Mofeez 23K-0800

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Assignment 02

QUESTION 1

a)

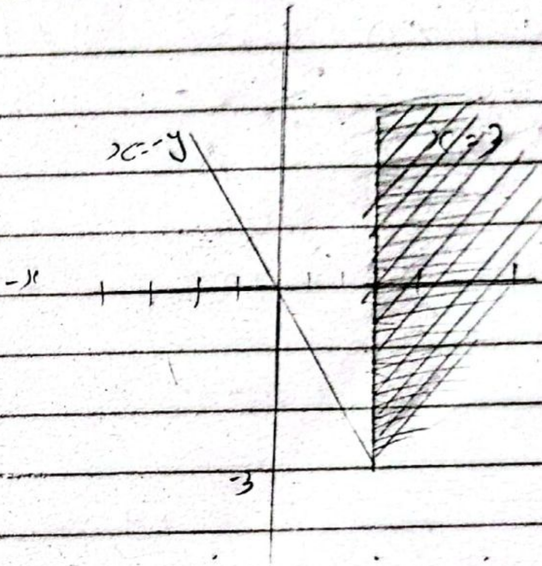
Sol

(i) $f(x,y) = \sqrt{x+y} - \sqrt{x-3}$

Sol

$x+y \geq 0 \Rightarrow x \geq -y$

$x-3 \geq 0 \Rightarrow x \geq 3$

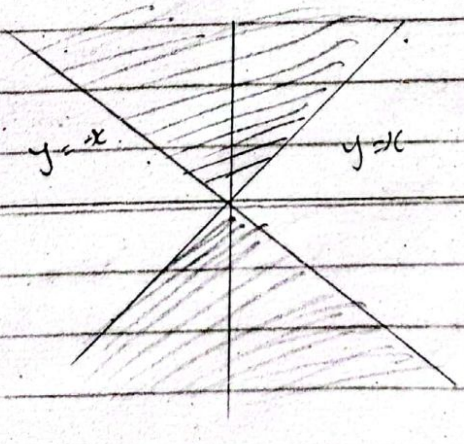


(ii) $f(x,y) = \sqrt{\frac{1}{x^2} - \frac{1}{y^2}}$

Sol

$\frac{1}{x^2} - \frac{1}{y^2} \geq 0 \Rightarrow \frac{y^2 - x^2}{x^2 y^2} \geq 0$

$y^2 - x^2 \geq 0 \Rightarrow y^2 \geq x^2 \Rightarrow \pm y \geq \pm x$



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(iii) $f(x,y,z) = \dfrac{1}{x+1} + \dfrac{1}{y-1} + \dfrac{1}{x+y-2}$

Sol

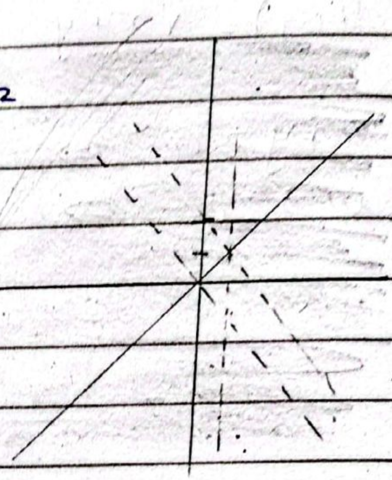
x

$x+1 \neq 0 \Rightarrow x \neq -1$

$y-1 \neq 0 \Rightarrow y \neq 1$

$x+y-2 \neq 0 \Rightarrow x+y \neq 2$

All values excluding dotted lines.

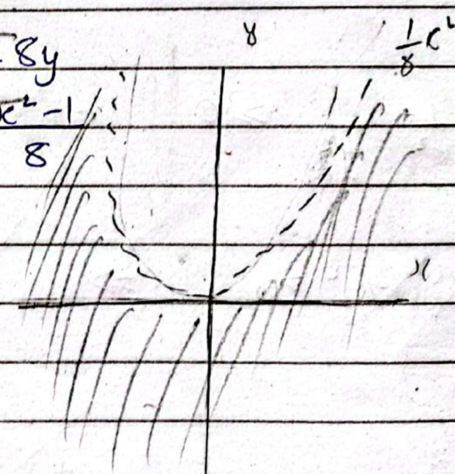


iv) $f(x,y) = \ln(x^2 - 8y)$

Sol

$\cancel{x^2 - 8y \geq 1 \Rightarrow x^2 - 1 \geq 8y}$

$\cancel{\Rightarrow 8y \leq x^2 - 1 \Rightarrow y \leq \frac{x^2-1}{8}}$



$x^2 - 8y > 0 \Rightarrow x^2 > 8y$

$\Rightarrow y < \dfrac{x^2}{8}$

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Question (b)

(i) $x^2 - 4z - y = 2$.

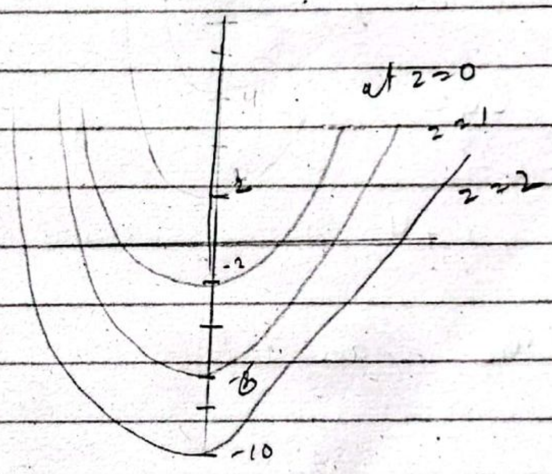
Sol

$\Rightarrow \quad x^2 - y - 2 = 4z \quad \Rightarrow \quad z = \frac{1}{4}(x^2 - y - 2)$.

At $z=0$: $\quad 0 = -2 - y + x^2 \Rightarrow \quad y = x^2 - 2$

$z=1 \Rightarrow 4 = -2 - y + x^2. \Rightarrow \quad y = x^2 - 6$

$z=2 \Rightarrow \quad 8 = x^2 - y - 2 \Rightarrow \quad y = x^2 - 10$



(ii) $z^2 + 4x^2 = 1 - y^2$

Sol

$z^2 = 1 - y^2 - 4x^2$

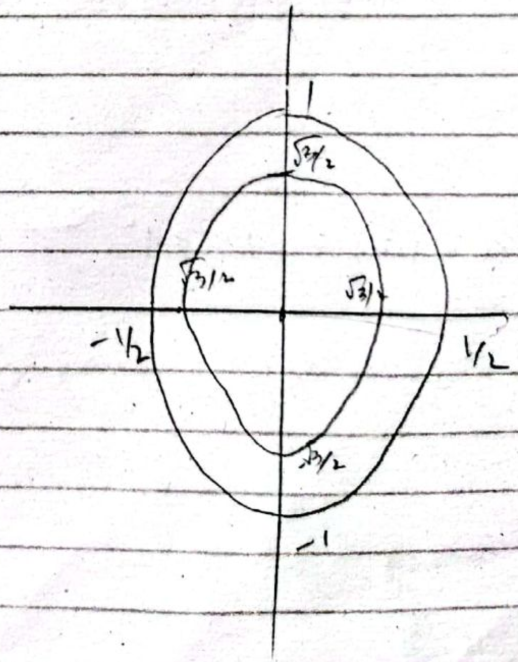
$z = \sqrt{1 - y^2 - 4x^2}$

$z=0 \quad 4x^2 + y^2 = 1$

$\Rightarrow \quad \frac{x^2}{(1/2)^2} + \frac{y^2}{(1)^2} = 1$

$z=1 \quad 4x^2 + y^2 = 0.$

$z=-1 \quad 4x^2 + y^2 = 2$



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iii) $2x - 6y + z = -2$

Sol

$z = -2 - 2x + 6y$

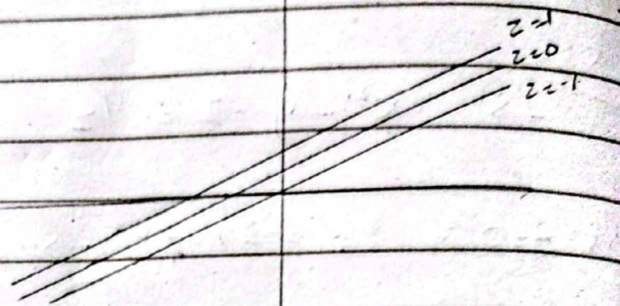
$\Rightarrow z = 6y - 2x - 2$

$z=-1$ $\quad 6y - 2x - 1 = 0$

$z=0$ $\Rightarrow$ $6y - 2x - 2 = 0$

$z=1$ $\quad 6y - 2x - 1 = 0$

$z=2$ $\quad 6y - 2x = 0$



(C)

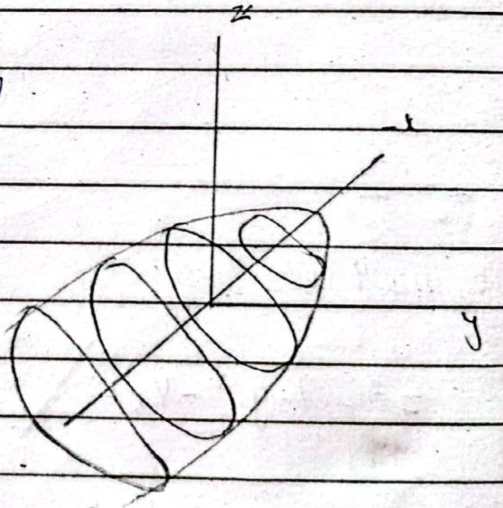
i) $f(x,y,z) = x - y^2 - z^2 + 1$, $k = -3$

$\Rightarrow -3 = x - y^2 - z^2 + 1$

$x^2 - y^2 - z^2 = -4$

$y^2 + z^2 = x + 4$

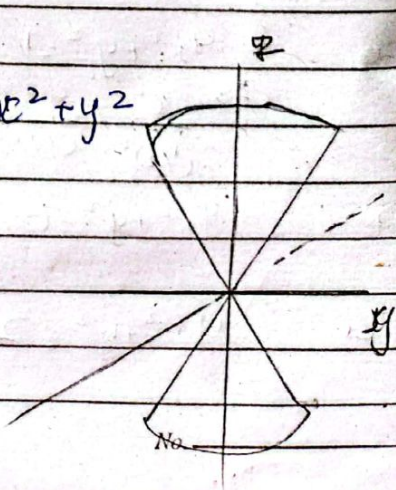
Elliptic paraboloid



ii) $f(x,y,z) = \dfrac{3x^2 + y^2}{z^2}$, $k = 9$

$9 = \dfrac{3x^2 + y^2}{z^2} \Rightarrow 9z^2 = 3x^2 + y^2$

Elliptic cone.



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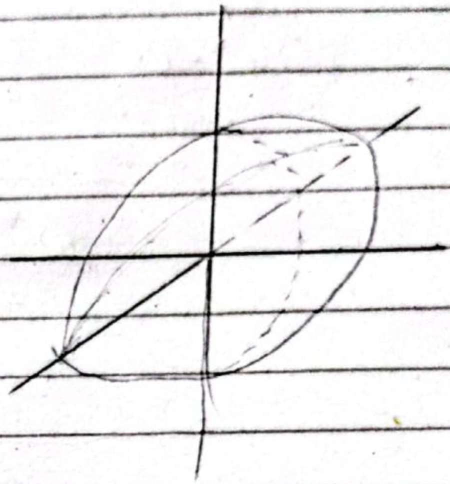
iii) $f(x,y,z) = 9x^2+4y^2+z^2$, $k=4$

Sol

$4 = 9x^2+4y^2+z^2$

$\frac{9}{4}x^2 + y^2 + \frac{z^2}{4} = 1.$

Ellipsoid



QUESTION 2

(a)

i) $\lim\limits_{(x,y)\to(0,0)} \frac{x^3y}{x^2+y^2}$

Sol

Using polar coordinates.

$x = r\cos\theta$

$y = r\sin\theta$.

Hence,

$\lim\limits_{r\to 0^+} \frac{(r\cos\theta)^3(r\sin\theta)}{(r\cos\theta)^2+(r\sin\theta)^2}$.

$\Rightarrow \lim\limits_{r\to 0^+} \frac{r^4\cos^3\theta\sin\theta}{\cancel{r^2}(r^2\cos^4\theta+\sin^2\theta)}$.

$\Rightarrow \lim\limits_{r\to 0^+} \frac{r^2\cos^3\theta\sin\theta}{r^2\cos^4\theta+\sin^2\theta}$.

Apply limit

$\Rightarrow \frac{0}{(0)\cos^4\theta+\sin^2\theta} \Rightarrow \frac{0}{\sin^2\theta} \Rightarrow 0$

Hence, limit exists.

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ii) $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2}$

Using polar coordinates.

$x^2+y^2=r^2 \quad x=r\cos\theta \quad y=r\sin\theta.$

$\Rightarrow \lim_{r\to 0^+} \frac{(r\cos\theta)^3-(r\sin\theta)^3}{r^2}$

$\Rightarrow \lim_{r\to 0^+} \frac{\cancel{r^3}(\cos^3\theta-\sin^3\theta)}{\cancel{r^2}}$

$\Rightarrow \lim_{r\to 0^+} r(\cos^3\theta-\sin^3\theta).$

Apply lim

$\Rightarrow 0$ limit exist at 0

Ans

iii) $\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{(x^2+y^2)^2}$

Sol

$r=x^2+y^2.$

$\Rightarrow \lim_{r\to 0^+} \frac{1-\cos r}{(r)^2} = \frac{0}{0}$

$\Rightarrow$ Applying L-Hopital

$\lim_{r\to 0^+} \frac{\sin r}{2r} : \frac{0}{0}$ ~~Apply limit~~ Again

$\Rightarrow \lim_{r\to 0} \frac{\cos r}{2} \Rightarrow \frac{\cos 0}{2} = \boxed{\frac{1}{2}}$ Ans

limit exist

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(b).

(i) $\frac{xy}{\sqrt{x^2+y^2}}$

Sol

Sol $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & , (x,y) \neq 0 \\ 0 & , (x,y) = (0,0) \end{cases}$.

$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$

Using polar coordinates.

$\Rightarrow \lim_{r\to 0^+} \frac{(r\cos\theta)(r\sin\theta)}{\sqrt{\cancel{(r\cos\theta)^2+(r\sin\theta)^2}\ r^2}}$

$\Rightarrow \lim_{r\to 0^+} \frac{r^{\cancel{2}}\cos\theta\sin\theta}{\cancel{r}}$

$\Rightarrow$ Apply limit

$\Rightarrow (\cancel{r}0)\ \cos\theta\sin\theta.$

$\Rightarrow 0.$

$\frac{xy}{\sqrt{x^2+y^2}}$ at $0 \Rightarrow 0$

Since, the ~~limit and~~ value of function at point (0,0) and $\lim_{(x,y)\to(0,0)}$ is 0 ~~and~~ ~~the~~ to equal, the function is continous. Answer

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(ii) $\frac{xy}{x^2+y^2}$

Sol

Using polar coordinates

$$\Rightarrow \lim_{r\to 0^+} \frac{(r\cos\theta)(r\sin\theta)}{r^2}$$

$$\Rightarrow \lim_{r\to 0^+} \frac{\cancel{r^2}\cos\theta\sin\theta}{\cancel{r^2}}$$

Apply limit $\Rightarrow$ $\sin\theta \cos\theta$

~~$f(x,y)$ at $(0,0)$. The function~~

$$\cancel{\Rightarrow \frac{(0)(0)}{0^2+0^2}} \Rightarrow$$

The value of limit of function and at point $(0,0)$ aren't equal, the function is discontinuous.

(iii) $\frac{x^4-y^2}{x^4+y^2}$

Sol

Using polar coordinates

$$\lim_{r\to 0^+} \frac{(r\cos\theta)^4 + (r\sin\theta)^2}{(r\cos\theta)^4+(r\sin\theta)^2}$$

$$\Rightarrow \lim_{r\to 0^+} \frac{r^4\cos^4\theta + r^2\sin^2\theta}{r^4\cos^4\theta + r^2\sin^2\theta}.$$

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$$\Rightarrow \lim_{r\to 0^+} \frac{\cancel{r^2}(r^2\cos^4\theta - \sin^2\theta)}{\cancel{r^2}(r^2\cos^4\theta + \sin^2\theta)}$$

$\Rightarrow$ Apply limit

$$\Rightarrow \frac{0-\sin^2\theta}{0+\sin^2\theta} \Rightarrow -1.$$

As the value of limit $(x,y)\to(0,0)$ of the function isn't equal to 0, function is discontinuous.

iv) $\dfrac{x^2 y}{x^4+y^2}$

Sol

Using polar coordinates

$$\Rightarrow \lim_{r\to 0^+} \frac{(r\cos\theta)^2(r\sin\theta)}{(r\cos\theta)^4+(r\sin\theta)^2}$$

$$\Rightarrow \lim_{r\to 0^+} \frac{(r^2\cos^2\theta)(r\sin\theta)}{r^4\cos^4\theta + r^2\sin^2\theta}$$

$$\Rightarrow \lim_{r\to 0^+} \frac{r^{\cancel{3}}\cos^2\theta\sin\theta}{\cancel{r^2}(r^2\cos^4\theta+\sin^2\theta)}$$

$\Rightarrow$ Apply limit

$$\Rightarrow \frac{(0)\cos^2\theta\sin\theta}{0+\sin^2\theta} = 0$$

$\Rightarrow$ The function is continuous as $\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$.

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QUESTION 3

PART I

(a) Sol

$$f(x,y) = xy\,\frac{x^2-y^2}{x^2+y^2} \quad : (x,y) \neq 0$$

To prove, $f_x(0,y) = -y$ $\quad f_y(x,0) = x$.

$$\Rightarrow f_x = \frac{x^3y - xy^3}{x^2+y^2}$$

$$f_x = \frac{(x^2+y^2)(3x^2y - y^3) - (x^3y - xy^3)(2x)}{(x^2+y^2)^2}$$

$$f_x = \frac{3x^4y - x^2y^3 + 3x^2y^3 - y^5 - 2x^4y + 2x^2y^3}{(x^2+y^2)^2}$$

$$f_x = \frac{x^4y + 4x^2y^3 - y^5}{(x^2+y^2)^2}.$$

$$f_x(0,y) = \frac{(0)y + 4(0)y^3 - y^5}{(0+y^2)^2}$$

$$f_x(0,y) \Rightarrow \frac{-\cancel{y^5}}{\cancel{y^4}} \Rightarrow -y \quad \text{Proved.}$$

Next,

$$f_y = \frac{x^3y - xy^3}{x^2+y^2}$$

$$f_y = \frac{(x^2+y^2)(x^3 - 3xy^2) - (x^3y - xy^3)(2y)}{(x^2+y^2)^2}$$

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$$\Rightarrow f_y = \frac{x^5 - 3x^3y^2 + x^3y^2 - 3xy^4 - 2x^3y^2 + 2xy^4}{(x^2+y^2)^2}$$

$$f_y = \frac{x^5 - 2xy^4 - 5x^3y^2 - 3xy^3}{(x^2+y^2)^2}$$

$$f_y(x,0) \Rightarrow \frac{x^5 - 2\cancel{(0)}(0)^4 - 5x^3(0) - 3x(0)}{(x^2+0)^2}$$

$$f(x,0) \Rightarrow \frac{x^5}{x^4} \Rightarrow x \quad \text{Proved.}$$

(b) Sol

From part (a)

$f_x(0,y) = -y$

$f_{xy}(0,0) = -1$ Proved.

Next,

$\cancel{f_y = x}$ $f_y(x,0) = x$

$f_{yx}(0,0) = 1$ proved.

(c) To prove if differentiable at $(0,0)$

Sol. Using polar coordinates.

$$\Rightarrow \lim_{r\to0^+} \frac{\cancel{(r\cos\theta)}(r\cos\theta)^3(r\sin\theta) - (r\cos\theta)(r\sin\theta)^3}{r^2}$$

$$\Rightarrow \lim_{r\to0^+} \frac{\cancel{r}\, r^4(\cos^3\theta\sin\theta - \cos\theta\sin^3\theta)}{\cancel{r^2}}$$

$$\Rightarrow \lim_{r\to0^+} r^2(\cos^3\theta\sin\theta - \cos\theta\sin^3\theta)$$

Apply limit $\Rightarrow (0)(\cos^3\theta\sin\theta - \cos\theta\sin^3\theta)$

$\Rightarrow 0$ $\therefore$ ~~Thus~~, Hence the function is ~~continuous~~ differentiable at $(0,0)$.

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PART II

Sol

Given that, $f_x(x,y) = 0$

Integrating w.r.t x

$$\int \frac{\partial f}{\partial x}\, dx = \int 0\, dx$$

$$\Rightarrow c$$

Integrating w.r.t y.

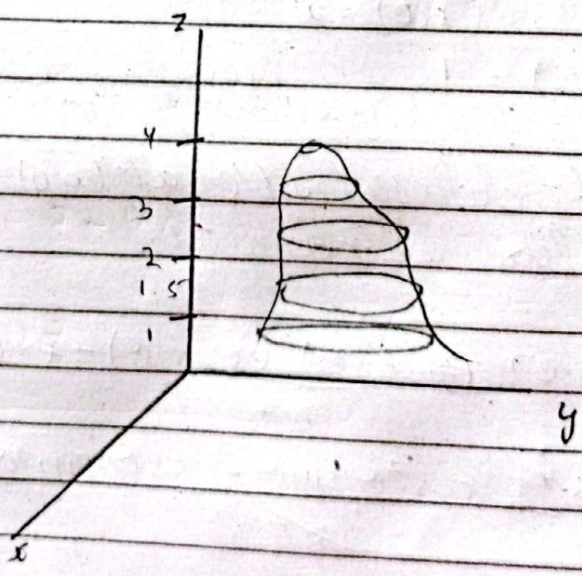
$$\int \frac{\partial f}{\partial y}\, dy = \int 0\, dy$$

$$= c$$

Hence, $\int f_x\, dx = \int f_y\, dy = c$

$f(x,y) = c$ Answer

PART III



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QUESTION 4

(a)

Sol

a To find out

$f_x(0,0) = ?$

$f_y(0,0) = ?$

$|\vec{u}| = \frac{\hat{i}+2\hat{j}}{\sqrt{1+4}} \Rightarrow \frac{\hat{i}+2\hat{j}}{\sqrt{5}}$.

$\Rightarrow \cancel{f_x} \Rightarrow \langle 1,2\rangle \langle f_x(0,0), f_y(0,0)\rangle = 1$ — ①

$\Rightarrow \langle 2,1\rangle \langle f_x(0,0), f_y(0,0)\rangle = 2$ — ②

$\Rightarrow$ From eq ① & ②

$f_x + 2f_y = 1$ — (A)

$2f_x + f_y = 2$

Multiply above eqn by 2

$4f_x + 2f_y = 4$. — (B)

Subtract (A) & (B)

$f_x + \cancel{2f_y} = 1$

$4f_x + \cancel{2f_y} = 4$

$(-) \quad (-) \quad (-)$

$-3f_x = -3$

$\boxed{f_x = 1}$ at $(0,0)$ Ans.

Put f_x in eq (A).

$1 + 2f_y = 1$

$2f_y = 0$

$\boxed{f_y = 0}$ Ans.

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(b)

Sol $z = f(x,y)$; $x = g(s,t)$, $y = h(s,t)$

$g(1,2) = 3$, $g_s(1,2) = -1$,

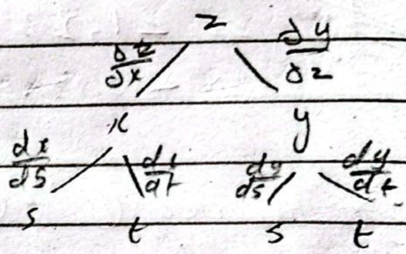
$g_t(1,2) = 4$.

$h(1,2) = 6$, $h_s(1,2) = -5$ $h_t(1,2) = 10$.

$f_x(3,6) = 7$, $f_y(3,6) = 8$.

To find

$\frac{\partial z}{\partial s} = ?$ & $\frac{\partial z}{\partial t} = ?$ at $s = 1$ & $t = 2$



For $\left.\frac{\partial z}{\partial s}\right|_{s=1}$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{dx}{ds} + \frac{\partial z}{\partial y}\frac{dy}{ds}.$$

$$= f_x(3,6)\, g_s(1,2) + f_y(3,6)\, h_s(1,2)$$

$$= (7)(-1) + (8)(-5)$$

$$\Rightarrow -7 - 40$$

$$\boxed{\frac{\partial z}{\partial s} = -47 \quad \text{Answer}}$$

For $\left.\frac{\partial z}{\partial t}\right|_{t=2}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{dz}{dy}\frac{dy}{dt}$$

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$$\Rightarrow f_x(3,6) \cdot g_t(1,2) + f_y(3,6)\, h_t(1,2)$$

$$\Rightarrow 7(4) + 8(10)$$

$$\boxed{\frac{\partial z}{\partial t} = 108 \quad \text{Answer}}$$

(C)

Sol: $z = y + f(x^2 - y^2)$

$$y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = x.$$

Sol.

Taking L.H.S

$$y\left(0 + f'(x^2-y^2)\, 2x\right] + x\left[1 + f'(x^2-y^2)(-2y)\right]$$

$$\Rightarrow \cancel{2xy\, f'(x^2-y^2)} + x - \cancel{2xy\, f'(x^2-y^2)}$$

$$\Rightarrow x$$

L.H.S = R.H.S Proved

$x = x$

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QUESTION 5 (a)

Sol:

$f(x,y,z) = x^3 \sqrt{y^2+z^2}$ at $(2,3,4)$

$\Rightarrow (1.98)^3 \sqrt{(3.01)^2 + (3.97)^2}$

$\therefore l(x,y,z) = f(x_0,y_0,z_0) + f_x(x_0,y_0,z_0)(x-x_0) + f_y(x_0,y_0,z_0)(y-y_0) + f_z(x_0,y_0,z_0)(z-z_0).$

$\Rightarrow 2^3\sqrt{3^2+4^2} + 3x^2\sqrt{y^2+z^2}\,(x-2) + \dfrac{x^3\, 2y_0}{2\sqrt{y_0^2+z_0^2}}(y-3) + \dfrac{x_0^3\, 2z_0}{2\sqrt{y_0^2+z_0^2}}(z-4)$

$\Rightarrow 8\sqrt{25} + 3(2)^2\sqrt{3^2+4^2}\,(x-2) + \dfrac{8(3)}{\sqrt{3^2+4^2}}(y-3) + \dfrac{8(4)}{\sqrt{3^2+4^2}}(z-4)$

$\therefore (x,y,z) = (1.98, 3.01, 3.97)$

$\Rightarrow 40 + 60(1.98-2) + \dfrac{24}{5}(3.01-3) + \dfrac{32}{5}(3.97-4)$

$\Rightarrow \boxed{L = 38.656}$ Answer

At

$f(1.98, 3.01, 3.97) = 38.672 \approx 38.656$

Ans

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(b)

(a) Sol

$$T_x(6,4) = \lim_{\Delta v \to 0} \frac{T(6+\Delta v, 4) - T(6,4)}{\Delta v}$$

let $\Delta v = 2$

$$T_x(6,4) \approx \frac{T(6+2,4) - T(6,4)}{2}$$

$$\Rightarrow \frac{86-80}{2} = 3$$

$$\boxed{T_x(6,4) \approx 3}$$

let $\Delta v = -2$

$$T_x(6,4) = \frac{T(6-2,4) - T(6,4)}{-2}$$

$$= \frac{72-80}{-2} = 4$$

$$\boxed{T_x(6,4) \approx \frac{3+4}{2} = 3.5\ c^\circ/m}$$ Answer

For T_y.

$$T_y(6,4) = \lim_{\Delta v \to 0} \frac{T(6,4+\Delta v) - T(6,4)}{\Delta v}$$

$$T_y(6,4) = \frac{T(6,4+2) - T(6,4)}{2}$$

$$T_y \approx -2.5$$

$$T_y(6,4) = \frac{T(6,4-2) - T(6,4)}{-2}$$

$$T_y = -3.5$$

$$\boxed{T_y(6,4) = \frac{-2.5-3.5}{2} = -3\ ^\circ c/m}$$

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(b)

Sol

$\therefore D_u T(x,y) = f_x(x,y)\, u_i + f_y(x,y)\, u_j$

From part A

$T_x = 3.5 \quad , T_y = -3$

$D_u T(6,4) = 3.5\left(\frac{1}{\sqrt{2}}\right) + (-3)\left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \frac{1}{2\sqrt{2}}$

$\boxed{D_u T(6,4) = 0.353 \ ^\circ C/m}$ Answer

(C)

Sol

From part (a)

$T_x(6,4) \approx 3.5 \ ^\circ C/m$

$\therefore T_{xy}(6,4) = \lim_{\Delta v \to 0} \frac{T_x(6, 4+\Delta v) - T_x(6,4)}{\Delta v}$

at $\Delta v = 2$

$T_{xy} \approx \frac{T_x(6,6) - T_x(6,4)}{2}$

For $T_x(6,6)$

Right Hand.

$T_x(6,6) \approx \frac{T(6+2,6) - T(6,6)}{2}$

$T_x = \frac{80-75}{2} \approx 2.5$

at $\Delta v = -2$

$T_{xy}(6,4) = \frac{T_x(6,2) - T(6,4)}{-2}$

For $T_x(6,6)$

left hand

$T_x(6,6) \approx \frac{T(6-2,6) - T(6,6)}{-2}$

$T(6,6) \Rightarrow \frac{68-75}{-2} \approx 3.5$

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$\Rightarrow T_x(6,6) \approx \frac{2.5+3.5}{2} = 3\ ^\circ C/m$

Now, for $T_x(6,2)$

Right Hand | left hand.

$T_x(6,2) = \frac{T(6+2,2) - T(6,2)}{2}$

$T_x(6,2) = \frac{90-87}{2} \approx 1.5$

$T_x(6,2) \approx \frac{T(6-2,2) - T(6,2)}{-2}$

$T_x(6,2) \approx \frac{74-87}{-2}$

$T_x(6,2) \approx 6.5$

$$T_x(6,2) \approx \frac{1.5+6.5}{2} = 4\ ^\circ C/m$$

Next.

$\Rightarrow T_{xy}(6,4) = \frac{3-3.5}{2}$

$T_{xy}(6,4) \approx -0.25$

$\Rightarrow T_{xy}(6,4) \approx \frac{4-3.5}{-2}$

$T_{xy}(6,4) \approx -0.25$

$$T_{xy}(6,4) \approx \frac{-0.25-0.25}{2} \approx -0.25\ C^\circ/m$$

Answer

(d)

linear approximation

$\therefore L = T(x_0,y_0) + T_x(x_0,y_0)(x-x_0) + T_y(x_0,y_0)(y-y_0)$

$L = T(6,4) + T_x(6,4)(x-6) + T_y(6,4)(y-4)$

$L = 80 + (3.5)(5-6) + (-3)(3.8-4)$

~~$L = 80$~~ $\boxed{L = 77.1^\circ C \text{ Answer}}$

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QUESTION 6.

(a) Sol

$x = 5m \qquad y = 12m.$

$\delta x = 0.2cm = 0.002m \qquad \delta y = 0.002m.$

(a) Max error in area of triangle

$$A = \frac{1}{2}xy \Rightarrow dA = \frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}dy$$

$$dA = \frac{1}{2}y\,dx + \frac{1}{2}x\,dy$$

$$= \frac{1}{2}(12)(0.002) + \frac{1}{2}(5)(0.002)$$

$$\boxed{dA = 0.017\,m^2 \text{ Answer}}$$

(b) error in hypotenuse

Using P.T.

$$H = \sqrt{B^2 + P^2} = \sqrt{x^2 + y^2}$$

$$H = \frac{\partial H}{\partial x}dx + \frac{\partial H}{\partial y}dy$$

$$dH = \frac{2x\,dx}{2\sqrt{x^2+y^2}} + \frac{2y}{2\sqrt{x^2+y^2}}dy.$$

$$dH = \frac{5(0.002)}{\sqrt{5^2+12^2}} + \frac{12(0.002)}{\sqrt{5^2+12^2}}$$

$$dH \Rightarrow 2.615 \times 10^{-3}$$

$$\boxed{\therefore dH = 0.002615m \text{ error Answer}}$$

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(b.

Sol :

$\frac{dx}{dt} = 3$ in/s $\quad \frac{dy}{dt} = -2$ in/s

$\frac{d\theta}{dt} = 0.05$ rad/s $\quad \frac{dA}{dt} = ?$

Sol

$$\frac{dA}{dt} = \frac{\partial A}{\partial x}\frac{dx}{dt} + \frac{\partial A}{\partial y}\frac{dy}{dt} + \frac{\partial A}{\partial \theta}\frac{d\theta}{dt}$$

$$= \frac{1}{2} y \sin\theta (3) + \frac{1}{2} x \sin\theta (-2) + \frac{1}{2} xy \cos\theta (0.05)$$

$$= \frac{3}{2}(50) \sin\frac{\pi}{6} + \frac{40}{\cancel{2}} \sin \pi/6 (-\cancel{2}) + \frac{1}{2}(50)(40)$$

$$\cos\left(\frac{\pi}{6}\right)(0.05)$$

$$\frac{dA}{dt} \Rightarrow 3.25\left(\frac{1}{2}\right) - \frac{40}{2} + 50\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{\left.\frac{dA}{dt}\right|_{\substack{x=40 \\ y=50 \\ \theta=\pi/6}} = 60.801 \text{ in}^2/\text{sec}}$$

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QUESTION 7.

(a)

Sol

$f(x,y,z) = ze^{xy}$ at $(0,1,2)$.

$\Rightarrow \nabla f(x,y,z) = f_x(x,y,z)\,\hat{i} + f_y(x,y,z)\,\hat{j} + f_z(x,y,z)\,\hat{k}$

$\nabla f(x,y,z) = (yze^{xy})\,\hat{i} + (xze^{xy})\,\hat{j} + (e^{xy})\,\hat{k}$

At $(0,1,2)$

$\nabla f(0,1,2) = (1)(2)\,e^{(0)(1)}\,\hat{i} + (0) + e^{(0)(1)}\,\hat{k}$

$\nabla f(0,1,2) = 2\hat{i} + \hat{k}$

$\Rightarrow \boxed{\nabla f = 2\hat{i} + \hat{k}} \rightarrow$ Direction.

Max rate.

$|\nabla f| = \sqrt{2^2 + 1^2}$

$|\nabla f| = \sqrt{5}$ Answer.

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b) $f(x,y,z)\ |r|^{-n}$ where $r = xi + yj + zk$

$\nabla f = \dfrac{-nr}{|r|^{n+2}}$

Sol.

$|r| = \sqrt{x^2+y^2+z^2}$

$f(x,y,z) = \left(\sqrt{x^2+y^2+z^2}\right)^{-n}$

$f(x,y,z) = (x^2+y^2+z^2)^{-n/2}$

$\therefore \nabla f = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$

$f_x(x,y,z) = \dfrac{-n}{2}\,(x^2+y^2+z^2)^{\frac{-n}{2}+1}\,(2x)$

$f_x(x,y,z) = \dfrac{-nx}{(x^2+y^2+z^2)^{\frac{n+2}{2}}}$

$f_y(x,y,z) = \dfrac{-n}{2}\,(x^2+y^2+z^2)^{\frac{-n}{2}-1}\,2y$

$f_y(x,y,z) = \dfrac{-ny}{(x^2+y^2+z^2)^{n+2/2}}$

$f_z(x,y,z) = -n/2\,(x^2+y^2+z^2)^{-n/2-1}\,(2z)$

$f_z = \dfrac{-nz}{(x^2+y^2+z^2)^{n+2/2}}$

$$\nabla f = \frac{-nx\hat{i}}{(x^2+y^2+z^2)^{\frac{n+2}{2}}} - \frac{ny\hat{j}}{(x^2+y^2+z^2)^{(n+2)/2}} - \frac{nz\hat{k}}{(x^2+y^2+z^2)^{(n+2)/2}}$$

$$\nabla f = -n\frac{(x\hat{i}+y\hat{j}+z\hat{k})}{[(x^2+y^2+z^2)^{1/2}]^{n+2}}$$

$\because r = x\hat{i}+y\hat{j}+z\hat{k}$

$|r| = \sqrt{x^2+y^2+z^2}$

$$\boxed{\nabla f = \frac{-nr}{|r|^{n+2}}}$$

proved

(Part c)

$$f(x,y,z) = x^2 e^{yz^2}$$

Sol.

$$\nabla f = f_x(x,y,z)\hat{i} + f_y(x,y,z)\hat{j} + f_z(x,y,z)\hat{k}$$

$f_x = 2xe^{yz^2}$

$f_y = x^2 e^{yz^2} z^2$

$f_z = x^2 e^{yz^2} 2yz$

$$\nabla f = 2xe^{yz^2}\hat{i} + x^2z^2e^{yz^2}\hat{j} + 2x^2yze^{yz^2}\hat{k}$$

Answer

i) When maximum?

When ∇f and u are in same direction $\theta = 0°$

$D_uf(x,y,z) = \nabla f \cdot u$

$= |\nabla f|\,|u| \cos\theta$

$= |\nabla f|\,(1) \cos 0°$

$D_uf(x,y,z) = |\nabla f|$ (maximum)

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ii) When $D_u f$ is min?

When $\theta = 180°$

$$D_u f(x,y,z) = |\nabla f|\,|u| \cos 180°$$
$$= -|\nabla f| \text{ minimum}$$

(iii) when zero

When ∇f and u are perpendicular i.e. $\theta = 90°$

$$D_u f(x,y,z) = |\nabla f|\,|u| \cos 90°$$

$$D_u f(x,y,z) = 0.$$

iv) When half of its max value:

$$D_u f(x,y,z) = \frac{1}{2} D_u f_{max}.$$

$$|\nabla f|\,|u| \cos\theta = \frac{1}{2} |\nabla f|$$

$$(1) \cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

$$\boxed{\theta = \frac{\pi}{3}}$$

When angle b/w ∇f and u is $\pi/3$, $D_u f(x,y,z)$ is half of its max value.

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