Roll # 23K-0860
Name: Golam Kumar

Date: 9-Mar-24 Multivariable Calculus Assig # 1

Question #1 (a) Find and sketch the domain of given functions

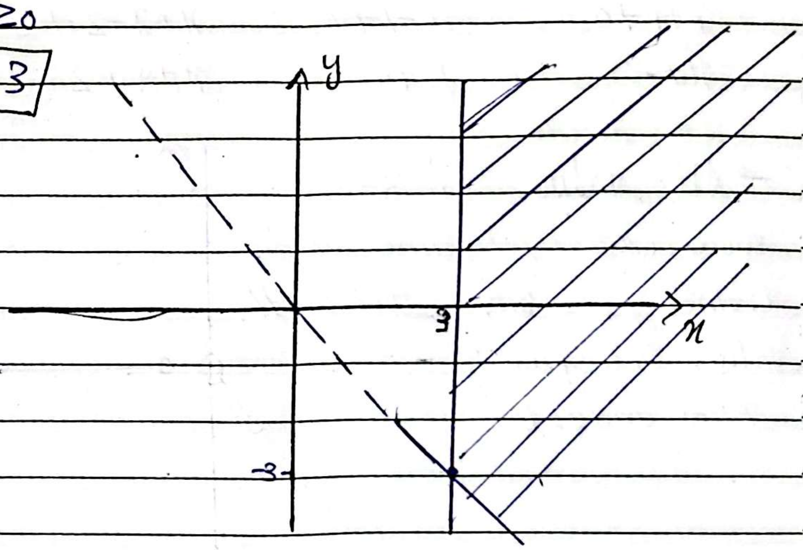
i) $f(x,y)=\sqrt{x+y}-\sqrt{x-3}$

Sol:-

Domain:

$x+y\geq 0$ $\quad$ $x-3\geq 0$

$\boxed{x\geq -y}$ $\quad$ $\boxed{x\geq 3}$



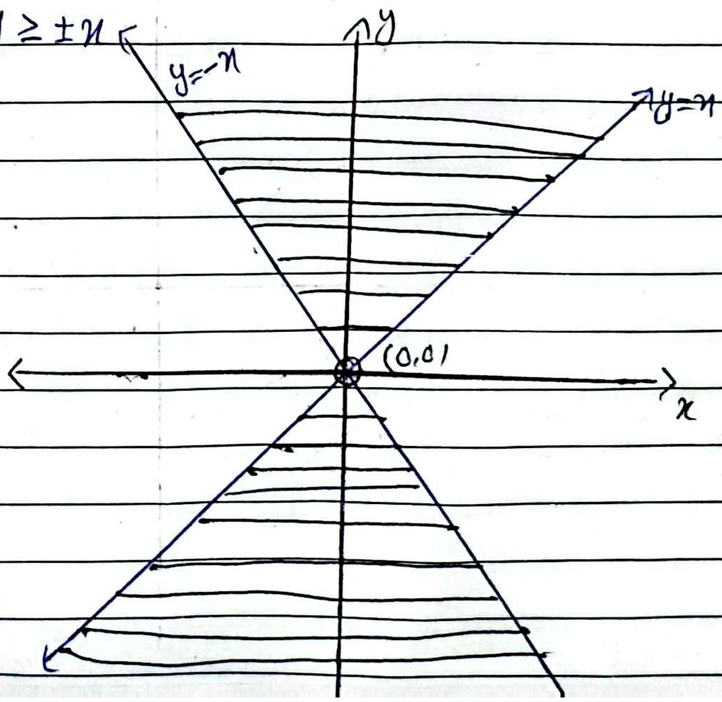
ii) $f(x,y)=\sqrt{\dfrac{1}{x^2}-\dfrac{1}{y^2}}$

Sol:-

$\dfrac{1}{x^2}-\dfrac{1}{y^2}\geq 0 \Rightarrow \dfrac{1}{x^2}\geq\dfrac{1}{y^2}$

$y^2\geq x^2 \Rightarrow y\geq \pm x$

$\Rightarrow \boxed{\begin{matrix} y\geq x \\ y\geq -x \end{matrix}}$



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III $f(x,y,z) = \frac{1}{x+1} + \frac{1}{y-1} + \frac{1}{x+y-z}$

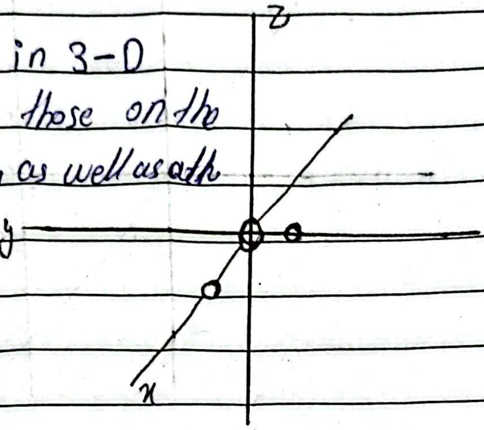
Sol:-

Domain

$x+1 \neq 0 =$ $\quad y-1 \neq 0$ $\quad x+y-z \neq 0$

$x \neq -1$ $\quad y \neq 1$ $\quad x+y \neq z$

→ All values in 3-D space, except for those on the boundary of planes as well as all values is 3-space if $x=-1$ and $y=1$



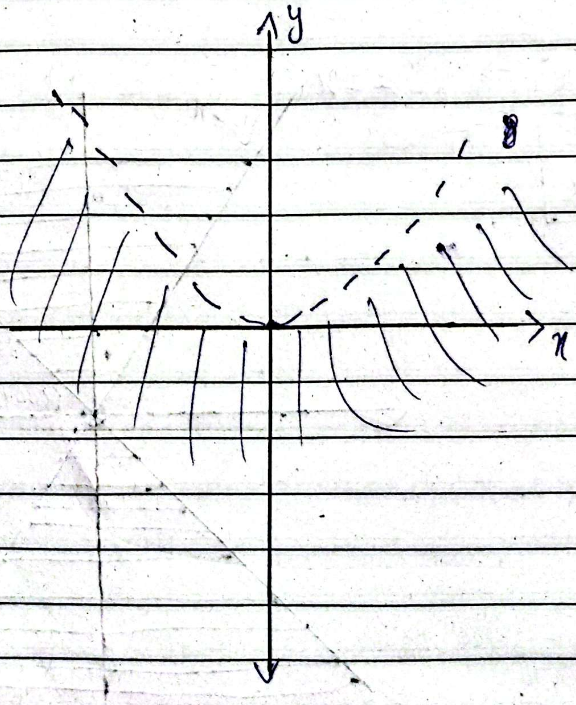
IV) $f(x,y) = \ln(x^2-8y)$

Sol:-

$x^2-8y > 0$

$x^2 > 8y$

$y < \frac{x^2}{8}$



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b) Identify and sketch the level curves (contour) for given function.

i) $x^2 - 4z - y = 2$

Sol:-

$y = x^2 - 4z - 2 \Rightarrow 4z = x^2 - y - 2 \Rightarrow z = \frac{1}{4}(x^2 - y - 2)$

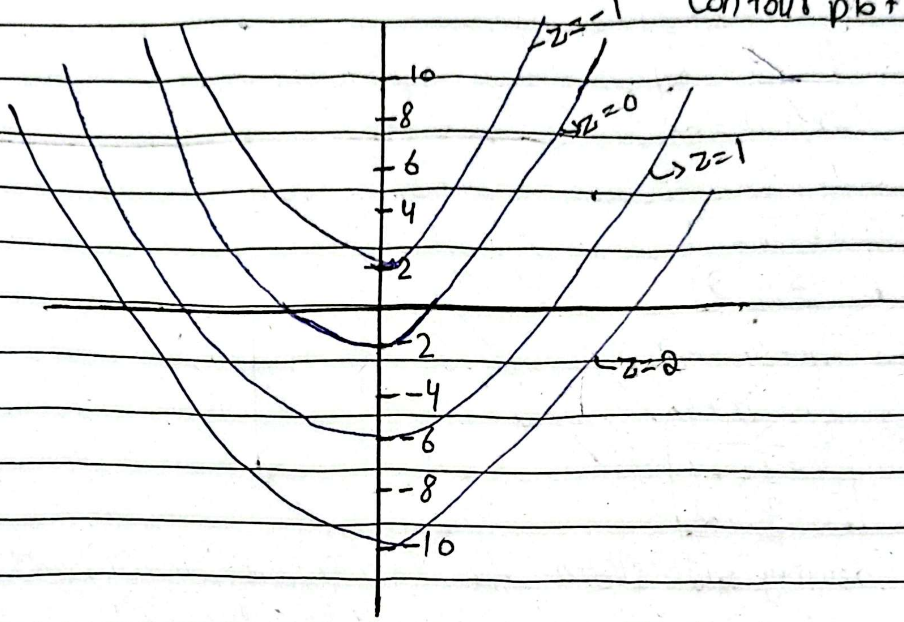
$z = 0 \Rightarrow 0 = x^2 - y - 2 \Rightarrow y = x^2 - 2$ (parabola)

$z = 1 \quad 4 = x^2 - y - 2 \Rightarrow y = x^2 - 6$

$z = -1 \Rightarrow -4 = x^2 - y - 2 \Rightarrow y = x^2 + 2$

$z = 2 \Rightarrow 8 = x^2 - y - 2 \Rightarrow y = x^2 - 10$

Contour plot



ii) $z^2 + 4x^2 = 1 - y^2$

Sol:-

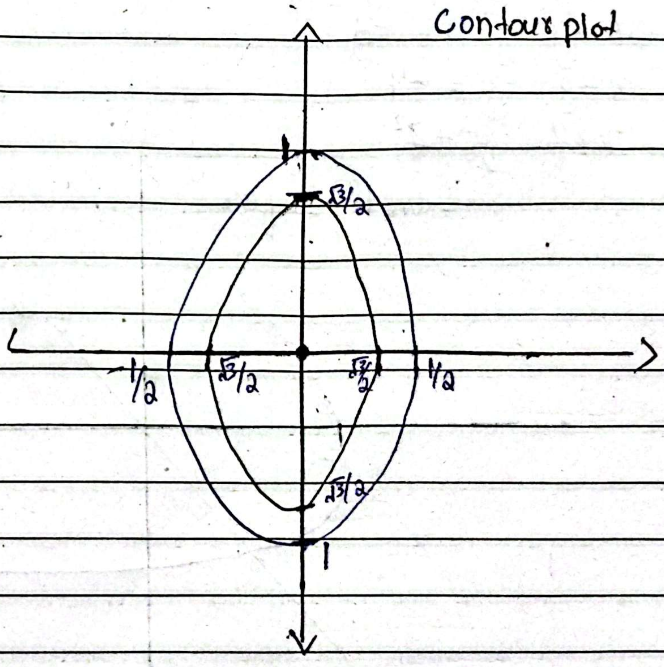
$4x^2 + y^2 = 1 - z^2$

$\boxed{z = \pm 1} \Rightarrow 4x^2 + y^2 = 0$

$\boxed{z = 0.5} \Rightarrow 4x^2 + y^2 = \frac{3}{4}$

$$\frac{x^2}{\left(\frac{\sqrt{3}}{16}\right)^2} + \frac{y^2}{\left(\frac{\sqrt{3}}{2}\right)^2} = 1$$

Contour plot



$\boxed{z = 0} \Rightarrow 4x^2 + y^2 = 1$

$$\frac{x^2}{(1/2)^2} + \frac{y^2}{(1)^2} = 1$$

$\boxed{z = -0.5} \Rightarrow 4x^2 + y^2 = 3/4$

$$\frac{16x^2}{3} + \frac{4y^2}{3} = 1$$

$$\frac{x^2}{\left(\frac{\sqrt{3}}{4}\right)^2} + \frac{y^2}{\left(\frac{\sqrt{3}}{2}\right)^2} = 1$$

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III) $2x-6y+z=-2$

Sol:

$2x-6y+z=-2$

$z=-2 \Rightarrow 2x-6y=0$

$2x=6y \Rightarrow \boxed{y=\frac{1}{3}x}$

$z=-1 \Rightarrow 2x-6y=-1$

$2x+1=6y$

$\boxed{y=\frac{1}{3}x+\frac{1}{6}}$

$z=0 \quad 2x-6y=-2$

$6y=2x+2$

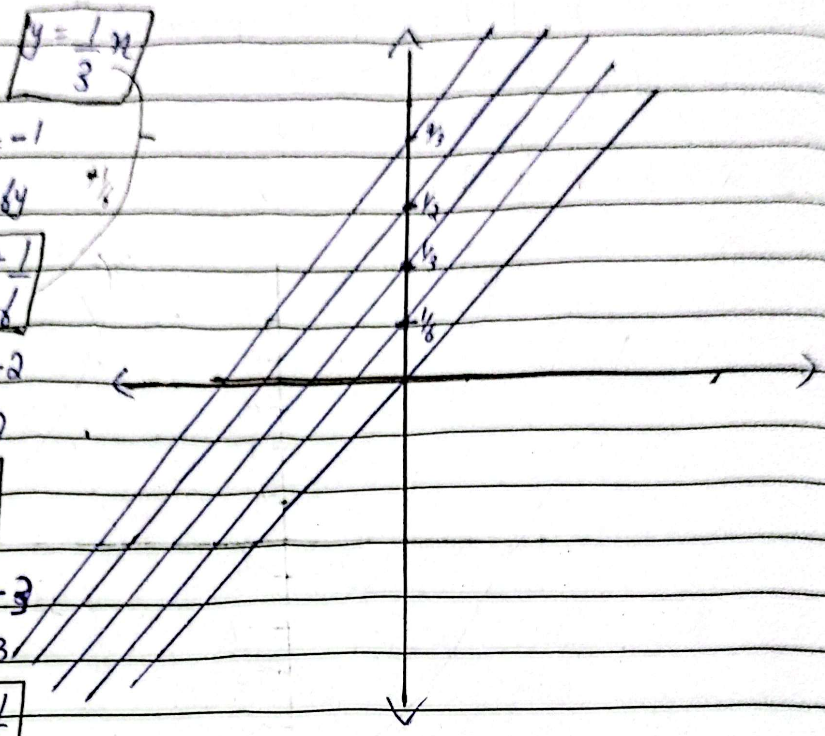
$\boxed{y=\frac{1}{3}x+\frac{1}{3}}$

$z=1 \Rightarrow 2x-6y=-3$

$6y=2x+3$

$\boxed{y=\frac{1}{3}x+\frac{1}{2}}$

Contour plot



c) Identify and sketch the level surfaces (or contours) for the given functions at the specified value of k

i) $f(x,y,z)=x-y^2-z^2+1 \quad k=-3$

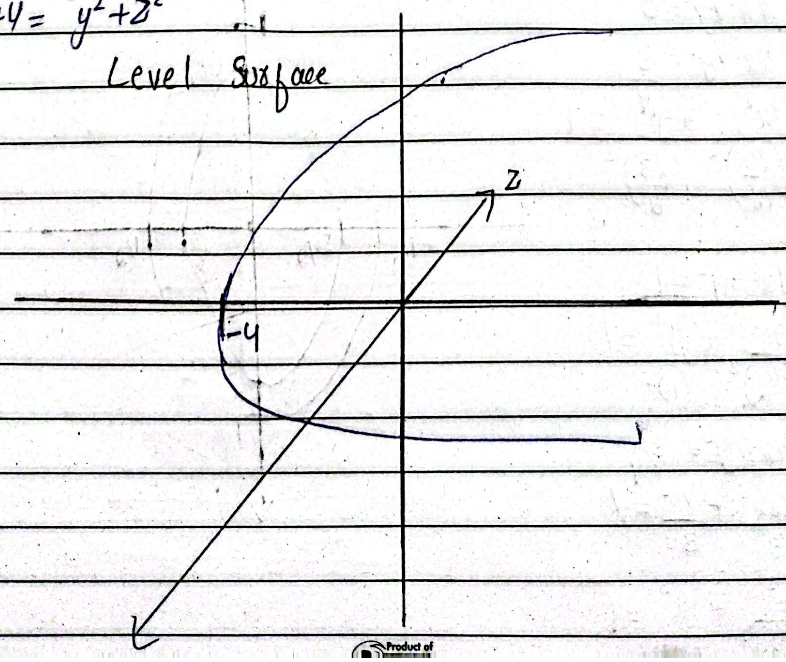
$x-y^2-z^2+1=-3$

$x-y^2-z^2=-4$

$x+4=y^2+z^2$

Level Surface

elliptic paraboloid in x-axis



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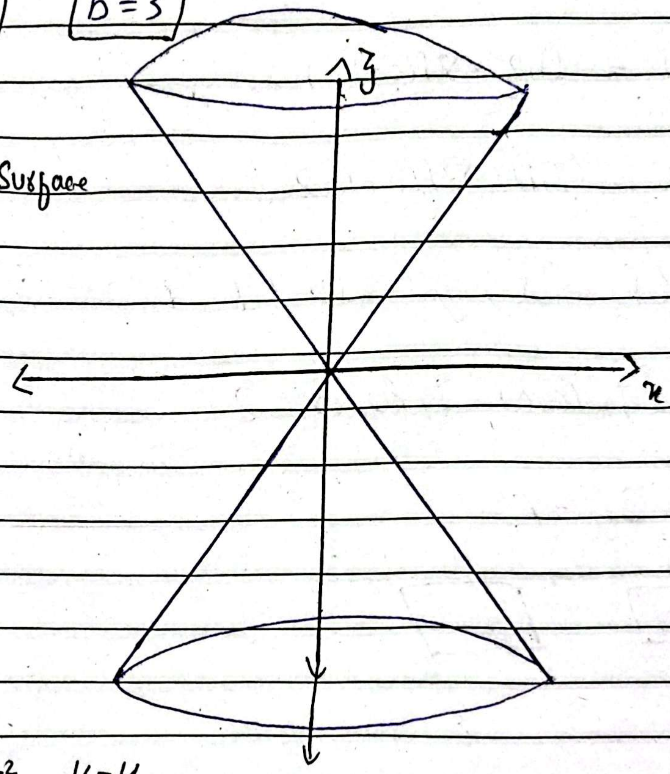
II) $f(x,y,z) = \frac{3x^2+y^2}{z^2}$ $\quad k=9$

$9z^2 = 3x^2+y^2 \Rightarrow z^2 = \frac{x^2}{3} + \frac{y^2}{9} \Rightarrow z^2 = \frac{x^2}{(\sqrt{3})^2} + \frac{y^2}{(3)^2}$

$\boxed{a=\sqrt{3}}$ $\quad \boxed{b=3}$

Level Surface



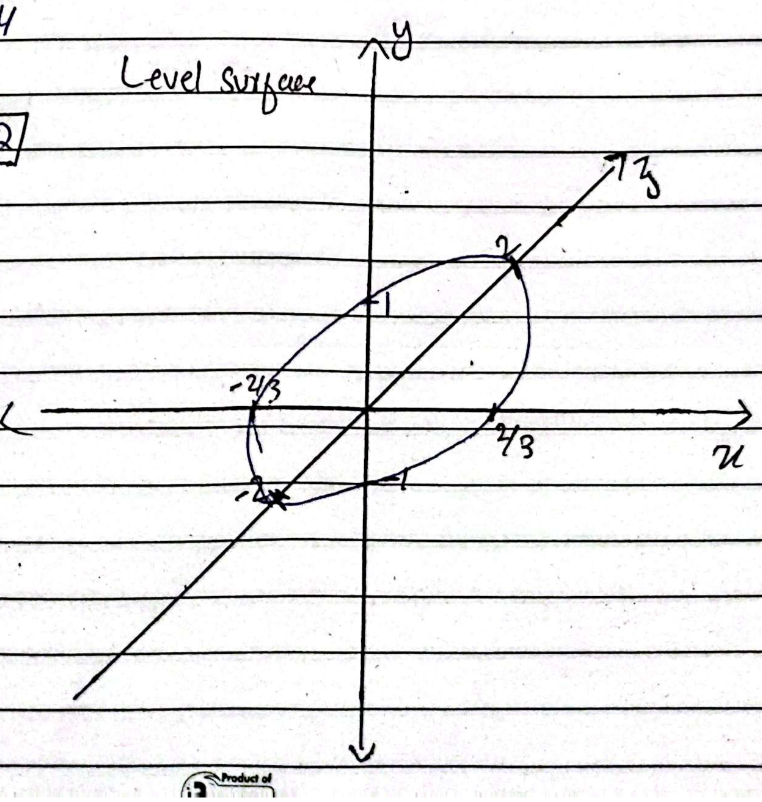
IV $9x^2+4y^2+z^2$ $\quad k=4$

$\frac{9x^2}{4}+y^2+\frac{z^2}{4}=1$ $\quad$ ellipsoid

$a^2=\frac{4}{9}$ $\quad b^2=1$ $\quad c^2=4$

$\boxed{a=2/3}$ $\boxed{b=1}$ $\boxed{c=2}$

Level surface



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Question #2

(a) Examine whether the following limits exists and find their values if they exist.

i) $\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^2}$

Solution

along x-axis $y=0$

$\lim_{x\to0} \frac{x^3y}{x^4+y^2} = \frac{0}{y^2} = \boxed{0}$

along $y=x$

$\lim_{y\to0} \frac{y^3(y)}{y^4+y^2} = \frac{y^4}{y^4+y^2}$

$\lim_{y\to} \frac{y^4}{y^{\cancel{4}}(y^2+1)} = \frac{y^2}{y^2+1}$

$y^2+1 \overline{)\, y^2 \,}$ $\;\; 1$

$\;\; y^2+1$

$1-\frac{1}{y^2+1}$

Apply limit

$\lim_{y\to0} 1-\frac{1}{y^2+1} = 1-\frac{1}{} = \boxed{0}$ Limit exist

ii) $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2}$

Solution

along y-axis $x=0$

$\lim_{y\to0} \frac{0-y^3}{0+y^2} = \frac{-y^3}{y^2} = -y$

Apply limit

$\lim_{y\to0} 0 = 0$

$\boxed{\text{along } x=y}$

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$$\lim_{x \to 0} \frac{x^3 - x^3}{x^2 + x^2} = \boxed{0}$$ Limit exist

III $$\lim_{(x,y) \to (0,0)} \frac{1 - \cos^2(x^2+y^2)}{(x^2+y^2)^2}$$

Solution

Let

$$r = x^2 + y^2$$

$$\lim_{r \to 0} \frac{1 - \cos^2(r)}{r^2} = \frac{0}{0}$$

Apply L-hopital

$$\lim_{r \to 0} \frac{\sin r}{2r} = \frac{0}{0}$$

Again apply L-hopital

$$\frac{\cos r}{2} = \frac{\phi}{2} =$$

$$\boxed{= \frac{1}{2} \text{ Ans}}$$ Limit exist

b)

i) $$\frac{xy}{\sqrt{x^2+y^2}}$$

Sol:-

along x-axis $y = 0$

$$\lim_{x \to 0} \frac{x(0)}{\sqrt{x^2+0}} = 0$$

along $y = x$

$$\lim_{y \to 0} \frac{x(x)}{\sqrt{x^2+x^2}} = \frac{x^2}{\sqrt{2x^2}} = \frac{x^2}{x\sqrt{2}}$$

$$\lim_{y \to 0} \frac{1x}{\sqrt{2}}$$ apply limit

∴ limit exits 0

$$f(0,0) = \lim_{(x,y) \to (0,0)} \frac{xy}{\ldots} = 0$$ ∴ $\boxed{\text{Continous}}$

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ii) $\frac{xy}{x^2+y^2}$

along x-axis $y=0$

$\lim_{x\to 0} \frac{x(0)}{x^2+0} = 0$

along $x=y$

$\lim_{y\to 0} \frac{y(y)}{y^2+y^2} = \frac{y^2}{2y^2} = \frac{1}{2}$

Limit D.N.E

→ function is Discontinous

iii) $\frac{x^4-y^2}{x^4+y^2}$

Sol:-

along y-axis $x=0$

$\lim_{y\to 0} \frac{0-y^2}{0+y^2} = -0$

along $x=y$

$\lim_{y\to 0} \frac{y^4-y^2}{y^4+y^2}$

$\lim_{y\to 0} = \frac{y^2(y^2-1)}{y^2(y^2+1)} = \frac{y^2-1}{y^2+1}$

Applimit

$\frac{0-1}{0+1} = -1$

Limit does not exist

$f(0,0) = \frac{x^4-y^2}{x^4+y^2} = 0$

$\boxed{\text{Discontinous because } f(0,0) \neq \lim_{x,y\to 0,0}}$

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iv) $\dfrac{x^2y}{x^4+y^2}$

along y-axis $x=0$

$$\lim_{y\to 0} \frac{0(y)}{0+y} = \boxed{0}$$

along $x=y$

$$\lim_{y\to 0} \frac{y^2(y)}{y^4+y^2} = \frac{y^3}{y^2(y^2+1)} = \frac{y}{y^2+1}$$

Apply Limit

$$= \frac{0}{0+1} = 0$$

$$f(0,0) = \frac{x^2y}{x^4+y^2} = \boxed{0} \quad \text{Limit exist}$$

→ $\boxed{\text{Continous Becase } f(0,0) = \lim_{x,y\to 0,0}}$

Q#3)

Let $f(x,y) = xy\dfrac{x^2-y^2}{x^2+y^2}$ if $(x,y) \neq (0,0)$

(a) $f_x(0,y) = -y$ and $f_y(x,0) = x$ for all x and y

$f_x(0,y) = -y$

$$\frac{\partial f}{\partial x} = (y)\frac{x^2-y^2}{x^2+y^2} - xy\left\{\frac{2x(x^2+y^2)-2x(x^2-y^2)}{(x^2+y^2)^2}\right\}$$

$$= \frac{y(x^2-y^2)}{x^2+y^2} - xy\left\{\frac{2x\,(x^2+y^2-x^2+y^2)}{(x^2+y^2)^2}\right\}$$

$$= \frac{y(x^2-y^2)}{x^2+y^2} - 2x^2y\left\{\frac{2y^2}{(x^2+y^2)^2}\right\}$$

$$= \frac{y(x^2-y^2)}{x^2+y^2} - \frac{4x^2y^3}{(x^2+y^2)^2}$$

$$\left.\frac{dz}{dx}\right|_{(0,y)} = \frac{y(0-y^2)}{0+y^2} - \frac{4(0)(y^2)^3}{(0+(y^2)^2)^3}$$

$$= -y - 0$$

$$\boxed{\left.\frac{dz}{dx}\right|_{(0,y)} = -y} \quad \text{hence Proved}$$

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and $f_y(x,0) = x$

$$\frac{\partial z}{\partial y} = xy\,\frac{x^2-y^2}{x^2+y^2}$$

$$\frac{\partial z}{\partial y} = \frac{x(x^2-y^2)}{x^2+y^2} - xy\left\{\frac{-2y(x^2+y^2) - 2y(x^2-y^2)}{(x^2+y^2)^2}\right.$$

$$\frac{\partial z}{\partial y} = \frac{x(x^2-y^2)}{x^2+y^2} - 2xy^2\left(\frac{x^2+y^2+x^2-y^2}{(x^2+y^2)^2}\right)$$

$$\frac{\partial z}{\partial x} = \frac{x(x^2-y^2)}{x^2+y^2} - 2xy^2\left(\frac{2x^2}{(x^2+y^2)^2}\right.$$

$$\left.\frac{\partial z}{\partial x}\right|_{(x,0)} = \frac{x(x^2-0)}{x^2} - 2(x)(0)\frac{2x^2}{(x^2+0)^2}$$

$$\boxed{\left.\frac{\partial z}{\partial x}\right|_{(x,0)} = x}$$ hence Proved

b) $f_{xy}(0,0) = -1$

$$f(x,y) = xy\,\frac{x^2-y^2}{x^2+y^2}$$

Firstly Differential w.r.t 'x'

$$\frac{\partial f}{\partial x} = y\left(\frac{x^2-y^2}{x^2+y^2}\right) + xy\left(\frac{2x(x^2+y^2) - 2x(x^2-y^2)}{(x^2+y^2)^2}\right.$$

$$= y\left(\frac{x^2-y^2}{x^2+y^2}\right) + xy\left(\frac{2x\,(x^2+y^2-x^2+y^2)}{(x^2+y^2)^2}\right)$$

According to partial $f_x(0,y) = -y$

$$= \frac{y(0-y^2)}{0+y^2} + 0$$

$\boxed{f_x = -y}$ This is obtain f_x Now we find f_{xy}

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$f_x(0,y) = -y$

Now Diff w.r.t y

$f_{xy}(0,y) = -1 \Rightarrow \boxed{f_{xy}(0,0) = -1}$

hence Proved

$f_{yx}(0,0) = 1$

$$f(x,y) = xy\frac{x^2-y^2}{x^2+y^2}$$

$$\frac{\partial f}{\partial y} = x\left(\frac{x^2-y^2}{x^2+y^2}\right) + xy\left(\frac{-2y(x^2+y^2) - 2y(x^2-y^2)}{(x^2+y^2)^2}\right)$$

$$= x\left(\frac{x^2-y^2}{x^2+y^2}\right) + 2xy^2\left(\frac{\cancel{x^2}+\cancel{y^2}+x^2-\cancel{y^2}}{(x^2+y^2)^2}\right)$$

$$= x\left(\frac{x^2-y^2}{x^2+y^2}\right) - 2xy^2\frac{(2x^2)}{(x^2+y^2)^2}$$

According to part(b)

$f_y(x,0) = x$

$$= x\left(\frac{x^2-0}{x^2+0}\right) - 2x(0)\frac{(2x^2)}{(x^2+0)^2}$$

$f_y(0,x) = x$

Now Diff w.r.t "y"

$f_{yx}(0,x) = 1 \Rightarrow f_{yx}(0,0) = 1$ all x

$$\boxed{f_{yx}(0,0) = 1}$$

hence Proved

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c) $f(x,y)$ is differentiable at $(0,0)$

$$f(x,y) = xy\,\frac{x^2-y^2}{x^2+y^2}$$

First find f_x

$$f_x(x,y) = y\left[\frac{x^2-y^2}{x^2+y^2}\right] + xy\left[2x(x^2+y^2) - 2x(\cancel{x^2+y^2})\right]$$

$$= y\left[\frac{x^2-y^2}{x^2+y^2}\right] + 2x^2y\left[\frac{x^2+y^2-x^2+y^2}{-(x^2+y^2)^2}\right]$$

$$= y\left[\frac{x^2-y^2}{x^2+y^2}\right] + \frac{4x^2y^3}{(x^2+y^2)^2}$$

$$f_x(0,0) = 0\left[\frac{0-0}{0+0}\right] + \frac{40(0)}{(0+0)^2}$$

$$\boxed{f_x(0,0) = 0}$$

Now f_y

$$f_y(x,y) = x\left[\frac{x^2-y^2}{x^2+y^2}\right] + xy\left[\frac{-2y(x^2+y^2) - 2y(x^2-y^2)}{(x^2+y^2)^2}\right]$$

$$= x\left[\frac{x^2-y^2}{x^2+y^2}\right] - 2xy^2\left[\frac{x^2+\cancel{y^2}+x^2-\cancel{y^2}}{(x^2+y^2)^2}\right]$$

$$= x\left[\frac{x^2-y^2}{x^2+y^2}\right] - 2xy^2\left[\frac{2x^2}{(x^2+y^2)^2}\right]$$

$$f_y(0,0) = 0\left[\frac{0-0}{0+0}\right] - 2(0)(0)^2\left[\frac{2(0)^2}{(0+0)^2}\right]$$

$$\boxed{f_y(0,0) = 0}$$

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Now calculate limit for check Continuously

$$\lim_{(x,y)\to 0,0} xy\,\frac{x^2-y^2}{x^2+y^2}$$

along $x=0$

$$\lim_{y\to 0} \frac{0\cdot y(0-y^2)}{0+y^2}$$

$$\boxed{\lim_{y\to 0} = 0}$$

Now

along $x=y$

$$\lim_{x,y\to 0,0} xy\,\frac{x^2-y^2}{x^2+y^2}$$

put $x=y$

$$\lim_{y\to 0} y\cdot y\cdot\frac{y^2-y^2}{y^2+y^2}$$

$$\boxed{\lim_{y\to 0} \frac{0\ \ 0}{0} = 0}$$

→ f_x and f_y are exist and $f(x,y)$ is continuase So it is also differentiable

II)

$f_x(x,y) = f_y(x,y) = 0$ for all (x,y). Then show that $f(x,y) = c$, a constant

Let

$f_x(x,y) = 0$

integrate bs w.r.t 'x'

$$\int \frac{\partial f}{\partial x}\,\partial x = \int 0\,dx$$

$f(x,y) = 0 + g'(y)$. ⟶ (i)

Diff w.r.t y

$f_y(x,y) = g'(y)$

$0 = g'(y)$ Because $f_y(x,y)=0$

$g(y)=0$

$\boxed{f(x,y)=c}$ hence Proved

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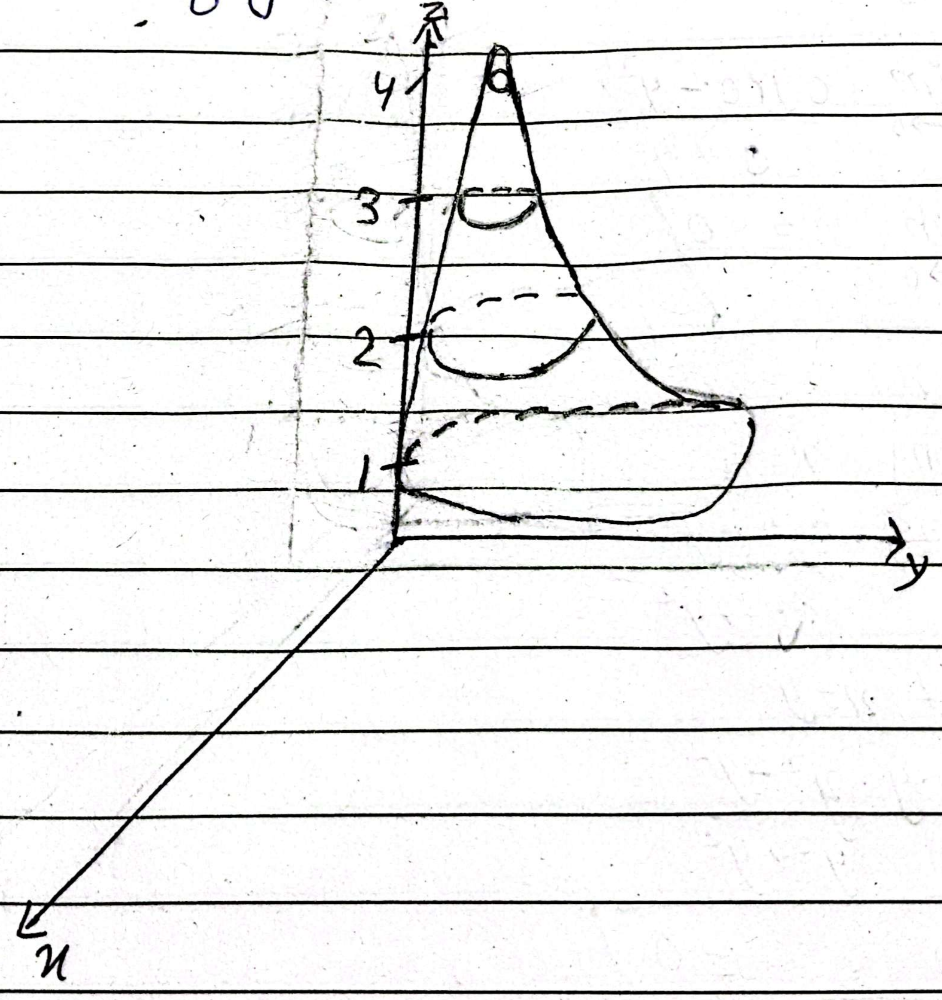
III. A contour map of a function f is shown. Use it make a rough sketch of the graph of f.

sketch of graph whose contour map is :



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Question #4

(a)

$\because D_u f(a,b) = u \cdot \nabla f(a,b)$

$D_{(1,2)} f(0,0) = (1,2) \cdot \nabla f(0,0) = 1$

$D_{(2,1)} f(0,0) = (2,1) \cdot \nabla f(0,0) = 2$

$\langle 1,2 \rangle \langle f_x(0,0), f_y(0,0) \rangle = 1$

$\langle 2,1 \rangle \langle f_x(0,0), f_y(0,0) \rangle = 2$

$f_x + 2f_y = 1$

$2f_x + f_y = 1$

$\boxed{f_x = 1}$ $\boxed{f_y = 0}$

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b) Suppose $z = f(x,y)$, where $x = g(s,t)$, $y = h(s,t)$
$g(1,2) = 3$, $g_s(1,2) = -1$, $g_t(1,2) = 4$ $h(1,2) = 6$
$h_s(1,2) = -5$, $h_t(1,2) = 10$, $f_x(3,6) = 7$ and $f_y(3,6) = 8$
Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ when $s=1$ and $t=2$

partial derivative of z w.r.t x

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial s}$$

Partial derivative of z w.r.t t

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$$

Functione

$x = g(s,t)$ $y = h(s,t)$
$z = f(x,y)$ $z_x = f_x$ $x_s = g_x$

$(s,t) = (1,2)$

$z_s(1,2) = f_x(g(1,2), h(1,2)) \cdot g_s(1,2) + f_y(g(1,2), h(1,2)) \cdot h_s(1,2)$
$g(1,2) = 3$ $h(1,2) = 6$ $g_s(1,2) = -1$
$z_s(1,2) = f_x(3,6) \cdot (-1) + f_y(3,6) \cdot -5$
$\because f_x(3,6) = 7$ $\because f_y(3,6) = 8$

$z_s(1,2) = 7(-1) + 8(-5)$

$$\boxed{z_s = -7 - 40 = -47}$$

$$\boxed{\frac{\partial z}{\partial x} = -47}$$

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$(s,t)=(1,2)$

$z_t(1,2) = f_x(g(1,2), h(1,2)) \cdot g_t(1,2) + f_y(g(1,2), h(1,2)) \cdot h_t(1,2)$

$\because g(1,2)=3 \quad h(1,2)=6$

$z_t(1,2) = f_x(3,6) \cdot g_t(1,2) + f_y(3,6) \cdot h_t(1,2)$

$\because f_x(3,6)=7 \quad g_t(1,2)=4$

$\because f_y(3,6)=8 \quad h_t(1,2)=10$

$z_t(1,2) = 7 \cdot (4) + 8(10)$

$= 28 + 80$

$z_s = 108$

$$\boxed{\frac{\partial z}{\partial s} = -47}$$

$$\boxed{\frac{\partial z}{\partial t} = 108}$$

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c) If $z = y + f(x^2 - y^2)$, where f is differentiable, show that

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x \quad \rightarrow (i)$$

Solution

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(y + f(x^2 - y^2))$$

$$= 0 + 2x - 0$$

$$\frac{\partial z}{\partial x} = 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(1 + 0 - 2y)$$

$$\frac{\partial z}{\partial y} = 1 - 2y$$

Now put f_x & f_y in (i)

$$y(2x) + x(1 - 2y) = x$$

$$\cancel{2xy} + x - \cancel{2xy} = x$$

$$\boxed{x = x}$$

hence Proved L.H.S = R.H.S

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Question #5

Find the linear approximation of the function $f(x,y,z) = x^3\sqrt{y^2+z^2}$ at the point $(2,3,4)$ and use it to estimate the number $(1.98)^3\sqrt{(3.01)^2+(3.9...}$

$$f(x,y,z) = x^3\sqrt{y^2+z^2}$$

$$L(x,y,z) = f(x,y,z) + f_x(x,y,z)(x-x_0) + f_y(x,y,z)(y-y_0) + f_z(x,y,z)(z-x_0)$$

$$(x,y,z) = (2,3,4)$$

Find f_x, f_y & f_z

$$f_x = x^3\sqrt{y^2+z^2}$$

$$\boxed{f_x = 3x^2\sqrt{y^2+z^2}}$$

$$\boxed{f_y = x^3 \cdot \frac{2y}{2\sqrt{y^2+z^2}} = \frac{x^3 y}{\sqrt{y^2+z^2}}}$$

$$\boxed{f_z = x^3 \frac{2z}{2\sqrt{y^2+z^2}} = \frac{x^3 z}{\sqrt{y^2+z^2}}}$$

Now find the value of partial derivative at $(2,3,4)$

$$f_x = 3(2)^2\sqrt{(3)^2+(4)^2} = \boxed{60}$$

$$f_y = \frac{(2)^3(3)}{\sqrt{(3)^2+(4)^2}} = \boxed{\frac{24}{5}}$$

$$f_z = \frac{(2)^3(4)}{\sqrt{(3)^2+(4)^2}} = \boxed{\frac{32}{5}}$$

$$f(2,3,4) = 2^3\sqrt{3^2+4^2} = \boxed{40}$$

$$L(x,y,z) = 40 + 60(x-2) + \frac{24}{5}(y-3) + \frac{32}{5}(z-4)$$

$Q(1.98, 3.01, 3.91)$ according this $(1.98)^3\sqrt{(3.01)^2+(3.91)^2}$

$x = 1.98 \quad y = 3.01 \quad z = 3.91$

$$L(Q) = 40 + 60(1.98-2) + \frac{24}{5}(3.01-3) + \frac{32}{5}(3.91-4)$$

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$$L(Q) = 40 + 60(-0.02) + \frac{24}{5}(0.01) + \frac{32}{5}(-0.04)$$

$$= 40 - 1.2 + 4.8(0.01) + 6.4(-0.04)$$

$$= 40 - 1.2 + 0.048 + (0.256)$$

$$= 38.592$$

$$\boxed{(1.98)^3 \sqrt{(3.01)^2 + (3.91)^2} \approx 38.54}$$

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b)
(a) Estimate the value of the partial derivative $T_x(6,4)$ and $T_y(6,4)$. What are the units

$$T_x'(6,4) = \lim_{\Delta x \to 0} \frac{(6+\Delta x, 4) - T(6,4)}{\Delta x}$$

$\Delta x = \pm 2$ ...

$$T_x(6,4) = \lim_{\Delta x \to 0} \frac{T(6+2,4) - T(6,4)}{2} = \lim_{\Delta x \to 0} \frac{T(8,4) - T(6,4)}{2}$$

$$T_x(6,4) = \frac{86 - 80}{2} = \boxed{3}$$

$$T_x(6,4) = \frac{T(6-2,4) - T(6,4)}{-2} = \frac{72 - 80}{-2} = \boxed{4}$$

value of $T_x(6,4)$ is the avg of both value

hence $T_x(6,4) = \frac{3+4}{2} = \frac{7}{2} = \boxed{3.5\,°C/m}$

Now $T_y(6,4)$

$\Delta y = 2 \quad \Delta y = -2$

$$T_y(6,4) = \frac{T(6,4+\Delta y) - T(6,4)}{\Delta y}$$

$$= \frac{T(6,4+2) - T(6,4)}{2}$$

$$= \frac{T(6,6) - T(6,4)}{2} = \frac{75 - 80}{2} = \frac{-5}{2} = -2.5\,°C/m$$

$\Delta y = -2$

$$T_y(6,4) = \frac{T(6,4-2) - T(6,4)}{-2} = \frac{87 - 80}{-2} = \boxed{-3.5\,°C/m}$$

Avg

$$T_y(6,4) = \frac{2.5 + 3.5}{2} = \frac{6}{2} = 3\,°C/m$$

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$$\boxed{\begin{array}{l} T_x(6,4) \approx 3.5°C/m \\ T_y(6,4) \approx -3°C/m \end{array}}$$

b) Estimate the value of $D_uT(6,4)$, where $u = \frac{i+j}{\sqrt{2}}$ interpret your result.

from part (a)

$T_x(6,4) \approx 3.5°C/m$

$T_y(6,4) \approx -3°C/m$

$u = \frac{i+j}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$$D_uT(6,4) = T_x(6,4) \cdot \frac{1}{\sqrt{2}} + T_y(6,4) \cdot \frac{1}{\sqrt{2}}$$

Now put value of T_x & T_y

$$= 3.5 \cdot \frac{1}{\sqrt{2}} - 3 \cdot \frac{1}{\sqrt{2}} =$$

$$= 2.47 - 2.12 = 0.35°C/m$$

$$\boxed{D_uT(6,4) = 0.35°C/m}$$

c) Estimate the value of $T_{xy}(6,4)$

$T_{xy}(6,4) = (T_x)_y(6,4)$

First we find T_x $\Delta x = +2, -2$

$$T_x(6,4) = \frac{T(6+\Delta x, 4) - T(6,4)}{\Delta x}$$

$$= \frac{T(6+2,4) - T(6,4)}{2} = \frac{T(8,4) - T(6,4)}{2}$$

$$= \frac{86-80}{2} = \frac{6}{2} = \boxed{3}$$

$$T_x(6,4) = \frac{T(6-2,4) - T(6,4)}{-2} = \frac{72-80}{-2} = \boxed{4}$$

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$T_x(6,4)$ Avg of Both value

$$T_x(6,4) = \frac{3+4}{2} = 3.5°C/m$$

Now $(T_x)_y(6,4)$

$$(T_x)_y(6,4) = \lim_{\Delta y \to 0} \frac{T_x(6,4+\Delta y) - T_x(6,4)}{\Delta y}$$

$$\Delta y = 2\ 6-2$$

$$(T_x)_y(6,4) = \frac{T_x(6,4+2) - T_x(6,4)}{2} =$$

$$= \frac{T_x(6,6) - T_x(6,4)}{2} = \frac{3-3.5}{2} = \boxed{-0.25°C/m}$$

$$(T_x)_y(6,4) = \frac{T_x(6,4-2) - T_x(6,4)}{-2} = \frac{4-3.5}{-2}$$

$$\boxed{(T_x)_y(6,4) = -0.25}$$

$(T_x)_y(6,4)$ as the Avg of Both

$$\boxed{(T_x)_y(6,4) \approx -0.25C/m^2}$$

d) Find a linear approximation to the Temp function $T(x,y)$ near the point $(6,4)$. Then use it to estimate the Temperature at the point $Q(5,3.8)$

$$f(x,y) = f(x,y) + f_x(x,y)(x-x_0) + f_y(x,y)(y-y_0)$$

from previou part we obtain

$$T_x(6,4) = 3.5 \qquad T_y = -3$$

$$T(x,y) = T(6,4) + T_x(6,4)(x-6) + T_y(6,4)(y-4)$$
$$= 80 + 3.5(x-6) - 3(y-4)$$

$$T(Q) = T(5,3.8) = 80 + 3.5(5-6) + 3(3.8-4)$$
$$= 80 - 3.5 + 0.6$$
$$= 77.1$$

Temp at $(5,3.8)$ is approximate 77.1

$$T(x,y) = 80 + 3.5(x-6) - 3(y-4)$$
$$T(5,3.8) = \boxed{77.1\,°C}$$

Ans

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Q#6

(a)

The two legs of a right triangle are measured as 5m and 12m with a possible error in measurement of at most 0.2cm in each. Use differentials to estimate the maximum error in the calculated value of:

a) the area of the triangle and

b) the length of hypotenuse.

$x = 5 \quad y = 12cm \qquad \partial x = \partial y = 0.2cm = 0.002m$

a) $\Delta = \frac{1}{2} xy$

$$\Delta = \frac{\partial}{\partial x}\left(\frac{1}{2}xy\right) + \frac{\partial}{\partial y}\left(\frac{1}{2}xy\right)$$

$$= \frac{1y}{2}\partial x + \frac{1x}{2}\partial y$$

$$= \frac{1}{2}\left(y\partial x + x\partial y\right)$$

$y = 12 \quad x = 5 \quad \partial x = \partial y = 0.002$

$$= \frac{1}{2}\left(12 \times 0.002 + 5 \times 0.002\right)$$

$$= \frac{1}{2}\left(0.024 + 0.01\right) \Rightarrow \frac{1}{2}\left(0.0341\right)$$

$$\boxed{\Delta = 0.017 \text{ sq m}}$$

b) $z = \sqrt{x^2 + y^2}$

$$\partial z = \frac{2x\partial x + 2y dy}{2\sqrt{x^2+y^2}} = \frac{2 \times 5 \times 0.002 + 2 \times 12 \times 0.002}{2\sqrt{(5)^2 + (12)^2}}$$

$$\partial z = \frac{0.02 + 0.048}{2\sqrt{169}} = \frac{0.068}{2(13)} = \boxed{0.00257}$$

→ the Maximum error calculated value of length hypoth is $\boxed{0.00257}$

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b) The length x of a side of a triangle is increasing at a rate of 3in/s, the length y side is decreasing at a rate of 2in/s and the contained angle θ is increasing at a rate of 0.05 radians. How fast is the area of the triangle changing when $x = 40$ in, $y = 50$ in, and $\theta = \frac{\pi}{6}$?

$$\frac{dy}{dx} = 3 \qquad \frac{dx}{dy} = -2 \qquad \frac{d\theta}{dt} = 0.05$$

$$\frac{dA}{dt} = ?$$

a) $A = \frac{1}{2} xy \sin\theta$

$$\frac{dA}{dt} = \frac{1}{2}\left[(y\sin\theta)\frac{dx}{dt} + (x\sin\theta)\frac{dy}{dt} + (xy\cos\theta)\frac{d\theta}{dt} \right]$$

So, $x = 40$, $y = 50$, $\theta = \frac{\pi}{6}$ put

$$\frac{dA}{dt} = \frac{1}{2}\left[\left(50 \sin\frac{\pi}{6}\right)(3) + \left(40 \sin\frac{\pi}{6}\right)(-2) + (40)(50)\cos\frac{\pi}{6}(0.05) \right]$$

$$= \frac{1}{2}\left(\left(\cancel{50}^{25} \times \frac{1}{\cancel{2}}\right)(3) + 40 \times \frac{1}{\cancel{2}}(-\cancel{2}) + (\cancel{40}^{20})(50)\frac{\sqrt{3}}{\cancel{2}}(0.05) \right)$$

$$= \frac{1}{2}\left(75 - 40 + (1000)(\sqrt{3})(0.05) \right)$$

$$= \frac{1}{2}\left[35 + 50\sqrt{3} \right] = \boxed{60.8 \text{ in}^2/\text{s}}$$

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Question #7

(a) Find the direction in which $f(x,y,z) = ze^{xy}$ increase most rapidly at the point $(0,1,2)$. what is the maximum rate of increase?

Maximum value of directional derivative $D_u f(x,y,z)$ is $|\nabla f(x,y,z)|$ and it occurs when u is parallel to $\nabla f(x,y,z)$

$$f(x,y,z) = ze^{xy}$$

$$f_x(x,y,z) = zye^{xy} \Rightarrow f_x(0,1,2) = 2e^{0(1)} = 2$$

$$f_y(x,y,z) = zxe^{xy} = f_y(0,1,2) = 2(0)e^{0(1)} = 0$$

$$f_z(x,y,z) = 1e^{xy} = f_z(0,1,2) = e^0 = 1$$

$$\boxed{\nabla f(0,1,2) = (2,0,1)}$$

Therefore, the direction required $(2,0,1)$ and the maximum rate of increase can be determined

$$|\nabla f(0,1,2)| = \sqrt{(2)^2+(0)^2+(1)^2}$$
$$= \sqrt{5}$$

$\boxed{\text{Direction } (2,0,1)}$

$\boxed{\text{Maximum rate increase } \sqrt{5}}$

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(b) Let $f(x,y,z) = |r|^{-n}$ where $r = xi + yj + zk$

Show that

$$\boxed{\nabla f = \frac{-nr}{|r|^{n+2}}}$$

$f(x,y,z) = |r|^{-n}$ $\quad r = xi + yj + zk$

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$$

$r = \sqrt{x^2+y^2+z^2}$

$$\frac{\partial (r^{-n})}{\partial x} = \frac{\partial}{\partial x} (x^2+y^2+z^2)^{-\frac{n}{2}}$$

$$= -\frac{n}{\cancel{2}} (x^2+y^2+z^2)^{\frac{-n}{2}-1} (\cancel{2}x)$$

$$\frac{d}{\partial x}(r^{-n}) = -nx (x^2+y^2+z^2)^{\frac{-n-2}{2}} \Rightarrow \boxed{-nx(\sqrt{x^2+y^2+z^2})^{-n-2}}$$

$$\frac{\partial}{\partial y}(r^{-n}) = \frac{\partial}{\partial y} (x^2+y^2+z^2)^{-\frac{n}{2}}$$

$$= -\frac{n}{\cancel{2}} (x^2+y^2+z^2)^{\frac{-n-2}{2}} (\cancel{2}y) = \cdot$$

$$= -ny (x^2+y^2+z^2)^{\frac{-n-2}{2}}$$

$$\frac{\partial}{\partial y} \boxed{= -ny(\sqrt{x^2+y^2+z^2})^{-n-2}}$$

$$\frac{\partial}{\partial z}(r^{-n}) = -\frac{n}{\cancel{2}} (x^2+y^2+z^2)^{\frac{-n-2}{2}} \cancel{2}z$$

$$\frac{d}{\partial z}(r^{-n}) = \boxed{-nz(\sqrt{x^2+y^2+z^2})^{-n-2}}$$

$$\therefore \nabla f = \partial/_{\partial x}(r^{-n})i + \partial/_{\partial y}(r^{-n})j + \partial/_{\partial z}(r^{-n})z$$

$$\Delta f = -nx(\sqrt{x^2+y^2+z^2})^{-n-2} + (-ny\sqrt{x^2+y^2+z^2})^{-n-2} + (-nz\sqrt{x^2+y^2+z^2})^{-2}$$

$\because r = \sqrt{x^2+y^2+z^2}$

$$\Delta f = -nx(r)^{-n-2} + (-ny\,\cancel{\partial}(r))^{-n-2} + (-nz\,|r|^{-n-2}),$$

$$= -n|r|^{-n-2}(x+y+z)$$

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$$\nabla f = -n|r|^{-n-2} (x+y+z)$$

$\because r = x+y+z$

$$\boxed{\nabla f = \frac{-n\,r}{|r|^{n+2}}}$$

Hence Proved

c) Find the gradient of the function $f(x,y,z) = x^2 e^{yz^2}$

$$f(x,y,z) = x^2 e^{yz^2}$$

The gradient is given by

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2 e^{yz^2})$$

$$= \boxed{2x e^{yz^2}}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2 e^{yz^2})$$

$$= \boxed{x^2 z^2 e^{yz^2}}$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(x^2 e^{yz^2})$$

$$= \boxed{2zx^2 e^{yz^2}},$$

The gradient of f is equal to

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = \left(2x e^{yz^2}, x^2 z^2 e^{yz^2}, 2x^2 z\, e^{yz^2}\right.$$

$$\boxed{\nabla f = (2x e^{yz^2}, x^2 z^2 e^{yz^2}, 2x^2 z e^{yz^2})}$$

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$$\nabla f = (2xe^{yz^2}, x^2z^2e^{yz^2}, 2xyz)$$

(i) When is the directional derivative of f a maximum?

i) The directional derivative of f is Maximum when the direction is the same as the gradient

$\theta = 0^\circ$

ii) When it is Minimum?

ii) The directional derivative of f is Minimum when the direction is opposite ~~of~~ to the gradient.

$\theta = \pi$

IV When it is ~~half of~~ it's maximum value?

IV The directional derivative of f is half of it's Maximum value when the direction is parallel to the unit vector in the direction of the gradient. $\theta = \pi/3$

III When it is zero?

The directional derivative of f is zero when the direction is perpendicular to the gradient. $\theta = 90^\circ$