$\cancel{x = r\sin\theta}$
$x = r\cos\theta$
$y = r\sin\theta$
$r^2 = x^2 + y^2$

Ex : 14.3

Q1) $\int_0^{\pi/2}\int_0^{\sin\theta} r\cos\theta \, dr\, d\theta$

- $\frac{r^2}{2}\cos\theta \Big|_0^{\sin\theta} \rightarrow \frac{\sin^2\theta\cos\theta}{2} \rightarrow \frac{1}{2}\int \cos\theta \cdot \sin^2\theta \cdot d\theta$

- $\frac{1}{2}\frac{\sin^3\theta}{3} \rightarrow \frac{1}{6}\sin^3\theta \Big|_0^{\pi/2} \Rightarrow \frac{1}{6}(1)^3 - 0 \rightarrow \frac{1}{6}$

Q2) $\int_0^{\pi}\int_0^{1+\cos\theta} r\, dr\, d\theta$

- $\frac{r^2}{2}\Big|_0^{1+\cos\theta} \rightarrow \frac{(1+\cos\theta)^2}{2} \rightarrow \cancel{\frac{1}{2}\int 1 \cdot d\theta + \frac{1}{2}\int \cos\theta\, d\theta}$

$\cancel{\frac{1}{2}\theta + \frac{1}{2}\sin\theta \Big|_0^{\pi} \Rightarrow \frac{1}{2}(\pi) + \frac{1}{2}\sin(\pi) - 0 - 0}$

- $\frac{1}{2}\int (1+\cos\theta)^2 \cdot d\theta \rightarrow (1 + 2\cos\theta + \cos^2\theta)\frac{1}{2}$

$\int \frac{1}{2} \cdot d\theta + \int \cos\theta \cdot d\theta + \frac{1}{2}\int \cos^2\theta \cdot d\theta \Big\} \frac{1}{2}\int \frac{\cos 2x + 1}{2}$

$\frac{1}{2}\theta + \sin\theta + \cancel{\frac{1}{2}\int \left[-1 + 2\cos^2 2\theta\right] d\theta}$

$\frac{1}{2}\theta + \sin\theta + \frac{1}{24}\left[\cancel{\theta + \frac{2\sin(2\theta)}{2}}\right]^{\cos(2x)+1} \Rightarrow \frac{1}{2}\theta + \sin\theta + \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta)$

$\cancel{= \theta + \sin\theta + \frac{1}{4}\sin(2\theta) \Big|_0^{\pi} \Rightarrow \pi + 0 +}$

$= \frac{1}{2}\theta + \sin\theta + \frac{1}{4}\int \cos(2\theta) \cdot d\theta + \frac{1}{4}\int \cdot d\theta$

$\frac{1}{2}\theta + \sin\theta + \frac{1}{4}\frac{\sin 2\theta}{2} + \frac{1}{4}\theta \Rightarrow \frac{1}{2}\pi + \frac{1}{4}\pi$

$$\int_{\alpha}^{\beta}\int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta \qquad \cos(2x) = \cos^2 x - \sin^2 x$$

$$= \frac{3}{4}\pi$$

Q3) $\int_0^{\pi/2}\int_0^{\alpha\sin\theta} r^2\, dr\, d\theta \rightarrow \frac{r^3}{3}\Big|_0^{\alpha\sin\theta} \rightarrow \alpha^3\sin^3\theta$

$\frac{\alpha^3}{3}\int \sin^3\theta \cdot d\theta \rightarrow \frac{\alpha^3}{3} \cancel{\cos}\left[-\frac{1}{3}\sin^2\theta\cos\theta + \frac{2}{3}\int \sin(\theta)\cdot d\theta\right]$

$\frac{\alpha^3}{3}\left(-\frac{1}{3}\sin^2\theta\cdot\cos\theta + \frac{2}{3}(-\cos\theta)\right) = -\frac{1}{9}\alpha^3\sin^2\theta\cdot\cos\theta - \frac{2}{9}\alpha^3\cos\theta$

$-\frac{\alpha^3}{9}\cos\theta(\sin^2\theta + 2)\Big|_0^{\pi/2} \rightarrow -\frac{\alpha^3}{9}\cos(\pi/2)(1+1) - \theta$

$\frac{\alpha^3}{9}\cos(0)(0+2) = \frac{2\alpha^3}{9}$

Q4) $\int_0^{\pi/6}\int_0^{\cos 3\theta} r\, dr\, d\theta \rightarrow \frac{r^2}{2}\Big|_0^{\cos 3\theta} \rightarrow \frac{(\cos 3\theta)^2}{2}$

$\frac{1}{2}\int_0^{\pi/6} (\cos 3\theta)^2 \cdot d\theta \Rightarrow \frac{1}{2}\int_0^{\pi/6} \frac{(\cos(2\times 3\theta) + 1)}{2} \cdot d\theta$

$\frac{1}{4}\int \cos 6\theta \cdot d\theta + \frac{1}{4}\int 1 \cdot d\theta \Rightarrow \frac{1}{4}\frac{\sin 6\theta}{6} + \frac{1}{4}\theta \Rightarrow \frac{1}{24}\sin 6\theta + \frac{1}{4}\theta$

$\Rightarrow \frac{1}{24}\sin\left(6\times\frac{\pi}{6}\right) + \frac{1}{4}\left(\frac{\pi}{6}\right) - 0 - 0 \Rightarrow 0 + \frac{\pi}{24}$

Q5) $\int_0^{\pi}\int_0^{1-\sin\theta} r^2\cos\theta \cdot dr\, d\theta \rightarrow \frac{r^3}{3}\cos\theta\Big|_0^{1-\sin\theta} \Rightarrow \frac{r^3}{3}\cos\theta(1-\sin\theta)^3 - \frac{r^3}{3}(1)$

$\cancel{\frac{r^3}{3}\int \cos\theta(1-\sin\theta)^3 d\theta - \frac{r^3}{3}\int 1\, d\theta}$

$\int_0^{\pi} \frac{(1-\sin\theta)^3}{3}\cos\theta \cdot d\theta \rightarrow -\frac{1}{3}\frac{(1-\sin\theta)^4}{4} \rightarrow -\frac{1}{12}(1-\sin\theta)^4\Big|_0^{\pi}$

$= -\frac{1}{12}(1-0)^4 + \frac{1}{12}(1) \Rightarrow 0$

Q6) $\int_0^{\pi/2}\int_0^{\cos\theta} r^3\,dr\,d\theta \rightarrow \frac{r^4}{4}\Big|_0^{\cos\theta} \rightarrow \frac{\cos^4\theta}{4} - 0$

$\frac{1}{4}\int_0^{\pi/2} \cos^4\theta \rightarrow \frac{1}{4}\left[\frac{1}{4}\cos^3\theta\sin\theta + \frac{3}{4}\int \cos^2\theta \cdot d\theta\right]$

$= \frac{1}{4}\left[\frac{1}{4}\cos^3\theta\cdot\sin\theta + \frac{3}{4}\int \frac{1+\cos 2\theta}{2}\cdot d\theta\right]$

$= \frac{1}{4}\left[\frac{1}{4}\cos^3\theta\cdot\sin\theta + \frac{3}{8}\int 1\cdot d\theta + \frac{3}{8}\int \cos 2\theta \cdot d\theta\right]$

$= \frac{1}{4}\left[\frac{1}{4}\cos^3\theta\cdot\sin\theta + \frac{3}{8}\theta + \frac{3}{8}\frac{\sin(2\theta)}{2}\right]$

$= \frac{1}{16}\cos^3\theta\sin\theta + \frac{3}{32}\theta + \frac{3}{32(2)}\sin 2\theta \Big|_0^{\pi/2}$

$= \frac{1}{16}(0) + \frac{3}{32}\left(\frac{\pi}{2}\right) + \frac{3}{64}(0) - \frac{1}{16}(0) + 0 + 0 \Rightarrow \frac{3\pi}{64}$

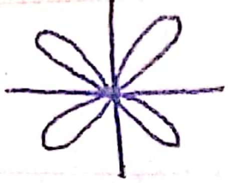
Q7) $\int_0^{2\pi}\int_0^{1-\cos\theta} r\,dr\,d\theta \rightarrow \frac{r^2}{2}\Big|_0^{1-\cos\theta} = \frac{(1-\cos\theta)^2}{2}$

$\frac{(1-2\cos\theta+\cos^2\theta)}{2} \rightarrow \int \frac{1}{2}\cdot d\theta - \int \cos\theta\, d\theta + \frac{1}{2}\int \cos^2\theta\, d\theta$

$\frac{1}{2}\theta - \sin\theta + \frac{1}{2}\left[\int \frac{1}{2} + \frac{\cos 2\theta}{2}\right] \rightarrow \frac{1}{2}\theta - \sin\theta + \frac{1}{4}\int 1\cdot d\theta + \frac{1}{4}\int$

$= \frac{1}{2}\theta - \sin\theta + \frac{1}{4}\theta + \frac{1}{4(2)}\sin 2\theta = \Big|\frac{3}{4}\theta - \sin\theta + \frac{1}{8}\sin 2\theta$

$= \frac{3}{4}(2\pi - 0) - \cancel{\sin(2\pi - 0)} + \cancel{\frac{1}{8}\sin(2(2\pi-0))} = \frac{3\pi}{2}$



88)

• $\sin 2\theta \to 4$ petals

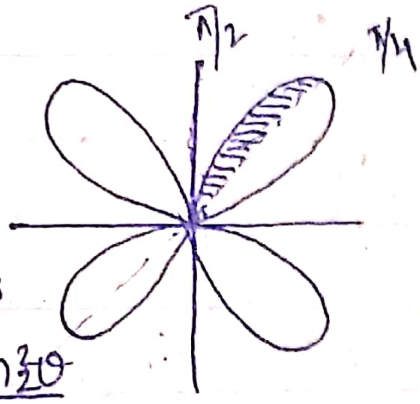
$$\int_0^{\pi/2}\int_0^{\sin 2\theta} r\,dr\,d\theta \to \frac{r^2}{2}\Big|_0^{\sin 2\theta} \to \frac{(\sin 2\theta)^2 - 0}{2}$$

$$\cancel{\frac{1}{(1)2}\int_0^{\pi/2}\sin^2 2\theta \,(2)\, d\theta \to \frac{1}{4}(-\cos 2\theta)\Big|_0^{\pi/2} \to -\frac{1}{4}\cos(\pi)+0}$$

$$\frac{1}{2}\int \frac{1-\cos 4\theta}{2}\,d\theta \to \frac{1}{4}\int 1\,d\theta - \frac{1}{4(4)}\int \cos 4\theta\,(4)\, d\theta$$

$$= \frac{1}{4}\theta - \frac{1}{16}\sin 4\theta\Big|_0^{\pi/2} \Rightarrow \frac{1}{4}\left(\frac{\pi}{2}\right) - \frac{1}{16}\sin\left(4\left(\frac{\pi}{2}-0\right)\right)$$

$$= \frac{\pi}{8} - 0 \to \frac{\pi}{\cancel{8}\,2}\times\cancel{4} \Rightarrow \frac{\pi}{2}$$



89)

• $\sin 2\theta \to 4$ petals, but 1st Quadrants

$$\int_{\pi/4}^{\pi/2}\int_{\sin 2\theta}^{1} r\,dr\,d\theta \to \frac{r^2}{2}\Big|_{\sin 2\theta}^{1} \to \frac{1}{2} - \frac{\sin^2 2\theta}{2}$$

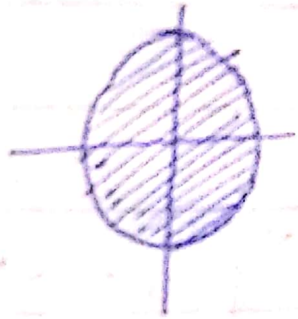
$$\frac{1}{2}\int 1\cdot d\theta - \frac{1}{2}\int \frac{1-\cos 4\theta}{2}\,d\theta \Rightarrow \frac{1}{2}\theta - \frac{1}{4}\theta + \frac{1}{4(4)}\sin 4\theta$$

$$= \frac{1}{4}\theta - \frac{1}{16}\sin 4\theta\Big|_{\pi/4}^{\pi/2} \Rightarrow \frac{1}{4}\left(\frac{\pi}{2}-\frac{\pi}{4}\right) + \frac{1}{16}\sin(\cancel{2\pi}+\pi) = \frac{\pi}{16}$$

$\cos(\theta)$

$$\int_0^{\pi/3}\int_{\sec\theta}^{2} r\,dr\,d\theta$$

~~Q1~~ Q23) $\iint \sin(x^2+y^2)\,dA$, $x^2+y^2=9$



$$\int_0^{2\pi}\int_0^{3} \sin\left(r^2\right)^{(2)} \cdot r\,dr\,d\theta$$

$$\frac{1}{2}\left(-\cos(r^2)\right)\Big|_0^3 \to -\frac{1}{2}\cos(9) + \frac{1}{2}\cancel{\cos(0)}^{\,1}$$

$$-\frac{1}{2}\cos(9)\int 1\cdot d\theta + \frac{1}{2}\int 1\cdot d\theta \to -\frac{1}{2}\cos(9)\theta + \frac{1}{2}\theta$$

$$\left(-\frac{1}{2}\cos 9 + \frac{1}{2}\right)\left(\frac{\pi}{2} - 0\right) \to \left(-\frac{\pi}{4}\cos 9 + \frac{\pi}{4}\right) \to \frac{\pi}{4}(1-\cos 9)$$

$$= \cancel{\frac{\pi}{4}}\cancel{(4)}(\cancel{\cos}\,1 - \cos 9) \to \pi(1-\cos 9)$$

14) $\iint \sqrt{9-x^2-y^2}\, dA$, $x^2+y^2=9$, first quadrant

$\cdot\ 9-(x^2+y^2) \to 9-r^2$

$\cdot \iint \sqrt{9-r^2}\cdot r\,dr\,d\theta \to \int_0^{\pi/2}\int_0^3 \sqrt{9-r^2}\cdot r\,dr\,d\theta$ (-2)

$-\frac{1}{\cancel{2}}\frac{(9-r^2)^{3/2}}{3/\cancel{2}} \to -\frac{1}{3}(9-r^2)^{3/2}\Big|_0^3 = -\frac{1}{3}(0)+\frac{1}{3}(9)^{3/2}$

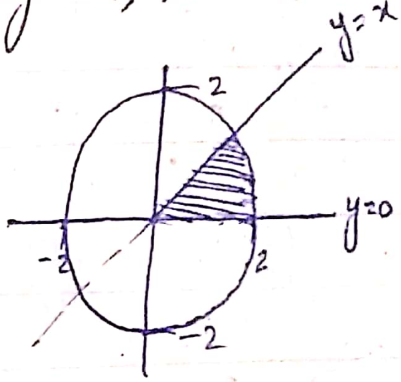
$= \frac{27}{3} \to 9$

$\cdot\ 9\int_0^{\pi/2} d\theta \to 9\theta\Big|_0^{\pi/2} \Rightarrow 9\left(\frac{\pi}{2}-0\right) \to \frac{9\pi}{2}$

Q25) $\iint \frac{1}{1+x^2+y^2}\, dA$, $y=0$, $y=x$, $x^2+y^2=4$, first Quad

$\iint \frac{1}{1+r^2}\cdot r\,dr\,d\theta$

$\int_0^{\pi/4}\int_0^2 \frac{1}{1+r^2}\, r\,dr\,d\theta$ (2)



$\frac{1}{2}\ln(1+r^2)\Big|_0^2 \to \frac{1}{2}\ln(5) - \cancel{\frac{1}{2}\ln(1)}$

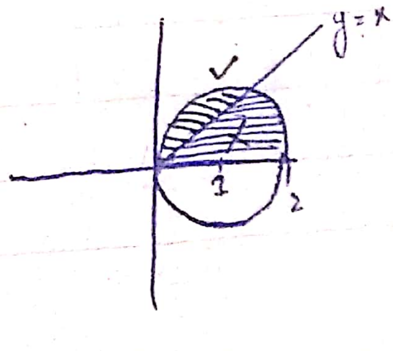
$\cdot\ \frac{1}{2}\ln 5 \int d\theta \to \frac{1}{2}\ln 5\,\theta\Big|_0^{\pi/4} \to \frac{1}{2}\ln 5\left(\frac{\pi}{4}\right) = \frac{\pi}{8}\ln 5$

26) $\iint 2y\, dA$, first Quad, $(x-1)^2+y^2=1$, below $y=x$

translate +1 in x

$\cdot\ (0 \leq \theta \leq \pi/4)$, (

$\int_{\pi/4}^{\pi/2}\int_0^{2\cos\theta} 2r\sin\theta\cdot r\,dr\,d\theta$



• $\int_0^{2\cos\theta} 2r^2\sin\theta \cdot dr\,d\theta \to 2\sin\theta \int_0^{2\cos\theta} r^2 \cdot dr$

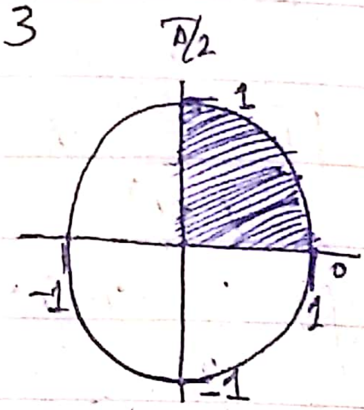
$2\sin\theta \cdot \frac{r^3}{3}\Big|_0^{2\cos\theta} = \frac{2}{3}\sin\theta\left((2\cos\theta)^3 - 0\right)$

$= \frac{2}{3}\sin\theta\left(8\cos^3\theta\right) \Rightarrow \frac{16}{3}\sin\theta \cdot \cos^3\theta$

• $\int_0^{\pi/4} (-)\frac{16}{3}\sin\theta\,(\cos\theta)^3 \to -\frac{16}{3}\,\frac{\cos^4\theta}{4} \to \frac{-\cancel{16}^{\,4}}{\cancel{12}_{\,3}}\cos^4\theta$

$-\frac{4}{3}\cos\left(\frac{\pi}{4}\right)^4 + \frac{4}{3}\cos(0) = \frac{1}{3}$

Q27) $\int_0^1\int_0^{\sqrt{1-x^2}} (x^2+y^2)\,dy\,dx$

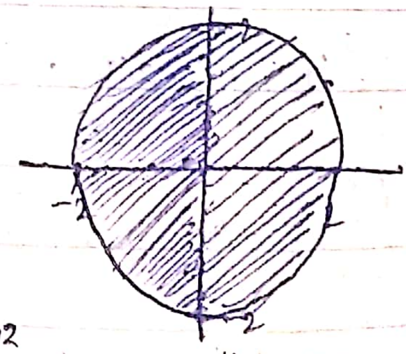


$\int_0^{\pi/2}\int_0^1 r^2\, r\,dr\,d\theta$

• $\int_0^1 r^3 \cdot dr \to \frac{r^4}{4}\Big|_0^1 = \frac{1}{4}$

• $\frac{1}{4}\theta\Big|_0^{\pi/2} \to \frac{1}{4}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{8}$

Q28) $\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-(x^2+y^2)} \cdot dx\,dy$



$x^2+y^2=4$

$\int_0^{2\pi}\int_0^2 e^{-r^2\,(-2)} \cdot r\,dr\,d\theta \to -\frac{1}{2}e^{-r^2}\Big|_0^2 \to -\frac{1}{2}\left(e^{-4} - e^0\right) =$

$= -\frac{1}{2}e^{-4}\int e^{-4}1 + \frac{1}{2}\int e 1 \to -\frac{1}{2}\theta e^{-4} + \frac{1}{2}\theta \to -\frac{1}{\cancel{2}}e^{-4}(\cancel{2}\pi) + \frac{1}{\cancel{2}}(\cancel{2}\pi)$

$= \pi\left(1-e^{-4}\right)$ $\qquad 2r\cos\theta - r^2\cos^2\theta$

Q19) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\; dy\,dx$

$\to r^2 = 2\cancel{r}\cos\theta$

$r = 2\cos\theta$

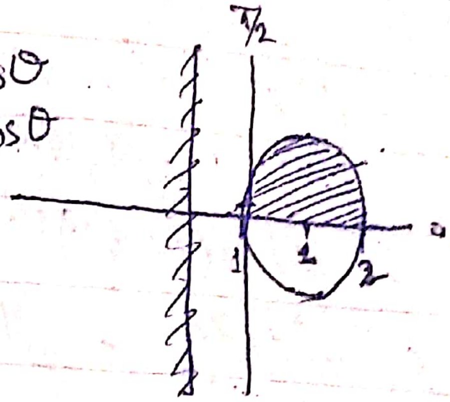
$y^2+x^2=2x$

$y^2+x^2-2x=0$

$y^2+(x)^2-2(x)(1)+(1)^2 = 1^2$

$y^2+(x-1)^2=1$

Translate



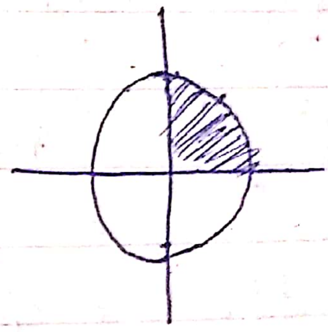
$\int_0^{\pi/2}\int_0^{2\cos\theta} r^2\,dr\,d\theta \to \frac{r^3}{3}\Big|_0^{2\cos\theta} \Rightarrow \frac{1}{3}\left(8\cos^3\theta - 0\right)$

$\frac{8}{3}\int_0^{\pi/2} \cos^3\theta\, d\theta \Rightarrow \frac{8}{3}\left[\cos^2 x\sin x + 2\int \sin^2 x\cos x\,dx\right]$

$= \frac{8}{3}\cos^2 x\sin x + \frac{16}{3}\,\frac{\sin^3 x}{3} \Rightarrow \frac{8}{3}\cos^2 x\sin x + \frac{16}{9}\sin^3 x\Big|_0^{\pi/2}$

$= \cancel{\frac{8}{3}\sin(\pi/2)} - \cancel{\frac{8}{3}\sin(0)} + \frac{16}{9}\left(\frac{\pi}{2}\right)^3 - \cancel{\frac{16}{9}(0)} = \frac{16}{9}$

Q30) $\int_0^1\int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\; dx\,dy$



$x^2+y^2=1 \to r^2=1$

$\int_0^{\pi/2}\int_0^1 \cos(r^2)\, r\,dr\,d\theta$

$\frac{1}{2}\sin(r^2)\Big|_0^1 \Rightarrow \frac{1}{2}\sin(1) - 0$

$\frac{1}{2}\sin 1\int(1)\cdot d\theta \to \frac{1}{2}\sin 1\,\theta\Big|_0^{\pi/2} \to \frac{1}{2}\sin 1\left(\frac{\pi}{2}-0\right) = \frac{\pi}{4}\sin 1$

$r^2\cos^2\theta = \cancel{r}\sin\theta$

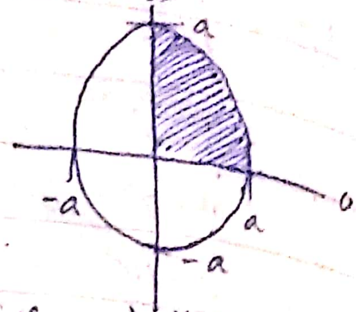
$r^2 - y^2 = y$ $\qquad$ $r = \underline{\sin\theta}$

$r^2 = y + y^2$

Q31) $\displaystyle\int_0^a\int_0^{\sqrt{a^2-x^2}} \frac{dy\,dx}{(1+x^2+y^2)^{3/2}}$ $(a>0)$

↳ +ve in x only $\pi/2$

$y^2+x^2=a^2 \to r^2=a^2 \to r=a$



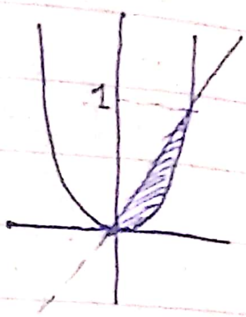
$\displaystyle\int_0^{\pi/2}\int_0^{a} \frac{(1)\,r\,dr\,d\theta}{(1+r^2)^{3/2}}$

$\displaystyle\frac{1}{2}\int \frac{2r\,dr\,d\theta}{(1+r^2)^{3/2}} \to \frac{1}{\cancel{2}}\,\frac{(1+r^2)^{-1/2}}{-\cancel{1/2}}$ ↳ $-1(1+r^2)^{-1/2}\Big|_0^a$

$-(1+a^2)^{-1/2} + (1+0)^{-1/2} \to -(1+a^2)^{-1/2}\displaystyle\int 1\cdot d\theta + \theta$

$-(1+a^2)^{-1/2}\left(\dfrac{\pi}{2}\right) + \dfrac{\pi}{2} \to \dfrac{\pi}{2}\left(1 - \dfrac{1}{(1+a^2)^{1/2}}\right)$

Q32) $\displaystyle\int_0^1\int_y^{\sqrt{y}} \sqrt{x^2+y^2}\,dx\,dy$



$x^2 = y \to r^{\cancel{2}}\cos^2\theta = \cancel{r}\sin\theta$

$x = y \to r = 0$ $\qquad$ $= r = \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\cos\theta}$

↳ $r = \tan\theta\sec\theta$

$\displaystyle\int_0^{\pi/4}\int_0^{\sec\theta\tan\theta} r\,r\,dr\,d\theta$

$\displaystyle\int_0^{\pi/4}\int_0^{\sec\theta\tan\theta} r^2\,dr\,d\theta \to \frac{r^3}{3}\Big|_0^{\sec\theta\tan\theta} = \frac{1}{3}\sec^3\theta\cdot\tan^3\theta$

$= \dfrac{1}{3}\displaystyle\int \sec\theta\tan\theta\cdot(\sec^2\theta\cdot\tan^2\theta)\cdot d\theta$

$= \dfrac{1}{3}\displaystyle\int \sec\theta\tan\theta\,\left(\overset{\sec^2\theta}{\cancel{1+\tan^2\theta}}\right)\left(\overset{1-\sec^2\theta}{\cancel{\tan^2\theta}}\right)$

$= \sec^3\theta\tan\theta - \sec^5\theta\cdot\tan\theta$

$$\int (\sec\theta)^2 \sec\theta\tan\theta \cdot d\theta - \int \sec\theta\cdot\tan\theta\,(\sec^4\theta)\cdot d\theta$$

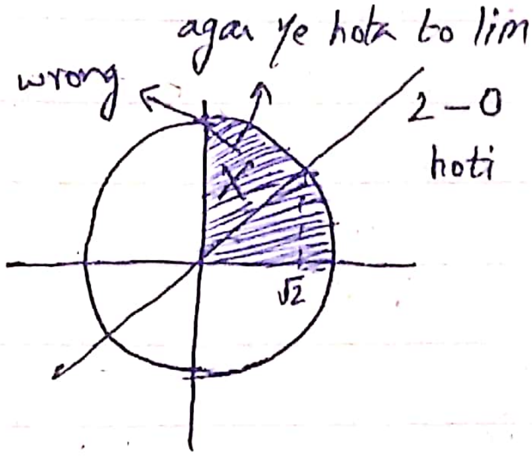
$\cdot d\theta$

$$\left[\frac{\sec^3\theta}{3} - \frac{\sec^5\theta}{5}\right] \Rightarrow \frac{1}{9}\sec^3\theta - \frac{1}{15}\sec^5\theta \cdot d\theta$$

$$\frac{1}{9}\left[\frac{1}{\cos\left(\frac{\pi}{4}\right)^3} - \frac{1}{\cos(0)^3}\right] - \frac{1}{15}\left[\frac{1}{\cos\left(\frac{\pi}{4}\right)^5} - \frac{1}{\cos(0)^5}\right]$$

$$= \frac{2\sqrt{2}}{9} - \frac{1}{9} + \frac{1}{15} -$$

3) $$\int_0^{\sqrt{2}}\int_y^{\sqrt{4-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\, dx\,dy$$



$x^2+y^2=4$

$r=2$

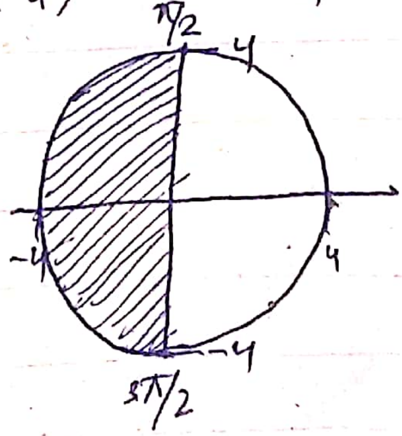
$$\int_0^{\pi/4}\int_0^{2} \frac{r\,dr\,d\theta}{\sqrt{1+r^2}}$$

$$\frac{1}{2}(1+r^2)^{-1/2} \to \frac{1}{\cancel{2}}\frac{(1+r^2)^{+1/2}}{\cancel{1/2}} \to (1+r^2)^{1/2}\Big|_0^2 \to 5^{1/2} - \cancel{0}\;1$$

$-1\int d\theta \qquad -\theta$

$$5^{1/2}\int 1\cdot d\theta \to 5^{1/2}\theta\Big|_0^{\pi/4} = 5^{1/2}\left(\frac{\pi}{4}\right) - \frac{\pi}{4}\;0 \Rightarrow \frac{\pi}{4}(\sqrt{5}-1)$$

4) $$\int_{-4}^{0}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} 3x\,dy\,dx$$



$y^2+x^2=16 \to r^2=16$

$r=4$

$$\int_{\pi/2}^{3\pi/2}\int_0^{4} 3r^2\cos\theta\,dr\,d\theta$$

$$3\cos\theta\int_0^4 r^2\cdot dr \to 3\cos\theta\,\frac{r^3}{\cancel{3}}\Big|_0^4 \to \cos\theta(64-0) = 64\cos\theta$$

$$64\int_{\pi/2}^{3\pi/2}\cos\theta\cdot d\theta \to 64\sin\theta\Big|_{\pi/2}^{3\pi/2} \Rightarrow 64\sin(3\pi/2) - 64\sin(\pi/2) = -128$$