FAST National University of Computer and Emerging Sciences
FALL 2024
MT-1004 Linear Algebra

Assignment 1

Systems of Linear Equations and Row Reduction

1. For each augmented matrix, find all solutions to the system of linear equations that it represents.

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -7 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -7 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} 1 & 3 & 6 & 1 \\ 0 & 2 & 1 & 7 \\ 0 & 0 & 3 & 9 \end{array}\right]$

2. Use row reduction to find solutions to each of the following systems of linear equations.

(a)
$$\begin{aligned} 3x_1 + 6x_2 + 3x_3 &= -3 \\ 5x_1 - 3x_2 + 18x_3 &= 8 \\ 7x_1 + 2x_2 + 19x_3 &= 5 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + 2x_2 &= 3 \\ 3x_1 - 6x_2 &= 9 \\ x_1 + x_2 &= 10 \end{aligned}$$

3. For what values of h is the following system consistent?

$$\begin{aligned} x_1 + hx_2 &= 1 \\ 2x_2 &= 2 \\ 3x_1 - x_3 &= 3 \end{aligned}$$

4. When doing row reduction, we are allowed to perform three types of operations: multiply any row by a nonzero scalar, swap two rows, and add a multiple of one row to another. In the first operation, why did we have to specify that the scalar is nonzero?

5. How many solutions does a system of linear equations have if the coefficient matrix in REF has:

   (a) A pivot in every row?
   (b) A pivot in every column?
   (c) A free variable (i.e. a column with no pivot)?
   (d) More columns than rows?
   (e) More rows than columns?

6. For what values of c are the following augmented matrices consistent?

(a) $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & c \end{array}\right]$ (b) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ c & 3 & -2 \\ 0 & 0 & 0 \end{array}\right]$

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Q:7

Matrices

1. For each of the following, either calculate the product of the matrix and the vector or state that the product is not defined.

(a) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 7 & -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Q:8 Determine whether the function

$$T: \boldsymbol{R^2} \to \boldsymbol{R^2} \quad T(x, y) = (x^2, y)$$

is a linear transformation or not?

Q:9 Let T: $\boldsymbol{R^2} \to \boldsymbol{R^2}$ be the linear transformation such that $T(1,1) = (0,2)$ and $T(1,-1) = (2,0)$. Compute $T(1,4)$.

Q:10

What is the determinant of the following matrix? Briefly justify your answer.

$$\begin{bmatrix} 1 & 7 & 8 & 1 & 2 & 3 \\ 2 & -9 & 81 & 2 & 7 & 0 \\ 3 & 4 & 7 & 3 & 7 & -1 \\ 4 & 1 & 1 & 4 & 1 & 1 \\ 5 & 7 & -3 & 5 & 13 & 788 \\ 6 & -1 & -2 & 6 & -4 & -5 \end{bmatrix}$$