

# REPORT

Laboratory work No. 6

Course: Cryptography and Security

Theme: Hash Functions and Digital Signatures

*Author:* Corneliu Catlabuga  
FAF-213

*Checked by:* univ. assist.  
Cătălin Mîțu

**Objective:**

Use RSA and ElGamal algorithms to check the digital signature of a text.

**Task:**

1. Hash a message, encrypt the hash with the private key and check it with the public key.
2. Hash a message, encrypt the hash with the shared secret generated through the ElGamal algorithm.

**Theoretical considerations:**

Text message: "each new setting of alberti's disk brought into play a new cipheralphabet, in which both the plaintext and the ciphertext equivalents are changed in regard to one another. there are as many of these alphabets as there are positions of his disk, and this multiplicity means that alberti here devised the first polyalphabetic cipher. this achievement—critical in the history of cryptology —alberti then adorned by another remarkable invention: enciphered code. it was for this that he had put numbers in the outer ring. in a table he permuted the numbers 1 to 4 in two-, three-, and four-digit groups, from 11 to 4444, and used these as 336 code groups for a small code. "in this table, according to agreement, we shall enter in the various lines at the numbers whatever complete phrases we please, for example, corresponding to 12, 'we have made ready the ships which we promised and supplied them with troops and grain.' " these code values did not change, any more than the mixed alphabet of the disk did. but the digits resulting from an encoding were then enciphered with the disk just as if they were plaintext letters. in alberti's words, "these numbers i then insert in my message according to the formula of the cipher, representing them by the letters that denote these numbers." these numbers thus changed their ciphertext equivalents as the disk turned. hence 341, perhaps meaning "pope," might become mnp at one position and fco at another. this constitutes an excellent form of enciphered code, and just how precocious alberti was may be seen by the fact that the major powers of the earth did not begin to encipher their code messages until 400 years later, near the end of the 19th century, and even then their systems were much simpler than this. alberti's three remarkable firsts—the earliest western exposition of cryptanalysis, the invention of polyalphabetic substitution, and the invention of enciphered code—make him the father of western cryptology. but although his treatise was published in italian in a collection of his works in 1568, and although his ideas were absorbed by papal cryptologists and perhaps influenced the science's development, they never had the dynamic impact that such prodigious accomplishments ought to have produced. symonds' evaluation of his work in general may both explain why and summarize the modern view of his cryptological contributions: "this man of many-sided genius came into the world too soon for the perfect exercise of his singular faculties. whether we regard him from the point of view of art, of science, or of literature, he occupies in each department the position of precursor, pioneer, and indicator. always original and always fertile, he prophesied of lands he was not privileged to enter, leaving the memory of dim and varied greatness rather than any solid monument behind him." polyalphabeticity took another step forward in 1518, with the appearance of the first printed book on cryptology, written by one of the most famous intellectuals of his day. this was johannes trithemius, a benedictine monk whose dabbling in alchemy and other mystic powers made him one of the most revered figures in occult science, while his more solid scholarship won him the title of "father of bibliography." in 1518, a year and a half after his

death, his polygraphiae libri sex, loannistrithemii abbatis peapolitani, quondam spanheimensis, ad maximilianumcaesarem ("six books of polygraphy, by johannes trithemius, abbot atwurzburg, formerly at spanheim, for the emperor maximilian") waspublished. by far the bulk of the volume consists of the columns ofwords printed in large gothic type that trithemius used in his systems ofcryptography. but in the work's book v appears, for the first time, thesquare table, or tableau. this is the elemental form of polyalphabeticsubstitution, for it exhibits all at once all the cipher alphabets in a particular system. these are usually all the same sequence of letters, butshifted to different positions in relation to the plaintext alphabet, as inalberti's disk the inner alphabet assumed different positions in regard tothe outer alphabet. the tableau sets them out in orderly fashion—thealphabets of the successive positions laid out in rows one below theother, each alphabet shifted one place to the left of the one above. eachrow thus offers a different set of cipher substitutes to the letters of theplaintext alphabet at the top. since there can be only as many rows asthere are letters in the alphabet, the tableau is square.the simplest tableau is one that uses the normal alphabet in variouspositions as the cipher alphabets. each cipher alphabet produces, inother words, a caesar substitution."

For task 2:

$p =$

323170060713110073001535134778251633624880571334890751745884341392698068341  
362100027920563626401646854585563579353308169288290230805734726252735547424  
612457410262025279165729728627063003252634282131457669314142236542209411113  
486299916574782680342305530863490506355577122191878903327295696961297438562  
417412362372251973464026918557977679768230146253979330580152268587307611975  
324364674758554607150438968449403661304976978128542959586595975670512838521  
327844685229255045682728791137200989318739591433741758378260002780349731985  
520606075332341226032546840881200311059074842810039949669561196969562486290  
32338072839127039

$g = 2$

## Implementation, practical results:

### 1. Generate p and q:

p =

244858544615956870056064079186546218397523481009022118156701643309245357386  
156449109190553675152495849826210603751356118024786816541752676760026873393  
293225137717251857615307650151312269752888471746034427123722588494775782325  
696027826903529712951894063204574322014089145453028750253222222581245091829  
493647720339290788052091307906205574463934811806245830093116869738655125183  
826917068541671221254120982574536023813057168882329629928219040347897343649  
732445525791529594882957156091395114161423307200289620827127015718691439688  
326137502740016757053979043907252357135837376116843321841022885476820270671  
42093734312924869

q =

310317574925389502612308909575219378962878811381964869032991785875769449437  
586840145927693138457296969884112799387535701852567680513623081466690134737  
995310649533442437206723922988765711716464500755769690670635579716444420716  
889031237597699490816880444314760266965365753412872933764911552653307803137  
19819827

### 2. Compute $n = p * q$

n =

759839097649840244169652267303445213133486536034323997585730596563340066149  
792663628777241702467326622831675073362793278314985972726319407565088639126  
225354800335053455387252587544922167589051734494152634736036780510440885901  
984214461623220248670037264922593974200452432159031217995969001450319473425  
155576025927894019545990898478353536225762695679379990585075790748886715909  
479831990340610923789210618894988353579388962584105149846483232637621581455  
605011155237996397954938311635414556368746298985076694485293900081503189279  
661652554927746511422616305598709806028936273783330550708657320764535272886  
927312605386510315315558918833265127225029454527779382057021572904217805918  
216043003268601905662028463911053153346087796800118420637520896621335557185  
806450376040935931203529573556130676484613917251180234260643199511148354296  
845287159710305315027455836455568363636515099524423608019228226220963706349  
692709364217540067577663

### 3. Compute $f = (p - 1) * (q - 1)$

f =

759839097649840244169652267303445213133486536034323997585730596563340066149  
792663628777241702467326622831675073362793278314985972726319407565088639126  
225354800335053455387252587544922167589051734494152634736036780510440885901  
984214461623220248670037264922593974200452432159031217995969001450319473425  
155576001442039557950303892871945617571140855927031889682863975078722384984  
944093374695700004733843103645403370958328587448493347367801578462353905452  
9176718259154826229752550104649541237519323696229519881851187709244339802  
083419985324963821069645010409303485571504072374416005405782295442313014762

418129656021738250354722621085184075373581050612360305144557425396043928753  
524937043631951292810891745729641050162788427083121593650987718542663471033  
763938733782909552519505131916171346672710202232278342585231041221477715364  
143346682654483352122656371287714891223234954464659299798320700831381782176  
510376891343492034832968

4. Generate  $e$  for which  $\gcd(f, e) = 1$

$e = 65537$

5. Generate  $d = e^{-1} \bmod f$

$d =$

583702326490092727661033361267558009295594544404657547010903869325439913793  
907436263344221333151007199390583663708284291267039516561736890212618635836  
394944648410337156285323275706076056689668577644813073466679459157394241432  
097826831719807489193784062476585648276878369425613419442560071074604176107  
991782867580137655751835596481958315403803750425659100753525288391874490319  
468550299053283133776726597998502108898745025483229283812685513033663539222  
532923204841768357073666663640669868221049336275488276522449883962065799278  
817916278761391329657312328136112180617015922664906446620292501396817808691  
486347216571011981233631541854732323339227276089526215153314060478261616996  
448005790036934675626964080424169227618682322375143150164929378672053839040  
463333621500840462968925734567033072130820683451852436905159784095935053176  
187448139802553738538766422211575235342383139028690242890984568768114436481  
322229040003175404636553

6. Hash the message

SHA-384 of the message:

eda19a7f481563d901f6cdacda0648d04637181db7f466f9fb7f1782b98a49f9c7db74cb7a914e  
b8b09a6e6edae91a2f

SHA-384 of the message decimal equivalent:

365747989233356570677807724547169274521594645995690861638741247180175411837  
24512787529252580357012737363224472132143

7. Sign the message using the private key  $\text{Sign} = h^d \bmod n$

$\text{Sign} =$

732742104862523608800898245158560239998430413787732335512298420469192221904  
042416888334065823158906573241967823270860377324185337839279458962653035297  
998774236465421897561432713380127998108366633611263607214920476989755975965  
930146103889054091988458742961298912568058435837739865851360985136645113963  
405410983288055052297316109624155473656064067563553029527986227643457395184  
367253142327478053590943967431142277335215989563135289351507849346907112279  
905364497429117617975138455611118314552394887169810545383019250896990467380  
099304278892620200518418526423563506588250662293022493249254343273559044432  
393707833999775038298560628206417672920213643264821933296312081520646810839  
832515877352983653487430338400645225871255464065716967395413818511918817608  
966825893595499964367113482207179867281610564875384777539448729120533601984

119808137329883922406178914669381798499307834576232208634535012084753218868  
548272490658123510264509

8. Check the signature using the public key  $h = \text{Sign}^e \bmod n$

$h =$

365747989233356570677807724547169274521594645995690861638741247180175411837  
24512787529252580357012737363224472132143

1.1. Party1 generates  $p(\text{prime})$ ,  $g(\text{generator})$ ,  $\text{PrivK1}$

$p =$

323170060713110073001535134778251633624880571334890751745884341392698068341  
362100027920563626401646854585563579353308169288290230805734726252735547424  
612457410262025279165729728627063003252634282131457669314142236542209411113  
486299916574782680342305530863490506355577122191878903327295696961297438562  
417412362372251973464026918557977679768230146253979330580152268587307611975  
324364674758554607150438968449403661304976978128542959586595975670512838521  
327844685229255045682728791137200989318739591433741758378260002780349731985  
520606075332341226032546840881200311059074842810039949669561196969562486290  
32338072839127039

$g = 2$

$\text{PrivK1} = 857$

1.2. Party1 computes  $\text{PubK1} = g^{\text{PrivK1}} \bmod p$ ; hash the message using SHA-256

$\text{PubK1} =$

960962154087001629436308185025184878247905227974336452694771122017616840015  
261658617268505418282416748614914455642299215255384405286687137505466996049  
907860485085046091635529899088428785810612776957410293491536147754283397719  
630991265870577566668501257551872

SHA-256 of the message:

29fa156225e639a107657503d5e62a8ffcdf623c786c52f3dcaeec2ca32ef5eb

SHA-256 of the message decimal equivalent:

189866861408478378983501955243535649742797531973342423095495103337642767784  
75

2. Party1 sends  $p$ ,  $g$ ,  $\text{PubK1}$  to Party2

3.1. Party2 generates  $\text{PrivK2}$

$\text{PrivK2} = 553$

3.2. Party2 computes  $\text{PubK2} = g^{\text{PrivK2}} \bmod p$ ; shared secret  $s2 = \text{PubK1}^{\text{PrivK2}} \bmod p$

$\text{PubK2} =$

294840814439182918143871451639708507102884470345034408466891117206689387686  
886629069228650404504591214177216799278425382794576924212874424418862050893  
17937841010900992

$s2 =$

990703868570605322415768597304519663557171412055020807739933742782170761215  
792589313205471832197102158624849480623470054046054059319826174139026821627  
076972317815566053341391410593022801172290120890907162496687406353708684438  
372834288315141633358445939940696497105483894931152410128302704373348086046  
333063893127240649972707640999003489734487982073098269163834079141538072629  
567712855989383120576792168051192470334373354571409180596024576980704128729

233042772016536898180937784127854069797026628124034104261517164225284613657  
018597474922146024310174257628799923123249679750134578491759885805262658262  
7337474593249904

4. Party2 sends PubK2 to Party1

5.1. Party1 computes the shared secret  $s1 = \text{PubK2}^{\text{PrivK1}} \bmod p$

$s1 =$

990703868570605322415768597304519663557171412055020807739933742782170761215  
792589313205471832197102158624849480623470054046054059319826174139026821627  
076972317815566053341391410593022801172290120890907162496687406353708684438  
372834288315141633358445939940696497105483894931152410128302704373348086046  
333063893127240649972707640999003489734487982073098269163834079141538072629  
567712855989383120576792168051192470334373354571409180596024576980704128729  
233042772016536898180937784127854069797026628124034104261517164225284613657  
018597474922146024310174257628799923123249679750134578491759885805262658262  
7337474593249904

5.2 Party1 signs the message using  $\text{Sign} = h * s1 \bmod p$

$\text{Sign} =$

165803906474281430618385825271720877554300741597054347859230972692565440156  
244113586582071969516709033359245940183177193126049720872320660290230440004  
382750067211539759931966223474703385744469538783098391349577518158903808458  
418101166130528719918584601996115588250823410777439944581716576064364012409  
868374323787671257750045616764073512766078301382599810064180734648290258974  
573595857482516399153296214976072662528399289934254677329807012895044658753  
292880425045822196282871873515499183331711745484065431301681047774931240009  
388301024165239277055737237945126751252102773623066254872581856067433555575  
9113252387512478

6. Party1 sends the signature to Party2

7. Party2 checks the signature using  $h = \text{Sign} * s2^{-1} \bmod p$

$h =$

189866861408478378983501955243535649742797531973342423095495103337642767784  
75



**Conclusions:**

1. The RSA algorithm can be used to sign a document with the private key, this way the signature cannot be forged and it can be checked with the public key.
2. The ElGamal algorithm for digital signatures works by generating a key pair consisting of a private key used for signing and a corresponding public key for verification. Signatures are created by combining the message with a random value and using modular arithmetic operations to produce a unique signature, while verification involves confirming the validity of the signature using the signer's public key and the original message.

**References:**

1. Wolfram alpha (used for random num. generation): <https://www.wolframalpha.com/>
2. Powermod calculator: <https://www.dcode.fr/modular-exponentiation>
3. Inverse powermod calculator: <https://www.dcode.fr/modular-inverse>
4. Big number multiplication calculator: <https://www.dcode.fr/big-numbers-multiplication>
5. Github repo: [https://github.com/muffindud/CS\\_Lab/tree/main/lab6](https://github.com/muffindud/CS_Lab/tree/main/lab6)