National University of Computer and Emerging Sciences, Karachi

FAST School of Computing

Final Examination, Fall 2023

December 21, 2023, 09:00 AM – 12:00 NOON

Course Code: CS1005	**Course Name: Discrete Structures**
Instructor Names: Dr. Waqas, Mr. Shoaib Raza, Ms. Bakhtawer, Ms. Safia, Mr. Fahad Hussain, Ms. Mafaza, Ms. Ayesha, Ms. Hania, and Ms. Yusra.	
Student Roll No:	**Section No:**

Instructions:

- Return the question paper together with the answer script. Read each question completely before answering it. There are **5 questions** written on **4 pages.**
- In case of any ambiguity, you may make assumptions. However, your assumptions should not contradict any statement in the question paper.
- **Attempt all the questions in the given sequence of the question paper. Show all steps properly in order to get full points.**

Allowed Time: 03 Hours **Maximum Points**: 80

Question # 1: **[CLO-3]** **[8 x 2 = 16 points]**

(a) Write the Converse of the statement (i) and Inverse of the statement (ii) given below:
i) It does not rain or it is not foggy only if the sailing race will be held but the lifesaving demonstration will go on.
ii) The trophy will be awarded when the sailing race is held and it is foggy.

(b) Prove that Resolution rule is valid using the laws of propositional logic and you can also use other rules of inference beside Resolution rule. (Hint: you will need one of the conditional identities from the laws of propositional logic).

(c) Let p: "you will succeed" and q: "you will work hard" be the propositions. Using truth table, prove or disprove that given below arguments are valid.
i) You will work hard if you succeed. You will work hard. Therefore, you will succeed.
ii) If you succeed, you will work hard. You don't succeed. Therefore, you will not work hard.

(d) Apply the negation on the statement $[(\neg R \vee \neg F) \rightarrow (S \wedge L)]$ and simplify using logical equivalence laws.

(e) Let Loves(x, y) mean "x loves y," Traveler(x) mean "x is a traveler, " City(x) mean "x is a city," Lives(x, y) mean "x lives in y.". Translate the given below predicates with quantifiers into equivalent statement in English.
i) $\exists x\ \forall y\ \forall z$ (city(x) $\wedge$ Traveler(y) $\wedge$ Lives(z, x)) $\rightarrow$ (Loves(y. x) $\wedge$ $\neg$ loves(z, x))
ii) $\forall x\ \forall y$((Traveler(x) $\wedge$ City(y) $\wedge$ Lives(x, y)) $\rightarrow$ $\neg$ loves(x, y))

(f) Use quantifiers to express each of these statements given below:
i) "The difference of a real number and itself is zero."
ii) "The product of two negative real numbers is not negative."

(g) Using Set Identities, prove or disprove that if A is a subset of a universal set U, then $A \oplus U = A^c$

(h) Out of 250 candidates who failed in an examination, it was revealed that 128 failed in Math, 87 in Physics and 134 in chemistry, 31 failed in math and in physics, 54 failed in chemistry and in math, 30 failed in chemistry and in physics. Find how many candidates failed in all the three subjects.

Question # 2: **[CLO-1]** **[8 x 2 = 16 points]**

(a) Draw the Venn diagrams for each of these combinations of the sets A, B, and C. $(A \cap B^c) \cup (A \cap C^c)$

(b) Consider the function g: Z→Z defined by g(n) = n^2+1. Find g (1) and g ({1}). Is g (1) = g ({1}) are equal if not why?

Suppose *f*: *A* → *B* and *g*: *B* → *C* where *A* = {2, 3}, *B* = {a, b, c}, *C* = {5}, and *f* and *g* are defined by *f* = {(2, a), (3, b)} and *g* = {(a,5), (b,5), (c,5)}. Now, answer the below part (c), (d), (e), and (f).

(c) Is Function f and g are invertible? If yes, provide the function. If not, provide the proper reason.

(d) Find f o g and g o f. If any does not exist, give proper reason.

(e) If f o g OR g o f obtained in part (d) is an onto function, must f be onto?

(f) Is the composition f o g OR g o f obtained in part (d) is POSET relation?

(g) Find the next term in the Sequence: 4,9,18,31,50, …

(h) Find a_6, the sequence is defined by $a_n = 2a_{n-1} - a_{n-2}$ with $a_0 = 3$ and $a_1 = 4$.

Question # 3: **[CLO-3]** **[8 x 2 = 16 points]**

(a) A graph has 26 vertices and 58 edges. There are five vertices of degree 4, six vertices of degree 5, and seven vertices of degree 6. If the remaining vertices all have the same degree, what is this degree?

(b) Find the union G = (G1 U G2) of the given pair of simple graphs G1 and G2 given in Figure 01.

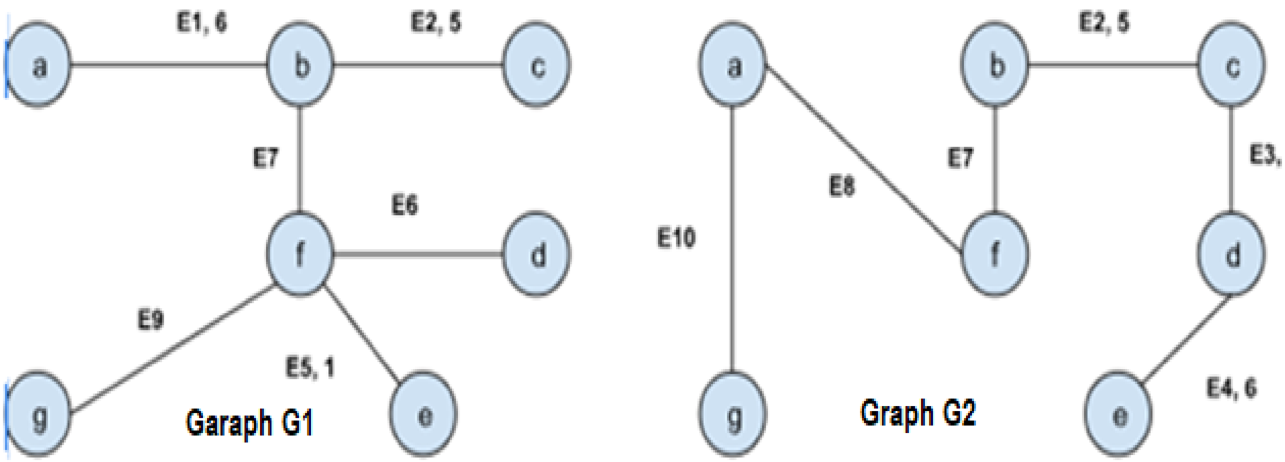

Figure 01: Pair of simple graphs G1 and G2

(c) In a Pseudo Random Number generator with a given seed value of edge (E5 = 1), a multiplier of 2, and an increment of 1 and modulo of 10. What are the weights of edges E6, E7, E8, E9, and E10 in the graph G = (G1 U G2) obtained in question # 3 part (b) and draw a graph with weights on all edges?

(d) Consider the final graph G with weight of all edges in question # 3 part (c), find the shortest path from node **a** to all other nodes. Use the table 01 given below for computations.

Table 01: Shortest path from node a to all other nodes

N	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)
a	-	-	-	-	-	-

(e) Consider the edges in graph G from question # 3 part (c), where each edge represents the sequence of book chapters, and the weight of each edge contributes to the creation of the ISBN-10 number for the corresponding book. Determine whether the obtained code is a valid ISBN-10 code or not.

(f) Determine whether the graph G = (G1 U G2) obtained in question 3 part (b) has an Euler Circuit and Hamiltonian path. If it does not exist then give the proper reason.

(g) Apply Prim's algorithm to find a Minimal Spanning Tree from node a for the graph G obtained in question # 3 part (c). Indicate the order in which edges are added to form the tree along with the final cost.

(h) Determine if the Tree obtained in question # 3 part (g) is a bipartite. If it is then draw the bipartition and show the set of vertices in each partition. If it is not, then show where the violation occurs.

Question # 4: **[CLO-2]** **[8 x 2 = 16 points]**

(a) Prove or disprove by Contradiction: There are no integers x and y such that $x^2 = 4y + 2$.

(b) Let d be a positive integer. Prove or disprove that the integers m and n are congruent modulo d if and only if there is an integer t such that m = n + dt.

(c) Prove using mathematical induction that for all $n \geq 1$; $1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}$.

(d) Prove or disprove: For every prime number P, 2P + 1 is also prime.

(e) Proof or disproof by Contraposition that if n = a*b then a ≤ $\sqrt{n}$ or b ≤ $\sqrt{n}$.

(f) Suppose you are going for a dinner. You can either get a crunchy or a spicy burger. You can choose either beef, chicken, or fish. Create a tree diagram to find the total number of possible burgers.

(g) Consider the tree obtained in question # 4 part (f). Show how many internal vertices and leaves the tree has? Determine the height of the tree?

(h) Consider the tree obtained in question # 4 part (f).
i) Determine whether it is a Full m-ary tree or not? Give reason. What is the value of m?
ii) Determine whether it is a balanced m-ary tree or not? Give reason.

Question # 5: **[CLO-2]** **[8 x 2 = 16 points]**

A woman has a farm with x animals.
When she groups the animals in groups of 11, 6 animals are left.
When she groups the animals in groups of 16, 13 animals are left.
When she groups the animals in groups of 21, 9 animals are left.
When she groups the animals in groups of 25, 19 animals are left.
How many minimum possible animals the woman must have to satisfy the given conditions.
In this problem, you are supposed to state and use of the following theorems:
(a) Apply:
 (i) Linear congruences
 (ii) The Euclidean Algorithm Lemma
(b) Apply:
 (i) Bézout's Theorem
 (ii) Chinese Remainder Theorem

(c) Use Fermat's little theorem to calculate the remainder of 7^{2019} when divided by 13.

(d) A message has been encrypted using the function f(p) = (p + 3) mod 26. If the message in coded form is "Hwlhqqh Ehcrxw", decode the message.

(e) How many permutations of the letters ABCDEFGH contain the strings BA and FGH?

(f) Thirteen people on a softball team show up for a game. How many ways are there to choose 10 players to take the field?

(g) Find the 51st term in the expansion of $(\frac{b}{2} + \frac{b}{2})^{100}$

(h) Six young workers received Rs.1500 wages altogether. Each of them wants to buy a cassette player costing RS. 320. Prove that at least one of them must wait for the next paycheck to make his purchase.

BEST OF LUCK ☺