(6) $f(x,y) = x^2 - y^2$ subject to constraint $x^2+y^2=25$.

$$\vec{\nabla} f = \lambda \vec{\nabla} g$$

which leads to the equations

$$\left.\begin{aligned} 2x &= \lambda 2x &&\rightarrow (1) \\ -2y &= \lambda 2y &&\rightarrow (2) \\ x^2+y^2 &= 25 &&\rightarrow (3) \end{aligned}\right\}$$

From equation (1)

If $x=0$, equation (3) $\Rightarrow y^2 = 25$

$\Rightarrow y = \pm 5$, $(0, \pm 5)$

If $x \neq 0$, eq (1) $\Rightarrow \lambda = \frac{2x}{2x} = 1$

it must satisfy eq (2) and eq (3).

thus $y=0$ (otherwise $\lambda = -1$).

eq (3) $\Rightarrow x^2 = 25$, $x = \pm 5$, $(\pm 5, 0)$

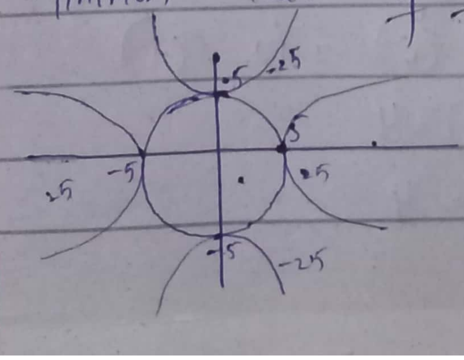
The possible values are

$(0,-5)$, $(0,5)$, $(-5,0)$, $(5,0)$

(x,y)	$(0,-5)$	$(0,5)$	$(-5,0)$	$(5,0)$
$f(x,y) = x^2-y^2$	-25	-25	25	25

Maximum value of f is 25 at $(\pm 5, 0)$.

Minimum value of f is -25 at $(0, \pm 5)$.



9) $f(x,y,z) = 2x+y-2z$ subject to constraint $x^2+y^2+z^2=4$

$$\vec{\nabla} f = \lambda \vec{\nabla} g$$

which leads to the equation.

$$2 = \lambda 2x \quad \to (1)$$
$$1 = \lambda 2y \quad \to (2)$$
$$-2 = \lambda 2z \quad \to (3)$$
$$x^2+y^2+z^2 = 4 \quad \to (4).$$

rewrite (1), (2), (3) as

$$\frac{2}{2x} = \lambda, \quad \frac{1}{2y} = \lambda, \quad \frac{-2}{2z} = \lambda$$

$$\frac{1}{x} = \lambda \ (a), \quad \frac{1}{2y} = \lambda \ (b), \quad \frac{-1}{z} = \lambda \ (c)$$

comparing (a) and (b)

$$\frac{1}{x} = \frac{1}{2y}$$

$$\Rightarrow x = 2y \quad \to (5).$$

comparing (b) and (c)

$$\frac{1}{2y} = \frac{-1}{z}$$

$$\Rightarrow z = -2y \quad \to (6)$$

substituting (5) and (6) into (4)

$$(2y)^2 + y^2 + (-2y)^2 = 4$$
$$4y^2 + y^2 + 4y^2 = 4$$

$y^2 = 4 - x^2$

$9y^2 = 4$

$\Rightarrow y^2 = \frac{4}{9}$, $y = \pm\frac{2}{3}$

From (5) $x = 2y$

$\Rightarrow x = 2\left(\frac{2}{3}\right) = \frac{4}{3}$

and $x = 2\left(\frac{-2}{3}\right) = \frac{-4}{3}$

From (6) $z = -2y$

$\Rightarrow z = -2\left(\frac{2}{3}\right) = \frac{-4}{3}$

$z = -2\left(\frac{-2}{3}\right) = \frac{4}{3}$

Test points are

$\left(\frac{4}{3}, \frac{2}{3}, -\frac{4}{3}\right)$, $\left(\frac{-4}{3}, -\frac{2}{3}, \frac{4}{3}\right)$.

$f\left(\frac{4}{3}, \frac{2}{3}, -\frac{4}{3}\right) = 6$

$f\left(-\frac{4}{3}, \frac{-2}{3}, \frac{4}{3}\right) = -6$

Maximum 6 at $(4/3, 2/3, -4/3)$

Minimum -6 at $(-4/3, -2/3, 4/3)$.