Use truth tables to determine whether the argument forms in 6–11 are valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. Your explanation should show that you understand what it means for a form of argument to be valid or invalid.

$$\begin{array}{l} p \vee q \\ p \to \sim q \\ p \to r \\ \therefore r \end{array}$$

			premises				conclusion
p	q	r	$\sim q$	$p \vee q$	$p \to \sim q$	$p \to r$	r
T	T	T	F	T	F	T	
T	T	F	F	T	F	F	
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	
F	T	T	F	T	T	T	T
F	T	F	F	T	T	T	F
F	F	T	T	F	T	T	
F	F	F	T	F	T	T	

This row shows that it is possible for an argument of this form to have true premises and a false conclusion. Thus this argument form is invalid.

Rules of Inference

Section 1.6

TABLE 1 **Rules of Inference.**

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism

TABLE 1 **Rules of Inference.**

Rule of Inference	*Tautology*	*Name*
$\begin{array}{rl} & p \\ \hline \therefore & p \vee q \end{array}$	$p \rightarrow (p \vee q)$	Addition
$\begin{array}{rl} & p \wedge q \\ \hline \therefore & p \end{array}$	$(p \wedge q) \rightarrow p$	Simplification
$\begin{array}{rl} & p \\ & q \\ \hline \therefore & p \wedge q \end{array}$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$\begin{array}{rl} & p \vee q \\ & \neg p \vee r \\ \hline \therefore & q \vee r \end{array}$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Modus Ponens

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Corresponding Tautology:

$(p \text{ A } (p \rightarrow q)) \rightarrow q$

Example:

Let *p* be “It is snowing.”
Let *q* be “I will study discrete math.”

“If it is snowing, then I will study discrete math.”
“It is snowing.”

“Therefore , I will study discrete math.”

Modus Ponens

- State which rule of inference is the basis of the following argument:
- “If it snows today, then we will go skiing”. “It is snowing today”, “therefore we will go skiing”.
- Let p be the proposition “It is snowing today” and q the proposition “We will go skiing” Then this argument is of the form

$$\begin{array}{l} p \rightarrow q \\ p \\ \therefore q \end{array}$$

- This is an argument that uses the Modus Ponens rule.

Modus Tollens

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Corresponding Tautology:
$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$

Example:
Let *p* be "it is snowing."
Let *q* be "I will study discrete math."

"If it is snowing, then I will study discrete math."
"I will not study discrete math."

"Therefore , it is not snowing."

Recognizing Modus Ponens and Modus Tollens

Use modus ponens or modus tollens to fill in the blanks of the following arguments so that they become valid inferences.

a. If there are more pigeons than there are pigeonholes, then at least two pigeons roost in the same hole.
   There are more pigeons than there are pigeonholes.
   $\therefore$ ______________________________.

b. If 870,232 is divisible by 6, then it is divisible by 3.
   870,232 is not divisible by 3.
   $\therefore$ ______________________________.

Solution

a. At least two pigeons roost in the same hole. by modus ponens

b. 870,232 is not divisible by 6. by modus tollens

Class Activity

Use modus ponens or modus tollens to fill in the blanks in the arguments of 1–5 so as to produce valid inferences.

1. If $\sqrt{2}$ is rational, then $\sqrt{2} = a/b$ for some integers a and b.
   It is not true that $\sqrt{2} = a/b$ for some integers a and b.
   $\therefore$ ________________________.

2. If $1 - 0.99999\ldots$ is less than every positive real number, then it equals zero.
   ________________________.
   $\therefore$ The number $1 - 0.99999\ldots$ equals zero.

3. If logic is easy, then I am a monkey's uncle.
   I am not a monkey's uncle.
   $\therefore$ ________________________.

4. If this figure is a quadrilateral, then the sum of its interior angles is $360°$.
   The sum of the interior angles of this figure is not $360°$.
   $\therefore$ ________________________.

Hypothetical Syllogism

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

Corresponding Tautology:
((p →q) A (q→r))→(p→ r)

Example:
Let *p* be “it snows.”
Let *q* be “I will study discrete math.”
Let *r* be “I will get an A.”

“If it snows, then I will study discrete math.”
“If I study discrete math, I will get an A.”

“Therefore , If it snows, I will get an A.”

Hypothetical Syllogism

- State which rule of inference is used in the argument:
- If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will have a barbecue tomorrow.
- Therefore, if it rains today, then we will have a barbecue tomorrow.

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\therefore p \rightarrow r$$

- Hence, this argument is a hypothetical syllogism.

What will be conclusion ?

If 18,486 is divisible by 18, then 18,486 is divisible by 9.

If 18,486 is divisible by 9, then the sum of the digits of 18,486 is divisible by 9.

$\therefore$ If 18,486 is divisible by 18, then the sum of the digits of 18,486 is divisible by 9.

Disjunctive Syllogism

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Corresponding Tautology:
$(\neg p \wedge (p \vee q)) \rightarrow q$

Example:
Let *p* be “I will study discrete math.”
Let *q* be “I will study English literature.”

“I will study discrete math or I will study English literature.”
“I will not study discrete math.”

“Therefore , I will study English literature.”

Addition

$$\frac{p}{\therefore p \vee q}$$

Corresponding Tautology:

$p \rightarrow (p \vee q)$

Example:

Let p be "I will study discrete math."
Let q be "I will visit Las Vegas."

"I will study discrete math."

"Therefore, I will study discrete math or I will visit Las Vegas."

Addition Rule

- State which rule of inference is the basis of the following argument:
- “It is below freezing now. Therefore, it is either below freezing or raining now.”
- Let p be the proposition “It is below freezing now” and q the proposition “It is raining now.” Then this argument is of the form

$$p$$
$$\therefore p \vee q$$

- This is an argument that uses the addition rule.

Simplification

$$\frac{p \wedge q}{\therefore\ q}$$

Corresponding Tautology:

$(p\mathrm{A}q) \rightarrow p$

Example:
Let *p* be “I will study discrete math.”
Let *q* be “I will study English literature.”

“I will study discrete math and English literature”

“Therefore, I will study discrete math.”

Simplification Rule

- State which rule of inference is the basis of the following argument:
- "It is below freezing and raining now. Therefore, it is below freezing now."
- Let p be the proposition "It is below freezing now," and let q be the proposition "It is raining now." This argument is of the form

pAq

∴p

- This argument uses the simplification rule.

Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Corresponding Tautology:
$((p) \wedge (q)) \rightarrow (p \wedge q)$

Example:
Let *p* be "I will study discrete math."
Let *q* be "I will study English literature."

"I will study discrete math."
"I will study English literature."

"Therefore, I will study discrete math and I will study English literature."

Resolution

Resolution plays an important role in AI and is used in Prolog.

$$\begin{array}{l} \neg p \vee r \\ p \vee q \\ \hline \therefore q \vee r \end{array}$$

Corresponding Tautology:

$((\neg p \vee r) \wedge (p \vee q)) \rightarrow (q \vee r)$

Example:
Let *p* be "I will study discrete math."
Let *r* be "I will study English literature."
Let q be "I will study databases."

"I will not study discrete math or I will study English literature."
"I will study discrete math or I will study databases."

"Therefore, I will study databases or I will English literature."

Fallacies

- Arguments are based on tautologies.
- Fallacies are based on contingencies.
- The proposition $((p \to q) \wedge q) \to p$ is not a tautology, because it is false when p is false and q is true.
- This type of incorrect reasoning is called the **fallacy of affirming the conclusion.**
- Example:
- If you do every problem in this book, then you will learn discrete mathematics. You learned discrete mathematics. Therefore, you did every problem in this book.

Fallacy of denying the hypothesis.

- The proposition $((p \rightarrow q) \wedge \neg p) \rightarrow \neg q$ is not a tautology, because it is false when p is false and q is true.
- Is it correct to assume that you did not learn discrete mathematics if you did not do every problem in the book, assuming that if you do every problem in this book, then you will learn discrete mathematics?
- It is possible that you learned discrete mathematics even if you did not do every problem in this book. This incorrect argument is of the form $p \rightarrow q$ and $\neg p$ imply $\neg q$,

which is an example of the fallacy of denying the hypothesis.

Using Rules of Inference

Fallacies

- Are the following arguments correct?
  - Example 1 (Fallacy of affirming the conclusion)

    Hypothesis
    - If you success, you work hard $\quad p \to q$
    - You work hard $\quad q$

    Conclusion
    - You success $\quad \therefore\ p$

  - Example 2 (Fallacy of denying the hypothesis)

    Hypothesis
    - If you success, you work hard $\quad p \to q$
    - You do not success $\quad \neg p$

    Conclusion
    - You do not work hard $\quad \therefore\ \neg q$

Class Activity

1	Not p
2	$r \longrightarrow p$

3	
5	Not $r \longrightarrow s$

6	
7	$s \longrightarrow t$

8	

TABLE 1 Rules of In

Rule of Inference	*Name*
p $p \to q$ $\therefore q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism
p $\therefore p \vee q$	Addition
$p \wedge q$ $\therefore p$	Simplification
p q $\therefore p \wedge q$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	Resolution

Using Rules of Inference

- Example 1:
  - **Given:**
    - It is not sunny this afternoon and it is colder than yesterday.
    - We will go swimming only if it is sunny
    - If we do not go swimming, then we will take a canoe trip
    - If we take a canoe trip, then we will be home by sunset
  - Can these propositions lead to the **conclusion** "We will be home by sunset" ?

Let	p:	It is sunny this afternoon
	q:	It is colder than yesterday
	r:	We go swimming
	s:	We take a canoe trip
	t:	We will be home by sunset

$\neg p \wedge q$	It is not sunny this afternoon and it is colder than yesterday
$r \to p$	We will go swimming only if it is sunny
$\neg r \to s$	If we do not go swimming, then we will take a canoe trip
$s \to t$	If we take a canoe trip, then we will be home by sunset
t	We will be home by sunset

Using Rules of Inference

Hypothesis:

$\neg p \wedge q$

$r \rightarrow p$

$\neg r \rightarrow s$

$s \rightarrow t$

Conclusion:

t

Step		Reason
1.	$\neg p \wedge q$	Premise
2.	$\neg p$	Simplification using (1)
3.	$r \rightarrow p$	Premise
4.	$\neg r$	Modus tollens using (2) and (3)
5.	$\neg r \rightarrow s$	Premise
6.	s	Modus ponens using (4) and (5)
7.	$s \rightarrow t$	Premise
8.	t	Modus ponens using (6) and (7)

Therefore, the propositions can lead to the conclusi
We will be home by sunset

TABLE 1 Rules of Ir

Rule of Inference	Name
p $p \rightarrow q$ $\therefore q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism
p $\therefore p \vee q$	Addition
$p \wedge q$ $\therefore p$	Simplification
p q $\therefore p \wedge q$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	Resolution

Using Rules of Inference

- Or, another presentation method:

Hypothesis:

$\neg p \wedge q$

$r \rightarrow p$

$\neg r \rightarrow s$

$s \rightarrow t$

Conclusion:

t

$(\neg p \wedge q) \wedge (r \rightarrow p) \wedge (\neg r \rightarrow s) \wedge (s \rightarrow t)$

$\Rightarrow \neg p \wedge (r \rightarrow p) \wedge (\neg r \rightarrow s) \wedge (s \rightarrow t)$ By Simplification

$\Rightarrow \neg r \wedge (\neg r \rightarrow s) \wedge (s \rightarrow t)$ By Modus Tollens

$\Rightarrow s \wedge (s \rightarrow t)$ By Modus Ponens

$\Rightarrow t$ By Modus Ponens

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EXAMPLE #2

- **Given:**
  - If you send me an e-mail message, then I will finish writing the program
  - If you do not send me an e-mail message, then I will go to sleep early
  - If I go to sleep early, then I will wake up feeling refreshed
- Can these propositions lead to the **conclusion** "If I do not finish writing the program, then I will wake up feeling refreshed."

Let p: you send me an e-mail message
q: I will finish writing the program
r: I will go to sleep early
s: I will wake up feeling refreshed

$p \to q$
- If you send me an e-mail message, then I will finish writing the program

$\neg p \to r$
- If you do not send me an e-mail message, then I will go to sleep early

$r \to s$
- If I go to sleep early, then I will wake up feeling refreshed

---

$\neg q \to s$
- If I do not finish writing the program, then I will wake up feeling refreshed

Hypothesis:

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

Conclusion:

$\neg q \rightarrow s$

Step		Reason
1.	$p \rightarrow q$	Premise
2.	$\neg q \rightarrow \neg p$	Contrapositive of (1)
3.	$\neg p \rightarrow r$	Premise
4.	$\neg q \rightarrow r$	Hypothetical Syllogism using (2) and (3)
5.	$r \rightarrow s$	Premise
6.	$\neg q \rightarrow s$	Hypothetical Syllogism using (4) and (5)

Therefore, the propositions can lead to the conclusion
If I do not finish writing the program,
then I will wake up feeling refreshed

TABLE 1 Rules of In

Rule of Inference	*Name*
p $p \rightarrow q$ $\therefore q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism
p $\therefore p \vee q$	Addition
$p \wedge q$ $\therefore p$	Simplification
p q $\therefore p \wedge q$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	Resolution

- Or, another presentation method:

Hypothesis:

$p \to q$

$\neg p \to r$

$r \to s$

Conclusion:

$\neg q \to s$

$$\boxed{(p \to q)} \land (\neg p \to r) \land (r \to s)$$

$$\Leftrightarrow \boxed{(\neg q \to \neg p) \land (\neg p \to r)} \land (r \to s)$$ Contrapositive

$$\Rightarrow \boxed{(\neg q \to r) \land (r \to s)}$$ By Hypothetical Syllogism

$$\Rightarrow (\neg q \to s)$$ By Hypothetical Syllogism

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Valid Arguments

Example 3: From the single proposition

$$p \wedge (p \to q)$$

Show that *q* is a conclusion.

Solution:

Step	**Reason**
1. $p \wedge (p \to q)$	Premise
2. p	Conjunction using (1)
3. $p \to q$	Conjunction using (1)
4. q	Modus Ponens using (2) and (3)

Any Questions ?