System of linear equation

Every system of linear equation has zero, one or infinitely many solution

a) One solution

$x - y = 1$

$2x + y = 6$

$+ \quad -$

$3x = 7$

$\boxed{x = \frac{7}{3}}$

$x - y = 1$

$\frac{7}{3} - y = 1$

$y = \frac{7}{3} - 1$

$\boxed{y = \frac{4}{3}}$

b) Zero solution or no solution

$x + y = 4 \longrightarrow (1)$

$3x + 3y = 6 \longrightarrow (2)$

Multiply (1) by 3 and then subtract with (2)

$3x + 3y = 12$

$3x + 3y = 6$

$- \quad -$

$0 \neq -6$

c) Infinitely many solution

$4x - 2y = 1 \longrightarrow (1)$

$16x - 8y = 4 \longrightarrow (2)$

Multiply (1) by 4 — (2)

$16x - 8y = 4$

$16x - 8y = 4$

$- \quad -$

$4 = 4 \quad 0$

$4x = 1 + 2y$

$x = 1/4 + y/2$

Let $y = t$; t is arbitrary values

$\therefore x = \frac{1}{4} + \frac{t}{2}$ Parametric

$x - y + 2z = 5$
$2x - 2y + 4z = 10$
$3x - 3y + 6z = 15$

Solve the system

$x - y + 2z = 5$
$2x - 2y + 4z = 10$
$3x - 3y + 6z = 15$

$x = 5 + y - 2z$
Let $y = t$ $z = u$

$x = 5 + t - 2u$

Augmented Matrix

$$\begin{bmatrix} 1 & -1 & 2 & 5 \\ 2 & -2 & 4 & 10 \\ 3 & -3 & 6 & 15 \end{bmatrix}$$

Simple Matrix

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}$$

$x + y + 2z = 9$
$2x + 4y - 3z = 1$
$3x + 6y - 5z = 0$

Q. 1 (a) Solve

a) $3x - 2y = 4 \rightarrow (1)$
$6x - 4y = 9 \rightarrow (2)$

$(1) \times 2 - (2)$

$6x - 4y = 4$
$6x - 4y = 9$
$- \quad + \quad -$
$4 = -9$
No solution

b) $2x - 4y = 1$
$4x - 8y = 2$

$4x - 8y = 2$
$4x - 8y = 2$
$- \quad + \quad -$
$2 - 2 = 0$
Infinite many Solution

$x = \frac{1}{2} + 2t$

c) $x - 2y = 0$
$x - 4y = 8$

$2x - 4y = 0$
$x - 4y = 8$
$- \quad + \quad -$
$x = -8$

$-8 - 2y = 0$
$2y = -8$
$y = -4$

$(-8, -4)$

Q.) Find the parametric equation

$$3x_1 - 5x_2 + 4x_3 = 7$$

Solve the system

$$\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

Echelon form

$$x - 3y + 4z = 7$$
$$y + 2z = 2$$
$$z = 5$$

$$y + 2(5) = 2$$
$$\therefore y = -8$$

$$\Rightarrow x - 3(-8) + 4(5) = 7$$
$$\Rightarrow x + 24 + 20 = 7$$
$$\Rightarrow x + 44 = 7$$
$$\therefore x = -37$$

ECHELON FORM

$$\begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Reduced Echelon form

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Augmented Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{bmatrix}$$

Solve

Q. 2) $\begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$

$x + 8z - 5t = 6$
$y + 4z - 9t = 3$
$z + t = 2$

Infinitely many solutions

$z = 2 - t \Rightarrow$

$y + 4(2-t) - 9t = 3$
$y + 8 - 4t - 9t = 3$
$y + 8 - 13t = 3$
$y - 13t = 3 - 8$
$y - 13t = -5 \Rightarrow y = 13t - 5$

$x + 8(2-t) - 5t = 6$
$x + 16 - 8t - 5t = 6$
$x - 13t = 6 - 16$
$x - 13t = -10 \Rightarrow x = 13t + 10$

$y - 13t = -5$
$x - 13t = -10$
$- \quad + \quad +$
$\therefore y - x = 5$

$\left\{ \begin{array}{l} x: 13t + 10 \\ y: 13t - 5 \\ z: 2 - t \\ t \end{array} \right\}$

Q.1 Solve the augmented matrix.

$$\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

Echelon form

$x - 3y + 4z = 7 \longrightarrow (1)$

$y + 2z = 2 \longrightarrow (2)$

$z = 5$

$y + 2(5) = 2$

$y = 2 - 10$

$y = -8$

$\Rightarrow x - 3(-8) + 4(5) = 7$

$\Rightarrow x + 24 + 20 = 7$

$\Rightarrow x = 7 - 24 - 20$

$\therefore x = -36$

b $$\begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

Q.2) Solve the system by Gaussian elimination.

$$x_1 + x_2 + 2x_3 = 8$$
$$-x_1 - 2x_2 + 3x_3 = 1$$
$$3x_1 - 7x_2 + 4x_3 = 10$$

Augmented matrix

$$= \begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 3 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{bmatrix} \quad R_1 + R_2$$

$$= \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix} \quad -3R_1 + R_2$$

Multiply R_2 by -1

$$= \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & 104 \end{bmatrix}$$

Multiply R_3 by $-1/52$

$$= \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$\cancel{z = 2}$

$$x + y + 2z = 8$$
$$y - 5z = -9$$
$$z = 2$$

Reduced Row Echelon

$R_3 \times 5 + R_2$

$$= \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_3 \times -2 + R_1$

$$= \begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_2 \times -1 + R_1$

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Q.3) $2x_1 + 2x_2 + 2x_3 = 0$
$-2x_1 + 5x_2 + 2x_3 = 1$
$8x_1 + x_2 + 4x_3 = -1$

The augmented matrix for the systems.

$$= \begin{bmatrix} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix}$$

Multiplying by $1/2$ to R_1

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix}$$

Adding $+2$ times the R_1 to R_2 & $-8R_1$ to R_3

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{bmatrix}$$

Multiplying R_2 by $1/7$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & -7 & -4 & -1 \end{bmatrix}$$

Adding $+7R_2$ to R_3

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 3 & 6 \end{bmatrix}$$

Dividing by $1/3$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1/3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$x + y + z = 0$
$y = 1/3$
$z = 2$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x + y + z = 0$
$y + \frac{4}{7} z = \frac{1}{7}$

$z = t$

$y = \frac{1}{7} - \frac{4}{7} t$

$x = -\frac{1}{7} - \frac{4}{7} t - t$

Q.1) Solve by Gauss - Jordan E Method

$$2x_1 + 2x_2 + 2x_3 = 0$$
$$-2x_1 + 5x_2 + 2x_3 = 1$$
$$8x_1 + x_2 + 4x_3 = -1$$

The augmented matrix for the system of equation is

$$= \begin{bmatrix} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix}$$

Multiplying by $+1/2$ to R_1

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix}$$

Adding $+2$ times the R_1 to R_2

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix}$$

Adding -8 times the $-8R_1 + R_2$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{bmatrix}$$

Multiplying by $1/7$ to R_2

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & -7 & -4 & -1 \end{bmatrix}$$

Adding $+7$ times the R_2 to R_3

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Adding -1 times the $-R_2$ to R_1

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 3/7 & -1/7 \\ 0 & 1 & 4/7 & 1/7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} 1-\frac{4}{7} \\ \\ \frac{3}{7} \end{matrix}$$

$$x_1 + 3/7\, x_3 = -1/7$$
$$x_2 + 4/7\, x_3 = 1/7$$

$$x_3 = t$$
$$x_2 = \frac{1}{7} - \frac{4}{7}t$$
$$x_1 = \frac{-1}{7} - \frac{3}{7}t$$

Q. 1) Find A^{-1}

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

$$= [\, A \mid I \,]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

Adding

= ~~Dividing~~ by -2 to R_1 to R_2

& -1 by R_1 to R_3

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

Adding $+2$ times the R_2 to R_3

$2R_2 + R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right]$$

Multiplying $+1$ to R_1

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & +5 & -2 & -1 \end{array}\right]$$

Adding $+3R_3$ to R_2 and $-3R_3$ to R_1

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & +6 & +3 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

Adding -2 times the R_2 to R_1

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & 2 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & 2 & -1 \end{bmatrix}$$

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Ex 1.5

Q.1) Find A^{-1}

$$= \begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$$

$$= [\, A \mid I \,]$$

$$= \left[\begin{array}{ccc|ccc} -1 & 3 & -4 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

= Multiplying R_1 by -1

$$= \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

Adding -2 times the R_1 to R_2 and $+4$ times the R_1 to R_3

$-2R_1 + R_2$

$4R_1 + R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 10 & -7 & 2 & 1 & 0 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right]$$

Multiplying by $1/10$ on R_2

$$= \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7/10 & 1/5 & 1/10 & 0 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right]$$

= Adding $+10$ times the R_2 to R_3

$10R_2 + R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7/10 & 1/5 & 1/10 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right]$$

- Adding $+3$ times the R_2 to R_1

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 19/10 & -1 & 0 & 0 \\ 0 & 1 & -7/10 & 1/5 & 1/10 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right]$$

Not invertible

$\frac{-21+4}{10}$

$\frac{-21+40}{10}$

$\frac{19}{10}$

Q. 1) Solve the system

$$x_1 + x_2 = 2$$
$$5x_1 + 6x_2 = 9$$

Solution :-

$$Ax = b; \longrightarrow (1)$$
$$x = A^{-1}b; \longrightarrow (2)$$

$$\begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$= \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 5 & 6 & 0 & 1 \end{array}\right]$$

Adding -5 times the first $-5R_1 + R_2$

$$= \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -5 & 1 \end{array}\right]$$

Adding -1 times the R_2 to R_1
$-1R_2 + R_1$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 6 & -1 \\ 0 & 1 & -5 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6(2) + (-1)(9) \\ -5(2) + 1(9) \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 - 9 \\ -10 + 9 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

Q.2) Solve the system

$$x_1 + 3x_2 + x_3 = 4$$
$$2x_1 + 2x_2 + x_3 = -1$$
$$2x_1 + 3x_2 + x_3 = 3$$

$Ax = B \longrightarrow (1)$

$x = A^{-1}B \longrightarrow (2)$

$$\therefore (1) \Rightarrow \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 4 & 1 & 0 & 0 \\ 2 & 2 & 1 & -1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\right.$$

Mully Adding -2 times the R_1 to R_3 & R_1 to R_2

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 4 & 1 & 0 & 0 \\ 0 & -4 & -1 & -9 & -2 & 1 & 0 \\ 0 & -3 & -1 & -5 & -2 & 0 & 1 \end{array}\right]$$

= Multiplying by $-1/4$ on R_2

$-2+3/2$

$-4+3/2$

$$= \left[\begin{array}{ccc} 1 & 3 & 1 \\ 0 & 1 & 1/4 \\ 0 & -3 & -1 \end{array}\right] \left[\begin{array}{ccc} 1 & 0 & 0 \\ 1/2 & 1/4 & 0 \\ -2 & 0 & 1 \end{array}\right]$$

Adding $+3$ times the R_2 to R_3

$$= \left[\begin{array}{ccc} 1 & 3 & 1 \\ 0 & 1 & 1/4 \\ 0 & 0 & -1/4 \end{array}\right] \left[\begin{array}{ccc} 1 & 0 & 0 \\ 1/2 & 1/4 & 0 \\ -1/2 & 3/4 & 1 \end{array}\right] \begin{array}{l} -1+\frac{3}{4} \\ \\ -4+\frac{3}{4} \end{array}$$

Multiplying by -4 on both sides of R_3

$$= \left[\begin{array}{ccc} 1 & 3 & 1 \\ 0 & 1 & 1/4 \\ 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc} 1 & 0 & 0 \\ 1/2 & 1/4 & 0 \\ 2 & -3 & -4 \end{array}\right]$$

Adding $-1/4$ on R_3 to R_2 & -1 times R_3 to R_1

$$= \left[\begin{array}{ccc} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc} -1 & 3 & 4 \\ 0 & 1 & 1 \\ 2 & -3 & -4 \end{array}\right]$$

= Adding -3 times the R_2 to R_1

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc} -1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & -3 & -4 \end{array}\right]$$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$Ax = B$

$x = A^{-1}B$

$$\begin{bmatrix} 1 & 3 & 1 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$$

$[A \mid I]$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 1 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right] \begin{array}{l} -2R_1+R_2 \\ -2R_1+R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right] -R_2+R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & -4 & -1 & -2 & 1 & 0 \end{array}\right] \text{Interchange } R_2 \leftrightarrow R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & -4 & -1 & -2 & 1 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -2 & -3 & 4 \end{array}\right] -4R_2+R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right] -R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right] -R_3+R_1$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 2 & 3 & -4 \end{array}\right] -3R_2+R_1$$

$$A^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}$$

$X = A^{-1}B$

$$= \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}$$

Q. 3)

$x_1 - 5x_2 = b_1$

$3x_1 + 2x_2 = b_2$

(I) $b_1 = 1, b_2 = 4$

(II) $b_1 = -2, b_2 = 5$

$$\left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 3 & 2 & 4 & 5 \end{array}\right]$$

$-3R_1 + R_2$

$$\left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 0 & 17 & 7 & 11 \end{array}\right]$$

$\div \frac{1}{17}$ by R_2

$$= \left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 0 & 1 & 1/17 & 11/17 \end{array}\right]$$

$+5R_2$ to R_1

$$= \left[\begin{array}{cc|c|c} 1 & 0 & 22/17 & 21/17 \\ 0 & 1 & 1/17 & 11/17 \end{array}\right]$$

$\frac{1+5}{17}$

$\frac{-2+55}{17}$

$\frac{-34+55}{17}$

$-x_1 + 4x_2 + x_3 = b_1$

$x_1 + 9x_2 - 2x_3 = b_2$

$6x_1 + 4x_2 - 8x_3 = b_3$

(I) $b_1 = 0, b_2 = 1, b_3 = 0$

$b_1 = -3, b_2 = 4, b_3 = -5$

Exercise 1.7

Inverse and Power of Diagonal Matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & +2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$$A^{3} = \begin{bmatrix} (1)^3 & 0 & 0 \\ 0 & (-3)^3 & 0 \\ 0 & 0 & (2)^3 \end{bmatrix}$$

$$A^{+3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -27 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

Triangular Matrix

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Symmetric Matrix

$$A = A^T$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$a_{12} = a_{21}$
$a_{13} = a_{31}$
$a_{23} = a_{32}$

Invertible matrix

all diagonal values are non-zero

Example:

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 4 \\ 5 & 7 & 6 \end{bmatrix}$$ Non-invertible

$$\begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & -0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 2 & 0 \\ 9 & 2 & 4 \end{bmatrix}$$

Q.1) Find all the values of unknown such that in which A is symmetric?

$$A = \begin{bmatrix} 2 & a-2b+c & 2a+b+c \\ 3 & 8 & a+c \\ 0 & -2 & 9 \end{bmatrix}$$

By symmetric matrices

$$a - 2b + c = 3$$
$$2a + b + c = 0$$
$$a + c = -2$$

$$\begin{bmatrix} 1 & -2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & -2 \end{bmatrix}$$

$-2R_1 + R_2 \longrightarrow R_2$ & $-R_1 + R_3 \to R_3$

$$= \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 5 & -1 & -6 \\ 0 & 2 & 0 & -5 \end{bmatrix}$$

Multiplying by $1/5$ to R_2

$$= \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & -1/5 & -6/5 \\ 0 & 2 & 0 & -5 \end{bmatrix}$$

$-2R_2 + R_3 \to R_3$

$$= \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & -1/5 & -6/5 \\ 0 & 0 & 2/5 & -13/5 \end{bmatrix}$$

$\frac{12-5}{5}$

$\frac{12-25}{5}$

$\frac{-13}{5}$

~~Adding~~ $5/2$ by R_3

$$= \begin{bmatrix} 1 & -2 & 1 & 3 \\ 0 & 1 & -1/5 & -6/5 \\ 0 & 0 & 1 & -13/2 \end{bmatrix}$$

$b - 1/5\, c = -6/5$

$c = -13/2$

$b - 1/5 = -6/5$

$b = \frac{-6}{5} + \frac{1}{5}\left(\frac{-13}{2}\right)$

$b = \frac{-6}{5} - \frac{13}{10}$

$b = \frac{-12-13}{10}$

$b = \frac{-26}{10}$

$c = 6$

$b = -6/5 + 1/5(6)$

$b = 0$

$a = 3 + 2(0) - 6$

Linear Transformation

If T is a function, with domain R^n and column R^m, then we say that T is a transformation from R^n to R^m

$$T: R^n \longrightarrow R^m$$

$$T: \text{variables} \longrightarrow eq^n$$

R^n = domain no. of variable

R^m = codomain no. of equation

Find domain, codomain, standard matrix and transformation

$$w_1 = 2x_1 - 3x_2 + x_3 - 5x_4$$
$$w_2 = 4x_1 + x_2 - 2x_3 + x_4$$
$$w_3 = 5x_1 - x_2 + 4x_3$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 & -5 \\ 4 & 1 & -2 & 1 \\ 5 & -1 & 4 & 0 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix}$$

Standard Matrix

Standard Matrix :

$$\begin{bmatrix} 2 & -3 & 1 & -5 \\ 4 & 1 & -2 & 1 \\ 5 & -1 & 4 & 0 \end{bmatrix}$$

Domain = 4

Codomain = 3

$$T: R^4 \longrightarrow R^3$$

Q. 1) Find all values

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 2x_1 + x_2 \\ x_1 - 3x_2 \\ -x_1 + x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & -3 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & -3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Domain = 2

Codomain = 3

Standard Matrix $\begin{bmatrix} 2 & 1 \\ 1 & 3 \\ -1 & 1 \end{bmatrix}$ $T: R^2 \rightarrow R^3$

Q.2) Find all values

$$T(x_1, x_2) = (3x_1 + x_2, 2x_1 - 4x_2)$$

$$T = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3x_1 + x_2 \\ 2x_1 - 4x_2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix}$$

Domain = 2

Codomain = 2

$T = R^2 \to R^2$

Reflection operator

① Reflection about x-axis

$$T(x, y) = T(x, -y)$$

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Standard Matrix $= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

② Reflection about y-axis

$$T(x, y) = (-x, y)$$

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

③ Reflection about line $y = x$

$$T(x, y) = (y, x)$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

④ Reflection about xy-plane

$$T(x, y, z) = (x, y, -z)$$

$$T\begin{bmatrix} x \\ y \\ +z \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

⑤ Reflection about xz-plane

$$T(x, y, z) = (x, -y, z)$$

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ -y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

⑥ Reflection about yz-plane

$$T(x, y, z) = (-x, y, z)$$

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

PROJECT OPERATOR

1-) ORTHOGONAL PROJECTION ONTO THE x-axis

$T(x,y) = (x,0)$

$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

↑

Standard Matrix

2-) ORTHOGONAL PROJECTION ONTO Y-axis

$T(x,y) = (0,y)$

$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

4 5 1 2 3

4 1 3 2 5

3-) ORTHOGONAL PROJECTION ONTO THE XY-PLANE

$T(x,y,z) = (x,y,0)$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

4-) ORTHOGONAL PROJECTION ONTO XZ-plane

$T(x,y,z) = (x,0,z)$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5-) ORTHOGONAL PROJECT ONTO YZ-plane

$T(x,y,z) = (0,y,z)$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rotational operator

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Q.1) Find standard Matrix and compute $T(x)$.

$$T(x_1, x_2, x_3) = (2x_1 - x_2 + x_3, \; x_2 + x_3, \; 0)$$

$$x = (2, 1, -3)$$

Solution:

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_2 + x_3 \\ x_2 + x_3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\text{Standard Matrix} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T(x) = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$$

$$T(x) = \begin{bmatrix} 2\times 2 + -1\times 1 + 1\times -3 \\ 0\times 2 + 1\times 1 + 1\times -3 \\ 0\times 2 + 0\times 1 + 0\times 3 \end{bmatrix} = \begin{bmatrix} 4-1-3 \\ 1-3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$$

Ex 1·8

Theorem 1·8·2

$T: R^n \longrightarrow R^m$ is a matrix transformation if and only if the following relation hold for all vectors u and v for scalar k

① $T(u+v) = T(u) + T(v)$

② $T(ku) = k\,T(u)$

Q.) Use theorem 1·8·2 to show that T is a matrix transformation

a.) $T(x,y) = (2x+y,\ x-y)$

Let

$u = (u_1, u_2)$,

$v = (v_1, v_2)$

$T(u+v) = (u_1+v_1,\ u_2+v_2)$

$= \cancel{(2u_1+v_1,\ u_2-v_2)}$

$= [2(u_1+v_1)+u_2+v_2,\ u_1+v_1-(u_2+v_2)]$

$= [(2u_1+u_2,\ u_1-u_2)+(2v_1+v_2,\ v_1-v_2)]$

$T(u) + T(v)$

(II) $T(ku) = T(ku_1, ku_2)$

$= T(2ku_1+ku_2,\ ku_1-ku_2)$

$= k(2u_1+u_2,\ u_1-u_2)$

$= k\,T(u)$

Use Matrix Multiplication to find the reflection of $(-1, 2)$ about

1. the x-axis
2. the y-axis
3. line $y = x$

Solution (a)

By the reflection about x-axis

The standard Matrix is

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$T_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$T_x = \begin{bmatrix} 1(-1) + 0(2) \\ 0(-1) + -1(2) \end{bmatrix}$$

$$T_x = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

b) By the reflection about y-axis

The standard Matrix is

$$= \begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix}$$

$$\Rightarrow T_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\Rightarrow T_y = \begin{bmatrix} -1(-1) + 0(2) \\ 0(-1) + 1(2) \end{bmatrix}$$

$$\Rightarrow T_y = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

c.)

By the reflection about $y = x$

The standard Matrix is

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0(-1) + 1(2) \\ 1(-1) + 0(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Use matrix to find the reflection of $(2, -5, 3)$ about ① xy-plane

② xz-plane
③ yz-plane

a -) By the reflection about xy-plane the standard matrix is :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$T_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix}$$

$$T_x = \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix}$$

Q.4) Use matrix to find the orthogonal projection $(-2, 1, 3)$ onto the

a.) xy-plane
b.) xz-plane
c.) yz-plane

By the orthogonal projection onto the standard matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

$$T_x = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

Q-5) Use matrix to find the image of the vector (3, −4) when it is rotated about the origin through angle of

(i) 30°
(ii) 60°
(iii) 45°

(i) By the rotational operation standard Matrix

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$T_x = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix}$$

$$T_x = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$T_x = \begin{bmatrix} 0.866 \times 3 - 1/2 \times -4 \\ \end{bmatrix}$$

$$T_x = \begin{bmatrix} 4.6 \\ 1.96 \end{bmatrix}$$

(ii) By the rotational operation
The standard Matrix is:

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & +\sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 \times 3 + \sqrt{3}/2 \times -4 \\ \sqrt{3}/2 \times 3 - 4 \times 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 - 3.464 \\ -2.598 - 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4.964 \\ +0.598 \end{bmatrix} \qquad \begin{bmatrix} -1.964 \\ 4.598 \end{bmatrix}$$

c.) By the relational operator

$$\text{Standard Matrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$T_x = \begin{bmatrix} \cos 45 & -\sin 45 \\ \sin 45 & \cos 45 \end{bmatrix}$$

$$T_x = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ +1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 0.707 \times 3 + 0.707 \times 4 \\ 0.707 \times 3 + 0.707 \times -4 \end{bmatrix}$$

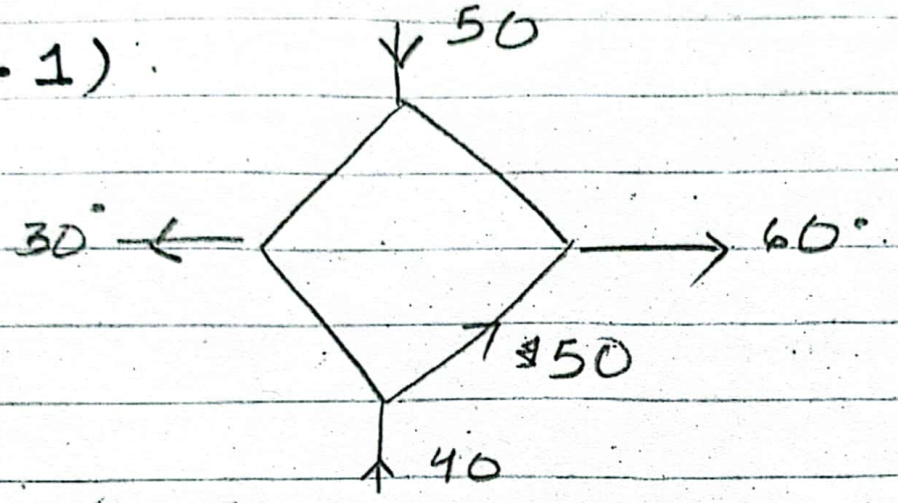
$$= \begin{bmatrix} 2.121 + 2.828 \\ 2.121 - 2.828 \end{bmatrix}$$

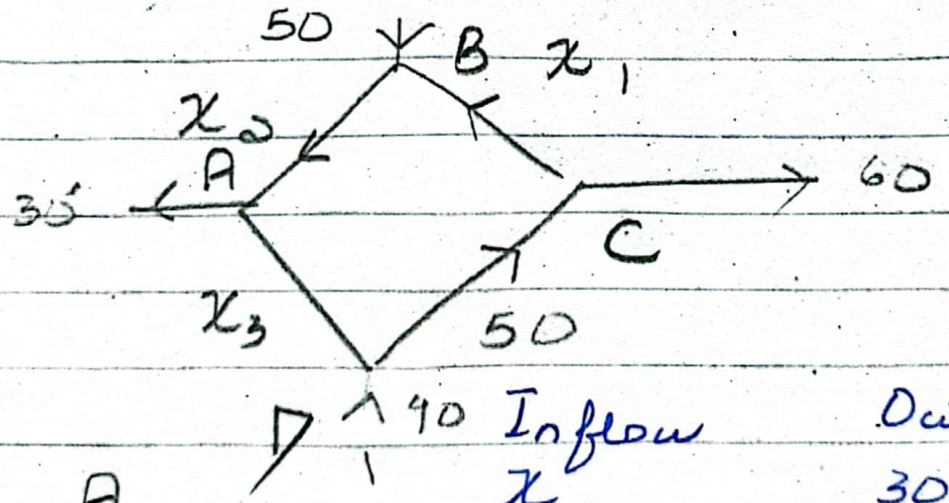
$$= \begin{bmatrix} 4.949 \\ -0.707 \end{bmatrix}$$

EXERCISE 1.10

APPLICATION OF LINEAR SYSTEM

Q.1)





	Inflow	Outflow
A	x_2	$30 + x_3$
B	$x_1 + 50$	x_2
C	50	$60 + x_1$
D	$x_3 + 40$	50

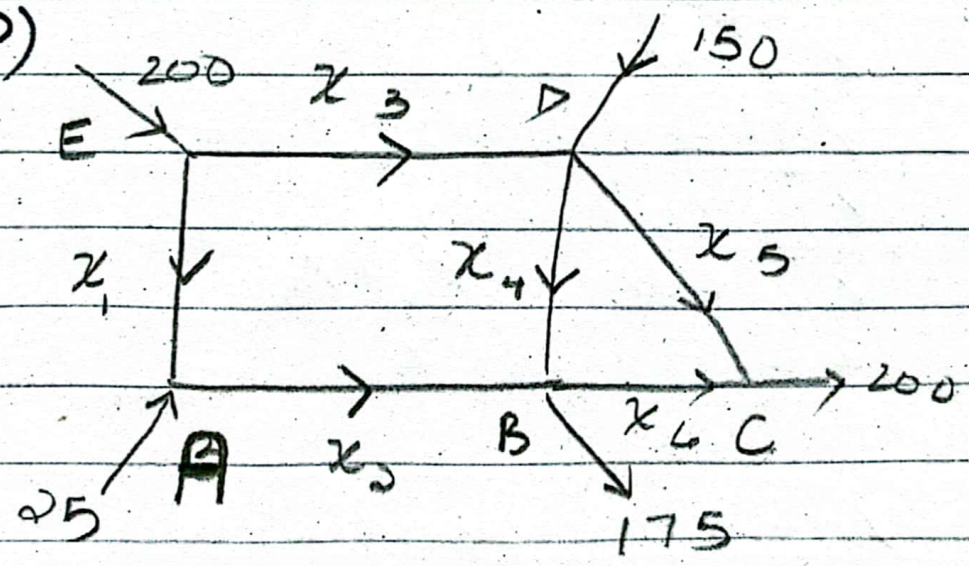
$x_2 - x_3 = 30$

$x_1 - x_3 = -50$

$x_1 = -10$
$x_3 = 10$

① $\Rightarrow x_2 = 30 + 10$
$x_2 = 40$

Q.2)



① Set up linear system
Solve
Find all values of $x_4 = 50$ & $x_6 = 0$

Node	Inflow	Outflow
A	$x_1 + 25$	x_2
B	$x_2 + x_4$	$175 + x_6$
C	$x_6 + x_5$	200
D	$x_3 + 150$	$x_4 + x_5$
E	200	$x_3 + x_1$

(1)

$$x_1 + 25 = x_2$$
$$x_2 + x_4 = 175 + x_6$$
$$x_6 + x_5 = 200$$
$$x_3 + 150 = x_4 + x_5$$
$$200 = x_3 + x_1$$

$$x_1 - x_2 = -25$$
$$x_2 + x_4 - x_6 = 175$$
$$x_6 + x_5 = 200$$
$$x_3 - x_4 - x_5 = -150$$
$$x_1 + x_3 = 200$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -25 \\ 0 & 1 & 0 & 1 & 0 & \\ & & & & & \\ & & & & & \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 1 & 0 & 1 & 0 & -1 & +175 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 1 & 0 & 1 & 0 & 0 & 0 & 200 \end{bmatrix}$$

Interchanging R_3 with R_5

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{bmatrix}$$

$R_4 - R_4 \Rightarrow R_4$

225 - 175

$$\begin{array}{r} 225 \\ -175 \\ \hline 50 \end{array}$$

Interchanging R_3 with R_4

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 1 & 0 & 1 & 0 & 0 & 0 & 200 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{bmatrix}$$

Adding $-R_1$ times to R_3

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & -1 & & & & & \\ & & & & & & \\ & & & & & & \end{bmatrix}$$

Adding $-R_1$ times to R_5

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 0 & 1 & 1 & 0 & 0 & 0 & 225 \end{bmatrix}$$

Adding $-R_2$ times the R_2 to R_5

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 0 & 0 & +1 & -1 & 0 & 1 & 50 \end{bmatrix}$$

Interchange R_2 with R_3

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 1 & -1 & 0 & 1 & 50 \end{bmatrix}$$

$-R_3 + R_5 \longrightarrow R_5$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -25 \\ 0 & 1 & 0 & 1 & 0 & -1 & 175 \\ 0 & 0 & 1 & -1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \end{bmatrix}$$

$-R_4 + R_5$

$$= \begin{bmatrix} & & & & & & \\ & & & & & & \\ & & & & & & \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$