Assignment/Experiment #05
Testing RC and LRC Models

Sarah Burke

1 Objective

This week, you will not have a lab session (other than make-up labs if needed). Instead we have an assignment that wraps up some of your work on the RC and LRC circuits and extends the analysis you did in Experiments 02 and 03 to answer the questions outlined in the instructions. Towards the end, you will likely find it helpful to also consider your frequency domain data from Experiment 04.

You will not be provided a template notebook this week: set up your own notebook and include the usual information expected from the labs (name, date, purpose) and sufficiently detailed descriptions of what you are doing to follow along. Two fitting codes are provided for a 1-parameter linear model (slope, no intercept) and a 2-parameter linear model (slope, and intercept). You will need to use and adapt these fitting codes to produce fits and plots that address the analysis questions outlined in the instructions below. You may also want to borrow other pieces of code that you used in some of the labs so far.

To get started and make sure you have all packages loaded, you may want to set up a cell at the beginning of your notebook that loads all the packages we have been using:

```
import numpy as np
import array
import pandas as pd
import matplotlib.pyplot as plt
import data_entry2
```

2 Learning Goals

After finishing the lab you will:

- learn about systematic errors in electronic instruments
- have practice fitting straight lines to data

- have practice linearizing data that follows a power law
- have practice comparing the quality of fits to different models

3 Analysis of RC Time Constant Data

In Experiment 02 you gathered data for the RC circuit's transient. Each of your data sets was fit to an exponential decay, from which you could extract the time constant, and an estimate of the uncertainty in the time constant. In Pre-lab 04, you pulled this data together to produce a plot of time constant τ versus resistance. Theoretically, this should be a straight line governed by

$$\tau = RC. \tag{1}$$

Tip 1

If you entered your values as variables in Experiment 2, you can copy these lines of code to help you calculate the new uncertainty

Your analysis is focussed on fitting a straight line to your data. We have provided you with two specialized fitting codes for straight lines. One is for fits with two parameters (slope and intercept) and one is for fits with one parameter (just a slope, intercept is zero). What is special about these is that there is an exact, analytical solution to the linear fitting problem, which means you don't need to start with an initial guess. Your general code is built for multi-parameter fits, so it will also do a two-parameter linear fit, but it can not do the one-parameter fit to just a slope.

Before using these codes, you need to consider what to use for the "x-axis" and the "y-axis". The standard chi-squared fitting that we are doing assumes the uncertainty is in the "y" quantity. You probably have found that the uncertainty in your values of τ are very small, much much less than 1%. For your resistance values though, you need to worry about the *accuracy*, not just resolution of the digitization. This is where the *accuracy*, which includes uncertainties in the calibration[1] of the instrument, quoted in your DMM manual comes into play. For example, if you were measuring resistance on the scale that reads up to 9.999 $k\Omega$, that measurement has a precision of .001, and if rounding was the only issue, that would correspond to a digital rounding uncertainty of $\pm 0.0005/\sqrt{(3)}$. However, we want to understand the real magnitude of things in this analysis. So we need to consider the instrument's accuracy. The accuracy is much worse. On this scale it is 0.5% + 3digits. For example, if you measured a resistance of 7.535 $k\Omega$, this accuracy would be $\pm(7.535 \times 0.005 + 0.003)$ = 0.041 $k\Omega$. So your knowledge of the actual value of R is 7.535 $\pm$ 0.041 $k\Omega$. If you haven't used this yet, you need to put these larger uncertainties into your CSV file containing τ and resistance.

Tip 2

Consider using `data_entry2` to enter your data, but make sure to use the order of x, δx, y, δy you want to fit (or you will need to modify where the fitting code assigns variables to the columns).

[1] Instruments like DMMs have a measurement model built in: they take a measurement that is *related* to the quantity you are interested in and apply that relationship. While these can be theoretically derived, when accuracy matters, they are usually individually "calibrated" where measurements are taken of a set of standards or compared to another already calibrated and more precise instrument. These relationships necessarily carry and propagate uncertainties!

Now, you will need to rearrange the equation for your model, so that Resistance is on the "y-axis" and τ is on the "x-axis".

First try the one-parameter linear fit. You will need to alter the code to get the correct quantities on the axes.

$$R = slope * \tau \tag{2}$$

Some things to consider:

- Is it a good fit? Use residuals and chi-squared to interpret this.
- What does the slope correspond to?

Next, try a a two-parameter fit.

$$R = slope * \tau + intercept \tag{3}$$

- Does the intercept improve the fit
- What does the slope correspond to? Does the value make sense?
- What is your interpretation of the intercept? Think about the units, and your assumptions about the component values you have measured.

4 Analysis of the LRC Resonant Frequency Data

You also measured the transient response of the LRC circuit in Experiment 03. There you have transients measured at different values of the capacitance C. In the limit of small damping (lots of oscillations before the signal decays) the resonant frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{4}$$

Much like the previous analysis, you have measurements of the resonant frequency in your transient analysis, taken at different values of capacitance C. The uncertainty in your resonant frequency derived from the fits is probably relatively low, so you will likely decide that your main uncertainty problem is in the measurements of C with the DMM (again, consider the stated accuracy, not just the digital rounding uncertainty). You can also rearrange this equation in a way that makes a linear relationship:

$$\underbrace{1/C}_{y} = slope * \underbrace{(\omega_0^2)}_{x} \tag{5}$$

A plot of $1/C$ versus ω_0^2 should appear as a straight line. Remember that you will need to propagate uncertainties in order to determine the uncertainties in $1/C$ versus ω_0^2. This can be done efficiently in the Python script; when you define *x, y,* and *y_sigma*.

Try a 1-parameter linear fit first. Again, comment on the quality of the fit. What does the slope correspond to? Does your value make sense?

The expression above is actually an approximation; in the presence of some damping there is a small correction, which would require an intercept in the plot.

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}. \tag{6}$$

So, you should also try the two parameter linear model. Some things to consider when you do these analyses.

- Does a simple one-parameter linearized fit function work?
- Does the one-parameter in the fit match the expectation for your measured inductance?
- Do you see any sign in the residuals that would suggest you are seeing the correction due to damping?
- Does the use of an intercept improve the fit significantly? In either case can your reconcile your conclusion from the data with what you expect from the circuit you have measured (consider using component values as well as your time-domain and frequency-domain measurements to think about this)?

Tip 3

Remember that smaller χ^2 is not necessarily better: ideal is close to ≈ 1. For interpreting χ^2 see the fitting guide.

5 Putting it all together

Finally, as a conclusion to the work you have done in Weeks 02 to 05, compare the characteristic parameters (e.g. τ for RC, ω_0 and γ for LRC) you obtained for the RC and LRC circuit measured in different ways: calculated from components, from the time-domain measurements, and from your frequency domain measurements. If there are disagreements, consider whether there are systematic uncertainties (biases introduced by calibrations or other measurement models, assumptions about your equipment and how it works, etc). Based on what you have learned over the course of this set of experiments, what would you consider when designing circuits and choosing components for particular filtering applications? (Insights you gain here may be helpful when working on your AM Radio project!)