QUIZ 2 - MACS 441
Fall 2010

You have to show your work to get credit for any problem!
One page of handwritten notes allowed
Please sign each page

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1. Consider the following four points in 2D below: $P_0 = (2, 2)$, $P_1 = (10, 10)$, $P_2 = (10, 20)$ and $P_3 = (2, 28)$. Find the cubic Bezier curve, B(t), corresponding to $\{P_0, P_1, P_2, P_3\}$ at $t = 0.5$.

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2. Let's say you have a regular grid of samples, located at points with integer coordinates, i.e. (x, y) with x and y integer. The value at sample (i, j) is $i^2 + j^2$. You are using bilinear interpolation to compute the value at $(1.5, 1.5)$. What is the result?

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3. Apply one subdivision step to the cubic B-spline curve with control points $(0, 0)$, $(4, 4)$, $(8, 8)$, $(12, 8)$, $(16, 4)$, $(20, 0)$. Write out the resulting sequence of control points.

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4. Write a pseudocode for a routine that would take a vertex v of a planar subdivision as the argument and output out all half-edges out of v efficiently (i.e. in linear time in the number of such half-edges). Use the standard half-edge notation. For a half edge h:
$h.o$ is the opposite half-edge
$h.p$ is the previous half-edge
$h.n$ is the next half-edge
$h.s$ is the starting vertex.
For a vertex v, $v.h$ is a half-edge out of v.
Output a half edge v with '`output` h'.

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5. Let's say you are given a 2010-triangle manifold surface of genus 0. You apply one Loop subdivision step to that surface (note that this does not change the genus). How many triangles, edges and vertices does the resulting surface have?

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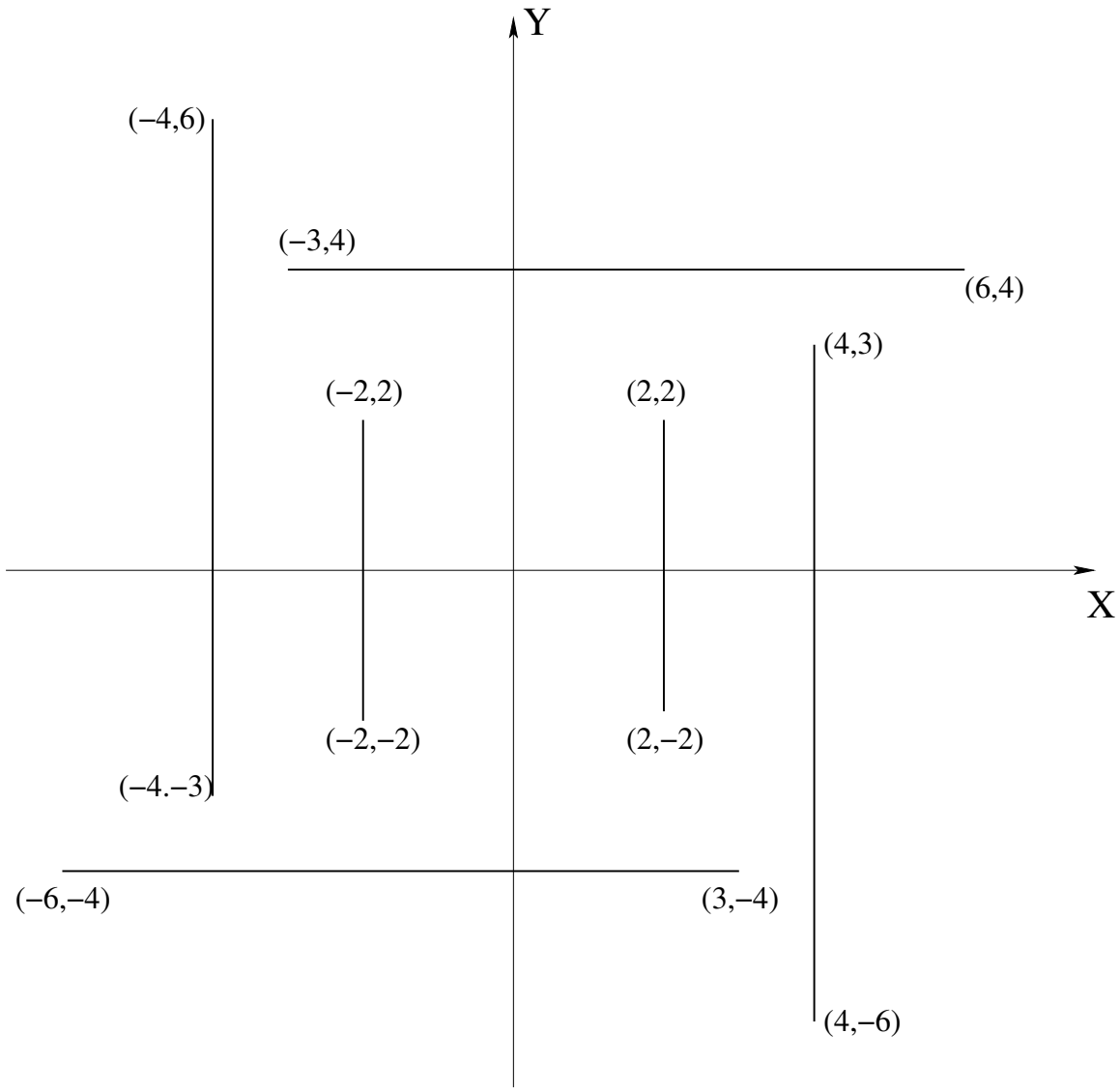
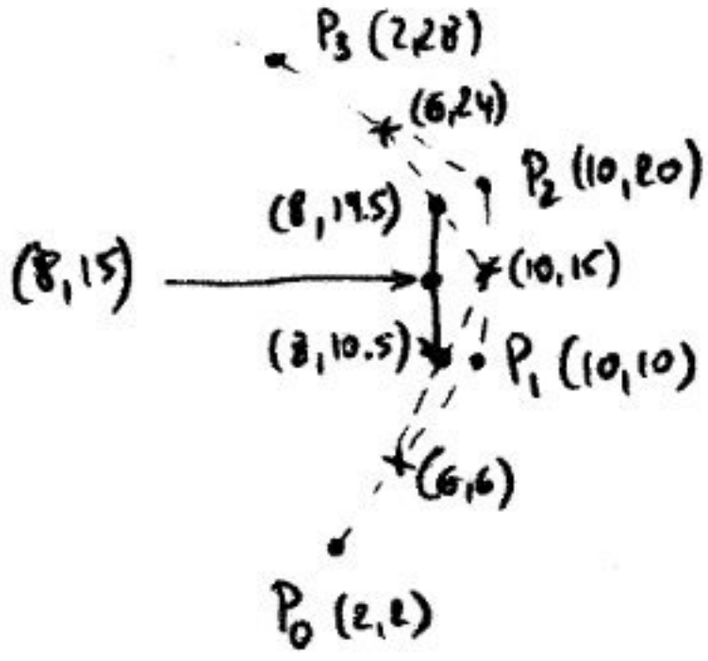
6. Draw a BSP tree for the set of following six segments. Your answer should be a tree with intervals at the nodes. Represent each interval as a pair of endpoints. See next page for a drawing of the segments. Note that each of them is either vertical or horizontal, which simplifies intersection calculations.


Y
X
(−4,6)
(−3,4)
(6,4)
(4,3)
(−2,2)
(2,2)
(−2,−2)
(2,−2)
(−4.−3)
(−6,−4)
(3,−4)
(4,−6)

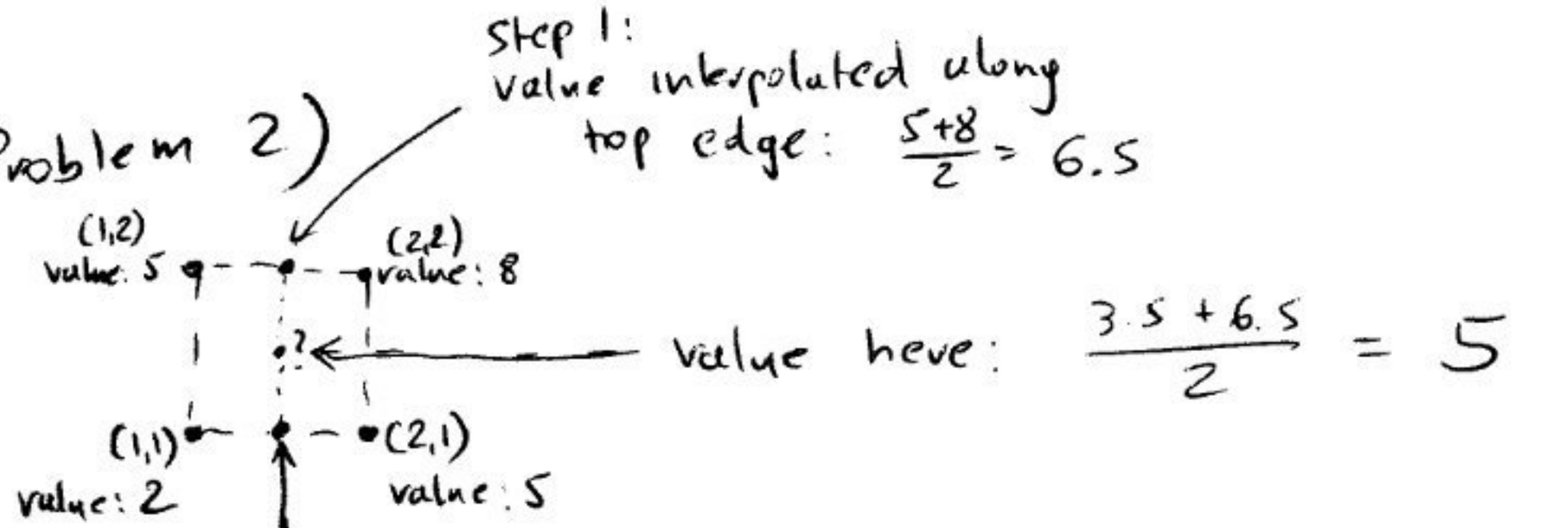
Problem 1)

Here is a solution by de Casteljau algorithm:



Answer: (8, 15)

Problem 2)



Answer: 5

Problem 3)

Subdivision for cubic B-splines = doubling followed by 3 iterations of averaging.

(0,0) (4,4) (8,8) (12,8) (16,4) (20,0) → double

(0,0) (0,0) (4,4) (4,4) (8,8) (8,8) (12,8) (12,8) (16,4) (16,4) (20,0) (20,0) → average

(90) (2,2) (4,4) (6,6) (8,8) (10,8) (12,8) (14,6) (16,4) (18,2) (20,0) → average

(1,1) (3,3) (5,5) (7,7) (9,8) (11,8) (13,7) (15,5) (17,3) (19,1) → average

(2,2) (4,4) (6,6) (8, 7.5) (10,8) (12, 7.5) (14,6) (16,4) (18,2) ← ANSWER.

Problem 4)

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h := v.h
do
  output h
  h := h.p.o
while h ≠ v.h.
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... or something equivalent

Problem 5) Subdivision increases the number of triangles by a factor of 4. So, the subdivided surface has 8040 triangles. Using the standard notation, the usual counting argument and the Euler's formula:

$$3T = 2E$$

$$T - E + V = 2$$

Since $T = 8040$, $E = 12060$ and $V = 2 + E - T = 4022$

triangles: 8040, # Edges: 12060, # vertices: 4022

Problem 6) There are many correct answers here. Here is one: