviour.

From a practical point of view, we will have you add tunable elements to the circuit. This will be important in your final project, because the LRC circuit can be used as a tunable passive filter - in a radio it is a way to tune between different broadcast stations. This general approach is adapted to an enormous number of applications.

As you did for Experiment 02, it is important to keep the components that you use. In the next two weeks you will be doing further measurements and analysis, making comparisons that rely on the components being the same. You also need to obtain enough datasets today to analyze later (MINIMUM 4 different capacitance values)

2 Learning Goals

After finishing the lab you will:

- know the theory behind LRC circuits
- take a further step in building multi-component circuits with tuneable elements
- know how to trigger an oscilloscope to capture transients
- know how to fit data with damped harmonic oscillations in Python

- begin to work with arrays

3 Introduction to LRC circuit

The properties of the RC circuit change significantly if you add in an inductor to make a series LRC circuit. As shown in the tutorial, the theory for the LRC circuit is closely related to that of a damped harmonic oscillator. A mass suspended on the end of a spring acts like a simple harmonic oscillator with a sinusoidal displacement around equilibrium. If the model is refined by adding a resistive force proportional to the velocity of the mass, the resulting motion is damped. A very similar theory can be used to model the current of a series LRC circuit (shown in Fig. 1). In the case of the LRC circuit a sudden drop in the applied voltage from the source (function generator) leads to oscillations in the current which are dampened by energy dissipation (loss) in the resistor.

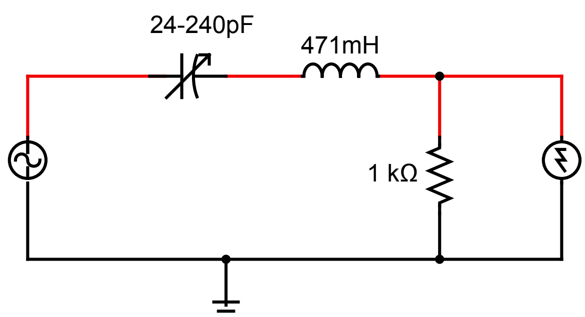

Figure 1: Circuit diagram for a series LRC circuit. The properties of the circuit can be measured by making a sudden change/decrease to an otherwise constant input voltage V_{in}^{0}. Alternatively one can drive the circuit with a sinusoidal input voltage $V_{in}(t)$. The voltage drop across the resistor can be used to directly monitor the time dependence of the current in the circuit.

We showed in tutorial 2 that the transient time response of the voltage across the resistor, $V_r(t)$, after the input voltage instantaneously switches from a constant (DC) value V_o to zero, is given by:

$$V_r(t) = RCV_o\omega_o \exp\left(-\gamma t/2\right) \cos(\omega_o t + \phi_o) \tag{1}$$

where $\gamma = R/2L$ is the so called the damping rate of the oscillation, $\omega_o = 1/\sqrt{LC}$ is the resonant *angular frequency* and $\phi_o = \pi/2$ is a phase shift between the current and the applied voltage. This expression is valid in the so called *under-damped* limit where $\omega_o >> \gamma/2$. Note the resonant frequency differs by a factor of 2π from ω_o: $f_o = \omega/2\pi = 1/(2\pi\sqrt{LC})$.

4 Measuring the Transient Response of the LRC circuit

Tip 1

Component measurements need to be made while not connected to applied voltages or other components. Either use the alligator clips directly on the component or use jumpers to probe *just* the one component in the breadboard.

The circuit diagram for the LRC series circuit is shown in Fig. 1. The circuit will use a variable capacitor, a 471 mH inductor and a 1 kOhm resistor, connected in series on the protoboard. Before assembling the circuit. Use your DMM to check the actual value of the resistor that you have been using. Don't forget to record the value and uncertainty in your notes. The lab also has an instrument that you can use to measure the inductance of the nominal 471 mH inductor, so measure that as well and record the value. The variable capacitor will be measured many times, since you will be changing it repeatedly, and remeasuring the transient. Start with it set in the middle of its range and record your starting value. Since the variable capacitors are a bit "touchy" you'll need to set the screw and then let go of it to measure. This might be easiest to do in the breadboard (just make sure nothing else is connected); this also has the advantage of accounting for any capacitance of the breadboard itself.

As we did in Experiment #02 on the RC circuit, the three circuit elements are connected in series, capacitor, inductor, and resistor. The complete loop of the circuit is made the same as last time as well. The ground side of the function generator is connected to the free end of the resistor this time as we'll be measuring the potential drop across the resistor. The voltage signal side of the function generator is connected to the free side of the capacitor, closing the loop. For your measurements, you attach the ground side of the oscilloscope to the ground side of the resistor and function generator. Then, to measure the voltage across the resistor, connect the other pin to the non-ground end of the resistor (so the two grounds are on the same side).

Now investigate the transient response. As in the RC circuit you can accomplish this by applying a low frequency square wave from the function generator so that the applied voltage switches up and down. Once again, it is convenient to use the square wave from the function generator for your triggering, so you should use a BNC Tee to connect the function generator to the oscilloscope, as well as to your circuit.

Discuss with your neighbours what the optimum square wave frequency is and explain your reasoning in your notebook. The idea is to pick a period that will allow sufficient time to see the transient, damped oscillations completely die down. Once you have decided that, you also need to pick a horizontal time scale on the oscilloscope that will allow you to see the damped oscillation in sufficient detail.

Give a detailed description of what you observe for the $V_r(t)$ immediately after the input voltage from the function generator jumps up or down.

Before saving your data, make sure that you have the vertical and horizontal scale and triggering adjusted so that you see the entire transient, from its start, to when it drops into the electrical noise. Also make sure you use the full vertical range of the oscilloscope.

Checkpoint 1

When you have captured a good transient call a TA and show them the signals on the scope as well as your circuit.

Save your data on a USB stick, upload to your Jupyter account, and don't forget to take notes and write down the filename in your Jupyter notebook. The data analysis guidance is embedded in your Jupyter notebook.

Two pieces of advice as you start in to fitting your data:

1. Estimating a few of the parameters for your guess either from the oscilloscope directly, or from the component values, may help you obtain good fits. With the number of parameters increasing, a poor initial guess may lead to a failure to fit the data.

2. You may need to revisit how you trimmed the data and/or estimated the uncertainty to obtain decent fits

Checkpoint 2

When you have a first set of data fit, call a TA over to discuss the fit quality and how you determined the region of interest, uncertainties, etc.

5 Adjusting the resonant frequency

Once you have tried analyzing your first set of data, take further measurements with different values of capacitance. Note that you should disconnect the generator when making the capacitance measurement, and make sure to measure capacitance as you take data as you won't be able to return the capacitor to the exact position again. So, you will be switching back and forth between measuring the capacitance, and acquiring a new transient at that capacitance value. Check the value both before and after saving the transient, to make sure it has not changed significantly.

As you take and analyze additional data, do you notice anything about the frequency of the oscillations? Is this consistent with your expectations?

Capture enough transients that you will eventually be able to produce a plot of resonant frequency versus C, over a wide range of C.

Before you leave the lab!

Make sure you:

- Put the resistor, inductor and variable capacitor that you used today in the box with your circuit board and keep it separated to use in the next labs.
- Had your oscilloscope signal checked (Checkpoint 1)

- Had your fit results checked (Checkpoint 2)
- Have all of your capacitance values measured and recorded (must be done for each dataset as you do them as you won't be able to return to the same position exactly!!)
- Have all of your datasets saved and filenames recorded in your notes to clearly associate them with the capacitance values
- Have sufficient sets of data to (eventually) produce a plot of the transient parameters as a function of capacitance (at least 4-5 different capacitance values, ideally more)