TEST 2 - MACS 441
Fall 2012
You have to show your work to get credit for any problem!

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1. Let the value at a sample point (i, j) with integer coordinates be i^2. Let f be the function obtained from these values using bilinear interpolation.

(i) Compute $f(4/3, 4/3)$, $f(4/3, 5/3)$, $f(5/3, 4/3)$ and $f(5/3, 5/3)$.

(ii) What kind of function is f? Is it piecewise linear, quadratic or cubic?

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2. In project 2, we used a photograph of a mirror sphere (taken from far away) to render fine-looking approximations of reflections from more interesting mirror objects, as seen from the camera's location. Can the photograph of a mirror sphere be replaced with:

(i) A photo of a mirror cube?

(ii) A photo of a mirror ellipsoid?

(iii) A photo of a mirror cylinder?

In all cases, assume that the orientation in space of the objects being photographed is precisely known. You don't have to explain the implementation details, just whether we have enough information in the texture to render something comparable to the result for the standard spherical environment map.

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3. Let's say you want to connect the following three Bezier segments into a smooth curve. Where does p need to be placed?

Segment A (cubic) : control points $(-1, -1)$, $(-1, 1)$, $(0, -1)$, $(0, 0)$

Segment B (quadratic) : control points $(0, 0)$, p, $(1, 0)$

Segment C (cubic) : control points $(1, 0)$, $(2, -2)$, $(3, 0)$, $(2012, 1221)$

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4. Let p be a 2D point and $\vec{v}$ a 2D vector. Define $P_i := p + i\vec{v}$ for $i = 0, 1, \ldots, 100$. Thus, P_i are equispaced points on a ray starting at p and with the direction vector $\vec{v}$. Describe the following curves:

(i) B-spline of degree 1 with control points $P_0, \ldots, P_{100}$

(ii) B-spline of degree 2 with control points $P_0, \ldots, P_{100}$

(iii) B-spline of degree 3 with control points $P_0, \ldots, P_{100}$

In particular, explain what $B_d(t)$ is and what is the domain of B_d (i.e. the interval of parameters t for which $B(t)$ is defined). Also, figure out where the curve starts and where it ends.

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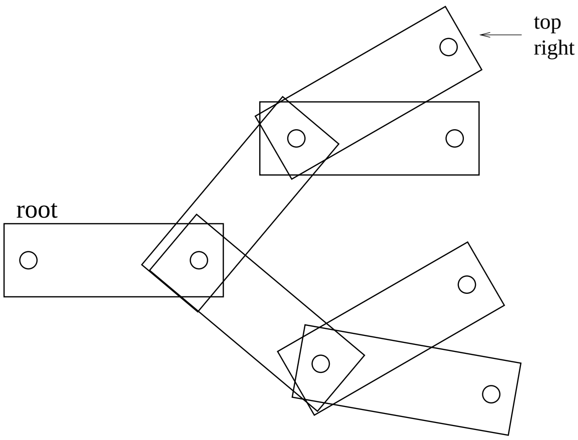
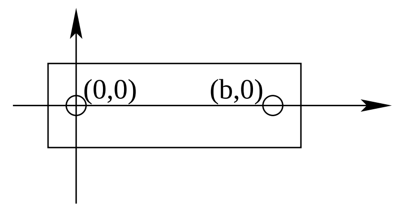
5. Let's say you want to model the articulated object shown on the left from rectangular parts (all of the same shape) shown on the left using a scene graph. Assume that the parts can rotate with respect to each other arbitrarily at the joints.





(i) Draw the scene graph you would use for this articulated object. Build it so that each of the 7 rigid pieces is represented by a separate node. Also, use the piece with "root" next to it as the root.

(ii) Include transformations at the edges of your scene graph, depending on reasonable parameters (most likely, representing relative poses of the pieces in some way). Make sure that all configurations can be represented using your parameters. When specifying transformations, use $T_{(s,t)}$ to denote translation by $[s,t]$ and R_α for rotation by α around the origin.

(iii) Explain how transformation that transforms the top right piece into the coordinate system of the root piece can be obtained from your scene graph. Write a formula for it (as a superposition of rotations and translations).

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6. Let's say you want to render a number of triangles with shadows using the shadow volume algorithm. Assuming the light source location is $l = (l_x, l_y, l_z)$, give a pseudocode of the algorithm that outputs a vertex stream suitable for rendering shadow polygons (in the triangle soup mode) for a single triangle with verteces $a = (a_x, a_y, a_z)$, $b = (b_x, b_y, b_z)$ and $c = (c_x, c_y, c_z)$. You can use standard vector operations (cross product, dot product etc) in your pseudocode. Also, use "output (x,y,z,h)" to output a vertex in homogenous coordinates.

What we want is a procedure that can be run for all triangles to produce a vertex stream that defines (if used with `GL_TRIANGLES` mode) correctly oriented shadow polygons for the entire scene.

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7. What is the result of one step of cubic B-spline subdivision for the sequence of control points $(0,0)$, $(0,8)$, $(8,8)$, $(8,0)$, $(16,0)$? Your answer should be a sequence of control points (that defines identical-looking B-spline curve).
