23K-0021

MVC-Ass 002

Q1) Evaluate

a) $\int_1^2 \int_4^6 \frac{x}{y^2}\, dx\, dy$

Sol

$\int_1^2 \int_4^6 \frac{x}{y^2}\, dx\, dy$

$\int_1^2 \frac{1}{y^2}\left[\frac{x^2}{2}\right]_4^6 dy$

$\int_1^2 \frac{1}{2y^2}\left[36 - 16\right]$

$\int_1^2 \frac{20}{2y^2}\, dy$

$\int_1^2 \frac{10}{y^2}\, dy$

$10\int_1^2 y^{-2}\, dy$

$10\left[\frac{y^{-2+1}}{-2+1}\right]_1^2$

$10\left[\frac{y^{-1}}{-1}\right]_1^2$

$-10\left[(2)^{-1} - (1)^{-1}\right]$

$-10\left[\frac{1}{2} - 1\right]$

$-10\left[-\frac{1}{2}\right] = +5$

Ans

b) $\int\int x^2+y^2\, dx\, dy$

$\int \left(\frac{x^3}{3} + xy^2\right) dy$

$\frac{x^3y}{3} + \frac{xy^3}{3} + c$

Ans

c) $\int_0^1 \int_1^2 \frac{xe^x}{y} \, dy \, dx$

Sol =

$\int_0^1 xe^x \left(\int_1^2 \frac{1}{y} dy \right) dx$

$\int_0^1 xe^x \left[\ln|y| \right]_1^2 dx$

$\int_0^1 xe^x \left[\ln|2| - \ln(1) \right] dx$

$\int_0^1 xe^x \ln|2| \, dx$

$\ln|2| \int_0^1 xe^x \, dx$ (1)

By using by-parts.

$\int u \, dv = uv - \int v \, du.$

$\boxed{u = x}$

$\frac{du}{dx} = 1$

$\boxed{du = dx}$

$\int dv = \int e^x dx$

$\boxed{v = e^x}$

$= \cancel{\frac{x^2}{2}} \int \frac{x^2}{2} dx$

$\cancel{\frac{x^3}{2}}$

$uv - \int v \, du$

$x \cdot e^x - \int e^x \cdot dx$

$xe^x - e^x$

$\boxed{e^x(x-1)}$ — put in (1)

$\ln|2| \left[e^x (x-1) \right]_0^1$

$\ln|2| \left[e^1(1-1) - e^0(0-1) \right]$

$\ln|2| \left[0 - 1(-1) \right]$

$\ln|2) (+1)$

$\ln|2|$

Ans:

280668.98

(Q:2)

$$P(L,K) = 70 L^{0.6} K^{0.4}$$

Output: $\int_{20,000}^{30,000}\int_{5000}^{6000} 70 L^{0.6} K^{0.4} \, dL \, dK$

$$70 \int_{20000}^{30000}\int_{5000}^{6000} L^{0.6} K^{0.4} \, dL \, dK$$

$$70 \int_{20000}^{30000} K^{0.4} \left[\frac{L^{1.6}}{1.6} \right]_{5000}^{6000} dK$$

$$\frac{70}{1.6} \int_{20000}^{30000} K^{0.4} \left[(6000)^{1.6} - (5000)^{1.6} \right] dK$$

$$\frac{70}{1.6} \int_{20000}^{30000} K^{0.4} (280688.98) \, dK$$

$$\frac{70}{1.6} \times 280688.98 \int_{20000}^{30000} K^{0.4} \, dK$$

$$12279250.33 \left[\frac{K^{1.4}}{1.4} \right]_{20000}^{30000}$$

$$\frac{12279250.33}{1.4}$$

$$8770893.125 \left((30000)^{1.4} - (20000)^{1.4} \right)$$

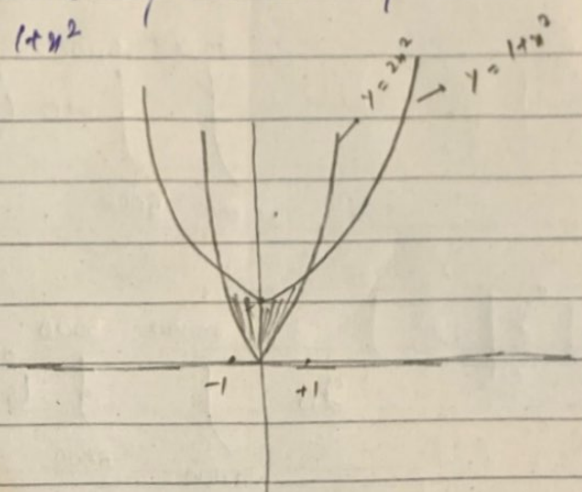
$= 7.04 \times 10^{12}$

Ans

Q:3)

a) $\iint_D (x+2y)\, dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1+x^2$

S:



$y = y$

$1+x^2 = 2x^2$

$x^2 = 1$

$x = \pm 1$

$x = +1$ $\quad$ $x = -1$

$y = 2$ $\quad$ $y = 2$

$$\int_{-1}^{+1} \int_{y=2x^2}^{y=1+x^2} x+2y \; dy\, dx$$

$$\int_{-1}^{+1} \left[xy + \frac{\cancel{2}y^2}{\cancel{2}} \right]_{y=2x^2}^{y=1+x^2} dx$$

$$\int_{-1}^{+1} \left[x(1+x^2) + (1+x^2)^2 \right] - \left[x(2x^2) + (2x^2)^2 \right] dx$$

$$\int_{-1}^{+1} x + x^3 + 1 + 2x^2 + x^4 - (2x^3 + 4x^4)\, dx$$

$$\int_{-1}^{+1} x + x^3 + 1 + 2x^2 + x^4 - 2x^3 - 4x^4 \; dx$$

$$\int_{-1}^{+1} -3x^4 - x^3 + 2x^2 + x + 1 \quad dx$$

$$\left[-\frac{3x^5}{5} - \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} + x \right]_{-1}^{+1}$$

$$\left(\frac{-3}{5}(1)^5 - \frac{1}{4}(1)^4 + \frac{2}{3}(1)^3 + \frac{1}{2}(1)^2 + 1\right) - \left(\frac{-3}{5}(-1)^5 - \frac{1}{4}(-1)^4 + \frac{2}{3}(-1)^3 + \frac{(-1)^2}{2} - 1\right)$$

$$\left(\frac{-3}{5} - \frac{1}{4} + \frac{2}{3} + \frac{1}{2} + 1\right) - \left(+\frac{3}{5} - \frac{1}{4} - \frac{2}{3} + \frac{1}{2} - 1\right)$$

$$-\frac{3}{5} \cancel{-\frac{1}{4}} + \frac{2}{3} \cancel{+\frac{1}{2}} + 1 - \frac{3}{5} \cancel{+\frac{1}{4}} + \frac{2}{3} \cancel{-\frac{1}{2}} + 1$$

$$-2(3/5) + 2(2/3) + 2$$

$$-6/5 + 4/3 + 2$$

$$\cancel{15} \quad 32/15 \quad \text{Ans}$$

b) $\displaystyle\int_0^4 \int_0^{\sqrt{y}} xy^2 \, dx \, dy$

$$\int_0^4 \left(\frac{x^2}{2} y^2\right)_0^{\sqrt{y}} dy = \frac{1}{2}\int_0^4 (\sqrt{y})^2 y^2 - (0) \, dy$$

$$\frac{1}{2}\int_0^4 y \cdot y^2 \, dy = \frac{1}{2}\int_0^4 y^3 \, dy$$

$$\frac{1}{2}\left[\frac{y^4}{4}\right]_0^4 = \frac{1}{8}\left[y^4\right]_0^4 = \frac{1}{8}(4)^4 - 0 = 32$$

A

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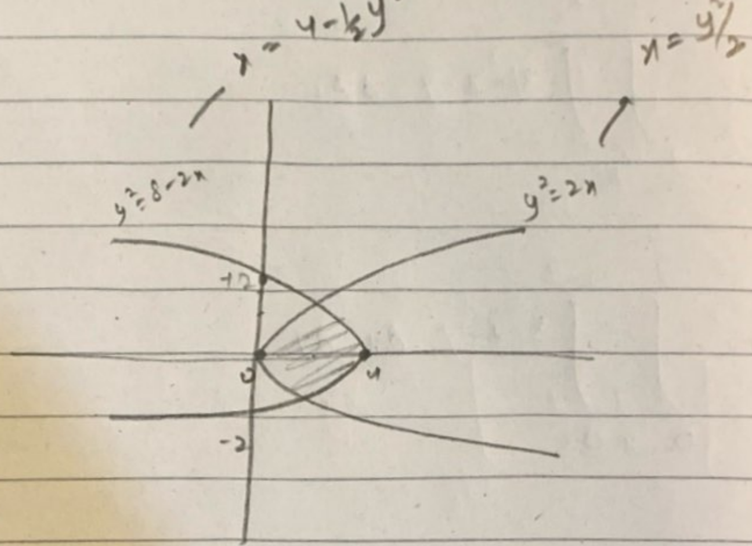
Q:3:c)

$\iint_R 4-y^2 \; dA$ over the region R which is bounded b/w $y^2 = 2x$ & $y^2 = 8-2x$

Sol:



$$= \int_{-2}^{+2}\int_{y^2/2}^{4-\frac{1}{2}y^2} 4-y^2 \; dx\, dy$$

$$\int_{-2}^{+2} \left[4-y^2(x)\right]_{y^2/2}^{4-\frac{1}{2}y^2} dy$$

$2x = 2x$

$y^2 = 8-y^2$

$2y^2 = 8$

$y^2 = 4$

$y = \pm 2$

$-2 \le y \le 2$

$$\int_{-2}^{+2} \left[4-y^2 \left[4-\frac{y^2}{2} - \frac{y^2}{2}\right]\right] dy$$

$$\int_{-2}^{+2} \left[(4-y^2)(4-y^2)\right] dy$$

$$\int_{-2}^{+2} 16-8y^2+y^4 \; dy \Rightarrow \left. \frac{y^5}{5} - \frac{8y^3}{3} + 16y \right|_{-2}^{+2}$$

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$\left[\frac{(2)^5}{5} - \frac{8}{3}(2)^3 + 16(2)\right] - \left[\frac{(-2)^5}{5} - \frac{8}{3}(-2)^3 + 16(-2)\right]$

$32/5 - 64/3 + 32 + 32/5 - 64/3 + 32$

$64/5 - \frac{128}{3} + 64$

$\frac{512}{15}$ ANS

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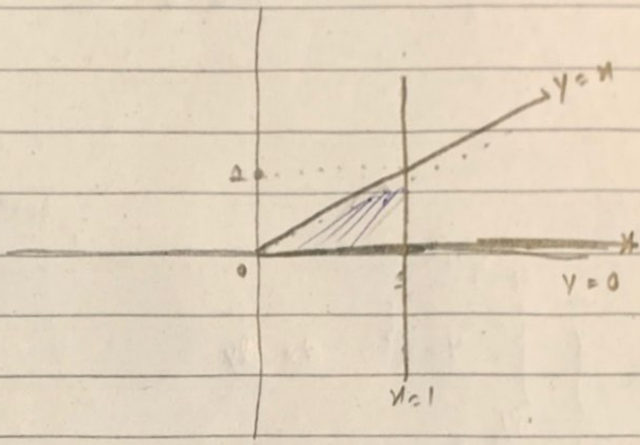
Q:4

$f(x,y) = 3 - x - y \quad , \quad y = x , \quad x = 1$



$$V = \iint_R 3 - x - y \; dA$$

$$V = \int_0^1 \int_{x=0}^{x=y} 3 - x - y \; dx \, dy$$

$x = 1$

$x = y$

$\boxed{y = 1}$ max limit

$y = 0$ - min limit

$$V = \int_0^1 \left. 3x - \frac{x^2}{2} - yx \right]_0^y dy$$

$$V = \int_0^1 \left[3y - \frac{y^2}{2} - y(y)\right] - [0]$$

$$V = \int_0^1 3y - \frac{y^2}{2} - y^2 \; dy$$

$$V = \int_0^1 3y - \frac{3y^2}{2} \; dy \Rightarrow \left[\frac{3y^2}{2} - \frac{3y^3}{2(3)}\right]_0^1$$

$$\left. \frac{3y^2}{2} - \frac{y^3}{2} \right)_0^1 = \left(\frac{3(1)^2}{2} - \frac{(1)^3}{2}\right) - 0$$

$$3/2 - 1/2 = 2/2 = 1$$

Q:5)

a) $$\int_{0}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y \, dx \, dy$$

x-int, 1, -1

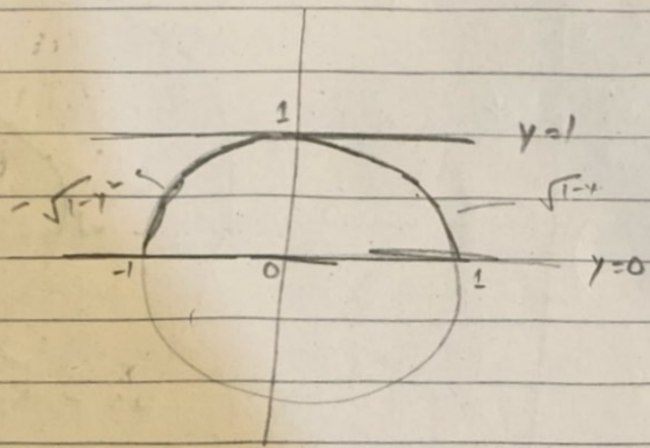
y-int = 0, 1

$y = 0$

$y = 1$

$x = \sqrt{1-y^2}$

$x = -\sqrt{1-y^2}$



$x = \sqrt{1-y^2}$

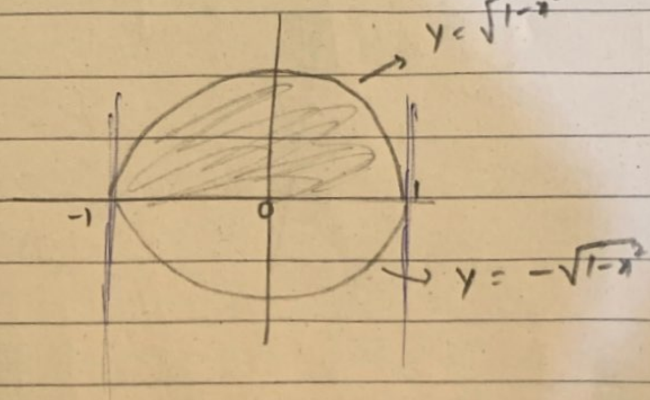
$x^2 = 1-y^2$

$y^2 = 1-x^2$

$y = +\sqrt{1-x^2}$

$y = -\sqrt{1-x^2}$

$$\int_{-1}^{+1}\int_{0}^{\sqrt{1-x^2}} 3y \, dy \, dx$$



$x = 1$, $y = 0$

$x = -1$, $y = 0$

$9-4x^2=0$

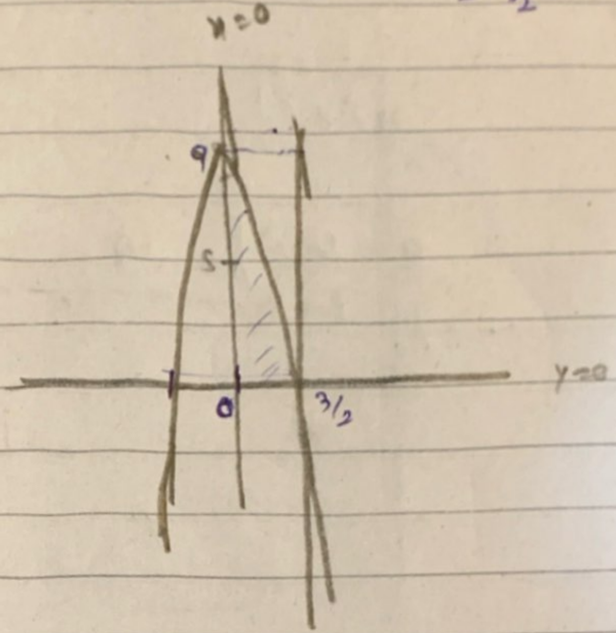
$4x^2=9$

$x^2=9/4$

$(3/2)$

$\pm 3/2$

b) $\displaystyle\int_0^{3/2}\int_0^{9-\cancel{4}x^2} 16x \; dy\, dx.$



$x=0$

$x=3/2$

$y=9-4x^2$

$y=0$

$y=9-\cancel{4}x^2$

$x\text{-int} = \pm 3/2$

$y\text{-int} = 9$

$4x^2=9-y$

$x^2=9/4 - \frac{1}{4}y$

$x = \pm\sqrt{\dfrac{9-y}{4}}$

$x = +\frac{1}{2}\sqrt{9-y}$

$\boxed{x = -\frac{1}{2}\sqrt{9-y}}$

Not included in the region.

$\displaystyle\int_0^{9}\int_0^{+\frac{1}{2}\sqrt{9-y}} 16x \; dx\; dy$

c) $\int_0^1 \int_{1-x}^{1-x^2} dy\,dx$

$\Rightarrow$

$0 \le x \le 1$

$1-x \le y \le 1-x^2$

$x=0$
$\hookrightarrow y=1$
$x=1$
$\hookrightarrow y=0$
range of reversed integral.

lower bound for x.

$y=1-x$

$\boxed{x=1-y}$

upper bound

$y=1-x^2$

$x=\pm\sqrt{1-y}$

$x=+\sqrt{1-y} \rightarrow$ included

$$\int_0^1 \int_{1-y}^{\sqrt{1-y}} dy\,dx$$

AN:

$\frac{\pi}{4} - \frac{\pi}{8}$

$\frac{2\pi - \pi}{8} = \frac{\pi}{8}$

Q:6) arc of lemniscate

$r^2 = 4\cos 2\theta$

$r = \sqrt{4\cos 2\theta}$

$r = 2\sqrt{\cos 2\theta}$

1. since there is r^2

so $r^2 \geq 0$

for this

$\cos 2\theta \geq 0$.

	θ	r
$-0,$	0	2
$-\pi/6$	$\pi/6$	1.86
$-\pi/4$	$\pi/4$	0
	$\pi/3$	$\times$ - iota

complex

$\cos 2\theta$ is positive in the range $\left(-\pi/4 \text{ to } +\pi/4\right)$

limit

$$A = \frac{1}{2}\int_{-\pi/4}^{\pi/4} r^2 \, d\theta$$

$$A = \frac{1}{2}\int_{-\pi/4}^{\pi/4} 4\cos 2\theta \, d\theta$$

$A = \frac{4}{2}$ let $u = 2\theta$

$\frac{du}{d\theta} = 2$

$\frac{du}{2} = d\theta$

$\theta = -\pi/4 \quad u = -\pi/2$

$\theta = \pi/4 \quad u = \pi/2$

$$A = \frac{4}{2}\int_{-\pi/2}^{\pi/2} \cos u \, \frac{du}{2}$$

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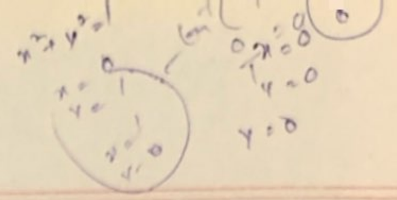


$$A = 1\left[-\sin u\right]_{-\pi/2}^{\pi/2}$$

$$A = -\sin(-\pi/2) - (-\sin(\pi/2))$$

$$A = +\sin(\pi/2) - (-\sin(\pi/2))$$

$$A = 1 - (-1)$$

$$1 + 1$$

$$A = 2$$

Q:7) Evaluate

a) $$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$$

$x = \sqrt{1-y^2} \Rightarrow x^2+y^2 = 1$

$r^2 = 1$

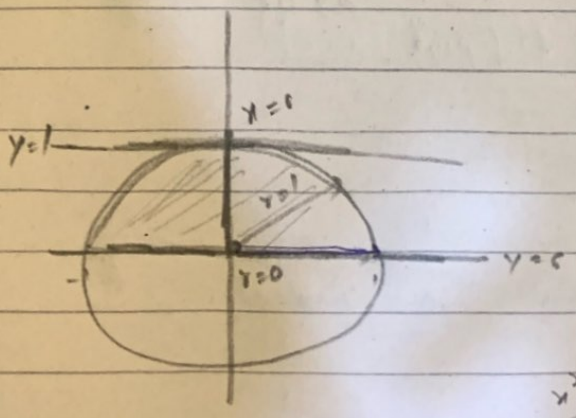
$\boxed{r = 1}$

$x = 0$

$y = 0$

$y = 1$

r range from 0 to 1



$x^2+y^2 \leq 1$

$r^2 \leq 1$ — $\theta = 0$ to $\pi/2$

$$\int_{\theta=0}^{\theta=\pi/2} \int_{r=0}^{r=1} r^2 \; r\,dr\,d\theta$$

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$$\int_0^{\pi/2}\int_0^1 r^3 \, dr \, d\theta$$

$$\int_0^{\pi/2} \left[\frac{r^4}{4}\right]_0^1 d\theta$$

$$\int_0^{\pi/2} \frac{1}{4} - 0 \, d\theta$$

$$\frac{1}{4}\int_0^{\pi/2} d\theta$$

$$\frac{1}{4}\left[\theta\right]_0^{\pi/2}$$

$$\frac{1}{4}\left[\pi/2 - 0\right]$$

$$\pi/8 \text{ Ans}$$

b) $$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}} \, dy \, dx$$

$y = 0$

$y = -\sqrt{1-x^2} \Rightarrow y^2 + x^2 = 1$

$x = 0 \qquad r^2 = 1$

$x = -1 \qquad r = 1$

$\theta = 3\pi/2$ to $\theta = \pi$

x ranges from -1

$$\int_{\pi}^{3\pi/2}$$

$$\int_{\pi}^{3\pi/2} 2$$

$$\frac{1+r}{1+r} + \frac{1}{1+r}$$

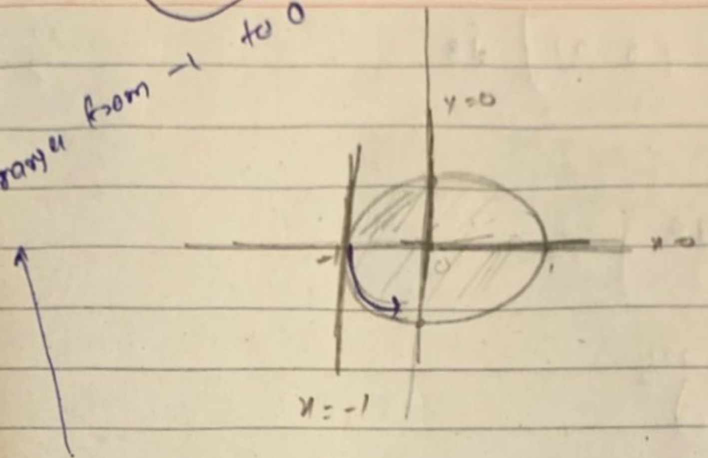
$x = r\cos\theta$

$r^{-1} = \pi$ — moving counterclockwise to 0

$\frac{-1}{\pi}$ to $3\pi/2$

$\frac{3 \times 180}{2}$ 270

x ranges from -1 to 0



$$\int_{\theta=\pi}^{\theta=3\pi/2} \int_0^1 \frac{2}{1+\sqrt{r^2}} \cdot r \, dr \, d\theta$$

$$\int_{\pi}^{3\pi/2} \int_0^1 \frac{2r}{1+r} \, dr \, d\theta$$

$$\int_{\pi}^{3\pi/2} 2 \int_0^1 \frac{r}{1+r} \, dr \, d\theta$$

$$\cancel{\int_{\pi}^{3\pi/2} 2 \int_0^1 1 - \frac{1}{1+r}}$$

$$\int_{\pi}^{3\pi/2} 2 \int_0^1 1 - \frac{1}{1+r} \, dr \, d\theta$$

$$\int_{\pi}^{3\pi/2} 2 \left[r - \ln|1+r| \right]_0^1 \, d\theta$$

$\frac{3\pi}{2} - \pi$

$\pi\left(\frac{3}{2} - 1\right)$

$\pi\left(\frac{3-2}{2}\right)$

$\pi/2$

$$\int_{\pi}^{3\pi/2} 2\left[1 - \ln(2) - (0-0)\right] d\theta$$

$$\int_{\pi}^{3\pi/2} 2\left[1 - \ln 2\right] d\theta$$

$$2\left[1-\ln 2\right] \theta \Big|_{\pi}^{3\pi/2}$$

$$2\left[1-\ln 2\right]\left[3\pi/2 - \pi\right]$$

$$2\left[1-\ln(2)\right]\left[\pi/2\right]$$

$$2\pi/2 - (\ln 2)\pi/2$$

Ans

e) $$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\, dy\, dx$$

$x = -1$ $\quad y = -\sqrt{1-x^2}$

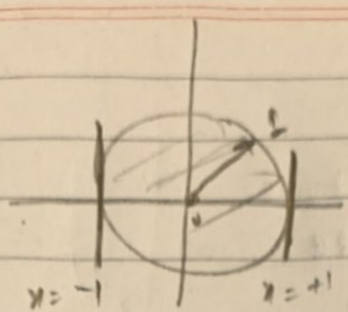
$x = +1$ $\quad y = +\sqrt{1-y^2}$

$y^2 = 1 + x^2$

$\boxed{x^2 + y^2 = 1}$

$r^2 = 1$

$\boxed{r = 1}$



$$\int_{\theta=\pi}^{\theta=2\pi}\int_{0}^{1} \frac{2}{(1+r^2)^2}\, r\,dr\,d\theta$$

$$\int_{\pi}^{2\pi}\int_{0}^{1} \frac{2}{(1+r^2)^2}\, r\,dr\,d\theta$$

Let $u = 1+r^2$

$\frac{du}{d\theta} = 2r$

$du = 2r\,d\theta$

$r = 0 \rightarrow u = 1$

$r = 1 \rightarrow u = 2$

$$\int_{\pi}^{2\pi}\int_{1}^{2} \frac{du}{\cancel{1+}(u)^2}\, d\theta$$

$$\int_{\pi}^{2\pi} \left[\frac{u^{-1}}{-1} \right]_{1}^{2} d\theta$$

$$\int_{\pi}^{2\pi} \left(-\frac{1}{4r}\right)_1^2 d\theta$$

$$\int_{\pi}^{2\pi} \left[-\frac{1}{2} - \left(-\frac{1}{1}\right)\right] d\theta$$

$$\int_{\pi}^{2\pi} \left[-\frac{1}{2} + 1\right] d\theta$$

$$\int_{\pi}^{2\pi} \frac{1}{2}\, d\theta$$

$$\frac{1}{2}\int_{\pi}^{2\pi} d\theta$$

$$\cancel{\frac{1}{2}\int} \quad \frac{1}{2}\left[\theta\right]_{\pi}^{2\pi}$$

$$\frac{1}{2}\left[2\pi - \pi\right]$$

$$\frac{\pi}{2}$$

AN.