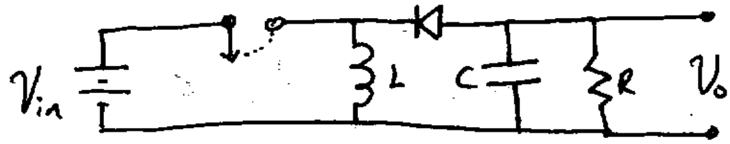
6.331 Pset 4 Josh Gordonson

1)




$L = 150\,\mu H$

$C = \cancel{8}20\,\mu F$

$R = 50\,\Omega$

a)

$\Delta I_{L_{op}} = -\Delta I_{L_{cl}}$ in steady state

$V_L = L\frac{dI_L(t)}{dt} \rightarrow i_L(t) = \int_0^t \frac{V_L}{L}dt \rightarrow \Delta i_{L_{open}} = \int_0^{DT} \frac{V_L}{L}dt = \frac{V_L DT}{L} = \frac{V_{in}DT}{L}$

$\Delta i_{L_{closed}} = \int_{DT}^{T} \frac{V_L}{L}dt = \frac{V_{out}(1-D)T}{L}$

$-\Delta i_{L_{closed}} = \Delta i_{L_{open}} \rightarrow \boxed{\frac{V_{out}}{V_{in}} = \frac{-D}{1-D}}$

Up Converter: $D > 0.5 \rightarrow \left|\frac{V_o}{V_i}\right| > 1$ Down Converter: $D < 0.5 \rightarrow \left|\frac{V_o}{V_i}\right| < 1$

b) $\langle P_{in}\rangle = \langle P_{out}\rangle$

$V_{in}\langle I_{in}\rangle = \frac{\langle V_{out}\rangle^2}{R}$

$\langle I_{in}\rangle = \frac{\langle V_{out}\rangle^2}{V_{in}R}$

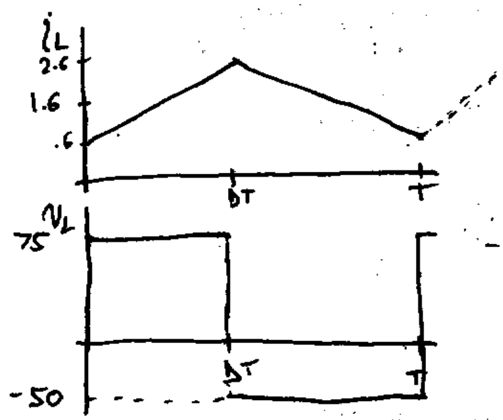
$\frac{DT\langle I_L\rangle}{T} = \langle I_{in}\rangle \rightarrow \langle I_L\rangle = \frac{\langle V_{out}\rangle^2}{V_{in}RD}$


$\boxed{\langle I_L\rangle = \frac{50^2}{75\cdot 50\cdot 0.4} = \frac{2}{3}\cdot\frac{1}{.4} = 1.6A}$

$\boxed{\langle I_{in}\rangle = .6\,A}$

$\boxed{\langle V_{out}\rangle = 50v}$

$\boxed{\Delta V_{out_{closed}} = \Delta V_{out_{open}} = \frac{\langle V_{out}\rangle}{RC}DT = 200mV}$



c)

$$L\frac{d\langle i_L\rangle}{dt} = \langle V_L\rangle = D V_i + (1-D)\langle V_c\rangle$$

$$C\frac{d\langle V_c\rangle}{dt} = \langle I_d\rangle - \frac{\langle V_c\rangle}{R_L} = (1-D)\langle i_L\rangle - \frac{\langle V_c\rangle}{R_L}$$

$$C\frac{d^2\langle V_c\rangle}{dt^2} = \frac{d\langle i_L\rangle}{dt} - \frac{dD}{dt}\langle i_L\rangle - D\frac{d}{dt}\langle i_L\rangle - \frac{1}{R_L}\frac{d}{dt}\langle V_c\rangle$$

$$C\frac{d^2\langle V_c\rangle}{dt^2} = \frac{(1-D)}{L}\left(DV_i + (1-D)\langle V_c\rangle\right) - \frac{dD}{dt}\langle i_L\rangle - \frac{1}{R}\frac{d}{dt}\langle V_c\rangle$$

$$\frac{d^2}{dt^2}\langle V_c\rangle + \frac{1}{RC}\frac{d}{dt}\langle V_c\rangle - \frac{(1-D)^2}{LC}\langle V_c\rangle = \frac{D(1-D)}{LC}V_{in} - \frac{\langle i_L\rangle}{C}\frac{dD}{dt}$$

$$\frac{d^2}{dt^2}v_c + \frac{1}{RC}\frac{d}{dt}v_c - \frac{(1-\bar{D}-d)^2}{LC}(V_c+v_c) = \frac{(\bar{D}+d)(1-\bar{D}-d)}{LC}V_{in} - \frac{(I_L+i_L)}{C}\frac{d}{dt}d$$

$$\frac{d^2 v_c}{dt^2} + \frac{1}{RC}\frac{d}{dt}v_c - \frac{(1-\bar{D})^2}{LC}(V_c+v_c) + \frac{2d(1-\bar{D})}{LC}V_c =$$

$$+\frac{\bar{D}(1-\bar{D})}{LC}V_{in} + \frac{d(1-2\bar{D})}{LC}V_{in} - \frac{I_L}{C}\frac{dd}{dt}$$

$\langle V_c\rangle = V_c + v_c$

$D = \bar{D} + d$

$\langle i_L\rangle = I_L + i_L$

ignore 2nd order terms

don't care about $i_L$

$1+\bar{D}^2+d^2-\bar{D}+dD-d+dD-\bar{D}-d$

$=(1-\bar{D})^2 - 2d(1-\bar{D})$

$\bar{D}-\bar{D}^2-\bar{D}d+d-d\bar{D}-d^2$

$-\bar{D}^2+\bar{D}-2\bar{D}d+d$

$\frac{V_o}{V_I} = \frac{V_c}{V_I} = \frac{-D}{1-D}$

$-DV_I = (1-D)V_c$

$V_c = \frac{-DV_I}{(1-\bar{D})}$

$$\frac{d^2 v_c}{dt^2} + \frac{1}{RC}\frac{dv_c}{dt} + \frac{(1-\bar{D})^2}{LC}v_c = \frac{2d(1-\bar{D})}{LC}V_c + \frac{\bar{D}(1-\bar{D})}{LC}V_I + \frac{\bar{D}(1-\bar{D})}{LC}V_{in} + \frac{d(1-2\bar{D})}{LC} - \frac{I_L}{C}\frac{dd}{dt}$$

$$\frac{d^2 v_c}{dt^2} + \frac{1}{RC}\frac{dv_c}{dt} + \frac{(1-\bar{D})^2}{LC}v_c = \frac{2d(1-\bar{D})}{LC}V_c + \frac{d(1-2\bar{D})}{LC}V_{in} - \frac{I_L}{C}\frac{dd}{dt} + \frac{2\bar{D}(1-\bar{D})}{LC}V_I$$

$$\left(s^2 + \frac{s}{RC} + \frac{(1-\bar{D})^2}{LC}\right)V_c(s) = \left(\frac{2(1-\bar{D})}{LC}V_c + \frac{(1-2\bar{D})V_{in}}{LC} - \frac{I_L}{C}s\right)D(s)$$

$$\frac{V_c(s)}{D(s)} = \frac{\frac{2(1-\bar{D})}{LC}\frac{(-\bar{D}V_I)}{1-\bar{D}} + \frac{(1-2\bar{D})V_{in}}{LC} - \frac{I_L}{C}s}{s^2 + \frac{s}{RC} + \frac{(1-\bar{D})^2}{LC}} =$$

$$\frac{d^2 v_c}{dt^2} + \frac{1}{RC}\frac{d}{dt}v_c - \frac{(1-\bar{D})^2}{LC}v_c + \frac{\bar{D}(1-\bar{D})}{LC}V_{In} = -\frac{2d(1-\bar{D})}{LC}V_c + \frac{\bar{D}(1-\bar{D})}{LC}V_{In} + \frac{d(1-2\bar{D})}{LC}V_{in} - \frac{I_L}{C}\frac{dd}{dt}$$

$$\frac{d^2 v_c}{dt^2} + \frac{1}{RC}\frac{dv_c}{dt} - \frac{(1-\bar{D})^2}{LC}v_c = d\left(\frac{+2\bar{D}}{LC}V_{In} + \frac{(1-2\bar{D})}{LC}V_{in}\right) - \frac{I_L}{C}\frac{dd}{dt}$$

$$\frac{V_c(s)}{D(s)} = \frac{\frac{1}{LC}V_{in} - \frac{I_L}{C}s}{s^2 + \frac{1}{RC}s - \frac{(1-\bar{D})^2}{LC}} = \boxed{\frac{\frac{V_{in}}{LC} - \frac{I_L}{C}s}{s^2 + \frac{s}{RC} - \frac{(1-\bar{D})^2}{LC}}}$$

← RHP zero, Identical to Boost Converter Transfer function.

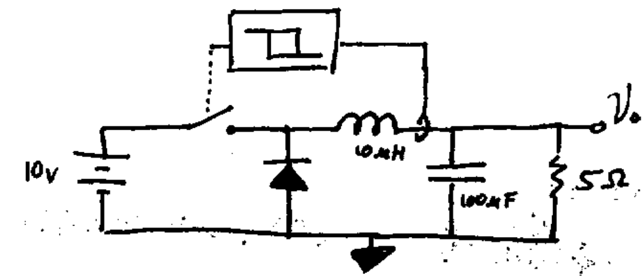
b)

$R_{on} = 1\Omega \quad V_D = 0.35V + 0.1\Omega\, i_D$

$\langle P_{sw} \rangle = D\frac{V_{sw}^2}{R} = D\, i_{L_{on}}^2 R = D\, 1.6A^2 \cdot 1\Omega = 1.025W$

$\langle P_{diode} \rangle = V_D i_D = (0.35V + 0.1\Omega \cdot i_D) i_D \rightarrow (1-D)\langle I_L \rangle (0.35V + .1\Omega \langle I_L \rangle) = .49W$

2)




a)

$$\Delta i_{L_{off}} = \int_{DT}^{T} \frac{V_{L_{off}}}{L}\,dt = \frac{(1-D)T\,V_o}{L} = \frac{0.5 \cdot 4\mu s \cdot 5}{10^{-5} H} = \boxed{1A}$$

for $f = 250\,kHz$, $T = 4\mu s$

for $\frac{V_o}{V_I} = .5$, $D = .5$

b)

$V_{out} = 7.5v \rightarrow D = \frac{7.5}{10} = .75$, $\Delta i_L = 1A \rightarrow f = \frac{(1-D)V_o}{L\,\Delta i_L} = 187.5\,kHz$

$V_{out} = 2.5v \rightarrow D = .25v$, $\Delta i_L = 1A \rightarrow f = \frac{(.75)2.5}{L\,\Delta i_L} = 187.5\,kHz$

If $V_{out} < 2.5v$, $I_{set} < 0.5A$, and the converter leaves continuous conduction mode (ccm) + enters DCM. In DCM, the hysteresis controller ~~fails~~ loses the ability to maintain a 1A Hysteresis band.

c) ~~Hysteresis Band = $I_{Band}$~~

~~$I_{set} \propto V_L = V_{in} - V_o = V_{in} - DV_{in}$~~

~~$V_{ref} \rightarrow (+) \rightarrow G(s) \xrightarrow{I_{set}} \frac{R}{V_{in}} \rightarrow \text{delay } T \xrightarrow{D} \frac{V_{in}}{D} \rightarrow \Delta I_L$~~

~~$I_L \rightarrow$ output filter $\rightarrow$~~

~~$I_{set} = \frac{V_{in} D}{R} = \frac{V_o}{R} = I_L$~~

~~$D = \frac{I_{set} R}{V_{in}}$~~

~~$\langle P_{in} \rangle = \langle P_{out} \rangle$~~

~~$V_{in}\langle I_{in} \rangle = \frac{\langle V_{out} \rangle^2}{R_L}$ $\quad V_{out} = DV_{in}$~~

~~$\langle I_{in} \rangle = \frac{D\langle I_L \rangle T}{T}$~~

~~$\langle I_L \rangle = \frac{\langle V_{out} \rangle^2 T}{R V_{in} D T} = \frac{D^2 V_{in}^2}{R D V_{in}}$~~

~~$\langle I_L \rangle = \frac{10D}{R}$~~

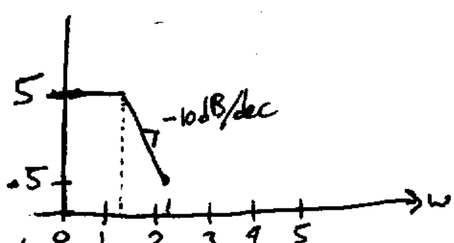
$I_{set} \rightarrow \boxed{e^{-sT}} \xrightarrow{I_L} \boxed{\text{Output filter}} \rightarrow V_o$

$Z(R \| C) \Rightarrow \frac{R}{RCs+1}$

$f = 250\,kHz$
$T = 4 \times 10^{-6} s$

$$\frac{V_o}{I_{set}} = e^{-sT}\frac{R}{RCs+1} = e^{-s(4\times10^{-6})}\frac{5}{0.5\times10^{-3}s+1}$$

$$L(s) = G(s)\frac{V_o}{I_{set}}(s) = G(s)e^{-sT}\frac{R}{RCs+1} \Rightarrow$$

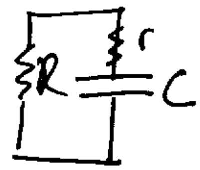



$\hookrightarrow$ G(s) needs an integrator, gain, + maybe a zero.

$$\boxed{G(s) = \frac{5\times10^{-4}s+1}{5\cdot10^{-5}s}} \rightarrow L(s) = \frac{e^{-s(4\times10^{-6})}}{10^{-5}s}$$

$$\boxed{\angle L(j10^5) = -90 - 22° = -112°}$$

2 b)

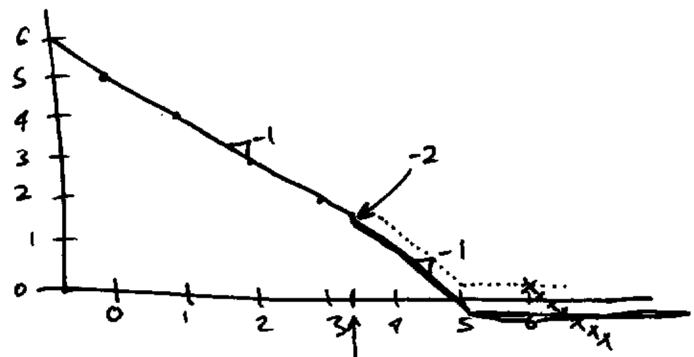



$$\cancel{R \| ESR}\quad R \| r + \frac{1}{Cs} = \frac{Rr + \frac{R}{Cs}}{R + r + \frac{1}{Cs}} = \frac{RrCs + R}{RCs + rCs + 1}$$

$$= \frac{R(rCs+1)}{(R+r)Cs+1}$$

$$L(s) = \frac{e^{-s(4\times10^{-6})}\cdot(10^{-5}s+1)(5\times10^{-4}s+1)}{10^{-5}\,\cancel{(5\times10^{-4}s+1)}\,s\,(5.1\times10^{-4}s+1)}$$

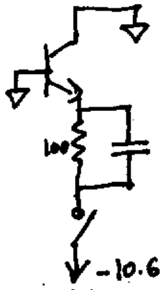
You throw in another zero & move the pole that you are trying to cancel.
Though, this isn't much of a problem:




Pole-zero doublet separated by ESR resistance causes us to cross over only slightly earlier. ($.98\times10^5$ instead of $10^5$)
So we can ignore this if we put the zero before that pole → put the zero at $\frac{1}{5.1\times10^{-4}}$ Hz. Then, put another pole a decade after crossover to make sure that we will cross over at some point.

$$\boxed{G'(s) = \frac{5.1\times10^{-4}s+1}{5\cdot10^{-5}s(10^{-6}s+1)}}$$

3) a)




$\beta_F = 100 \quad \beta_R = 5 \quad \tau_F = 0.5ns \quad \tau_R = 1ns \quad \tau_S = 755ns$

~~$V_e = i_c(100) + V_{BE} = i_c(100) = 10.6V$~~

$-i_e = \frac{10V}{100\Omega} = .1A\, u(t) = q_F\left(\frac{1}{\tau_F} + \frac{1}{\tau_{BF}}\right) + \frac{dq_F}{dt}$

$\boxed{i_c = \frac{q_F(t)}{\tau_F} \cong .1(1 - e^{-2.02\times10^9 t})}$

$q_F(t) = \left(\frac{\tau_F \tau_{BF}}{\tau_F + \tau_{BF}}\right).1A\left(1 - e^{-\frac{t(\tau_F+\tau_{BF})}{\tau_F\tau_{BF}}}\right) = \boxed{4.9\times10^{-11}(1 - e^{-2.02\times10^9 t})}$

b) $-i_e = \frac{10V}{100\Omega} u(t) + i_{Cap} = .1A\, u(t) + C \cdot 10V\, \delta(t)$ $\qquad \tau = \frac{\tau_F \tau_{BF}}{\tau_F + \tau_{BF}} = 4.95\times10^{-10}$

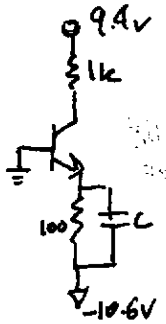
$-i_e = q_F(t)\left(\frac{1}{\tau_F} + \frac{1}{\tau_{BF}}\right) + \frac{dq_F}{dt}$

$\boxed{q_F(t) = \tau(0.1A)(1 - e^{-t/\tau}) + C \cdot 10V\, e^{-t/\tau}}$ $\qquad \boxed{i_c = \frac{q_F(t)}{\tau_F} = 0.1A(1 - e^{-2\times10^9 t}) + 5\times10^{-11} e^{-\frac{t}{\tau}}}$

$10_V C = \tau(0.1A)$

$\boxed{C = \frac{0.1\tau}{10} F = \frac{\tau}{100}} \boxed{\cong 5pF}$

c)




$-i_e = \frac{10V}{100\Omega} u(t) = .1A u(t) = q_F\left(\frac{1}{\tau_F} + \frac{1}{\tau_{BF}}\right) + \frac{dq_F}{dt}$ $\qquad \tau = \frac{\tau_F \tau_{BF}}{\tau_F + \tau_{BF}} \cong 5\times10^{-10}$

$q_F = \tau(0.1A)(1 - e^{-t/\tau})$

$i_c = \frac{q_F(t)}{\tau_F} = \frac{\tau_{BF}}{\tau_F + \tau_{BF}}(0.1A)(1 - e^{-t/\tau})$ $\qquad V_{CE,sat} = 0.2V$

$V_c = 9.4V - i_c(1K) = 9.4V - \frac{\tau_{BF}}{\tau_F + \tau_{BF}}^{\approx 1}(0.1A)(1 - e^{-t/\tau}) \cdot 1K$

enters sat @ $V_c = -.6V \cong 9.4V - (0.1A)(1 - e^{-2\times10^9 t}) \cdot 1K$

~~$t = \frac{1}{2\times10^9} \ln\left(1 - \frac{10V}{100V}\right) = 5.1\times10^{-11}$ Seconds~~ $\ln\left(1 - \frac{10V}{100V} \cdot \frac{\tau_F + \tau_{BF}}{\tau_{BF}}\right) = -t/\tau \rightarrow \boxed{t = 5.1\times10^{-11}s}$

d) $q_{BO} = \tau_F I_{Csat}$ $\qquad I_{Csat} = \frac{10V}{1000\Omega} = 10mA \rightarrow q_{BO} = 0.5ns \cdot 10mA = \boxed{5pC}$

$q_{BO} \cong 5pC$ $\qquad I_{BO} = \frac{I_{Csat}}{\beta} = 100\mu A$

D) continued:

$I_{Csat}$ is all you'll get from the collector, so the rest has to come from the base.

Steady State: $I_e = .1A$, $I_e = I_c + I_B$ $I_C = I_{C_{sat}} = 10mA$ $I_B = I_e - I_{C_{sat}} + \frac{I_{Csat}}{\beta}$

$I_B = .09A + \frac{.01}{100}$

Since $i_b = \frac{q_F}{\tau_{BF}} + \cancel{\frac{dq_F}{dt}}^{0} + \frac{q_R}{\tau_{BR}} + \cancel{\frac{dq_R}{dt}}^{0}$

$i_C = \frac{q_F}{\tau_F} - \cancel{\frac{dq_R}{dt}}^{0} - q_R\left(\frac{1}{\tau_R} + \frac{1}{\tau_{BR}}\right)$

$i_e = i_b + i_c = 100mA$

$i_c = 10mA = \frac{q_F}{\tau_F} - q_R\left(\frac{1}{\tau_R} + \frac{1}{\tau_{BR}}\right) \longrightarrow 10mA = \frac{\left(90mA - \frac{q_R}{\tau_{BR}}\right)\tau_{BF}}{\tau_F} - \frac{q_R}{\tau_R} - \frac{q_R}{\tau_{BR}} \rightarrow q_R = \frac{(9-0.1)}{\left(\frac{1}{\tau_R} + \frac{1}{\tau_{BR}} + \frac{100}{\tau_{BR}}\right)}$

$i_b = 90mA = \frac{q_F}{\tau_{BF}} + \frac{q_R}{\tau_{BR}} \rightarrow q_F = \left(90mA - \frac{q_R}{\tau_{BR}}\right)\tau_{BF}$

$\boxed{q_R = 429pC}$

$\boxed{q_F = 260pC}$

$i_b = 90mA$; $\underbrace{i_B - I_{BO}}_{89.9mA} = \frac{q_s}{\tau_s} + \frac{dq_s}{dt} \rightarrow 0$

$89.9mA = \frac{q_s}{\tau_s} \rightarrow \boxed{q_s = 679pC}$

$q_F + q_R \overset{?}{=} q_{BO} + q_s \rightarrow 684 = 684$

E)

$-i_e = \frac{10V}{100\Omega}u(t) + i_{cap} = 0.1A\,u(t) + C\cdot 10v\,\delta(t) = \frac{q_F}{\tau_F} + \frac{q_F}{\tau_{BF}} + \frac{dq_F}{dt}$

$q_F = 0.1A\tau(1-e^{-t/\tau}) + C10v\,e^{-t/\tau}$, $\tau \cong 500\times10^{-12}$

$i_c = \frac{q_F}{\tau_F} = 0.1A\frac{\tau}{\tau_F}(1-e^{-t/\tau}) + \frac{10C}{\tau_F}e^{-t/\tau}$

Saturates @ $V_{CE}=0$, $V_C = -0.6v = 9.4_v - R_L i_c \Rightarrow 1k\left(0.1A\frac{\tau}{\tau_F}(1-e^{-t/\tau}) + \frac{10C}{\tau_F}e^{-t/\tau}\right) = 10v$

$\frac{10v}{1000}\cdot\tau_F - \frac{\tau}{10} = e^{-t/\tau}\left(-\frac{\tau}{10} + 10C\right)$ $t = -\tau\ln\left(\frac{\frac{10}{1000}\tau_F - \frac{\tau}{10}}{10C - \frac{\tau}{10}}\right)$

Solve $t=0$ for a real $C$, $\boxed{C = .5pC}$

F)

$i_e = 0 \rightarrow \boxed{i_b = -i_{C_{sat}} \cong -.01A}$

G) ~~$i_b = \frac{q_s}{\tau_s} + \frac{dq_s}{dt}$~~ $i_b - I_{BO} = \frac{q_s}{\tau_s} + \frac{dq_s}{dt} = -10mA - \frac{10mA}{100} = \frac{q_s}{\tau_s} + \frac{dq_s}{dt}$

$q_s(\infty) = -10.1mA\cdot\tau_s = -76pC$ $q_s(t_{SD}) = -76pC + 679pC\left(e^{-t_{SD}/\tau_s}\right) = 0$

$t_{SD} = \ln\left(\frac{679pC}{76pC}\right)\tau_s = \boxed{16.5ns}$

3H)