MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering

6.331 Advanced Circuit Techniques

Fall Term 2011 — Issued : September 22, 2011
Problem Set 3 — Due : Thursday, September 29, 2011

Problem 1 Feedback systems with variable parameters are among the most difficult ones to compensate because of the compromises frequently required. Consider the sample-and-hold circuit shown below as an example of this type of system.

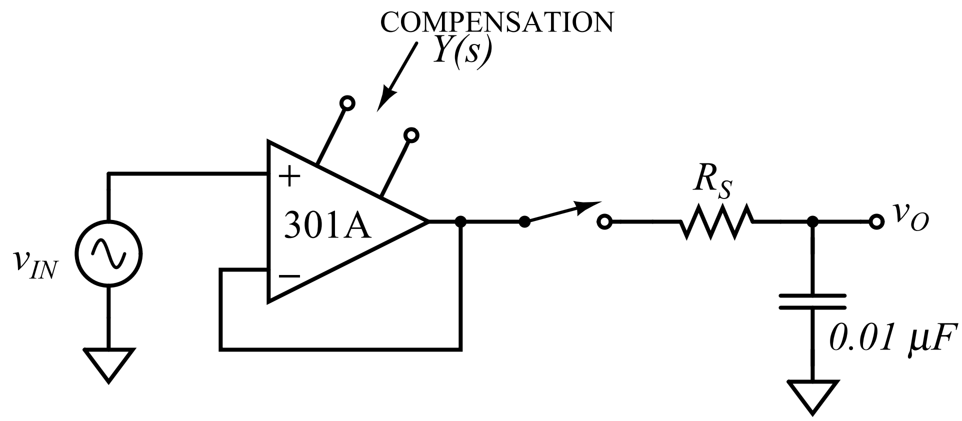


Figure 2.1: Sample and Hold Circuit

The resistance $R_S = 5\Omega$ models the switch resistance. Assume that the open-loop output resistance of the 301A is 75 ohms, and that its open-loop transfer function is found from minor-loop feedback arguments

$$A(s) = \frac{G_M R(s)}{1 + R(s)Y(s)}$$

where $Y(s)$ is the short-circuit transfer admittance of the compensating network, G_M is the transconductance of the input stage (2×10^{-4} ℧), and $R(s)$ is the second stage transresistance

$$R(s) = \frac{10^9\ \Omega}{(\tau s + 1)^2}$$

with $\tau = 70$ microseconds.

(a) The "741 mentality" suggests that the optimum compensation is always 30pF. Demonstrate the difficulty with this type of compensation for our sample-and-hold by calculating its crossover frequency and phase margin with the switch both open and closed.

(b) Try to find an alternative type of compensation that improves stability in the sample mode without significantly compromising crossover frequency. Note that while stability is necessary when the switch is open, the dynamics of the circuit in this mode are relatively unimportant since the amplifier output is not used except when sampling. Include rough sketches of the loop transmission dynamics.

Problem 2 Consider the following loop transfer functions:

$$L_1(s) = \frac{10^6}{s} \qquad L_2(s) = \frac{10^6}{s+1} \qquad L_3(s) = \frac{10^{10}(10^{-4}s+1)}{s^2} \qquad L_4(s) = \frac{10^6(10^{-4}s+1)}{(10^{-2}s+1)^2}$$

For each loop transfer function:

(a) Plot an asymptotic Bode Plot.

(b) Find the open loop DC gain, the crossover frequency ω_c, and the phase margin ϕ_M.

(c) Find the error transfer function, assuming that the above loop transfer functions describe op amp circuits with unity feedback.

(d) Find the steady state error to a 1 V step input.

(e) Use synthetic division to find the first three error coefficients of the error series, e_0, e_1, and e_2.

(f) For a unit ramp input, the steady state error grows as

$$e_{ss} = e_0 t + e_1$$

Find the steady state error e_{ss} to an input ramp with a slope of 1 V/μs. Comment on the relative magnitude and measurability of these errors.

(g) Use MATLAB to simulate (with `lsim`) the actual error response to the above ramp. Show and comment on the fast transient versus the slow transient.

Problem 3 An operational amplifier connected as a unity-gain non-inverting amplifier is excited with an input signal

$$v_i(t) = 5\arctan(10^5 t)$$

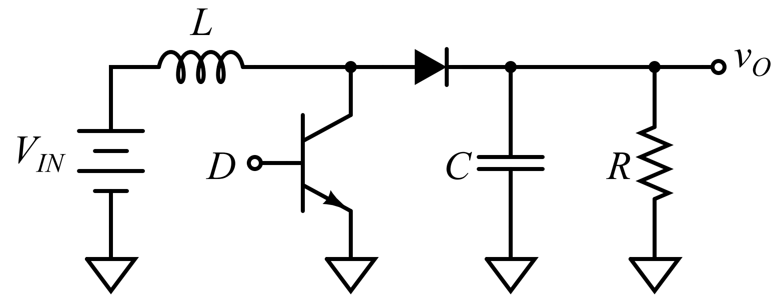
Estimate the error between the actual and ideal outputs assuming that the open-loop transfer function can be approximated as indicated below. (Note that these transfer functions all have identical values for unity-gain frequency.)

(a) $a(s) = 10^7/s$

(b) $a(s) = 10^{13}(10^{-6}s+1)/s^2$

(c) $a(s) = 10^{19}(10^{-6}s+1)^2/s^3$

Problem 4 Consider the following boost converter.



(a) Show that the following state equations describe its operation

$$L\frac{d}{dt}\langle i_L\rangle = V_{IN} - (1-D)\langle v_C\rangle$$

$$C\frac{d}{dt}\langle v_C\rangle = (1-D)\langle i_L\rangle - \frac{\langle v_C\rangle}{R}$$

where the angle brackets $\langle v_C\rangle$ denote an average value, defined as

$$\langle v_C\rangle = \frac{1}{T}\int_0^T v_C(t)\,dt$$

(b) Use the above equations to write a single second-order state equation for the capacitor voltage (differentiate the second state equation and substitute in for derivatives of the inductor current). Remember that v_C, i_L, and D are all functions of time.

(c) Linearize the above result using the following substitutions

$$\langle v_C\rangle = V_C + v_c \qquad \langle i_L\rangle = I_L + i_l \qquad D = D_0 + d$$

Drop any second order terms involving d^2, dv_c, etc. Ignore the small variations in the inductor current by dropping the terms involving i_l. Subtract off the nominal equation (the result from part (b), written in terms of V_C, I_L, and D_0).

(d) Take the Laplace transform of this result and solve for the transfer function from changes in duty cycle to capacitor voltage. You can eliminate the input voltage by using $V_{IN} = V_C(1-D_0)$. The result you should find is

$$\frac{v_c(s)}{d(s)} = \frac{-s(I_L/C) + (1-D_0)V_C/LC}{s^2 + s/RC + (1-D_0)^2/LC}$$

Marvel at the right half plane zero.