Quiz 3

Multivariable Calculus

Q 1. Find the absolute extrema of the given function on the indicated closed and bounded set R.

$f(x, y) = xy^2$; R is the region that satisfies the inequalities $x \geq 0$, $y \geq 0$, and $x^2 + y^2 \leq 1$. [5 marks]

Solution:

$f_x = y^2 = 0$, $f_y = 2xy = 0$; no critical points in the interior of R. Along $y = 0$: $u(x) = 0$; along $x = 0$: $v(y) = 0$; along $x^2 + y^2 = 1$: $w(x) = x - x^3$ for $0 \leq x \leq 1$; critical point $\left(1/\sqrt{3}, \sqrt{2/3}\right)$.

(x, y)	$(0,0)$	$(0,1)$	$(1,0)$	$\left(1/\sqrt{3}, \sqrt{2/3}\right)$
$f(x, y)$	0	0	0	$2\sqrt{3}/9$

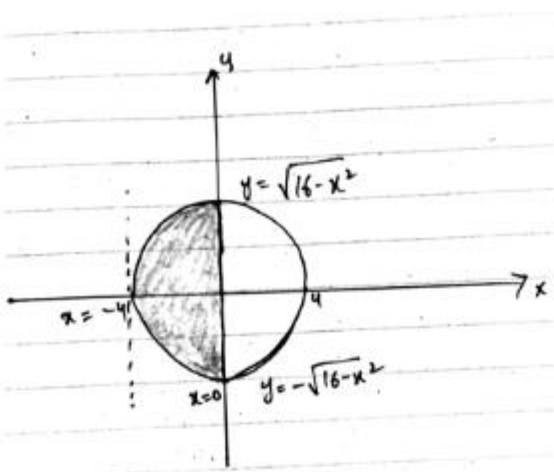
Absolute maximum value is $\frac{2}{9}\sqrt{3}$, absolute minimum value is 0.

Q 2. Evaluate the iterated integral by converting to polar co-ordinates. [5 marks]

$$\int_{-4}^{0} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} 3x \, dy \, dx$$

Solution:

$$\int_{\pi/2}^{3\pi/2} \int_{0}^{4} 3r^2 \cos\theta \, dr \, d\theta = \int_{\pi/2}^{3\pi/2} 64 \cos\theta \, d\theta = -128.$$



Q 3. Use a double integral to find the area of the region. [5 marks]

One loop of the rose $r = \cos 3\theta$

Solution:

One loop is given by the region

$D = \{(r, \theta) \mid -\pi/6 \le \theta \le \pi/6, 0 \le r \le \cos 3\theta\}$, so the area is

$$\iint_D dA = \int_{-\pi/6}^{\pi/6} \int_0^{\cos 3\theta} r\, dr\, d\theta = \int_{-\pi/6}^{\pi/6} \left[\frac{1}{2} r^2\right]_{r=0}^{r=\cos 3\theta} d\theta$$

$$= \int_{-\pi/6}^{\pi/6} \frac{1}{2} \cos^2 3\theta\, d\theta = 2 \int_0^{\pi/6} \frac{1}{2} \left(\frac{1 + \cos 6\theta}{2}\right) d\theta$$

$$= \frac{1}{2} \left[\theta + \frac{1}{6} \sin 6\theta\right]_0^{\pi/6} = \frac{\pi}{12}$$