Rough Work

21384

Q: 1

Surface $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$

x - intercepts

$y = z = 0$, $x^2 = 1 \Rightarrow x = \pm 1$

$(\pm 1, 0, 0)$.

y - intercepts

$x = z = 0$, $\frac{y^2}{9} = 1 \Rightarrow y^2 = 9$

$\Rightarrow y = \pm 3$

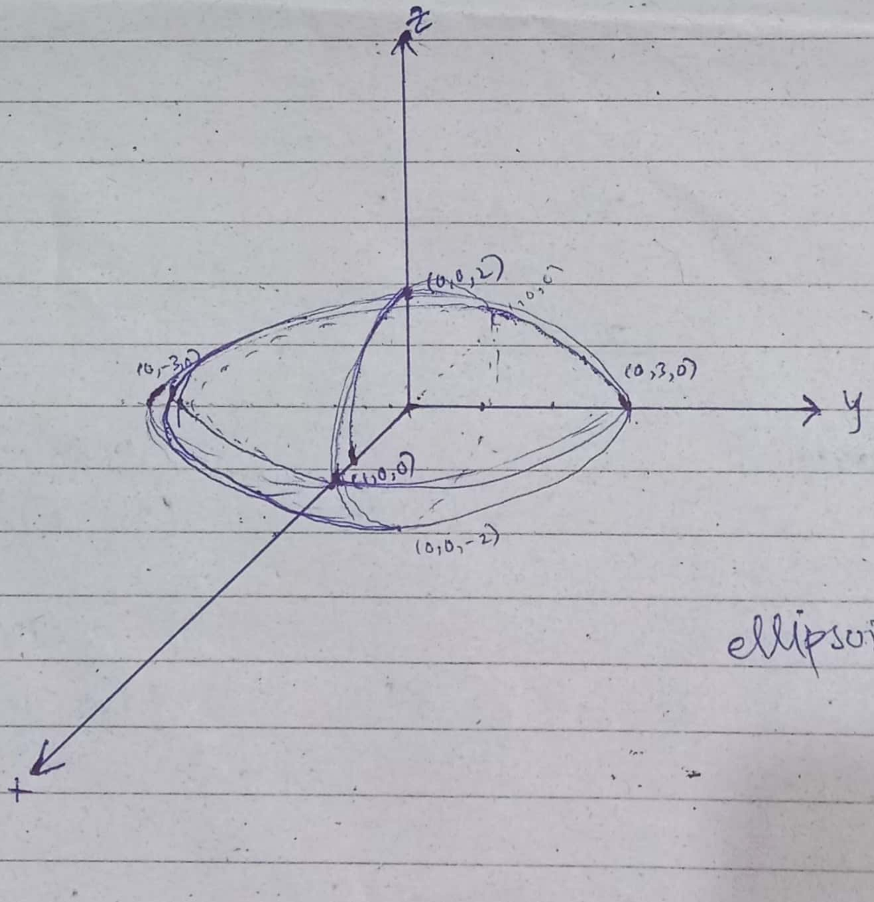
$(0, \pm 3, 0)$.

z - intercepts:

$x = y = 0$, $\frac{z^2}{4} = 1 \Rightarrow z^2 = 4$

$\Rightarrow z = \pm 2$

$(0, 0, \pm 2)$.



ellipsoid.

Level curves: $z = k$

$$x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

$$x^2 + \frac{y^2}{9} + \frac{k^2}{4} = 1$$

$$x^2 + \frac{y^2}{9} = 1 - \frac{k^2}{4}$$

For $k = 0$,

$$x^2 + \frac{y^2}{9} = 1$$

x-intercepts.

$y = 0$ $\quad x = \pm 1$ $\quad , (\pm 1, 0)$

y-intercepts.

$x = 0$ $\quad , y^2 = 9 \quad , y = \pm 3 \quad , (0, \pm 3)$

For $k = 1$

$$x^2 + \frac{y^2}{9} = 1 - \frac{1}{4}$$

$$x^2 + \frac{y^2}{9} = \frac{3}{4}$$

$$\frac{x^2}{3/4} + \frac{y^2}{27/4} = 1$$

x-intercepts. $\qquad (0.8660)$

$y = 0 \quad , \quad x^2 = \frac{3}{4} \quad \Rightarrow \quad x = \pm \frac{\sqrt{3}}{2} \qquad \left(\pm \frac{\sqrt{3}}{2}, 0\right)$

y-intercepts $\qquad (2.598)$

$x = 0 \quad , \quad y^2 = \frac{27}{4} \quad \Rightarrow \quad y = \pm \frac{3\sqrt{3}}{2} \qquad \left(0, \pm \frac{3\sqrt{3}}{2}\right)$

For $k = -1$.

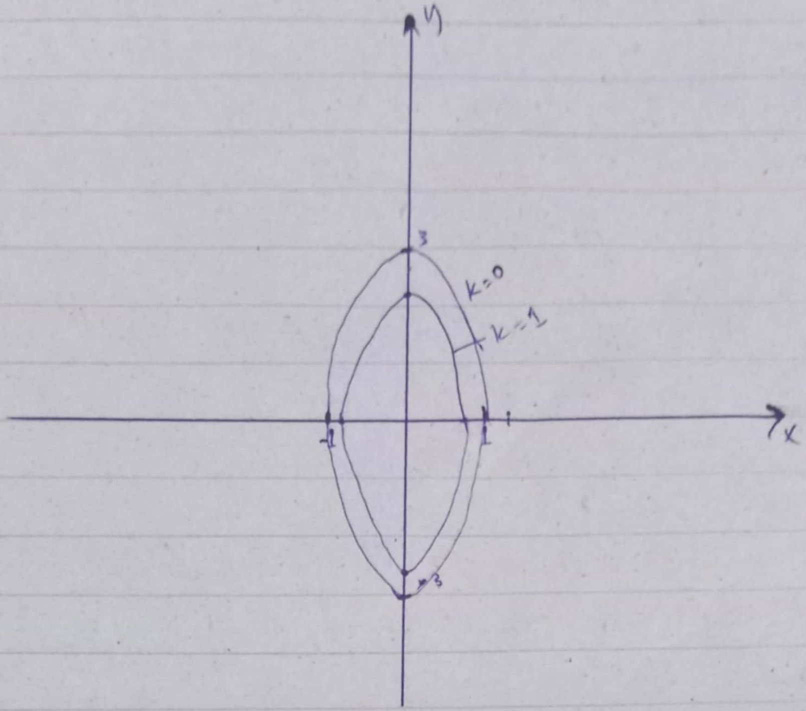
~~x-intercepts.~~

~~$y = 0$~~ ,

$$x^2 + \frac{y^2}{9} = 1 - \frac{(-1)^2}{4}$$

$$x^2 + \frac{y^2}{9} = 1 - \frac{1}{4}$$

$$= x^2 + \frac{y^2}{9} = \frac{3}{4} \quad \text{(same as 1)}$$



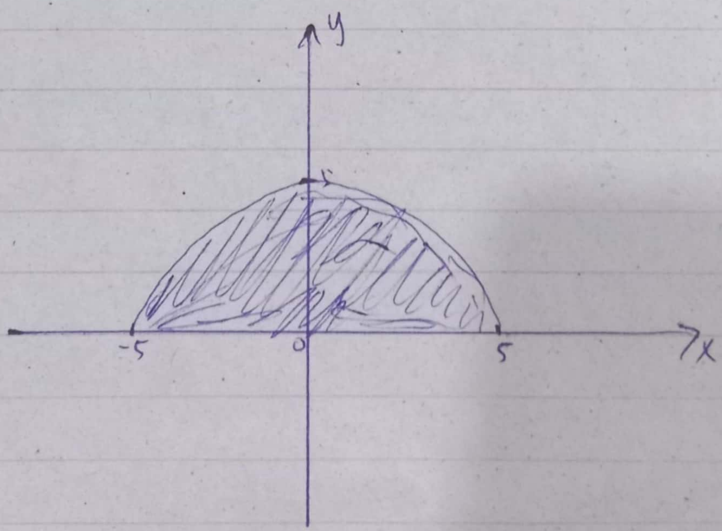
In words:

$k = -1, 0, 1$ family of ellipses
or ellipses.

Q# 2.

$$\sqrt{y} + \sqrt{25 - x^2 - y^2}$$

$$y \geq 0, \quad 25 - x^2 - y^2 \geq 0$$
$$\Rightarrow x^2 + y^2 \leq 25$$



Q#3.

checking at $(0,0)$.

$f(0,0) = 0$.

checking limit at $(0,0)$.

along x-axis

$y=0$. $\quad f(x,0) = \frac{0}{x^2} = 0$

$\Rightarrow f(x,y) \to 0$ as $(x,y) \to (0,0)$ along x-axis.

along $y=x$

$$f(x,x) = \frac{x^2}{3x^2} = \frac{1}{3}$$

$\Rightarrow f(x,y) \to 0$ as $(x,y) \to (0,0)$ along $y=$

Hence, ~~limit~~

$\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist

so, f is not continuous at $(0,0)$

Q#4.

$$f_x = \frac{\partial f}{\partial x} = 3x^2e^{-y} + \frac{y^3}{2}x^{-1/2}\sec\sqrt{x}\tan\sqrt{x}$$

$$f_y = \frac{\partial f}{\partial y} = -x^3e^{-y} + 3y^2\sec\sqrt{x}$$

$$\frac{\partial f}{\partial x}(1,3) = 3(1)e^{-3} + \frac{27}{2}(1)^{-1/2}\sec\sqrt{1}\tan\sqrt{1}$$

$$= 3e^{-3} + \frac{27}{2}\sec\sqrt{1}\tan\sqrt{1}$$