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BCS-2J

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Q1)

a) $\int_1^2 \int_4^6 \frac{x}{y^2} \cdot dxdy$

- $\int_4^6 xy^{-2} \cdot dx \rightarrow y^{-2} \int_4^6 x \cdot dx \rightarrow y^{-2} \frac{x^2}{2} \Big|_4^6 \Rightarrow \frac{36}{2}y^{-2} - \frac{16}{2}y^{-2}$

$= 10y^{-2}$

- $\int_1^2 10y^{-2} \cdot dy \rightarrow 10 \int_1^2 y^{-2} \cdot dy \rightarrow 10 \frac{y^{-2+1}}{-1} \rightarrow -10y^{-1} \Big|_1^2$

$= -10(2^{-1} - 1^{-1}) \Rightarrow -\frac{10}{2} + 10 = 5$

b) $\int \int x^2 + y^2 \cdot dxdy$

- $\int x^2 + \int y^2 \cdot dx \rightarrow \frac{x^3}{3} + y^2 x$

- $\frac{1}{3}x^3 \int 1 \cdot dy + x \int y^2 \cdot dy \rightarrow \frac{x^3}{3}y + \frac{xy^3}{3} + c$

c) $\int_0^1 \int_1^2 xe^x y^{-1} \cdot dydx$

- $xe^x \int_1^2 y^{-1} \cdot dy \rightarrow xe^x \int \frac{1}{y}(1) \cdot dy \rightarrow xe^x \ln y \Big|_1^2 \Rightarrow xe^x \ln 2 - xe^x \ln 1$

$\therefore \ln 1 = 0$, so $\rightarrow xe^x \ln 2 - 0$

- $\ln 2 \int_0^1 \underset{u}{x} \underset{dv}{e^x} \cdot dx = uv - \int v \cdot du$ $\qquad u = x$, $\int dv = \int e^x$, $du = 1dx$, $v = e^x$

$\ln 2 \left[ (x)(e^x) - \int e^x \cdot dx \right] \rightarrow (xe^x - e^x) \Big|_0^1 \Rightarrow e^1(1) - e - 0 - e^0$

$= \cancel{e} - \cancel{e} - 1 = \boxed{1 \ln 2}$

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$$= \frac{8}{15}$$

Q2) $\frac{1}{A(R)} \iint f(x,y)\, dA$

$$\frac{1}{10{,}000} \int_{20}^{30} \int_{5000}^{6000} 70L^{0.6} K^{0.4}\, dL\, dK$$

$$\frac{1}{10000} \left. \frac{70L^{1.6} K^{0.4}}{1.6} \right|_{5000}^{6000} \Rightarrow \frac{70}{1.6} K^{0.4} \left(6000^{1.6} - 5000^{1.6}\right)$$

$$\frac{1}{10000} \int_{20}^{30} 12279250.57 K^{0.4}\, dK$$

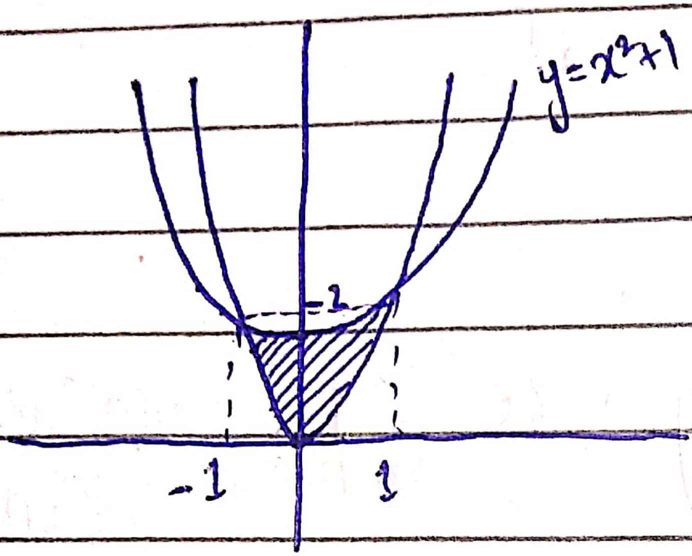
$$\left. 1227.9250 \frac{K^{1.4}}{1.4} \right|_{20}^{30} \Rightarrow 44426.95351$$

$= 2.500 \times 10$ ...

Q3) a) $\iint (x+2y) \cdot dA$, $y = 2x^2$, $y = 1 + x^2$

$\hookrightarrow y = x^2$ (stretch by factor $1/2$)

$\hookrightarrow y = x^2$ (shifted 1↑ on y axis)




• Finding common pt:

$2x^2 = x^2 + 1 \Rightarrow x^2 = 1$

$x = 1, -1$

$y = 2, 2$

$$\int_{-1}^{1}\int_{2x^2}^{1+x^2} x+2y \cdot dy\,dx \Rightarrow x\int 1 \cdot dy + 2\int y \cdot dy \rightarrow xy + \frac{2y^2}{2} \Rightarrow xy + y^2$$

$$= xy + y^2 \Big|_{2x^2}^{1+x^2} = x(1+x^2) + (1+x^2)^2 - x(2x^2) - (2x^2)^2$$

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$= x + \cancel{x^3} + 1 + 2x^2 + x^4 + -\cancel{2x^3} - 4x^4$

$= -x^3 - 3x^4 + 2x^2 + x + 1$

$= \left(-\frac{3x^5}{5} - \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} + x\right)\Big|_{-1}^{1}$

$= -\frac{3}{5}\left(1^5-(-1)^5\right) - \frac{1}{4}\left(1^4-(-1)^4\right) + \frac{2}{3}\left(1^3-(-1)^3\right) + \frac{1}{2}\left(1^2-(-1)^2\right) + 1-(-1)$

$= -\frac{3(2)}{5} - \cancel{\frac{1}{4}(0)} + \frac{2}{3}(2) + \frac{1}{2}(0) + 1 + 1 = \frac{32}{15}$

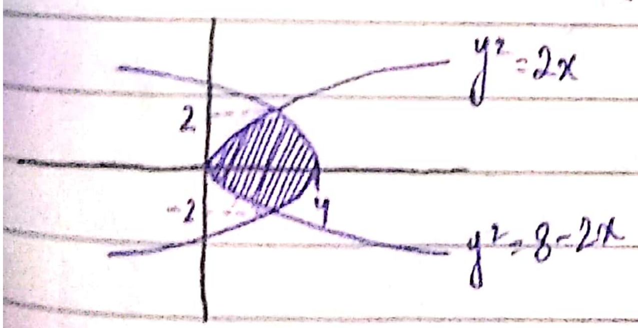
$= \frac{32}{15}$ units

b) $\int_0^4 \int_0^{\sqrt{y}} xy^2\,dx\,dy$

- $y^2 \int_0^{\sqrt{y}} x \cdot dx \to y^2 \frac{x^2}{2}\Big|_0^{\sqrt{y}} \Rightarrow \frac{y^2}{2}(\sqrt{y}^2 - 0^2) = \frac{y^3}{2}$
- $\frac{1}{2}\int_0^4 y^3 \cdot dy \to \frac{1}{2}\cdot\frac{y^4}{4}\Big|_0^4 = \frac{1}{2(4)}(4^4 - 0^4) = \frac{1}{8}(256)$

$= \frac{256}{8} = 32$ units

c) $\iint (4-y^2)\,dA$, $y^2 = 2x$, $y^2 = 8-2x$




$y^2 = 8-(0)$ | $0 = 8-2x$

$y = \pm\sqrt{8}$ | $x = 4$

$2x = 8 - 2x$

$4x = 8 \to x = 2$

$y^2 = 2(2) \Rightarrow +2, -2$

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$$\therefore y^2 = 2x \rightarrow x = y^2/2$$

$$y^2 = 8 - 2x \rightarrow x = \frac{8 - y^2}{2}$$

$$\cdot\ 4\int_{-2}^{2}\int_{y^2/2}^{4-1/2 y^2} 1 \cdot dx\,dy = \int_{-2}^{2}\int_{y^2/2}^{4-\frac{1}{2}y^2} 1 \cdot dx\,dy$$

$$\cdot\ (4x - y^2x)\Big|_{y^2/2}^{4-\frac{1}{2}y^2} \Rightarrow 4\left(4 - \frac{1}{2}y^2 - \frac{y^2}{2}\right) - y^2\left(4 - \frac{1}{2}y^2 - \frac{y^2}{2}\right)$$

$$= 4(4-y^2) - y^2(4 - y^2)$$

$$= 16 - 4y^2 - 4y^2 + y^4$$

$$= 16 - 8y^2 + y^4$$

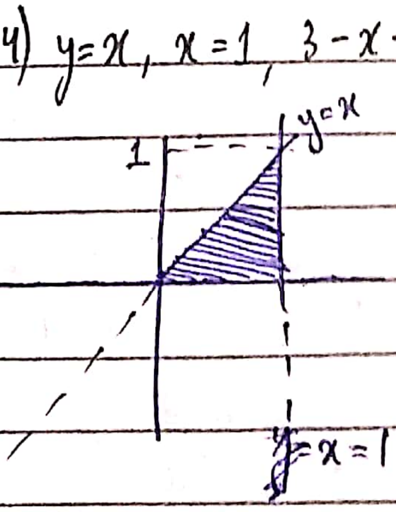
$$\cdot\ 16\int 1 \cdot dy - 8\int y^2 \cdot dy + \int y^4 \cdot dy$$

$$\left(16y - \frac{8y^3}{3} + \frac{y^5}{5}\right)\Big|_{-2}^{2} = 16(2-(-2)) - \frac{8}{3}(2^3 - (-2)^3) + \frac{1}{5}(2^5 - (-2)^5)$$

$$= 16(4) - \frac{8}{3}(8+8) + \cancel{\frac{1}{5}(32+32)} + \frac{1}{5}(32+32)$$

$$= \frac{512}{15}$$

Q4) $y = x,\ x = 1,\ 3 - x - y$




$\Rightarrow x + y = 3$

$x + (0) = 3$

$(0) + y = 3$

$$\int_0^1\int_0^x 3 - x - y \cdot dy\,dx$$

$y = x,\ x = 1$

$y = 1$

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$\left(3y - xy - \frac{y^2}{2}\right)\Big|_0^x = \int 3x - x^2 - \frac{x^2}{2} - 0 + 0 + 0$

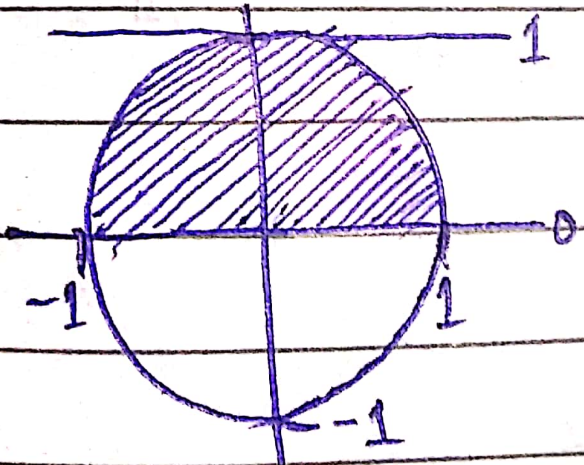
$\frac{3x^2}{2} - \frac{x^3}{3} - \frac{1}{2}\frac{x^3}{3} \Rightarrow \frac{3}{2}(1-0) - \frac{x^3}{3}(1-0) - \frac{1}{6}(1-0)$

$= 1$ unit

Q5) a) $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y \cdot dx\,dy$

$\therefore x^2 = 1 - y^2$

$x^2 + y^2 = 1$ (circle $r=1$)




$\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} 3y \cdot dy\,dx$

$\cancel{3yx \Big|^{\sqrt{1-x^2}} = 3(1-x^2)^{1/2} x - 3y}$

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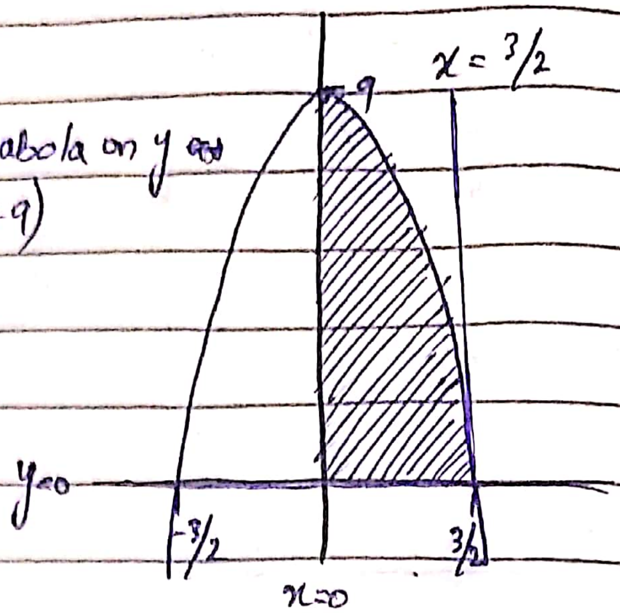
b) $\int_0^{3/2}\int_0^{9-4x^2} 16x \, dy\,dx$




$y = 9 - 4x^2 \rightarrow$ (-ve parabola on y axis transl +9)

$y = 0$

$x = 3/2$

$x = 0$

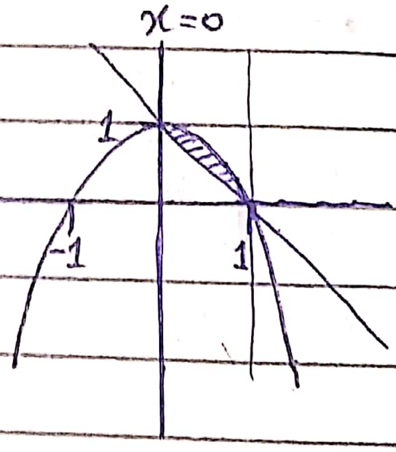
$y = 9 - 4(x^2)$

$0 = 9 - 4x^2$

$x = \pm 3/2$

$\therefore \int_0^9 \int_0^{+\sqrt{\frac{9-y}{4}}} 16x\,dy\,dx$

c) $\int_0^1 \int_{1-x}^{1-x^2} dy\,dx$




$y = 1 - x^2 \rightarrow \sqrt{1-y}$

$y = 1 - x \rightarrow x = 1 - y$

$x = 1,\ x = 0$

$(y + x = 1)$

$1 - x^2 = 0$

$x = \pm 1$

$\therefore \int_0^1 \int_{1-y}^{\sqrt{1-y}} \cdot\, dy\,dx$

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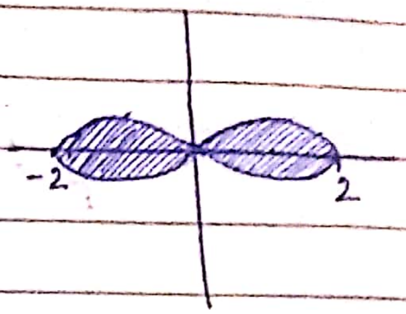
Q6) $r^2 = 4\cos 2\theta$

| $r$ | $\theta$ | |
|---|---|---|
| 2 | 0 | $r^2 = 4 \to r = 2$ |
| 0 | $\pi/4$ | |
| ? | $\pi/2$ | $r^2 = -4x$ |
| 0 | $3\pi/4$ | |
| -2 | $\pi$ | $r^2 = 4 \to -2$ |




$r = 2\sqrt{\cos 2\theta}$

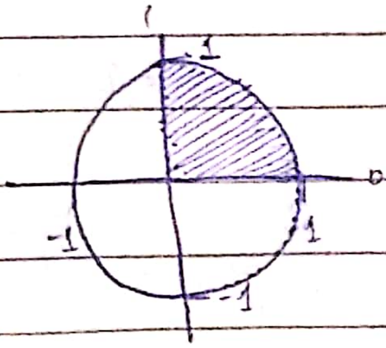
$$\int_{-\pi/4}^{\pi/4}\int_0^{2\sqrt{\cos 2\theta}} r\,dr\,d\theta \to ① \frac{r^2}{2}\Big|_0^{2\sqrt{\cos 2\theta}} \Rightarrow \frac{1}{2}(4\cos^2 2\theta) - 0$$

$$② \; 2\int \cos(2\theta)\cdot d\theta \to \int 2\cos(2\theta)\cdot d\theta \to \sin(2\theta)\Big|_{-\pi/4}^{\pi/4}$$

$$= \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{-\pi}{2}\right) = 2$$

③ $2(2) = 4$ Ans

Q7) a) $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$




- $x^2+y^2 = 1 \to r = 1$

  $y = 1,\; y = 0,\; x = 0$

$$\int_0^{\pi/2}\int_0^1 r^2(dA) \to \int_0^{\pi/2}\int_0^1 r^2\, r\,dr\,d\theta = r^3\,dr\,d\theta$$

- $\int_0^1 r^3\,dr \to \frac{r^4}{4}\Big|_0^1 = \frac{1}{4}(1-0) \Rightarrow \frac{1}{4}$

- $\frac{1}{4}\int d\theta \to \frac{1}{4}\theta\Big|_0^{\pi/2} \to \frac{1}{4}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{8}$

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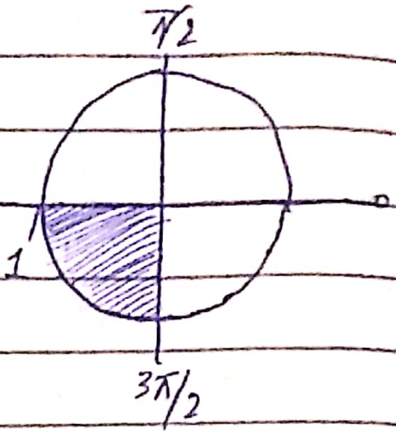
b) $\displaystyle\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}} \cdot dy\,dx \longrightarrow \frac{2\, r\,dr\,d\theta}{1+r}$

$-ve$ region in $y$ $\quad y^2+x^2=1$




• $\displaystyle\int_{\pi}^{3\pi/2}\int_{0}^{1} \frac{2r}{1+r}\,dr\,d\theta \rightarrow$ improper fraction 2

$\frac{1}{r} + \frac{\cancel{r}}{\cancel{r}}$ $\qquad 1+r \,\overline{)\,2r\,}$ quotient $2$; $2r+2$; remainder $0 \; -2$

• $\displaystyle 2\int 1 \cdot dr - 2\int \frac{1}{1+r} \cdot dr$

$\displaystyle 2r\Big|_0^1 - 2\ln \cancel{r}^{(1+r)}\Big|_0^1 \rightarrow 2(1-0) - 2(\ln(2) - \cancel{\ln 0})$

$= 2 - 2\ln 2$

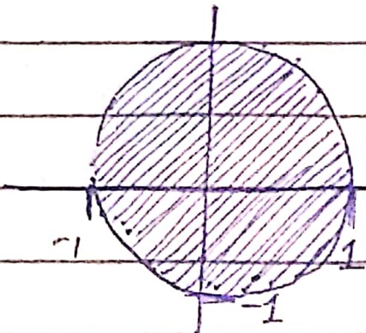
• $\displaystyle 2\int_{\pi}^{3\pi/2} d\theta - 2\ln 2\int_{\pi}^{3\pi/2} d\theta \Rightarrow 2\theta\Big|_{\pi}^{3\pi/2} - 2\ln 2\,\theta\Big|_{\pi}^{3\pi/2}$

$= 2\left(\frac{3\pi}{2} - \pi\right) - 2\ln 2\left(\frac{3\pi}{2} - \pi\right) = \frac{\pi}{\cancel{2}}(\cancel{2}) - \cancel{2}\ln 2\left(\frac{\pi}{\cancel{2}}\right)$

$= \pi - \pi\ln 2 \rightarrow \pi(1-\ln 2)$

c) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\,dy\,dx$

$y^2+x^2=1$

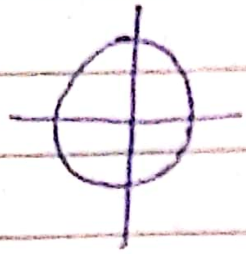



$\displaystyle\int_{0}^{2\pi}\int_{0}^{1} \frac{2}{(1+r^2)^2} \cdot r\,dr\,d\theta \rightarrow \iint 2(1+r^2)^{-2} \cdot r\,dr\,d\theta$

$\cdot \int 2r(1+r^2)^{-2}\,dr\,d\theta \rightarrow \dfrac{(1+r^2)^{-1}}{-1} \rightarrow -(1+r^2)^{-1}\Big|_0^1$

$= -(1+(-1)^2)^{-1} + (1)^{-1} \Rightarrow -\frac{1}{2}+1 = \frac{1}{2}$

$\cdot \int_0^{2\pi} \frac{1}{2}\,d\theta \rightarrow \frac{1}{2}\theta \Big|_0^{2\pi} = \frac{1}{2}(2\pi) - \frac{1}{2}(0) \Rightarrow \pi$



Q8) a) $z = x^2+y^2$, $z=9$

$x^2+y^2 = 9$

$\hookrightarrow r = \pm 3$

$A = \iint \sqrt{1+(f_x)^2+(f_y)^2}\cdot dx\,dy$

$= \int_0^{2\pi}\int_0^3 \sqrt{1+(2x)^2+(2y)^2}\cdot dx\,dy \rightarrow \int_0^{2\pi}\int_0^3 \sqrt{1+4(x^2+y^2)}\cdot dx\,dy$

$= \int_0^{2\pi}\frac{1}{8}\int_0^3 \sqrt{1+4r^2}\;r\,dr\,d\theta = \frac{1}{4\cdot 8}\dfrac{(1+4r^2)^{3/2}}{3/2}\Big|_0^3$ (8)

$= \frac{1}{12}(37^{3/2}-1) \rightarrow \int \frac{1}{12}(37^{3/2}-1)\,d\theta$

$\frac{1}{\cancel{12}\,6}(37^{3/2}-1)\left[2\pi\right] \Rightarrow \frac{1}{6}\pi(37^{3/2}-1)$

b) $x^2+y^2+z=4$ above $xy$-plane

- Above $xy$ plane, so $z=0$, $x^2+y^2=4$ $(r=2)$
  $\hookrightarrow$ circle

$\int_0^{2\pi}\int_0^2 \sqrt{1+(2x)^2+(2y)^2} \longrightarrow \int_0^{2\pi}\int_0^2 \sqrt{1+4r^2}\;r\,dr\,d\theta$

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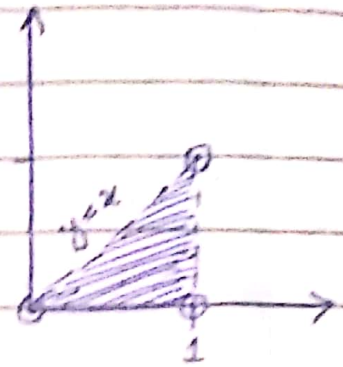
$\Rightarrow \frac{1}{8} \int \frac{(1+4r)^{3/2}}{3/2} \Rightarrow \frac{1}{12}\left[(1+16)^{3/2} - (1)\right] \Rightarrow \frac{1}{12}\left[17^{3/2} - 1\right] \theta \Big|_0^{2\pi}$

$\Rightarrow \frac{\pi}{6}\left[17^{3/2} - 1\right]$

d) $z = x^2 + 2y$, vertices $(0,0)$, $(1,0)$, $(1,1)$

$\int_0^1 \int_0^x \sqrt{1+(2x)^2+(2)^2}\, dy\,dx$




$\int_0^1 \int_0^x \sqrt{4x^2+5}\, dy\,dx$

$(\sqrt{4x^2+5})x - 0 \Rightarrow \int \sqrt{4x^2+5}\,(x)(8)$

$= \frac{1}{4 \cdot 8} \frac{(4x^2+5)^{3/2}}{3/2} \Rightarrow \frac{1}{12}(9^{3/2} - 5^{3/2})$

$= \frac{1}{12}(27 - 5\sqrt{5})$

Q7) $\int_0^{\sqrt{2}} \int_0^{\sqrt{2-x^2}} \int_{x^2+y^2}^{2} x\, dz\,dy\,dx$

$\cdot\ xz \Big|_{x^2+y^2}^{2} \Rightarrow x(2) - x(x^2+y^2)$

$2x - x(x^2+y^2) \Rightarrow 2x - x^3 - xy^2$

$\cdot \left(2xy - x^3y - \frac{xy^3}{3}\right)\Big|_0^{\sqrt{2-x^2}}$

$\Rightarrow 3/2\,(1+1/2)$

$= (2x)\sqrt{2-x^2} - x^3(\sqrt{2-x^2}) - \frac{x}{3}(\sqrt{2-x^2})^3 - 0$

$\cdot \int_0^{\sqrt{2}} 2x\sqrt{2-x^2} - x^3\sqrt{2-x^2} - \frac{1}{3}(2x-x^3)(\sqrt{2-x^2})$

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$\int 2x\sqrt{2-x^2} - \int x^3\sqrt{2-x^2} - \frac{2x}{3}\sqrt{2-x^2} + \frac{x^3}{3}\sqrt{2-x^2}$

$\Rightarrow \int_0^{\sqrt{2}} \frac{4}{3}2x\sqrt{2-x^2} - \cancel{x^3} \cdot \frac{2x^3\sqrt{2-x^2}}{3}$

$= \frac{4}{3}\int x(2)\sqrt{(2-x^2)}\cdot dx - \frac{2}{3}\int x^3\sqrt{2-x^2}\cdot dx$

$= \frac{24}{3}\frac{(2-x^2)^{3/2}}{3/2} \Rightarrow \frac{48}{9}(2-x^2)^{3/2}\Big|_0^{\sqrt{2}} - \frac{2}{3}\int x^3\sqrt{2-x^2}\cdot dx$

$= \frac{84}{9}(0-2\sqrt{2}) \Rightarrow \frac{8\sqrt{2}}{9}$

$\cdot -\frac{2}{3}\int x^3\cdot\sqrt{2-x^2}\,dx$

$\left.\begin{array}{l} u = 2-x^2 \\ \frac{du}{dx} = -2x \end{array}\right\}$ $+\frac{2}{3}\int -2x\cdot x^2\cdot\sqrt{2-x^2}\,dx = \frac{1}{3}\int\sqrt{u}(2-u)\,du$

$\frac{1}{3}\int 2u^{1/2} - u^{3/2} \rightarrow \frac{2}{3}\frac{u^{3/2}}{3/2} - \frac{1}{3}\frac{u^{5/2}}{5/2}\Big|_2^0$

$u = 2-(\sqrt{2})^2 = 0$

$u = 2-(0) = 2$ $\quad -\frac{2}{3}\frac{u^{3/2}}{3/2} + \frac{1}{3}\frac{u^{5/2}}{5/2}\Big|_0^2 =$

$\cancel{\frac{8\sqrt{2}}{9}} - \cancel{\frac{4}{9}}(2)^{3/2} + \frac{62}{15}(2)^{2.5} \Rightarrow \frac{8\sqrt{2}}{15}$

b) $\int_0^3\int_0^2\int_0^1 (xyz)^2 \to x^2y^2z^2$

$\int_0^3\int_0^2\int_0^1 x^2y^2z^2 \cdot dx\,dy\,dz$

• $\frac{x^5}{3} y^2 z^2 \Big|_0^1 = \frac{1}{3} y^2 z^2 - 0$

• $\frac{1}{3}\int y^2 z^2 \cdot dy \Rightarrow \frac{1}{3} z^2 \frac{y^3}{3} \Rightarrow \frac{1}{9} z^2 y^3 \Big|_0^2 = \frac{1}{9} z^2 (8) \Rightarrow \frac{8z^2}{9}$

• $\frac{8}{9}\int z^2 \cdot dz \to \frac{z^3}{3}\left(\frac{8}{9}\right)\Big|_0^3 \Rightarrow \frac{27(8)}{3\cdot 9} \Rightarrow 8$

c) $\int_0^{\pi/4}\int_0^{\ln\sec t}\int_{-\infty}^{2s} e^r\,dr\,ds\,dt$

• $e^r\Big|_{-\infty}^{2s} \Rightarrow e^{2s} - e^{-\infty} \to e^{2s} - \frac{1}{e^\infty} \to e^{2s} - \cancel{\frac{1}{\infty}}^{0}$

• $\frac{1}{2}\int e^{2s}(2)\cdot ds \to \frac{1}{2} e^{2s}\Big|_0^{\ln\sec t} \Rightarrow \frac{1}{2} e^{2\ln\sec t} - \frac{1}{2} e^0 \Rightarrow \frac{1}{2} e^{2\ln\sec t} - \frac{1}{2}$

• $\frac{1}{2}\int e^{2\ln\sec t} - \int\frac{1}{2}\cdot dt \to \int\frac{1}{2} e^{2\ln\sec t} - \int\frac{1}{2}\,dt$

$= \frac{1}{2} e^{\ln\sec^2 t} \to \frac{1}{2}\ln e\,\sec^2 t - \frac{1}{2} \Rightarrow \frac{1}{2}\int\sec^2 t - \frac{1}{2}\int dt$

$= \frac{1}{2}\tan t\Big|_0^{\pi/4} - \frac{1}{2} t\Big|_0^{\pi/4} \Rightarrow \frac{1}{2}\tan\left(\frac{\pi}{4}\right) - \cancel{\frac{1}{2}\tan(0)} - \frac{1}{2}\left(\frac{\pi}{4}\right) + 0$

$= \frac{1}{2} - \frac{\pi}{8} \to \frac{4-\pi}{8}$

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a) $\iiint yz^2 \sin(xyz) \cdot dx\,dy\,dz$

- $\int yz \cdot z \sin(xyz)\, dx \rightarrow \int z \sin(xyz) \cdot dxyz$

$U = xyz \quad , \quad \frac{du}{dx} = yz \rightarrow du = dxyz$

$z\int \sin(u) \cdot du \rightarrow -z\cos u \} \rightarrow -z\cos(xyz) + c$

- $\int -z\cos(u) \cdot \cancel{dy(xz)}\,du + \int c \cdot dy \qquad \overset{U}{\cancel{d}} = xyz \rightarrow \frac{du}{dy} = xz$

$= -\frac{z}{xz}\int \cos(u) \cdot du \rightarrow -\frac{z}{x}\sin(u)\} \rightarrow -\frac{\cancel{z}}{x}\overset{\sin}{\cancel{\cos}}(xyz) + cy$

$\cancel{-z\int \sin(u) \cdot dz + \int cy \cdot dz} \qquad U = xyz, \quad \frac{du}{dz} = xy$

- $\cancel{z\sin(u)} \cdot dz$

$-\frac{1}{x}\int \cancel{z}\sin(xyz) + \int cy \cdot dz + \int c \cdot dz$

$\qquad\qquad\qquad\qquad\qquad \downarrow cyz \qquad \downarrow cz$

$-\frac{1}{x(xy)}[-\cos(xyz)] + cyz + cz = 7 + \frac{1}{x^2y}\cos(xyz) + cyz + cz + c$

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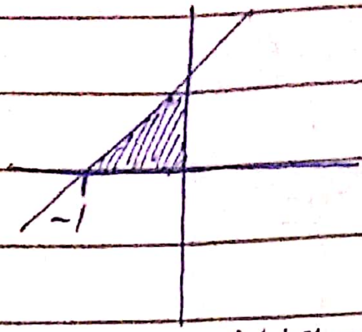
Q10) a) $y=0, x=0, y-x+z=1$

$y = x+1$




$$\int_{-1}^{0}\int_{0}^{1+x}\int_{0}^{1+x-y} dz\,dy\,dx$$

$\cdot\ z\Big|_{0}^{1+x-y} \to 1+x-y-0$

$\cdot\ \int 1\cdot dy + x\int 1\cdot dy - \int y \cdot dy$

$\cdot\ y + xy - \frac{y^2}{2}\Big|_{0}^{1+x} \Rightarrow (1+x) + x(1+x) - \frac{(1+x)^2}{2}$

$1+x+x+x^2$

$\cdot\ \cancel{\int 1\cdot dz + 2x\int 1\cdot dz + x^2\int 1\cdot dz - \frac{1}{2}\int (1+x^2)^2\cdot dz}$

$\cancel{\left(z + 2xz + x^2 z - \frac{1}{2}(1+x)^2 z\right)\Big|_{-1}^{0}}$

$(-1+2x(-1)$

$\cdot\ \int 1\cdot dx + \int 2x\cdot dx + \int x^2\cdot dx - \frac{1}{2}\int (1+x)^2\cdot dx$

$x + \frac{2x^2}{2} + \frac{x^3}{3} - \frac{1}{2}\frac{(1+x)^3}{3}$

$-\frac{1}{6}$

$x + x^2 + \frac{x^3}{3} - \frac{1}{6}(1+x)^3\Big|_{-1}^{0} \Rightarrow -1 + (-1)^2 - \frac{(-1)^3}{3} - \frac{1}{6}(0)$

$= -\frac{1}{6} \cancel{+1} \cancel{-1} + \frac{1}{3} \Rightarrow \frac{1}{6}$

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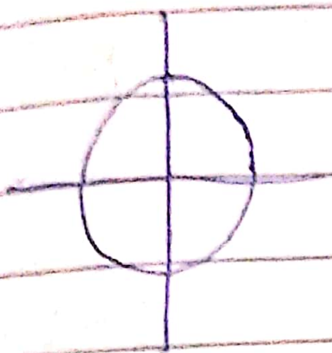
b) $x^2+y^2=1$, $x+y+z=3$, $xy$ plane

↳ $x+y=3$



$$\int_0^{2\pi}\int_0^1\int_0^{3-x-y} dz\, r\,dr\,d\theta$$

↳ $z\Big|_0^{3-x-y}$

$$\int_0^{2\pi}\int_0^1 \Big\{$$

$$z\Big|_0^{3-r\cos\theta - r\sin\theta} \Rightarrow (3 - r\sin\theta - r\cos\theta)r$$

$$\int_0^1 (3r - r^2\sin\theta - r^2\cos\theta)\,dr$$

$$\frac{3r^2}{2} - \sin\theta\,\frac{r^3}{3} - \cos\theta\cdot\frac{r^3}{3}\Big|_0^1 \Rightarrow \frac{3(1)}{2} - \frac{\sin\theta}{3} - \frac{\cos\theta}{3}$$

$$\frac{3}{2}\int d\theta - \frac{1}{3}\int \sin\theta\cdot d\theta - \frac{1}{3}\int\cos\theta\cdot d\theta$$

$$\frac{3}{2}\theta + \frac{1}{3}\cos\theta - \frac{1}{3}\sin\theta\Big|_0^{2\pi} \Rightarrow \frac{3}{\cancel{2}}(\cancel{2}\pi) + \frac{1}{3}(1) - \frac{1}{3}(0)$$

$$- \frac{3(0)}{2} - \frac{1}{3}(1) + \frac{1}{3}(0)$$

$$\Rightarrow 3\pi$$

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c) $y + z = 1$, $y = x^2$

$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dz\,dy\,dx \rightarrow z\Big|_0^{1-y} \Rightarrow 1-y$

$\int 1\,dy - \int y \cdot dy \rightarrow y - \frac{y^2}{2}\Big|_{x^2}^{1} \rightarrow 1 - \frac{1}{2} - x^2 + \frac{x^4}{2} = \frac{1}{2} - x^2 + \frac{x^4}{2}$

$-\int x^2 dx + \frac{1}{2}\int x^4 \cdot dx + \frac{1}{2}\int dx$

$-\frac{x^3}{3} + \frac{1}{2}\frac{x^5}{5} + \frac{1}{2}x\Big|_{-1}^{1} \Rightarrow -\frac{(1)}{3} + \frac{1}{10}(1) + \frac{1}{2}(1) + \frac{(-1)^3}{3} - \frac{(-1)^5}{10} - \frac{(-1)}{2}$

$= \frac{8}{15}$

Q2) 1 $\iint f(x,y)\,dA$