1)

β large

$I = 1mA = I_c$

$$I_C = I_S e^{\frac{qV_{BE}}{KT}} \rightarrow V_{BE} = \frac{-KT}{q} \ln\left(\frac{I_S}{I_C}\right)$$

$$\frac{dV_{BE}}{dT} = -\frac{K}{q} \ln\left(\frac{I_S}{I_C}\right) - \frac{KT}{q} \frac{1}{I_S} \frac{dI_S}{dT}$$

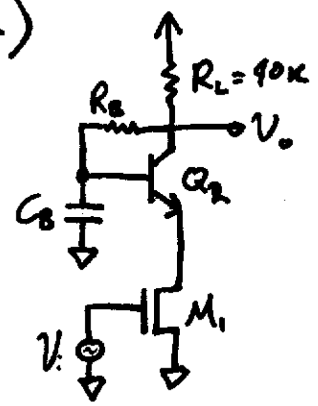
$$= \frac{V_{BE}}{T} - \frac{KT}{q} \frac{1}{I_S} \frac{dI_S}{dT} \rightarrow \frac{1}{I_S} \frac{dI_S}{dT} = \frac{q}{KT}\left(\frac{V_{BE}}{T} - \frac{dV_{BE}}{dT}\right)$$

$$V_{BE} = V_T \ln\left(\frac{0.1mA}{10^{-15}A}\right) @ 300°K = 0.656v \; @ 300°K$$

$$\frac{dV_{BE}}{dT} = -2mV/°C \; @ 300°K$$

$$\hookrightarrow \frac{1}{I_S} \frac{dI_S}{dT} = \frac{1}{V_T}\left(\frac{0.656v}{300°K} + 2mV/°C\right) = 16.16\% /°C$$

2)



$M_1: V_T = 0.6V$
$\mu_n C_{ox} W/L = 200\mu A/V^2, \lambda = 0$

$Q_2: I_S = 10^{-15}A, V_A = \infty, V_{CEsat} = 0.2V$
$\beta_F = \beta_o = 200$
$r_b = r_c = r_e = 0$

a) $I_{D_1} = I_{E_2} = \left(\frac{1+\beta_F}{\beta_F}\right)\underbrace{I_{C_2}}_{50\mu A} = 50.25\mu A$

In Saturation,

$$I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{GS}-V_T)^2 = \frac{1}{2}(200\mu A/V^2)(V_I - 0.6V)^2 = 50.25\mu A$$

$V_I = 1.31V$

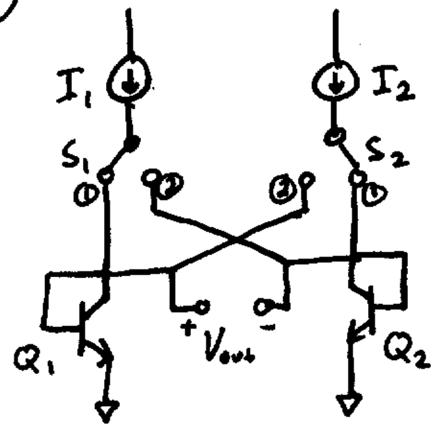
b) $\cancel{V_o = V_{CC} - I_{C_2}R_L \cong 3V}$ $V_o = V_{CC} - (I_{C_2} + I_{B_2})R_L = 2.99V \cong 3V$

c) M_1 is saturated so long as $V_{DS} \geq V_{GS} - V_T \geq 1.31V - 0.6V \geq 0.71V$

$$V_{DS_1} = V_o - R_B I_{B_2} - V_{BE_2} = 2.99V - R_B \frac{50\mu A}{\beta} - 0.6V \geq 0.71V$$

$$\frac{2.99V - 0.71V - 0.6V}{.25\mu A} > R_B \rightarrow R_B < 6.72M\Omega$$

3)



$\frac{I_1}{I_2} = 100 \qquad \beta_F \to \infty$

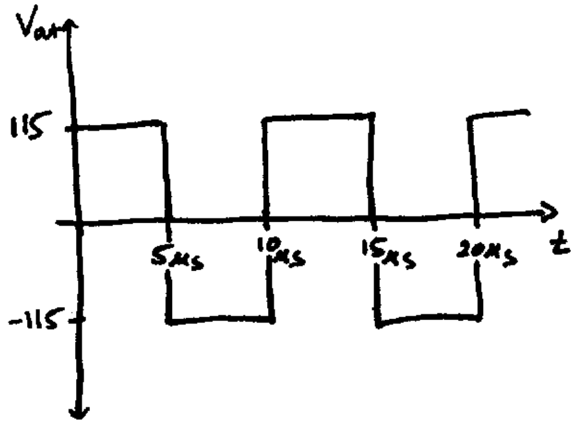
a) $V_{out} = V_+ - V_-$

$V_+ = V_{BE_1} = \frac{KT}{q} \ln\left(\frac{I_{C1}}{I_S}\right)$, $V_- = V_{BE_2} = \frac{KT}{q} \ln\left(\frac{I_{C2}}{I_S}\right)$

$V_{out} = \frac{KT}{q}\left(\ln\left(\frac{I_{C1}}{I_S}\right) - \ln\left(\frac{I_{C2}}{I_S}\right)\right) = \frac{KT}{q} \ln\left(\frac{I_{C1}}{I_{C2}}\right)$

In pos. 1, $V_{out} = \frac{KT}{q} \ln(100) = 115\,mV$

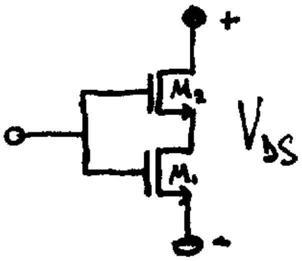
In pos. 2, $V_{out} = \frac{KT}{q} \ln(.01) = -115\,mV$



b) $V_{rms} = \sqrt{\frac{1}{\tau}\int_0^{\tau} (V_{out})^2 dt} = \cancel{\sqrt{\frac{1}{\tau}\int_0^{\tau} (115\,mV)^2 dt}} \; \sqrt{\frac{1}{\tau}\int_0^{\tau} \left(\frac{KT}{q}\ln\left(\frac{I_{C1}}{I_{C2}}\right)\right)^2 dt} = \left|\frac{KT}{q} \ln\left(\frac{I_{C_1}(t)}{I_{C_2}(t)}\right)\right|$

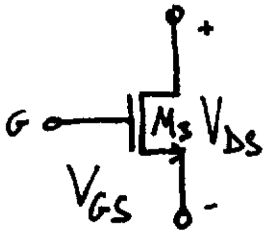
$= \frac{KT}{q} \ln(100)$

4)



$M_1 \to \frac{W}{L_1}$

$M_2 \to \frac{W}{L_2}$



$M_3 \to \frac{W}{L_1+L_2}$

M_1 should remain in the triode region of operation, since $V_{DS_1} = V_{DS} - V_{DS_2}$ + $V_{GS_2} = V_{GS} - V_{DS_1}$, so V_{DS_1} needs to be small to keep $V_{GS_2} > V_T$.

$$I_{D_1} = K' \frac{W}{L_1}\left((V_{GS_1}-V_T)V_{DS_1} - \frac{V_{DS_1}^2}{2}\right) \to V_{DS_1} = V_{GS} - V_T \pm \sqrt{(V_{GS}-V_T)^2 - \frac{2I_1}{K'(\frac{W}{L_1})}}$$

In triode region, $V_{DS} < V_{GS} - V_T$ $\to \boxed{V_{DS_1} = V_{GS} - V_T - \sqrt{(V_{GS}-V_T)^2 - \frac{2I_1}{K'\frac{W}{L_1}}}}$ 1, $I_1 = I_2$

$\underline{M_2}$ may be in saturation or triode region, depending on V_{DS_2}

Saturation: $V_{DS_2} > V_{GS_2} - V_T$

$\underbrace{V_{DS} - V_{DS_1}}_{V_{DS_2}} > \underbrace{V_{GS} - V_{DS_1}}_{V_{GS_2}} - V_T \to V_{DS} > V_{GS} - V_T \Rightarrow$ If M_2 is in saturation, so is M_3.

$\hookrightarrow \boxed{I_3 = \frac{K'}{2}\left(\frac{W}{L_1+L_2}\right)(V_{GS}-V_T)^2}$

$\hookrightarrow$ See how $M_1 + M_2$ compare to M_3

$$\hookrightarrow I_2 = \frac{K'}{2}\left(\frac{W}{L_2}\right)(V_{GS_2}-V_T)^2 = \frac{K'}{2}\left(\frac{W}{L_2}\right)(V_{GS} - \overset{1}{V_{DS_1}} - V_T)^2 = \frac{K'}{2}\frac{W}{L_2}\left((V_{GS}-V_T)^2 - \frac{2I_2}{K'\frac{W}{L_1}}\right) \quad I_1 = I_2$$

$\hookrightarrow \boxed{I_2 = \frac{K'}{2}\left(\frac{W}{L_1+L_2}\right)(V_{GS}-V_T)^2 = I_3}$ 3, when M_2 + M_3 are in saturation + M_1 is in the triode region.

Triode: $V_{DS_2} < V_{GS_2} - V_T$

$\hookrightarrow V_{DS} < V_{GS} - V_T \Rightarrow$ If M_2 is in the triode region, so is M_3

$\hookrightarrow \boxed{I_3 = K'\frac{W}{L_1+L_2}\left((V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right)} \to$ Compare to $M_1 + M_2$

$$\hookrightarrow I_2 = K'\frac{W}{L_2}\left((V_{GS_2}-V_T)V_{DS_2} - \frac{V_{DS_2}^2}{2}\right) = K'\frac{W}{L_2}\left((V_{GS}-V_{DS_1}-V_T)(V_{DS}-V_{DS_1}) - \frac{(V_{DS}-V_{DS_1})^2}{2}\right)$$

$$I_2 = K'\frac{W}{L_2}\left((V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2} - (V_{GS}-V_T)V_{DS_1} + \frac{V_{DS_1}^2}{2}\right)$$

$$I_2 = I_1 = K'\frac{W}{L_1}\left((V_{GS}-V_T)V_{DS_1} - \frac{V_{DS_1}^2}{2}\right) \to \boxed{\frac{I_2}{K'\frac{W}{L_1}} = \left((V_{GS}-V_T)V_{DS_1} - \frac{V_{DS_1}^2}{2}\right)} \; 2$$

$$\hookrightarrow I_2 = K'\frac{W}{L_2}\left((V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2} - \frac{I_2}{K'\frac{W}{L_1}}\right)$$

$$\to \boxed{I_2 = I_3 = K'\frac{W}{L_1+L_2}\left((V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right)} \; 4$$

3 + 4

$M_1 + M_2$ and M_3 behave the same