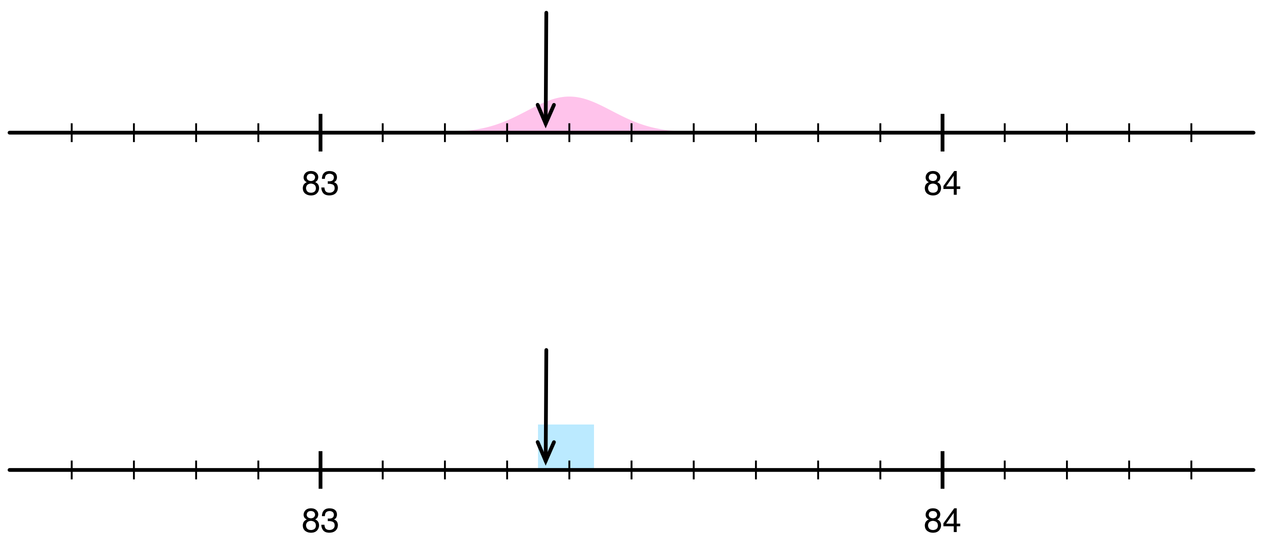
PHYS 219 – Week 2

Fits, noise and, RC and LRC transients

Dealing with Noise

Digital vs. Analog uncertainty

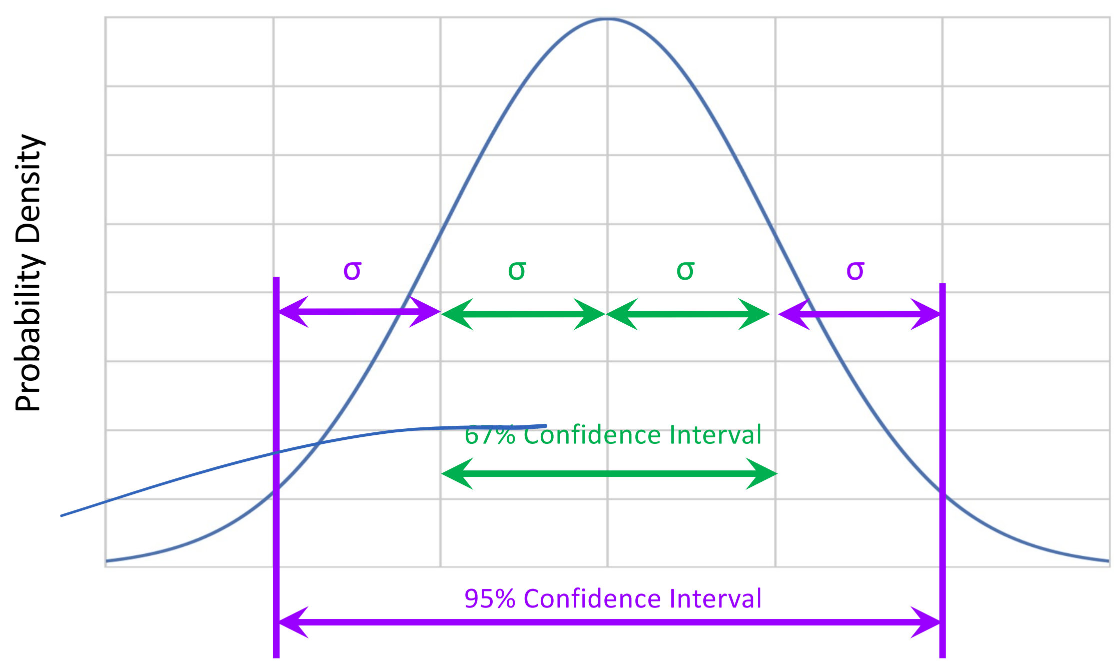


From: uncertainty basics

Uncertainty & statistics

Measurements are often assumed to follow a normal/Gaussian distribution.

i.e. If we make a measurement many, many times, we expect to find this distribution

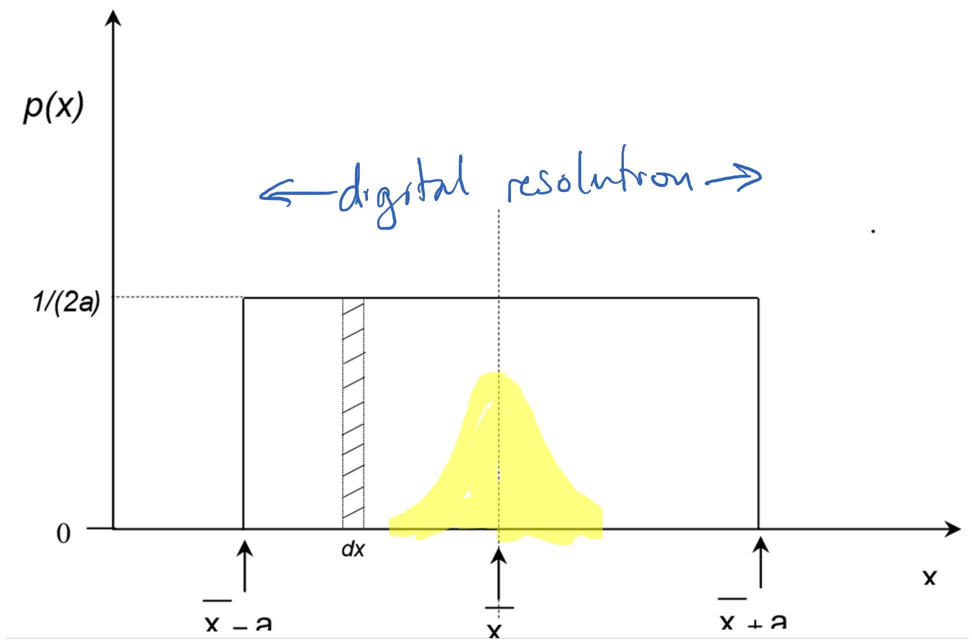


Rectangular/uniform PDF

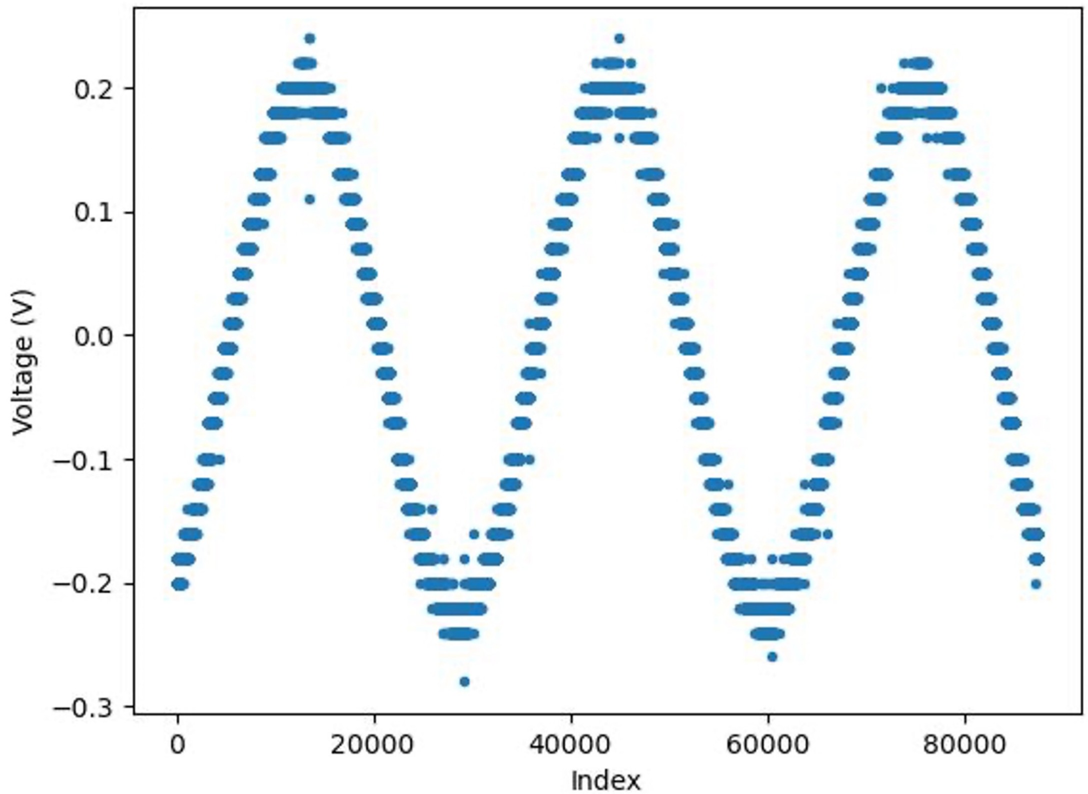
Used when a digital measurement device (electronic mass scale, digital multimeter, etc) limits the precision of the measurement.

The standard uncertainty for a rectangular/uniform PDF is

$$u[x] = \frac{a}{\sqrt{3}}$$

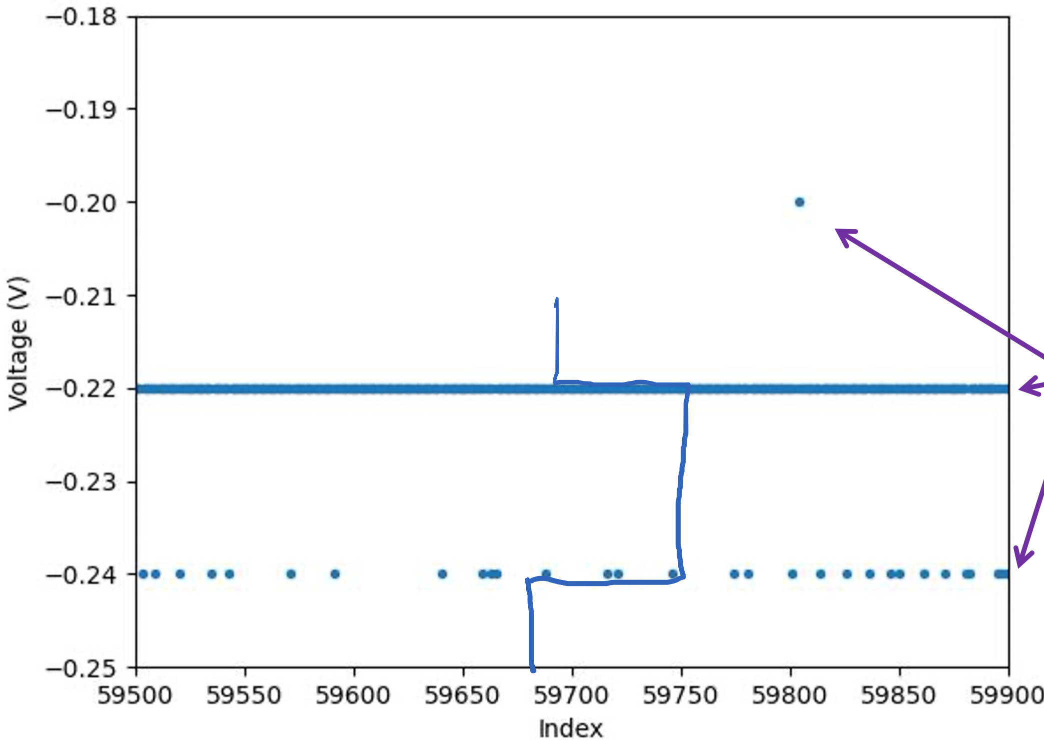


Low Resolution Example



Avoid this by using full screen of the oscilloscope!

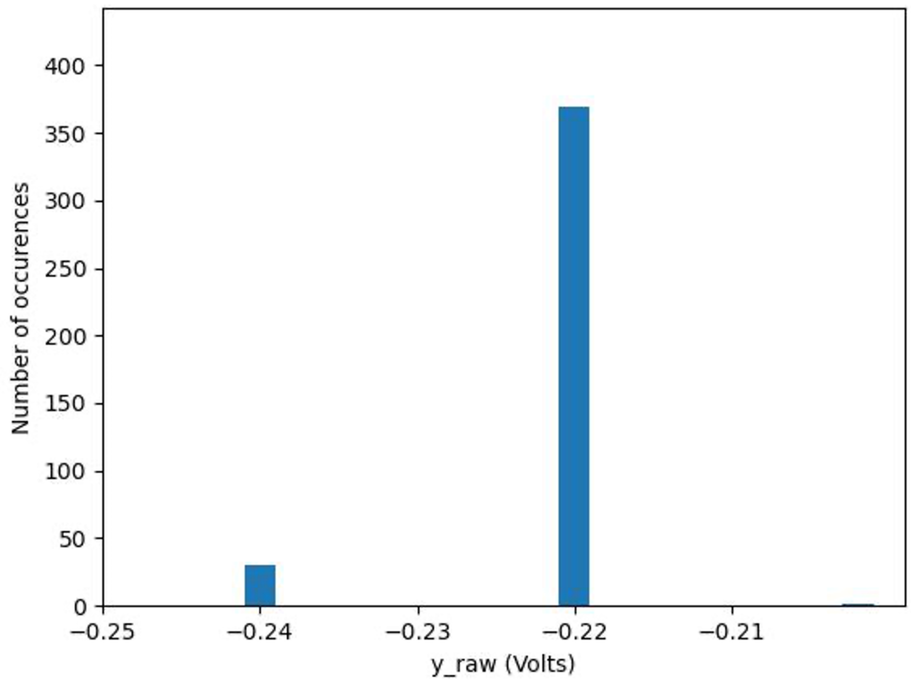
Low Resolution - zoomed in



”Bit noise”: Gaussian distributed noise filtered into discrete bins set by the digital resolution

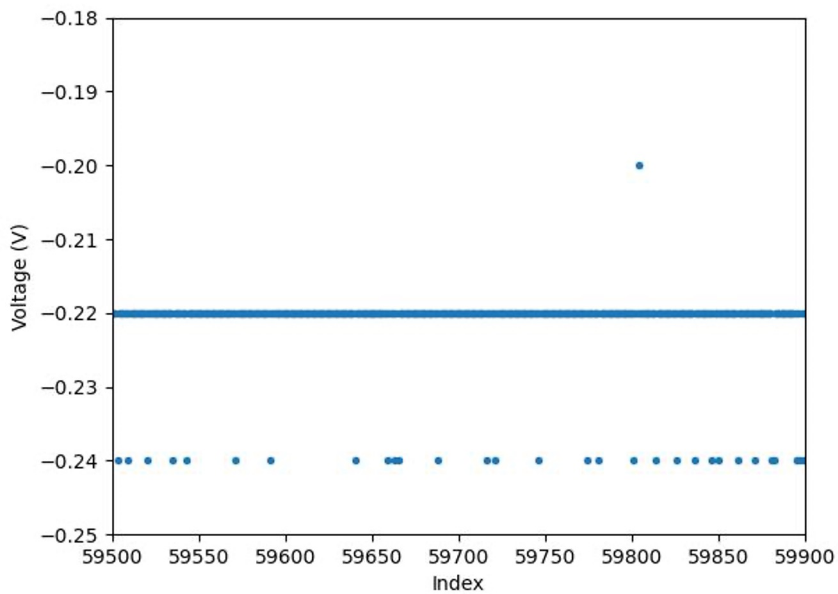
Effect of digital resolution - 0.02 Volt steps in the data

Low Resolution



One dominant number, so rounding is a problem

Low Resolution

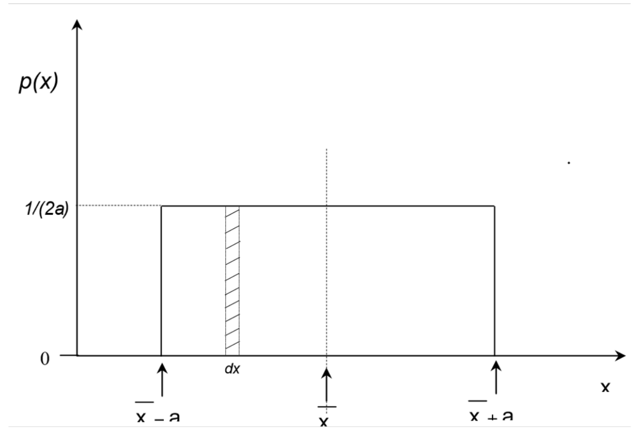


standard deviation
σ = 0.0052 Volts

digital resolution is
0.02 Volts

Rectangular/uniform PDF

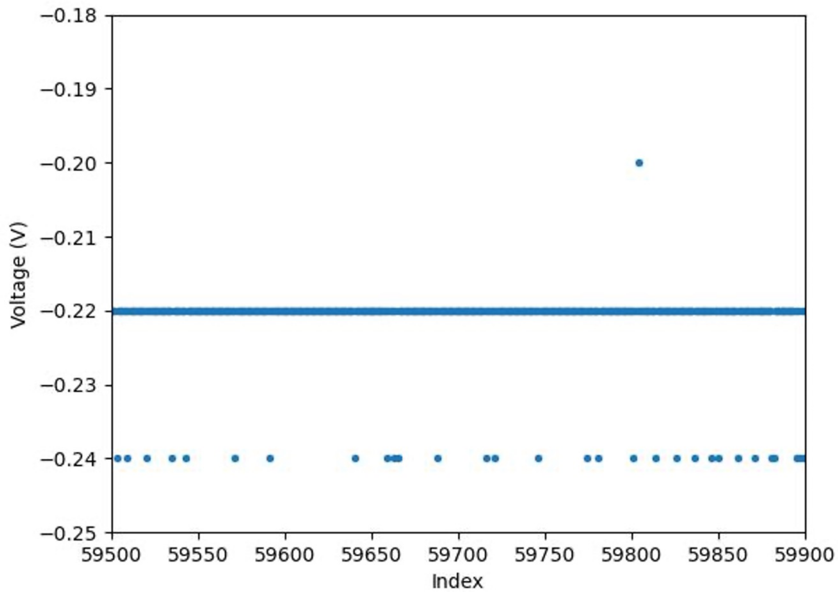
Used when a digital measurement device (electronic mass scale, digital multimeter, etc) limits the precision of the measurement.



The standard uncertainty for a rectangular/uniform PDF is

$$u[x] = \frac{a}{\sqrt{3}}$$

Low Resolution



For This Data

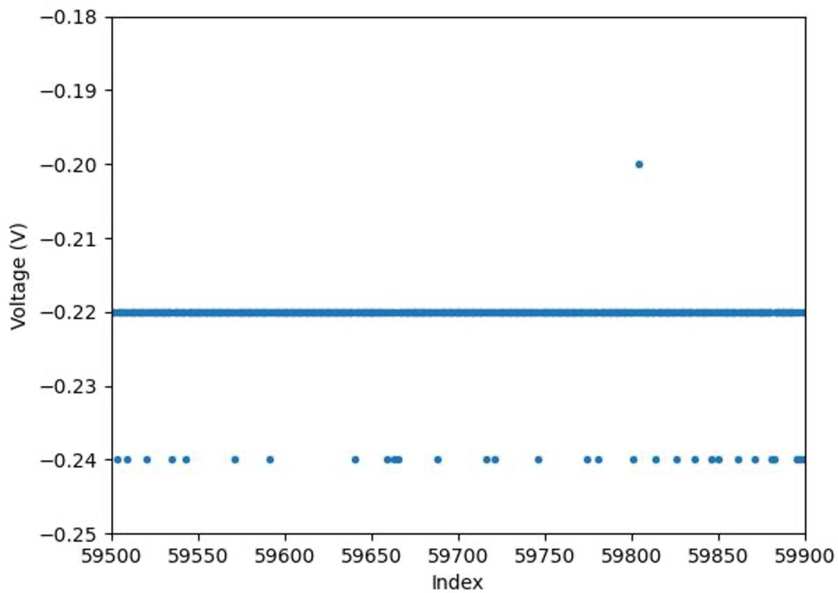
standard deviation
σ = 0.0052 Volts

digital resolution is
0.02 Volts

a = 0.01 volts

$a/\sqrt{3}$ = 0.0058

Low Resolution



For This Data

standard deviation
σ = 0.0052 Volts

digital resolution is
0.02 Volts

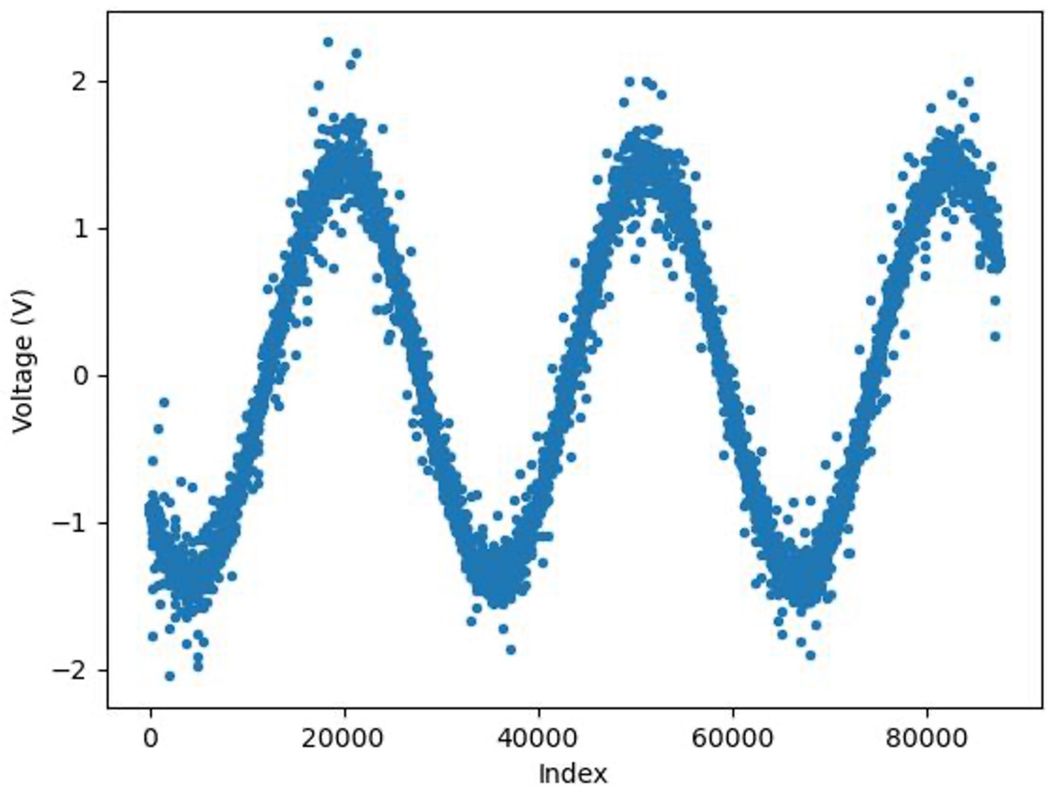
a = 0.01 volts

$a/\sqrt{3}$ = 0.0058

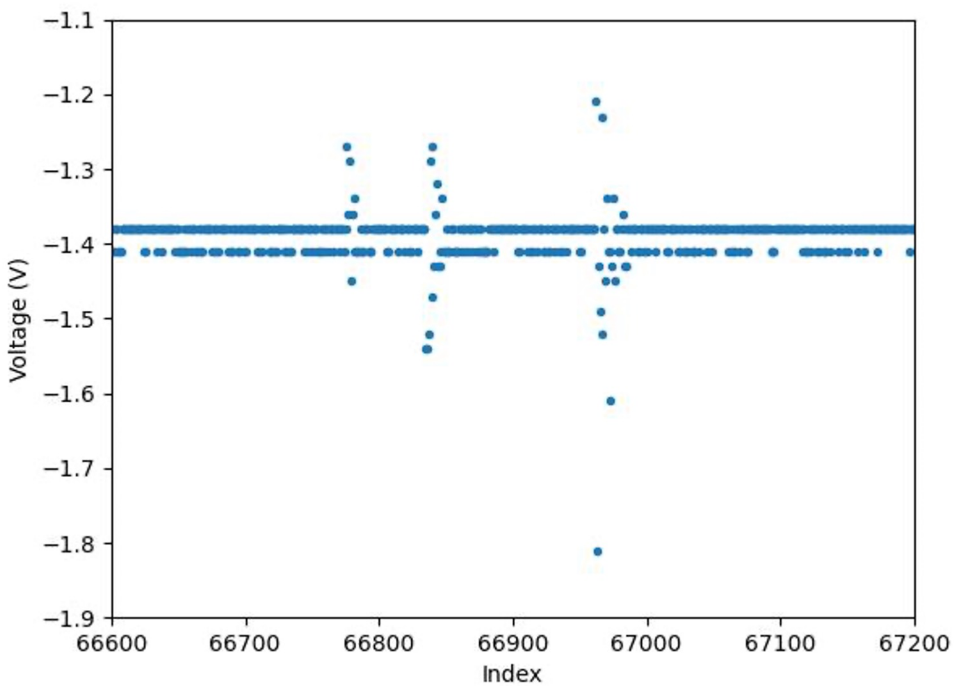
Total uncertainty estimate for each measured data point is

uncertainty = sqrt(0.0052^2 + 0.0058^2) = 0.0078 ± 0.008

Data dominated by noise

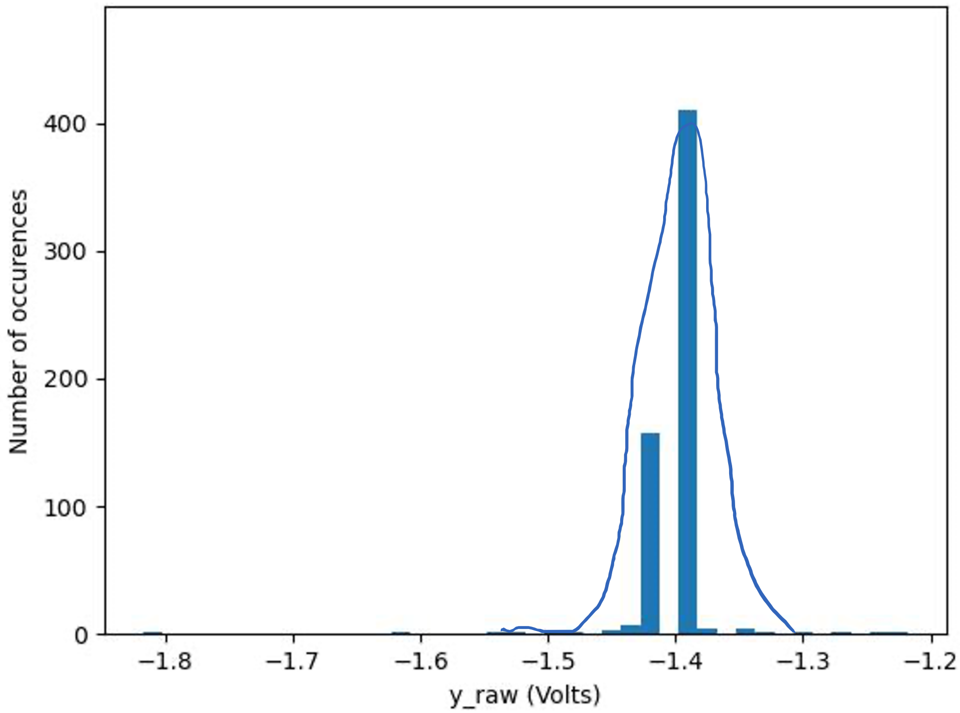


Less dominated by resolution/rounding



Note: noise is not always even in time - here we have sharp spikes of noise

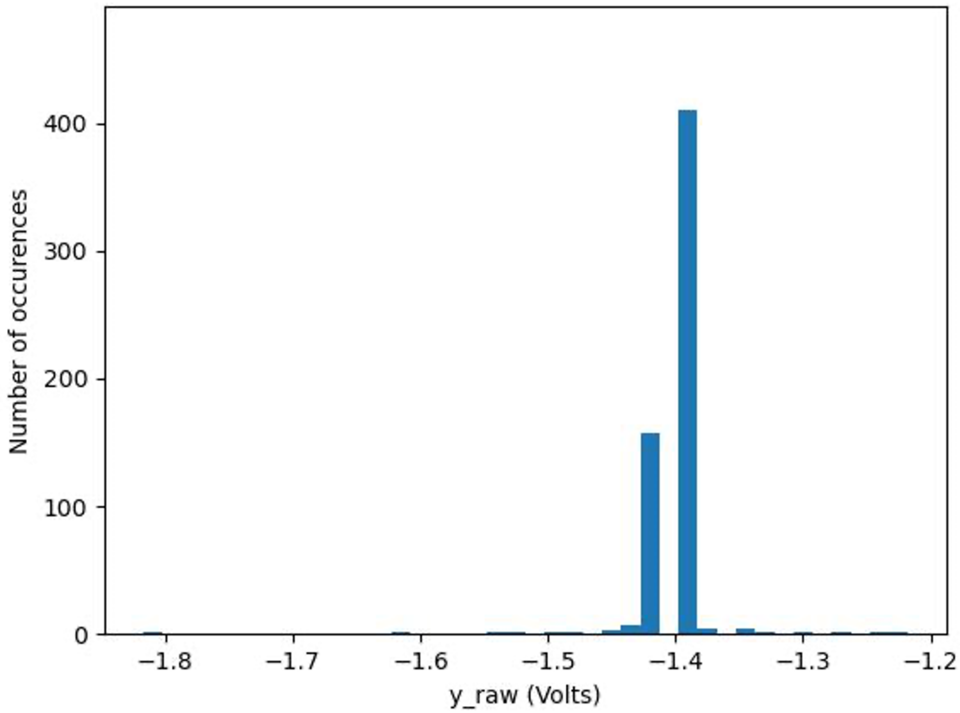
Higher Resolution Data



The distribution of the data is more apparent now when the noise is larger than the resolution.

Note that it is far from being Gaussian noise

Higher Resolution Data

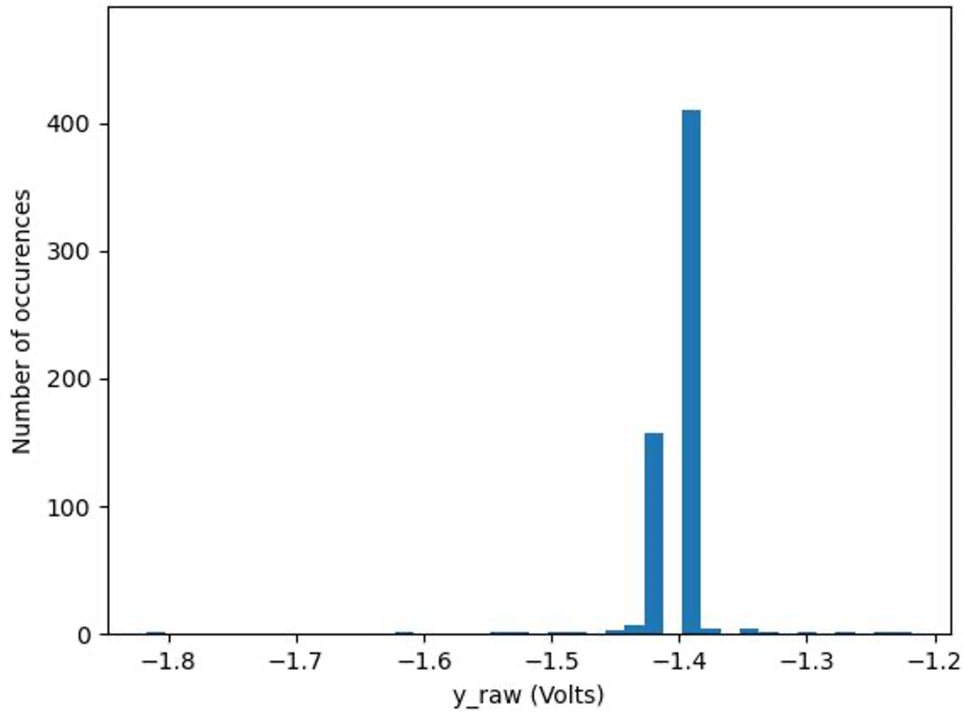


For This Data

standard deviation σ = 0.031 Volts

digital resolution is 0.03 Volts
a = 0.015 volts
$a/\sqrt{3}$ = 0.0086

Higher Resolution Data



For This Data

standard deviation σ = 0.031 Volts

digital resolution is 0.03 Volts
a = 0.015 volts
$a/\sqrt{3}$ = 0.0086

Total uncertainty estimate for each measured data point is
uncertainty = sqrt(0.031^2 + 0.0086^2) = 0.032
Note the larger source of uncertainty soon wins!

Fitting a model

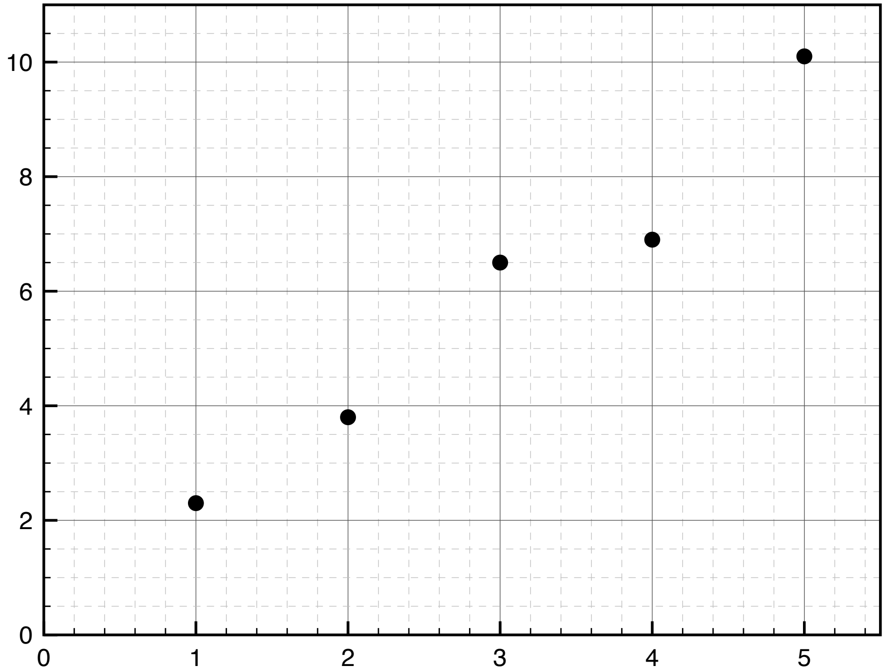
Fitting a model

We can try a linear model: m & b are adjustable

$$y = mx + b$$

We change them to minimize the distance between the data and the line:

$$\chi^2_R = \underbrace{\frac{1}{n-p}}_{degrees\ \ of\ \ freedom} \overbrace{\sum_{i=1}^{n} \frac{(O_i - E_i)^2}{\sigma_i^2}}^{weighted\ \ sum\ \ of\ \ squares}$$

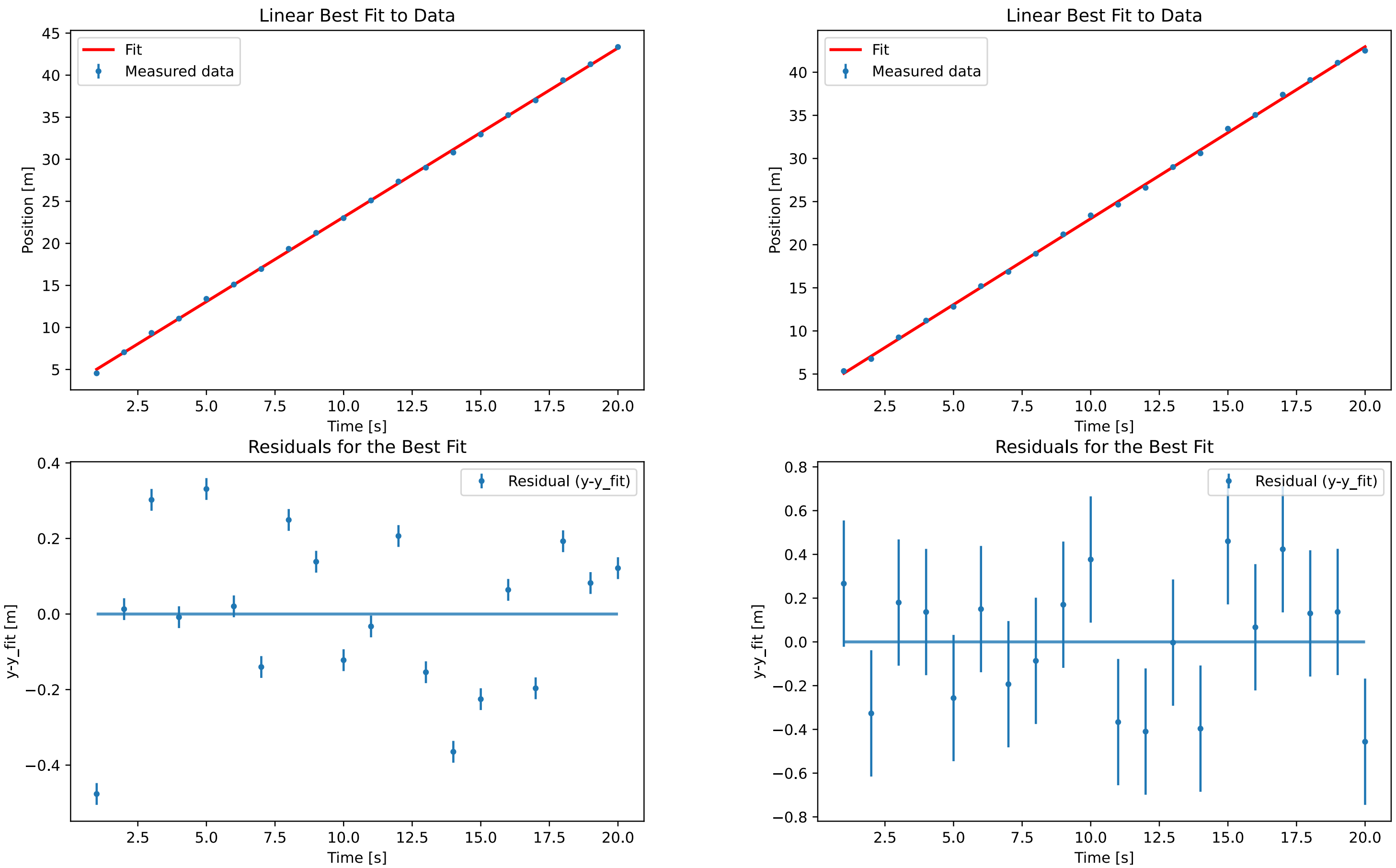


Interpreting χ^2

Like t'-score, gives a measure of how probable it is that the model and data differ (large number, more likely different)

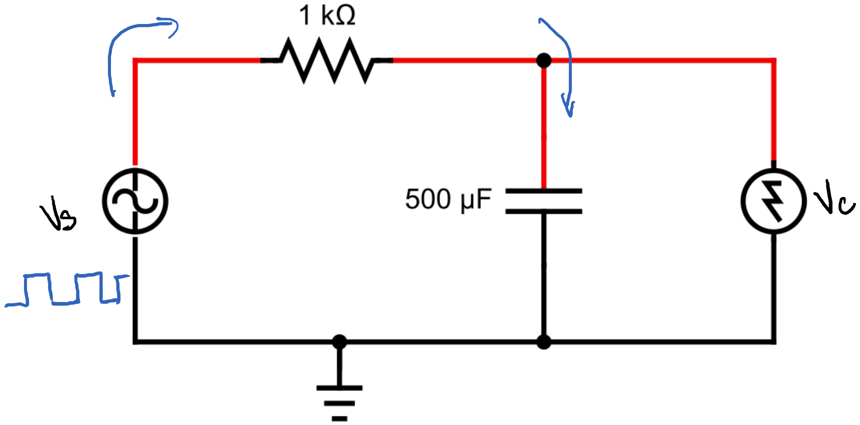
- $\boldsymbol{\chi^2}$ **<< 1**: the differences between the data and the model are small (indistinguishable), but this isn't a "good" fit; the uncertainties have been overestimated, or the model is "overfitting" with too many parameters that captures noise and spurious features
- $\boldsymbol{\chi^2}$ **~ 1**: the model and data cannot be distinguished within the uncertainty
- $\boldsymbol{\chi^2}$ **>> 1**: the differences between the data and the model are large given the level of uncertainty estimated; either the model poorly describes the data or the uncertainties are underestimated

Linear fit example: uncertainty

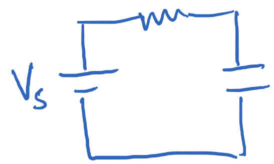


The RC circuit

This week's circuit: RC



CHARGING



@ $t=0$

$V_c = 0$

$$V_s = IR + \frac{q}{C}$$

$$V_s = \frac{dq}{dt}R + \frac{q}{C}$$

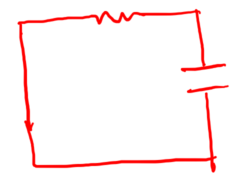
$$q - V_s C = -RC\frac{dq}{dt}$$

$$\int (q - V_s C)\,dt = \int -RC\,dq$$

$$q(t) = CV_0\left(1 - e^{-t/RC}\right)$$

$$V_c = \frac{q}{C}$$

DISCHARGING



$$0 = V_c + IR$$

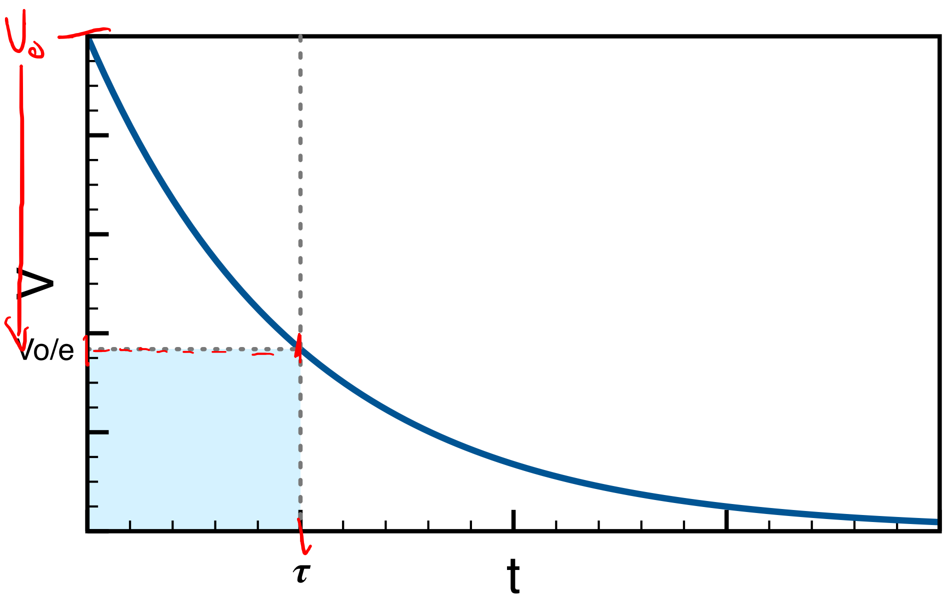
$$0 = \frac{q}{c} + \frac{dq}{dt}R$$

$$\int \frac{-dt}{RC} = \int \frac{dq}{q}$$

$$\frac{q}{Q_0} = e^{-t/RC}$$

$$V_c(t) = V_0 e^{-t/RC}$$

RC time-constant



$\tau = RC$

$V_c(t) = V_0 e^{-t/RC}$

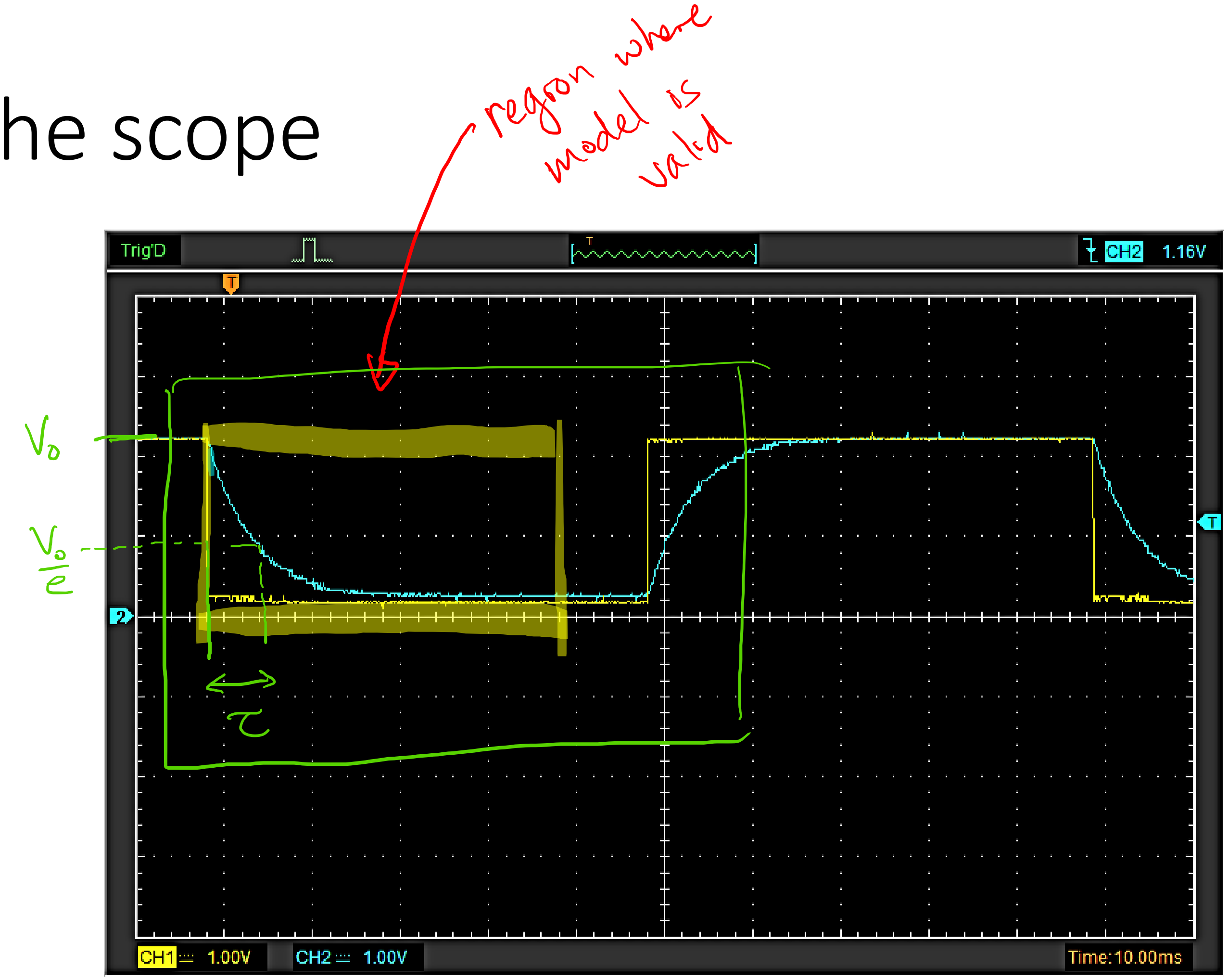
$\tau = RC$

$V_c(t) = V_0 e^{-t/\tau}$

$V_c(\tau) = V_0 e^{-\tau/\tau}$

$= \frac{V_0}{e}$

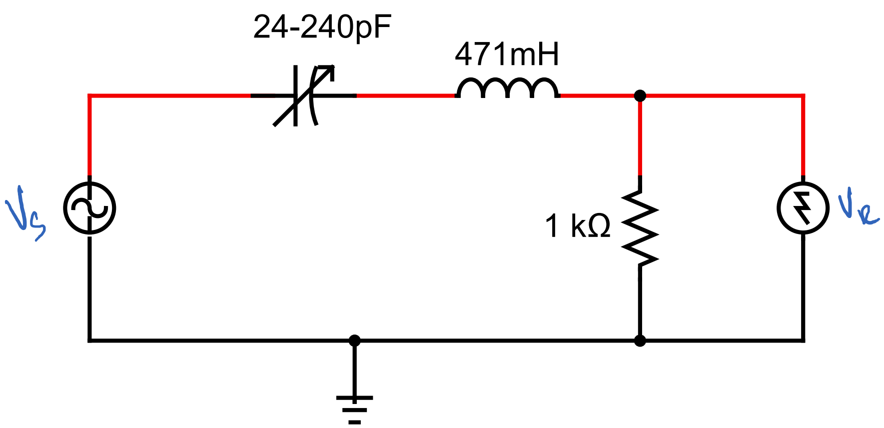
On the scope



region where model is valid
Trig'D
CH2 1.16V
V_o
$\frac{V_o}{e}$
τ
CH1 1.00V
CH2 1.00V
Time:10.00ms

The LRC circuit

Next week's circuit: LRC



$$V_s = V_c + V_L + V_R$$

$$= i\frac{q}{C} + iL\frac{dI}{dt} + IR$$

$$V_s = 1L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + 1\frac{q}{C}$$

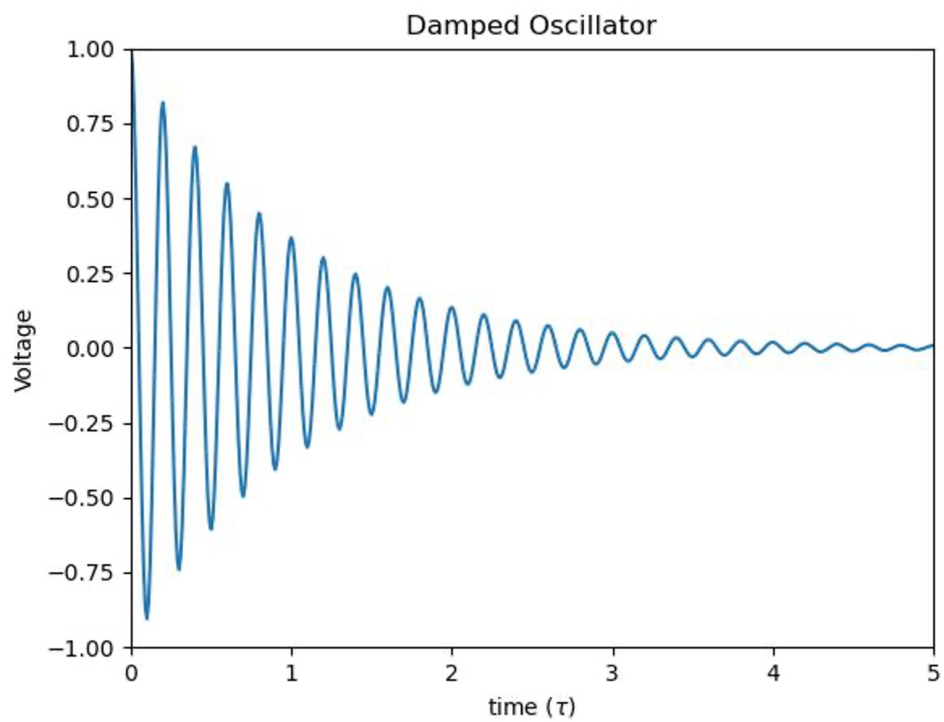
$$F_{drive} = m\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + kx$$

After any sudden change, the circuit exhibits damped oscillations - an example of a simple harmonic oscillator.

For example, suddenly dropping the applied voltage from some value V_0 to zero results in a voltage following

$$V_R(t) = V_0 e^{-t/\tau} \cos(\omega_0 t + \varphi)$$

LRC Circuit



Oscillates at resonant frequency ω_0 = 1/sqrt(LC).

For every step in time $\boldsymbol{\tau}$ = 2L/R, voltage falls 1/e .

Fit example

- Linear least squares fits have an analytic solution, but non-linear models do not
- Initial guesses need to be “close enough” to find a reasonable local minimum
- More parameters, and/or parameters that can trade off each other leads to less robust fits and larger uncertainty in parameters

DMM lies: resolution & accuracy

Resolution vs. accuracy

9.990 ± 0.005 kΩ

Resolution vs. accuracy

uncertainty

Resistance	99.99Ω	0.01Ω	±(1.0%+3)
	999.9Ω	0.1Ω	±(0.5%+3)
	9.999kΩ	0.001kΩ	
	99.99kΩ	0.01kΩ	
	999.9kΩ	0.1kΩ	
	9.999MΩ	0.001MΩ	±(1.5%+3)
	99.99MΩ	0.01MΩ	±(3.0%+5)

$9.99\,k\Omega \times 0.005 + 0.03\,k\Omega = 0.08\,k\Omega$

$9.99 \pm 0.08\,k\Omega$

Uncertainty language

Precision: how close the measurements are to each other (usually what we report as an uncertainty)

Accuracy: how close are the measurements to the true value (often difficult to determine a priori! May require convergence of many measurements done different ways)

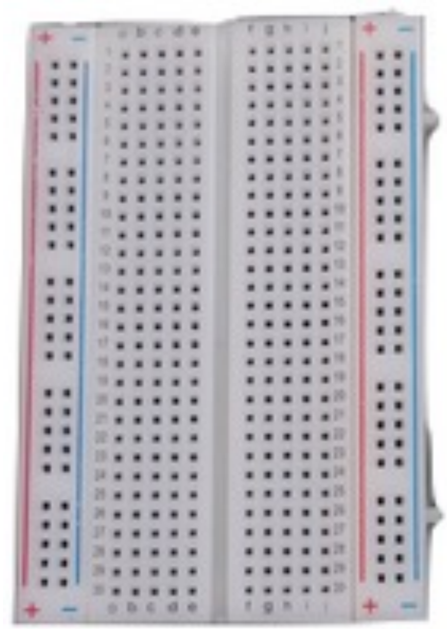
Tolerance: the spread in values for a manufactured specification – individual components will vary within this range (e.g. resistance of a resistor)

Breadboards

”Breadboards” or proto-boards are circuit boards designed for quick prototyping (as opposed to using a PCB and soldering).

The “legs” of components are pushed into the holes, which are connected underneath.

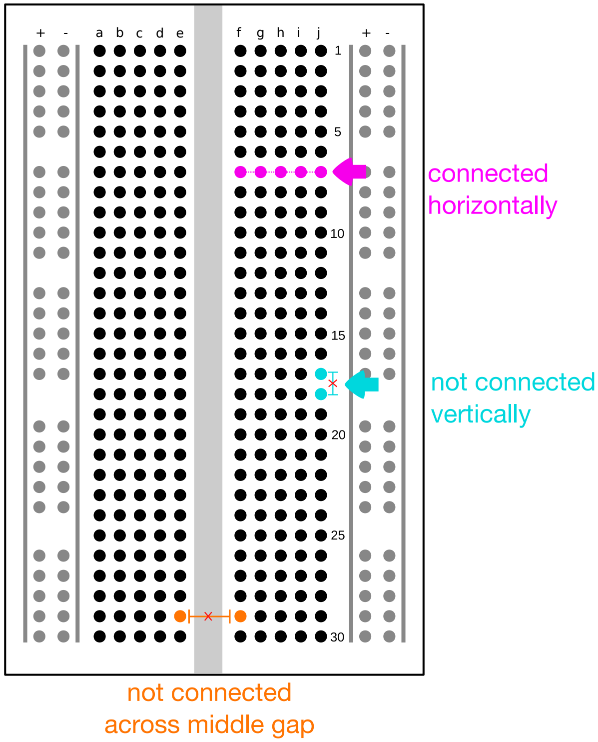
It is **VERY IMPORTANT** to know how the holes are connected or you will end up with a disconnected circuit, or worse a short circuit.



See: Resources/Circuit Basics/Breadboard-intro.pdf

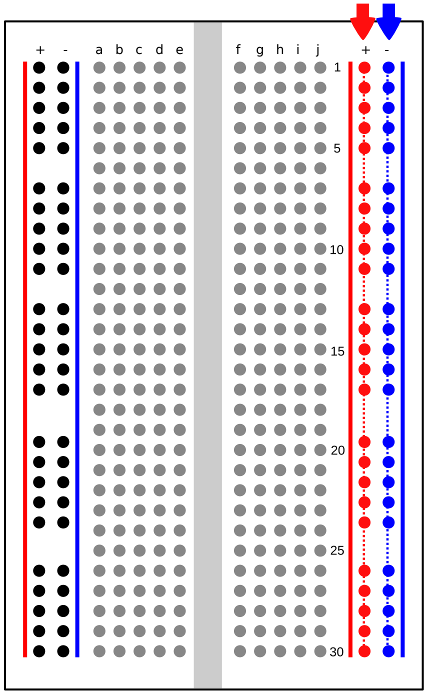
Breadboards

Middle: “terminal block”



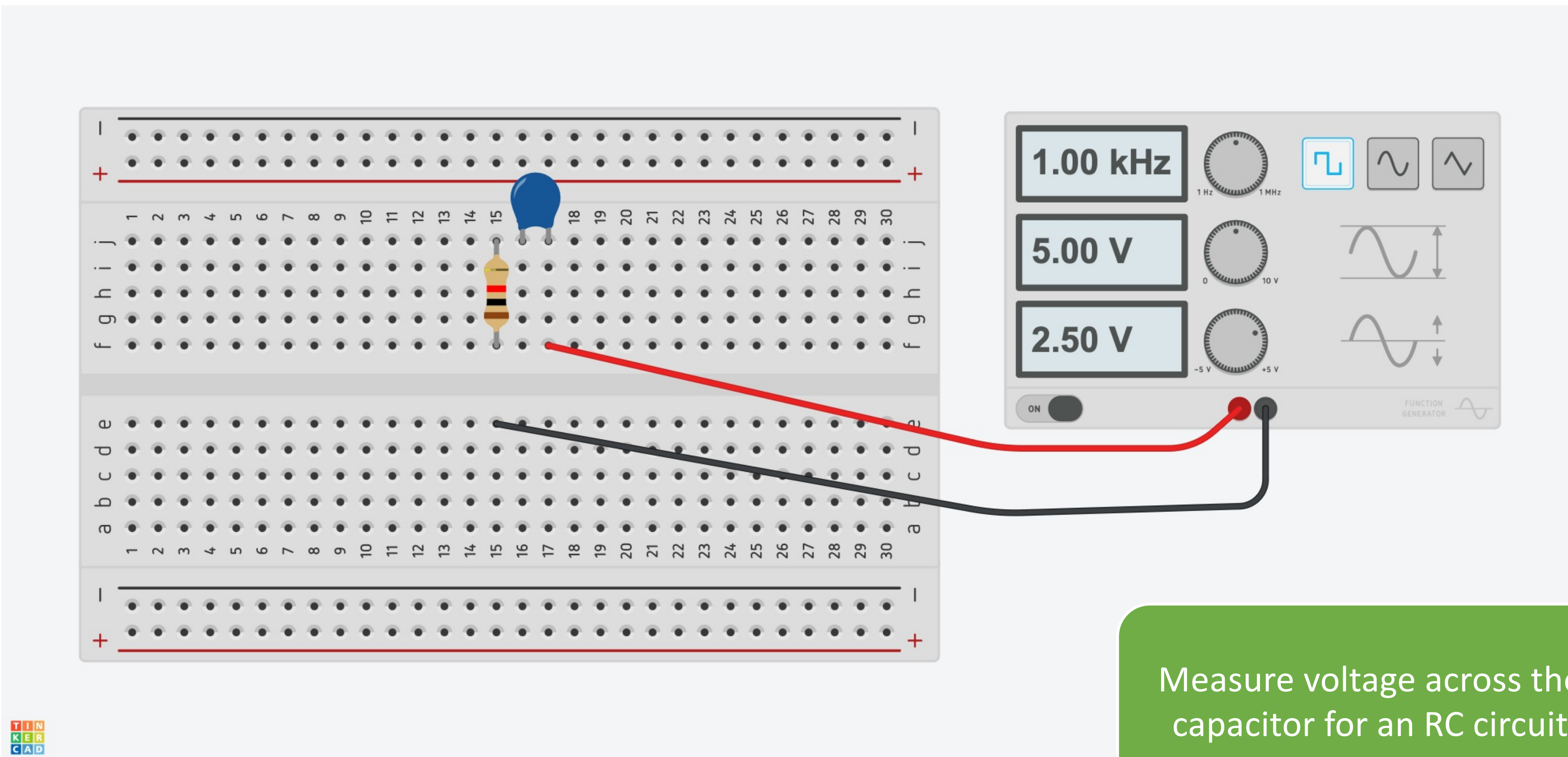
DC power + connected vertically

DC power – connected vertically



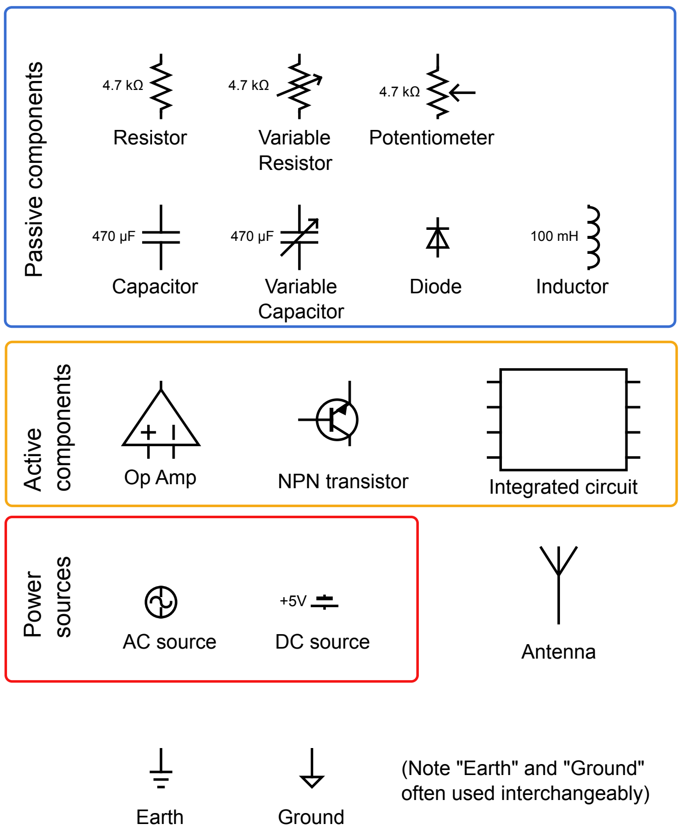
Edges: “Power rails”

Breadboard troubleshooting



Measure voltage across the capacitor for an RC circuit

Circuit diagrams & components



One of the things you'll be doing is reading schematic circuit diagrams: both to analyze/understand the expected circuit behaviour, and to turn it into a real circuit on a breadboard that you can measure.

Some of the symbols will be familiar, some a bit less, but they'll be introduced as we go.

SparkFun (again) has a good article on this.
https://learn.sparkfun.com/tutorials/how-to-read-a-schematic/all

Reminders

- Prelab due before lab (hard cutoff this week and going forward!)
- Make sure you review the final checklist and check-out with a TA (we want to ensure you have everything you need!)
- Read the instructions ***before*** lab
- Take notes during the lab