* Ex: 14.1 (1-16)

1) $\int_0^1\int_0^2 (x+3)\,dy\,dx$

$\int_0^1\left[\int_0^2 (x+3)\,dy\right]dx \to (x+3)y\Big|_0^2 \to 2(x+3)-0$

$2\int_0^1 (x+3)\cdot dx \to \frac{\cancel{2}x^2}{\cancel{2}} + \overset{(2)}{3x}\Big|_0^1 \Rightarrow 1+\overset{6}{\cancel{3}}(1)-0 = \cancel{4}7$

2) $\int_{1}^3\int_{-1}^1 (2x-4y)\cdot dy\cdot dx$

$\int_1^3\left[\int_{-1}^1 (2x-4y)\cdot dy\right]\cdot dx = 2xy - \frac{4y^2}{2} = 2xy-2y^2\Big|_{-1}^1$

$= 2x-\cancel{2}+2x+\cancel{2} \Rightarrow 4x$

$\int_1^3 4x\cdot dx \to \frac{\overset{2}{\cancel{4}}x^2}{\cancel{2}} \to 2x^2\Big|_1^3 \Rightarrow 18-2=16$

3) $\int_2^4\int_0^1 x^2y\cdot dx\cdot dy$

$\cdot \int_0^1 x^2y\cdot dx \to y\frac{x^3}{3}\Big|_0^1 \Rightarrow \frac{y}{3}-0=\frac{y}{3}$

$\cdot \frac{1}{3}\int_2^4 y\,dy \to \frac{1}{3}\frac{y^2}{2}\Big|_2^4 \Rightarrow \frac{1}{6}(4)^2-\frac{1}{6}(2)^2 \Rightarrow \frac{16-4}{6} \Rightarrow \frac{\overset{2}{\cancel{12}}}{\cancel{6}}$

$\Rightarrow 2$

4) $\int_{-2}^0\int_{-1}^2 (x^2+y^2)\,dx\cdot dy$

$\cdot \int_{-1}^2 (x^2+y^2)\cdot dx \Rightarrow \frac{x^3}{3}+y^2x\Big|_{-1}^2 \Rightarrow \left(\frac{8}{3}+2y^2\right)-\left(\frac{-1}{3}+-y^2\right)$

$\frac{8}{3}+\frac{1}{3}+2y^2+y^2 = \frac{\overset{3}{\cancel{9}}}{\cancel{3}}+3y^2 \Rightarrow 3+3y^2$

$\int_{-2}^0 3+3y^2\cdot dy \to 3y+\frac{\cancel{3}y^3}{\cancel{3}}\Big|_{-2}^0 \Rightarrow -(3(-2)+(-2)^3)$

$\Rightarrow -(-6-8) \Rightarrow 14$

$\rightarrow \frac{(2y+1)^{-2+1}}{-1}$

5) $\int_0^{\ln 3}\int_0^{\ln 2} e^{x+y}\, dy \cdot dx$

$\int_0^{\ln 2} (e^{x+y}) \cdot dy \Rightarrow e^{x+y}\Big|_0^{\ln 2} \Rightarrow e^{x+\ln 2} - e^{x}$

$\int_0^{\ln 3} e^{x+\ln 2} - e^{x} \cdot dx \Rightarrow e^{x+\ln 2} - e^{x}\Big|_0^{\ln 3}$

$e^{\ln 3+\ln 2} - e^{\ln 3} - e^{0+\ln 2} + e^{0}$

$e^{\ln 3}\cdot e^{\ln 2} - e^{\ln 3} - e^{0}e^{\ln 2} + e^{0}$

$(3)(2) - (3) - (1)(2) + 1 = 6-3-2+1 \Rightarrow 7-5=2$

6) $\int_0^{2}\int_0^{1} y\sin x \cdot dy \cdot dx$

$\int_0^{1} y\sin x \cdot dy \rightarrow \sin x\left(\frac{y^2}{2}\right)\Big|_0^{1} \rightarrow \frac{1}{2}\sin x - 0 = \frac{1}{2}\sin x$

$\frac{1}{2}\int_0^{2} \sin x \cdot dx \rightarrow \frac{1}{2}(-\cos x)\Big|_0^{2} \Rightarrow \frac{1}{2}(-\cos(2)) - \frac{1}{2}(-\cos(0$

$= -\frac{1}{2}\cos(2) + \frac{1}{2}(1) \Rightarrow \frac{1}{2}(1-\cos 2)$

7) $\int_{-1}^{0}\int_{2}^{5} dx\, dy$

$\int_2^{5} 1 \cdot dx \rightarrow x\Big|_2^{5} \Rightarrow 5-2 \Rightarrow 3$

$\int_{-1}^{0} 3 \cdot dy \rightarrow 3y\Big|_{-1}^{0} \Rightarrow 3(0) - 3(-1) \Rightarrow 3$

8) $\int_4^{6}\int_{-3}^{7} dy \cdot dx$

$\int_{-3}^{7} 1 \cdot dy \rightarrow y\Big|_{-3}^{7} \Rightarrow 7-(-3) = 10$

$\int_4^{6} 10 \cdot dx \rightarrow 10x\Big|_4^{6} \Rightarrow 60 - 40 = 20$

9) $\int_0^{1}\int_0^{1} \frac{x}{(xy+1)^2} \cdot dy \cdot dx \rightarrow x(xy+1)^{-2} \rightarrow \frac{(\cancel{2}y+1)^{-2+1}}{-1}\Big|_0^{1}$

$-(\cancel{2}+1)^{-1} \rightarrow -1(1)^{-1} \Rightarrow -\frac{1}{\cancel{3}} - (x+1)^{-1} + 1$

$$\int_0^1 \left(1 - \frac{1}{(x+1)}\right) \cdot dx \to x - \ln(x+1)\Big|_0^1 \Rightarrow (1 - \ln 2) - 0 + \ln 1$$
$$= 1 - \ln 2$$

10) $\int_{\pi/2}^{\pi} \int_1^2 x\cos(xy) \cdot dy \cdot dx$

$$\int x\cos(xy) \cdot dy \to \sin(xy)\Big|_1^2 \Rightarrow \sin(2x) - \sin(x)$$

$$\int_{\pi/2}^{\pi} \sin 2x - \int_{\pi/2}^{\pi} \sin x \to \left|\frac{1}{2}(-\cos(2x)) + \cos x\right|_{\pi/2}^{\pi}$$

$$-\frac{1}{2}\cos(2\pi) + \cos(\pi) + \frac{1}{2}\cos(\pi) - \cos(\pi/2)$$

$$-\frac{1}{2}(1) + (-1) + \frac{1}{2}(-1) - 0 \Rightarrow -2$$

11) $\int_0^{\ln 2} \int_0^1 xy e^{y^2 x} dy \cdot dx$

$$\left(\frac{1}{2}\right) \int_0^1 xy e^{y^2 x} (2) \cdot dy \to \frac{1}{2} e^{y^2 x}\Big|_0^1 \Rightarrow \frac{1}{2} e^x - \frac{1}{2} e^0$$
$$\Rightarrow \frac{1}{2} e^x - \frac{1}{2}$$

$$\int_0^{\ln 2} \frac{1}{2} e^x - \frac{1}{2} x \to \frac{1}{2} e^x - \frac{1}{2} x \Big|_0^{\ln 2} \to \frac{1}{2} e^{\ln 2} - \frac{1}{2}(\ln 2) - \frac{1}{2} e^0 + 0$$

$$\Rightarrow \cancel{\frac{1}{2}(2)} - \frac{1}{2}\ln 2 - \frac{1}{2} \Rightarrow \frac{1}{2} - \frac{1}{2}\ln 2 \Rightarrow \frac{1}{2}[1 - \ln 2]$$

12) $\int_2^4 \int_1^2 \frac{1}{(x+y)^2} \cdot dy \cdot dx$

$$\int_1^2 (x+y)^{-2} \cdot dy \to \frac{(x+y)^{-2+1}}{-1}\Big|_1^2 = -(x+y)^{-1}\Big|_1^2$$

$$-(x+2)^{-1} + (x+1)^{-1}$$

$$-\int_3^4 \frac{1}{(x+2)}\cdot dx + \int_3^4 \frac{1}{(x+1)}\cdot dx$$

$$\left. -1\ln(x+2) + 1\ln(x+1) \right|_3^4 \Rightarrow -\ln(6) + \ln(5) + \ln(\cancel{4}^{5}) - \ln(\tfrac{4}{3})$$

$$= \ln(\cancel{30}^{25}) + -\ln(\cancel{18}^{24})$$

$$= \ln\left(\frac{\cancel{20}}{\cancel{18}}\right) + \cancel{\ln\left(\frac{10}{9}\right)}\ \ln\left(\frac{25}{24}\right)$$

$$\iint 4xy^3 \cdot dA \quad R = \{-1 \le x \le 1, -2 \le y \le 2\}$$

$$\int 4xy^3 \cdot dy \rightarrow 4x\int y^3 \cdot dy \rightarrow \frac{\cancel{4}xy^4}{\cancel{4}} \rightarrow \left. xy^4 \right|_{-2}^{2} \Rightarrow 16x - 16x \Rightarrow 0$$

$$\iint \frac{xy}{\sqrt{x^2+y^2+1}} \cdot dA \rightarrow R = \{0 \le x \le 1, 0 \le y \le 1\}$$

$$\frac{y}{2}\int (2)x(x^2+y^2+1)^{-1/2} \cdot dx \rightarrow \frac{y}{2}\left.(x^2+y^2+1)^{1/2}\right|_0^1$$

$$= \frac{1}{2}y(xy^2+2)^{1/2} - 0 \Rightarrow \frac{1}{2}(xy^2+2)^{1/2}\Big|$$

$$\frac{1y}{2(2)}\int (xy^2+2)^{1/2} \cdot dy\,(2) \Rightarrow \frac{1}{4}\left.(y^2+2)^{3/2}\right|_0^1 \Rightarrow$$

Ex: 14.2

Q1) $\int_0^1\int_{x^2}^{x} xy^2\,dy\,dx \to \frac{y^3}{3}x\Big|_{x^2}^{x}\cdot dx\Big|_0^1 \to \int\frac{1}{3}(x^4-x^7)\cdot dx$

$= \frac{1}{3}\int x^4\cdot dx - \frac{1}{3}\int x^7\cdot dx$

$= \frac{1}{3}\frac{x^5}{5} - \frac{1}{3}\frac{x^8}{8} \Rightarrow \Big|\frac{1}{15}x^5 - \frac{1}{24}x^8\Big|_0^1 \Rightarrow \frac{1}{15} - \frac{1}{24} = \frac{1}{40}$

Q2) $\int_1^{3/2}\int_y^{3-y} y\cdot dx\,dy \to \int yx\Big|_y^{3-y}\cdot dx \to y(3-y) - y(y)$

$3y - y^2 - y^2$

$\int_1^{3/2} 3y\cdot dy - \int_1^{3/2} 2y^2\cdot dy$

$= \Big|\frac{3y^2}{2} - \frac{2y^3}{3}\Big|_1^{3/2} \Rightarrow \frac{3}{2}(3/2)^2 - \frac{2}{3}(3/2)^3 - \frac{3}{2}(1) + \frac{2}{3}(1)$

$= \frac{7}{24}$

Q3) $\int_0^3\int_0^{\sqrt{9-y^2}} y\,dx\,dy \to \int_0^3 yx\Big|_0^{\sqrt{4-y^2}}\cdot dy \Rightarrow y(9-y^2)^{1/2}$

$\frac{1}{2}\int_0^3 y(9-y^2)^{1/2}\cdot dy\ (2) \to \frac{1}{2}\frac{(9-y^2)^{1/2+1}}{3/2} \to \frac{2}{3}(9-y^2)^{3/2}\frac{1}{2}$

$\frac{1}{2}\times\left[\frac{2}{3}(9-9)^{3/2} - \frac{2}{3}(9-0)^{3/2}\right] \Rightarrow \frac{2}{3}\frac{(9)^{3/2}}{2} \to \frac{\cancel{27}^{\,9}\times\cancel{2}}{\cancel{2}\ \cancel{3}} \to 9$

Q4) $\int_{1/4}^{1}\int_{x^2}^{x}\left(\frac{x}{y}\right)^{1/2}\cdot dy\cdot dx \to \cancel{\int_{x^2}^{x} xy^{-1}\,dy \to x\ln y\Big|_{x^2}^{x}}$

$= (xy^{-1})^{1/2}\cdot dy \to \int_{x^2}^{x} x^{1/2}y^{-1/2}\cdot dy \to x^{1/2}\frac{y^{1/2}}{1/2}\Big|_{x^2}^{x}$

$2x^{1/2}x^{1/2} - 2x^{1/2}x \Rightarrow 2x - 2x^{5/2}$

$-\frac{1}{x}\cos\left(\frac{y}{x}\right)\Big|_0 \to x - \frac{1}{x}\cos\left(\frac{x}{x}\right) \ldots \frac{1}{x}\cos(x^2)$

$= \frac{\cancel{2}x^2}{\cancel{2}} - \frac{2x^{5/2}}{5/2} \Rightarrow x^2 - \frac{4}{5}x^{5/2} \Rightarrow 1 - \frac{4}{5} - \left(\frac{1}{4}\right)^2 + \frac{4}{5}\left(\frac{1}{4}\right)^{5/2}$

$\Rightarrow \frac{13}{80}$

Q5) $\int_{\sqrt{\pi}}^{\sqrt{2\pi}}\int_0^{x^3} \sin(y/x)\cdot dy\cdot dx$

$u = \frac{y}{x} \to \frac{du}{dy} = \frac{1}{x} \to \frac{du}{dy} = \frac{1}{x} \to dy = x\,du$

$\iint x\sin(u)\cdot du\cdot dx \to \cancel{\int \frac{x}{\cancel{x}}\sin(u)\cdot\frac{1}{\cancel{x}}\cdot dx}$

$-\int x\cos u\Big|_0^{x^3} \to -x\cos\left(\frac{y}{x}\right)\Big|_0^{x^3} \to -x\cos\left(\frac{x^3}{x}\right) + x$

$-\frac{1}{2}\int x\cos(x^2)^{(2)} + \int x \to -\frac{1}{2}\sin(x^2) + \frac{x^2}{2}\Big|_{\sqrt{\pi}}^{\sqrt{2\pi}}$

$= \cancel{-\frac{1}{2}\sin(2\pi)} + \frac{\cancel{2}\pi}{\cancel{2}} \cancel{+ \frac{1}{2}\sin(\pi)} - \frac{\pi}{2} \Rightarrow \frac{\pi}{2}$

Q6) $\int_{-1}^{1}\int_{-x^2}^{x^2}(x^2-y)\,dy\,dx$

$\left|x^2y - \frac{y^2}{2}\right|_{-x^2}^{x^2} \to \cancel{\frac{1}{2}(x^4 - x^4)} = x^4 - \frac{x^4}{2} + x^4 + \frac{x^4}{2}$

$= 2x^4$

$2\int_{-1}^{1}x^4 \to \frac{2}{5}x^5\Big|_{-1}^{1} \to \frac{2(1)}{5} - \frac{2(-1)}{5} = \frac{4}{5}$

Q7) $\int_0^1\int_0^x y\sqrt{x^2-y^2}^{\,-(2)}\cdot dy\cdot dx \to -\frac{1}{\cancel{2}}\frac{(x^2-y^2)^{3/2}}{3/\cancel{2}}\Big|_0^x$

$-\frac{1}{3}(x^2-y^2)^{3/2}\Big|_0^x \to -\frac{1}{3}(0)^{3/2} + \frac{1}{3}(x^2)^{3/\cancel{2}} = \frac{1}{3}x^3$

$$\int_{const}^{const}\int_{f(x)}^{f(x)} \cdot dx\,dy$$

$$\frac{1}{3}\int_0^1 x^3 \cdot dx \to \frac{x^4}{4(3)}\Big|_0^1 \to \frac{x^4}{12}\Big|_0^1 \to \frac{1}{12}$$

8) $\int_1^2\int_0^{y^2} e^{x/y^2} \cdot dx\,dy \to e^{xy^{-2}}$

$xy^{-2} = u \to \frac{du}{dx} = y^{-2}(1) \to du = y^{-2} \cdot dx$

$$\int_1^2\int_0^{y^2} e^{x/y^2} \cdot \frac{du}{y^{-2}} \cdot dy \to y^2 e^u \cdot du \cdot dy$$

$$y^2 e^u \to y^2 e^{x/y^2}\Big|_0^{y^2} \to y^2 e^{\frac{y^2}{y^2}} - y^2 e^0$$

$$= y^2 e^{\frac{y^2}{y^2}} - y^2(1)$$

$$\int_1^2 y^2 e - \int_1^2 y^2 \Rightarrow e\frac{y^3}{3}\Big|_1^2 - \frac{y^3}{3}\Big|_1^2 \Rightarrow \frac{e}{3}\left(\frac{8}{3}\right) - \frac{8}{3} \mp \frac{e}{3} + \frac{1}{3}$$

$$\Rightarrow \frac{8e}{3} - \frac{e}{3} - \frac{8}{3} + \frac{1}{3} = \frac{7e}{3} - \frac{7}{3}$$

$$\Rightarrow \frac{7(e-1)}{3}$$

Q:9) a) $\int_0^2\int_0^{x^2} \cdot dy\,dx$ b) $\int_0^4\int_{\sqrt{y}}^2 \cdot dx\,dy$

Q10) a) $\int_0^1\int_{x^2}^{\sqrt{x}} \cdot dy\,dx$ b) $\int_0^1\int_{y^2}^{\sqrt{y}} \cdot dx\,dy$

$y = x^2$

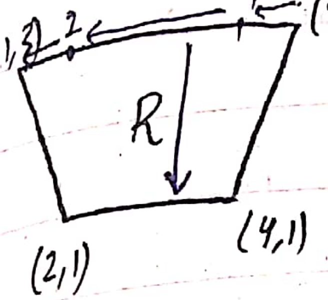
$x = \sqrt{y}$

$x^2 = \sqrt{x}$

$x^4 = x \to x^4 - x = 0$

$x(x^3 - 1) \to x = 0, 1, -1$

Q12) $(1,3)$ 2 4 $(5,3)$



a) $\int_1^2\int_{-2x+5}^{3} \cdot dydx + \int_2^4\int_1^3 \cdot dydx + \int_4^5\int_{2x-7}^{3} \cdot dydx$

① $(1,3),(2,1) \to y-3=-2(x-1)$

$m = \frac{3-1}{1-2} = \frac{2}{-1}$ $\quad y=-2x+5$

② Ez tha

③ $(5,3)(4,1) \to y-1=2(x-4)$

$m = \frac{1-3}{4-5} = \frac{\cancel{-}2}{\cancel{-}1}$ $\quad y=2x-7$

Q12)

b) $\int_1^3\int_{y-5/2}^{y+7/2} \cdot dxdy$

① $y=-2x+5$

$\frac{y-5}{-2}=x$

② $y=2x-7$

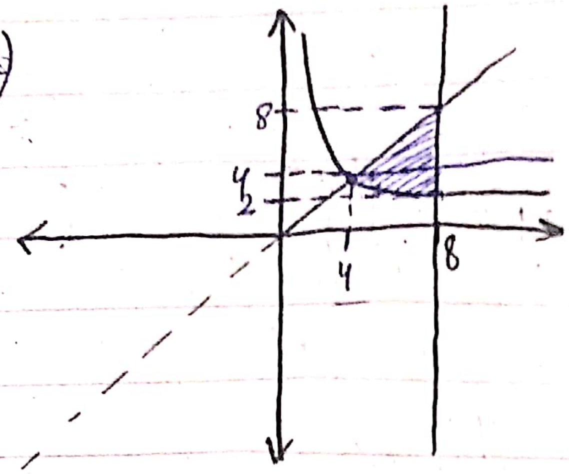
$\frac{y+7}{2}=x$

Q12) a) $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \cdot dydx$

b) $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}$

Q15) $\iint x^2 \cdot dA$, $y=\frac{16}{x}$, $y=x$, $x=8$

a)



$y=\frac{16}{8}\to 2$

$x^2=16\to\pm4$

$y=8$

a) $\int_4^8\int_{16/x}^{x} x^2\cdot dy\,dx \to x^2y\Big|_{16/x}^{x} \to \int x^3 - \int 16x$

$\left(\frac{x^4}{4} - \frac{\cancel{16}^{8}x^2}{\cancel{2}}\right)\Big|_4^8 \Rightarrow \frac{(8)^4}{4} - 8(8)^2 - \frac{(4)^4}{4} + 8(4)^2$

≈ 576

b) $\int_2^4\int_{16/y}^{8} x^2 \cdot dx\,dy + \int_4^8\int_y^8 x^2\cdot dx\,dy$

$\frac{x^3}{3}\Big|_{16/y}^{8} \to \frac{512}{3} - \frac{4096/y^3}{3}$

$\int_2^4 \frac{512}{3}\cdot dy - \frac{4096}{3}\int_2^4 y^{-3}\cdot dy \Rightarrow \frac{512}{3}y - \frac{4096}{3}\frac{y^{-2}}{-2}$

$\frac{512}{3}(4-2) - \frac{4096}{-6}\left(\frac{1}{16}-\frac{1}{4}\right) \Rightarrow \frac{640}{3}$

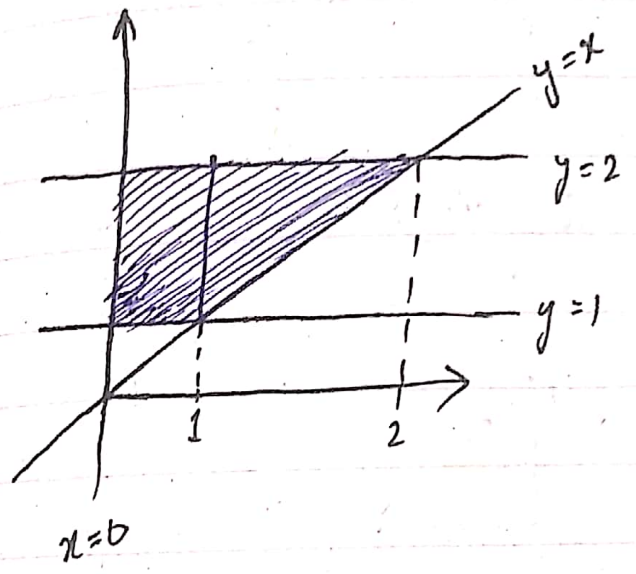
$\frac{x^3}{3}\Big|_y^8 \to \frac{512}{3} - \frac{y^3}{3}$

$\int \frac{512}{3}\cdot dy - \frac{1}{3}\int y^3\cdot dy \Rightarrow \left(\frac{512}{3}y - \frac{1}{3}\frac{y^4}{4}\right)\Big|_4^8$

$\frac{512}{3}(8-4) - \frac{1}{12}(4096-256) \Rightarrow \frac{2048}{3} - 320 \Rightarrow \frac{1088}{3}$

$\Rightarrow \frac{640}{3} + \frac{1088}{3} \Rightarrow 576$

Q16) $\iint xy^2 dA$, $y=1, y=2, x=0, y=x$



a) $\iint xy^2\cdot dydx \to \int_0^1\int_1^2 xy^2\cdot dydx + \int_1^2\int_x^2 xy^2\cdot dydx$

$\frac{xy^3}{3}\Big|_1^2 \Rightarrow \frac{8x}{3} - \frac{x}{3} \Rightarrow \int \frac{7}{3}x\cdot dx \to \frac{7x^2}{3\,2} \to \frac{7}{6}(1-0) = \frac{7}{6}$

$\frac{x}{3}y^3\Big|_x^2 \to \frac{x}{3}(8) - \frac{x^4}{3} \Rightarrow \int_1^2 \frac{8}{3}x - \int_1^2 \frac{x^4}{3}$

$\cancel{\frac{8x^2}{3\,2} - \frac{1}{3} = \frac{8}{3} - \frac{1}{3} \Rightarrow \frac{7}{3}}$

$= \frac{8}{3}\frac{x^2}{2} - \frac{1}{3}\frac{x^5}{5} \Rightarrow \frac{8}{6}x^2 - \frac{1}{15}x^5 \Rightarrow \frac{4}{3}x^2 - \frac{1}{15}x^5$

$\frac{4}{3}(2^2-1) - \frac{1}{15}(2^5-1^5) \Rightarrow \frac{4(3)}{3} - \frac{1}{15}(32-1)$

$\Rightarrow \frac{29}{15}$

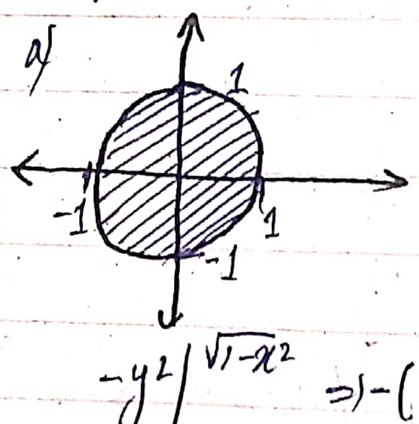
$\frac{29}{15} + \frac{7}{6} = \frac{31}{10}$

b) $\int_1^2\int_0^y xy^2 \cdot dx\,dy \Rightarrow y^2\frac{x^2}{2}\Big|_0^y \Rightarrow \frac{y^4}{2} - 0$

$\frac{1}{2}\frac{y^5}{5} \to \frac{1}{10}y^5\Big|_1^2 \Rightarrow \frac{32}{10} - \frac{1}{10} \Rightarrow \frac{31}{10}$

Q17) $\iint 3x - 2y \cdot dA$, $x^2+y^2=1$

a)



$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3x - 2y \cdot dy\,dx \Rightarrow -\frac{\cancel{2}y^2}{\cancel{2}}$

$-y^2\Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \Rightarrow -(1-x^2) + (1-x^2) \Rightarrow \cancel{-1}\cancel{+x^2}\cancel{+1}\cancel{-x^2} = 0$

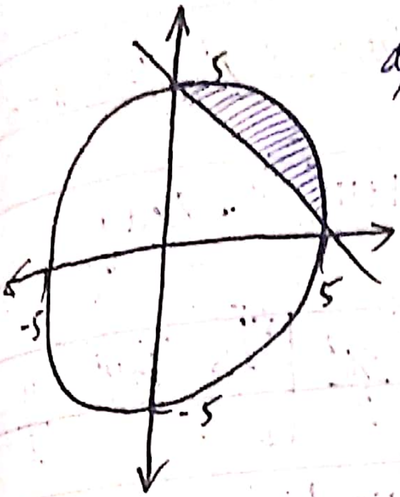
b) $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3x - 2y \cdot dx\,dy \Rightarrow \frac{3x^2}{2}$

$\cancel{\frac{3}{2}(1-y^2)} - \cancel{\frac{3}{2}(1-y^2)} = 0$

$$-\cancel{\frac{25}{2}} + \frac{10x}{2} - \frac{x^2}{2} + \cancel{\frac{25}{2}} - \frac{x^2}{2}$$

$$5x - x^2$$

8.18) $\iint\limits_{x+y=5} y\,dA$, $x^2+y^2=25$ (only 1st Quadrant) &



a) $\displaystyle\int_0^5\int_{5-x}^{\sqrt{25-x^2}} y\cdot dy\,dx \to \frac{y^2}{2}\Bigg|_{5-x}^{\sqrt{25-x^2}}$

$$\frac{1}{2}(25-x^2) - \frac{1}{2}(5-x)^2$$

$$-(5-x^2)^2/2$$

$$\frac{25}{2} - \cancel{\frac{8}{2}} - \frac{x^2}{2} + \cancel{\frac{x}{2}} \Rightarrow 10 - \frac{x^2}{2} + \frac{x}{2}$$

$$\cancel{10x - \frac{1}{2}\frac{x^3}{3} + \frac{1}{2}\frac{x^2}{2} \to \left(10x - \frac{1}{6}x^3 + \frac{1}{4}x^2\right)\Bigg|_0^5}$$

$$\cancel{10(5) - \frac{1}{6}(5)^3 + \frac{1}{4}(5)^2 - 0}$$

$$\cancel{\frac{25}{2}x - \frac{x^3}{6} - \frac{1}{26}(5-x)^3\Bigg|_0^5 \Rightarrow \frac{25}{2}(5) - \frac{(5)^3}{6} - \frac{1}{6}(0) - 0 - 0}$$

$$+\frac{1}{6}(5)^3$$

$$\int_0^5 5x\cdot dx - \int_0^5 x^2\cdot dx$$

$$\frac{5x^2}{2} - \frac{x^3}{3}\Bigg|_0^5 \Rightarrow \frac{5}{2}(5-0)^2 - \frac{(5-0)^3}{3} \Rightarrow \frac{125}{2} - \frac{125}{3} \Rightarrow \frac{125}{6}$$

b) $\displaystyle\int_0^5\int_{5-y}^{\sqrt{25-y^2}} y\cdot dx\,dy \to yx\Bigg|_{5-y}^{\sqrt{25-y^2}} \Rightarrow y\left(\sqrt{25-y^2} - 5 + y\right)$

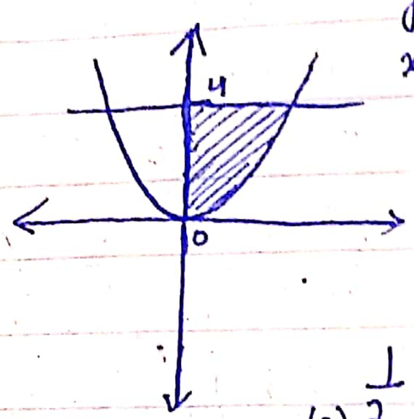
$$-\frac{1}{2}\int y(25-y^2)^{1/2}(-2)\,dy - 5\int y\,dy + \int y^2\,dy$$

$$-\frac{1}{2}\frac{(25-y^2)^{3/2}}{3/2} - \frac{5y^2}{2} + \frac{y^3}{3} \Rightarrow -\frac{(25-y^2)^{3/2}}{3} - \frac{5y^2}{2} + \frac{y^3}{3}\Bigg|_0^5$$

$$\cancel{= 0 - 25 + \frac{125}{2} - \frac{125}{3} - 0 + 0 \Rightarrow}$$

$$= 0 - \frac{125}{2} + \frac{125}{3} + \frac{125}{3} + 0 + 0 \Rightarrow \frac{125}{6}$$

Q19) $\iint x(1+y^2)^{-1/2}\, dA$, first Quadrant $y=x^2$, $y=4$, $x=0$



$y = x^2$

$x = \sqrt{y}$ $\quad \int_0^4 \int_0^{\sqrt{y}} x(1+y^2)^{-1/2} \, dx\, dy$

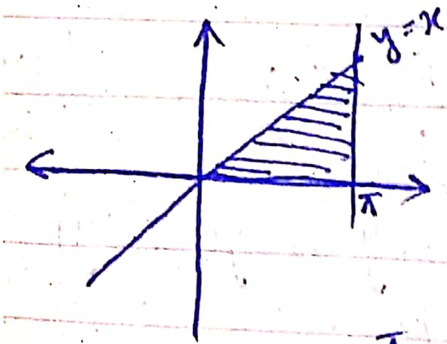
$$(1+y^2)^{-1/2} \frac{x^2}{2}\Big|_0^{\sqrt{y}} \Rightarrow (1+y^2)^{-1/2} \frac{y}{2} - 0$$

$$\frac{(2)}{(2)}\frac{1}{2}\int_0^4 y (1+y^2)^{-1/2} (2) \cdot dy \Rightarrow \frac{1}{2 \cdot 4}\frac{(1+y^2)^{1/2}}{1/2}$$

$$= \frac{1}{2}(1+y^2)^{1/2}\Big|_0^4 = \frac{1}{2}(1+16)^{1/2} - \frac{1}{2}(1)$$

$$= \frac{\sqrt{17} - 1}{2}$$

Q20) $\iint x\cos y \, dA$, $y = x$, $y = 0$, $x = \pi$



$$\int_0^{\pi}\int_0^{x} x\cos y \cdot dy\, dx$$

$$x\int_0^x \cos y \cdot dy \rightarrow x \sin y\Big|_0^x \rightarrow x\sin x - 0$$

$$\int_0^{\pi} \underset{u}{\underset{\downarrow}{x}} \, \underset{dv}{\underset{\downarrow}{\sin x \cdot dx}}$$

$u = x$ $\quad \int dv = \int \sin x$

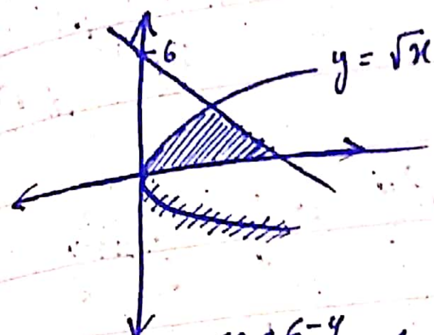
$\frac{du}{dx} = 1$ $\quad v = -\cos x$

$uv - \int v \cdot du \Rightarrow (x)(-\cos x) + \int \cos x \cdot dx$

$(-x\cos x + \sin x)\Big|_0^{\pi} \Rightarrow -\pi\cos\pi + \sin\pi + 0 - 0$

$= -\pi(-1) + 0 = 1 + \pi$

Q21) $\iint xy \, dA$, $y = \sqrt{x}$, $y = 6 - x$, $y = 0$



$\cdot\ 6 - x = \sqrt{x}$

$(6-x)^2 = x \Rightarrow 36 - 2(6)x + x^2 =$

$= 36 - 12x + x^2 = x$

$x = 4,\ x = 9$

$y = 2,\ y = 3$

$= \int_0^2 \int_{y^2}^{6-y} xy \cdot dx\,dy$

$\cdot\ y\int x \cdot dx \rightarrow y\frac{x^2}{2}\Big|_{y^2}^{6-y} = y\frac{(6-y)^2}{2} - y\frac{(y^2)^2}{2}$

$\frac{y}{2}(6-y)^2 - \frac{y^5}{2}$

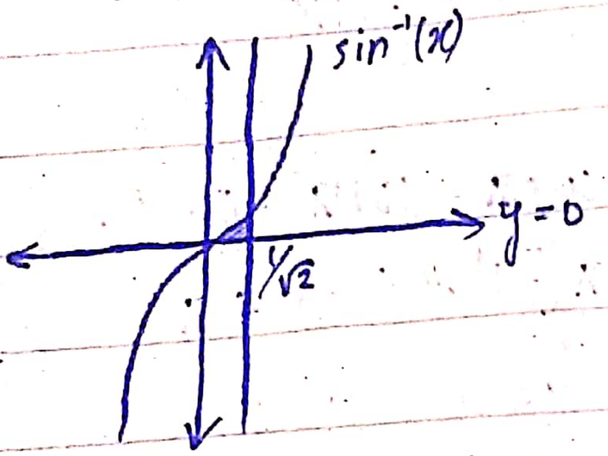
$\cdot\ \frac{y}{2}(36 - 12y + y^2) \Rightarrow 18y - 6y^2 + \frac{1}{2}y^3 - \frac{y^5}{2}$

$\cdot\ 18\int y \cdot dy - 6\int y^2 \cdot dy + \frac{1}{2}\int y^3 - \frac{1}{2}\int y^5$

$= 18\frac{y^2}{2} - 6\frac{y^3}{3} + \frac{1}{2}\frac{y^4}{4} - \frac{1}{2}\frac{y^6}{6} \Rightarrow 9y^2 - 2y^3 + \frac{1}{8}y^4 - \frac{1}{12}y^6$

$= 9(2-0)^2 - 2(2-0)^3 + \frac{1}{8}(2-0)^4 - \frac{1}{12}(2-0)^6 = \frac{50}{3}$

Q22) $\iint x\,dA$, $y=\sin^{-1}(x)$, $x = 1/\sqrt{2}$, $y=0$



$$\int_0^{1/\sqrt{2}}\int_0^{\sin^{-1}(x)} x \cdot dy\,dx$$

$$xy\Big|_0^{\sin^{-1}(x)} \to x\sin^{-1}(x)$$

OR

$\because \sin^{-1}(x) = 1/\sqrt{2}$

$\sin^{-1}(1/\sqrt{2}) = \pi/4$

$$\int_0^{\pi/4}\int_{\sin y}^{1/\sqrt{2}} x \cdot dx\,dy$$

$$\frac{x^2}{2}\Big|_{\sin y}^{1/\sqrt{2}} \to \frac{(1/2)}{2} - \frac{\sin^2 y}{2} \Rightarrow \frac{1}{4} - \frac{1}{2}\sin^2 y$$

$$\cancel{\frac{1}{4}\int dy - \frac{1}{2}\int \sin^2 y \to \left(\frac{1}{4}y + \frac{1}{2}\cos y\right)\Big|_0^{\pi/4} \Rightarrow \frac{1}{4}(\pi/4) + \frac{1}{2}\left(\cos\frac{\pi}{4}\right)}$$

$$\cancel{-0 - \frac{1}{2}\cos(0)}$$

$$\cancel{= \frac{\pi}{8} + \frac{1}{4}\sqrt{2} - \frac{1}{2}(1)}$$

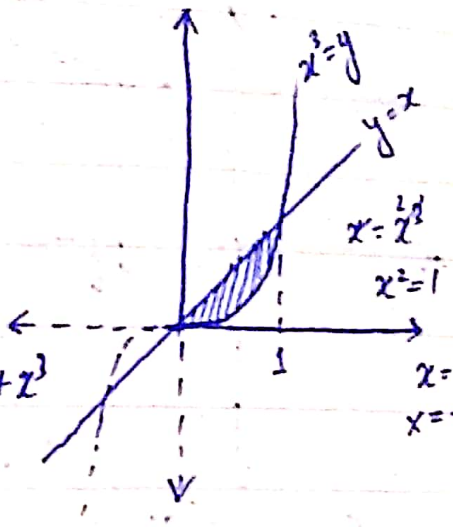
$$\frac{1}{4}\left(1 - 2\sin^2 y\right) \to \frac{1}{4}\cos(2y)\cdot dy$$

$$\frac{1}{4}\int_0^{\pi/4}\cos(2y)\cdot dy \to \frac{1}{4(2)}\,\cancel{\cos}\,\sin(2y) \Rightarrow \frac{1}{8}(1) - 0 \Rightarrow \frac{1}{8}$$

Q23) $\iint (x-1)\cdot dA$, $y=x$, $y=x^3$, in first Quadrant

$\int_0^1\int_{x^3}^{x}(x-1)\cdot dy\,dx \rightarrow$

$\cancel{(x^2 - x)}\Big|_{x^3}^{x} \cancel{\Rightarrow x^2}$

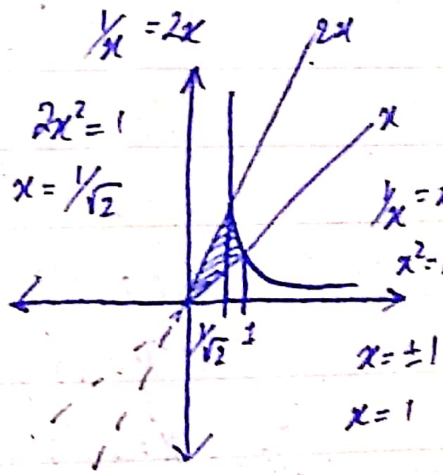


$(xy - y)\Big|_{x^3}^{x} \Rightarrow x(x) - x\cancel{(x^3)} - x(x^3) + x^3$

$-x + x^2 - x^4 + x^3\Big|_0^1$

$-\frac{x^2}{2} + \frac{x^3}{3} - \frac{x^5}{5} + \frac{x^4}{4}\Big|_0^1 \Rightarrow -\frac{1}{2} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} \Rightarrow -\frac{7}{60}$

Q24) $\iint x^2\cdot dA$, $xy=1$, $y=x$, $y=2x$



$\int_0^{1/\sqrt{2}}\int_x^{2x} x^2\cdot dy\,dx + \int_{1/\sqrt{2}}^{1}\int_x^{1/x} x^2\,dy\,dx$

$\frac{xy^{\cancel{3}}}{\cancel{3}}\Big|_x^{2x} \Rightarrow \cancel{\frac{(2x)^3}{3}}\ x^2(2x) - x^2(x)$

$\Rightarrow 2x^3 - x^3 \Rightarrow x^3$

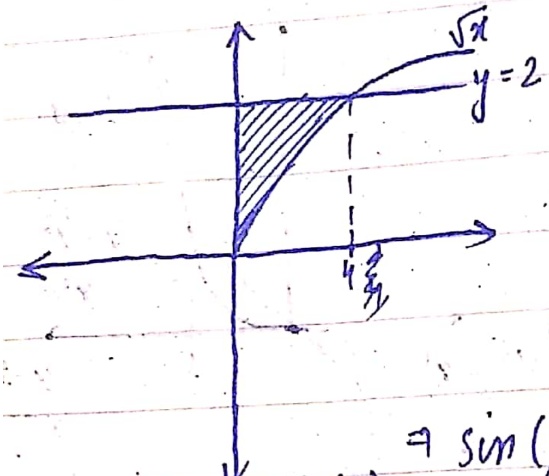
$\int x^3\cdot dx \rightarrow \frac{x^4}{4}\Big|_0^{1/\sqrt{2}} \rightarrow \frac{(1/\sqrt{2})^4}{4} \rightarrow \frac{1}{16}$

$x^2y\Big|_x^{1/x} \rightarrow x^2\left(\frac{1}{x}\right) - x^2x \rightarrow x - x^3$

$\int_{1/\sqrt{2}}^{1} x\cdot dx - \int_{1/\sqrt{2}}^{1} x^3\cdot dx \rightarrow \left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_{1/\sqrt{2}}^{1} \Rightarrow \frac{(1-1/\sqrt{2})^2}{2} - \frac{(1-1/\sqrt{2})^4}{4}$

$\frac{1}{2} - \frac{1}{4} - \frac{1/2}{2} + \frac{1/4}{4} = \frac{1}{16}$ $\quad \cancel{\frac{3-2\sqrt{2}}{4}} \rightarrow \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$

Q25) $\iint \sin(y^3)\cdot dA$ $\quad y=\sqrt{x},\ y=2,\ x=0$



$2=\sqrt{x} \rightarrow x=4$

$\int_0^2\int_0^{y^2} \sin(y^3)\cdot dx\,dy$

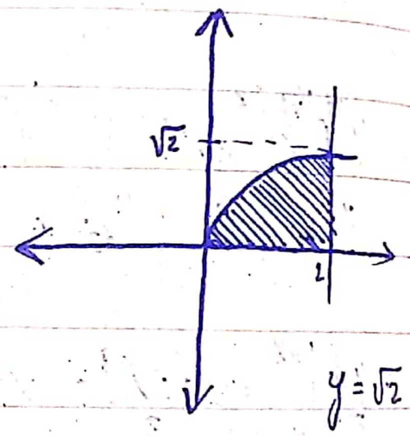
$\int_0^{y^2} \sin(y^3)\cdot dx \rightarrow \sin(y^3)x\Big|_0^{y^2}$

$\Rightarrow \sin(y^3)(y^2)-0 \Rightarrow y^2\sin(y^3)$

$\frac{1}{3}\int_0^2 y^2\sin(y^3)\cdot dy \rightarrow \frac{1}{3}\left(-\cos(y^3)\right)$ (3)

$=\frac{1}{3}\left(-\cos(8)+\cos(0)\right) \rightarrow \frac{1-\cos(8)}{3}$

Q47) $\int_0^2\int_0^{\sqrt{x}} f(x,y)\cdot dy\,dx$



Extracting eq from limits:

① $y=\sqrt{x},\ y=0$

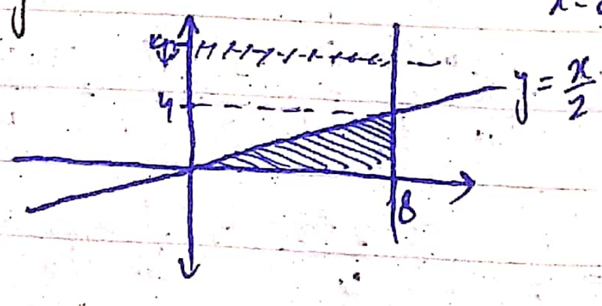
② $x=2,\ x=0$

$\int_0^{\sqrt{2}}\int_{y^2}^{2} f(x,y)\cdot dx\,dy$

$y=\sqrt{2}$

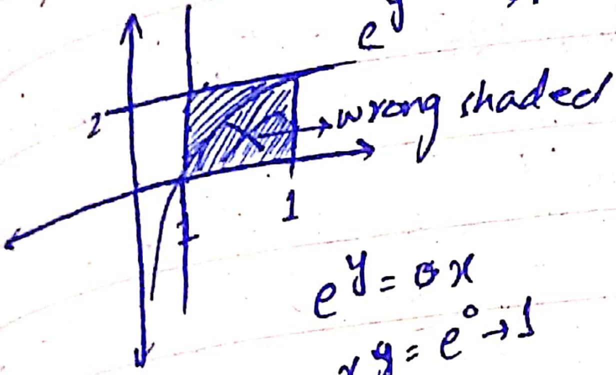
Q48) $\int_0^4\int_{2y}^8 f(x,y)\cdot dx\,dy$

$x=8,\ x=2y$



$=\int_0^8\int_0^{x/2} dy\,dx$

Q49) $\int_0^2 \int_1^{e^y} f(x,y) \cdot dx\,dy$ $\qquad$ $\int_{0}^{e^2} \int_{\ln x}^{2} f(x,y) \cdot dy\,dx$



$e^y = x$

$x = e^0 \to 1$

$x = e^2$

$x = e^y$

$y = \ln x$

Q50) $\int_1^e \int_0^{\ln x} f(x,y)\,dy\,dx$ $\qquad$ $\int_0^1 \int_{e^y}^{e} dx\,dy$