Test Your Understanding: function

Test yourself by filling in the blanks.

1. A function f from a set X to a set Y is a relation between elements of X (called inputs) and elements of Y (called outputs) such that ____ input element of X is related to ____ output element of Y.
2. Given a function f from a set X to a set Y, $f(x)$ is ____.
3. Given a function f from a set X to a set Y, if $f(x) = y$, then y is called ____ or ____ or ____ or ____.
4. Given a function f from a set X to a set Y, the range of f (or the image of X under f) is ____.
5. Given a function f from a set X to a set Y, if $f(x) = y$, then x is called ____ or ____.
6. Given a function f from a set X to a set Y, if $y \in Y$, then $f^{-1}(y) =$ ____ and is called ____.
7. Given functions f and g from a set X to a set Y, $f = g$ if, and only if, ____.
8. Given positive real numbers x and b with $b \neq 1$, $\log_b x =$ ____.
9. If F is a function from a set X to a set Y, then F is one-to-one if, and only if, ____.
10. If F is a function from a set X to a set Y, then F is not one-to-one if, and only if, ____.
11. If F is a function from a set X to a set Y, then F is onto if, and only if, ____.
12. If F is a function from a set X to a set Y, then F is not onto if, and only if, ____.
13. The following two statements are ____:
    $\forall\, u,v \in U$, if $H(u) = H(v)$ then $u = v$.
    $\forall\, u,v \in U$, if $u \neq v$ then $H(u) \neq H(v)$.
14. Given a function F: $X \to Y$ (where X is an infinite set or a large finite set), to prove that F is one-to-one, you suppose that ____ and then you show that ____.
15. Given a function F: $X \to Y$) (where X is an infinite set or a large finite set), to prove that F is onto, you suppose that ____ and then you show that ____.
16. Given a function F: $X \to Y$, to prove that F is not one-to-one, you ____.
17. Given a function F: $X \to Y$, to prove that F is not onto, you ____.
18. A one-to-one correspondence from a set X to a set Y is a ____ that is ____.
19. If F is a one-to-one correspondence from a set X to a set Y and y is in Y, then $F^{-1}(y)$ is ____.
20. The pigeonhole principle states that ____.

21. The generalized pigeonhole principle states that ____.

22. If X and Y are finite sets and f is a function from X to Y then f is one-to-one if, and only if, ____.

23. If f is a function from X to Y and g is a function from Y to Z, then $g \circ f$ is a function from ____ to ____, and $(g \circ f)(x)$ ____ for all x in X.

24. If f is a function from X to Y and i_X and i_Y are the identity functions from X to X and Y to Y, respectively, then $f \circ i_X =$ ____ and $i_Y \circ f =$ ____.

25. If f is a one-to-one correspondence from X to Y, then $f^{-1} \circ f =$ ____ and $f \circ f^{-1} =$ ____.

26. If f is a one-to-one function from X to Y and g is a one-to-one function from Y to Z, you prove that $g \circ f$ is one-to-one by supposing that ____ and then showing that ____.

27. If f is an onto function from X to Y and g is an onto function from Y to Z, you prove that $g \circ f$ is onto by supposing that ____ and then showing that ____.

28. A set is finite if, and only if, ____.

29. To prove that a set A has the same cardinality as a set B you must ____.

30. Given a set A, the reflexive property of cardinality says that ____.

31. Given sets A and B, the symmetric property of cardinality says that ____.

32. Given sets A, B, and C, the transitive property of cardinality says that ____.

33. A set is called countably infinite if, and only if, ____.

34. A set is called countable if, and only if, ____.

35. In each of the following, fill in the blank with the word countable or the word uncountable.

    (a) The set of all integers is ____.
    (b) The set of all rational numbers is ____.
    (c) The set of all real numbers between 0 and 1 is ____.
    (d) The set of all real numbers is ____.
    (e) The set of all computer programs in a given computer language is ____.
    (f) The set of all functions from the set of all positive integers, $\mathbf{Z}^+$, to $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is ____.

Answers

1. each, one and only one
2. the unique output element y in Y that is related to x by f
3. the value of f at x; the image of x under f; the output of f for the input x
4. the set of all y in Y such that $f(x) = y$
5. an inverse image of y under f; a preimage of y
6. $\{x \in X \mid f(x) = y\}$; the inverse image of y
7. $f(x) = g(x)$ for all $x \in X$
8. the exponent to which b must be raised to obtain x
   Or: $\log_b y = x \Leftrightarrow b^y = x$
9. for all x_1 and x_2 in X, if $F(x_1) = F(x_2)$ then $x_1 = x_2$
10. there exist elements x_1 and x_2 in X such that $F(x_1) = F(x_2)$ and $x_1 \neq x_2$
11. for all y in Y, there exists at least one element x in X such that $f(x) = y$
12. there exists an element y in Y such that for all elements x in X, $f(x) \neq y$
13. logically equivalent ways of expression what it means for H to be a one-to-one function (The second way is the contrapositive of the first.)
14. x_1 and x_2 are any *[particular but arbitrarily chosen]* elements in X with the property that $F(x_1) = F(x_2)$; $x_1 = x_2$
15. y is any *[particular but arbitrarily chosen]* element in Y; there exists at least one element x in X such that $F(x) = y$
16. show that there are concrete elements x_1 and x_2 in X with the property that $F(x_1) = F(x_2)$ and $x_1 \neq x_2$
17. show that there is a concrete element y in Y with the property that $F(x) \neq y$ for any element x in X
18. function from X to Y; one-to-one and onto
19. the unique element x in X such that $F(x) = y$ (in other words, $F^{-1}(y)$ is the unique preimage of y in X)
20. if n pigeons fly into m pigeonholes and $n > m$, then at least two pigeons fly into the same pigeonhole
    Or: given any function from a finite set to a smaller finite set, there must be at least two elements in the function's domain that have the same image in the function's co-domain
    Or: a function from one finite set to a smaller finite set cannot be one-to-one
21. if n pigeons fly into m pigeonholes and, for some positive integer k, $n > mk$, the at least one pigeonhole contains $k + 1$ or more pigeons
    Or: for any function f from a finite set X to a finite set Y and for any positive integer k, if $N(X) > k \cdot N(Y)$, then there is some $y \in Y$ such that y is the image of at least $k + 1$ distinct elements of Y
22. f is onto

23. X; Z; $g(f(x))$
24. f; f
25. i_X; i_Y
26. x_1 and x_2 are any *[particular but arbitrarily chosen]* elements in X with the property that $(g \circ f)(x_1) = (g \circ f)(x_2)$; $x_1 = x_2$
27. z is any *[particular but arbitrarily chosen]* element in Z; there exists at least one element x in X such that $(g \circ f)(x) = z$
28. it is the empty set or there is a one-to-one correspondence from $\{1, 2, \ldots, n\}$ to it, where n is a positive integer
29. show that there exists a function from A to B that is one-to-one and onto;
    Or: show that there exists a one-to-one correspondence from A to B
30. A has the same cardinality as A
31. if A has the same cardinality as B, then B has the same cardinality as A
32. if A has the same cardinality as B and B has the same cardinality as C, then A has the same cardinality as C
33. it has the same cardinality as the set of all positive integers
34. it is finite or countably infinite
35. countable; countable; uncountable; uncountable; countable; uncountable