

# Test Your Understanding: propositional logic

Test yourself by filling in the blanks.

1. A statement is \_\_\_\_.
2. If  $p$  and  $q$  are statements, the statement “ $p$  but  $q$ ” is symbolized \_\_\_\_.
3. If  $p$  and  $q$  are statements, the statement “neither  $p$  nor  $q$ ” is symbolized \_\_\_\_.
4. An *and* statement is true if, and only if, both components are \_\_\_\_.
5. An *or* statement is false if, and only if, both components are \_\_\_\_.
6. An *if-then* statement is false if, and only if, its hypothesis is \_\_\_\_ and its conclusion is \_\_\_\_.
7. A statement of the form  $p \leftrightarrow q$  is true if, and only if, \_\_\_\_.
8. If a logical expression includes the symbols  $\sim$ ,  $\wedge$  or  $\vee$ , and  $\rightarrow$  or  $\leftrightarrow$  and the expression does not include parentheses, then the first operation to be performed is \_\_\_\_, the second is \_\_\_\_, and the third is \_\_\_\_\_. To indicate the order of operations for an expression that includes both  $\wedge$  and  $\vee$  or both  $\rightarrow$  and  $\leftrightarrow$ , it is frequently necessary to add \_\_\_\_.
9. A tautology is \_\_\_\_ for every substitution of statements for the statement variables.
10. A contradiction is \_\_\_\_ for every substitution of statements for the statement variables.
11. Two statement forms are logically equivalent if, and only if, their truth values are \_\_\_\_ for every substitution of statements for the statement variables.
12. *Less formal version:* Two statement forms are logically equivalent if, and only if, they always have \_\_\_\_.
13. Two statement forms are not logically equivalent if, and only if, \_\_\_\_.
14. De Morgan’s laws say that \_\_\_\_ and \_\_\_\_.
15. The negation of  $p \rightarrow q$  is \_\_\_\_.
16. The contrapositive of “if  $p$  then  $q$ ” is \_\_\_\_.
17. The converse of “if  $p$  then  $q$ ” is \_\_\_\_.
18. The inverse of “if  $p$  then  $q$ ” is \_\_\_\_.
19. A conditional statement and its contrapositive are \_\_\_\_.
20. A conditional statement and its converse are not \_\_\_\_.
21. If  $r$  and  $s$  are statements,  $r$  only if  $s$  can be expressed in if-then form as \_\_\_\_ or as \_\_\_\_.
22. If  $t$  and  $u$  are statements,  $t$  is a sufficient condition for  $u$  can be expressed in if-then form as \_\_\_\_.
23. If  $v$  and  $w$  are statements,  $v$  is a necessary condition for  $w$  can be expressed in if-then form as \_\_\_\_ or as \_\_\_\_.

## Answers

1. a sentence that is true or false but not both
2.  $p \wedge q$
3.  $\sim p \wedge \sim q$
4. true
5. false
6. true, false
7. both  $p$  and  $q$  are true or both  $p$  and  $q$  are false
8.  $\sim$ ;  $\wedge$  or  $\vee$ ;  $\rightarrow$  or  $\leftrightarrow$ ; parentheses
9. true
10. false
11. identical
12. the same truth values
13. there exist statements with the property that when the statements are substituted for the statement variables, one of the resulting statements is true and the other is false
14.  $\sim (p \wedge q) \equiv \sim p \vee \sim q$ ;  $\sim (p \vee q) \equiv \sim p \wedge \sim q$
15.  $p \vee \sim q$
16. if  $\sim q$  then  $\sim p$
17. if  $q$  then  $p$
18. if  $\sim p$  then  $\sim q$
19. logically equivalent
20. logically equivalent
21. if  $\sim s$  then  $\sim r$ ; if  $r$  then  $s$
22. if  $t$  then  $u$
23. if  $\sim v$  then  $\sim w$ ; if  $w$  then  $v$