Course Code: CS1005	Course Name: Discrete Structures
Instructor Names: Mr. Shoaib Raza, Ms. Safia, Mr. Fahad Hussain and Ms. Naz Memon	
Student Roll No:	Section No:

Instructions:

- Return the question paper together with the answer script. Read each question completely before answering it. There are **5 questions** written on **4 pages.**
- In case of any ambiguity, you may make assumptions. However, your assumptions should not contradict any statement in the question paper.
- **Attempt all the questions in the given sequence of the question paper. Show all steps properly in order to get full points.**

Total Time: 03 Hours   **Maximum Points**: 80

Question # 1:   [CLO -3]   **[8 x 2 = 16 points]**

(a) Let P, Q, and R be the propositions

   P: The exam date is announced.   **Q**: The papers have been ready.   **R**: 20% students expected to be failed.

Write these propositions using P, Q, and R and logical connectives (including negations):
   (i) 20% students expected to be failed if the papers have been ready.
   (ii) The papers have not been ready only if the exam date is not announced.
   (iii) The paper have been ready but 20% students are not expected to be failed.

(b) Using the premises (statements) from part (a), apply rules of inference to obtain conclusion(s) from these premises. Name the rules of inference used to obtain each conclusion from the premises.

(c) Using a truth table, prove or disprove that the premises (statements) from part (a) and conclusion(s) from part (b) form a tautology.

(d) Determine using laws of logic if the following statement is a tautology, contradiction or a contingency.
   $((p \lor q) \land (p \to r)) \to (q \lor r)$

(e) Let C(x) be the statement "x is a comedian" and F(x) be the statement "x is funny," where the domain for the x consists of all the humans. Use quantifiers to express each of these statements.
   (i) If a human is comedian then this human is funny.
   (ii) There exists a human that is comedian and funny.

(f) Write the English statement using the predicates from part (e) and any needed quantifiers:
   (i) $\exists x (C(x) \to F(x))$
   (ii) $\neg \exists x (C(x) \land F(x))$

(g) Using logical equivalences, prove or disprove the following set operations:
   A ∩ (B U C) = (A∩B) U (B∩C)

(h) Of 21 typists in an office, 5 use all manual typewriters (M), electronic typewriters (E) and word processors (W); 9 use E and W; 7 use M and W; 6 use M and E; but no one uses M only. Represent this information in a Venn diagram.

Question # 2: [CLO -2] **[8 x 2 = 16 points]**

(a) Let f: R → R be defined by $f(x) = 2x - 3$, Find an inverse of a function f if exists, otherwise give the reason.

(b) Consider the childhood game Rock, Paper, and Scissors. "rock beats scissors, scissors beat paper, and paper beats rock." Construct the adjacency matrix that represents the "beats" relation.

(c) Is "beats" relation in part (b) is a Partial Order Relation? Show all properties.

(d) Find the sum of numbers between 10 and 50 which are divisible by 3 or 7.

(e) Express in sigma notation the sum of the first n terms of the series 1 + 8 + 27 + 64 + 125 +. ...

(f) Proof the expression obtained in part (e) with the help of mathematical induction.

(g) Prove or disprove by counterexample: For every positive integer n, $n! \leq n^2$.

(h) Find the minimum number of students needed to guarantee that 3 of them have last names which begin with the same first letter.

Question # 3: [CLO -2] **[8 x 2 = 16 points]**

A rider is travelling from Shahrah-e-Faisal to Lucknow Society to deliver an order. He needs to reach the destination in no more than 30 minutes. He has limited fuel so it is necessary to stop at some petrol pump in order to refill. He can use following available points for his route. Distances between these points are given in table 01.

| Location A	Location B	Distance
Shahrah-e-Faisal	Attock Pump	8.5km
Shahrah-e-Faisal	Brooks Chowrangi	4.8km
Brooks Chowrangi	Attock Pump	2.1km
Attock Pump	Lucknow Society	1.3km
Brooks Chowrangi	Lucknow Society (Directly)	3.3km

Table 01: Distances between the points

(a) Represent this scenario using a connected graph.

(b) Determine whether the given graph obtained in part (a) has a Euler Circuit or Path. Construct such a circuit or path when one exists. If no provide valid reason.

(c) Determine whether the given graph obtained in part (a) has a Hamilton Circuit or Path. Construct such a circuit or path when one exists. If no provide valid reason.

(d) Determine the Minimum Cost Route from Shahrah-e-Faisal to Lucknow Society using Prim's Algorithm. Write the order of edges added and the cost of Minimum Spanning Tree.

(e) Consider the tree obtained in part (d). Determine the names of node(s) at level 2 and height of the tree.

(f) Consider the tree obtained in part (d).
     (i) Determine whether it is a full m-ary tree or not? Give reason.
     (ii) Determine whether it is a balanced m-ary tree or not? Give reason.

(g) Draw a bipartite graph from the set X = {1, 3, 7, 15, 31} to the set Y = {1, 3, 6, 10, 15} with edges defined by the relatively prime relation, that is, the greatest common denominator of x and y, gcd (x, y), is not equal to 1.

(h) Determine that whether given pair of graph G and H in figure 01 are isomorphic or not. If they are, give function F: V(G) → V(H) that define the isomorphism. If they are not, give reason.

Figure 01
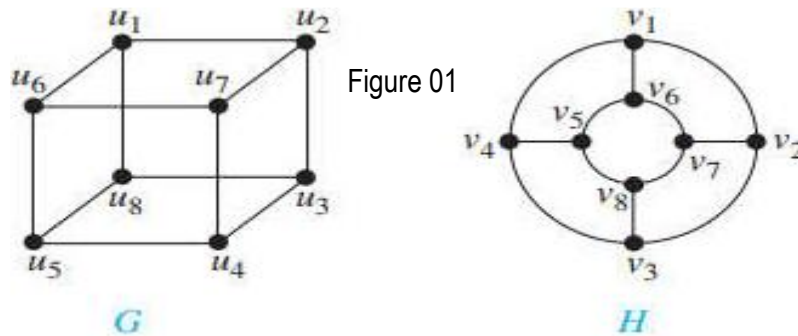
Question # 4:   **[CLO -3]**   **[8 x 2 = 16 points]**

Suppose that a store sells six varieties of soft drinks: Cola, Ginger ale, Orange, Root beer, Lemonade, and Cream soda. if all varieties are available in 12-ounce bottles, all but lemonade are not available in 20-ounce bottles, only cola and ginger ale are available in 32-ounce bottles, and all but neither lemonade nor cream soda are available in 64-ounce bottles.

(a) Use a tree diagram to determine the number of different types of bottles the store must stock to have all varieties available in all size bottles.

(b) Show pre-order traversal of the tree obtained in Part (a).

(c) Using direct proof method, prove that If n is an integer such that n – 2 is divisible by 3, then $n^2 - 1$ is divisible by 3.

(d) Using contradiction method, prove that there are no integer x and y such that $x^2 = 4y + 2$.

(e) Prove by contraposition that for all integers a and b, if a + b is odd then a is odd or b is odd.

(f) There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

(g) Thirteen people on a softball team show up for a game. How many ways are there to choose 10 players to take the field?

(h) What is the co-efficient of $x^7 y^2$ in the expansion of $(x + 3y)^9$.

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Question # 5:   [CLO -2]   **[8 x 2 = 16 points]**

(a) A data entry operator is entering books in a library system with their ISBNs, while entering the data he saw a previously added ISBN '*1259731286*' which he feels is erroneous. How can he verify if the previously entered ISBN is correct or not.

(b) Find the check digit of the following Universal Product Code (UPC): 69277198116.

(c) Suppose that a computer has only the memory locations 0, 1, 2. . . 64. Use the hashing function $h(x) = (x + 9)$ mod 65 to determine the memory locations at which the following values are stored:
63, 509, 197, 832, and 652.

(d) List all integers between −100 and 100 that are congruent to −1 modulo 25.

(e) Find the greatest common divisor, d, of 250 and 29 and determine integers x and y such that $d = 250x + 29y$.

(f) A message has been encrypted using the function $f(x) = (x + 7)$ mod 26. If the message in coded form is XBLZAPVU WHWLY, decode the message.

(g) Use Fermat's little theorem to calculate the remainder of $2^{1,000,00}$ mod 127.

(h) Encrypt the message "*Solution*" using the RSA system with $n = 5 \cdot 7$ and $e = 11$. Translate each letter into integers and write in the form of Cipher text equation.

BEST OF LUCK ☺