Repeated Integrals

$\boxed{14.2}$

⇒ For $f(x,y)$, The volume under the surface, over region "R", is given by:

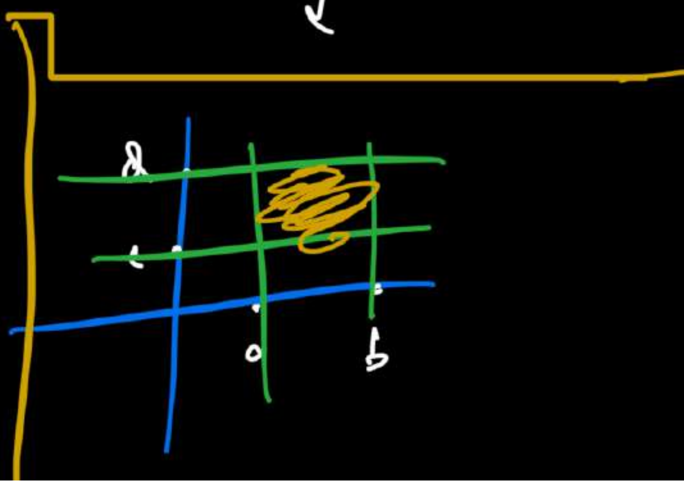
$$V = \iint_R f(x,y)\, dA$$

$\because$ $dA = dx\,dy$

$dA = dy\,dx$

For Rectangular regions

$$R: \{(x,y) \mid a \le x \le b,\ c \le y \le d\}$$



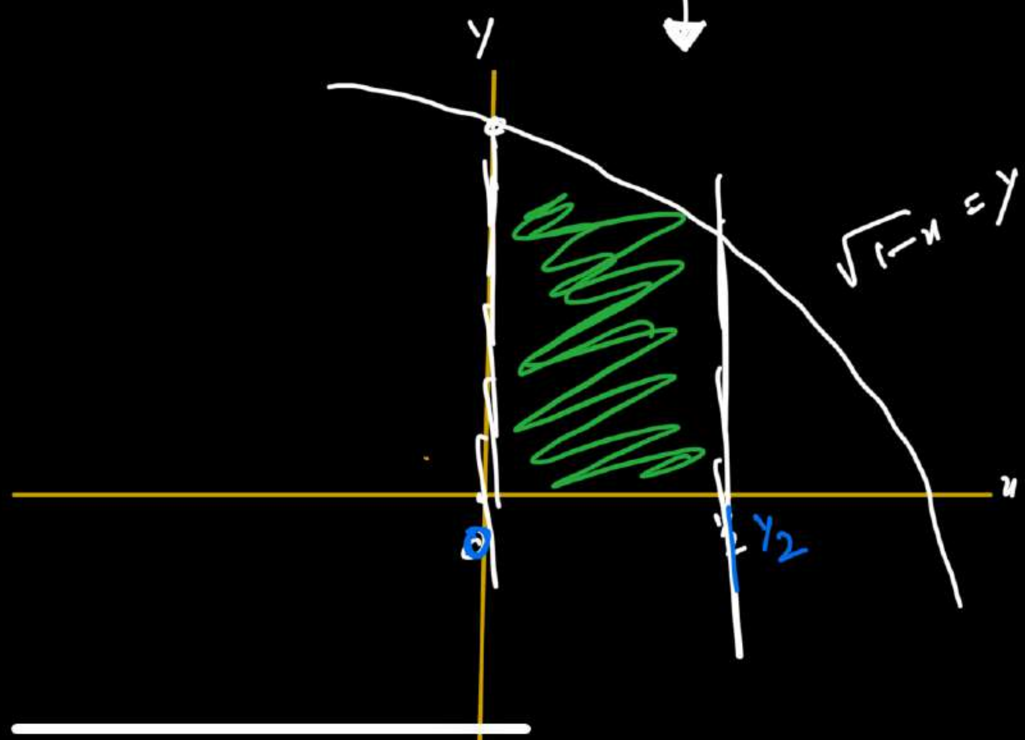
$$\boxed{\int_c^d \int_a^b f(x,y)\, dx\, dy}$$

Or

$$\boxed{\int_a^b \int_c^d f(x,y)\, dy\, dx}$$

Eg

$$\iint_R 2xy\, dA \quad , \quad R : \{ (x,y) \mid 0 \le x \le 1/2 , 0 \le y \le \sqrt{1-x} \}$$



Sol

$$V = \int_0^{1/2} \int_0^{\sqrt{1-x}} 2xy \, dy \, dx$$

$$V = \int_0^{1/2} \cancel{2}x \left(\frac{y^2}{\cancel{2}} \right) \Big|_0^{\sqrt{1-x}} dx$$

$$V = \int_0^{1/2} x \left[(\sqrt{1-x})^2 - (0)^2 \right] dx$$

$$\int_0^{1/2} x (1-x) \, dx$$

$$\int_0^{1/2} x - x^2 \, dx$$

$$\left. \frac{x^2}{2} - \frac{x^3}{3} \right|_0^{1/2}$$

$$\frac{(1/2)^2}{2} - \frac{(1/2)^3}{3} - 0$$

$$\frac{1}{4\times 2} - \frac{1}{8\times 3}$$

$$\frac{1}{8} - \frac{1}{24}$$

$$\frac{3-1}{24} = 2/24 = 1/12$$

ANS

Fubini's theorem for "general" regions

$$\int_a^b \int_{y=g_1(x)}^{y=g_2(x)} f(x,y)\,dy\,dx = \int_c^d \int_{x=h_1(y)}^{x=h_2(y)} f(x,y)\,dx\,dy$$

↓

Bound by two functions: "y equals" & x-constant

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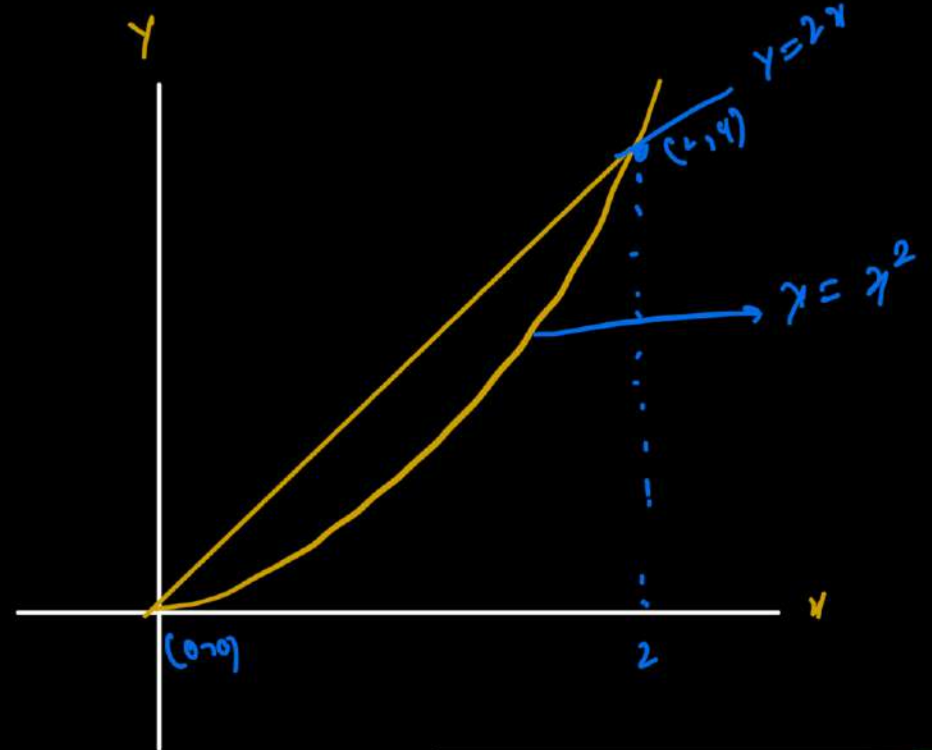
Bounds by two functions: "x equals" & y-constant

(x) Ex: R : Region bounds by $y = 2x$ & $y = x^2$



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$$V = \boxed{\int_0^2 \int_{y=x^2}^{y=2x} f(x,y)\, dy\, dx} \equiv \boxed{V = \int_0^4 \int_{x=y/2}^{x=\sqrt{y}} f(x,y)\, dx\, dy}$$

Type - 1 Type - 2

Eg

$$= \iint_R \frac{x}{1+xy}\, dA \qquad R: \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 1\}$$

$$= \int_0^1 \int_0^1 \frac{x}{1+xy}\, dy\, dx$$

∴ choose that dx or dy first in which there is no switch from one function to another

let $u_y = 1 + xy$

$$\frac{du_y}{dy} = x$$

$$du = x\, dy$$

Q) $$\iint_R \frac{1}{xy}\, dA \quad ; R:\{(x,y) \mid 1 \le y \le e,\ y \le x \le y^2\}.$$

Sol

$$\int_1^e \int_y^{y^2} \frac{1}{xy}\, dx\, dy$$

$$\int_1^e \frac{1}{y}\Big[\ln x\Big]_y^{y^2} dy$$

$$\int_1^e \frac{1}{y}\left[\ln y^2 - \ln y\right] dy$$

$$\int_1^e \frac{1}{y}\left[2\ln y - \ln y\right] dy$$

$$\int^e \frac{1}{y} \cdot \ln y$$

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$$\int_0^1 \ln(1+x)\,dx$$

$$x\ln(1+x) - x + \ln(1+x)\Big|_0^1$$

$$(\ln(2) - 1 + \ln(2)) - 0$$

$$\boxed{2\ln 2 - 1}$$

ANS

$\ln(1+x) = u$ $\int du \int dv$

$du = \frac{1}{1+x}\,dx$ $x = v$

$du = \frac{dx}{1+x}$

$$\int u\,dv = uv - \int v\,du$$

$$\ln(1+x)\cdot x - \int \frac{x\cdot dx}{1+x}$$

$$x(\ln(1+x)) - \left(\frac{x+1}{1+x} - \frac{1}{1+x}\right)$$

$$x\ln(1+x) - \int 1 - \frac{1}{1+x}\,dx$$

$$x\ln(1+x) - (x - \ln(1+x))$$

$$\boxed{x\ln(1+x) - x + \ln(1+x)}$$

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$y = 1 \Rightarrow u = 1+x$

$y = 0 \Rightarrow u = 1$

$$\int_0^1 \int_\phi^{1+x} \frac{du}{u}\, dx$$

$$\int_0^1 \ln|u| \Big|_\phi^{1+x}$$

$$\int_0^1 \left[\ln|1+x| - \ln(1)\right] dx$$

$$\int_0^1 \ln(1+x) - 0 \; dx$$

$$\int_0^1 \ln(1+x)\, dx \rightarrow$$ can solve it by using $\boxed{\text{byparts}}$

$\ln(1+x) = u$ $\qquad \int du \int dv$

$du = \frac{1}{1+x} dx$ $\qquad x = v$

$du = \frac{dx}{1+x}$

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$$(x-2)(x+1)=0$$

$\boxed{x=2}$ $\boxed{x=-1}$

$y=1$ $y=4$

$\therefore$ dy because the region is always b/w the same functions that's why we choose dy and not dx

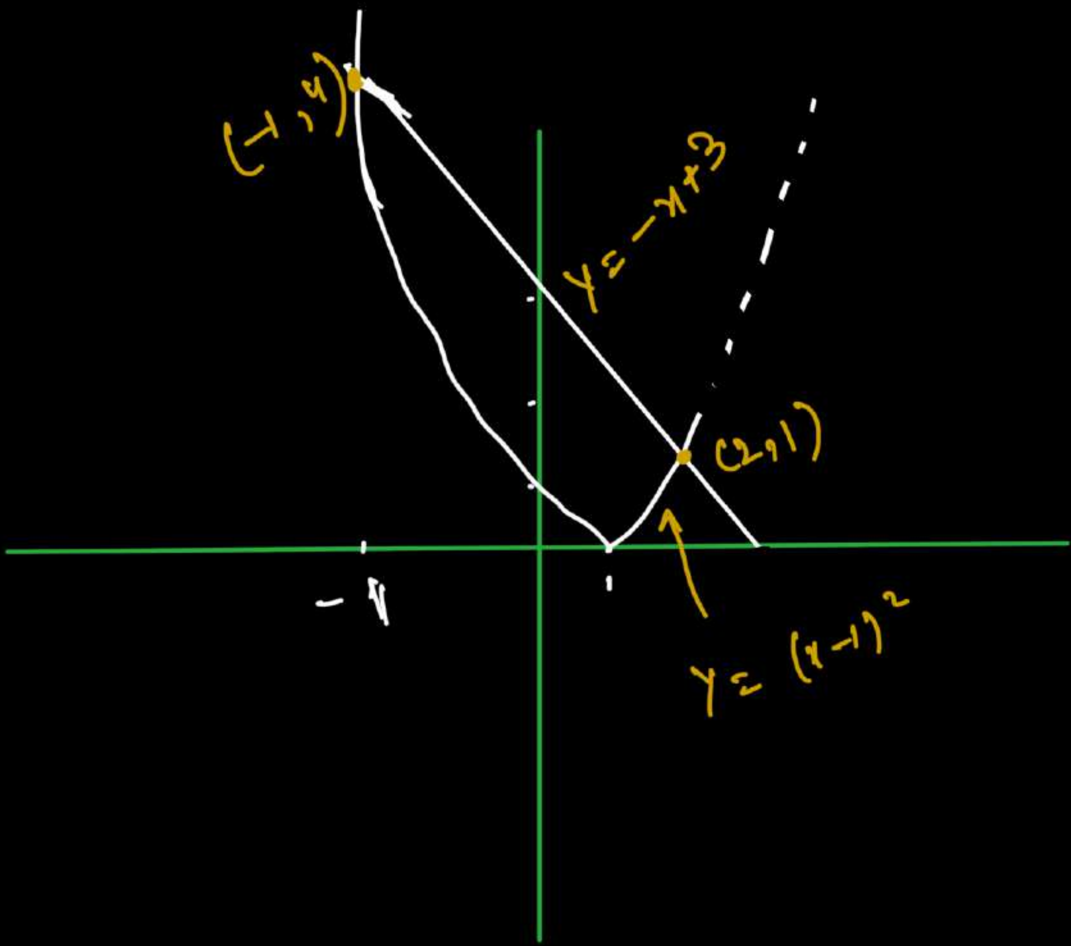
$$\int_{x=-1}^{x=2}\int_{y=(x-1)^2}^{y=-x+3} 4x^3 \, dy \, dx$$

$$= \frac{72}{5}$$

Q)

$$\iint_R 4x^3 \, dA$$

R: The region bound by $y = (x-1)^2$, $y = -x+3$



$$y = y$$

$$-x+3 = (x-1)^2$$

$$-x+3 = x^2-2x+1$$

$$x^2 - x - 2 = 0$$

$$\int_1^e \frac{\ln y}{y}\, dy$$

$$\left.\frac{(\ln y)^2}{2}\right|_1^e$$

$$\frac{(\ln e)^2}{2} - \frac{(\ln (1))^2}{2}$$

$$\frac{1}{2} - 0$$

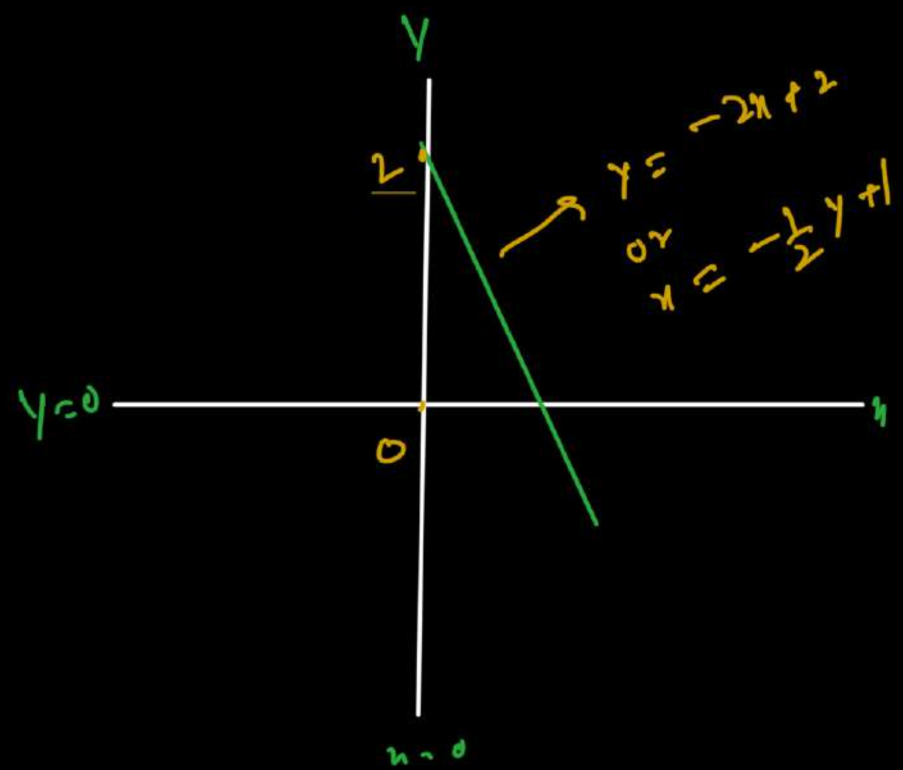
$$\frac{1}{2}$$

ANS//

Q. Find the volume of the solid created by

$y^2+z^2=9$, $\underbrace{z=0,\ y=0,\ x=0\ \&\ 2x+y=1}_{\text{planes.}}$

sol



$f(x,y) = z$

$z^2 = 9-y^2$

$z = \sqrt{9-y^2}$

$$\boxed{\int_0^2 \int_0^{x=-\frac{1}{2}y+1} \sqrt{9-y^2}\, dx\, dy}$$

$+\sqrt{5}$

$$x = y$$

$$y = -\frac{1}{2}y + 6$$

$$y + \frac{1}{2}y = 6$$

$$\frac{\cancel{3}}{2}y = 6 \cdot 2$$

$$y = 4$$

$$\boxed{\int_{0}^{4} \int_{x=y}^{x=-\frac{1}{2}y+6} f(x,y)\,dx\,dy}$$

4 WEIRD STUFF:

"R" where you're given points:

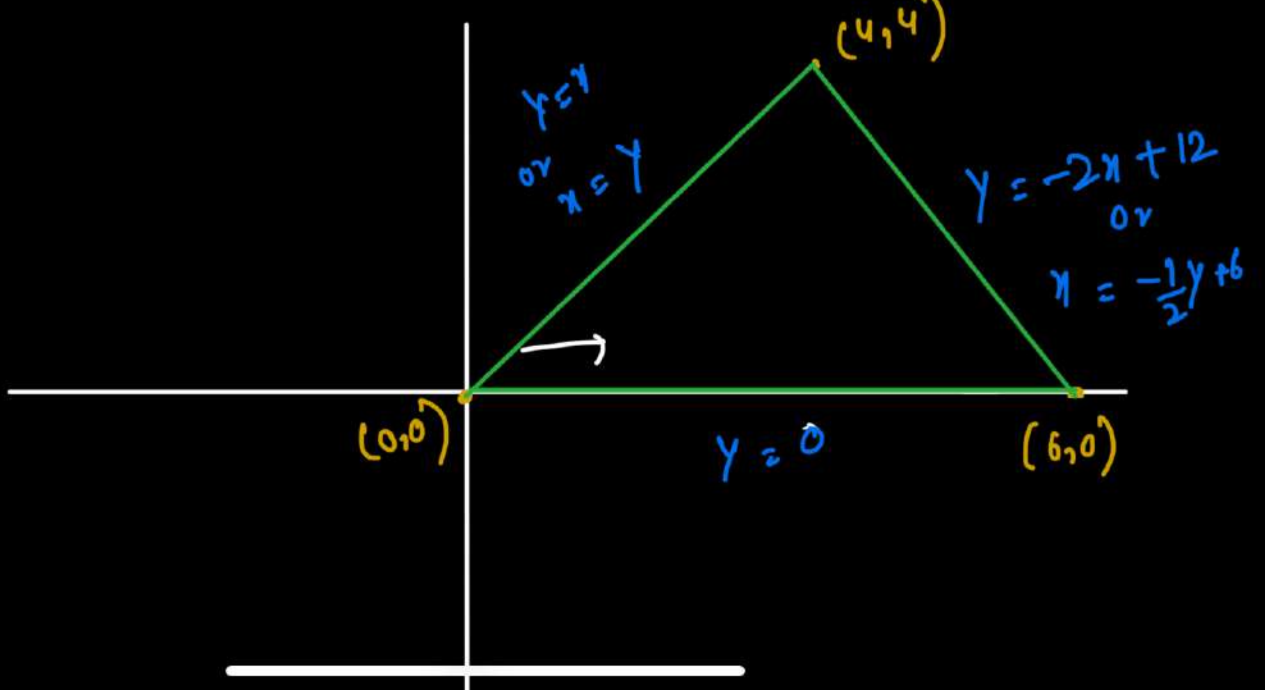
(0,0) (4,4) (6,0)

sol:

Draw Pic

use "m" & $y-y_1=m(x-x_1)$

To create Lines for your region



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$x=0 \Rightarrow u=0$

$x=2 \quad u=-4$

$$\frac{1}{2}\int_{0}^{-4} e^{u}\,\frac{du}{-2}$$

$$\frac{1}{2}\times -\frac{1}{2}\int_{0}^{-4} e^{u}\,du$$

$$-\frac{1}{4}(-1)\int_{-4}^{0} e^{u}\,du$$

$$\frac{1}{4}\times\left[e^{u}\right]_{-4}^{0}$$

$$\frac{1}{4}\left(e^{0}-e^{-4}\right)$$

$$\frac{1}{4}\left[1-\frac{1}{e^{4}}\right] \quad \text{Ans}$$

$$\int_0^2 \int_0^{x/2} e^{-x^2} \, dy \, dx$$

Sol
=

$$\int_0^2 \left[y e^{-x^2} \right]_0^{x/2}$$

$$\int_0^2 \left[\frac{x}{2} e^{-x^2} - 0 \right] dx$$

$$\frac{1}{2} \int_0^2 x e^{-x^2} \, dx$$

let $u = -x^2$

$$\frac{du}{dx} = -2x$$

$$\frac{du}{-2} = x\,dx$$

0 0

$$= \boxed{-\frac{1}{6}\left(27 - 5\sqrt{5}\right) + \frac{9}{2}\sin^{-1}(2/3) + \sqrt{5}}$$

Eg

$$\int_0^1 \int_{2y}^{2} e^{-x^2}\,dx\,dy$$

→ when you solve it you have to go through Mclarian series and all those stuffs :

$x = 2y$
$x = 2$
$y = 0$
$y = 1$

so

acc: to fubini's theorm :)