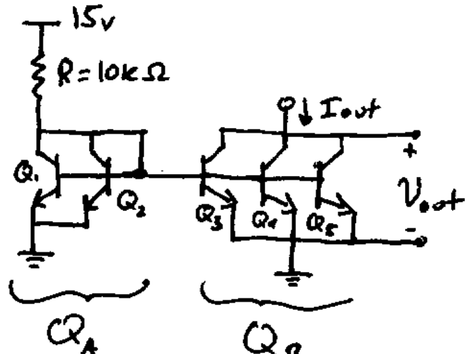
Pset 5 Solutions

1) GHLM 4.1



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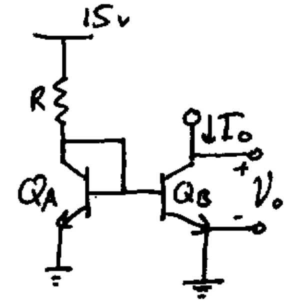


$I_{S_A} = 2I_S \qquad I_{S_B} = 3I_S$

Ignoring Base Currents,

$$I_o = \frac{3I_S}{2I_S} I_{C_A}\left(1 + \frac{V_{CE_B} - V_{CE_A}}{V_A}\right)$$

$$I_o = \frac{3}{2}\,\frac{14.4V}{10k\Omega}\left(1 + \frac{V_o - 0.6V}{130V}\right)$$

$$= 2.16mA\left(1 + \frac{V_o - 0.6V}{130V}\right)$$

$V_{CE_A} = V_{BE_A} \cong 0.6V$

$V_{CE_B} = V_o$

$V_A = 130V$

$$\boxed{\begin{aligned} I_o(V_o = 1V) &= 2.167mA \\ I_o(V_o = 5V) &= 2.233mA \\ I_o(V_o = 20V) &= 2.482mA \end{aligned}}$$

$$\tilde{R}_o = \frac{dV_o}{dI_o} = \frac{d}{dI_o}\left(\left(\frac{I_o}{2.16mA} - 1\right)130V + 0.6V\right)$$

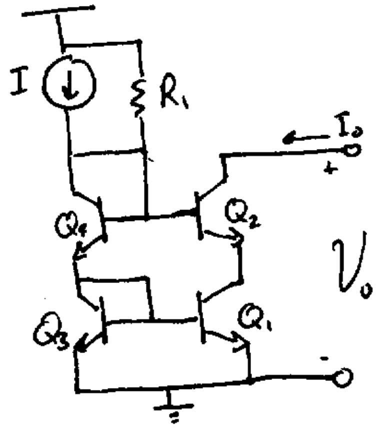
$$\boxed{R_o = 60.2k\Omega}$$

You should find lower currents than calculated in the spice model
Because base currents were neglected.

2) GHLM 4.4

$r_{\pi_1} = r_{\pi_2} = r_{\pi_3} = r_{\pi_4} = r_\pi$

$r_{o_1} = r_{o_2} = r_{o_3} = r_{o_4} = r_o$



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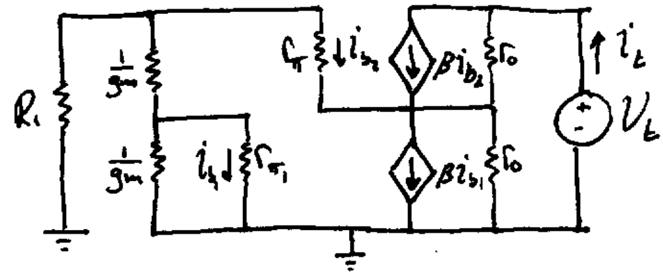


$$i_o = \beta i_{b_2} + \frac{V_o - V_{e_2}}{r_o}$$

$$i_{b_2}(1+\beta) - \beta i_{b_1} + \frac{V_o - 2V_{e_2}}{r_o} = 0$$

$$V_{b_2} = -i_{b_2}\left(R_1 \| \left(\tfrac{1}{g_m} + \tfrac{1}{g_m} \| r_{\pi_1}\right)\right) = V_{e_2} + i_{b_2} r_\pi$$

$$i_{b_1} = \frac{V_{b_2}}{\frac{1}{g_m} + \frac{1}{g_m} \| r_\pi} \cdot \frac{\frac{1}{g_m} \| r_\pi}{r_\pi} = \frac{V_{b_2}}{2 r_\pi} = \frac{-i_{b_2}\left(R_1 \| \frac{2}{g_m}\right)}{2 r_\pi}$$

$$V_e = V_{b_2} - i_{b_2} r_\pi = -\left(\left(R_1 \| \tfrac{2}{g_m}\right) + r_\pi\right) i_{b_2}$$

$$\left(1 + \beta\left(1 + \frac{R_1 \| \frac{2}{g_m}}{2 r_\pi}\right)\right) i_{b_2} r_o + V_o + 2\left(R_1 \| \tfrac{2}{g_m} + r_\pi\right) i_{b_2} = 0$$

$$i_{b_2} = \frac{-V_o}{\left(1 + \beta + \frac{g_m}{2}\left(R_1 \| \frac{2}{g_m}\right)\right) r_o + 2\left(R_1 \| \frac{2}{g_m} + r_\pi\right)}$$

$$r_o i_o = \frac{-\left(\beta r_o + \left(R_1 \| \frac{2}{g_m} + r_\pi\right)\right) V_o}{\left(1 + \beta + \frac{g_m}{2}\left(R_1 \| \frac{2}{g_m}\right)\right) r_o + 2\left(R_1 \| \frac{2}{g_m} + r_\pi\right)} + V_o$$

$$= \frac{\left(1 + \frac{g_m}{2}\left(R_1 \| \frac{2}{g_m}\right)\right) r_o + R_1 \| \frac{2}{g_m} + r_\pi}{\left(1 + \beta + \frac{g_m}{2}\left(R_1 \| \frac{2}{g_m}\right)\right) r_o + 2\left(R_1 \| \frac{2}{g_m} + r_\pi\right)} V_o$$

$$\tilde{R}_o = \frac{V_o}{i_o} = \frac{\left(1 + \beta + \frac{g_m}{2}\left(R_1 \| \frac{2}{g_m}\right)\right) r_o + 2\left(R_1 \| \frac{2}{g_m} + r_\pi\right)}{\left(1 + \frac{g_m}{2}\left(R_1 \| \frac{2}{g_m}\right)\right) r_o + R_1 \| \frac{2}{g_m} + r_\pi} r_o \quad , \; R_1 >> \frac{1}{g_m}$$

$$= \frac{(2+\beta) r_o + 2 r_\pi + \frac{\frac{2}{g_m}}{r_o}}{2 r_o + \frac{2}{r_o g_m} + \frac{r_\pi}{r_o}}$$

$$\approx \frac{(2+\beta) r_o + 2 r_\pi}{2} \approx \frac{(2+\beta) r_o}{2} \approx \boxed{\frac{\beta r_o}{2}}$$

$V_{cc} = 5\text{V}$
$R_1 = 10\text{k}$
$\beta = 200$
$V_A = 130$

$\boxed{R_o = 34\,\text{M}\Omega}$

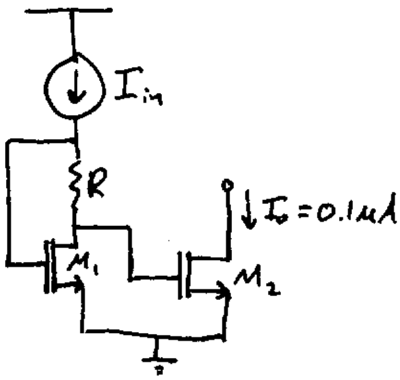
Q_2 saturates before Q_1

↳ Saturation limit for $Q_2 = 0.2\text{V}$

↳ $V_{ce_2} = V_o - V_{BE_1}$

$\boxed{V_o > 0.8\text{V}}$

Problem 3) GHLM 4.24



a) $I_{in} = 1\mu A$ $\quad n=1.5$

$I_o \approx I_{in}\, e^{-\left(\frac{I_{in} R}{n V_{TH}}\right)}$

$R = -\frac{n V_{TH}}{I_{in}} \ln\left(\frac{I_o}{I_{in}}\right)$

$\boxed{R = 86.3\,k\Omega}$

$I_D < \frac{W}{L} I_t$

M_1 draws 10x more current than M_2

$\boxed{\frac{W}{L} > \frac{I_{in}}{I_t} = \frac{1\mu A}{0.1\mu A} = 10}$

b) $R = 10k$

$I_o = I_{in}\, e^{-\left(\frac{I_{in} R}{n V_{TH}}\right)} = 0.1\mu A$ Solve.

$\boxed{I_{in} = 19.8\,mA}$

$\boxed{\frac{W}{L} > 198}$

Problem 4) GHLM 4.25

$\beta = 200$

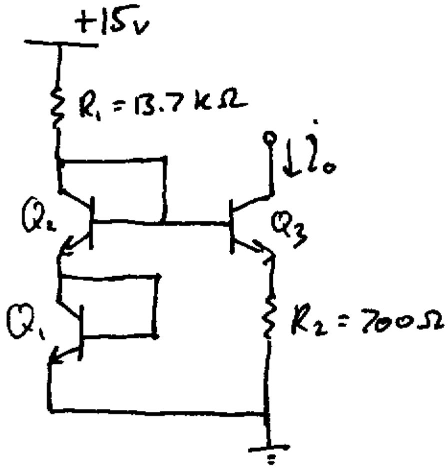
$r_o = \frac{V_A}{I_o} = \frac{130V}{0.97mA} = 134\,M\Omega$

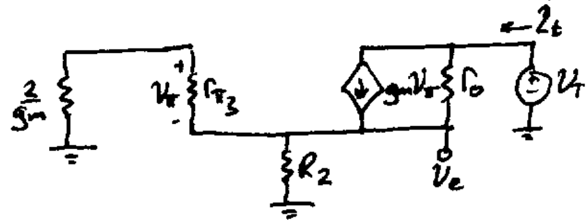
$r_\pi = \frac{\beta V_{Th}}{I_o} = 5.2\,k\Omega$

$I_{in} \simeq \frac{(15-1.2)V}{13.7k\Omega} \cong 1\,mA$

$I_o \cong \frac{0.6V}{700\Omega} \cong 857\,\mu A$

$I_o = \frac{V_{Th}}{700\Omega} \ln\left(\frac{I_{in}^2}{I_S I_o}\right) = .97\,mA$





$i_t = g_m v_\pi + \frac{v_t - v_e}{r_o} \qquad i_b = \frac{v_\pi}{r_{\pi 3}}$

$i_t = \beta i_b + \frac{v_t - v_e}{r_o}$

$(\beta+1) i_b + \frac{v_t - v_e}{r_o} - \frac{v_e}{R_2} = 0$

$v_e = -i_b\left(\frac{2}{g_m} + r_{\pi 3}\right) \simeq -i_b r_{\pi 3}$

$i_t = \beta i_b + \frac{v_t + i_b r_{\pi 3}}{r_o}$

$i_b(\beta+1) + i_b \frac{r_{\pi 3}}{r_o} + i_b \frac{r_{\pi 3}}{R_2} = \frac{-v_t}{r_o}$

$i_b\left(\beta + 1 + \frac{r_{\pi 3}}{r_o} + \frac{r_{\pi 3}}{R_2}\right) = \frac{-v_t}{r_o}$

$i_b = \frac{-v_t}{(\beta+1) r_o + r_{\pi 3} + \frac{r_{\pi 3} r_o}{R_2}}$

$i_t = \frac{-\beta v_t}{(\beta+1) r_o + r_{\pi 3} + \frac{r_{\pi 3} r_o}{R_2}} + \frac{v_t}{r_o} - \frac{r_{\pi 3} v_t}{\left((\beta+1) r_o + r_{\pi 3}\left(1 + \frac{r_o}{R_2}\right)\right) r_o}$

$i_t r_o = v_t \left(\frac{r_o + \frac{r_{\pi 3} r_o}{R_2}}{(\beta+1) r_o + \left(1 + \frac{r_o}{R_2}\right) r_{\pi 3}} \right)$

$$\boxed{R_o = \frac{v_t}{i_t} = \frac{(\beta+1) r_o + \left(1 + \frac{r_o}{R_2}\right) r_{\pi 3}}{1 + \frac{r_{\pi 3}}{R_2}} = 3.3\,M\Omega}$$