National University of Computer and Emerging Sciences

Karachi Campus

Probability and Statistics (MT2005)

Date: April 4th , 2024

Course Instructor(s)

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Sessional-II Exam

Total Time: 1 Hours

Total Marks: 30

Total Questions: 02

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Roll No Section Student Signature

Attempt all the questions.

CLO 1: Describe the fundamental concepts in probability and statistics

Q1: [10 marks]

(a) Strands of copper wire from a manufacture are analyzed for strength and conductivity. The result of 100 strands are as follows:

	Strength	
	High	Low
High conductivity	74	8
low conductivity	15	3

A strand is randomly selected,

i. What is the probability of selected strand having high conductivity or low strength? [2 Marks]

ii. If a strand has low conductivity, what is the probability that its strength is high? [2 Marks]

iii. Are high strength, low conductivity and high conductivity events mutually exclusive? [1 Marks]

Solution:

	Strength		
	High	Low	Total
High conductivity	74	8	82
low conductivity	15	3	18
Total	89	11	100

i. $P(HC\ or\ LS) = P(HC) + P(LS) - P(HC\ and\ LS) = \frac{82}{100} + \frac{11}{100} - \frac{8}{100} = 0.85$

ii. $P(HS|LC) = \frac{P(HS\ and\ LC)}{P(LC)} = \frac{\frac{15}{100}}{\frac{18}{100}} = \frac{15}{18} = \frac{5}{6} = 0.8333$

iii. No

(b) An insurance company classifies drivers as low-risk, medium-risk, and high risk. of those insured, 60% are low-risk, 30% are medium-risk, and 10% are high-risk. After a study, the company finds that during a 1-year period, 1% of the low-risk drivers had an accident, 5% of the medium-risk drivers had an accident, and 9% of the high-risk drivers had an accident. If a driver is selected at random, find the probability that the driver will have had an accident during the year? [3 Marks]

Solution:

$$P(A) = P(LR)P(A|LR) + P(MR)P(A|MR) + P(HR)P(A|HR)$$
$$= 0.6 * 0.01 + 0.3 * 0.05 + 0.1 * 0.09 = 0.006 + 0.015 + 0.009 = 0.03$$

CLO 2: Analyze the data and produce probabilistic models for different problems

Q2: [22 marks]

(a) On a laboratory assignment, if the equipment is working, the density function of the observed outcome, X, is

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

i. Prove that $f(x)$ is a valid density function [2 Marks]
ii. Calculate P(X ≤ 1/3) [2 Marks]
iii. Given that X ≥ 0.5, what is the probability that X will be less than 0.75**?** [2 Marks]

Solution:

i. $\int_0^1 f(x)dx = \int_0^1 2(1-x)dx = 2\left(x - \frac{x^2}{2}\right)(limit\ x{:}\ 0\ to\ 1) = 2\left(1 - \frac{1}{2}\right) = 1\ (proved)$

ii. $P\left(X \le \frac{1}{3}\right) = \int_0^{1/3} 2(1-x)dx = \frac{5}{9}$

iii. $P(X < 0.75 | X \ge 0.5) = \frac{P(0.5 \le X < 0.75)}{P(X \ge 0.5)} = \frac{\int_{0.5}^{0.75} f(x)dx}{\int_{0.5}^{1} f(x)dx} = \frac{\int_{0.5}^{0.75} 2(1-x)dx}{\int_{0.5}^{1} 2(1-x)dx} = \frac{3}{4}$

(b) Two cards are drawn without replacement from the 12 face cards (jacks, queens and kings) of an ordinary deck of 52 playing cards. Find

i. Joint probability distribution of number of kings (X) and number of jacks (Y) selected; [5 Marks]
ii. $P[(X, Y) \in A]$, where A is the region given by $\{(x, y) \mid x + y \ge 1\}$ [2 Marks]
iii. Find the marginal distributions of x and y [2 Marks]
iv. Compute $P(y \ge 1 | x = 1)$ [2 Marks]
v. Compute coefficient of correlation of x and y [4 Marks]
vi. Are x and y independent? [1 Marks]

Solution:

i.

x/y	0	1	2	Total
0	1/11	8/33	1/11	14/33
1	8/33	8/33	0	16/33
2	1/11	0	0	1/11
Total	14/33	16/33	1/11	1

ii. $P[(X,Y) \in A], where\ A\ is\ the\ region\ given\ by\ \{(x,y) \dashv |\ x+y \geq 1\}$

$$= f(1,0) + f(0,1) + f(1,1) = \frac{8}{33} + \frac{8}{33} + \frac{8}{33} = \frac{24}{33} = 0.727273$$

iii.

x	0	1	2
$g(x)$	$\frac{14}{33}$	$\frac{16}{33}$	$\frac{1}{11}$

y	0	1	2
$h(y)$	$\frac{14}{33}$	$\frac{16}{33}$	$\frac{1}{11}$

iv. $P(Y \geq 1 | X = 1) = \frac{f(1,1)+f(1,2)}{g(1)} = \frac{\frac{8}{33}+0}{\frac{16}{33}} = \frac{1}{2} = 0.5$

v.

$$E(X) = \sum xg(x) = 0 * \frac{14}{33} + 1 * \frac{16}{33} + 2 * \frac{1}{11} = \frac{2}{3}$$

$$E(X^2) = \sum x^2 g(x) = 0^2 * \frac{14}{33} + 1^2 * \frac{16}{33} + 2^2 * \frac{1}{11} = \frac{28}{33}$$

$$E(Y) = \sum yh(y) = 0 * \frac{14}{33} + 1 * \frac{16}{33} + 2 * \frac{1}{11} = \frac{2}{3}$$

$$E(Y^2) = \sum y^2 h(y) = 0^2 * \frac{14}{33} + 1^2 * \frac{16}{33} + 2^2 * \frac{1}{11} = \frac{28}{33}$$

$$E(XY) = \sum_x \sum_y xyf(x,y) = 1 * 1f(1,1) = \frac{8}{33}$$

$$\sigma_x = \sqrt{E(X^2) - \left(E(X)\right)^2} = \sqrt{\frac{28}{33} - \frac{4}{9}} = 0.63564$$

$$\sigma_y = 0.63564$$

$$\sigma_{xy} = E(XY) - E(X)E(Y) = \frac{8}{33} - \frac{2}{3} * \frac{2}{3} = -0.20202$$

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = -\frac{0.20202}{0.63564^2} = -0.5$$

vi. No because $\rho_{xy} \neq 0$