In Exercises 25–26, determine the values of a *for which the system has no solutions, exactly one solution, or infinitely many solutions.*

25.
$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

Solution:

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{bmatrix}$$

← The augmented matrix for the system.

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{bmatrix}$$

← -3 times the first row was added to the second row and -4 times the first row was added to the third row.

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2-16 & a-4 \end{bmatrix}$$

← -1 times the second row was added to the third row.

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & a^2-16 & a-4 \end{bmatrix}$$

← The second row was multiplied by $-\frac{1}{7}$.

The system has no solutions when $a = -4$ (since the third row of our last matrix would then correspond to a contradictory equation $0 = -8$).

The system has infinitely many solutions when $a = 4$ (since the third row of our last matrix would then correspond to the equation $0 = 0$).

For all remaining values of a (i.e., $a \neq -4$ and $a \neq 4$) the system has exactly one solution.