How Regimes Affect Asset Allocation

Andrew Ang and Geert Bekaert

International equity returns are characterized by episodes of high volatility and unusually high correlations coinciding with bear markets. This article provides models of asset returns that match these patterns and illustrates their use in asset allocation. The presence of regimes with different correlations and expected returns is difficult to exploit within a framework focused on global equities. Nevertheless, for global all-equity portfolios, the regime-switching strategy dominated static strategies in an out-of-sample test. In addition, substantial value was added when an investor switched between domestic cash, bonds, and equity investments. In a persistent high-volatility market, the model told the investor to switch primarily to cash. Large market-timing benefits are possible because high-volatility regimes tend to coincide with periods of relatively high interest rates.

International equity returns are more highly correlated with each other in bear markets than in normal times. Longin and Solnik (2001) have shown that this asymmetric correlation is statistically significant, and we have shown previously (Ang and Bekaert 2002a) that regime-switching (RS) models replicate well the degree of asymmetric correlations observed in the data.[1]

RS models build on the seminal work of Hamilton (1989). In its simplest form, an RS model allows the data to be drawn from two or more possible distributions (regimes). At each point of time, there is a certain probability that the process will remain in the same regime or that it will transition to another regime for the next period. We previously found (Ang and Bekaert 2002a) that international equity returns can be characterized by two regimes—a normal regime and a bear market regime when returns are, on average, lower and much more volatile than in normal times. In the bear market, the correlations between various returns are higher than in the normal regime.

RS strategies need not be restricted to equity returns: There is strong evidence that regimes exist in U.S. and international short-term interest rates.[2] Short-term rates are characterized by high persistence and low volatility at low interest rate levels but lower persistence and much higher volatility at higher interest rate levels. Again, RS models perfectly capture these features of the data. The regimes in interest rates and equity returns are correlated and related to the business cycle.

Surprisingly, quantitative asset allocation research usually ignores these salient features of international equity returns and interest rate data. The presence of asymmetric correlations in equity returns has so far primarily raised doubts about the benefits of international diversification, in that these benefits may not be forthcoming when an investor needs them the most. The presence of regimes should be exploitable, however, in an active asset allocation program. The optimal equity portfolio in a high-volatility regime is likely to be very different (for example, more home biased) than the optimal portfolio in a normal regime. Optimal regime-switching asset allocation may require shifting assets into bonds or cash when a bear market is expected. In this article, we illustrate how the presence of regimes can be incorporated in a global asset allocation setting (with six equity markets and potential cash) and a market-timing setting for U.S. cash, bonds, and equity.

In previous work, Clarke and de Silva (1998) showed how the existence of two regimes ("states" in their terminology) affects mean–variance asset allocation, but they did not consider how the return characteristics in the two states may be extracted from the data. Ramchand and Susmel (1998) estimated a number of RS models from international equity return data but did not explore how the regimes affect portfolio composition. Das and Uppal (2001) used a continuous-time jump model and investigated the implications of their findings for asset allocation. The jumps in their model are only transitory, however, and cannot fully capture the

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persistent nature of bear markets. Guidolin and Timmermann (2002) also considered the asset allocation implications of an RS model, but they restricted their attention to allocating wealth between a risk-free asset and domestic equity. The work reported here builds on a framework we developed while investigating optimal asset allocation when returns follow various RS processes (Ang and Bekaert 2002a). Our previous work was limited, however, to returns from U.S., U.K., and German assets; the current article greatly expands the list of markets and assets and illustrates the market-timing benefits of regime switching in a practical setup.

Data

Our first application involves a universe of developed equity markets and a U.S.-based investor. In addition to North America (Canada and the United States), we considered the United Kingdom and Japan, Europe (which we split into large markets and small markets), and the Pacific ex Japan region. **Exhibit 1** lists the countries. All data are from Morgan Stanley Capital International, and the sample period is February 1975 through the end of 2000. We measured all returns as simple net returns expressed in U.S. dollars.

Exhibit 1. Composition of International Universe

Europe: Small Countries	*Europe: Large Countries*	*Pacific ex Japan*
Austria	France	Australia
Belgium	Germany	New Zealand
Denmark	Italy	Singapore
Finland		
Ireland	*North America*	*United Kingdom*
Netherlands	Canada	
Norway	United States	*Japan*
Spain		
Sweden		
Switzerland		

In our second application, we focused on U.S. returns. We allowed a U.S. investor to implement a strategy of switching between cash (one-month U.S. T-bills), 10-year bonds (the Ibbotson Associates constant-maturity series), and the U.S. stock market (proxied by the S&P 500 Index). In this application, we used a longer sample period—January 1952 to the end of 2000.

RS Model for Equity Portfolios

In this section, we describe the RS model for the universe of developed equity markets and show how it can be estimated and applied to asset allocation.

The Model. To build a quantitative model for the six international asset classes, we started with the familiar capital asset pricing model (CAPM). We denoted the world market return at time t measured in excess of the T-bill rate by y_t^w and assumed

$$y_t^w = \mu^w + \sigma^w \varepsilon_t^w, \tag{1}$$

where μ^w is the world market expected excess return and σ^w is the conditional volatility. For modeling purposes, we assumed that the unexpected return, or shock, ε_t^w, is drawn from a standard normal distribution.

The world CAPM implies a linear security market line: The expected excess return on any security is linear in its beta with respect to the world market. Let individual excess returns for security j be denoted y^j; then,

$$\begin{aligned} y_t^j &= (1-\beta^j)\mu^z + \beta^j\mu^w + \beta^j\sigma^w\varepsilon_t^w + \bar{\sigma}^j\varepsilon_t^j \\ &= \mu^z + \beta^j(\mu^w - \mu^z) + \beta^j\sigma^w\varepsilon_t^w + \bar{\sigma}^j\varepsilon_t^j, \end{aligned} \tag{2}$$

where μ^z is the zero-beta excess return. The unexpected return on security or portfolio j is now determined by the security's sensitivity to the world market return and by an idiosyncratic term, which has volatility $\bar{\sigma}^j$.

The term $(1 - \beta^j)\mu^z$ does not appear in the standard CAPM. The constant μ^z allows a flatter security market line, for which there is strong empirical evidence.[3] The model in Equation 2 is a version of Black's (1972) zero-beta CAPM, which can theoretically be motivated by the presence of differential borrowing and lending rates.

Regime switching in the world market. From an asset allocation perspective, nothing could be more boring than a CAPM that prescribes simply holding the market portfolio. By making one critical change in the setup of Equations 1 and 2, however, we can create a model that not only fits the empirical patterns in international equity returns but also makes quantitative asset allocation potentially fruitful.

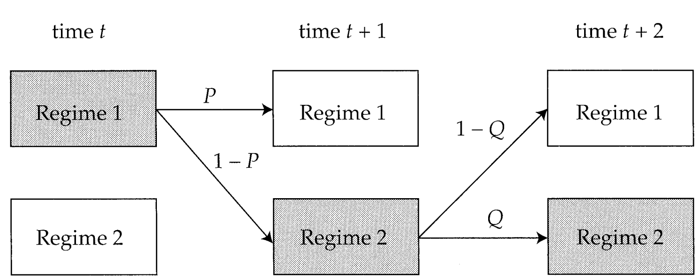
Suppose the world expected return and conditional volatility can take on two values, which depend on the realization of a regime variable that reflects the world market regime. An economic mechanism behind world market regimes is the world business cycle (expansions or recessions). Denote the world conditional expected return and volatility, which depend on regime i, as, respectively, $\mu^w(i)$ and $\sigma^w(i)$. Then, stock markets will be characterized by larger uncertainty and lower returns when a global recession is anticipated, as was the case in 2001.

Assume that the portfolio manager knows which regime is being realized at each point of time but does not know which regime will be realized next month. (Later, we discuss how the identity of the regime can be determined in practice.) So, if the current state is Regime 1, the probability of remaining in that regime is P (and the probability of transitioning to the other regime is $1 - P$). Similarly, if the current state is Regime 2, Q denotes the probability of staying in the second regime. Technically, the regime variable follows a Markov process with constant transition probabilities P and Q. **Figure 1** depicts the regime probabilities and the conditional expected return and variance expressions for Regime 1 and Regime 2. Figure 1 shows three dates, and although at each point of time either regime could be realized, we assumed for illustration that the sequence is Regime 1, Regime 2, and then Regime 2 again.

With this change in the model, expected returns and variances now vary through time. Consider the situation today at time t in Figure 1. The portfolio manager knows that today the world market is in Regime 1. The expected return for next period depends on the manager's expectations for the regime realization at time $t + 1$; consequently, the manager weights the two possible realizations of μ^w with their relevant probabilities. The expected excess return in Regime 1 is denoted $e^w(1)$. Whenever Regime 1 is realized, the portfolio manager assesses the expected return to be $e^w(1)$. When the portfolio manager finds the world market in Regime 2, as is the case at time $t + 1$, the manager uses $e^w(2)$ as the expected return. To compute this expected return, the manager now uses the $1 - Q$ and Q probabilities (rather than the $1 - P$ and P probabilities) to weight $\mu^w(1)$ and $\mu^w(2)$. Note that if $P = 1 - Q$, then the regime structure is inconsequential for the expected returns because they are the same across regimes. Empirical estimations of RS models, such as those of Gray (1996) and Ang and Bekaert (2002b, 2002c), found that both P and Q are well over 50 percent, indicating that regimes are persistent.

Like the conditional mean, the conditional variance also depends on the regime. When the portfolio manager is in Regime 1, as at time t in Figure 1, the manager anticipates that the first regime will continue with probability P and that the volatility of world market shocks will be $\sigma^w(1)$ and assigns a probability $1 - P$ of transitioning to the, perhaps more volatile, second regime with volatility $\sigma^w(2)$. Not surprisingly, the conditional variance is a weighted average of the conditional variances in the two regimes. The conditional variance has an additional jump component, however, that arises

Figure 1. A Regime-Switching Model for the World Market



	time t	time $t + 1$	time $t + 2$
Conditional Expected Return	$\underbrace{P\mu^w(1) + (1 - P)\mu^w(2)}_{e^w(1)}$	$\underbrace{(1 - Q)\mu^w(1) + Q\mu^w(2)}_{e^w(2)}$	$\underbrace{(1 - Q)\mu^w(1) + Q\mu^w(2)}_{e^w(2)}$
Conditional Variance	$\underbrace{P[\sigma^w(1)]^2 + (1 - P)[\sigma^w(2)]^2 + P(1 - P)[\mu^w(1) - \mu^w(2)]^2}_{\Sigma^w(1)}$	$\underbrace{(1 - Q)[\sigma^w(1)]^2 + Q[\sigma^w(2)]^2 + Q(1 - Q)[\mu^w(1) - \mu^w(2)]^2}_{\Sigma^w(2)}$	$\underbrace{(1 - Q)[\sigma^w(1)]^2 + Q[\sigma^w(2)]^2 + Q(1 - Q)[\mu^w(1) - \mu^w(2)]^2}_{\Sigma^w(2)}$

Note: Regime realizations are shaded for illustration.

because the conditional mean is also different in the two regimes. The conditional variances in Figure 1 are denoted $\Sigma^w(1)$ and $\Sigma^w(2)$ for, respectively, Regimes 1 and 2.

Country expected returns and volatilities. For the individual assets, the model in Equation 2 is maintained except that the world market parameters, $\mu^w(i)$ and $\sigma^w(i)$, now vary across regimes. Because the mean of the world excess return switches between regimes, the expected excess return of country j is given by $(1 - \beta^j)\mu^z + \beta^j e^w(i)$ for the current regime, i, where the $e^w(i)$ are given in Figure 1. Expected returns of individual equity markets differ only through their different betas with respect to the world market.

The conditional variance for the individual assets is complex. Intuitively, the conditional variance depends on three components. First, as in a standard CAPM, an asset's conditional variance depends on the asset's exposure to systematic risk through the asset's beta. In the world CAPM, however, the world market return switches regimes, so the market conditional variance now also depends on the regime prevailing at time t. Second, also as in a standard CAPM, each asset has an idiosyncratic volatility term unrelated to its systematic (beta) exposure. Finally, the variance of an individual asset depends not only on the realization of the current regime but also on the jump component, which arises because the conditional means differ across regimes.[4]

Although the model structure is parsimonious, the model generates rich patterns of stochastic volatility and time-varying correlations. In particular, the model captures the asymmetric correlation structure in international equity returns that motivated our analysis. In any factor model, correlations are higher when factor volatility is higher. Hence, if one regime is more volatile than the other regime, the correlation between the different asset returns increases in the more volatile regime.

Model Estimation and Results. This model requires only the estimation of P and Q, the world market return process, the μ^z parameter, and a beta and idiosyncratic volatility term for each country. Because the regime is not observable, the estimation involves inferring from the data which regime prevails at each point in time.[5]

Table 1 contains the estimation results for the RS equity model. Panel B indicates that the first regime is a normal, quiet regime, where world excess returns are expected to yield 0.90 percent a month, with volatility 2.81 percent a month. The other regime is a volatile regime, with standard deviation 5.04 percent a month and with a lower but imprecisely estimated mean of 0.13 percent a month. The estimate of μ^z is larger than the expected excess equity return in the low-volatility regime. The country betas in Panel C were estimated precisely, and their magnitudes seem economically appealing. The

Table 1. RS Equity Model Parameter Estimates, 1975–2000

A. Transition probabilities and μ_z

Measure	P	Q	μ^z
Estimate	0.8917	0.8692	0.74
Standard error	0.0741	0.1330	0.68

B. World market

Measure	$\mu(1)$	$\mu(2)$	$\sigma(1)$	$\sigma(2)$
Estimate	0.90	0.13	2.81	5.04
Standard error	0.32	0.62	0.44	0.55

C. Country betas, β

Measure	North America	United Kingdom	Japan	Europe: Large	Europe: Small	Pacific ex Japan
Estimate	0.88	1.03	1.21	0.90	0.89	0.92
Standard error	0.03	0.06	0.07	0.05	0.04	0.07

D. Idiosyncratic volatilities, $\bar{\sigma}$

Measure	North America	United Kingdom	Japan	Europe: Large	Europe: Small	Pacific ex Japan
Estimate	2.40	4.50	4.62	3.87	2.72	4.99
Standard error	0.09	0.18	0.19	0.16	0.11	0.20

Notes: All parameters are monthly; the mean, μ, and standard deviation, σ, parameters are expressed in percentages.

only surprise is that Japan, which has a low average return in the data, has been assigned a high beta. This is because Japan had the highest volatility of all the equity returns we considered, which the model fits through a high beta and high idiosyncratic volatility (the highest idiosyncratic volatility of all the markets).

Panel A of **Table 2** reports the implied expected excess returns for the six markets. Because the betas are close to 1, expected returns are close to each other in the normal regime. In the bear market, Regime 2, expected excess returns are dramatically lower and more dispersed, with the United Kingdom and Japan having the lowest expected excess returns. In this regime, as Table 1 indicates, the zero-beta excess return, μ^z, is higher than the excess return of the world market, which causes the high-beta countries to have lower expected returns from Equation 2. In fact, the expected return for Japan implied by the model is the highest of all markets in the normal regime but by far the lowest in the bear market. North America and the small countries of Europe have the lowest idiosyncratic volatility implied by the model. In the data, these two groups also had the lowest overall volatility.

Panel B of Table 2 shows the covariance and correlation matrix in each regime. Given that Regime 2 is a high-volatility regime, we expected the model to generate asymmetric correlations, with correlations higher in the second regime. This is indeed the case: The correlations in Regime 2 are, on average, some 20 percent higher than those in Regime 1.

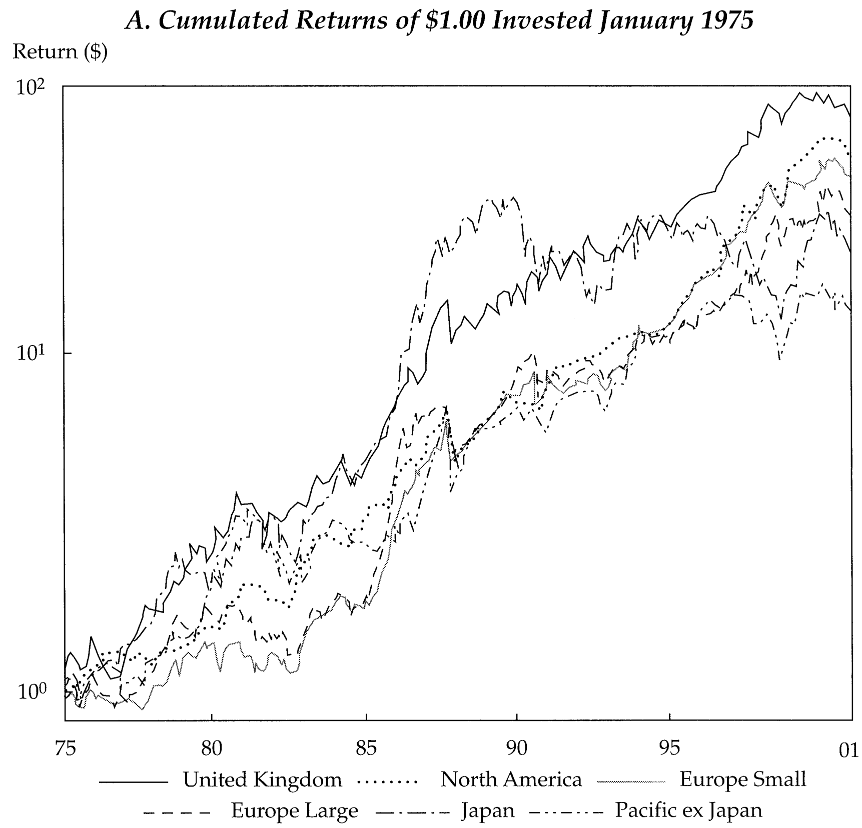
The estimation procedure also yielded inferences about classifying the prevailing regime in each month. **Figure 2** shows the cumulative (total) returns of the six markets over the sample period in Panel A and, in Panel B, the *ex ante* and smoothed regime probabilities. The *ex ante* probability is the probability, given current information, that the regime next month will be the low-volatility world market regime; the smoothed probability is the probability, given all of the information present in the data sample, that the regime next month will be the low-volatility regime. High-volatility bear markets are notable in the early 1980s, the period right

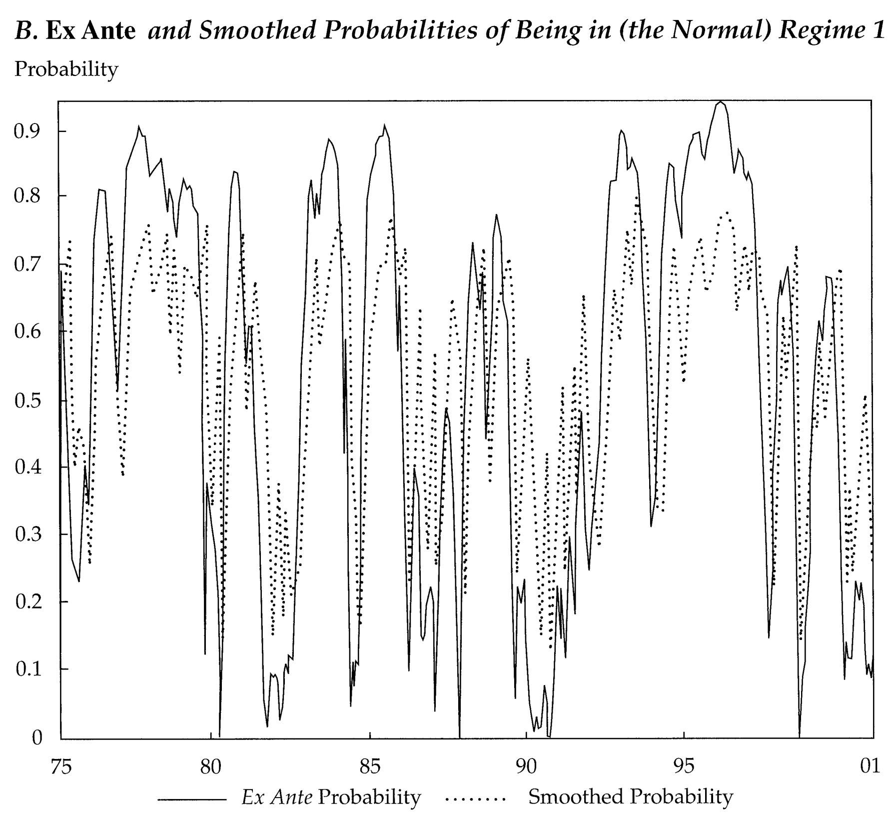
Table 2. RS Equity Model Estimation Results, 1975–2000

Regime	North America	United Kingdom	Japan	Europe: Large	Europe: Small	Pacific ex Japan
A. Regime-dependent excess returns						
Regime 1	9.64	9.76	9.90	9.65	9.65	9.67
Regime 2	3.47	2.54	1.42	3.36	3.39	3.22
B. Regime-dependent covariances and correlations						
Regime 1						
North America	1.35	0.44	0.48	0.45	0.54	0.38
United Kingdom	0.90	3.08	0.37	0.35	0.42	0.29
Japan	1.06	1.25	3.60	0.38	0.46	0.32
Europe: Large	0.79	0.92	1.08	2.30	0.43	0.30
Europe: Small	0.78	0.91	1.07	0.80	1.53	0.36
Pacific ex Japan	0.81	0.94	1.11	0.82	0.82	3.33
Regime 2						
North America	2.37	0.64	0.68	0.65	0.73	0.58
United Kingdom	2.10	4.49	0.58	0.55	0.63	0.49
Japan	2.47	2.89	5.53	0.58	0.66	0.52
Europe: Large	1.83	2.14	2.52	3.36	0.63	0.49
Europe: Small	1.82	2.13	2.50	1.85	2.58	0.56
Pacific ex Japan	1.88	2.20	2.58	1.91	1.90	4.45
C. Tangency portfolio weights						
Regime 1	0.42	0.06	–0.01	0.15	0.31	0.08
Regime 2	0.79	–0.14	–0.55	0.25	0.54	0.10
Unconditional	0.52	0.04	–0.16	0.18	0.37	0.09
Average market cap	0.50	0.09	0.22	0.08	0.08	0.02

Notes: All numbers are annualized. Correlations in Panel B are shaded. Variances and covariances are in percentages and correspond to annual volatility numbers. For Panel C, we computed the portfolio weights by using an interest rate of 7.67 percent, which is the average one-month T-bill rate over the sample period. The "Average market cap" is the monthly market cap averaged across the sample.

Figure 2. Cumulated Historical Returns and *Ex Ante* and Smoothed Probabilities of the International Equities Model, 1975–2000





after the October 1987 crash, the early 1990s, and a period in 1999. Overall, the stable (or unconditional) probability of the normal regime is 53 percent.

Asset Allocation. This section describes mean–variance asset allocation that takes into account regime switching and illustrates how to implement an RS model in practice for the developed equity markets.

Mean–variance optimization under regime switching. To implement an asset allocation strategy, we used mean–variance optimization with monthly rebalancing, which is consistent with the data frequency.[6] The standard optimal mean–variance portfolio vector in regime i, $w(i)$, is given by

$$w(i) = \frac{1}{\gamma}\sum(i)^{-1}e(i), \tag{3}$$

where

γ = investor's risk aversion

$\sum(i)$ = covariance matrix associated with regime i

$e(i)$ = vector of conditional means for regime i

We can implement mean–variance optimization in a number of ways. The first issue is to specify the risk-free rate. In this application, for each month, we took the one-month T-bill rate to be the risk-free rate. Hence, the risk-free rate varied over time as we implemented the asset allocation program. Our RS model provides two optimal tangency (all-equity) portfolios the investor would choose, one for each regime. (An extension of this framework is to add state dependence by using predictor variables for equity returns. The second application in this article illustrates this possibility.) In the results we report, we focus on these all-equity portfolios, which did not depend on the risk-free rate or risk aversion.

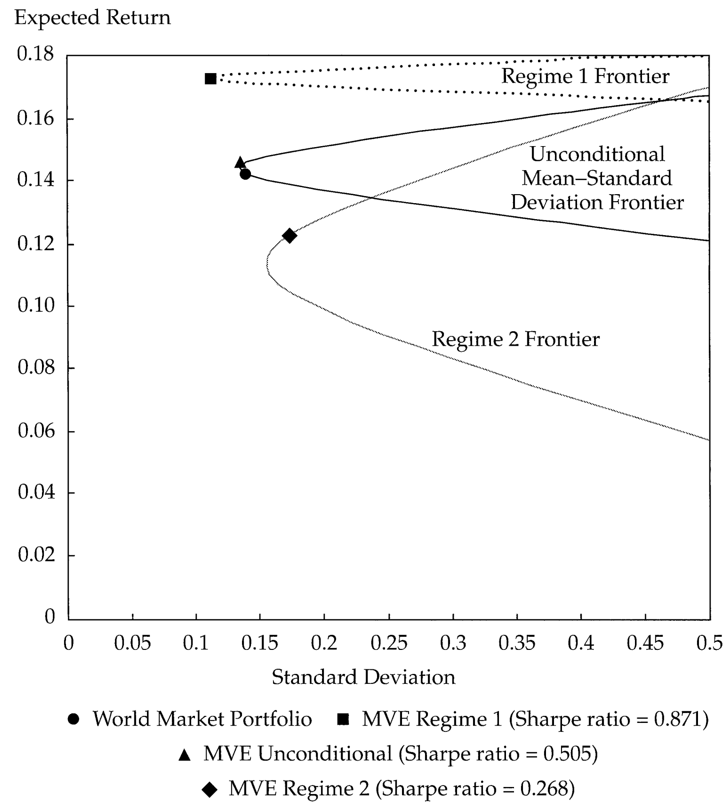
The second issue is that mean–variance portfolios based on historical data may be quite unbalanced, as Green and Hollifield (1992) and Black and Litterman (1992) emphasized. Practical asset allocation programs, therefore, impose constraints (short-sale constraints, for example) or keep asset allocations close to market-capitalization weights. Although constraints can be imposed in our application, we chose not to do so but to show how the performance of our RS model compares with the performance of a mean–variance asset allocation program based on historical moments in an out-of-sample exercise.

Panel C of Table 2 shows the tangency portfolios in Regime 1 and Regime 2 based on the returns, volatilities, and covariances/correlations in Panels A and B. In the normal Regime 1, the model tells the investor to place 42 percent of portfolio wealth in North American assets, which is not far from the average relative market cap for the sample period. The European and Pacific indexes are overweighted relative to their market caps, but the U.K. and Japanese markets are underweighted because of the implied high volatility of these markets. (The allocation even calls for a small short position in the Japanese market in Regime 1.) In Regime 2, the investor following our model resolutely switches toward the less-volatile markets, which include North America. The portfolio is not home biased for a U.S. investor, however, because the investor also invests heavily in the European markets, allocating more than 50 percent of wealth to the small countries of Europe. The short position in Japan is now substantial, exceeding 50 percent.

Figure 3 shows the essence of the implications of RS for asset allocation. The dark solid line represents the mean–standard deviation frontier when the unconditional moments are used and regime switches are ignored. The other frontiers are the ones applicable in the two regimes. The frontier near the top of Figure 3 is for the normal regime, Regime 1. The risk–return trade-off is generally better in the normal regime than for the unconditional frontier because the investor is taking into account that, because the regime is persistent, the likelihood of a bear market with high volatility in the next period is small. The Sharpe ratio available along the capital allocation line (the line that would emanate from the risk-free rate on the vertical axis tangent to the Regime 1 frontier) is 0.871. In the bear market, the risk–return trade-off (represented by the shaded line) deteriorates, so the investor selects a very different portfolio that has a Sharpe ratio of only 0.268. When the moments in the two regimes are averaged, the result is the unconditional frontier implied from the RS model (the dark solid line). The best possible Sharpe ratio for this frontier is 0.505. Note that the world market portfolio (based on average market-cap weights) is inefficient; it is inside the unconditional frontier.

Theoretically, the presence of two regimes and two frontiers means that the RS investment opportunity set dominates the investment opportunity set offered by a single unconditional frontier. In particular, in Regime 1, the unconditional tangency portfolio yields a Sharpe ratio of 0.619. The investor could improve this trade-off to 0.871 by holding the optimal tangency portfolio for this low-variance regime. In Regime 2, the unconditional tangency portfolio yields a Sharpe ratio of only 0.129, which could be improved to 0.268 by holding the optimal tangency portfolio for the high-variance regime.

Figure 3. Mean–Standard Deviation Frontiers, 1975–2000 Data



Note: Mean and standard deviation annualized.

Practical implementation. We carried out an out-of-sample demonstration of asset allocation based on the RS model. The RS model was estimated up to time t, and the RS-dependent weights were computed from information available only up to time t. The test started with a $1.00 investment in 1985. The model was reestimated every month. For comparison with a standard mean–variance optimization, we investigated a non-regime-dependent strategy that incorporated means and covariances estimated from data up to time t. The performance criterion was the *ex post* Sharpe ratio realized by the various strategies.

The RS strategy required the risk-free rate and the realization of the regime. For the risk-free rate, we simply used the available one-month T-bill rate. To infer the regime, we had the investor compute the regime probability from current information, which is a by-product of estimating the RS model. If the regime probability was larger than 1/2 for Regime 1, the investor classified the regime as 1, otherwise it was classified as 2. This calculation did not require any further data input.

Table 3 reports the portfolio results. Over the out-of-sample period, the RS portfolio's Sharpe ratio was more than double the Sharpe ratio for the out-of-sample world market portfolio and also higher than the Sharpe ratio for the non-regime-dependent portfolio. The RS strategy did so well because in this sample period, the U.S. market recorded huge returns. In fact, U.S. equity's Sharpe ratio for the period was 0.65! At the same time, Japan performed poorly, and the world market portfolio featured a relatively large Japanese equity

Table 3. Out-of-Sample Performance of All-Equity Portfolios, 1985–2000

Measure	World Market	North America	Regime-Dependent Performance	Non-Regime-Dependent Performance
Mean return (%)	13.73	15.84	21.46	20.04
Standard deviation (%)	14.86	15.21	14.51	15.67
Sharpe ratio	0.52	0.65	1.07	0.90

Note: Returns and standard deviations are annualized.

allocation. In the normal regime, the all-equity portfolio for the RS model had a large weight in North American equity (see Panel C of Table 2). In the bear market, the RS strategy had a large short position in Japanese equities.

Figure 4 shows how wealth accumulated over time in these strategies. The large North American and the short Japanese positions led both the RS strategy and the non-regime-dependent strategy to outperform the world market and the North American market consistently. The outperformance is particularly striking for the last five years. It is also over the last five years that the RS strategy outperformed the non-regime-dependent strategy particularly successfully.

Given that our results in this example may be intimately linked to a (perhaps special) historical period, we do not want to claim that the success of the RS strategy shown here is necessarily a good indicator of future success. For example, not all investors would feel comfortable with the relatively large short positions implied by the model. The important conclusion is that RS strategies have the potential to outperform because they set up a defensive portfolio in a bear market that hedges against high correlations and low returns. This conclusion remains valid in the presence of short-sale constraints because this portfolio essentially tilts the allocations toward the lowest-volatility assets. Moreover, the RS portfolio need not be completely home biased; in our example, it still involved substantial net international positions. In any practical implementation of an RS model, which would rely less on historical moments or be based on a different sample period, the optimal portfolios are likely to be even more internationally diversified. This was the case in our earlier study (see Ang and Bekaert 2002a).

RS Market-Timing Model

When a volatile bear market is expected, the optimal asset allocation may be to switch to a safe asset or a bond. The model we explore in this section considers asset allocation among three assets—cash, a 10-year (constant-maturity) bond, and an equity index (all for the United States). We formulate the model in excess returns. We use r_t to denote the risk-free rate (the nominal T-bill rate), r_t^b to denote the excess bond return, and r_t^e to denote the excess return on U.S. equity.

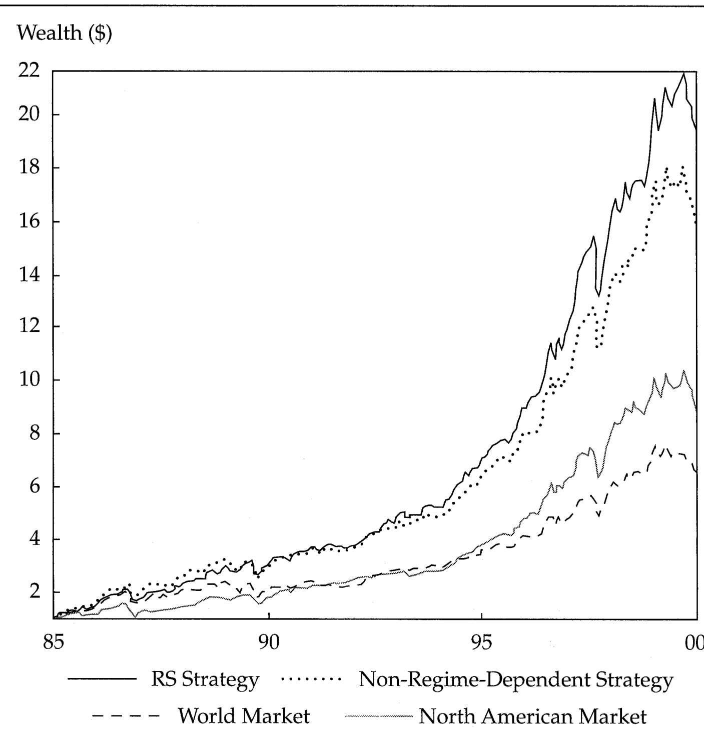
The market-timing model is

$$r_t = \mu^r(i) + \rho(i)\, r_{t-1} + \varepsilon_t^1; \quad (4a)$$

$$r_t^b = \mu^b + \varepsilon_t^2; \quad (4b)$$

$$r_t^e = \mu^e + \varepsilon_t^3. \quad (4c)$$

Figure 4. Out-of-Sample Wealth for Various Markets or Strategies, 1985–2000



Note: Accumulation of $1.00 invested on 1 January 1985.

This model allows the short rate to exhibit different behavior during each regime *i*. The error terms, $\varepsilon_t = (\varepsilon_t^1 \varepsilon_t^2 \varepsilon_t^3)'$, are drawn from a normal distribution with zero mean but with a covariance that switches between the regimes, so the conditional volatility of all assets is regime dependent.

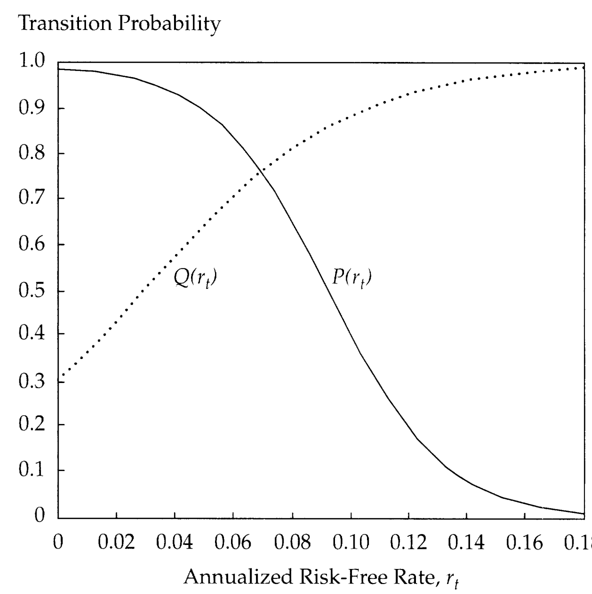
The short rate follows an autoregressive process, but the constant term, $\mu^r(i)$, and the autoregressive parameter, $\rho(i)$, depend on the regime. Many articles (such as Ang and Bekaert 2002b) have demonstrated that the data support such a model, in which interest rates are highly persistent [$\rho(1)$ is close to 1] and not very variable in a normal regime and another regime captures times of higher, volatile interest rates that revert quickly to lower rates [$\rho(2) < \rho(1)$].

In the RS equity model described in the section "RS Model for Equity Portfolios," the transition probabilities between the regimes, P and Q, were constant. In our market-timing model, we allow the interest rate to influence the transition probabilities. Hence, P_t and Q_t are now time varying.[7] This means that the probability of staying in the "normal" or "bad" regime may be different when interest rates are high from when interest rates are low. Consequently, the short rate predicts transitions in the regime, and hence, it implies time variation in expected returns. The predictive power of nominal interest rates for equity premiums has a long tradition in finance—going back at least to Fama and Schwert (1977): When short-term interest rates are low, subsequent equity returns tend to be high. Most studies, however, allow only linear predictability, entering through the conditional mean. If we allow the conditional means of excess bond and equity returns to become regime dependent and also allow the lagged short rate to enter the conditional mean, these coefficients are estimated with little precision. We cannot reject our model relative to this more intricate specification.

Estimation Results. The first regime is a normal regime in which the short rate is nearly a random walk [$\rho(1)$ is approximately 0.99], shocks to the interest rate are not very variable (standard deviation of 0.02 percent a month), and shocks to excess bond and equity returns have little volatility (standard deviations of, respectively, 1.75 percent and 3.41 percent a month). In the second regime, interest rates are high, rapidly mean reverting, and volatile. Here, the short rate is less persistent [$\rho(2)$ of 0.94] and interest rates have a conditional volatility of 0.09 percent a month. Shocks to bond and equity returns are also much more volatile, with standard deviations of, respectively, 3.98 percent and 5.55 percent a month. The mean for the excess bond return is 0.07 percent a month, and the mean excess equity return is 0.68 percent a month.

Figure 5 graphs the transition probability functions as a function of the annualized risk-free rate. Keep in mind that P is the probability, given that the markets are currently in Regime 1, of staying in Regime 1. As interest rates rise, the probability of transitioning into the high-volatility market increases. The other probability, Q, is the probability, given that one is currently in Regime 2, of remaining in Regime 2. In the high-volatility regime, as interest rates move higher, the probability of staying in this regime increases. A constrained model in which P and Q are constant is strongly statistically rejected. Hence, nonlinear predictability is an important feature of the data. The long-run probability of the normal regime implied by the model is 0.7014.

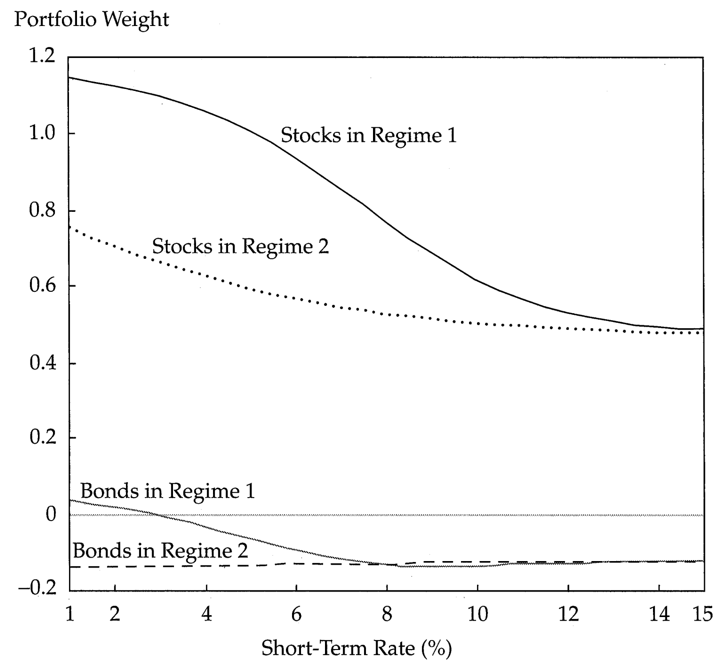
Figure 5. Transition Probabilities of the Market-Timing Model



Mean–Variance Market Timing. In this part of the implementation, we follow the same mean–variance strategy as for RS asset allocation among international equities, except that the optimal asset allocation vector is now a function of the expected excess returns on the two risky assets, bonds and equity, and their covariance.[8]

To obtain intuition on the asset allocation weights for this model, **Figure 6** graphs the optimal asset allocations to bonds and stocks (which add to 1 minus the weight assigned to the risk-free asset) as a function of the short-term rate at the estimated parameters. We set the risk aversion level, γ, to 5. In Regime 1, if interest rates are low enough, the

Figure 6. Asset Allocation of the Market-Timing Model as a Function of the Short Rate



Note: Risk aversion = 5.

investor borrows at the risk-free rate and invests a small fraction of the portfolio in bonds and more than 100 percent in equities. As interest rates rise, equities become less attractive as the probability of switching to the high-variance regime increases. Bonds also become less attractive, and because the bond premium is small, it quickly becomes optimal to short bonds. In the second regime, the investor always shorts bonds but the investment in equities is never higher than 80 percent. The main hedge for volatility is clearly the risk-free asset, not a bond investment.

Because the interest rate is so important in this model, the optimal asset allocation varies over time with different realizations of the interest rate. **Figure 7** shows optimal asset allocation weights, under the assumption that the investor uses the moments implied by the full sample estimation for all three assets across time for 1952–2000. Note that during the 1987 crash, the investor would have been heavily invested in equity. After the crash, the investor would have shifted this equity portion into risk-free holdings. Importantly, the model-recommended weights show only infrequent large changes in asset allocation. These changes coincide with regime changes. Because interest rates are relatively smooth and persistent, the month-to-month changes in asset allocation are generally modest.

Out-of-Sample Model Performance. In this implementation of the market-timing model, as in the out-of-sample test of the model's performance in international equity allocation, the investor begins with $1.00 in 1985. We show the mean return, volatility, and Sharpe ratio that result from following the optimal RS strategies for the market-timing model and compare them with a strategy that simply uses unconditional moments. The results are reported in **Table 4**. The market-timing model's strategy is more volatile but delivers higher average returns than a non-regime-dependent strategy. The market-timing model is the best performing model in terms of Sharpe ratios, but Sharpe ratios are quite low for the highly risk averse investors.

Figure 8 shows that the superior performance is not because of a few isolated months in the sample but that the last five years did play an important role in giving the RS strategies an edge. During these years, the market-timing model allocated more money than the non-regime-dependent strategy to equity and benefited handsomely from the U.S. bull market. The RS strategy's positions were more leveraged, however, and although they had higher returns, they also had higher volatility.

Figure 7. Asset Allocation of the Market-Timing Model over Time, 1952–2000

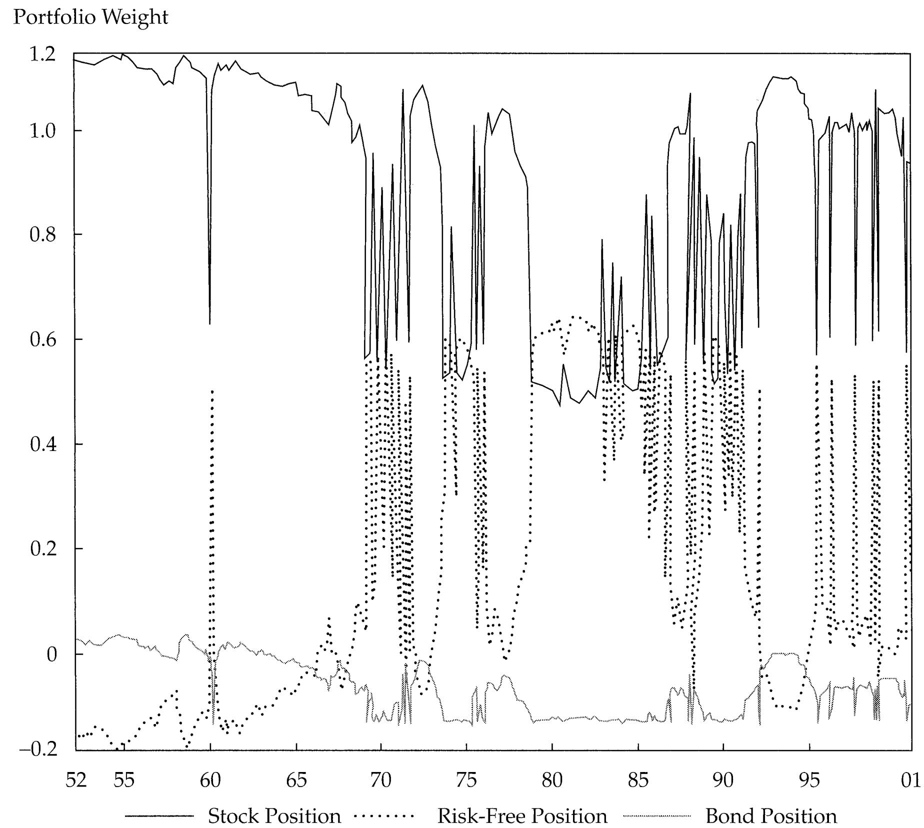


Table 4. Out-of-Sample Portfolio Allocations and Performance of Market-Timing Example, 1985–2000

	Risk Aversion, γ				
Measure	2	3	4	5	10
A. Regime-dependent allocations					
Mean return (%)	25.29	17.69	13.89	11.61	7.05
Standard deviation (%)	34.53	23.02	17.27	13.82	6.91
Sharpe ratio	0.58	0.54	0.50	0.47	0.27
B. Non-regime-dependent allocations					
Mean return (%)	17.65	12.60	10.07	8.55	5.52
Standard deviation (%)	26.25	17.50	13.13	10.50	5.26
Sharpe ratio	0.48	0.42	0.37	0.32	0.07

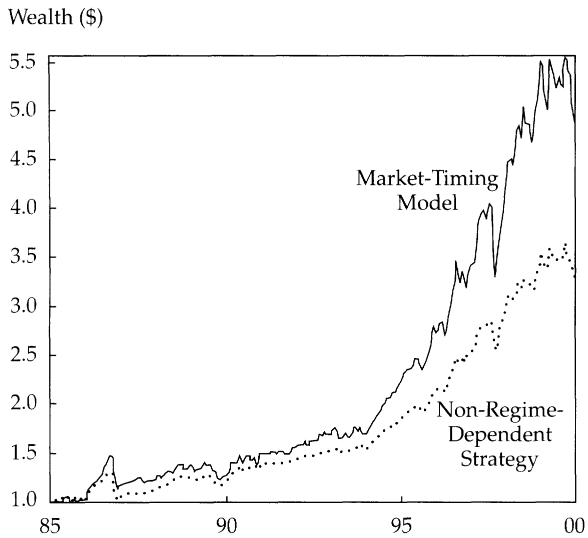
Note: All returns are annualized.

Conclusion

The academic literature contains much evidence that expected returns and volatility vary through time. Moreover, in high-volatility environments across the world, not only do equity returns perform poorly but they also become more highly correlated. Therefore, active portfolio managers should be able to exploit these changes between low- and high-volatility regimes to add value.

We showed how such exploitation can be formally accomplished through a regime-switching model. Our results are meant to be illustrative. On the one hand, we exaggerated the performance of the models because we did not take transaction costs into account. Of course, the RS strategies are relatively robust to transaction costs because they are designed to exploit low-frequency changes in expected returns and volatilities. Because the

Figure 8. Out-of-Sample Wealth for the Market-Timing Model, 1985–2000



Notes: Accumulation of $1.00 invested on 1 January 1985. Risk aversion = 5.

probability of staying within the same regime is relatively high, portfolio turnover is low. On the other hand, we greatly undersold the potential of RS models because we did not try to estimate the best possible model, do an extensive model search, or incorporate performance-enhancing constraints.

Our current results point to two robust conclusions. First, a global manager can add value in all-equity portfolios; the presence of a bear market (a high-correlation regime) does not negate the benefits of international diversification. Although recommended portfolios in that regime are more home biased, they still involve significant international exposure. Second, RS models are very valuable in tactical asset allocation programs that allow switching to a risk-free asset.

A long list of extensions that are likely to improve performance can be accommodated in this framework. First, managers of equity portfolios are typically compensated on the basis of tracking error relative to an index. Therefore, active managers often start from a benchmark, as in Black and Litterman, and deviate from the benchmark allocations toward the predictions of a proprietary model. Instead of expected returns reverse-engineered from an index, we used only historical data.

Second, in international asset allocation, managers often hedge equity benchmarks against currency risk. We have shown (Ang and Bekaert 2002a) that the RS equity model can be extended to allow both currency-hedged and nonhedged returns. The asset allocation model yields the optimal currency hedge ratio.

Third, we used only one regime variable, but it would be interesting to accommodate country-specific regimes and imperfectly correlated regimes in short-term rates and equity returns.

Finally, in the optimization here, we focused on only first and second moments (expected returns and standard deviations), but many investors prefer positive skewness and dislike kurtosis. RS models have nontrivial higher-order moments (the model can be interpreted as a mixture of normal models with time-varying weights). So, for investors with preferences involving higher-order moments of returns, RS models are a viable alternative.

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Notes

1. See also Erb, Harvey, and Viskanta (1994); Campbell, Koedijk, and Kofman (2002).
2. See Gray (1996); Bekaert, Hodrick, and Marshall (2001); Ang and Bekaert (2002b, 2002c).
3. See Black, Jensen, and Scholes (1972) for an early example.
4. The expected return for asset j with beta β^j in regime i is $e^j(i) = (1 - \beta^j)\mu^z + \beta^j e^w(i)$. There are two possible variance matrixes for unexpected returns next period; they are given by $\Omega^j(i) = (\beta^j)^2[\sigma^w(i)]^2 + [\bar{\sigma}^j]^2$ for $i = 1, 2$. The conditional variance of asset j in Regime 1 is then $[\sigma^j(1)]^2 = P\Omega^j(1) + (1 - P)[\Omega^j(2)] + P(1 - P)[e^j(2) - e^j(1)]^2$, and the conditional variance of asset j in Regime 2 is $[\sigma^j(2)]^2 = (1 - Q)\Omega^j(1) + Q[\Omega^j(2)] + Q(1 - Q)[e^j(2) - e^j(1)]^2$.
5. See Hamilton (1994) and Gray (1996) for estimation methods for RS models that use maximum-likelihood techniques.
6. Because the first and second moments of our model vary through time, investors with different horizons might hold different portfolios. Brandt (1999) and Ang and Bekaert (2002a) showed, however, that the differences among these portfolios are not large, so we have ignored them here.
7. Specifically, we set P_t equal to $[\exp(a_1 + b_1 r_t)]/[1 + \exp(a_1 + b_1 r_t)]$ and Q_t equal to $[\exp(a_2 + b_2 r_t)]/[1 + \exp(a_2 + b_2 r_t)]$.
8. The determination of conditional expected returns and variances is similar to the procedure in the section that describes the RS model for international equities, except that the transition probabilities now vary over time.

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