Assignment Report

a) The problem formulations specified mathematically

Variation 1: A driver can only take one order at a time

Parameters:

N: set of nodes (pickup locations, drop locations, vehicle start locations)

S: set of start locations

PD: pickup delivery pairs

V: set of vehicles

$\boldsymbol{d_{ijv}}$: distance between node i and node j for vehicle v

$\boldsymbol{c_{ijv}}$: cost of travel between node i and node j for vehicle v

$\boldsymbol{Q_v}$: Capacity of vehicle v

$\boldsymbol{q_i}$: demand at node i

$\boldsymbol{h_i}$**:** order count addition at node i

$\boldsymbol{s_v}$: starting node for vehicle v

$\boldsymbol{a_i}$, $\boldsymbol{b_i}$: Earliest and latest arrival times at node i

$\boldsymbol{m}$: penalty/second on violation of upper time window of locations

Decision Variables:

$\boldsymbol{x_{ijv}}$: Binary variable which takes value 1 if vehicle $\boldsymbol{v}$ goes from location $\boldsymbol{i}$ to location $\boldsymbol{j}$

$\boldsymbol{t_{iv}}$: Time at which vehicle v arrives at node i

Objective Function:

$$Minimize$$

$$\sum_{i,j\in N,v\in V} c_{ijv} * x_{ijv} \quad + \quad m * \sum_{i\in N,v\in V} Max(0, t_{iv} - b_i)$$

Constraints:

1. Vehicle leaves node that it enters.

$$\sum_{i\in N-S} x_{ijv} \; = \; \sum_{i\in N-S} x_{jiv} \quad \forall\, j \in (N-S), v \in V$$

2. Ensure that every node is entered only once

$$\sum_{j\in N-S,v\in V} x_{ijv} = 1 \;\; \forall\, i \in N - S$$

3. Vehicles must leave from their respective start locations

$$\sum_{j\in N} x_{s_v jv} \; = 1 \;\; \forall\, v \in V$$

4. Capacity of Vehicles must be respected at all times

$$\sum_{j\in N} q_i \cdot x_{ijv} \; \leq Q_v \quad \forall\, i \in N, v \in V$$

5. One driver can have one order at a time

$$\sum_{j\in N} h_i \cdot x_{ijv} \; \leq Q_v \quad \forall\, i \in N, v \in V$$

6. Lower bound of time window must be respected

$$a_i \leq t_{iv} \; \forall\, i \in N, v \in V$$

7. Pickup and Delivery constraints
    a. Pickup must be done before delivery

$$t_{pv} \leq t_{dv} \ \ \forall \ (p, d) \in PD$$

    b. Same vehicle must visit both pickup and delivery locations

$$\sum_{j \in N} x_{pjv} \ = \ \sum_{i \in N} x_{idv} \ \forall \ (p, d) \in PD \ , v \in V$$

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Variation 2: A driver can combine orders

Parameters:

N: set of nodes (pickup locations, drop locations, vehicle start locations)

S: set of start locations

PD: pickup delivery pairs

V: set of vehicles

$\boldsymbol{d_{ijv}}$: distance between node i and node j for vehicle v

$\boldsymbol{c_{ijv}}$: cost of travel between node i and node j for vehicle v

$\boldsymbol{Q_v}$: Capacity of vehicle v

$\boldsymbol{q_i}$: demand at node i

$\boldsymbol{h_i}$**:** order count addition at node i

$\boldsymbol{s_v}$: starting node for vehicle v

$\boldsymbol{a_i}$, $\boldsymbol{b_i}$: Earliest and latest arrival times at node i

$\boldsymbol{m}$: penalty/second on violation of upper time window of locations

Decision Variables:

$\boldsymbol{x_{ijv}}$: Binary variable which takes value 1 if vehicle $\boldsymbol{v}$ goes from location $\boldsymbol{i}$ to location $\boldsymbol{j}$

$\boldsymbol{t_{iv}}$: Time at which vehicle v arrives at node i

Objective Function:

$Minimize$

$$\sum_{i,j \in N, v \in V} c_{ijv} * x_{ijv} \quad + \quad m * \sum_{i \in N, v \in V} Max(0, t_{iv} - b_i)$$

Constraints:

1. Vehicle leaves node that it enters.

$$\sum_{i \in N-S} x_{ijv} \ = \sum_{i \in N-S} x_{jiv} \quad \forall\, j \in (N - S), v \in V$$

2. Ensure that every node is entered only once

$$\sum_{j \in N-S, v \in V} x_{ijv} = 1 \ \ \forall\, i \in N - S$$

3. Vehicles must leave from their respective start locations

$$\sum_{j \in N} x_{s_v j v} \ = 1 \ \ \forall\, v \in V$$

4. Capacity of Vehicles must be respected at all times

$$\sum_{j \in N} q_i \cdot x_{ijv} \leq Q_v \quad \forall\, i \in N, v \in V$$

5. Lower bound of time window must be respected

$$a_i \leq t_{iv} \ \forall\, i \in N, v \in V$$

6. Pickup and Delivery constraints
    a. Pickup must be done before delivery

$$t_{pv} \leq t_{dv} \ \ \forall\, (p, d) \in PD$$

    b. Same vehicle must visit both pickup and delivery locations

$$\sum_{j \in N} x_{pjv} \ = \ \sum_{i \in N} x_{idv} \ \forall\, (p, d) \in PD\,, v \in V$$