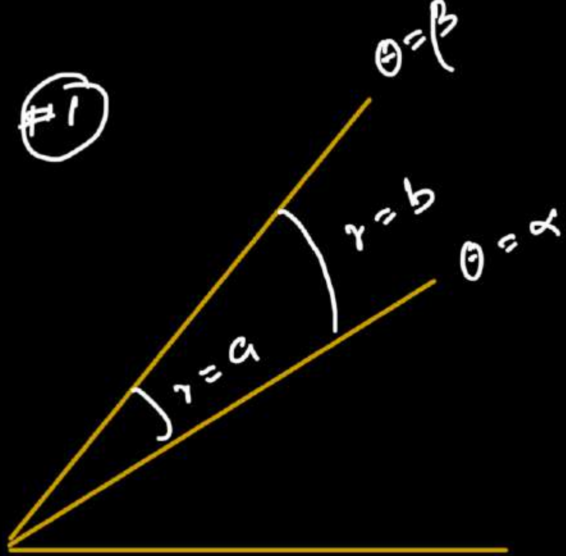
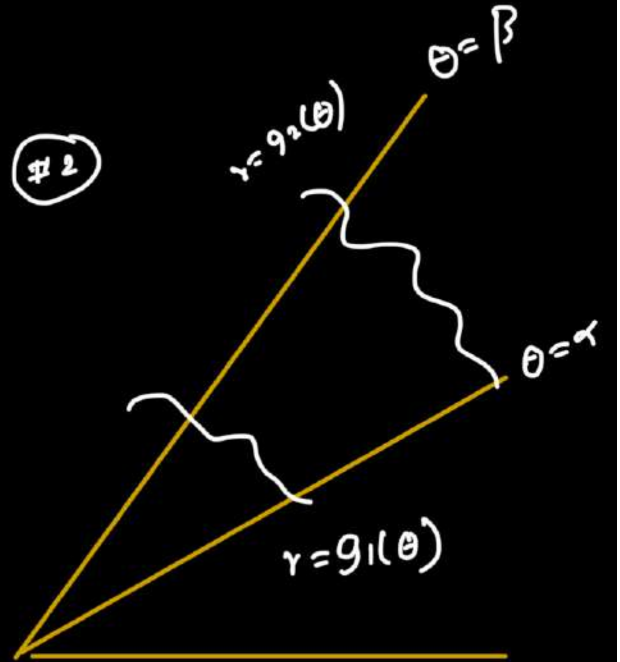
Polar co-ordinates in Double integration :-

$$\boxed{A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, d\theta}$$

#1



#2



Polar rectangle

$$\iint_R f(x,y)\, dA$$

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$\tan\theta = \frac{y}{x}$

$$\boxed{dA = r\,dr\,d\theta}$$

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∴ always take the constants as outer limits

∴ across the region along the radius

figure ①

$$\int_{\theta=\alpha}^{\theta=\beta} \int_{r=a}^{r=b} f(r\cos\theta, r\sin\theta)\, dr\, d\theta$$

figure ②

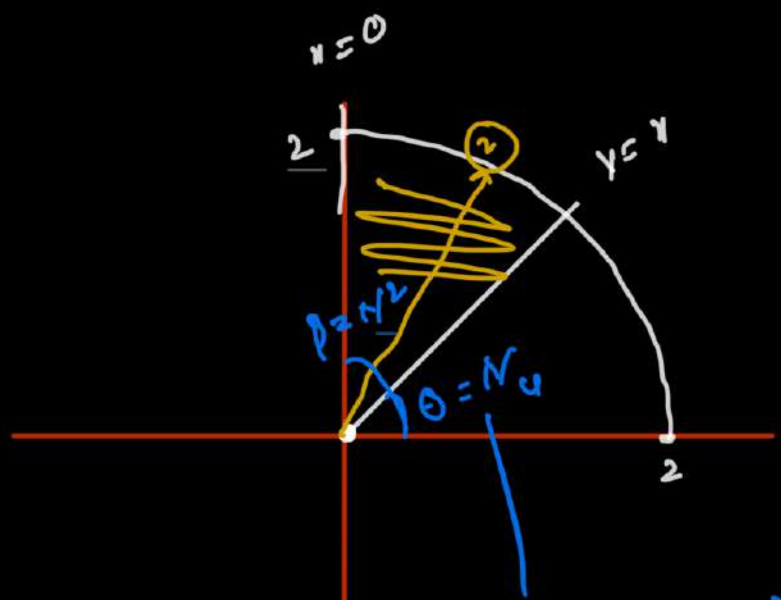
$$\int_{\theta=\alpha}^{\theta=\beta} \int_{r=g_1(\theta)}^{r=g_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

eg $\iint_R xy \; dA$, R: The region in Q. bound by $x^2+y^2=4$, $x=y$ & $x=0$

Sol:

$r^2=4$

$\boxed{r=2}$



$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\theta = \tan^{-1}(1)$

$\boxed{\theta = \pi/4}$

$$\int_{\theta=\pi/4}^{\theta=\pi/2}\int_{r=0}^{r=2} r\cos\theta \cdot r\sin\theta \;\; r\,dr\,d\theta$$

$$\int_{\pi/4}^{\pi/2}\int_{0}^{2} r^3\cos\theta\sin\theta \;\; dr\,d\theta$$

4 $\qquad 2$

$$\int_{\pi/4}^{\pi/2} \left[\frac{r^4}{4} \cos\theta \sin\theta \right]_0^2 d\theta$$

$$\int_{\pi/4}^{\pi/2} \left[\frac{16}{4} \cos\theta \sin\theta \right] - 0 \quad d\theta$$

$$4\int_{\pi/4}^{\pi/2} \cos\theta \sin\theta \quad d\theta$$

$$4 \left[\frac{\sin^2\theta}{2} \Big)_{\pi/4}^{\pi/2} \right]$$

$$4 \left[\frac{\sin^2(\pi/2)}{2} - \frac{\sin^2(\pi/4)}{2} \right]$$

$$4 \left[\frac{1}{2} - \frac{1}{4} \right]$$

$$4 \left[\frac{1}{4} \right] = 1$$

ANS

$$\boxed{\because \int f(x) \cdot f'(x) \, dx = \frac{[f(x)]^{n+1}}{n+1} + C}$$

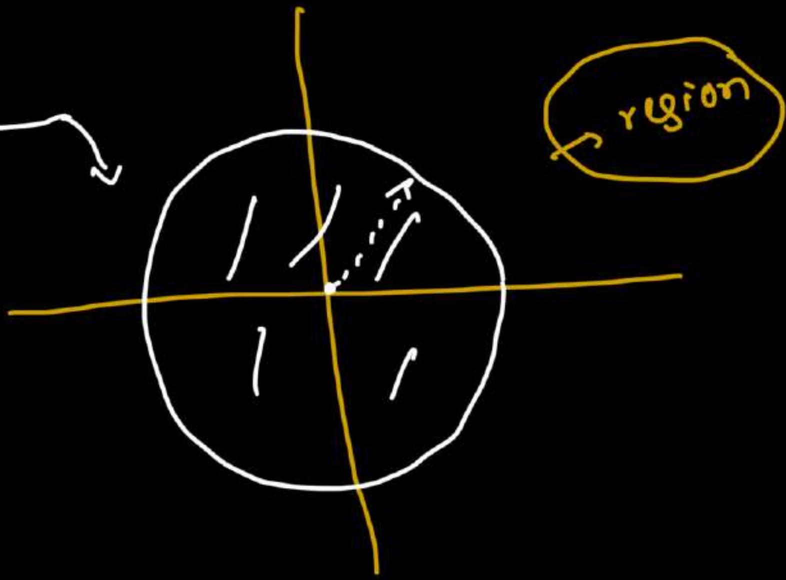
Q) Find volume between $z = 9 - x^2 - y^2$ & the xy-plane as bound by the cylinder

$x^2 + y^2 = 1$

$\downarrow$

$r^2 = 1$

$r = 1$



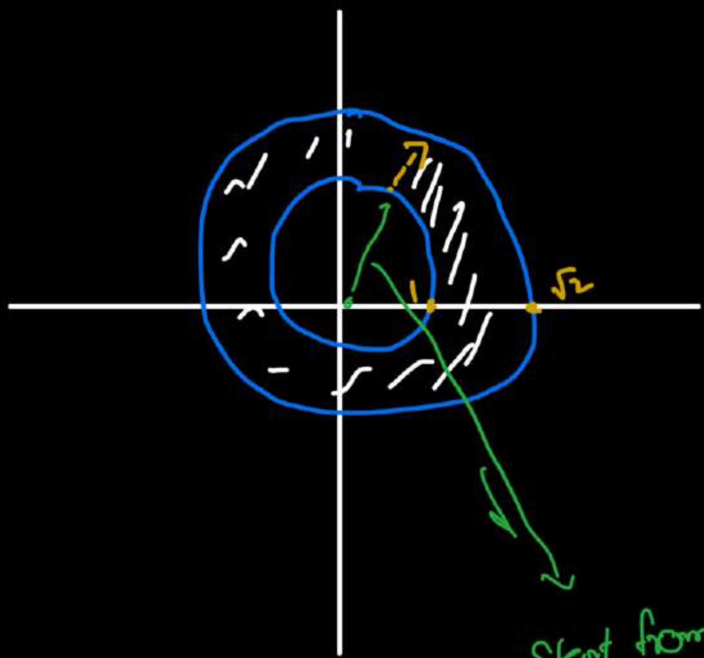
$$\int_0^{2\pi} \int_0^1 (9 - r^2) r \, dr \, d\theta$$

$$\int_0^{2\pi} \int_0^1 9r - r^3 \, dr \, d\theta$$

$$\int_0^{2\pi} \left[\frac{9r^2}{2} - \frac{r^4}{4} \right]_0^1 d\theta$$

Q) Volume b/w $z = \dfrac{y^2}{x^2+y^2}$, above xy-plane

& b/w cylinders $x^2+y^2=1$ & $x^2+y^2=2$



Start from origin, as the arrow touches the shaded region that will become our $\boxed{\text{initial } r}$

$$\int_{\theta=0}^{\theta=2\pi}\int_{1}^{\sqrt{2}} \frac{\cancel{r^2}\sin^2\theta}{\cancel{r^2}} \cdot r\,dr\,d\theta$$

$$\int_{0}^{2\pi}\int_{1}^{\sqrt{2}} \sin^2\theta \, r\,dr\,d\theta$$

$$\int_0^{2\pi} \sin^2\theta \left[\frac{r^2}{2}\right]_1^{\sqrt{2}} d\theta$$

$$\int_0^{2\pi} \sin^2\theta \left[\frac{2}{2} - \frac{1}{2}\right] d\theta$$

$$\frac{1}{2}\int_0^{2\pi} \sin^2\theta \, d\theta$$

$$\frac{1}{2}\int_0^{2\pi} \frac{1-\cos 2\theta}{2} \, d\theta$$

$$\frac{1}{4}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$\frac{1}{4}\left[(2\pi - 0) - (0-0)\right]$$

$$\frac{2\pi}{4} = \pi/2$$

ANS

$\equiv$

$\iint_R y\,dA$, R: The region bound by $x^2+y^2=2x$ & $y=x$

$x^2-2x+y^2=0$

$(x-1)^2-1+y^2=0$

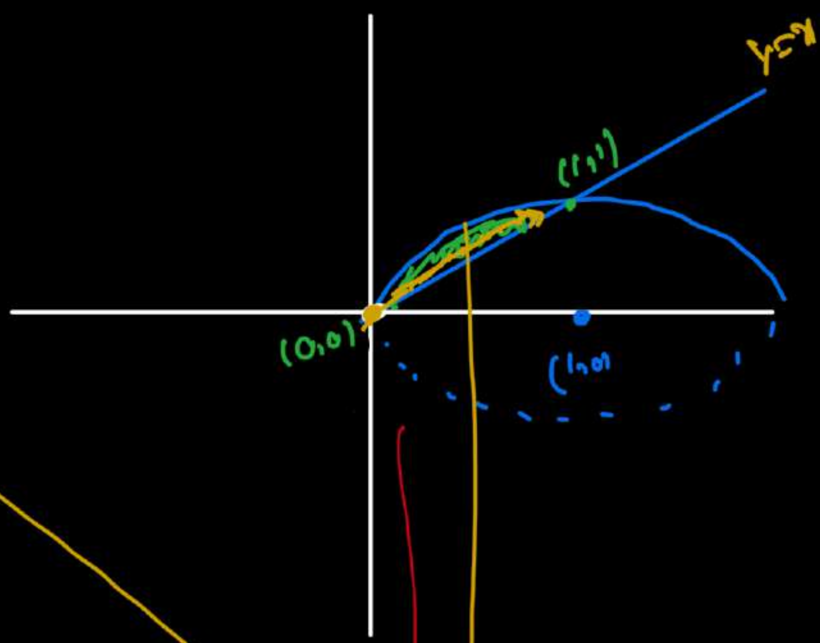
$\boxed{(x-1)^2+y^2=1}$

$\boxed{y=x}$

$x^2+y^2=2x$

$r^{\cancel{2}}=2\cancel{r}\cos\theta$

$\boxed{r=2\cos\theta}$



$$\int_{\theta=\pi/4}^{\theta=\pi/2}\int_{r=0}^{r=2\cos\theta} r\sin\theta \; r\,dr\,d\theta$$

$$\int^{\pi/2}\int^{2\cos\theta} \sin\theta \; r^2\,dr\,d\theta$$

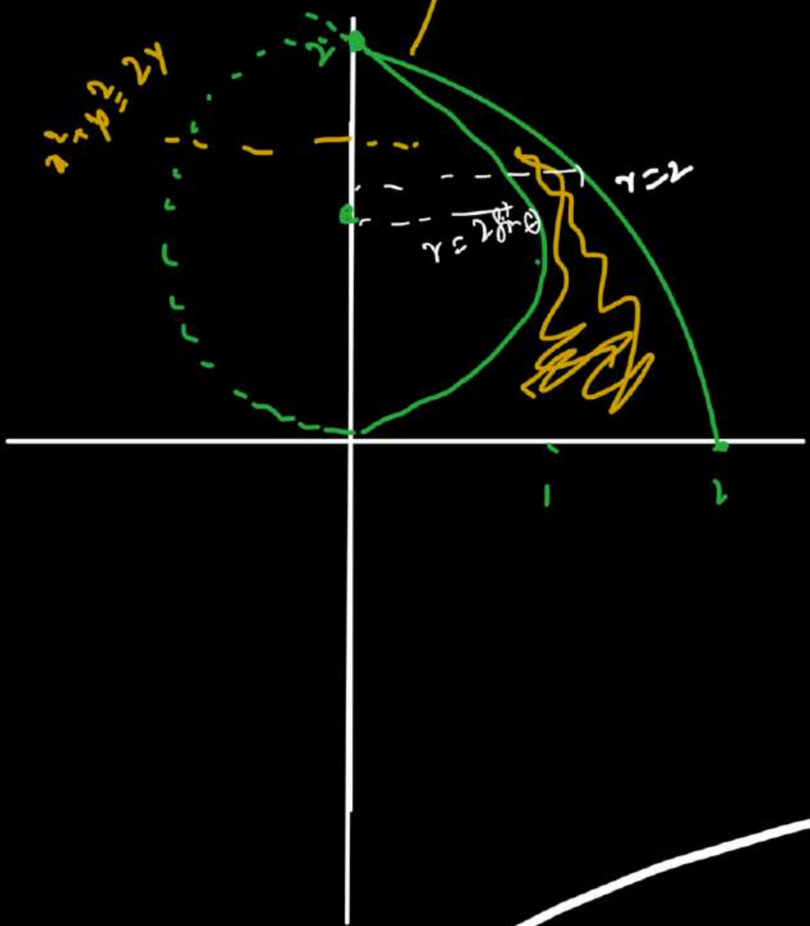
Eq: $x^2+y^2=2x$

we need to convert it in the form of polar to make it in the integral

Eg 2 Set the region

$R: \; x^2+y^2=4 \quad , \quad x^2+y^2=2y$ in QuaD:

$x^2+y^2=4 \;\downarrow\; r=2$



$x^2+y^2=2y$

$\hookrightarrow x^2+y^2-2y=0$

$x^2+(y-1)=1$

$\hookrightarrow r^2=2r\sin\theta$

$\boxed{r=2\sin\theta}$

$$\int_{\theta=0}^{\theta=\pi/2}\int_{r=2\sin\theta}^{r=2} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

AN

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Limits:

$\theta = \pi/4 \quad u = 1/\sqrt{2}$

$\theta = \pi/2 \quad u = 0$

$\boxed{\theta = \pi/4}$ $\boxed{\theta = \pi/2}$

$\pi/2 > \pi/4$

$$+\frac{8}{3}\int_0^{1/\sqrt{2}} u^3 + du$$

$$\frac{8}{3}\left[\frac{u^4}{4}\right]_0^{1/\sqrt{2}}$$

$$\frac{8}{3}\left[\frac{1}{16}\right]$$

$$\frac{1}{6}$$

Ans

$\theta = \pi/4$ $r = 0$

Ex: $x^2+y^2=2x$

we need to convert it in the form of polar to make it in the integral

$$\int_{\pi/4}^{\pi/2}\int_0^{2\cos\theta} \sin\theta \; r^2 \, dr \, d\theta$$

$$\int_{\pi/4}^{\pi/2} \sin\theta \left[r^3/3 \right]_0^{2\cos\theta} d\theta$$

$x^2+y^2=2x$ $y=x$

$x^2+x^2=2x$

$2x^2=2x$

$x^2-x=0$

$$\int_{\pi/4}^{\pi/2} \sin\theta \left[\frac{8\cos^3\theta}{3} - 0 \right] d\theta$$

$x=1$ $x=0$

$y=1$ $y=0$

$(1,1)$ $(0,0)$

$$\frac{8}{3}\int_{\pi/4}^{\pi/2} \sin\theta \; \cos^3\theta \; d\theta$$

$\tan\theta = \frac{y}{x}$ $\tan\theta = \frac{y}{x}$

let $u = \cos\theta$

$du = -\sin\theta d\theta$

$\tan\theta = \frac{0}{0}$

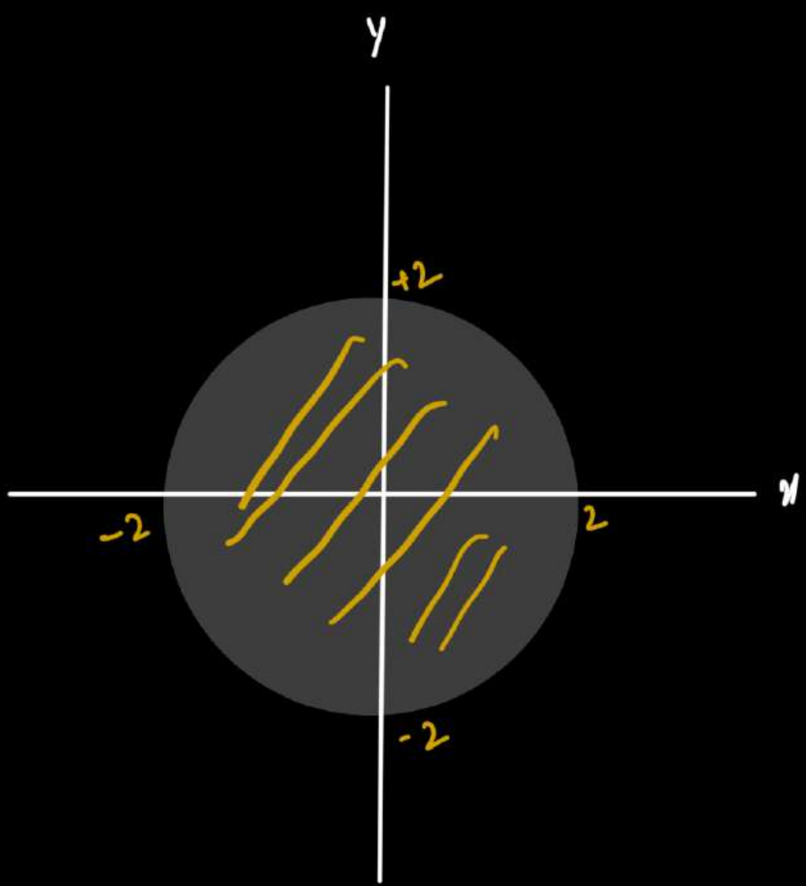
$\tan\theta = \frac{1}{1}$

$\sin\theta d\theta = -du$

Intersection — $\boxed{4 = x^2+y^2}$

$4 = r^2$

$\boxed{r = 2}$



$$V = \iint_R f \, dA \qquad f = 9-2x^2-2y^2 - 1 = 8-2(x^2+y^2) = 8-2r^2$$

top — bottom

$$V = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=2} (8-2r^2)\, r \, dr \, d\theta$$

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(π) b/c
~~when $\theta = 0$, $r = 2$~~

and $\theta = \pi/2$
$r = 0$

but when
$\theta = \pi$
$\boxed{r = -2}$ — and in polar when r is -ve we flip it means r becomes positive

$r = 2$

full rotation
$\boxed{0 \to \pi}$

These two will intersect and it gives the level curve

(Top function - Bottom function)

Ex 2 Volume b/w $z = 9 - 2x^2 - 2y^2$ & $z = 1$

Top — Bottom

Sol:

$9 - 2x^2 - 2y^2 = 1$

$8 = 2(x^2 + y^2)$

Intersection — $\boxed{4 = x^2 + y^2}$

$$z = 12 - 3x - 4y$$

$$f(x,y) = 12 - 3x - 4y$$

Eg Volume below $3x + 4y + z = 12$ bound by region b/w $x^2 + y^2 = 2x$ & above xy-plane

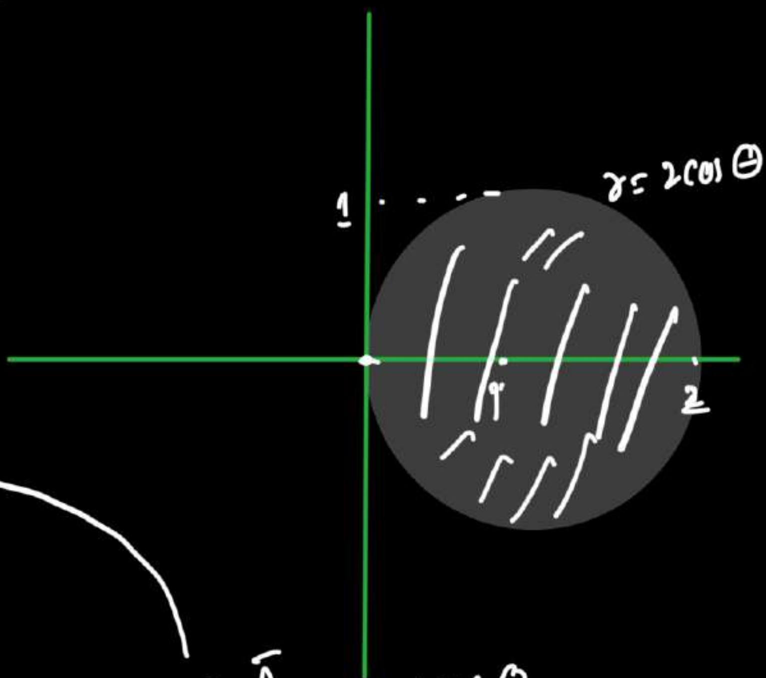
$$x^2 + 2x + y^2 = 0$$

$$\boxed{(x-1)^2 + y^2 = 1}$$

polar

$$r^2 = 2r\cos\theta$$

$$\boxed{r = 2\cos\theta}$$



$$\iint_R f(x,y)\, dA = \int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=2\cos\theta} (12 - 3r\cos\theta - 4r\sin\theta)\, r\, dr\, d\theta$$

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$$V = \int_0^{2\pi} \int_0^1 r\sqrt{2-r^2} - r^2 \, dr \, d\theta$$

$$V = \int_0^{2\pi} \int_0^1 r\sqrt{2-r^2} \, dr \, d\theta - \int_0^{2\pi} \int_0^1 r^2 \, dr \, d\theta$$

$\downarrow$

let $u = 2 - r^2$

$\frac{du}{dr} = -2r$

$\frac{du}{-2} = r\,dr$

$r = 0 \rightarrow u = 2$

$r = 1 \rightarrow u = 1$

Q volume b/w $x^2+y^2+z^2=2$ & $z=\sqrt{x^2+y^2}$

Sol

$z=\sqrt{2-x^2-y^2}$ $\qquad$ $z=\sqrt{x^2+y^2}$

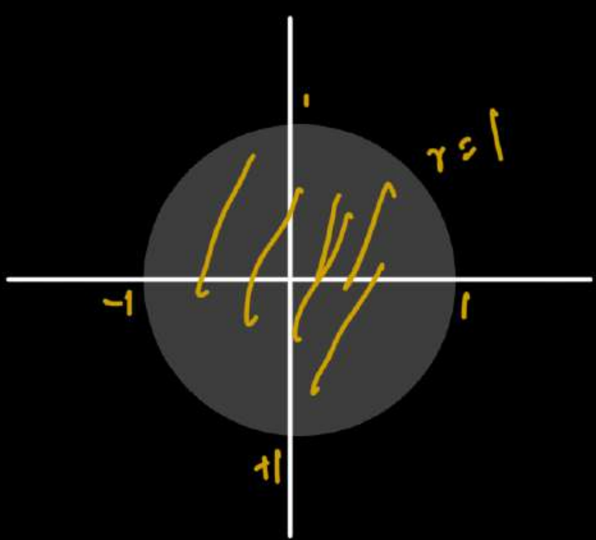
$$\sqrt{2-x^2-y^2}=\sqrt{x^2+y^2}$$

$$2-x^2+y^2=x^2+y^2$$

$$2=2x^2+2y^2$$

$$\cancel{2}=\cancel{2}(x^2+y^2)$$

intersection: $\leftarrow$ $\boxed{x^2+y^2=1}$



1. let P(0,0)
↳ to Decide top & bottom

$\sqrt{2-r^2}-\sqrt{r^2}$

$\sqrt{2-r^2}-r$

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$$V = \int_0^{2\pi} \left[\frac{8r^2}{2} - \frac{2r^4}{4} \right]_0^2 d\theta$$

$$V = \int_0^{2\pi} 4(2)^2 - \frac{2(2)^4}{4} \, d\theta$$

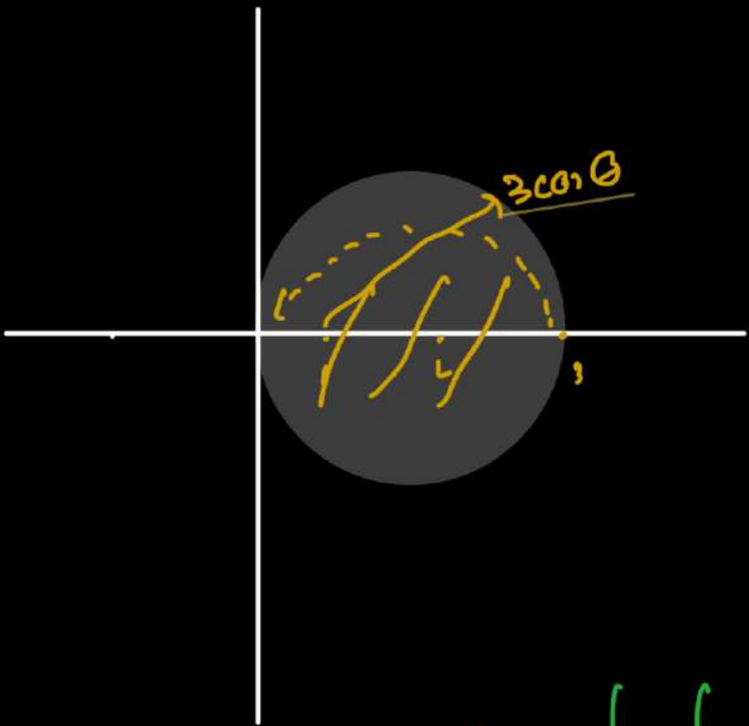
$$V = \int_0^{2\pi} (16 - 8) \, d\theta$$

$$V = 8 \int_0^{2\pi} d\theta \Rightarrow 8 \left[\theta \right]_0^{2\pi}$$

$$8 \left[2\pi - 0 \right] = 16\pi$$

ANS

Eg find Area of 'R' where 'R' is the region bound by $r = 3\cos\theta$



$$A = \iint_R 1 \, dA$$

$$A = 2\int_{\theta=0}^{\theta=\pi/2} \int_{r=0}^{r=3\cos\theta} r \, dr \, d\theta \Rightarrow \frac{9\pi}{4}$$

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$$V = \frac{4\pi(\sqrt{2}-1)}{3}$$

Ans

(b)

$$V = \iint_R f(x,y)\,dA \quad , \quad f(x,y) = \text{Height}$$

$$V = A \cdot H$$

$$A = \frac{V}{H}$$

$$A = \frac{\iint_R \cancel{f(x,y)}\,dA}{\cancel{f(x,y)}}$$

∵ volume is Equal to Area when height is Equal to one

$$\boxed{A = \iint_R dA}$$ Given area of a Region : R

$$V = \int_0^{2\pi} + \int_1^2 \sqrt{u}\left(+\frac{du}{2}\right) d\theta \quad - \int_0^{2\pi} \frac{r^3}{3}\Big|_0^1 \, d\theta$$

$$V = \int_0^{2\pi} \frac{1}{2}\int_1^2 u^{1/2}\, du\, d\theta \quad - \int_0^{2\pi} \frac{1}{3}\, d\theta$$

$$V = \frac{1}{2}\int_0^{2\pi} \frac{2}{3}\left[u^{3/2}\right]_1^2 d\theta \quad - \frac{1}{3}(2\pi)$$

$$V = \frac{1}{3}\int_0^{2\pi} \left(2^{3/2} - 1\right) d\theta \quad - \frac{2\pi}{3}$$

$$V = \frac{1}{3}\left[2\sqrt{2} - 1\right]\left[\theta\right]_0^{2\pi}$$

$$V = \frac{1}{3}\left[2\sqrt{2} - 1\right]\left[2\pi\right]$$

$$V \quad \frac{4\sqrt{2}}{3}\pi - \frac{2\pi}{3} \quad - \frac{2\pi}{3}$$

eg

$$\int_{-1}^{+1}\int_{0}^{\sqrt{1-y^2}} \frac{1}{1+x^2+y^2}\,dx\,dy$$

$x=0$

$x=\sqrt{1-y^2}$ — right side semi circle

$y=-1$

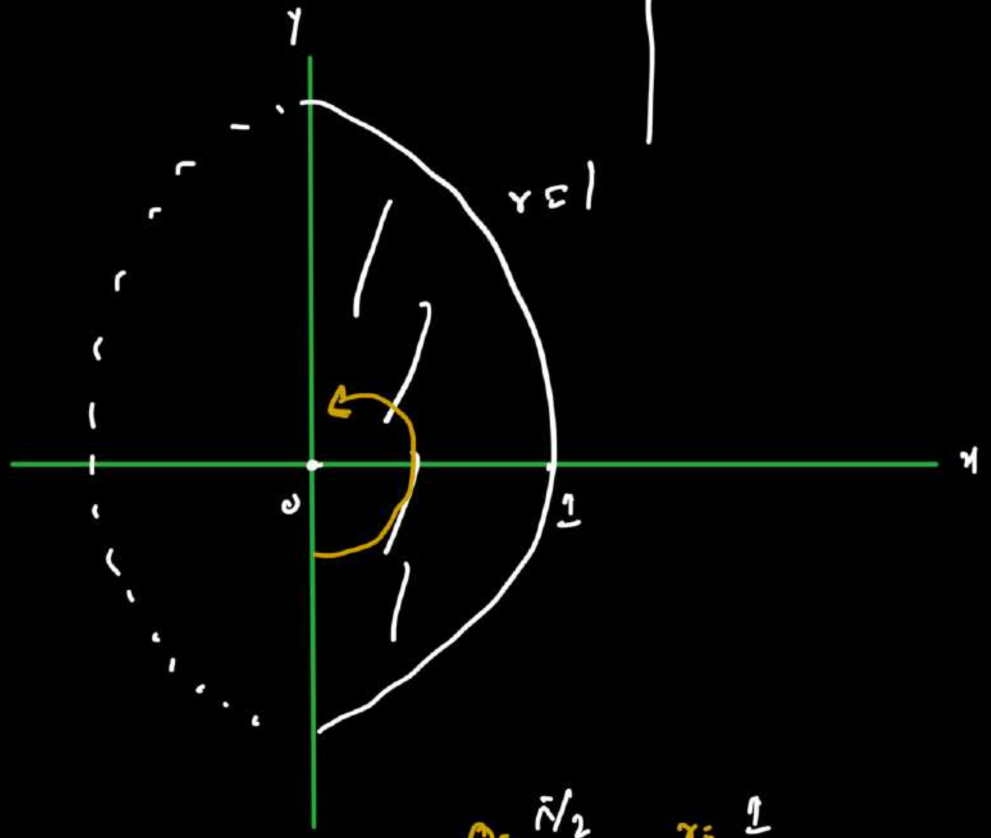
$y=1$

$x^2+y^2=1$

$\boxed{r=1}$

sol

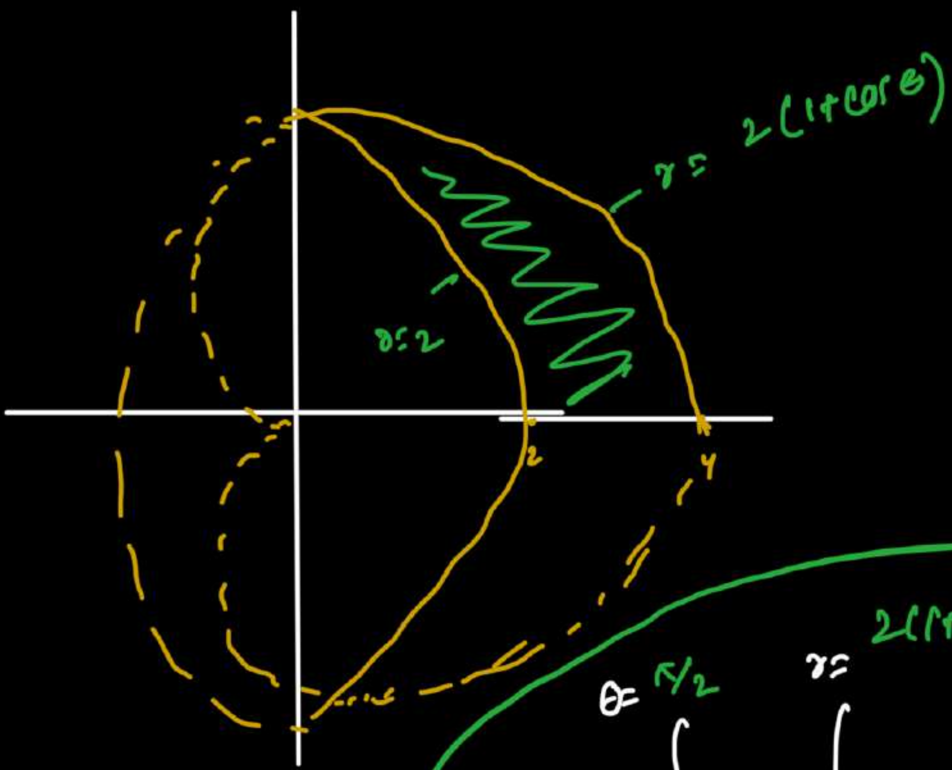
$y=0$



$$\int_{\theta=-\pi/2}^{\theta=\pi/2}\int_{r=0}^{r=1} \frac{1}{1+r^2}\; r\,dr\,d\theta$$

circle cardiod.

Q) Find Area of the region bound by $r = 2$ & $r = 2(1+\cos\theta)$ on QUAD-I



Area =

$$\int_{\theta=0}^{\theta=\pi/2} \int_{r=2}^{r=2(1+\cos\theta)} r \, dr \, d\theta$$

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//

$\because$ The θ range from 0 to $\pi/2$ b/c if we get our range from 0 to 2π which means the upper positive area will cancel the lower negative area which results in total area to be zero so for avoiding this we find the area of our upper circle by θ range: 0 to $\pi/2$ and double the answer b/c both are $\boxed{\text{symmetric}}$.

$\because$ whenever you are given a region in the polar form like: $r = x\cos\theta$ or $r = y\sin\theta$

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$$\int_{-\pi/2}^{\pi/2} \frac{1}{2} \left[\ln a - 0\right] d\theta$$

$$\frac{\ln a}{2} \int_{-\pi/2}^{\pi/2} d\theta$$

$$\frac{\ln a}{2} \left[\theta\right]_{-\pi/2}^{\pi/2}$$

$$\frac{\ln a}{2} \left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \frac{\ln a}{2} (\pi)$$

$$\frac{\pi \ln a}{2}$$

Ans

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$-\pi/2$

let $u = 1 + r^2$

$\frac{du}{dr} = 2r$

$du/2 = r\,dr$

$r = 0 \quad u = 1$

$r = 1 \quad u = 2$

$$\int_{-\pi/2}^{\pi/2} \int_{1}^{2} \frac{du/2}{u} \, d\theta$$

$$\int_{-\pi/2}^{\pi/2} \frac{1}{2} \Big[\ln u \Big]_{1}^{2} \, d\theta$$

$$\int_{-\pi/2}^{\pi/2} \frac{1}{2} \left[\ln 2 - 0 \right] d\theta$$

$\boxed{3\pi/2 \text{ to } \pi/2}$

↑

wrong

b/c

we measure angle only in clockwise direction.

(θ)

↓

$y = 1 \rightarrow y = r\sin\theta$

$1 = r\sin\theta$

$\therefore r = 1$

$\sin\theta = 1/1 = \theta = \pi/2$

$y = -1 \rightarrow y = r\sin\theta$

$-1 = r\sin\theta$

$\theta = \frac{-1}{1} = -\pi/2$ (or $3\pi/2$)

$$\int_{-\pi/2}^{\pi/2}\int_{0}^{1} \frac{r}{1+r^2}\, dr\, d\theta$$

$$\int_0^{\pi/4} 2\sec\theta \, d\theta$$

$$2\left[\ln|\sec\theta - \tan\theta|\right]_0^{\pi/4}$$

$$2\left[\ln|\sqrt{2} - 1| - \ln|1-0|\right]$$

$$2\left[\ln(\sqrt{2}-1) - 0\right]$$

$$\boxed{2\ln(\sqrt{2}-1)}$$

Ans

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$$\int_{\theta=0}^{\theta=\pi/4}\int_{r=\sec\theta}^{r=3\sec\theta} \frac{1}{\sqrt{r^2}} \, r\,dr\,d\theta$$

Sol

$$\int_{0}^{\pi/4}\int_{\sec\theta}^{3\sec\theta} \frac{1}{\cancel{r}} \cdot \cancel{r} \, dr\,d\theta$$

$$\int_{0}^{\pi/4} \Big[r \Big]_{\sec\theta}^{3\sec\theta} d\theta$$

$$\int_{0}^{\pi/4} (3\sec\theta - \sec\theta)\, d\theta$$

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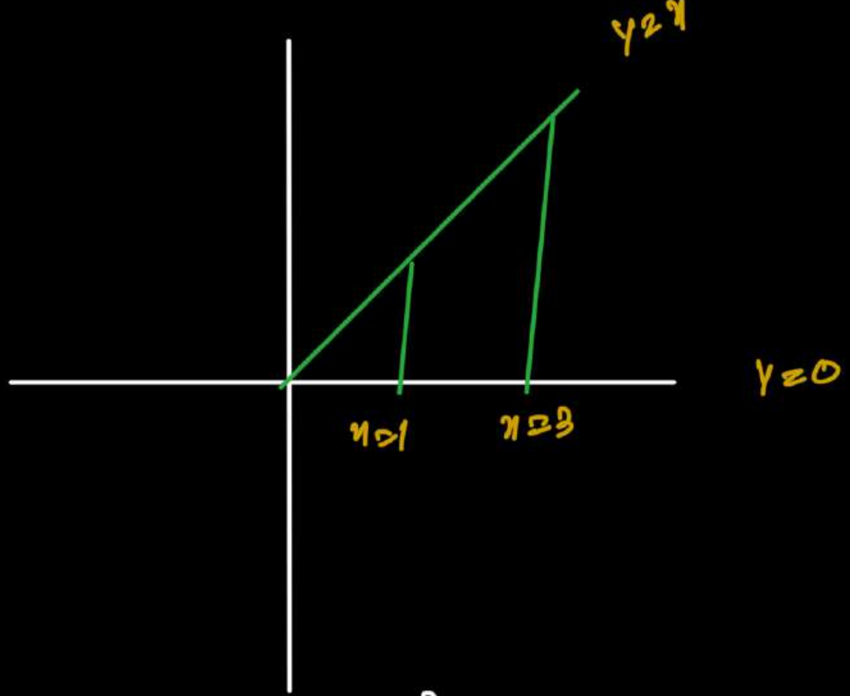
Q) $\displaystyle\int_1^3 \int_0^x \frac{1}{\sqrt{x^2+y^2}}\, dy\, dx$



$x = 3$

$r\cos\theta = 3$

$r = \frac{3}{\cos\theta} = 3\sec\theta$

$x = 1$

$r\cos\theta = 1$

$r = \frac{1}{\cos\theta}$

$r = \sec\theta$

$y = x \Rightarrow \tan(y/x) = \theta$

$\tan 1 = \theta$

$\theta = \pi/4$

$y = 0 \rightarrow \tan(0) = \theta \rightarrow \theta = 0^\circ$