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BCS-3J

Q1)

	P	T/F
a)	✓	T
d)	✓	F

	P	T/F
b)	✓	F
e)	×	-

	P	T/F
c)	✓	T
f)	×	-

Q2) a) p: Smartphone B has most RAM of these three smartphones

- Statement : p
- T/F : I

b) p: Smartphone C has more ROM than Smartphone B
q: Smartphone C has ~~more~~ higher resolution camera than Smartphone B

- Statement: $p \vee q$
- T/F : $F \vee T = I$

c) p: Smartphone B has more RAM than Smartphone A
q: Smartphone B has more ROM than Smartphone A
r: Smartphone B has more resolution camera than Smartphone A

- Statement : $p \wedge q \wedge r$
- T/F : $T \wedge T \wedge F = F$

d) p: Smartphone B has more RAM than smartphone C
q: Smaphone B has more ~~camera resolution~~ than smartphone C
r: Smartphone B has more camera ROM Resolution than smartphone C

- Statement : $(p \wedge q) \rightarrow r$
- T/F : $(T \wedge T) \rightarrow F \Rightarrow T \rightarrow F = F$

e) p: Smartphone A has more RAM than Smartphone B
q: Smartphone B has more RAM than Smartphone A

- Statement: $p \leftrightarrow q$
- T/F : $F \leftrightarrow T = F$

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Q3) a) p: Quixote Media had largest ~~revenue~~ annual revenue.
- Statement: p
- T/F: F

b) p: Nadir Software had lowest net profit
q: Acme computer had largest Annual Revenue
- Statement: $p \wedge q$
- T/F: $T \wedge T = T$

c) p: Acme computer had largest net profit
q: Quixote had largest net profit
- statement: $p \vee q$
- $F \vee T = T$

d) p: Quixote media had smallest net profit
q: Acme computer had largest annual revenue
- statement: $p \rightarrow q$
- $F \rightarrow T : T$

e) p: Nadir Software had smallest net profit
q: Acme computer had largest Annual revenue
- statement: $p \leftrightarrow q$
- $T/F : T \leftrightarrow T = T$

Q4) a) If you have flue, then you will miss final examination
b) You won't miss the final examination if & only if you pass the course
c) If you miss the final examination, then you will not be passing the course
d) You have the flu or you miss the final exam or you pass the course.
e) If you have the flue, then you will not pass the course or if you miss the final examination, then you will not pass the course.
f) You have the flu & you miss the examination or you will not miss the final examination and you pass the course.

Q5) p: You get an A in final exam
q: You do every exercise in this book
r: You get an A in this class

a) $r \wedge \sim q$
b) $p \wedge q \wedge r$
c) $r \rightarrow p$
d) $p \wedge \sim q \wedge r$
e) $(p \wedge q) \rightarrow r$
f) $r \leftrightarrow (q \wedge p)$

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Q9) a) exclusive, as staying home & going out are not achievable at the same time

b) inclusive, as late payment and or incorrect payment are achievable together and alternatively too.

c) inclusive, as no scheduling and no hotel rooms both can together and singularly implies failure of my trip

d) inclusive, no sense if you wont wear a shirt or shoes or both together

Q10) a) $(p \wedge (\sim(\sim p \vee q))) \vee (p \wedge q) \equiv p$

$(p \wedge (\sim\sim p \wedge \sim q)) \vee (p \wedge q)$ (DeMorgan)

$(p \wedge (p \wedge \sim q)) \vee (p \wedge q)$ (Double Negation)

$((p \wedge p) \wedge \sim q) \vee (p \wedge q)$ (Associative)

$(p \wedge \sim q) \vee (p \wedge q)$ (Idempotent)

$p \wedge (\sim q \vee q)$ (Distributive)

$p \wedge (t) \rightarrow p$ (Negation)
(Identity)

b) $\sim(p \leftrightarrow q) \equiv (p \leftrightarrow \sim q)$

$\sim[(p \rightarrow q) \wedge (q \rightarrow p)]$ (Biimplication equivalent)

$\sim[(\sim p \vee q) \wedge (\sim q \vee p)]$ (Implication ")

$\sim(\sim p \vee q) \veebar \sim(\sim q \vee p)$ (De Morgan)

$(\sim\sim p \wedge \sim q) \vee (\sim\sim q \veebar \sim p)$ (De Morgan)

$(p \wedge \sim q) \vee (q \veebar \sim p)$ (Double Negation) x 2

~~$(p \wedge \sim q) \vee (q \wedge \sim p) \vee (p \wedge \sim p) \vee (q \wedge \sim q)$~~

~~$(q \rightarrow p) \vee (p \rightarrow q) (p \wedge \sim q) (q \wedge \sim p) \vee (p \wedge \sim p)$~~ ~~(Implication Equivalent)~~

~~$\sim[(q \rightarrow p) \wedge (p \rightarrow q)] \vee (q \wedge \sim q) \equiv (\sim p \vee q) \wedge (q \vee p)$~~ ~~(De Morgan)~~

~~$\sim p \leftrightarrow \sim q$~~ ~~$(p \wedge q) \vee (\sim p \wedge \sim q)$~~

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$(p \wedge \sim q) \vee (q \wedge \sim p) \vee (p \wedge \sim p) \vee (q \wedge \sim q)$ — Adding $(p \wedge \sim p) \& (q \wedge \sim q)$ makes no diff as F

$(\sim p \vee \sim q) \wedge (q \vee p)$ — Distributive

$(p \rightarrow \sim q) \wedge (\sim q \rightarrow p)$ — $\rightarrow$ equivalent

$(p \leftrightarrow \sim q)$ — $\leftrightarrow$ equivalent

c) $\sim p \leftrightarrow q \equiv p \leftrightarrow \sim q$

$(\sim p \rightarrow q) \wedge (q \rightarrow \sim p)$ — $\leftrightarrow$ equivalent

$(\sim\sim p \vee q) \wedge (\sim q \vee \sim p)$ — $\rightarrow$ equivalent

$(p \vee q) \wedge (\sim q \vee \sim p)$ — Double Negation

~~$(p \wedge \sim q) \vee (p \wedge \sim p) \vee (q \wedge \sim q) \vee (q \wedge \sim p)$~~ — Distributive

~~$(p \wedge \sim q) \vee \mathbf{c} \vee \mathbf{c} \vee (q \wedge \sim p)$~~ — Negation

~~$(p \wedge \sim q) \vee (q \wedge \sim p)$~~

~~$(p \vee \sim q) \hat{\vee} (q \vee p)$~~ — Distributive

$(\sim q \rightarrow p) \wedge (p \rightarrow \sim q)$ — $\rightarrow$ equivalent

$(p \leftrightarrow \sim q)$ — $\leftrightarrow$ equivalent

d) $(p \wedge q) \rightarrow (p \rightarrow q) \equiv T$

$\sim(p \wedge q) \vee (p \rightarrow q)$ — $\rightarrow$ equivalent

$\sim(p \wedge q) \vee (\sim p \vee q)$ — $\rightarrow$ equivalent

$(\sim p \vee \sim q) \vee (\sim p \vee q)$ — De Morgan

$(\sim p \vee \sim p) \vee (q \vee \sim q)$ — Associative

$(\sim p) \vee T$ — Domination + Idempotent

$\equiv T$ — Domination

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e) $\neg(p \vee \sim(p \wedge q)) \equiv F$

$(\sim p \wedge \sim\sim(p \wedge q))$ — De Morgan

$(\sim p \wedge (p \wedge q))$ — Double Negation

$((\sim p \wedge p) \wedge q)$ — Associative

$(c \wedge q)$ — Domination

(c) — Domination

Q(ii) a) $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

p	q	r	$p \to r$	$q \to r$	$\wedge$ ✓	$(p \vee q)$	$(p \vee q) \to r$ ✓
F	F	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	T	F	T	F	F	T	F
F	T	T	T	T	T	T	T
T	F	F	F	T	F	T	F
T	F	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	T	T	T	T	T	T	T

∴ Equivalent

b) $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

p	q	r	$p \to q$	$p \to r$	$\vee$ ✓	$q \vee r$	$p \to (q \vee r)$ ✓
F	F	F	T	T	T	F	T
F	F	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F
T	F	T	F	T	T	T	T
T	T	F	T	F	T	T	T
T	T	T	T	T	T	T	T

∴ Equivalent

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c) $(p \to q) \to (r \to s) \equiv (p \to r) \to (q \to s)$

p	q	r	s	$p \to q$	$r \to s$	① $\to$	$p \to r$	$q \to s$	② $\to$
F	F	F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T	T
F	F	T	F	T	F	F	T	T	T
F	F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F	F
F	T	F	T	T	T	T	T	T	T
F	T	T	F	T	F	F	T	F	F
F	T	T	T	T	T	T	T	T	T
T	F	F	F	F	T	T	F	T	T
T	F	F	T	F	T	T	F	T	T
T	F	T	F	F	F	T	T	T	T
T	F	T	T	F	T	T	T	T	T
T	T	F	F	T	T	T	F	F	T
T	T	F	T	T	T	T	F	T	T
T	T	T	F	T	F	F	T	F	F
T	T	T	T	T	T	T	T	T	T

∴ Not Equivalent

Q(2) $\frac{m}{n}$

a) $\frac{5}{4}$ (False)

b) $\frac{2}{4} \to \frac{2}{2(2)}$ (True)

c) Not All +ve integers are divisible by +ve integers (False)

d) Some values of m can divide every n $(\frac{m}{1(m)})$ (True)

e) n not divisible by all m (False)

f) 1 can divide all (True)

Q(3) a) ∃ is true $(\pm\sqrt{2})^2 = 2$ ~~(False)~~ (True)

b) ∃ is even false as $(-)^2$ inside can never yield -ve (False)

c) ∀ is true as +2 and $()^2$ will always yield +ve so least is $(0)^2+2 = 2 \geq +1$ (True)

d) ∃ is true in case of $(0)^2$ and $(1)^2$ (True)

Q14) $F(x,y)$: x can fool y

a) $\forall x\, F(x, Bob)$

b) $\forall y\, F(Alice, y)$

c) $\forall x \exists y F(x,y)$

d) $\neg \exists x \forall y\, F(x,y)$

e) $\forall y . \exists x\, F(x,y)$

Q15) $P(x)$: x can speak russian
$Q(x)$: x knows computer language C++

a) $\exists x\, (P(x) \wedge Q(x))$

b) $\exists x\, (P(x) \wedge \sim Q(x))$

c) $\forall x\, (P(x) \vee Q(x))$

d) ~~$\sim[\forall x (P(x), Q(x))]$~~ $\forall x \sim (P(x) \vee Q(x))$

Q16) a) There are some students in your class who has sent an email message to some students in your class.

b) There are some students in your class who has sent an email message to all students in your class

c) Every student in your class has sent an email-message to atleast one student in your class

d) There are some student in your class who was sent an email message by every student in your class

e) Every student in your class has been sent a email message by atleast one student of your class

f) Every student in your class has sent an email message to every student in your class

Q17) a) Atleast one student has taken atleast one computer science class

b) Atleast one student has taken all computer science classes

c) All student has taken atleast one computer science class

d) Atleast one computer science class ~~was~~ is taken by all students

e) All computer science classes is taken by atleast one student

f) All students have taken all computer science classes

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Q18) a) $\frac{p}{\therefore p \vee q}$ (Addition)

b) $\frac{p \wedge q}{\therefore p}$ (Simplification)

c) $\frac{p \to r \quad p}{\therefore r}$ (Modus Ponen)

d) $\frac{p \to q \quad \sim q}{\therefore \sim p}$ (Modus Tollen)

e) $\frac{p \to q \quad q \to r}{\therefore p \to r}$ (Hypothetical Syllogism)

Q19) a) p: Today is twesday
q: I have a test in maths ~~or~~
r: I have a test in economics
s: My economics profs is sick

$p \to (q \vee r)$
$s \to \sim r$
$\frac{p \wedge s}{\therefore q}$

i) $p \wedge s$
2) p /s (Simplification)
$p \to (q \vee r)$
3) $q \vee r$ (Modus Ponen)
s
~~$s \to \sim r$~~
$s \to \sim r$
$\sim r$ (Modus Ponen)
$q \vee r$
$\therefore q$ (Elimination)

b) p: Ali is lauger
q: Ali is Ambitious
r: Ali is an early riser
s: Ali doesnot like chacolate

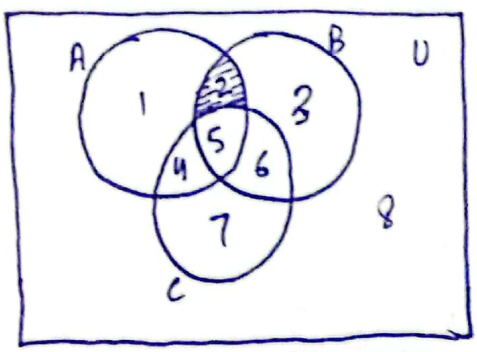
$p \to q$
$r \to s$
$\frac{q \to r}{\therefore p \to s}$

i) $p \to q$
$q \to r$ (Hypothetical Syllogism)
$p \to r$
$r \to s$ (Hypothetical Syllogism)
$\therefore p \to s$

Q20)

a) $(A \cap B) \cap \bar{C}$

- $A \cap B = \{2, 5\}$
- $\bar{C} = \{1, 2, 3, 8\}$
- $(A \cap B) \cap \bar{C} = \{2\}$



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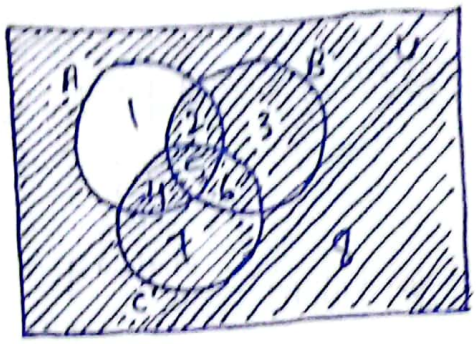
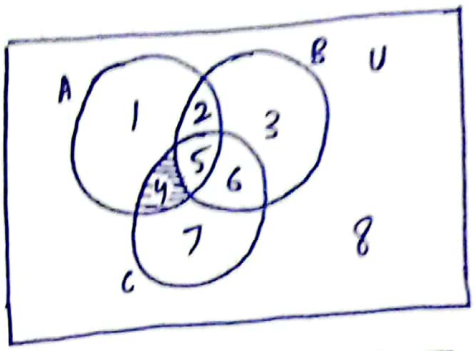
b) $\bar{A} \cup (B \cup C)$

- $\bar{A} = \{3,6,7,8\}$
- $B \cup C = \{2,3,4,5,6,7\}$
- $\bar{A} \cup (B \cup C) = \{2,3,4,5,6,7,8\}$



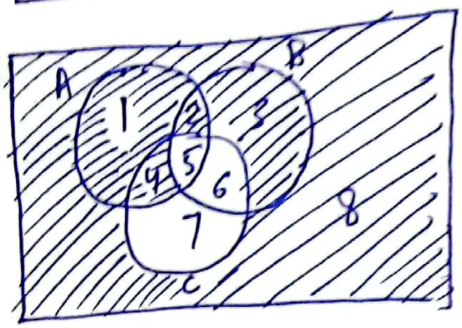
c) $(A-B) \cap C$

- $A-B \rightarrow A \cap \bar{B} = \{1,4\}$
  - $\bar{B} = \{1,4,7,8\}$
- $(A-B) \cap C = \{1,4\} \cap \{4,5,6,7\}$
  $= \{4\}$



d) $(A \cap \bar{B}) \cup \bar{C}$

- $\bar{B} = \{1,4,7,8\}$
- $A \cap \bar{B} = \{1,4\}$
- $(A \cap \bar{B}) \cup \bar{C} = \{1,2,3,4,8\}$
  - $\bar{C} = \{1,2,3,8\},4\}$



Q21) $(A-(A \cap B)) \cap (B-(A \cap B)) \equiv \emptyset$

a) $(A \cap \overline{(A \cap B)}) \cap (B \cap \overline{(A \cap B)})$

$(A \cap (\bar{A} \cup \bar{B})) \cap (B \cap (\bar{A} \cup \bar{B}))$ — equivalent $(A-B \equiv A \cap \bar{B})$

$((A \cap \bar{A}) \cup (A \cap \bar{B})) \cap ((B \cap \bar{A}) \cup (B \cap \bar{B}))$ — De Morgan

$(\emptyset \cup (A \cap \bar{B})) \cap (B \cap \bar{A}) \cup \emptyset)$ — Distributive, $A \cap \bar{A} = \emptyset$

$(A \cap \bar{B}) \cap (B \cap \bar{A})$ — Associative

$(A \cap \bar{A}) \cap (\bar{B} \cap B) \Rightarrow \emptyset \cap \emptyset = \emptyset$

b) $(A-B) \cup (A \cap B) = A$

$(A \cap \bar{B}) \cup (A \cap B)$ — equivalent

$A \cap (\bar{B} \cup B)$ — Distributive, $\bar{B} \cup B = U$

$A \cap U = A$ — $A \cap U = A$

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c) $(A-B)-C \equiv (A-C)-B$

$(A \cap \bar{B}) - \bar{C} = (A \cap \bar{B}) \cap \bar{C}$ - equivalent

$(A \cap \bar{C}) \cap \bar{B}$ Associative

$(A-C)-B$

d) $\overline{(\bar{B} \cup (\bar{B}-A))} \equiv B$

$\bar{\bar{B}} \cap (\overline{\bar{B}-A})$

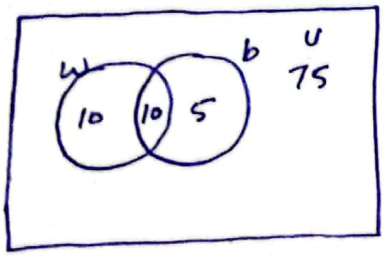
$B \cap (\overline{\bar{B} \cap \bar{A}})$ De Morgan

$B \cap (\bar{\bar{B}} \cup \bar{\bar{A}}) =$ Double compliment + - equivalent

$(B \cap B) \cup (B \cap A) = B \cup (B \cap A)$ De Morgan + Double Compliment

$= B$ Apsorption law

Q22) a) $U = 100$
$n(w) = 20$
$n(b) = 15$
$n(w \cap b) = 10$



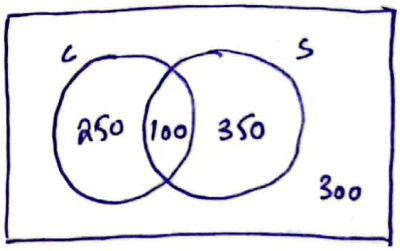
Sold = $U - n(w \cup B)$

$(w \cup B) = n(w) + n(B) - n(w \cap B)$

• $= 20 + 15 - 10 \Rightarrow 25$

• $100 - 25 = 75$

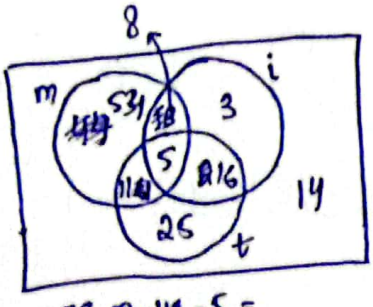
b) $U = 1000$
$n(c) = 350$
$n(s) = 450$
$n(c \cap s) = 100$
$\sim n(c \cup s) = ?$
$n(c \cup s) = ?$



① $n(c \cup s) = n(c) + n(s) - n(c \cap s)$
$= 350 + 450 - 100 \Rightarrow 700$

② $\sim n(c \cup s) = 1000 - 700 = 300$

c) $U = ?$
$n(m) = 78$
$n(I) = 32$
$n(t) = 57$
$n(t \cap I) = 21$
$n(m \cap I) = 13$
$n(t \cap m) = 16$
$n(t \cap m \cap I) = 5$
$\sim n(t \cup m \cup I) = 14$



• $78 - 38 - 116 - 5 =$

$78 + 32 + 57 - 16 - 21 - 13 + 14$

• $534 + 3 + 25 + 14 + 5 + 16 + 11 + 8$

$\Rightarrow 136$

d) $A \times (B \cap C) \equiv (A \times B) \cap (A \times C)$

$\{(x,y) : \{(x \in A) \wedge (y \in B)\} \cap \{(x,y) : \{(x \in A) \wedge (y \in C)\}$

$\{(x,y) : (x \in A) \wedge (x \in A) \wedge (y \in B) \wedge (y \in C)\}$

$\{(x,y) : (x \in A) \wedge (y \in B) \wedge (y \in C)\}$

$\{(x,y) : (x \in A) \wedge (y \in B \cap C)\}$

$= A \times (B \cap C)$

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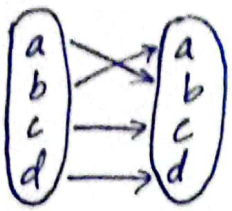
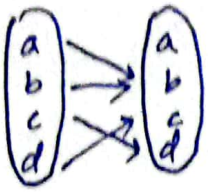
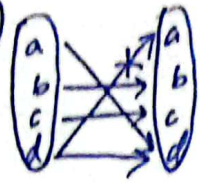
Q23) a)



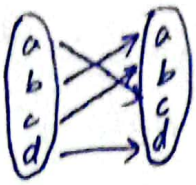
b)



c)



d)



(i) a) domain: {a,b,c,d}
Codomain: {a,b,c,d}
Range: {a,b,c,d}

b) domain: {a,b,c,d}
Codomain: {a,b,c,d}
Range: {b,c,d}

c) domain: {a,b,c,d}
Codomain: {a,b,c,d}
Range: {b,c,d}

d) domain: {a,b,c,d}
Codomain: {a,b,c,d}
Range: {a,b,c,d}

(ii) a) Bijective
(Injective & Surjective)

b) Neither

c) Neither

d) Bijective
(Injective & Surjective)

(iii) If bijective, inverse is possible, so,

a) Exists
$f^{-1}(b)=a, f^{-1}(a)=b,$
$f^{-1}(c)=c, f^{-1}(d)=d$

b) Non-Existant

c) Non-Existant

d) Exists
$f^{-1}(a)=b, f^{-1}(b)=c,$
$f^{-1}(c)=a, f^{-1}(d)=d$

Q24) a) $-2,-1,0,1,2,3$

- $\left\lfloor \frac{(-2)^2}{3} \right\rfloor = \frac{4}{3}\downarrow \to 1$
- $\left\lfloor \frac{(-1)^2}{3} \right\rfloor = \frac{1}{3}\downarrow \to 0$
- $\left\lfloor \frac{(0)^2}{3} \right\rfloor = 0$
- $\left\lfloor \frac{(1)^2}{3} \right\rfloor = \frac{1}{3}\downarrow \to 0$
- $\left\lfloor \frac{(2)^2}{3} \right\rfloor = \frac{4}{3}\downarrow \to 1$
- $\left\lfloor \frac{3^2}{3} \right\rfloor = \frac{9}{3} \to 3$
- $\{1,0,0,0,1,3\}$

b) $0,1,2,3,4,5$

- $\left\lfloor \frac{0}{3} \right\rfloor = 0$
- $\left\lfloor \frac{1}{3} \right\rfloor = 0$
- $\left\lfloor \frac{2^2}{3} \right\rfloor = \frac{4}{3}\downarrow \to 1$
- $\left\lfloor \frac{3^2}{3} \right\rfloor = 3$
- $\left\lfloor \frac{4^2}{3} \right\rfloor = \frac{16}{3}\downarrow = 5$
- $\left\lfloor \frac{5^2}{3} \right\rfloor = 8.33\downarrow = 8$
- $\{0,0,1,3,5,8\}$

c) $1,5,7,11$

- $\left\lfloor \frac{1}{3} \right\rfloor \downarrow = 0$
- $\left\lfloor \frac{5^2}{3} \right\rfloor = \frac{25}{3}\downarrow = 8$
- $\left\lfloor \frac{7^2}{3} \right\rfloor = \frac{49}{3}\downarrow = 16$
- $\left\lfloor \frac{11^2}{3} \right\rfloor = \frac{121}{3}\downarrow = 40$
- $\{0,8,16,40\}$

d) $2,6,10,14$

- $\left\lfloor \frac{2^2}{3} \right\rfloor = \frac{4}{3}\downarrow = 1$
- $\left\lfloor \frac{6^2}{3} \right\rfloor = \frac{36^{12}}{3} \Rightarrow 12$
- $\left\lfloor \frac{10^2}{3} \right\rfloor = \frac{100}{3}\downarrow = 33$
- $\left\lfloor \frac{14^2}{3} \right\rfloor = \frac{196}{3}\downarrow = 65$
- $\{1,12,33,65\}$

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b) i) $\left\lceil \frac{3}{4} \right\rceil = 0.75 \uparrow = 1$ ii) $\left\lfloor \frac{7}{8} \right\rfloor = 0.875 \downarrow = 0$ iii) $\left\lceil -\frac{3}{4} \right\rceil = -0.75 \uparrow = 0$

iv) $\left\lfloor -\frac{7}{8} \right\rfloor = -0.875 \downarrow = -1$ v) $\lceil 3 \rceil = 3$ vi) $\lfloor -1 \rfloor = -1$

vii) $\left\lfloor \frac{1}{2} + \left\lceil \frac{3}{2} \right\rceil \right\rfloor$

$\left\lfloor \frac{1}{2} + 2 \right\rfloor = 2.5 \downarrow = 2$

viii) $\left\lfloor \frac{1}{2} \cdot \left\lfloor \frac{5}{2} \right\rfloor \right\rfloor$

$\left\lfloor \frac{1}{2} \cdot 2 \right\rfloor = 1$

c) $\lfloor -x \rfloor = -\lceil x \rceil$

Taking arbitary value: 1.5

- $\lfloor -1.5 \rfloor \rightarrow -1.5 \downarrow = -2$
- $-\lceil 1.5 \rceil \rightarrow 1.5 \uparrow = -2$

$-2 = -2$

Proved !

$\lceil -x \rceil = -\lfloor x \rfloor$

Taking arbitary value: 1.5

- $\lceil -1.5 \rceil = -1.5 \uparrow = -1$
- $-\lfloor 1.5 \rfloor = 1.5 \downarrow = -1$

Q.25) a). fog

$2(3a+2)+3$

$6a+4+3 \Rightarrow \underline{6a+7}$

. gof

$3(2a+3)+2$

$6a+9+2 \Rightarrow \underline{6a+11}$

b)