Test Your Understanding: predicate and quantifier

Test yourself by filling in the blanks.

1. A predicate is ____.
2. The truth set of a predicate $P(x)$ with domain D is ____.
3. A statement of the form "$\forall x \in D$, $Q(x)$" is true if, and only if, ____.
4. A statement of the form "$\exists x \in D$, $Q(x)$" is true if, and only if, ____.
5. A universal conditional statement is a statement of the form ____.
6. A negation of a universal statement is an ____ statement.
7. A negation of an existential statement is a ____ statement.
8. A statement of the form "All A are B" can be written with a quantifier and a variable as ____.
9. A statement of the form "Some A are B" can be written with a quantifier and a variable as .
10. A statement of the form "No A are B" can be written with a quantifier and a variable as .
11. A negation for a statement of the form "$\forall x \in D$, $Q(x)$" is ____.
12. A negation for a statement of the form "$\exists x \in D$ such that $Q(x)$" is ____.
13. A negation for a statement of the form "$\forall x \in D$, if $P(x)$ then $Q(x)$" is ____.
14. For a statement of the form "$\forall x \in D$, $Q(x)$" to be vacuously true means that ____.
15. Given a statement of the form "$\forall x$, if $P(x)$ then $Q(x)$," the contrapositive is ____, the converse is ____, and the inverse is ____.
16. If you want to establish the truth of a statement of the form "$\forall x \in D$, $\exists y \in E$ such that $P(x, y)$," your challenge is to allow someone else to pick ____, and then you must find ____ for which $P(x, y)$ ____.
17. If you want to establish the truth of a statement of the form "$\exists x \in D$ such that $\forall y \in E$, $P(x, y)$," your job is to find ____ with the property that no matter what ____, $P(x, y)$ will be ____.
18. A negation for a statement of the form "$\forall x \in D$, $\exists y \in E$ such that $P(x, y)$" is ____.
19. A negation for a statement of the form "$\exists x \in D$ such that $\forall y \in E$, $P(x, y)$" is ____.
20. The rule of universal instantiation says that ____.
21. Universal modus ponens is an argument of the form ____, and universal modus tollens is an argument of the form ____.

Answers

1. a sentence that contains a finite number of variables and becomes a statement when specific values are substituted for the variables
2. the set of all x in D such that $P(x)$ is true
3. $Q(x)$ is true for each individual x in D
4. there is at least one x in D for which $Q(x)$ is true
5. $\forall x$, if $P(x)$ then $Q(x)$, where $P(x)$ and $Q(x)$ are predicates
6. existential
7. universal
8. $\forall x$, if x is an A then x is a B
9. $\exists x$ such that x is an A and x is a B
10. $\forall x$, if x is an A then x is not a B (Or: $\forall x$, if x is an B then x is not a A)
11. $\exists x \in D$ such that $\sim Q(x)$
12. $\forall x \in D, \sim Q(x)$
13. $\exists x \in D$ such that $P(x)$ and $\sim Q(x)$
14. there are no elements in D
15. $\forall x$, if $\sim Q(x)$ then $\sim P(x)$;
    $\forall x$, if $Q(x)$ then $P(x)$
    $\forall x$, if $\sim P(x)$ then $\sim Q(x)$
16. whatever element x in D they wish; an element y in E; is true
17. an element x in D; element y in E anyone might choose; true
18. $\exists x \in D$ such that $\forall y \in E, \sim P(x, y)$
19. $\forall x \in D, \exists y \in E$ such that $\sim P(x, y)$
20. if a property is true of everything in a domain, then it is true of any particular thing in the domain
21.

$$\begin{array}{ll} & \forall x, \text{ if } P(x) \text{ then } Q(x) \\ & P(a), \text{ for a particular } a \\ \therefore & Q(a) \end{array} \qquad \begin{array}{ll} & \forall x, \text{ if } P(x) \text{ then } Q(x) \\ & \sim Q(a), \text{ for a particular } a \\ \therefore & \sim P(a) \end{array}$$