23K-0800

Assignment #02

Date

Q1

a) $\int_1^2 \int_4^6 \frac{x}{y^2} \, dx \, dy$

Sol

$\int_1^2 \frac{1}{y^2} \frac{x^2}{2} \Big]_4^6 dy \Rightarrow \int_1^2 \frac{1}{2y^2} [36 - 16] \, dy$

$\Rightarrow \int_1^2 \frac{20}{2y^2} \, dy \Rightarrow \int_1^2 \frac{10}{y^2} \, dy$

$\Rightarrow 10 \frac{y^{-2+1}}{-2+1} \Big]_1^2 \Rightarrow -10 \frac{1}{y} \Big]_1^2$

$\Rightarrow -10 \left\{ \frac{1}{2} - 1 \right\} \Rightarrow +10(-1/2) \Rightarrow 5$ Answer

b) $\iint x^2 + y^2 \, dx \, dy$

$\Rightarrow$ Sol $\Rightarrow \int \frac{x^3}{3} + xy^2 \, dy$

$\Rightarrow \frac{x^3 y}{3} + \frac{x y^3}{3} + c$ Answer

c) $\int_0^1 \int_1^2 \frac{x e^x}{y} \, dy \, dx$

Sol

$\Rightarrow \int_0^1 x e^x \ln|y| \Big]_1^2 dx \Rightarrow \int_0^1 x e^x [\ln|2| - \ln|1|] \, dx$

$\Rightarrow \int_0^1 x e^x \ln|2| \, dx$

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$\Rightarrow \int_0^1 x e^x dx$. Using By parts

$\Rightarrow$
$x \searrow e^x$
$1 \searrow e^x \Rightarrow x e^x - e^x$
$0 \quad e^x \Rightarrow e^x(x-1)$

$\Rightarrow \ln|2| \left[e^x(x-1) \right]_0^1$

$\Rightarrow \ln|2| \{ e^1(1-1) - e^0(0-1) \}$

$\Rightarrow \ln|2| \cdot [0 - 1(-1)]$

$\Rightarrow \ln|2| (1) \Rightarrow \ln|2|$ Answer

QUESTION 2

Sol

$P(L, K) = 70 L^{0.6} K^{0.6}$

Output = ?

$\Rightarrow \int_{20,000}^{30,000} \int_{5000}^{6000} 70 L^{0.6} K^{0.6} \, dL \, dk$

$\Rightarrow 70 \int_{20000}^{30000} \int_{5000}^{6000} L^{0.6} K^{0.4} \, dL \, dK$

$\Rightarrow 70 \int_{20000}^{30000} K^{0.4} \left. \frac{L^{1.6}}{1.6} \right]_{5000}^{6000} dk$

$\Rightarrow \frac{70}{1.6} \int_{20000}^{30000} 280688.5 \, K^{0.4} \, dk$

$$\Rightarrow 12279250.33 \left.\frac{K^{1.4}}{1.4}\right]_{20000}^{30000}$$

$$\Rightarrow \frac{12279250.33}{1.4}\left\{(30000)^{1.4} - (20000)^{1.4}\right\}$$

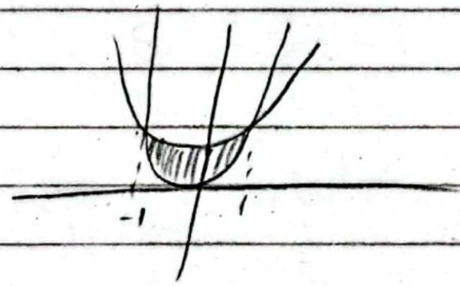
$$\Rightarrow 7.04 \times 10^{12}$$ Answer

QUESTION 3

a) $\iint_D (x+2y)\,dA \quad ; y = 2x^2 \; \} \; y = 1+x^2$

Sol

$$\int_{-1}^{1}\int_{2x^2}^{1+x^2} x+2y \; dy \; dx$$



$$\cancel{\Rightarrow \int_{-1}^{1} \left.\frac{x^2}{2} + 2xy\right]_{2x^2}^{1+x^2} dx}$$

$$\Rightarrow \int_{-1}^{1} \left. xy + y^2 \right]_{2x^2}^{1+x^2} dx$$

$2x^2 = x^2+1$

$x^2 = 1 \quad x = \pm 1$

$$\Rightarrow \int_{-1}^{1} x(1+x^2) + (1+x^2)^2 - 2x^3 - 4x^4 \; dx.$$

$$\Rightarrow \int_{-1}^{1} x + x^3 + 1 + 2x^2 + x^4 - 2x^3 - 4x^4 \; dx$$

$$\Rightarrow \int_{-1}^{1} -3x^4 - x^3 + 2x^2 + x + 1 \; dx$$

$$\Rightarrow \left[-\frac{3x^5}{5} + \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} + x \right]_{-1}^{1}$$

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$\Rightarrow \left\{ \frac{-3}{5}(1)^3 - \frac{1}{4}\cdot(1)^4 + \frac{2}{3}(1)^3 + \frac{1}{2}(1)^2 + 1 \right\}$

$- \left\{ \frac{-3}{5}(-1)^3 - \frac{1}{4}(-1)^4 + \frac{2}{3}(-1)^3 + \frac{1}{2}(-1)^2 - 1 \right\}$

$\Rightarrow \left(\frac{-3}{5} - \frac{1}{4} + \frac{2}{3} + \frac{1}{2} + 1\right) - \left(\frac{3}{5} - \frac{1}{4} - \frac{2}{3} + \frac{1}{2} - 1\right)$

$\Rightarrow \frac{-3}{5} - \cancel{\frac{1}{4}} + \frac{2}{3} + \cancel{\frac{1}{2}} + 1 - \frac{3}{5} + \cancel{\frac{1}{4}} + \frac{2}{3} - \cancel{\frac{1}{2}} + 1$

$\Rightarrow -\frac{3}{5} - \frac{3}{5} + \frac{2}{3} + \frac{2}{3} + 2$

$\Rightarrow \frac{32}{15}$ Answer

n) $\int_0^4 \int_0^{\sqrt{y}} xy^2 \, dx\, dy$

Sol

$\Rightarrow \int_0^4 \frac{y^2}{2} \left. x^2 \right]_0^{\sqrt{y}} dy$

$\Rightarrow \int_0^4 \frac{y^2 (\sqrt{y})^2}{2} dy \Rightarrow \frac{1}{2}\int_0^4 y^3 \, dy$

$\Rightarrow \frac{1}{2} \left. \frac{y^4}{4} \right]_0^4 \Rightarrow \frac{1}{8} \left\{ (4)^4 - (0)^4 \right\}$

$\Rightarrow \frac{256}{8} = 32$ Answer

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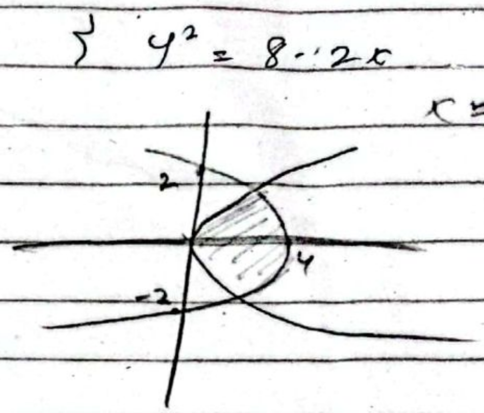
c) $\iint_R (4-y^2)$; $y^2 = 2x$ } $y^2 = 8-2x$

$x = y^2/2$

Sol

$$\Rightarrow \int_{-2}^{2}\int_{y^2/2}^{4-y^2/2} 4-y^2 \, dx\, dy$$



$$\Rightarrow \int_{-2}^{2} 4x - xy^2 \Big]_{y^2/2}^{4-y^2/2} dy$$

$x = -\frac{y^2}{2} + 4$

$\frac{y^2}{2} + \frac{y^2}{2} = 4$

$\cancel{2}y^2 = \cancel{8}\,4$

$y = \pm 2$

$$\cancel{\Rightarrow \int_{-2}^{2} 4\left\{\frac{4-y^2}{2}\right\} - y^2 x}$$

$$\Rightarrow \int_{-2}^{2} 4-y^2 \left[4 - \frac{y^2}{2} - \frac{y^2}{2}\right] dy$$

$$\Rightarrow \int_{-2}^{2} (4-y^2)(4-y^2)\, dy \Rightarrow \int_{-2}^{2} 16 - 4y^2 + y^4 \, dy$$

$$\Rightarrow 16y - \frac{4y^3}{3} + \frac{y^5}{5}\Big]_{-2}^{2}$$

$$\Rightarrow 16(2) - \frac{4(8)}{3} + \frac{32}{5} - 16(-2) + \frac{4(-2)^3}{3} - \frac{(-2)^5}{5}$$

$$\Rightarrow \frac{32}{5} - \frac{64}{3} + 32 + \frac{32}{5} - \frac{64}{3} + 32$$

$$\Rightarrow \frac{64}{5} - \frac{138}{3} + 64 \Rightarrow \frac{512}{15}$$ Answer

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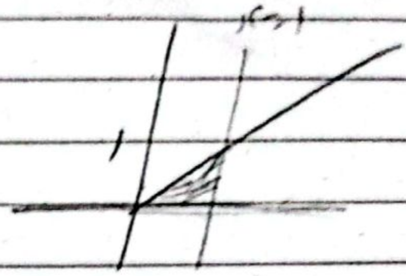
QUESTION 4

Q $y = x$, $x = 1$ $f(x,y) = 3 - x - y$.

Sol $\cancel{\int_0^1 \int_0^x 3 - x - y \, dy \, dx}$



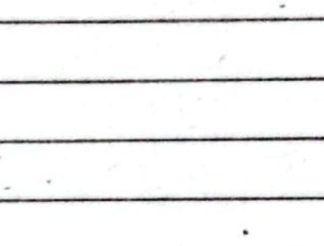
$\Rightarrow \int_0^1 \int_0^y 3 - x - y \, dx \, dy$

$\Rightarrow \int_0^1 3x - \frac{x^2}{2} - xy \Big]_0^y \, dy$

$\Rightarrow \int_0^1 3y - \frac{y^2}{2} - y^2 \, dy$

$\Rightarrow \frac{3y^2}{2} - \frac{y^3}{6} - \frac{y^3}{3} \Big]_0^1 \Rightarrow \frac{3}{2} - \frac{1}{6} - \frac{1}{3} \Rightarrow 1$ Ans

Q5) (a) $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y \, dx \, dy$



Sol

$x = \sqrt{1-y^2}$

$x^2 = 1 - y^2$

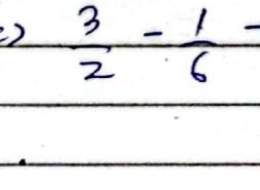
$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$

$y^2 = 1$

$y = \pm 1$

$\sqrt{1-x^2}$



$\int_{-1}^1 \int_{-\sqrt{1-x^2}} 3y \, dy \, dx$

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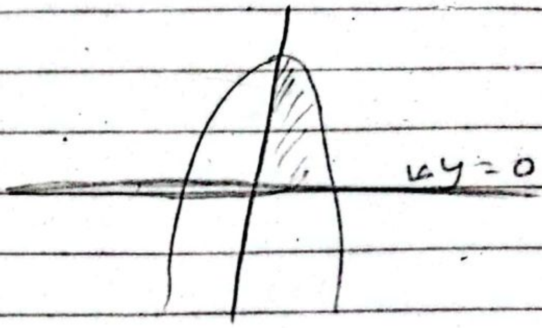
(b) $\int_0^{3/2}\int_0^{9-4x^2} 16x \, dy \, dx$

Sol $y=0$
$y = 9-4x^2$



put $y=0$

$0 = 9-4x^2$

$9 = 4x^2$

$x^2 = \frac{9}{4}$

$x = \pm\frac{3}{2}$

$4x^2 = 9-y$

$x^2 = \frac{9}{4} - \frac{y}{4}$

Put $x = \pm 3/2$

$y =$

$x = \frac{3}{2} - \frac{\sqrt{y}}{2}$

$$\int_0^9 \int_0^{\frac{1}{2}\sqrt{9-y}} 16y \, dx \, dy$$

c) $\int_0^1 \int_{1-x}^{1-x^2} dy\, dx$

$0 \le x \le 1$

$1-x \le y \le 1-x^2$

$x=0$
$y=1$
$x=1$
$y=0$

lower bound
$x = 1-y$
$y=1-x^2$
$x = \sqrt{1-y}$

$$\int_0^1 \int_{1-y}^{\sqrt{1-y}} dx\, dy$$

Ans

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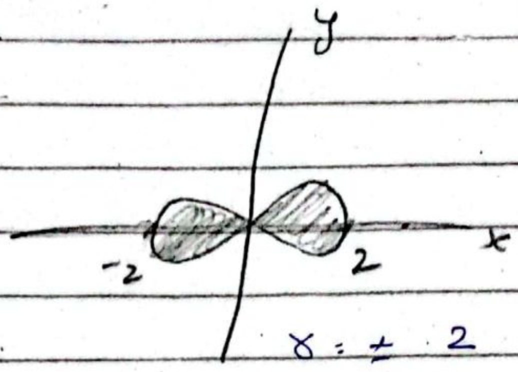
QUESTION 6.

Sol

$r^2 = 4\cos 2\theta$



$\theta = 0, \quad r = \pm 2.$

$\theta = \pm \frac{\pi}{6} \quad ; \quad r = \pm\sqrt{2}$

$\theta = \pm \frac{\pi}{4} \quad ; \quad r = 0.$

$$\therefore \quad A = \int_{-\pi/4}^{\pi/4} \int_{0}^{2\sqrt{\cos 2\theta}} r \, dr \, d\theta$$

$$A = \int_{-\pi/4}^{\pi/4} \frac{r^2}{2} \Big]_{0}^{\cancel{r}\,2\sqrt{\cos 2\theta}} d\theta$$

$$A = \int_{-\pi/4}^{\pi/4} \frac{\cancel{4}^{\,2}\cos 2\theta}{\cancel{2}} \, d\theta$$

$$A = 2\int_{-\pi/4}^{\pi/4} \cos 2\theta \, d\theta \quad \Rightarrow$$

$$A = \sin 2\theta \Big]_{-\pi/4}^{\pi/4} \quad \Rightarrow \quad \sin 2\left(\frac{\pi}{4}\right) - \sin 2\left(\frac{-\pi}{4}\right)$$

$$\Rightarrow \quad 1 - (-1) = 2$$

Area for both halfs $= 2(2) = 4$ Ans

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QUESTION 7

a) $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\, dx\, dy$

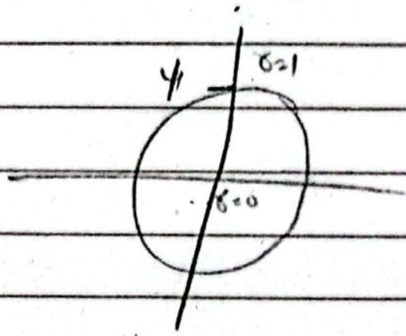
Sol

$y=0 \quad ; y=1 \quad ; \quad x=0$

$x = \sqrt{1-y^2} \quad \Rightarrow \quad x^2+y^2=1$

$\because x^2+y^2 = r^2$

$\boxed{r^2=1} \Rightarrow r=1$



$\Rightarrow \int_{0}^{\theta=\pi/2} \int_0^1 r \cdot r^2\, dr\, d\theta$

$\int_0^{\pi/2} \int_0^1 r^3\, dr\, d\theta \Rightarrow \int_0^{\pi/2} \left.\frac{r^4}{4}\right]_0^1 d\theta$

$\Rightarrow \int_0^{\pi/2} \frac{1}{4}\, d\theta \Rightarrow \frac{1}{4} \left.\theta\right]_0^{\pi/2} \Rightarrow \frac{1}{4}\left\{\frac{\pi}{2}-0\right\}$

$\Rightarrow \frac{\pi}{8}$ Answer

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(b) $\displaystyle\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}\, dy\, dx$

-1 to 0

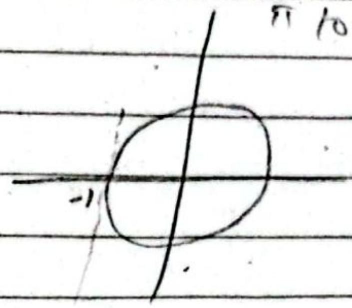
π to $3\pi/2$

Sol

$y = -\sqrt{1-x^2}$

$x^2+y^2 = 1$

$r^2 = 1;\ \boxed{r = 1}$



$x = 0\ ;\ x = -1\ ;\ y = 0$

$$\Rightarrow \int_{\pi}^{3\pi/2}\int_{0}^{1} \frac{2}{1+\sqrt{r^2}} \cdot r\, dr\, d\theta.$$

$$\int_{\pi}^{3\pi/2}\int_{0}^{1} \frac{2r}{1+r}\, dr\, d\theta$$

$$\Rightarrow \int_{\pi}^{3\pi/2} 2\int_{0}^{1} \frac{r}{1+r}\, dr\, d\theta$$

$$\Rightarrow \int_{\pi}^{3\pi/2} 2\int_{0}^{1} 1 - \frac{1}{1+r}\, dr\, d\theta.$$

$$\int_{\pi}^{3\pi/2} 2 \left\{ r + \ln|1+r| \right\}_{0}^{1} d\theta$$

$$\int_{\pi}^{3\pi/2} 2\left(1+\ln|2|\right) d\theta.$$

$$\Rightarrow 2\left(1+\ln|2|\right)\ \theta \Big]_{\pi}^{3\pi/2} \Rightarrow 2\left(1+\ln|2|\right)\left(\frac{3\pi}{2} - \pi\right).$$

$$\Rightarrow 2\left(1+\ln|2|\right)\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \pi + \pi \ln|2| \quad \underline{\text{Answer}}$$

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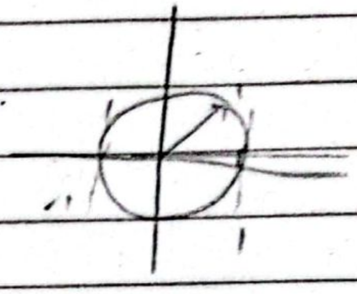
① $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\, dy\, dx$

Sol $x = 1 \quad , \quad x = -1$

$y = \sqrt{1-x^2} \quad , \quad y = -\sqrt{1-x^2}$

$x^2 + y^2 = 1$

$r^2 = 1 \quad , \quad r = 1$



$\Rightarrow \int_{\pi}^{2\pi} \int_{0}^{1} \frac{2}{(1+r^2)^2}\, r\, dr\, d\theta$

$\Rightarrow \int_{\pi}^{2\pi} \int_{0}^{1} 2r\,(1+r^2)^{-2}\, dr\, d\theta$

$\Rightarrow \int_{\pi}^{2\pi} -(1+r^2)^{-1} \Big]_0^1 d\theta \Rightarrow -\int_{\pi}^{2\pi} \frac{1}{(1+r^2)} \Big]_0^1 d\theta$

$\Rightarrow -\int_{\pi}^{2\pi} \frac{1}{2} - 1\, d\theta \Rightarrow -\int_{\pi}^{2\pi} -\frac{1}{2}\, d\theta$

$\Rightarrow \frac{1}{2}\, \theta \Big]_{\pi}^{2\pi} \Rightarrow \frac{1}{2}(2\pi - \pi) \Rightarrow$

$\Rightarrow \pi$ Answer

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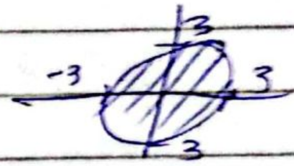
QUESTION 8.

a) $z = x^2+y^2$ $\qquad z = 9.$

Sol

$$x^2+y^2 = 9 \qquad r^2 = 9$$
$$r = 3$$

$$A = \iint_D \sqrt{1+(f_x)^2+(f_y)^2}\, dx\,dy$$



~~Converting to polars~~

~~A =~~

$$A = \int_0^{2\pi} \int_0^{3} \sqrt{1+(2x)^2+(2y)^2}$$
D

Converting to Polars

$$A = \int_0^{2\pi} \int_0^{3} \sqrt{1+4r^2}\; r\,dr\,d\theta$$

$$A = \int_0^{2\pi} \int_0^{3} \frac{8r}{8} \sqrt{1+4r^2}\; dr\,d\theta.$$

$$A = \int_0^{2\pi} \frac{1}{8} \cdot \frac{2}{3} \left\{ (1+4r^2)^{3/2} \right]_0^3 d\theta$$

$$= \int_0^{2\pi} \frac{1}{12} \left[(37)^{3/2} - 1 \right] d\theta$$

$$= \frac{1}{12} \left(37^{3/2} - 1\right)\left(2\pi - 0\right)$$

$$\Rightarrow \frac{\pi}{6} \left(37^{3/2} - 1\right)$$

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b) $x^2+y^2+z=4$ xy-plane abve.

$z=0$.

$x^2+y^2-y=0$

$x^2+y^2=4$

$r^2=4 \Rightarrow r=2$

$$\int_0^{2\pi}\int_0^2 \sqrt{1+(2x)^2(2y)^2}\; r\,dr\,d\theta.$$

$$\int_0^{2\pi}\int_0^2 \sqrt{1+4(x^2+y^2)}\; r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^2 \sqrt{1+4r^2}\,dr\,d\theta$$

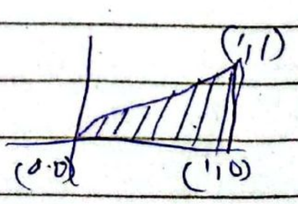
$$\int_0^{2\pi} \frac{\cancel{2}}{\cancel{8}}\cdot\frac{2}{3}(1+4r^2)^{3/2}\Big]_0^2 d\theta \Rightarrow \int_0^{2\pi}\frac{1}{12}\{17^{3/2}-1\}\,d\theta.$$

$$\Rightarrow \frac{1}{12}\left[17^{3/2}-1\right]\theta\Big]_0^{2\pi} \Rightarrow \frac{1}{12}\left[17^{3/2}-1\right](2\pi)$$

$\Rightarrow \frac{\pi}{6}(17^{3/2}-1)$ Answer

(c) $z=x^2+2y$ above $\triangle$ in xy-plane

(0,0), (1,0) & (1,1)



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$$\int_0^1 \int_0^x \sqrt{1+(2x)^2+(2)^2}\; dy\, dx.$$

$$\Rightarrow \int_0^1 \int_0^x \sqrt{4x^2+5}\; dy\, dx.$$

$$\Rightarrow \int_0^1 \sqrt{4x^2+5}\; y\Big]_0^x dx \Rightarrow \int_0^1 x\sqrt{4x^2+5}\; dx$$

$\times \div$ by 8

$$\Rightarrow \frac{1}{8}\int_0^1 8x\sqrt{4x^2+5}\; dx \Rightarrow \frac{1}{8}\, \frac{(4x^2+5)^{3/2}}{3/2}\Big]_0^1$$

$$\Rightarrow \frac{1}{12}\left\{(4+5)^{3/2} - 5^{3/2}\right)$$

$$\Rightarrow \frac{1}{12}\left\{9\sqrt{3} - 5\sqrt{5}\right) \Rightarrow 1.01$$

Answer

QUESTION 9

(a) $$\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}}\int_{x^2+y^2}^{2} x \, dz\, dy\, dx$$

Sol:

$$\Rightarrow \int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} xz\Big]_{x^2+y^2}^{2} dy\, dx$$

$$\Rightarrow 2x - x^3 - xy^2$$

$$\Rightarrow \int_0^{\sqrt{2}} 2xy - x^3y - \frac{xy^3}{3}\Big]_0^{\sqrt{2-x^2}} dx$$

$$\Rightarrow 2x\sqrt{2-x^2} - x^3\sqrt{2-x^2} - \frac{x}{3}\left(\sqrt{2-x^2}\right)^3 - 0$$

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$$\Rightarrow \int_0^{\sqrt{2}} 2x\sqrt{2-x^2} - x^3\sqrt{2-x^2} - \frac{1}{3}(2x-x^3)(\sqrt{2-x^2})\,dx.$$

$$= \int_0^{\sqrt{2}} 2x\sqrt{2-x^2} - x^3\sqrt{2-x^2} - \frac{2x}{3}\sqrt{2-x^2} + \frac{x^3}{3}\sqrt{2-x^2}\,dx$$

$$\Rightarrow \int_0^{\sqrt{2}} \frac{4}{3}\cdot 2x\sqrt{2-x^2} - \frac{2}{3}x^3\sqrt{2-x^2}\,dx.$$

$$\Rightarrow \frac{4}{3}\int_0^{\sqrt{2}} 2x\sqrt{2-x^2}\,dx - \frac{2}{3}\int_0^{\sqrt{2}} x^3\sqrt{2-x^2}\,dx.$$
(i) (ii)

Consider (i)

$$\Rightarrow \frac{24}{3}\cdot\frac{2}{3}(2-x^2)^{3/2}\Big]_0^{\sqrt{2}} \Rightarrow \frac{48}{9}(2-x^2)^{3/2}\Big]_0^{\sqrt{2}}$$

$$\Rightarrow \frac{84}{9}(0-2\sqrt{2}) = \frac{8\sqrt{2}}{9}$$

Consider (ii).

$$\Rightarrow -\frac{2}{3}\int x^3\cdot\sqrt{2-x^2}\,dx.$$

let $u = 2-x^2$

$\frac{du}{dx} = -2x$

$$\Rightarrow \frac{2}{3}\int -2x\cdot x^2\sqrt{2-x^2}\,dx \Rightarrow \frac{1}{3}\int \sqrt{u}(2-u)\,du.$$

$$\Rightarrow \frac{1}{3}\int 2u^{1/2} - u^{3/2} \Rightarrow \frac{2}{3}\frac{u^{3/2}}{3/2} - \frac{1}{3}\frac{u^{5/2}}{5/2}\Big]_2^0$$

$u = 2-(\sqrt{2})^2 = 0$

$u = 2-0 = 2.$

$$\Rightarrow -\frac{2}{3}\frac{u^{3/2}}{3/2} + \frac{1}{3}\cdot\frac{u^{5/2}}{5/2}\Big]_0^2$$

$$\Rightarrow \frac{8}{9}\sqrt{2} - \frac{4}{9}(2)^{3/2} + \frac{2}{15}(2)^{5/2} \Rightarrow \boxed{\frac{8\sqrt{2}}{15}}$$ Ans

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b) $\int_0^3 \int_0^2 \int_0^1 (xyz)^2 \, dx\,dy\,dz$

Sol $\int_0^3 \int_0^2 \frac{x^3}{3} y^2 z^2 \Big]_0^1 \Rightarrow \frac{1}{3} y^2 z^2$

$\Rightarrow \frac{1}{3} \int_0^3 \frac{z^2 y^3}{3} \Big]_0^2 \Rightarrow \frac{1}{9} \int_0^3 z^2 \, 8 \, dz.$

$\Rightarrow \frac{8}{9} \frac{z^3}{3} \Big]_0^3 \Rightarrow \frac{8}{\cancel{27}} \cancel{27} \Rightarrow \boxed{8 \text{ Answer}}$

(c) $\int_0^{\pi/4} \int_0^{\ln \sec t} \int_{-\infty}^{2s} e^{r} \, dr\,ds\,dt$

$\Rightarrow e^{r} \Big]_{-\infty}^{2s} \Rightarrow e^{2s} - e^{-\infty} \Rightarrow e^{2s} - \frac{1}{e^{\infty}}$

$\Rightarrow \frac{1}{2} \int_0^{\ln \sec t} e^{2s} (2) \, ds \Rightarrow \frac{1}{2} e^{2s} \Big]_0^{\ln \sec t}$

$\Rightarrow \frac{1}{2} e^{2 \ln \sec t} - \frac{1}{2} e^{0} \Rightarrow \frac{1}{2} e^{2 \ln \sec t} - \frac{1}{2}$

$\Rightarrow \frac{1}{2} \int_0^{\pi/4} e^{2 \ln \sec t} - \frac{1}{2} \int_0^{\pi/4} dt$

$\Rightarrow \frac{1}{2} \int_0^{\pi/4} e^{\ln \sec^2 t} - \frac{1}{2} \int_0^{\pi/4} dt$

$\Rightarrow \frac{1}{2} \int_0^{\pi/4} \sec^2 t - \frac{1}{2} \int_0^{\pi/4} dt$

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$$\Rightarrow \frac{1}{2} \tan t \Big]_0^{\pi/4} - \frac{1}{2} t \Big]_0^{\pi/4}$$

$$\Rightarrow \frac{1}{2} \tan\left(\frac{\pi}{4}\right) - \frac{1}{2} \tan(0) - \frac{1}{2}\left(\frac{\pi}{4}\right) + 0$$

$$\Rightarrow \frac{1}{2} - \frac{\pi}{8} = \boxed{\frac{4-\pi}{8}}$$ Ans

(d) $\iiint yz^2 \sin(xyz)\, dx\, dy\, dz$

Sol

$\iiint z \cdot yz \sin(xyz)\, dx\, dy\, dz \Rightarrow \int z \sin(xyz)\, dx\, y$

let $u = xyz$ $\quad \frac{du}{dx} = yz \quad du = dx\, yz.$

$\Rightarrow z \int \sin(u)\, du \Rightarrow -z \cos u$

$\Rightarrow \int -z \cos(xyz)\, dy + \int c \cdot dy \Rightarrow$

let $u = xyz$ $\qquad \Rightarrow \frac{-\cancel{z}}{x\cancel{z}} \int \cos(u)\, du$

$\frac{du}{dy} = xz$; $\frac{du}{xz} = dy$

$\Rightarrow -\frac{1}{x} \sin u + cy + c.$

$\Rightarrow -\frac{1}{x} \int \sin(xyz)\, dz + c \int y\, dz + c \int dz$

$\Rightarrow -\frac{1}{x(xy)} \left(-\cos(xyz)\right) + cyz + cz.$

$$\Rightarrow \boxed{\frac{1}{x^2 y} \cos(xyz) + cyz + cz + c}$$ Ans

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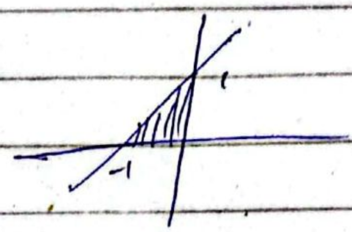
QUESTION 10

a) $y=0,\ z=0,\ x=0 \qquad y-x+z=1$

Sol)



$$\Rightarrow \int_{-1}^{0}\int_{0}^{1+x}\int_{0}^{1+x-y} dz\,dy\,dx$$

$$\Rightarrow z\Big]_{0}^{1+x-y} \Rightarrow 1+x-y$$

$$\Rightarrow \int_{0}^{1+x} dy + \int_{0}^{1+x} x\,dy - \int_{0}^{1+x} y\,dy \Rightarrow y + xy - \frac{y^2}{2}\Big]_{0}^{1+x}$$

$$\Rightarrow \int_{-1}^{0} (1+x) + x + x^2 - \frac{(1+x)^2}{2}\,dx$$

$$\Rightarrow x + \frac{\cancel{2}x^2}{\cancel{2}} + \frac{x^3}{3} - \frac{(1+x)^3}{6}\Big]_{-1}^{0}$$

$$\Rightarrow \cancel{0+0+0+0 = 1+1-\frac{1}{3},-0} \Rightarrow$$

$$\Rightarrow \cancel{0+0+0-0-(-1)-(-1)^2-(-1)^3-\frac{(1-1)^3}{6}}$$

$$\Rightarrow \cancel{1-1+1\,0} =$$

$$\Rightarrow -1-(-1)^2-\frac{(-1)^3}{3}-\frac{1}{6}(0) \Rightarrow \frac{1}{6} \text{ Answer}$$

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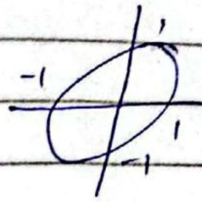
b) $x^2+y^2=1$. $x+y+z=3$.
$x+y=3$.



$$\Rightarrow \int_0^{2\pi}\int_0^{1}\int_0^{3-x-y} dz\, r\,dr\,d\theta$$

$= x = r\cos\theta$
$y = r\sin\theta$.

$$\Rightarrow z\Big]_0^{3-x-y} \Rightarrow (3 - r\cos\theta - r\sin\theta)\, r.$$

$$\Rightarrow \int_0^1 (3r - r^2\cos\theta - r^2\sin\theta)\, dr$$

$$\Rightarrow \frac{3r^2}{2} - \frac{\sin\theta\, r^3}{3} - \frac{\cos\theta\, r^3}{3}\Big]_0^1$$

$$\Rightarrow \frac{3}{2}(1) - \frac{\sin\theta}{3} - \frac{\cos\theta}{3}$$

$$\Rightarrow \frac{3}{2}\int d\theta - \frac{1}{3}\int \sin\theta\, d\theta - \frac{1}{3}\int \cos\theta\, d\theta$$

$$\Rightarrow \frac{3}{2}\theta + \frac{1}{3}\cos\theta - \frac{1}{3}\sin\theta\Big]_0^{2\pi}$$

$$\Rightarrow \frac{3}{\cancel{2}}(\cancel{2}\pi) + \frac{1}{3}\cancel{\cos(2\pi)} - \frac{1}{3}\sin(2\pi) - \cancel{\frac{1}{3}(1)}$$

$$\boxed{\Rightarrow 3\pi \quad \text{Answer}}$$

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c) $y + z = 1$, $y = x^2$

Sol $\int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} dz\,dy\,dx$

$\Rightarrow z \Big]_{0}^{1-y} \Rightarrow 1-y$

$\Rightarrow y - \frac{y^2}{2} \Big]_{x^2}^{1} \Rightarrow 1 - \frac{1}{2} - x^2 - \frac{x^4}{2}$

$\Rightarrow x - \frac{x}{2} - \frac{x^3}{3} - \frac{x^5}{10} \Big]_{-1}^{1}$

$\Rightarrow 1 - \frac{1}{2} - \frac{1}{3} - \frac{1}{10} - 1 + \frac{(-1)}{2} + \frac{(-1)^3}{3} + \frac{(-1)^5}{10}$

$\Rightarrow \boxed{\frac{8}{15} \quad \text{Answer}}$