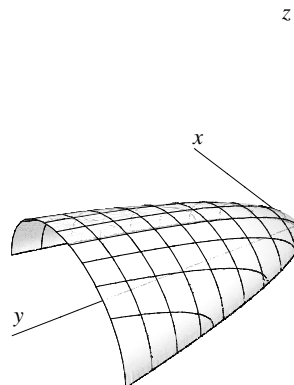
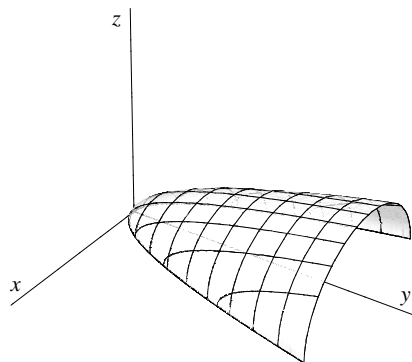
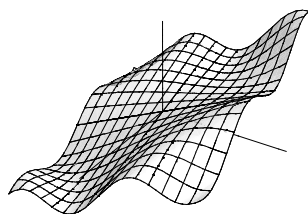
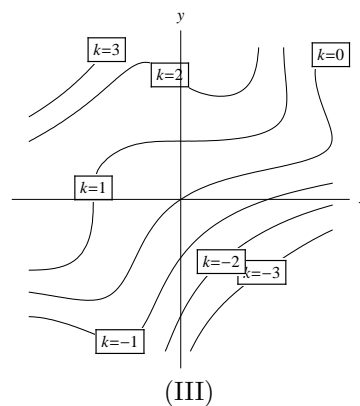
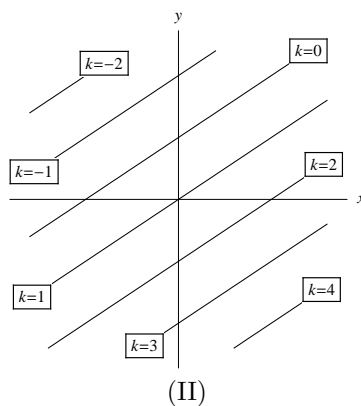
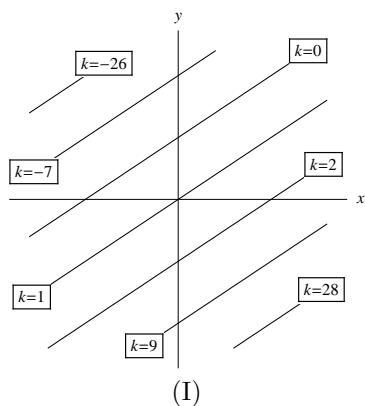


## Functions and Graphs

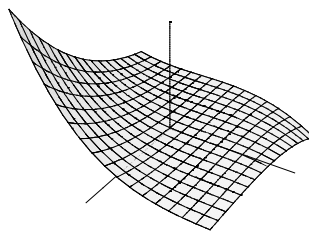
Here is the graph  $z = \sqrt{y - x^2}$  of the function  $f(x, y) = \sqrt{y - x^2}$ , shown from two different angles.



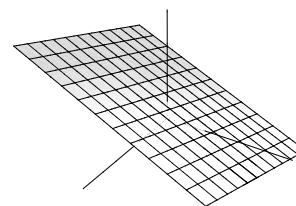
1. The first row shows traces of three graphs  $z = f(x, y)$  in the planes  $z = k$ . (Traces of the graph  $z = f(x, y)$  in  $z = k$  are also known as level sets of  $f(x, y)$ .) Match each diagram with the graph of the function.



(a)



(b)

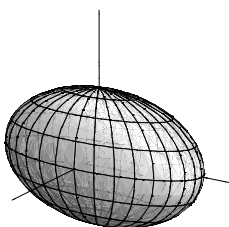


(c)

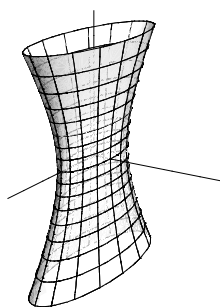
## Quadric Surfaces

Six basic types of quadric surfaces:

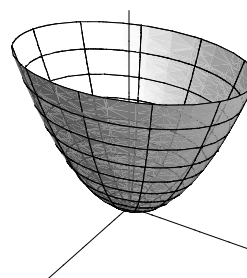
- ellipsoid
- cone
- elliptic paraboloid
- hyperboloid of one sheet
- hyperboloid of two sheets
- hyperbolic paraboloid



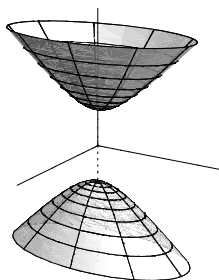
(A)



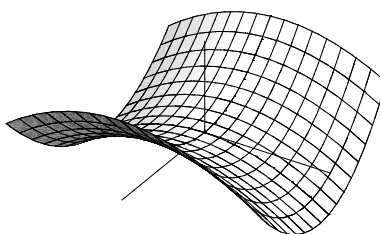
(B)



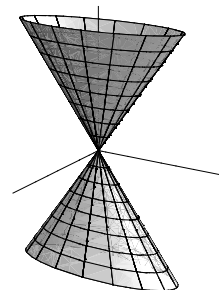
(C)



(D)



(E)



(F)

Quick reminder:

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = r$  describes ...
  - an ellipse if  $r > 0$ .
  - a point if  $r = 0$  (we consider this a “degenerate” ellipse).
  - nothing if  $r < 0$ .
- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = r$  describes ...
  - a hyperbola if  $r \neq 0$ .
  - a pair of lines if  $r = 0$  (we consider this a “degenerate” hyperbola).
- $y = ax^2 + b$  describes a parabola.

1. For each surface, describe the traces of the surface in  $x = k$ ,  $y = k$ , and  $z = k$ . Then pick the term from the list above which seems to most accurately describe the surface (we haven't learned any of these terms yet, but you should be able to make a good educated guess), and pick the correct picture of the surface.

(a)  $\frac{x^2}{9} - \frac{y^2}{16} = z.$

- Traces in  $x = k$ :
- Traces in  $y = k$ :
- Traces in  $z = k$ :

$$\frac{k^2}{9} - \frac{y^2}{16} = z \Rightarrow$$

$$z + \frac{y^2}{16} = \frac{k^2}{9}$$

$$y^2 = -16z + \frac{16k^2}{9}$$

→ parabola  
?

(b)  $\frac{x^2}{4} + \frac{y^2}{25} + \frac{z^2}{9} = 1.$

- Traces in  $x = k$ :
- Traces in  $y = k$ :
- Traces in  $z = k$ :

$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$

Hyperbola

→ ellipse

→ not possible because i g.

(c)  $\frac{x^2}{4} + \frac{y^2}{9} = \frac{z}{2}.$

→ ellipsoid hi he shayad,

- Traces in  $x = k$ :
- Traces in  $y = k$ :
- Traces in  $z = k$ :

(d)  $\frac{z^2}{4} - x^2 - \frac{y^2}{4} = 1.$

- Traces in  $x = k$ :
- Traces in  $y = k$ :
- Traces in  $z = k$ :

(e)  $x^2 + \frac{y^2}{9} = \frac{z^2}{16}.$

- Traces in  $x = k$ :
- Traces in  $y = k$ :
- Traces in  $z = k$ :

(f)  $\frac{x^2}{9} + y^2 - \frac{z^2}{16} = 1.$

- Traces in  $x = k$ :
- Traces in  $y = k$ :
- Traces in  $z = k$ :

2. Sketch the surface  $9y^2 + 4z^2 = 36$ . What type of quadric surface is it?

me keh rha hun ye concentric cylinder  
he along x-axis but copileot keh rha k  
ellipsoid h ye

3. Sketch the surface  $y^2 + 2y + z^2 = x^2$ . What type of quadric surface is it?

Elliptic paraboloid

4. What type of quadric surface is  $4x^2 - y^2 + z^2 + 9 = 0$ ?

eliptic cone