20.2. Exercises

(1) Suppose that A is a 3×3 matrix such that $\langle A\mathbf{x}, \mathbf{x}\rangle = {x_1}^2 + 5{x_2}^2 - 3{x_3}^2 + 6x_1x_2 - 4x_1x_3 + 8x_2x_3$ for all $\mathbf{x} \in \mathbb{R}^3$. Then $A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$ where $a =$ ___ , $b =$ ___ , $c =$ ___ , $d =$ ___ , $e =$ ___ , and $f =$ ___ .

(2) A curve C is given by the equation $2x^2 - 72xy + 23y^2 = 50$. What kind of curve is C?
Answer: It is a(n) ______________________________ .

(3) The equation $5x^2 + 8xy + 5y^2 = 1$ describes an ellipse. The principal axes of the ellipse lie along the lines $y =$ _____ and $y =$ _____ .

(4) The graph of the equation $13x^2 - 8xy + 7y^2 = 45$ is an ellipse. The length of its semimajor axis is _____ and the length of its semiminor axis is _____ .

(5) Consider the equation $2x^2 + 2y^2 - z^2 - 2xy + 4xz + 4yz = 3$.

(a) The graph of the equation is what type of quadric surface?
Answer: ____ .

(b) In standard form the equation for this surface is
$u^2 + v^2 +$ _____ $w^2 =$ _____ .

(c) Find three orthonormal vectors with the property that in the coordinate system they generate, the equation of the surface is in standard form.
Answer: $\frac{1}{\sqrt{6}}$ (1 , ___ , ___), $\frac{1}{\sqrt{2}}$ (___ , ___ , 0), and $\frac{1}{\sqrt{3}}$ (1 , ___ , ___).

(6) Determine for each of the following matrices whether it is positive definite, positive semidefinite, negative definite, negative semidefinite, or indefinite.

(a) The matrix $\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$ is ______________________________ .

(b) The matrix $\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & -1 & -2 \end{bmatrix}$ is ______________________________ .

(7) Determine for each of the following matrices whether it is positive definite, positive semidefinite, negative definite, negative semidefinite, or indefinite.

(a) The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 9 \end{bmatrix}$ is ______________________________ .

(b) The matrix $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 6 & -2 & 0 \\ 0 & -2 & 5 & -2 \\ 0 & 0 & -2 & 3 \end{bmatrix}$ is ______________________________ .

(c) The matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}^2$ is ______________________________ .

(8) Let $B = \begin{bmatrix} 2 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{bmatrix}$. For what range of values of b is B positive definite?
Answer: ______________________________ .

(9) Let $A = \begin{bmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{bmatrix}$. For what range of values of a is A positive definite?

Answer: ______________________________ .