Test Your Understanding: proof

Test yourself by filling in the blanks.

1. An integer is even if, and only if, it ____.
2. An integer is odd if, and only if, it ____.
3. An integer is prime if, and only if, ____.
4. An integer is composite if, and only if, ____.
5. If $n = 2k + 1$ for some integer k, then ____.
6. Given integers a and b, if there exists an integer k such that $b = ak$, then ____.
7. To find a counterexample for a statement of the form "$\forall x \in D$, if $P(x)$ then $Q(x)$" you find ____.
8. According to the method of generalizing from the generic particular, to prove that every element of a domain satisfies a certain property, you suppose that ____ and you show that ____.
9. According to the method of direct proof, to prove that a statement of the form "$\forall x$ in D, if $P(x)$ then $Q(x)$" is true, you suppose that ____ and you show that ____.
10. Proofs should always be written in ____ sentences, and each assertion made in a proof should be accompanied by a ____.
11. The fact that a universal statement is true in some instances does not imply that it is ____.
12. When writing a proof, it is a mistake to use the same letter to represent ____.
13. A real number is rational if, and only if, ____.
14. An integer a divides an integer b if, and only if, ____.
15. If a and b are integers, the notation $a \mid b$ stands for ____, and the notation a/b stands for ____.
16. According to the theorem about divisibility by a prime number, given any integer $n > 1$, there is a ____.
17. The unique factorization theorem (fundamental theorem of arithmetic) says that given any integer $n > 1$, n can be written as a ____ in a way that is unique, except possibly for the ____ in which the numbers are written.
18. The quotient-remainder theorem says that given any integer n and any positive integer d, there exist unique integers q and r such that ____.
19. If n is a nonnegative integer and d is a positive integer, then n *div* $d =$ ____ and n *mod* $d =$ ____ where ____.
20. The parity property says that any integer is either ____.

Answers

1. equals twice some integer
2. equals twice some integer plus 1
3. it is greater than 1, and if it is written as a product of positive integers, then one of the integers is 1
4. it is greater than 1, and it can be written as a product of positive integers neither of which is 1
5. n is an odd integer
6. a divides b (or $a \mid b$, or a is a factor of b; or a is a divisor of b; or b is divisible by a; or b is a multiple of a)
7. an element of D for which $P(x)$ is true and $Q(x)$ is false
8. you have a particular but arbitrarily chosen element of the domain
   that element satisfies the property
9. x is any *[particular but arbitrarily chosen]* element of D for which $P(x)$ is true
   $Q(x)$ is true
10. complete; reason that justifies the assertion
11. true in all instances
12. two different quantities
13. it can be written as a ratio of integers with a nonzero denominator
14. there is an integer, say k, such that $b = ak$
15. the sentence "a divides b"; the real number a divided by b (if $b \neq 0$)
16. prime number that divides n
17. product of prime numbers, order
18. $n = dq + r$ and $0 \leq r < d$
19. q; r; $n = dq + r$ and $0 \leq r < d$
20. even or odd