National University of Computer and Emerging Sciences, Karachi
FAST School of Computing
Mid 1 Examination, Spring 2023
28th February, 2023, 11:30 AM – 12:30 PM

Course Code: CS1005	Course Name: Discrete Structures
Instructor Names:	
Student Roll No:	Section No:

Instructions:

- Return the question paper along with the answer script. Read each question completely before answering it. There are **3 questions on 2 pages.**
- In case of any ambiguity, you may make assumptions, however, your assumption should not contradict any statement in the question paper.
- Answer all the questions in the given sequence of the question paper.

Total Time: 60 minutes **Maximum Points**: 24

Question # 1 (Propositional Logic, Rules of Inference and Set Theory) **[CLO-3 C3]** **[2 x 6 = 12 Points]**

(a) Let **P**, **Q**, and **R** be the propositions.

P: You get an A in the final exam. **Q:** You do every exercise in this book. **R:** You get an A in this class.

Write these propositions using P, Q, and R and logical connectives (including negations):

i. If You do not get an A in this class, you do not get an A in the final exam.

ii. It is not the case that You get an A in the final exam if you do every exercise in this book.

iii. You do every exercise in this book when you do not get an A in the final exam.

iv. Getting an A in the final exam and doing every exercise in this book is sufficient and necessary for getting an A in this class.

(b) Write in words the contrapositive of Statement (i) and Converse of Statement (iii) given in part (a) from question # 1.

(c) Using rules of inference, show that the following argument is valid:

$$((\neg b \rightarrow (c \rightarrow \neg d)) \wedge (\neg b \vee f) \wedge (\neg a \rightarrow c) \wedge (\neg f)) \rightarrow (d \rightarrow a)$$

(d) Using laws of Logic, prove or disprove the following statement:

$$((p \vee q) \rightarrow (q \rightarrow (p \wedge q)) = \neg q \vee p$$

(e) Consider the following assumptions:

S1: All dictionaries are useful. S2: Mary owns only fiction novels. S3: No fiction novel is useful.

Use a Venn diagram to determine the validity of each of the following conclusions:

(i) Fiction novels are not dictionaries.

(ii) Mary does not own a dictionary.

(iii) All useful books are dictionaries.

(f) Construct a membership table to prove or disprove that:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Question # 2 (Predicates, Quantifiers and Functions) **[CLO-2 C2]** **[2 x 6 = 12 Points]**

(a) Let Q(x, y, z) be the statement "$x + y = z$", where 'x' and 'y' and 'z' consists of all real numbers. Determine the truth value of the following statements.

i. ∀x ∀y ∃z Q (x, y, z)
ii. ∃x ∀y ∀z Q (x, y, z)

(b) Translate each of the following statements into English where P(x, y) is the predicate "x saw y", Q(x, y) is the predicate "x liked y" and R(y) is the predicate "y is a comedy". The universe of discourse of x is the set of people and the universe of discourse for y is the set of movies.

i. ∃y ∀x Q (x, y)
ii. ∃x ∀y [R(y) → P (x, y)]

(c) Let Q(a) be the statement "a spends more than five hours every weekday in class," and R(a) be the statement "a lives in a hostel" where the domain for a consists of all students. Use quantifiers to express each of these statements.

i. Every hostel student does not spend more than five hours every weekday in class.
ii. No one in this class lives in a hostel and spends more than five hours every weekday.

(d) Suppose f: R→Z where $f(x) = \lceil \frac{x}{2} \rceil$. Determine whether the function is an onto (surjective) and/or a one-to-one (injective) or both (bijective).

(e) Given $g(x) = x^3 + 18$ and $f(x) = 4x + 1$, find (g o f) (x).

(f) Let f: R→R be defined by f(x) = 3x - 7. Is this function invertible? If yes, provide one, If no, provide reason?

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