Exploring Robustness in Dynamic Graphs

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Abstract. A *dynamic graph* is a graph "in motion;" it changes over time. A static graph is one which never changes. A dynamic graph may be additive in nature, where vertices or edges are added to the graph. Or, a dynamic graph may be subtractive in nature, where elements are removed. These natures are not mutually exclusive, as a dynamic graph may demonstrate both additive and subtractive behaviors. The *robustness* of a graph is a measure of how well the graph maintains its structure, form, and integrity when undergoing drastic (perhaps unexpected) change. Common measures of robustness are related to connectivity, path length, and clustering. This project aims to examine robustness in dynamic graphs by constructing models of behavior (both normal and unexpected), and executing them on dynamic graphs. The measure of robustness in these experiments will be effectiveness of a spreading process. The work will be performed in a Jupyter Notebook environment with Gephi for graph visualization.

1. Introduction

The objective of this project is to determine if the effectiveness of a *spreading process* can be used as a measure of *robustness* in *dynamic graphs*. The general idea is to model a dynamic graph with some additive behavior and different detractive behaviors. The detractive processes will be modelled as random failures and targeted attacks. While these structural changes are taking place on the network, a spreading process will be working to "infect" the entire network. The spreading processes is considered effective, or successful, if and when the entire network becomes infected. The measure indicating success will be expressed as either a rate over time or as an amount of time to fully spread (i.e., reach saturation). This experimental apporach will be applied to exponential (Erdos-Renyi model) networks, and scale-free (Barabasi-Albert model) networks, and the resuls will be compared.

1.1 Dynamic Graphs

A *graph* G is a pair (V, E), where V is a finite set of vertices (or nodes), and E is a set of edges (or links). In an *undirected graph*, each edge is an unordered pair $\{u, v\}$ of distinct nodes. In a *directed graph*, each edge is an ordered pair $\{u, v\}$ of distinct nodes; this is an indication of a direction which reflects the origin and destination for the relationship represented by the edge. For *weighted graphs*, the notion of functions mapping values to numbers (the weight of a graph component) is added to the graph definition; $f()$: map vertices to numbers, and $g()$: map edges to numbers. The full model can be expressed as:

$$G = (V, E, f, g)$$

A *dynamic graph* is obtained when any of the graph components changes over time [1]. Graphs are subject to discrete changes, such as insertions or deletions of vertices or edges, or changes in weights. By dynamic graph we denote a graph that is subject to a sequence of updates [2].

Dynamic graphs are often typed or classified by the characteristics of their dynamic behavior. Depending on perspective, one of the following categorizations may be employed to assist in characterizing a dynamic graph.

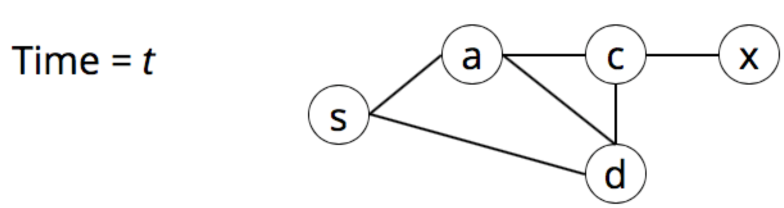
A component-based typing may be used where the dynamic graph is designated as [1]:

- **node-dynamic**: the set of V varies over time; some nodes may be added or removed.
- **edge-dynamic**: the set of E varies over time; edges may be added or removed.
- **weight-dynamic**: the weights on vertices or edges changes
- **fully-dynamic**: all aspects of the graph may change

Or, the emphasis may be placed on generally trying to qualify the type of change [3]:

- **incremental**: vertices or edges are only added
- **decremental**: vertices or edges are only removed
- **partially dynamic**: only certain components can be added or removed
- **fully dynamic**: any component can be added, removed, or changed

> *A common point in all these types and classification is that the system structure - the network topology - varies in time. Furthermore the rate and/or degree of the changes is generally too high to be reasonably modeled in terms of network faults or failures: in these systems changes are not anomalies but rather integral part of the nature of the system.*
>
> A.Casteigts, *Time-Varying Graphs and Dynamic Networks*



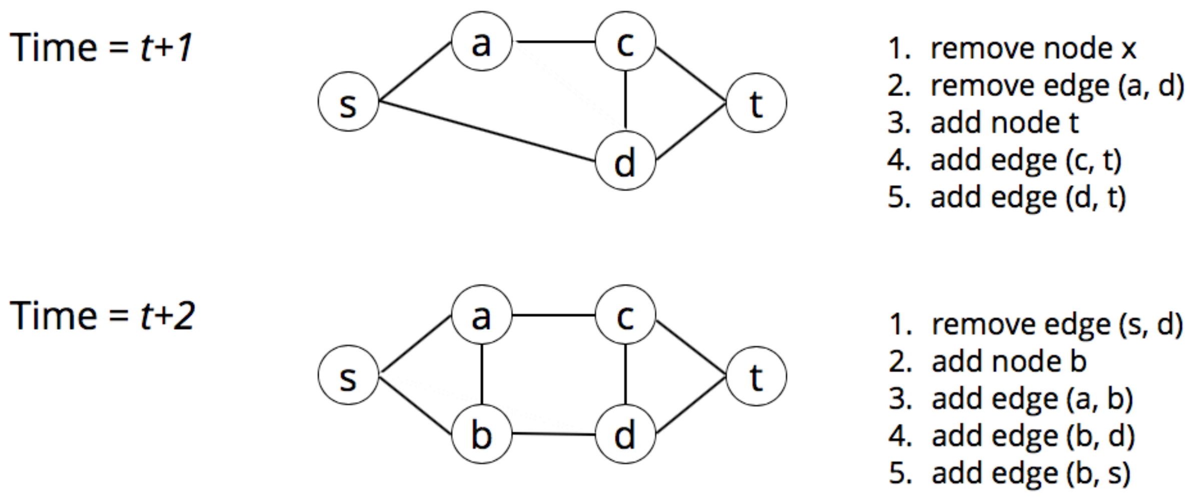


Figure 1 - Example Dynamic Network. A dynamic graph will change across a span of time. At each measured point in time (i.e., a snapshot) the amount of change effected, or the catalog of changes, on the graph can be accounted.

1.1.1 Models

Some formal models for dynamic graphs have emerged as the field has been studied. What follows is a brief overview of two that were interesting. One is presented in a formal model fashion, and the second is presented in a more descriptive manner. These presentations are for informational and study purposes only. While they were instructive and influential, they are not an integral part of the experiment.

1.1.1.1 TVG

Casteigts et al [4] set out with the goal of building a unified model and framework for dynamic networks. The research team explored many papers and journals to gather as much information as they could on the various published models. Ultimately they came to formally define a time-varying graph (TVG) and describe it by:

$$\mathcal{G} = (V, E, \mathcal{T}, \rho, \zeta, \psi, \varphi)$$

where

V	is the set of vertices
E	is the set of edges
$\mathcal{T}$	is a time span representing the lifetime of the system, where $\mathcal{T} \subseteq \mathbb{T}$
$\rho : E \times \mathcal{T} \rightarrow \{0, 1\}$	is a *presence function* that indicates whether an edge is available at a given time
$\zeta : E \times \mathcal{T} \rightarrow \mathbb{T}$	is a *latency function* that indicates the latency (i.e., weight or cost) of an edge at a given time

$\psi : V \times \mathcal{T} \to \{0, 1\}$ is a node presence function

$\varphi : V \times \mathcal{T} \to \mathbb{T}$ is a node latency function

1.1.1.2 Barabasi-Albert (extended)

Acknowledging that the original form of the Barabasi-Albert model has some shortcomings, several extensions were developed in order to captures a wide range of phenomena that shape the topology of real networks [7]. The extensions are briefly described here; the mathematics and complete description and definition can be found in the reference material.

- **Initial Attractiveness**: Developed to overcome the fact that nodes with degree = 0 will never gain any edges under *preferential attachment*. This extension adds a constant feature to the probability such that every node has at least a minimal likelihood of some attachment. Given this, nodes with degree = 0 will have some chance of gaining a connection to the broader network.
- **Internal Links**: Each cycle of the normal additive process in the BA model will add new nodes to the network and then add links to from those new nodes to existing nodes. In real world scenarios new links are added between existing nodes (not just involving new nodes). So the model was extended such that after the m new nodes are added, n new links are added between existing nodes.
- **Node Deletion**: This capability acknowledges the fact that nodes are removed in real networks.
- **Accelerated Growth**: Instead of growing the number of links linearly with the number of nodes, this extension provides the capability to grow the number of links faster than the normal (model defined) rate.
- **Aging**: Nodes, or edges, may naturally age over time and lose "strength" or effectiveness or drive or interest (for example, in the context of social or citation networks). This acts as a growth rate and expresses that a node's adding of edges may either slow or decline, or even become detractive (edges are removed) over time.

1.1.2 Representation

Various approaches have been developed for representing dynamic graphs. Given that a dynamic graph is not static, maintaining a simple edge list will not be sufficient. The temporal aspect of dynamic graphs must be addressed and accounted for, and this is generally achieved in one of two fashions.

1.1.2.1 Time-ordered snapshots

The *time-ordered snapshots* approach is to take a snapshot of the dynamic graph at either uniform periods of time, or after each change (or after some accumulated amount of change). The snapshot is simply a static representation of the dynamic graph at some point in time. These snapshots are bound into a collection called a sequence which are ordered by time. The format of the snapshot can be any valid form of representation: e.g., edge list, adjacency matrix.

A significant advantage of a sequence of time-ordered snapshots is that the state of the graph can be easily retrieved for any point in time. The main disadvantage of this approach is the amount of storage space required, especially when the graph is large and the number of snapshots increases.

1.1.2.2 Stream of time-ordered events

We first consider that every graph is essentially constructed by a set of ordered steps. At inception the first node in a graph appears, then a second node, and perhaps a link is established between those two initial nodes. The graph then continues to grow in each dimension -- adding nodes, and adding edges -- until construction is "complete." Up to this point, only an `add` operation has been employed. Beyond the initial contruction phase, the `update` and `delete` operation may be employed based on the behavior in the dynamic graph. Modelling adds, updates, and deletes as events is a common operation in software systems, and fits very naturally with dynamic graphs.

A major advantage of this approach is we are only required to store each event (add, update, delete) in time order, as they occur. For this we could employ a log or journaling mechanim, which has built-in ordering semantics. This approach could dramatically reduce the amount of storage space needed, even for large graphs with a lot of dynamic behavior.

This approach does come with some disadvantages. If we have no active representation of the graph (in computer memory), then in order to obtain the state of the graph at a given point in time, all events must be played from the beginning of time up to the desired point in time. For a large network with a lot of dynamic activity, this means processing a very large number of events which could take a long time. However, once a graph image (snapshot) is loaded in memory, if events can be played from that given point in time either forward or backward, then navigating from one point in time to another could be rather quick. This "play and rewind" approach could also permit very interesting active visualization of the graph, and provide live observations of the changes occuring.

1.1.2.3 Combined Approach

We could combine the stream of time-ordered events with time-ordered snapshots. The goal would be to achieve the benefits of both, and limit the disadvatages as much as possible. As with most algorithms and storage systems, there may be a need to choose between time and space tradeoffs. A perfect dynamic graph system would permit the user to choose the storage and representation approach based on their needs and computer system capacities.

1.2 Robustness

> *Many natural and social systems have a remarkable ability to sustain their basic functions even when some of their components fail.*
>
> Barabasi, *Network Science*

Robustness is an indicator of a network's ability to tolerate change. How is robustness or tolerance measured? The simplest approach is to compare a network's ability to function across different scenarios. The first scenario in which the perfect environemnt exists (i.e., no change, or no unexpected behavior) is designated the *baseline*. Comparison of other scenarios, with varying conditions and environments, to that baseline should indicate relatively how well or how poorly the network performed. With some form of threshold on the measurement, the indicator of tolerance for a network can then be distilled down and labeled as either *tolerant* or *vulnerable* under certain conditions.

Given that change is inherant in a dynamic network, then the concept of robustness should obviously apply as a first principle. But how can we effectively measure robustness in an ever-changing environment? The answer is to carefully apply the same baselining and scenario-based approach:

- Pick a measure
- Establish a baseline
- Run controlled experiments
- Analyze and compare
- Draw a conclusion
- Seek peer review
- Repeat

1.3 Hypothesis

Error and attack tolerance of complex networks [9] reports what we might consider the generally expected behavior for exponential (Erdos-Renyi model) and scale-free (Barabasi-Albert model) networks when experiencing random failures and when under targeted attack (Table 1). What about when the network is a dynamic network?

Network Type	Failure	Attack
Exponential	tolerant	tolerant
Scale-Free	tolerant	vulnerable

Table 1 - Expected Tolerance. The exponential and scale-free networks exhibit tolerance to random failures, but differ in their tolerances to targeted attacks.

Hypothesis 1

Given that exponential networks are tolerant to random failures and attacks, we suggest that a dynamic network formed from an Erdos-Renyi model and following a growth pattern consistent with that model (in terms of average node degree) will also be tolerant to random failures and attacks.

Hypothesis 2

Given that scale-free networks are tolerant to random failures but vulnerable to attacks, we suggest that a dynamic network formed from a Barabasi-Albert model and following a growth pattern consistent with that model (in terms of average node degree) will demonstrate the same tolerances.

In order to test these hypotheses, we use a spreading process to measure the network performance. The effectiveness of the spreading process will be measured in two ways. First, by the rate of spread (number of reached nodes over a period of time). Second, by the total elapsed time (simulated time cycles) to reach saturation.

2. Methods

The simulations for these experiments are written in Python and are run in the Jupyter Notebook environment. This affords us with the ability to rapidly try things and make changes. It also provides a means to give immediate visual feedback through plotting with Matplotlib.

NetworkX is used for creating, storing and manipulating the dynamic networks. Other libraries and packages were evaluated, but given the desired approach NetworkX was the best choice.

The networks are visualized in Gephi in real time as they are constructed and as the dynamic and spreading processes are run. This is achieved using the Gephi Graph Streaming plug-in [13] and refactoring an implementation which utilized the Gephi Graph Streaming API [14].

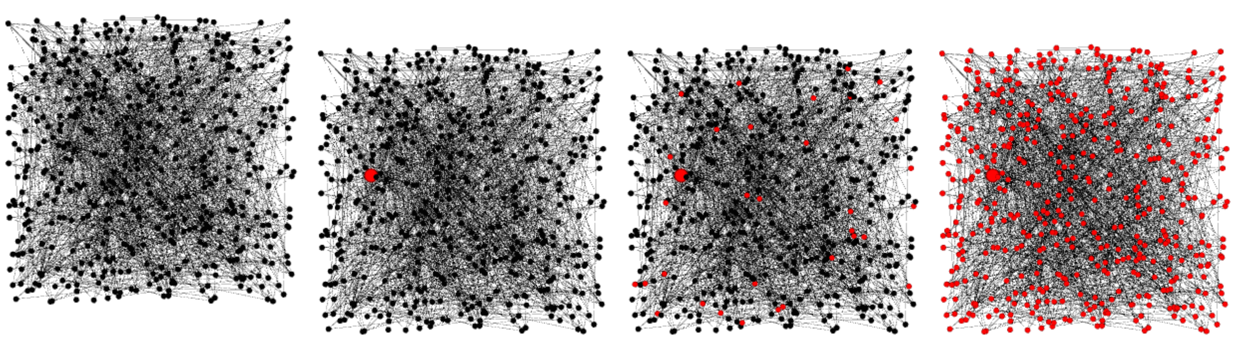


Figure 2 - Dynamic Graph Visualization in Gephi (a-d, left to right). (a) an Erdos-Renyi graph with 500 nodes has been rendered. (b) the origin of the spreading process has been randomly selected. (c) the spreading process is progressing. (d) the spreading process has completed, infecting all reachable nodes.

The spreading and dynamic processes are implemented as Python classes with functions and methods which perform the additive, detractive, and infection processes on a graph. The simulation itself is driven from a Python function which takes as parameters: the graph, the dynamic model, and the maximum number of time cycles (iterations) to run. The simulation will cycle until either the spreading processes completes (i.e., zero nodes are infected during an interation) or the maximum number of time cycles has elapsed.

The source code for this project and all results are provided as an appendix to this Jupyter Notebook, and should be executable given the necessary versions of software packages are installed.

3. Results

A series of six experiments were conducted, a baseline on static graphs and five additional runs with varying dynamic process. During each experiment a spreading process was run which initially targeted one node (the origin, or seed node). During each simulated time cycle the spreading process expanded by infecting any uninfected neighbors of all infected nodes.

The six experiments are identified and described below (note, the lettered list label is used to identify the sceanrio with the plots in the following sections):

a. *Baseline (Spreading Only)*. In the baseline scenario we are running just the spreading process. The graphs have no dynamic activity; no adds, no deletes, nothing changes.

b. *Additive Process Only*. In this scenario an additive process was applied to the graphs, thus making them dynamic graphs. 20 nodes were added to the graph at each simulated time cycle. Edges were randomly added to the new nodes giving them a degree comparable to the average degree of the entire graph. The spreading process was run simultaneously.

c. *Random Detractive Process*. In addition to the additive process (adding 20 nodes per time cycle), a detractive process was applied to the graphs. The detractive process removed 20 randomly selected nodes each simulated time cycle. The spreading process was run simultaneously.

d. *Targeted Detractive Process*. A targeted detractive process was deployed with the additive process (adding 20 nodes per time cycle) and the spreading process. At each time cycle, the detractive process removed the node with the highest degree.

e. *Invasive Targeted Detractive Process*. The targetted detractive process was made more invasive. At each time cycle, the detractive process removed 2.5% of nodes by highest degree. This was run along with the additive process (adding 20 nodes per time cycle) and the spreading process.

f. *Extremely Invasive Detractive Process*. The targetted detractive process was made even more invasive. At each time cycle, the detractive process removed 10% of nodes by highest degree. In order to offset the invasive detractive process and keep the graph size approximately the same (in terms of number of nodes), the effectiveness of the additive process was increased to adding 50 nodes per simulated time cycle.

The graphs presented in the following sections show the two spreading metrics of interest:

- **spreading rate** (number of nodes infected over time), as red dashes
- **total spread** which indicates *time to reach saturation*, as solid blue lines

3.1 Exponential graphs (Erdos-Renyi model)

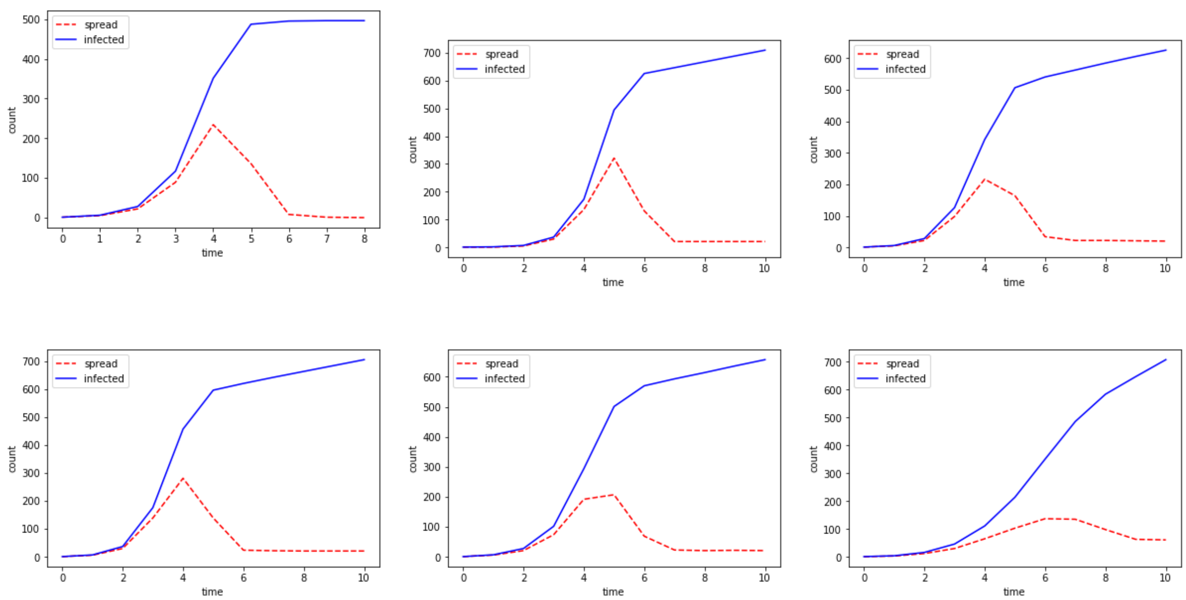


Figure 3 - Results on Exponential Graphs (a-e, top left to bottom right). The peak of the red lines indicate the point at which the spreading process was moving the fastest. When the blue line levels off saturation (total spread) has been reached. Note that in (b) nodes are being added at a constant rate, and so total spread continues to grow at a slow linearly pace after time = 6 and the spreading rate never drops to zero; this also occurs in all subsequent scenarios. In (c), (d), and (e) the continued slow linearly growth is due to the oscilation of adding and removing nodes at the same rate. In (f) we observe that the higher amount of churn is causing the total spread to condinue to grow at a slightly greater linear rate, but the spreading rate indicates a leveling off.

3.2 Scale-Free graphs (Barabasi-Albert model)

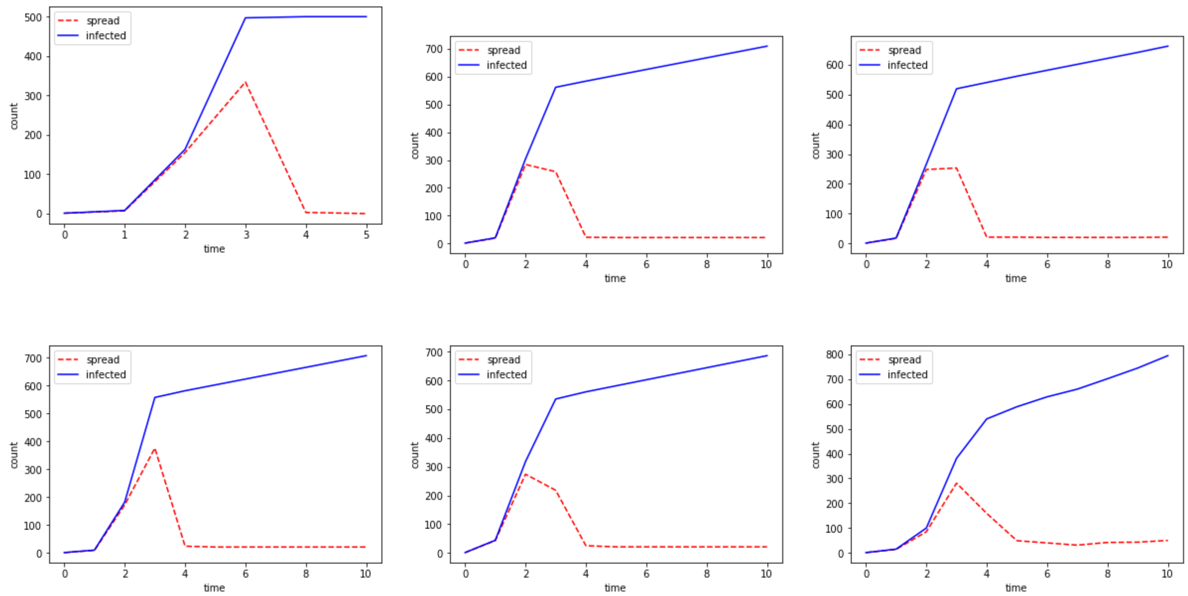


Figure 4 - Results on Scale-Free Graphs (a-e, top left to bottom right). All scenario results are approximately consistent. We observe a sharp spike in spreading at time = 2 or time = 3 which indicates spreading from hubs to a large number of uninfected neighbors. After the spike the spreading rate drops dramatically, almost to zero, indicating that total saturation is reached nearly immediately after the hubs are infected.

4. Discussion

4.1 Spreading Rate

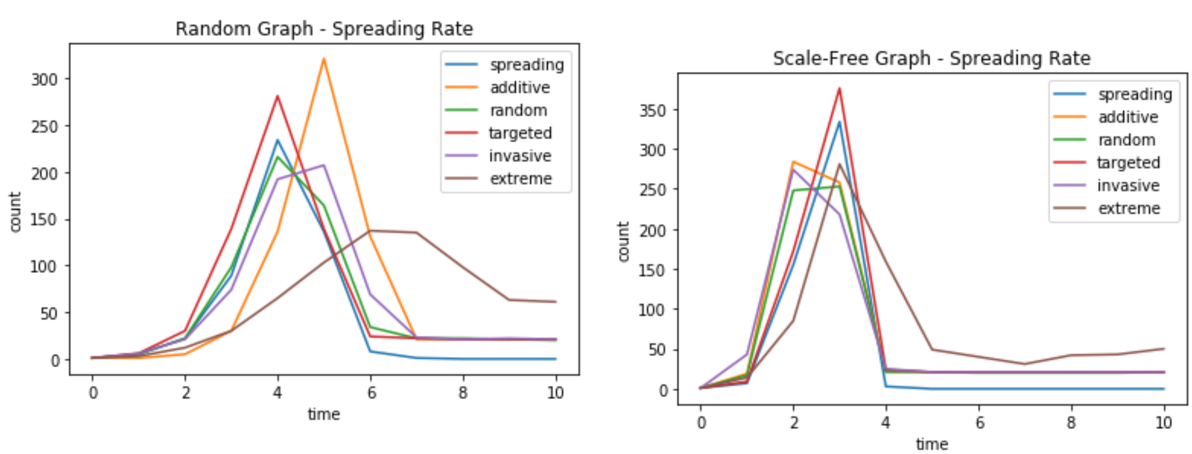


Figure 5 - Spreading Rate: by graph type, and scenario. In the random graphs (figure (a), left) the spreading rate for the spreading (no dynamic activity), random detractive, and targeted detractive all peak at time = 4. The additive process peaks at time = 5, suggesting that the extra nodes added delay the sharp rise in spreading rate. Delayed and slower spreading are observed in the invasive and extreme scenarios, where many more nodes are removed each cycle. The

scale-free graphs (figure (b), right) show approximately the same spreading rate across all scenarios, with peaks at time = 2 or time = 3, and then sharp drop off which indicates reaching saturation. Only in the extreme scenario does the decay of spreading rate linger to time = 5.

4.2 Total Spread (Saturation)

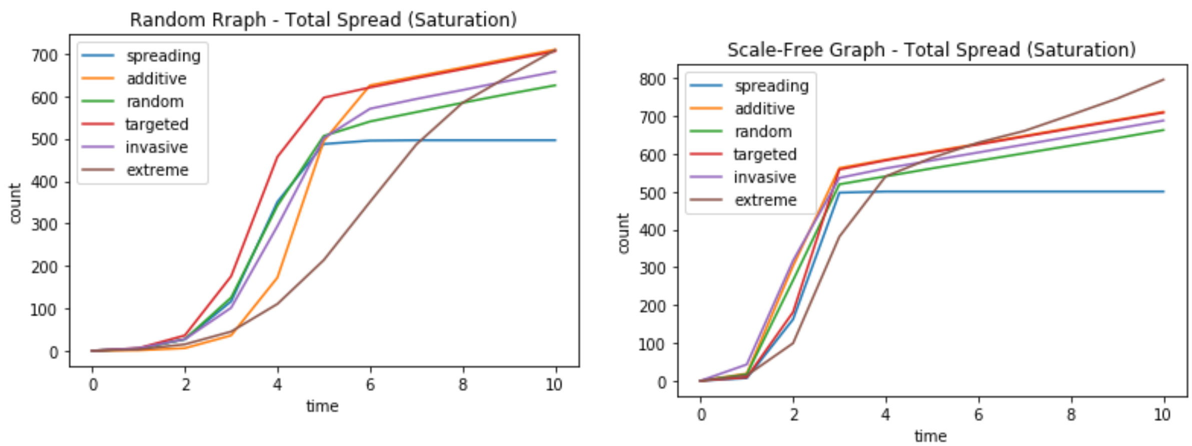


Figure 6 - Total Spread (Saturation): by graph type, and scenario. For the random graphs (figure (a), left) we observe that the point of saturation is reached by time = 5 for all scenarios except extreme. In the extreme case we note that the rate of growth is much slower until about time = 4, and then the rate increases. But saturation for extreme must be assumed at time = 10 when the scenario complete. In the final few time cycles the churn in the graph was oscilating between adding uninfected nodes and removing infected notes at an approximately equal rate. In the scale-free graphs the time at which we reach the point of saturation is consistent with the observed peak spreading rate observations (figure 5(b)). Again, only in the case of the extreme scenario is saturation reached later than time = 3.

5. Conclusion

Network Type	Failure (random)	Attack (targeted)	Invasive Attack
Exponential	tolerant	tolerant	vulnerable
Scale-Free	tolerant	tolerant	tolerant

Table 2 - Observed Tolerance in Dynamic Networks. Both exponential and scale-free dynamic networks exhibit tolerance to random failures and targeted attacks, which is a deviation from the observations in *Error and attack tolerance of complex networks*. In the invasive attack scenarios, the exponential network showed vulnerability as the spreading rate was hindered and saturation was almost not reached. However, perhaps most surprising, is that the scale-free network was very tolerant under the invasive attack scenario.

In this experiment we are not able to conclude that either hypothesis is confirmed true. We suggested that exponential networks would be tolerant to all dynamic activity, but under invasive attack we saw evidence of vulnerability. Also, scale-free networks did not align with what was suggested. In fact, under all attack scenarios no detrimental effect was observed by the removal of hubs. This suggests that there is a greater underlying role provided by the low-degree nodes in dynamic networks [12].

One of the preliminary objectives when we set out was to determine if *the effectiveness of a spreading process can be used as a measure of robustness in dynamic graphs*. We were able to examine two metrics of spreading -- spreading rate, and total spreading (saturation) -- and both measures provided insight into the behavior of different graph models under different dynamic processes. Hence, the results from these experiments support this approach and show the effectiveness of using a spreading process as a measure of robustness in dynamic graphs.

6. What's Next?

The logical next step in this area of exploration (or research) would be to enhance the fidelity of the simulation, and perhaps consider other measures of robustness (e.g., diameter, shortest paths, effects on community structures).

Some additional thoughts on enhancements:

- add fairness by varying the order of execution steps in the dynamic processes
- truly run the dynamic processes concurrently (or in a more fair manner)
- build an event-based mechanism for dynamic processes
- vary the amount of cycle-to-cycle activity of the dynamic processes, while still achieving the desired goal over a longer time span
- run longer simulations, and run with models of real world networks and behavior
- explore dynamic behavior with respect to edges (links)
- consider different spreading phenomena, including models with concepts like reinforcement and resilience (vaccination)
- investigate further into the role of low-degree nodes
- build richer tracking of dynamic processes and finer grained results, in order to gain deeper understanding through detailed "what happened?" analysis
- visualize dynamic graphs with Cubix [15]

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Appendix

Exploring Gephi Graph Streaming

Sources of information and inspiration:

Gephi - Graph Streaming (https://github.com/gephi/gephi/wiki/GraphStreaming)

Visualization of Large Dynamic Networks (http://www.eecs.wsu.edu/~yyao/DynamicGraph.html)

Versions of software, packages, and libraries used:

- Python 3.6
- Jupyter Notebook 5.4.1
- Gephi 0.92
- requests 2.18.4
- numpy 1.14.2
- NetworkX 2.1
- Matplotlib 2.2.2

Notes:

- NetworkX: Migration guide from 1.X to 2.0 (https://networkx.github.io/documentation/stable/release/migration_guide_from_1.x_to_2.0.htr

Setup and Foundation

Gephi Graph Streaming plugin

The Gephi Graph Streaming plugin must be installed. To do this, start Gephi and open the Plugins page (Tools -> Plugins). Switch to the Available Plugins panel, find and select Graph Streaming in the list and click Install. Once complete Gephi may need to restart to enable the plugin.

Turning on Gephi Master (Streaming)

- Run the Gephi application
- Create an empty workspace (File -> New Project, or Workspace -> New)
- Add some nodes and edges to your graph
- Go to the Streaming tab (next to Appearance in the lower left)
- right-click on the “Master Server” and select “Start”

Start a Graph Stream

You can obtain a stream of graph events by making a `getGraph` request to Gephi.

```
curl "http://localhost:8080/workspace1?operation=getGraph"
```

This will not terminate, it is a live stream of Graph events that will run "forever" (that is, until Gephi Streaming is turned off, Gephi terminates, or the client requesting the stream is stopped).

Note: this is not a required step to run this Notebook, nor is it necessary to use the Gephi Graph Streaming API. It is simply an example, and an effective means to observe a Gephi Graph Stream.

Gephi API

Below is a simple implementation of the Gephi Graph Streaming API based on [`GephiJsonClient` (http://www.eecs.wsu.edu/~yyao/DirectedStudyII/src/Citation/GephiJsonClient.py)](http://www.eecs.wsu.edu/~yyao/DirectedStudyII/src/Citation/GephiJsonClient.py); it is modified for newer version of Python and newer version of Gephi. Also it was changed to use the [`requests` (http://docs.python-requests.org/en/master/)](http://docs.python-requests.org/en/master/) library, which also has the benefit of simplifying the implementation significantly.

Parameters

Where is Gephi running?

- `host` = the name of the machine where Gephi is running
- `port` = the HTTP port for the Gephi REST API
- `workspace` = number of the Gephi workspace

In [82]:
```
host = "localhost"
port = 8080
workspace = 1
```

Build a URL from the parameters

This URL provides access to manipulate the graph in Gephi, note the `operation=updateGraph`. There are other operations which allow querying Gephi for vertex and edge information, and probably others which were not explored in this project.

Each *workspace* in Gephi is a separate space and must be addresed directly; note the `/workspace` part of the URL with a trailing number. Gephi *Graph Streaming* must be turned on for each workspace that we want to acccess.

In [83]:
```
def gephiURL(host, port, workspace):
    return "http://{}:{}/workspace{}?operation=updateGraph".format(host,
```

In [84]:
```
url = gephiURL(host, port, workspace)
print(url)
```

http://localhost:8080/workspace1?operation=updateGraph (http://localhost:8080/workspace1?operation=updateGraph)

JSON formatted messages

A JSON document with the specific graph operation is POST'ed to the above URL:

- Vertex operations:
  - `an` : add node
  - `cn` : update node
  - `dn` : delete node
- Edge operations:
  - `ae` : add edge
  - `ce` : update edge
  - `de` : delete edge

Example: *Add Node*

```
{
  "an":{
        "A":{
             "label":"Node A",
             "size":2
            }
  }
}
```

Example: *Add Edge*

```
{
  "ae":{
        "AB":{
              "source":"A",
              "target":"B",
              "directed":false,
              "weight":2
             }
  }
}
```

Gephi direct access functions

In [85]:
```
import requests

def post(content):
    #print(content)
    response = requests.post(url, json=content)
    return response
```

In [86]:
```
def addVertex(node, attr={}):
    content = {"an":{""+node:attr}}
    return post(content)

def removeVertex(node):
    content = {"dn":{""+node:{}}}
    return post(content)

def changeVertex(node, attr={}):
    content = {"cn":{""+node:attr}}
    return post(content)

def addEdge(eid, source, target, directed=False, weight=1.0):
    attributes = {}
    attributes['source'] = source;
    attributes['target'] = target;
    attributes['directed'] = directed;
    attributes['weight'] = weight
    content = {'ae': {eid:attributes}}
    return post(content)

def removeEdge(eid):
    content = {'de':{eid:{}}}
    return post(content)

def changeEdge(eid, attr={}):
    content = {'ce':{eid:attr}}
    return post(content)
```

Gephi convenience functions

In [87]:
```
def addNode(node):
    return addVertex(str(node), {'label':str(node)})

def addLink(source, target, directed=False, weight=1.0):
    return addEdge("{}-{}".format(str(source), str(target)), str(source),

def addLinks(links):
    for link in links:
        addLink(link[0], link[1])

def connect(node, neighbors):
    '''
    connect(node, neighbors)
      connect a node to list of neighbors

      Inputs:
        node = the source node
        neighbors = a list of target neighbors
    '''
    for neighbor in neighbors:
```

```
        addLink(node, neighbor)

def newNode(node, neighbors):
    '''
    newNode(node, neighbors)
      Create a node node and connect to a list of neighbors

      Inputs:
        node = the new node to be added
        neighbors = a list of neighbors
    '''
    addNode(node)
    connect(node, neighbors)

def deleteNode(node):
    removeVertex(str(node))

BLACK = '#000000'
WHITE = '#ffffff'
RED   = '#ff0000'
GREEN = '#00ff00'
BLUE  = '#0000ff'

def setNodeColor(node, color):
    node = str(node)
    attr = {'color':color}
    return changeVertex(node, attr)

DEFAULT = 10.0
SMALL   = 5.0
MEDIUM  = 15.0
LARGE   = 25.0
XLARGE  = 50.0

def setNodeSize(node, size):
    node = str(node)
    attr = {'size':size}
    return changeVertex(node, attr)
```

Create a graph in Gephi

custom format -> Gephi

This custom format is a multi-level Python dictionary. The top-level keys represent the vertices of the graph. The values are dictionaries which designate directed edges where the keys are the target vertices and the values are the weights.

In [88]:
```python
graph = {
    's': {'a': 2, 'b': 1},
    'a': {'s': 3, 'b': 4, 'c': 8},
    'b': {'s': 4, 'a': 2, 'd': 2},
    'c': {'a': 2, 'd': 7, 't': 4},
    'd': {'b': 1, 'c': 11, 't': 5},
    't': {'c': 3, 'd': 5}
}
```

In [89]:
```python
'''
gephiGraphCuston(graph)
  Create a graph in Gephi from custom graph format

  Inputs:
    graph = graph in custom format

  Notes:
    Creates an undirected and unweighted graph, despite the fact
    that the custom format allows specifying directed and weighted.
'''
def gephiGraphCuston(graph):
    for node in graph:
        response = addNode(node)
    for node in graph:
        for neighbor in graph[node]:
            response = addLink(node, neighbor)
```

NetworkX -> Gephi

This takes a NetworkX `Graph` and will send a series of commands (for all of the nodes and links) to construct and visualize the graph in Gephi.

In [90]:
```python
'''
gephiGraphNx(graph)
  Create a graph in Gephi from a NetworkX graph

  Inputs:
    graph = a NetworkX graph

  Notes:
    Creates an undirected and unweighted graph.
'''
def gephiGraphNx(graph):
    for node in graph.nodes():
        result = addNode(str(node))
    for edge in graph.edges():
        source = edge[0]
        target = edge[1]
        result = addLink(str(source), str(target))
```

Network Creation functions

In [91]:
```
import numpy as np
import networkx as nx

'''
barabasi_albert_graph(n, m=1):
    a function that takes `n` the number of nodes for the graph,
    and `m` the initial number of nodes, as arguments and returns a netwo
    graph with a scale-free degree distribution.
'''
def barabasi_albert_graph_without_using_degree(n, m=1):
    # create a complete graph with m initial nodes.
    # (note: we could use nx.complete_graph() here, but opt to do the cre
    # to keep the `type` of the graph created as Graph instead of Complet
    g = nx.Graph()
    for i in range(m):
        g.add_node(i)
        for j in range(i+1,m):
            g.add_edge(i,j)

    # while network has less than n nodes,
    for i in range(m, n):
        # get the list of edges
        edges = [e for e in g.edges]
        # randomly select m of the edges with equal probability
        selected = np.random.choice(len(edges), m, replace=False)
        # create a new node
        g.add_node(i)
        for s in selected:
            # for each of the selected edges, randomly pick one of the no
            linkTo = np.random.choice(edges[s], 1)[0]
            # and add an edge from the new node to that node
            g.add_edge(i, linkTo)

    return g
```

Plotting and Supporting funtions

Graph Calculations

In [92]:
```
import networkx as nx

def averageDegree(graph):
    return sum([x[1] for x in list(nx.degree(graph))])/len(nx.nodes(graph
```

In [93]:
```
def probabilityForDegree(N, k):
    return k/(N-1)
```

In [94]:
```
import networkx as nx

def graphDegreeTopN(graph, n=10):
    nodes = sorted(list(nx.degree(graph)), key=lambda x: x[1], reverse=Tr
    print("{0:4s} {1:8s} {2:8s}".format("N","Node","Degree"))
    print("---- -------- --------")
    for i in range(n):
        print("{0:4d} {1:8d} {2:8d}".format(i+1,nodes[i][0],nodes[i][1]))
```

Plot Degree Distribution

In [95]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline

def showDegreeDistribution(graph):
    degrees = sorted([d[1] for d in graph.degree()])
    (v, e, p) = plt.hist(degrees)

def showDegreeDistributionWithFit(graph):
    degrees = sorted([d[1] for d in graph.degree()])
    (y, x, p) = plt.hist(degrees)
    y = np.append(y, [0])
    z = np.polyfit(x, y, 3)
    f = np.poly1d(z)
    x_new = np.linspace(x[0], x[-1], 50)
    y_new = f(x_new)
    plt.plot(x_new, y_new, "r")
    plt.xlim([x[0]-1, x[-1] + 1 ])
    plt.show()

def showDegreeDistributionLogScale(graph):
    degrees = sorted([d[1] for d in graph.degree()], reverse=True)
    x = np.arange(0, len(degrees))
    plt.plot(x, degrees, "r")
    plt.xscale('log')
    plt.show()

def showDegreeDistributionLogLogScale(graph):
    degrees = sorted([d[1] for d in graph.degree()], reverse=True)
    x = np.arange(0, len(degrees))
    plt.loglog(x, degrees, "r")
    plt.show()
```

Method

Overview

The exploration is essentailly these steps: create a graph, run a simulation which performs a spreading process. The exploration will be run on static graphs where this is no dynamic behavior: no nodes or edges are added or removed. This will act as a baseline from which we can compare the results on dynamic graphs.

The exploration will be performed on:

- static graphs: no dynamic graph activity, just a spreading process
- dynamic graphs: spreading with an addative process
- dynamic graphs: spreading with an addative and random detractive process
- dynamic graphs: spreading with an addative and targeted detractive process
- dynamic graphs: spreading with an addative and more invasive targetted detractive process
- dynamic graphs: spreading with a larger addative and extreme invasive targetted detractive process

Discussion

- what will be the starting point?
- probably a BA graph (Power Law degree distribution)
- what is the normal dynamic nature?
- adding vertices, and edges
- at what rate?
- how will we control time?
- what are the different fault models?
    - what are the targets? (vertices or edges)
    - failure: random target selected
    - attack: specific target selected
- how are the fault models controlled?
- what is the measure for robustness?
- how will we show the robustness measure?

Ultimately I would like to define, model, and run the simulation from the Jupyter Nodebook. The activity should be visible in the Jupyter Notebook (i.e., printing messages about actions taking place), observe the live activity in Gephi, and at periodic intervals export data which can be displayed in Cubix (separately, after the simulation completes).

Implementation

Simulation Driver

In [96]:

```
import time

def simulate(graph, model, max_iterations = 100):
    spreader = model.spreader()
```

```
grower = model.grower()
destroyer = model.destroyer()

def init():
    return {'spread':      {'count': {interval: 0}, 'total': {interva
            'growth':      {'count': {interval: 0}, 'total': {interva
            'destruction': {'count': {interval: 0}, 'total': {interva

def track(tracking, key, count):
    if interval > 0:
        total = tracking[key]['total'][interval-1]
    else:
        total = 0
    tracking[key]['count'][interval] = count
    tracking[key]['total'][interval] = (total + count)
    return

def spreadingProcess():
    spread = spreader.spread(graph)
    track(tracking, 'spread', spread)
    return spread

def growingProcess():
    growth = grower.grow(graph)
    track(tracking, 'growth', growth)
    return growth

def destroyingProcess():
    destruction = destroyer.destroy(graph)
    track(tracking, 'destruction', destruction)
    return destruction

# start...
interval = 0
tracking = init()
# initial contact to a single 'origin' node
spread = spreader.spread(graph)
track(tracking, 'spread', spread)
# pause a little so we can observe the 'origin' selection
time.sleep(10)

# run until spreading stops, or the max number of iterations has elap
while (spread != 0) and (interval < max_iterations):
    interval += 1
    # Step 1: run growing
    if grower != None: growth = growingProcess()
    # Step 2: run destruction
    if destroyer != None: destruction = destroyingProcess()
    # Step 3: run spreading
    spread = spreadingProcess()
    # pause so we can observe each iteration's activity
    time.sleep(2)

return tracking
```

Spreading Process

In [97]:

```
import networkx as nx

class Spreader:
    contagion='infected'
    indicator=RED
    interval = 0

    def __init__(self, contagion='infected', indicator=RED):
        self.contagion = contagion
        self.indicator = indicator

    def config(self):
        print("Spreader() contagion: {} indicator: {}".format(self.contag

    def infect(self, graph, node, size=DEFAULT):
        graph.nodes[node].update({self.contagion: True})
        setNodeColor(node, self.indicator)
        setNodeSize(node, size)

    def initial(self, graph):
        # add the contagion attribute to all nodes
        nx.set_node_attributes(graph, name=self.contagion, values=False)
        # randomly select the origin
        origin = np.random.choice(list(graph.nodes()), 1)[0]
        self.infect(graph, origin, size=LARGE)
        return 1

    def incremental(self, graph, infected):
        self.interval += 1
        count = 0
        # infect any uninfected neighbors of the infected nodes
        for node in infected:
            for neighbor in nx.neighbors(graph, node):
                if not graph.nodes.data()[neighbor][self.contagion]:
                    print("time: {0:4d} spread from: {1} -> {2}".format(s
                    self.infect(graph, neighbor)
                    count += 1
        return count

    def uninitialized(self, graph):
        nodes = [node[0] for node in filter(lambda x: x[1] == None, list(
        for node in nodes:
            graph.nodes[node].update({self.contagion: False})

    def spread(self, graph):
        infected = [node[0] for node in filter(lambda x: x[1] == True, li
        if len(infected) == 0:
            return self.initial(graph)
        else:
            self.uninitialized(graph)
            return self.incremental(graph, infected)
```

Graph trait for additive and detractive processes

In [98]:
```
class Grapher:
    def __init__(self):
        pass

    def graphAdd(self, graph, node, edges, color=BLACK, size=DEFAULT):
        self.graphAddNode(graph, node, color, size)
        self.graphAddEdges(graph, edges)

    def graphAddNode(self, graph, node, color=BLACK, size=DEFAULT):
        # add node to NetworkX graph
        graph.add_node(node)
        # add node to Gephi, and set color
        addNode(node)
        setNodeColor(node, color)

    def graphAddEdge(self, graph, source, target):
        # add edge to NetworkX graph
        graph.add_edge(source, target)
        # add edge to Gephi
        addLink(source, target)

    def graphAddEdges(self, graph, edges):
        # add edges to NetworkX graph
        graph.add_edges_from(edges)
        # add edges to Gephi
        addLinks(edges)

    def graphDeleteNode(self, graph, node):
        # delete node from NetworkX graph
        graph.remove_node(node)
        # delete node in Gephi
        deleteNode(node)
```

Additive Process

In [99]:
```
import numpy as np

class Expander(Grapher):
    interval = 0
    nodes = 0
    links = 0
    next_node = 0

    def __init__(self, nodes = 10, links = 10):
        self.nodes = nodes
        self.links = links

    def expand(self, graph):
        self.interval += 0
        nodes = list(graph.nodes())
```

```
        nodes = list(graph.nodes())
        if self.next_node == 0:
            self.next_node = max(nodes)
        for i in range(self.nodes + 1):
            self.next_node += 1
            neighbors = np.random.choice(nodes, self.links, replace=False
            edges = list(zip(np.full(len(neighbors), self.next_node), nei
            self.graphAdd(graph, self.next_node, edges, color=BLUE)
        return self.nodes

class HubCreator(Grapher):
    interval = 0
    number = 3
    threshold = 0.25

    def __init__(self, number = 3, threshold = 0.25):
        self.number = number
        self.threshold = threshold

    def create(self, graph):
        self.interval += 0
        created = 0
        # find top `number` of nodes by degree
        # count how many have degree >= threshold?
        # if `count` is less than `number`
        # then create `number` - `count` hubs with degree (`threshold` *
        return created

class Grower:
    interval = 0
    expander = None
    hubCreator = None

    def __init__(self, expander = None, hubCreator = None):
        self.expander = expander
        self.hubCreator = hubCreator

    def grow(self, graph):
        self.interval += 1
        growth = 0
        if self.expander != None:
            growth += self.expander.expand(graph)
        if self.hubCreator != None:
            growth += self.hubCreator.create(graph)
        return growth
```

Detractive Process

In [100]:

```
import numpy as np

class Target:
    number = 10
    percent = 0.0

    def __init__(self, number=10, percent=0.0):
        self.number = number
        self.percent = percent

class TargetRandomNodes(Target):
    def select(self, graph):
        nodes = list(graph.nodes())
        # randomly select a percent of all nodes, or a fixed number of nc
        if self.percent > 0.0:
            targets = np.random.choice(nodes, int(self.percent * len(node
        else:
            targets = np.random.choice(nodes, self.number, replace=False)
        return targets

class TargetHighDegreeNodes(Target):
    def select(self, graph):
        # get list of nodes ordered by degree, descending (highest -> low
        nodes = [y[0] for y in sorted(list(nx.degree(graph)), key=lambda
        # select targetted nodes, starting from the head of the list
        if self.percent > 0.0:
            target = nodes[0:int(self.percent * len(nodes))]
        else:
            target = nodes[0:self.number]
        return target

class Destroyer(Grapher):
    interval = 0
    target = None

    def __init__(self, target=None):
        self.target = target

    def destroy(self, graph):
        self.interval += 1
        count = 0
        if self.target != None:
            selected = self.target.select(graph)
            for node in selected:
                count += 1
                self.graphDeleteNode(graph, node)
        return count
```

Model

In [101]:

```
'''
Model

  Contains the three components of the dynamic model
  - Spreader: responsible for the spreading process
  - Grower: responsibile for dynamically growing the graph
  - Destroyer: destroys elements of the graph

'''
class Model:
    _spreader = None
    _grower = None
    _destroyer = None

    def spreader(self):
        return self._spreader

    def grower(self):
        return self._grower

    def destroyer(self):
        return self._destroyer
```

In [102]:

```
'''
SpreadingOnly

  Provides only a Spreader.

'''
class SpreadingOnly(Model):
    def __init__(self, graph = None):
        self._spreader = Spreader()
        self._grower = None
        self._destroyer = None
```

In [103]:
```
'''
AdditiveOnly

  Provides a Spreader, and a Grower with an Expander that will add nodes
  on the `nodes` parameter, and links based on the `links` parameter or t
  average degree of the graph.

'''
class AdditiveOnly(Model):
    def __init__(self, graph, nodes=20, links=-1):
        if links == -1:
            links = round(averageDegree(graph))
        self._spreader = Spreader()
        self._grower = Grower(expander = Expander(nodes = nodes,
                                                  links = links),
                              hubCreator = None)
        self._destroyer = None
```

In [104]:
```
'''
AddativeWithFixedRandomDetractive

  Provides a Spreader, and a Grower with an Expander, and also provides a
  Destroyer that will TargetRandomNodes on a fixed number basis defined
  by the `remove` parameter.

'''
class AddativeWithFixedRandomDetractive(Model):
    def __init__(self, graph, nodes=20, links=-1, remove=20):
        if links == -1:
            links = round(averageDegree(graph))
        self._spreader = Spreader()
        self._grower = Grower(expander = Expander(nodes = nodes,
                                                  links = links),
                              hubCreator = None)
        self._destroyer = Destroyer(target=TargetRandomNodes(number = rem
```

In [105]:
```
'''
AddativeWithPercentRandomDetractive

  Provides a Spreader, and a Grower with an Expander, and also provides a
  Destroyer that will TargetRandomNodes on a top-n percentage basis defin
  by the `percent` parameter.

'''
class AddativeWithPercentRandomDetractive(Model):
    def __init__(self, graph, nodes=20, links=-1, percent=0.05):
        if links == -1:
            links = round(averageDegree(graph))
        self._spreader = Spreader()
        self._grower = Grower(expander = Expander(nodes = nodes,
                                                  links = links),
                              hubCreator = None)
        self._destroyer = Destroyer(target=TargetRandomNodes(number = 0,
```

In [106]:
```
'''
AddativeWithTargetedDetractive

  Provides a Spreader, and a Grower with an Expander, and a Destroyer tha
  TargetHighDegreeNodes by number specified by the `remove` parameter..

'''
class AddativeWithTargetedDetractive(Model):
    def __init__(self, graph, nodes=20, links=-1, remove=1):
        if links == -1:
            links = round(averageDegree(graph))
        self._spreader = Spreader()
        self._grower = Grower(expander = Expander(nodes = nodes,
                                                  links = links),
                              hubCreator = None)
        self._destroyer = Destroyer(target=TargetHighDegreeNodes(number =
```

In [107]:
```
'''
AddativeWithInvasiveDetractive

  Provides a Spreader, and a Grower with an Expander, and a Destroyer tha
  TargetRandomNodes on a top-n percent basis.

'''
class AddativeWithInvasiveDetractive(Model):
    def __init__(self, graph, nodes=20, links=-1, percent=0.025):
        if links == -1:
            links = round(averageDegree(graph))
        self._spreader = Spreader()
        self._grower = Grower(expander = Expander(nodes = nodes,
                                                  links = links),
                              hubCreator = None)
        self._destroyer = Destroyer(target=TargetHighDegreeNodes(number =
```

In [108]:
```
'''
AddativeWithExtremeInvasiveDetractive

  Provides a Spreader, and a Grower with an Expander that has a higher no
  and a Destroyer that will TargetRandomNodes with a high top-n percent v

'''
class AddativeWithExtremeInvasiveDetractive(Model):
    def __init__(self, graph, nodes=50, links=-1, percent=0.10):
        if links == -1:
            links = round(averageDegree(graph))
        self._spreader = Spreader()
        self._grower = Grower(expander = Expander(nodes = nodes,
                                                  links = links),
                              hubCreator = None)
        self._destroyer = Destroyer(target=TargetHighDegreeNodes(number =
```

Results Processing

In [109]:

```python
import pandas as pd
import matplotlib.pyplot as plt

%matplotlib inline

class Results:
    results = {}
    data = {}

    def __init__(self):
        pass

    def store(self, tag, results):
        self.results[tag] = results
        self.data[tag] = self.resultsDataFrame(results)

    def showResultsTable(self, tag):
        return self.data[tag]

    def showSpreading(self, tag):
        results = self.results[tag]
        x = list(results['spread']['count'].keys())
        y1 = list(results['spread']['count'].values())
        y2 = list(results['spread']['total'].values())
        plt.plot(x, y1, "r--", label='spread')
        plt.plot(x, y2, "b", label='infected')
        plt.xlabel('time')
        plt.ylabel('count')
        plt.legend()
        plt.show()

    def resultsDataFrame(self, results):
        df = pd.DataFrame(columns=['time',
                                   'spread_count', 'spread_total',
                                   'growth_count', 'growth_total',
                                   'destruction_count', 'destruction_tota
        times = list(results['spread']['count'].keys())
        df.time = times
        if len(list(results['spread']['count'].values())) == len(times):
            df.spread_count = list(results['spread']['count'].values())
            df.spread_total = list(results['spread']['total'].values())
        if len(list(results['growth']['count'].values())) == len(times):
            df.growth_count = list(results['growth']['count'].values())
            df.growth_total = list(results['growth']['total'].values())
        if len(list(results['destruction']['count'].values())) == len(tim
            df.destruction_count = list(results['destruction']['count'].v
            df.destruction_total = list(results['destruction']['total'].v
        return df
```

Graph Creation

In [110]:
```
# Erdos-Renyi graph model
class ErdosRenyiGraph:
    N = 500
    p = 0.01
```

In [111]:
```
import networkx as nx

def randomGraph():
    return nx.erdos_renyi_graph(ErdosRenyiGraph.N, ErdosRenyiGraph.p)
```

In [112]:
```
# Barabasi-Albert graph model
class BarabasiAlbertGraph:
    N = 500
    m_o = 7
```

In [113]:
```
def baGraph():
    return barabasi_albert_graph_without_using_degree(BarabasiAlbertGraph
```

Simulation Parameters and Run-time

In [114]:
```
class Simulation:
    max_iterations = 10
```

In [115]:
```
results = Results()
```

Exploration

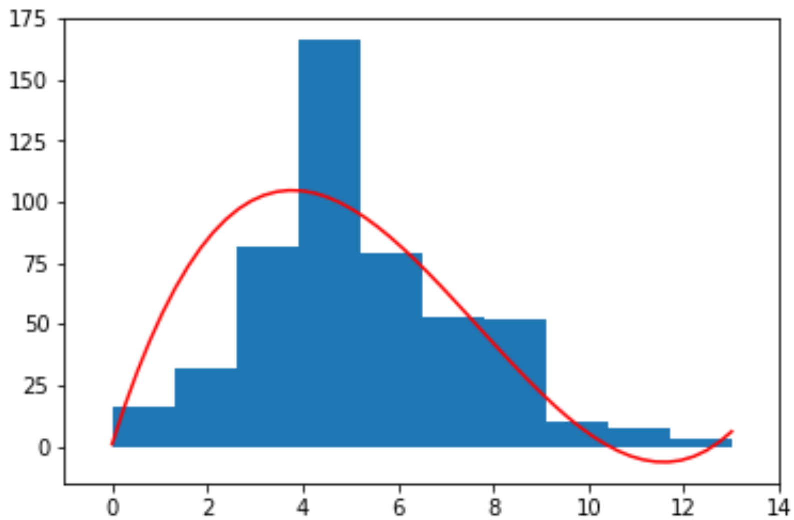
Static Graphs - spreading process only

Erdos-Renyi (random) graph

In [116]:
```
url = gephiURL(host, port, 1)
tag = "static-er-spreading-only"
```

In [117]:
```
graph1 = randomGraph()
```

In [118]:
```
showDegreeDistributionWithFit(graph1)
```



In [119]:
```
%%time
gephiGraphNx(graph1)
```

```
CPU times: user 4.41 s, sys: 1.01 s, total: 5.42 s
Wall time: 7.05 s
```

In [120]:
```
averageDegree(graph1)
```

Out[120]:
```
5.12
```

In [121]:
```
results.store(tag, simulate(graph1, SpreadingOnly(), Simulation.max_itera
```

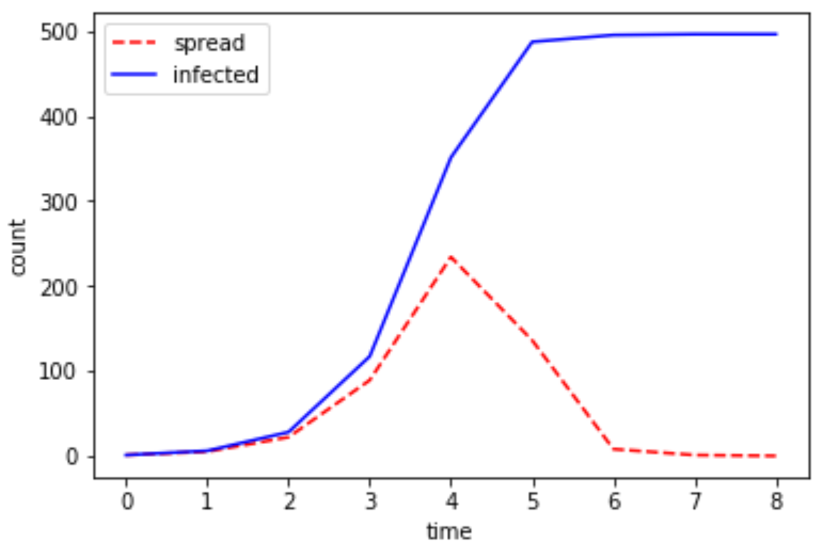
```
time:    1 spread from: 57 -> 173
time:    1 spread from: 57 -> 256
time:    1 spread from: 57 -> 269
time:    1 spread from: 57 -> 345
time:    1 spread from: 57 -> 484
time:    2 spread from: 173 -> 29
time:    2 spread from: 173 -> 60
time:    2 spread from: 173 -> 70
time:    2 spread from: 173 -> 84
time:    2 spread from: 173 -> 136
time:    2 spread from: 173 -> 164
time:    2 spread from: 173 -> 200
time:    2 spread from: 173 -> 326
time:    2 spread from: 173 -> 373
time:    2 spread from: 173 -> 435
time:    2 spread from: 256 -> 115
time:    2 spread from: 256 -> 182
time:    2 spread from: 256 -> 230
time:    2 spread from: 256 -> 396
```

In [122]:
```
results.showResultsTable(tag)
```

Out[122]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_t
0	0	1	1	NaN	NaN	NaN	I
1	1	5	6	NaN	NaN	NaN	I
2	2	22	28	NaN	NaN	NaN	I
3	3	89	117	NaN	NaN	NaN	I
4	4	234	351	NaN	NaN	NaN	I
5	5	136	487	NaN	NaN	NaN	I
6	6	8	495	NaN	NaN	NaN	I
7	7	1	496	NaN	NaN	NaN	I
8	8	0	496	NaN	NaN	NaN	I

In [123]:
```
results.showSpreading(tag)
```



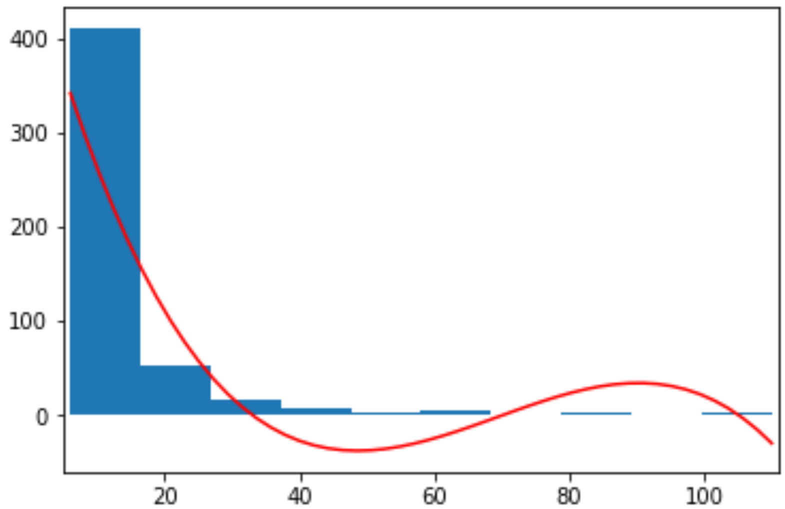
In [ ]:
```
```

Barabasi-Albert graph

In [127]:
```
url = gephiURL(host, port, 2)
tag = "static-ba-spreading-only"
```

In [128]:
```
graph2 = baGraph()
```

In [129]:
```
showDegreeDistributionWithFit(graph2)
```



In [130]:
```
%%time
gephiGraphNx(graph2)
```

```
CPU times: user 9.11 s, sys: 1.99 s, total: 11.1 s
Wall time: 14.9 s
```

In [131]:
```
averageDegree(graph2)
```

Out[131]:
```
13.508
```

In [132]:
```
graphDegreeTopN(graph2)
```

```
N     Node      Degree
----  --------  --------
   1         3       110
   2        11       109
   3         5        96
   4         1        85
   5         6        80
   6        24        70
   7        17        66
   8         0        63
   9         7        62
  10         4        61
```

In [133]:
```
results.store(tag, simulate(graph2, SpreadingOnly(), Simulation.max_itera
```

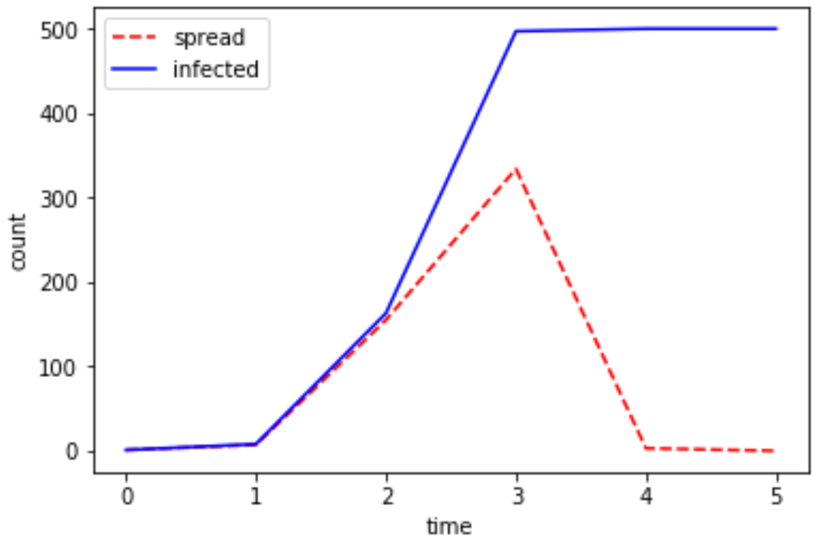
```
time:    1 spread from: 403 -> 57
time:    1 spread from: 403 -> 11
time:    1 spread from: 403 -> 311
time:    1 spread from: 403 -> 293
time:    1 spread from: 403 -> 99
time:    1 spread from: 403 -> 363
time:    1 spread from: 403 -> 479
time:    2 spread from: 11 -> 3
time:    2 spread from: 11 -> 1
time:    2 spread from: 11 -> 10
time:    2 spread from: 11 -> 5
time:    2 spread from: 11 -> 4
time:    2 spread from: 11 -> 7
time:    2 spread from: 11 -> 14
time:    2 spread from: 11 -> 15
time:    2 spread from: 11 -> 17
time:    2 spread from: 11 -> 20
time:    2 spread from: 11 -> 21
time:    2 spread from: 11 -> 23
```

In [134]:
```
results.showResultsTable(tag)
```

Out[134]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_t
0	0	1	1	NaN	NaN	NaN	I
1	1	7	8	NaN	NaN	NaN	I
2	2	155	163	NaN	NaN	NaN	I
3	3	334	497	NaN	NaN	NaN	I
4	4	3	500	NaN	NaN	NaN	I
5	5	0	500	NaN	NaN	NaN	I

In [135]:
```
results.showSpreading(tag)
```



In [ ]:

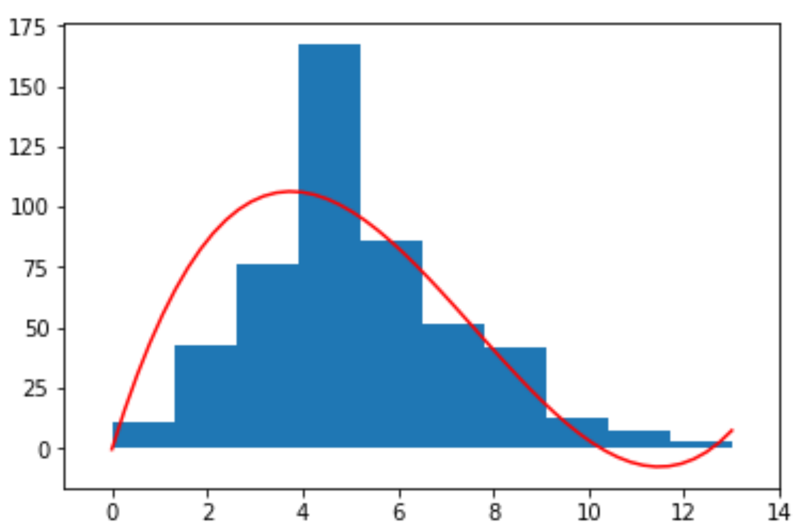
Dynamic Graphs - spreading with additive process

Erdos-Renyi (random) graph

In [136]:
```
url = gephiURL(host, port, 3)
tag = "dynamic-er-additive-only"
```

In [137]:
```
graph3 = randomGraph()
```

In [138]:
```
showDegreeDistributionWithFit(graph3)
```



In [139]:
```
%%time
gephiGraphNx(graph3)
```

```
CPU times: user 3.96 s, sys: 831 ms, total: 4.79 s
Wall time: 6.12 s
```

In [140]:
```
averageDegree(graph3)
```

Out[140]:
```
5.1
```

In [141]:
```
results.store(tag, simulate(graph3, AdditiveOnly(graph3), Simulation.max_
```

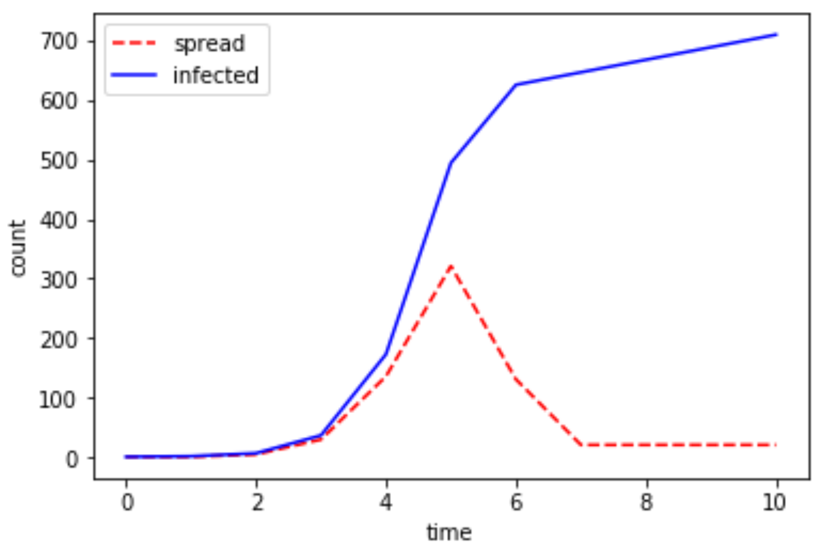
```
time:     3 spread from: 439 -> 490
time:     3 spread from: 439 -> 502
time:     3 spread from: 526 -> 82
time:     3 spread from: 526 -> 275
time:     3 spread from: 526 -> 17
time:     3 spread from: 526 -> 561
time:     4 spread from: 17 -> 215
time:     4 spread from: 17 -> 245
time:     4 spread from: 17 -> 267
time:     4 spread from: 17 -> 517
time:     4 spread from: 17 -> 574
time:     4 spread from: 45 -> 110
time:     4 spread from: 45 -> 221
time:     4 spread from: 45 -> 310
time:     4 spread from: 45 -> 375
time:     4 spread from: 45 -> 393
time:     4 spread from: 45 -> 495
time:     4 spread from: 45 -> 560
time:     4 spread from: 45 -> 563
time:     4 spread from: 65 -> 18
```

In [142]:
```
results.showResultsTable(tag)
```

Out[142]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	0	0	NaN	
1	1	1	2	20	20	NaN	
2	2	5	7	20	40	NaN	
3	3	30	37	20	60	NaN	
4	4	136	173	20	80	NaN	
5	5	321	494	20	100	NaN	
6	6	131	625	20	120	NaN	
7	7	21	646	20	140	NaN	
8	8	21	667	20	160	NaN	
9	9	21	688	20	180	NaN	
10	10	21	709	20	200	NaN	

In [143]:
```
results.showSpreading(tag)
```



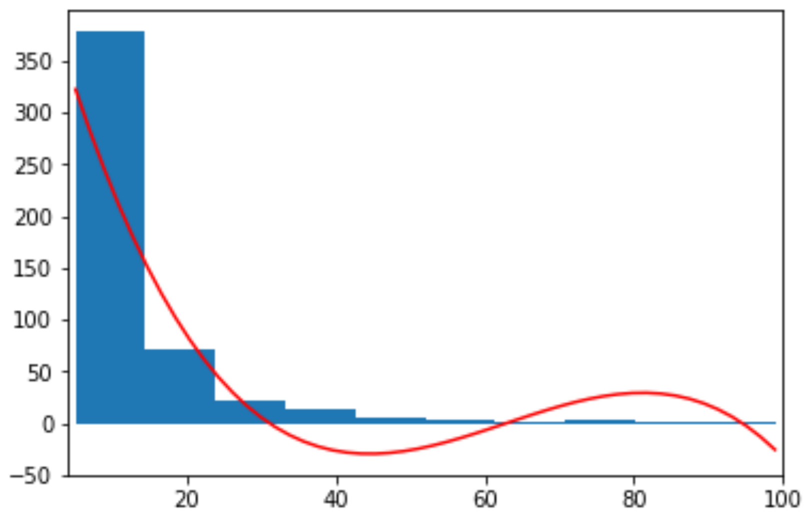
In [ ]:

Barabasi-Albert graph

In [144]:
```
url = gephiURL(host, port, 4)
tag = "dynamic-ba-additive-only"
```

In [145]:
```
graph4 = baGraph()
```

In [146]:
```
showDegreeDistributionWithFit(graph4)
```



In [147]:
```
%%time
gephiGraphNx(graph4)
```

```
CPU times: user 8.62 s, sys: 1.85 s, total: 10.5 s
Wall time: 12.9 s
```

In [148]:
```
averageDegree(graph4)
```

Out[148]: 13.4

In [149]:
```
graphDegreeTopN(graph4)
```

```
N     Node      Degree
----  --------  --------
   1         6        99
   2         2        84
   3         3        79
   4         4        75
   5         5        71
   6         9        67
   7        15        62
   8         0        61
   9         1        59
  10        12        52
```

In [150]:
```
results.store(tag, simulate(graph4, AdditiveOnly(graph4), Simulation.max_
```

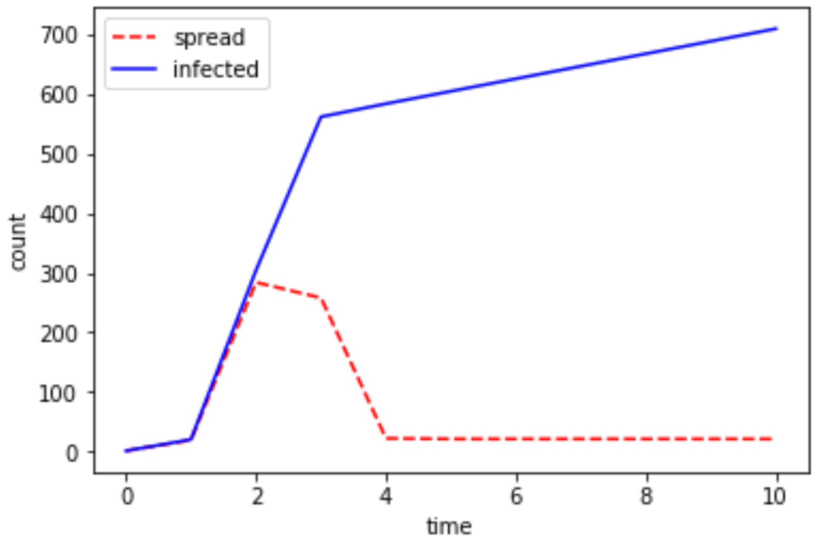
```
time:    1 spread from: 205 -> 76
time:    1 spread from: 205 -> 28
time:    1 spread from: 205 -> 80
time:    1 spread from: 205 -> 39
time:    1 spread from: 205 -> 3
time:    1 spread from: 205 -> 2
time:    1 spread from: 205 -> 7
time:    1 spread from: 205 -> 206
time:    1 spread from: 205 -> 209
time:    1 spread from: 205 -> 223
time:    1 spread from: 205 -> 286
time:    1 spread from: 205 -> 307
time:    1 spread from: 205 -> 356
time:    1 spread from: 205 -> 373
time:    1 spread from: 205 -> 390
time:    1 spread from: 205 -> 407
time:    1 spread from: 205 -> 416
time:    1 spread from: 205 -> 436
time:    1 spread from: 205 -> 508
```

In [151]:
```
results.showResultsTable(tag)
```

Out[151]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	0	0	NaN	
1	1	19	20	20	20	NaN	
2	2	284	304	20	40	NaN	
3	3	258	562	20	60	NaN	
4	4	22	584	20	80	NaN	
5	5	21	605	20	100	NaN	
6	6	21	626	20	120	NaN	
7	7	21	647	20	140	NaN	
8	8	21	668	20	160	NaN	
9	9	21	689	20	180	NaN	
10	10	21	710	20	200	NaN	

In [152]:
```
results.showSpreading(tag)
```



In [ ]:
```

```

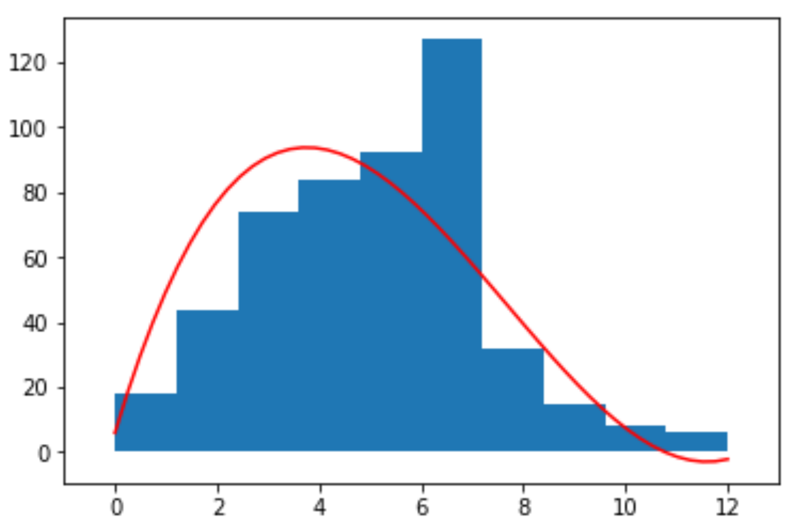
Dynamic Graphs - spreading with addative and random detractive

Erdos-Renyi (random) graph

In [153]:
```
url = gephiURL(host, port, 5)
tag = "dynamic-er-additive-with-random-detractive"
```

In [154]:
```
graph5 = randomGraph()
```

In [155]:
```
showDegreeDistributionWithFit(graph5)
```



In [156]:
```
%%time
gephiGraphNx(graph5)
```

```
CPU times: user 3.94 s, sys: 820 ms, total: 4.76 s
Wall time: 5.93 s
```

In [157]:
```
averageDegree(graph5)
```

Out[157]:
```
4.968
```

In [158]:
```
results.store(tag, simulate(graph5, AddativeWithFixedRandomDetractive(gra
```

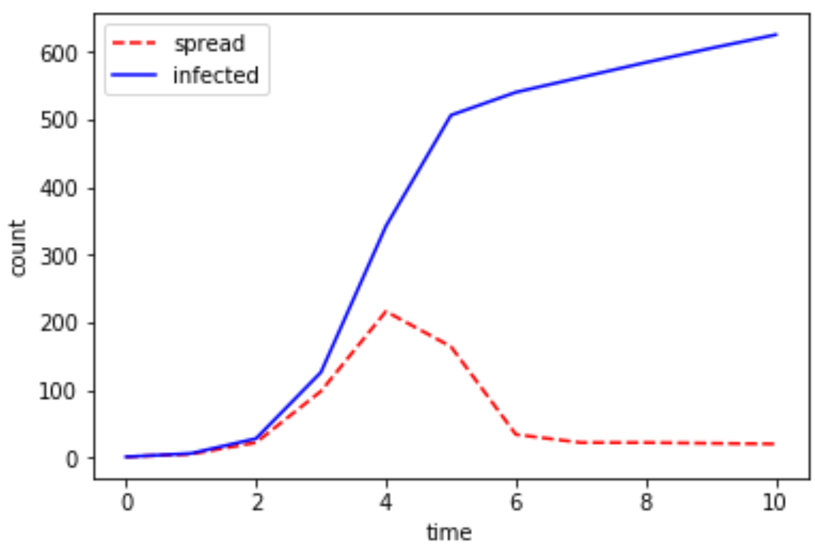
```
time:    1 spread from: 262 -> 40
time:    1 spread from: 262 -> 141
time:    1 spread from: 262 -> 301
time:    1 spread from: 262 -> 426
time:    1 spread from: 262 -> 507
time:    2 spread from: 40 -> 125
time:    2 spread from: 40 -> 541
time:    2 spread from: 141 -> 1
time:    2 spread from: 141 -> 111
time:    2 spread from: 141 -> 233
time:    2 spread from: 141 -> 361
time:    2 spread from: 262 -> 529
time:    2 spread from: 301 -> 76
time:    2 spread from: 301 -> 181
time:    2 spread from: 301 -> 384
time:    2 spread from: 301 -> 458
time:    2 spread from: 426 -> 46
time:    2 spread from: 426 -> 65
time:    2 spread from: 426 -> 89
```

In [159]:
```
results.showResultsTable(tag)
```

Out[159]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	0	0	0	
1	1	5	6	20	20	20	
2	2	22	28	20	40	20	
3	3	98	126	20	60	20	
4	4	216	342	20	80	20	
5	5	164	506	20	100	20	
6	6	34	540	20	120	20	
7	7	22	562	20	140	20	
8	8	22	584	20	160	20	
9	9	21	605	20	180	20	
10	10	20	625	20	200	20	

In [160]:
```
results.showSpreading(tag)
```



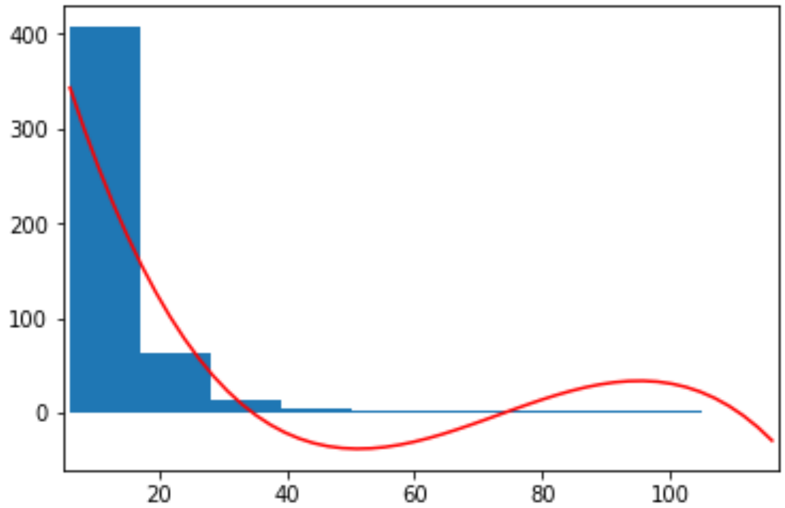
In [ ]:

Barabasi-Albert graph

In [161]:
```
url = gephiURL(host, port, 6)
tag = "dynamic-ba-additive-with-random-detractive"
```

In [162]:
```
graph6 = baGraph()
```

In [163]:
```
showDegreeDistributionWithFit(graph6)
```



In [164]:
```
%%time
gephiGraphNx(graph6)
```

```
CPU times: user 8.91 s, sys: 1.9 s, total: 10.8 s
Wall time: 13.5 s
```

In [165]:
```
averageDegree(graph6)
```

Out[165]: 13.452

In [166]:
```
graphDegreeTopN(graph6)
```

```
N     Node      Degree
---- -------- --------
   1        3      116
   2        6       98
   3        7       94
   4        1       93
   5       11       90
   6        4       79
   7       13       76
   8        8       70
   9        2       65
  10       24       51
```

In [167]:
```
results.store(tag, simulate(graph6, AddativeWithFixedRandomDetractive(gra
```

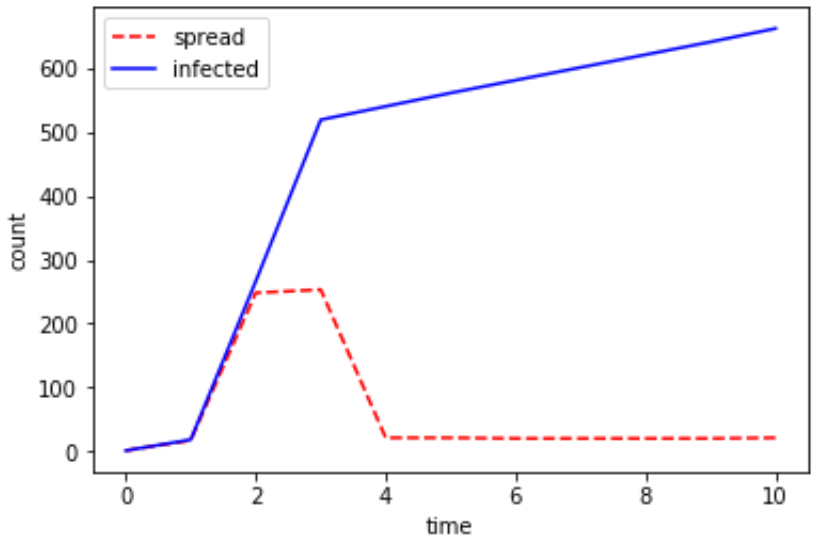
```
time:    1 spread from: 59 -> 50
time:    1 spread from: 59 -> 1
time:    1 spread from: 59 -> 48
time:    1 spread from: 59 -> 7
time:    1 spread from: 59 -> 11
time:    1 spread from: 59 -> 0
time:    1 spread from: 59 -> 18
time:    1 spread from: 59 -> 69
time:    1 spread from: 59 -> 111
time:    1 spread from: 59 -> 124
time:    1 spread from: 59 -> 192
time:    1 spread from: 59 -> 299
time:    1 spread from: 59 -> 301
time:    1 spread from: 59 -> 337
time:    1 spread from: 59 -> 366
time:    1 spread from: 59 -> 408
time:    1 spread from: 59 -> 451
time:    2 spread from: 0 -> 2
time:    2 spread from: 0 -> 3
```

In [168]:
```
results.showResultsTable(tag)
```

Out[168]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	0	0	0	
1	1	17	18	20	20	20	
2	2	248	266	20	40	20	
3	3	253	519	20	60	20	
4	4	21	540	20	80	20	
5	5	21	561	20	100	20	
6	6	20	581	20	120	20	
7	7	20	601	20	140	20	
8	8	20	621	20	160	20	
9	9	20	641	20	180	20	
10	10	21	662	20	200	20	

In [169]:
```
results.showSpreading(tag)
```



In [170]:
```
graphDegreeTopN(graph6)
```

```
N     Node      Degree
---- -------- --------
   1        6       68
   2        7       63
   3       11       63
   4        4       56
   5       13       55
   6        8       53
   7        2       46
   8       24       40
   9       26       35
  10       17       30
```

In [ ]:
```

```

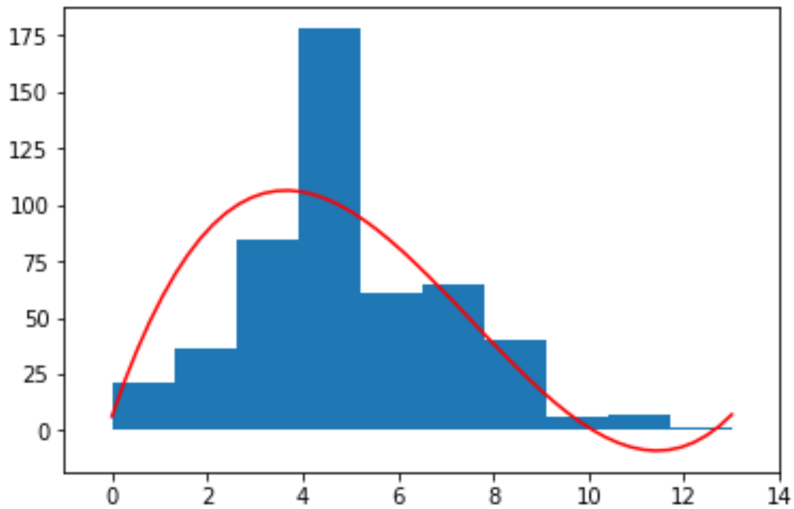
Dynamic Graphs - addative and targeted detractive

Erdos-Renyi (random) graph

In [171]:
```
url = gephiURL(host, port, 7)
tag = "dynamic-er-additive-with-targeted-detractive"
```

In [172]:
```
graph7 = randomGraph()
```

In [173]:
```
showDegreeDistributionWithFit(graph7)
```



In [174]:
```
%%time
gephiGraphNx(graph7)
```

```
CPU times: user 4.2 s, sys: 621 ms, total: 4.82 s
Wall time: 6.15 s
```

In [175]:
```
averageDegree(graph7)
```

Out[175]:
```
4.904
```

In [176]:
```
graphDegreeTopN(graph7)
```

```
N     Node      Degree
---- -------- --------
   1      102       13
   2       57       11
   3       95       11
   4      196       11
   5      284       11
   6      328       11
   7      338       11
   8      491       11
   9        8       10
  10      121       10
```

In [177]:
```
results.store(tag, simulate(graph7, AddativeWithTargetedDetractive(graph7
```

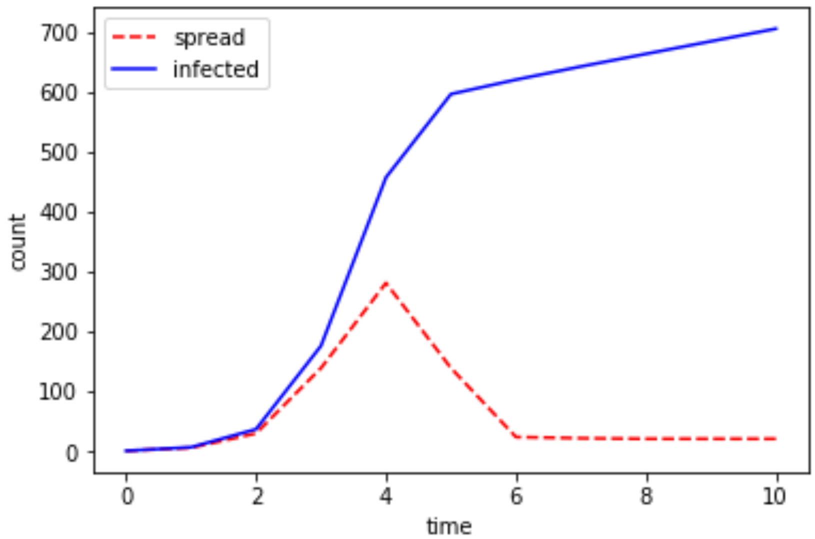
```
time:    1 spread from: 28 -> 56
time:    1 spread from: 28 -> 144
time:    1 spread from: 28 -> 197
time:    1 spread from: 28 -> 324
time:    1 spread from: 28 -> 381
time:    1 spread from: 28 -> 491
time:    2 spread from: 56 -> 61
time:    2 spread from: 56 -> 255
time:    2 spread from: 56 -> 308
time:    2 spread from: 56 -> 328
time:    2 spread from: 56 -> 464
time:    2 spread from: 197 -> 168
time:    2 spread from: 197 -> 250
time:    2 spread from: 197 -> 291
time:    2 spread from: 197 -> 435
time:    2 spread from: 197 -> 476
time:    2 spread from: 324 -> 193
time:    2 spread from: 324 -> 244
time:    2 spread from: 324 -> 333
```

In [178]:
```
results.showResultsTable(tag)
```

Out[178]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	0	0	0	
1	1	6	7	20	20	1	
2	2	30	37	20	40	1	
3	3	139	176	20	60	1	
4	4	281	457	20	80	1	
5	5	139	596	20	100	1	
6	6	24	620	20	120	1	
7	7	22	642	20	140	1	
8	8	21	663	20	160	1	
9	9	21	684	20	180	1	
10	10	21	705	20	200	1	

In [179]:
```
results.showSpreading(tag)
```



In [180]:
```
graphDegreeTopN(graph7)
```

```
N     Node      Degree
---- -------- --------
   1      237       13
   2       32       12
   3       57       12
   4      196       12
   5      200       12
   6      284       12
   7      338       12
   8      445       12
   9       48       11
  10       69       11
```

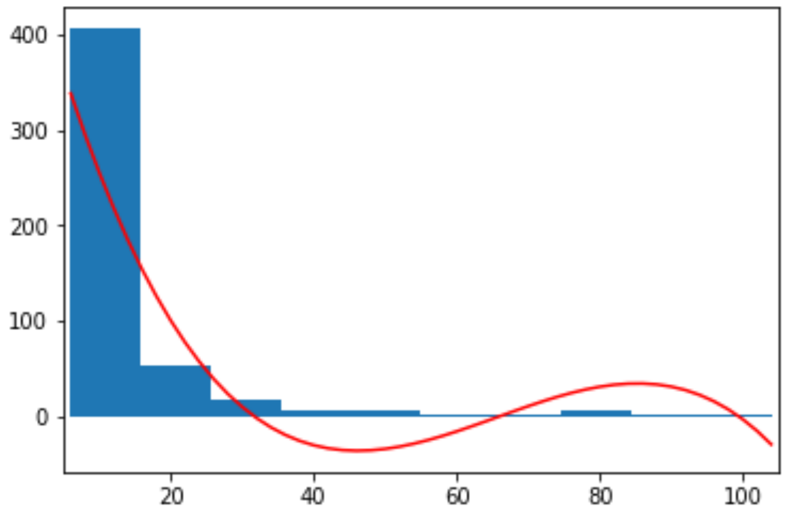
In [ ]:

Barabasi-Albert graph

In [181]:
```
url = gephiURL(host, port, 8)
tag = "dynamic-ba-additive-with-targeted-detractive"
```

In [182]:
```
graph8 = baGraph()
```

In [183]:
```
showDegreeDistributionWithFit(graph8)
```



In [184]:
```
%%time
gephiGraphNx(graph8)
```

```
CPU times: user 8.64 s, sys: 1.26 s, total: 9.9 s
Wall time: 12.4 s
```

In [185]:
```
averageDegree(graph8)
```

Out[185]:
```
13.464
```

In [186]:
```
graphDegreeTopN(graph8)
```

```
N     Node      Degree
---- -------- --------
   1        0      104
   2        3       88
   3        1       81
   4        4       81
   5        6       81
   6        5       75
   7        9       75
   8       12       70
   9       10       69
  10        7       60
```

In [187]:
```
results.store(tag, simulate(graph8, AddativeWithTargetedDetractive(graph8
```

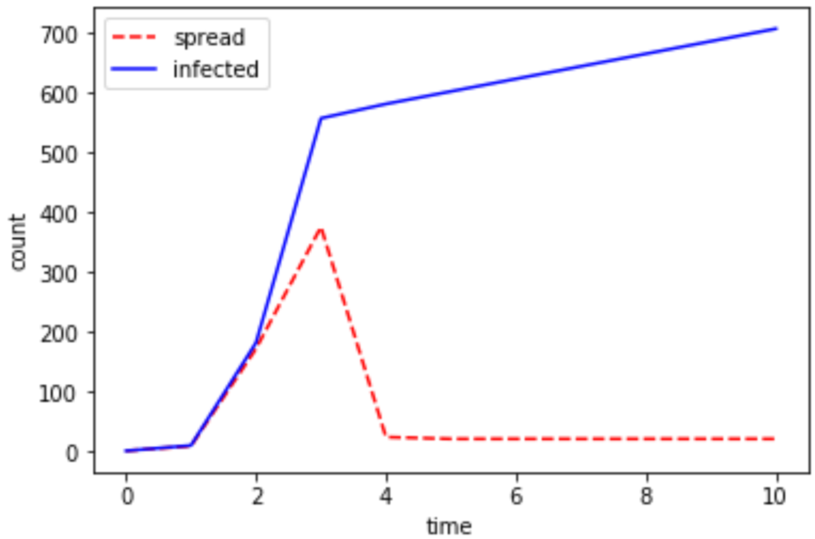
```
time:    1 spread from: 339 -> 245
time:    1 spread from: 339 -> 60
time:    1 spread from: 339 -> 2
time:    1 spread from: 339 -> 5
time:    1 spread from: 339 -> 276
time:    1 spread from: 339 -> 331
time:    1 spread from: 339 -> 187
time:    1 spread from: 339 -> 396
time:    1 spread from: 339 -> 478
time:    2 spread from: 2 -> 1
time:    2 spread from: 2 -> 4
time:    2 spread from: 2 -> 6
time:    2 spread from: 2 -> 8
time:    2 spread from: 2 -> 9
time:    2 spread from: 2 -> 16
time:    2 spread from: 2 -> 17
time:    2 spread from: 2 -> 19
time:    2 spread from: 2 -> 20
time:    2 spread from: 2 -> 21
```

In [188]:
```
results.showResultsTable(tag)
```

Out[188]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	0	0	0	
1	1	9	10	20	20	1	
2	2	172	182	20	40	1	
3	3	376	558	20	60	1	
4	4	24	582	20	80	1	
5	5	21	603	20	100	1	
6	6	21	624	20	120	1	
7	7	21	645	20	140	1	
8	8	21	666	20	160	1	
9	9	21	687	20	180	1	
10	10	21	708	20	200	1	

In [189]:
```
results.showSpreading(tag)
```



In [190]:
```
graphDegreeTopN(graph8)
```

```
N     Node      Degree
---- -------- --------
   1       34       57
   2        2       54
   3       13       52
   4       31       52
   5       30       49
   6       22       48
   7       15       47
   8       33       43
   9       24       41
  10       35       41
```

In [ ]:

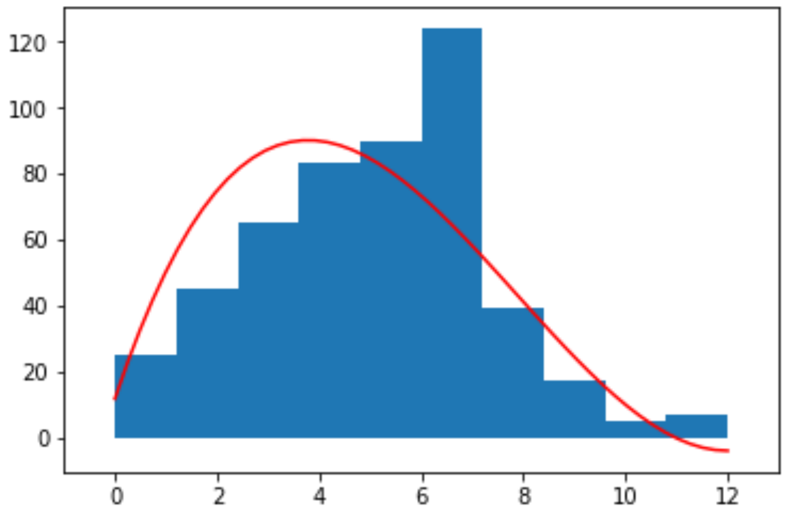
Dynamic Graphs - addative and invasive detractive

Erdos-Renyi (random) graph

In [191]:
```
url = gephiURL(host, port, 9)
tag = "dynamic-er-additive-with-invasive-detractive"
```

In [192]:
```
graph9 = randomGraph()
```

In [193]:
```
showDegreeDistributionWithFit(graph9)
```



In [194]:
```
%%time
gephiGraphNx(graph9)
```

```
CPU times: user 3.91 s, sys: 560 ms, total: 4.47 s
Wall time: 5.62 s
```

In [195]:
```
averageDegree(graph9)
```

Out[195]:
```
4.936
```

In [196]:
```
graphDegreeTopN(graph9)
```

```
N     Node      Degree
---- -------- --------
   1      296       12
   2      131       11
   3      190       11
   4      259       11
   5      305       11
   6      322       11
   7      362       11
   8      181       10
   9      214       10
  10      265       10
```

In [197]:
```
results.store(tag, simulate(graph9, AddativeWithInvasiveDetractive(graph9
```

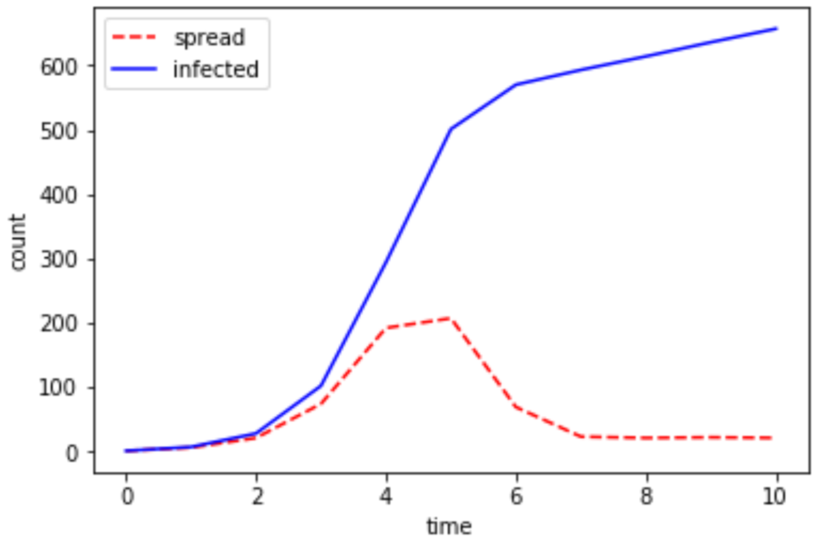
```
time:     1 spread from: 293 -> 104
time:     1 spread from: 293 -> 197
time:     1 spread from: 293 -> 207
time:     1 spread from: 293 -> 281
time:     1 spread from: 293 -> 325
time:     1 spread from: 293 -> 464
time:     2 spread from: 104 -> 39
time:     2 spread from: 104 -> 40
time:     2 spread from: 104 -> 218
time:     2 spread from: 104 -> 292
time:     2 spread from: 104 -> 461
time:     2 spread from: 104 -> 522
time:     2 spread from: 197 -> 310
time:     2 spread from: 197 -> 483
time:     2 spread from: 281 -> 178
time:     2 spread from: 281 -> 402
time:     2 spread from: 281 -> 448
time:     2 spread from: 325 -> 46
time:     2 spread from: 325 -> 112
```

In [198]:
```
results.showResultsTable(tag)
```

Out[198]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	0	0	0	
1	1	6	7	20	20	13	
2	2	21	28	20	40	13	
3	3	74	102	20	60	13	
4	4	192	294	20	80	13	
5	5	207	501	20	100	13	
6	6	69	570	20	120	14	
7	7	23	593	20	140	14	
8	8	21	614	20	160	14	
9	9	22	636	20	180	14	
10	10	21	657	20	200	14	

In [199]:
```
results.showSpreading(tag)
```



In [200]:
```
graphDegreeTopN(graph9)
```

```
N     Node      Degree
---- -------- --------
   1      350        7
   2      377        7
   3      444        7
   4      447        7
   5      448        7
   6      449        7
   7      455        7
   8      456        7
   9      470        7
  10      472        7
```

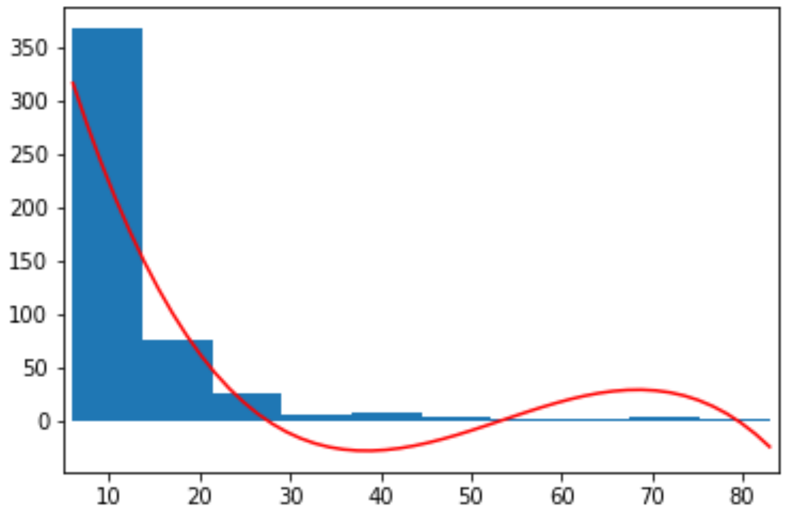
In [ ]:
```

```

Barabasi-Albert graph

In [201]:
```
url = gephiURL(host, port, 10)
tag = "dynamic-ba-additive-with-invasive-detractive"
```

In [202]:
```
graph10 = baGraph()
```

In [203]:
```
showDegreeDistributionWithFit(graph10)
```



In [204]:
```
%%time
gephiGraphNx(graph10)
```

```
CPU times: user 8.79 s, sys: 1.28 s, total: 10.1 s
Wall time: 12.6 s
```

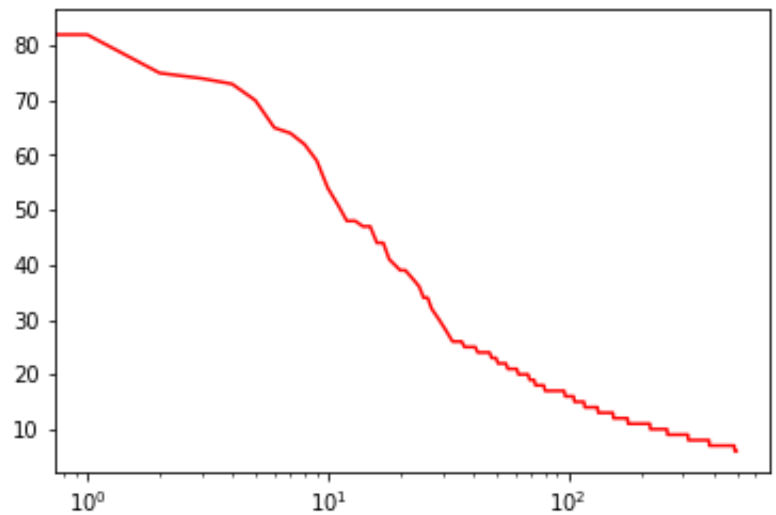
In [205]:
```
averageDegree(graph10)
```

Out[205]:
```
13.432
```

In [206]:
```
graphDegreeTopN(graph10)
```

```
N    Node      Degree
---- -------- --------
   1        5       83
   2       11       82
   3        1       75
   4        2       74
   5        4       73
   6        3       70
   7        9       65
   8        7       64
   9        6       62
  10        8       59
```

In [207]:
```
showDegreeDistributionLogScale(graph10)
```



In [208]:
```
results.store(tag, simulate(graph10, AddativeWithInvasiveDetractive(graph
```

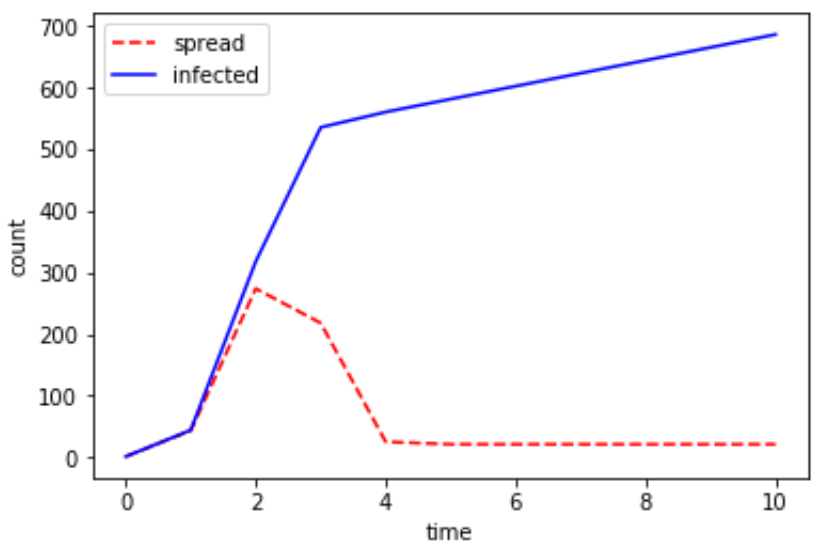
```
time:     1 spread from: 21 -> 18
time:     1 spread from: 21 -> 16
time:     1 spread from: 21 -> 23
time:     1 spread from: 21 -> 27
time:     1 spread from: 21 -> 36
time:     1 spread from: 21 -> 38
time:     1 spread from: 21 -> 45
time:     1 spread from: 21 -> 50
time:     1 spread from: 21 -> 59
time:     1 spread from: 21 -> 62
time:     1 spread from: 21 -> 63
time:     1 spread from: 21 -> 67
time:     1 spread from: 21 -> 85
time:     1 spread from: 21 -> 88
time:     1 spread from: 21 -> 95
time:     1 spread from: 21 -> 97
time:     1 spread from: 21 -> 110
time:     1 spread from: 21 -> 121
time:     1 spread from: 21 -> 130
```

In [209]:
```
results.showResultsTable(tag)
```

Out[209]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	0	0	0	
1	1	43	44	20	20	13	
2	2	274	318	20	40	13	
3	3	218	536	20	60	13	
4	4	25	561	20	80	13	
5	5	21	582	20	100	13	
6	6	21	603	20	120	14	
7	7	21	624	20	140	14	
8	8	21	645	20	160	14	
9	9	21	666	20	180	14	
10	10	21	687	20	200	14	

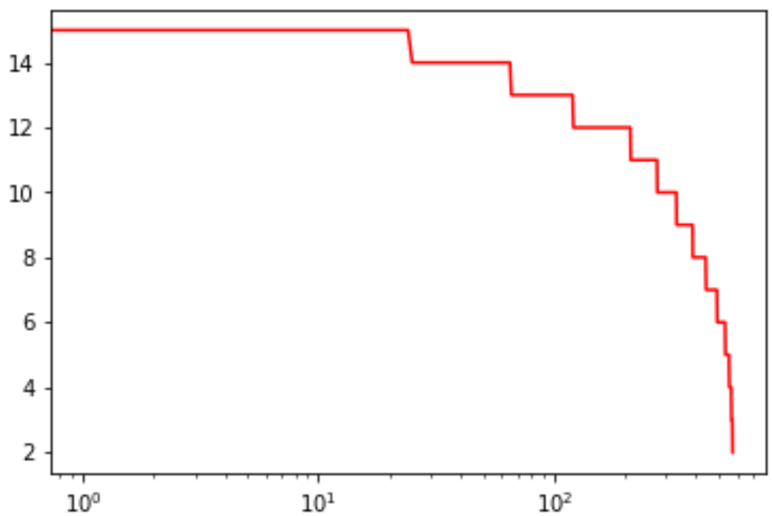
In [210]:
```
results.showSpreading(tag)
```



In [211]:
```
graphDegreeTopN(graph10)
```

```
N     Node      Degree
---- -------- --------
   1       73       15
   2       74       15
   3      139       15
   4      146       15
   5      220       15
   6      252       15
   7      254       15
   8      500       15
   9      512       15
  10      515       15
```

In [212]:
```
showDegreeDistributionLogScale(graph10)
```



In [ ]:
```

```

Dynamic Graphs - addative and extremely invasive detractive

Erdos-Renyi (random) graph

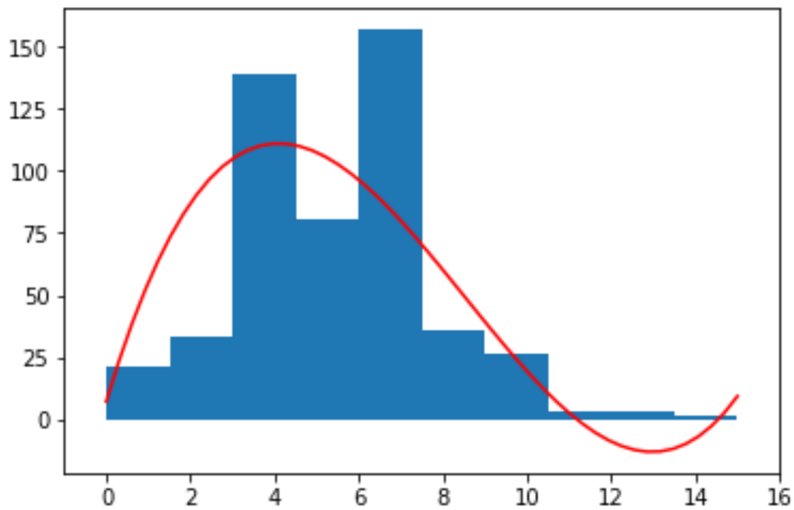
In [213]:
```
url = gephiURL(host, port, 11)
tag = "dynamic-er-additive-with-extremely-invasive-detractive"
```

In [214]:
```
graph11 = randomGraph()
```

In [215]:
```
print(nx.info(graph11))
```

```
Name:
Type: Graph
Number of nodes: 500
Number of edges: 1303
Average degree:   5.2120
```

In [216]:
```
showDegreeDistributionWithFit(graph11)
```



In [217]:
```
%%time
gephiGraphNx(graph11)
```

```
CPU times: user 4.06 s, sys: 587 ms, total: 4.64 s
Wall time: 5.87 s
```

In [218]:
```
graphDegreeTopN(graph11)
```

```
N     Node      Degree
---- -------- --------
   1       83       15
   2       84       13
   3      355       13
   4      127       12
   5      157       11
   6      255       11
   7      491       11
   8        5       10
   9       37       10
  10       86       10
```

In [219]:
```
results.store(tag, simulate(graph11, AddativeWithExtremeInvasiveDetractiv
```

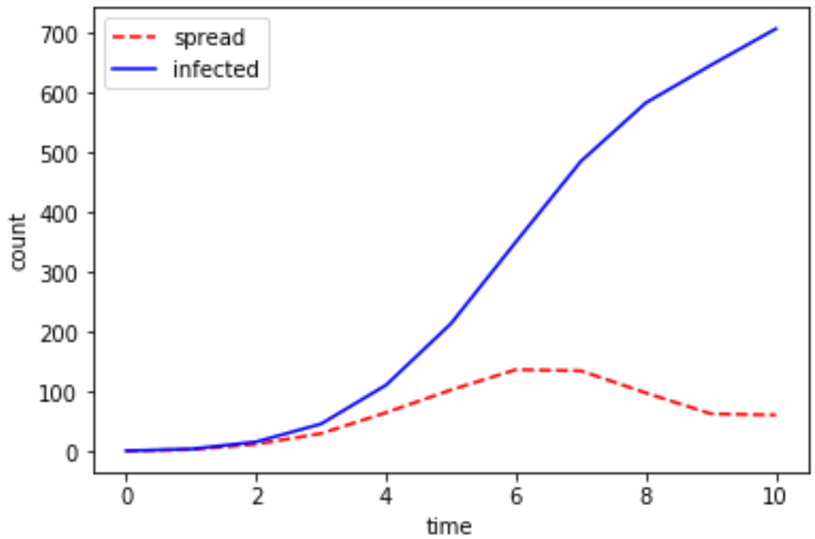
```
time:     1 spread from: 310 -> 111
time:     1 spread from: 310 -> 120
time:     1 spread from: 310 -> 276
time:     2 spread from: 111 -> 252
time:     2 spread from: 111 -> 363
time:     2 spread from: 111 -> 399
time:     2 spread from: 111 -> 576
time:     2 spread from: 120 -> 266
time:     2 spread from: 120 -> 454
time:     2 spread from: 120 -> 582
time:     2 spread from: 276 -> 285
time:     2 spread from: 276 -> 391
time:     2 spread from: 276 -> 480
time:     2 spread from: 276 -> 511
time:     2 spread from: 276 -> 552
time:     3 spread from: 120 -> 621
time:     3 spread from: 252 -> 551
time:     3 spread from: 285 -> 98
time:     3 spread from: 285 -> 219
```

In [220]:
```
results.showResultsTable(tag)
```

Out[220]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	0	0	0	
1	1	3	4	50	50	55	
2	2	12	16	50	100	54	
3	3	30	46	50	150	54	
4	4	65	111	50	200	54	
5	5	103	214	50	250	53	
6	6	137	351	50	300	53	
7	7	135	486	50	350	53	
8	8	98	584	50	400	53	
9	9	63	647	50	450	53	
10	10	61	708	50	500	52	

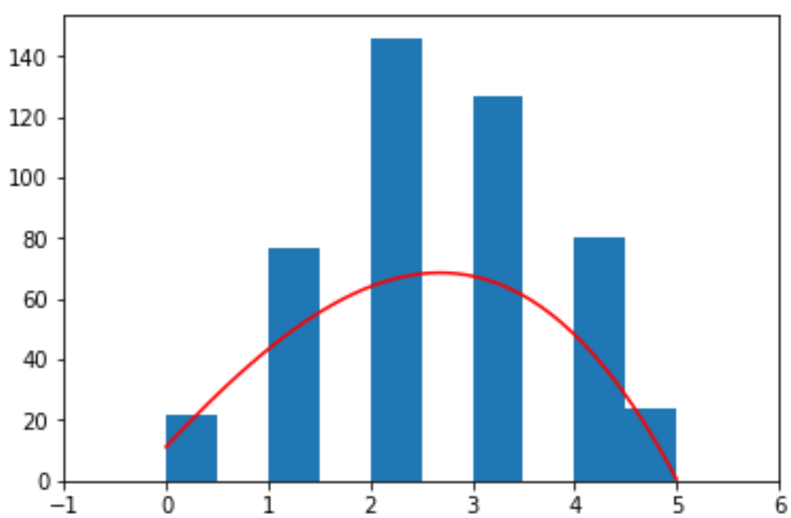
In [221]:
```
results.showSpreading(tag)
```



In [222]:
```
print(nx.info(graph11))
```

```
Name:
Type: Graph
Number of nodes: 476
Number of edges: 595
Average degree:   2.5000
```

In [223]:
```
showDegreeDistributionWithFit(graph11)
```



In [224]:
```
graphDegreeTopN(graph11)
```

```
N     Node      Degree
---- -------- --------
   1      775        5
   2      785        5
   3      816        5
   4      851        5
   5      854        5
   6      867        5
   7      869        5
   8      876        5
   9      880        5
  10      881        5
```

In [ ]:

Barabasi-Albert graph

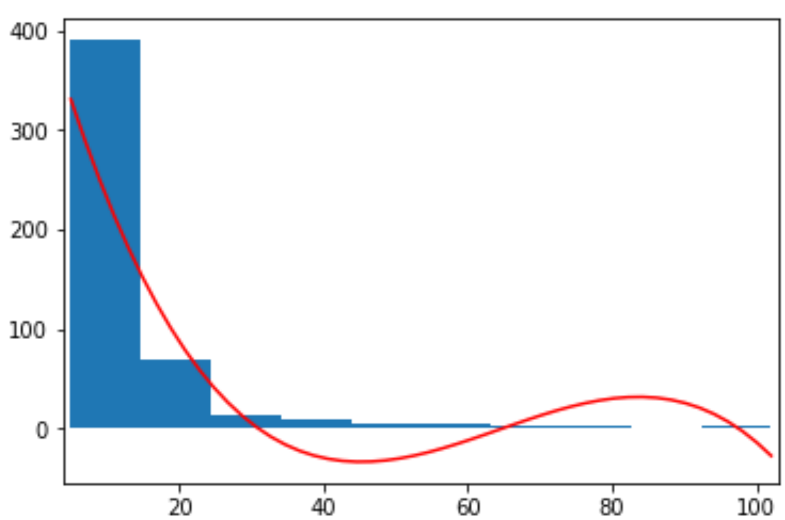
In [225]:
```
url = gephiURL(host, port, 12)
tag = "dynamic-ba-additive-with-extremely-invasive-detractive"
```

In [226]:
```
graph12 = baGraph()
```

In [227]:
```
print(nx.info(graph12))
```

```
Name:
Type: Graph
Number of nodes: 500
Number of edges: 3376
Average degree:  13.5040
```

In [228]:
```
showDegreeDistributionWithFit(graph12)
```



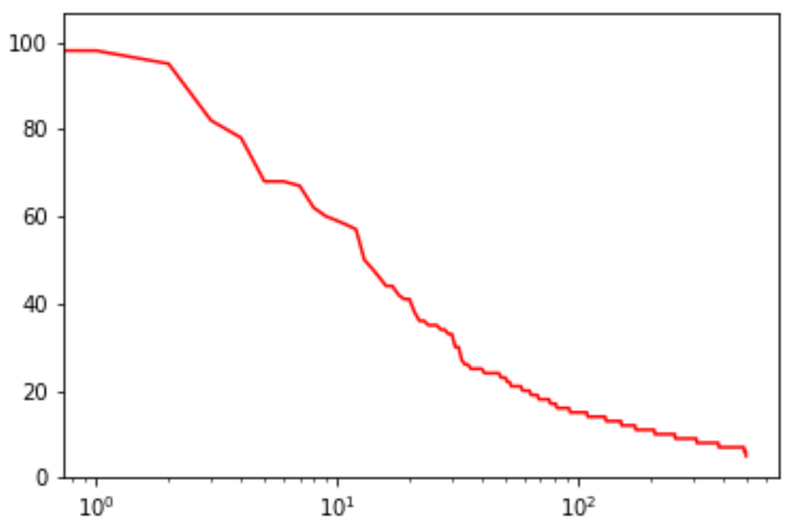
In [229]:
```
%%time
gephiGraphNx(graph12)
```

```
CPU times: user 8.59 s, sys: 1.27 s, total: 9.86 s
Wall time: 12.5 s
```

In [230]:
```
graphDegreeTopN(graph12)
```

N	Node	Degree
1	4	102
2	2	98
3	1	95
4	0	82
5	5	78
6	6	68
7	11	68
8	17	67
9	8	62
10	7	60

In [231]:
```
showDegreeDistributionLogScale(graph12)
```



In [232]:
```
results.store(tag, simulate(graph12, AddativeWithExtremeInvasiveDetractiv
```

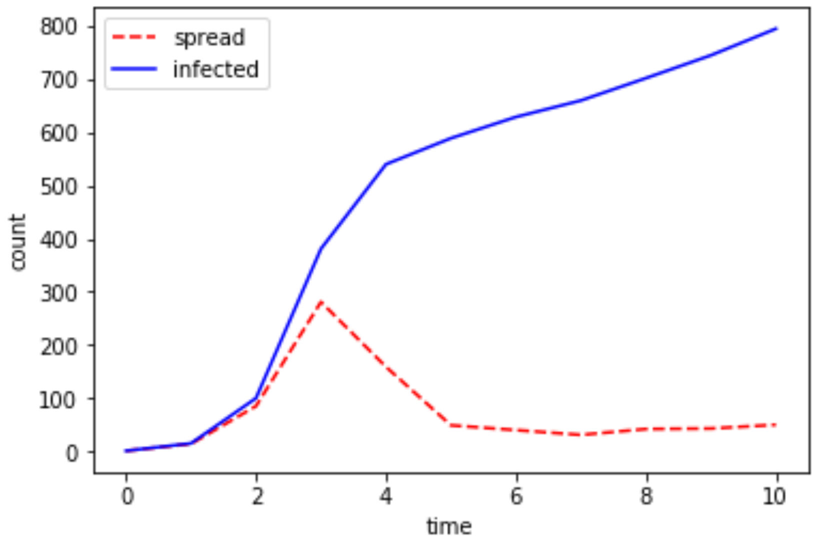
```
time:    1 spread from: 156 -> 157
time:    1 spread from: 156 -> 194
time:    1 spread from: 156 -> 230
time:    1 spread from: 156 -> 272
time:    1 spread from: 156 -> 274
time:    1 spread from: 156 -> 316
time:    1 spread from: 156 -> 328
time:    1 spread from: 156 -> 395
time:    1 spread from: 156 -> 466
time:    1 spread from: 156 -> 487
time:    1 spread from: 156 -> 507
time:    1 spread from: 156 -> 512
time:    1 spread from: 156 -> 532
time:    1 spread from: 156 -> 534
time:    2 spread from: 157 -> 281
time:    2 spread from: 157 -> 336
time:    2 spread from: 157 -> 352
time:    2 spread from: 157 -> 374
time:    2 spread from: 157 -> 377
```

In [233]:
```
results.showResultsTable(tag)
```

Out[233]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	0	0	0	
1	1	14	15	50	50	55	
2	2	85	100	50	100	54	
3	3	281	381	50	150	54	
4	4	159	540	50	200	54	
5	5	49	589	50	250	53	
6	6	40	629	50	300	53	
7	7	31	660	50	350	53	
8	8	42	702	50	400	53	
9	9	43	745	50	450	53	
10	10	50	795	50	500	52	

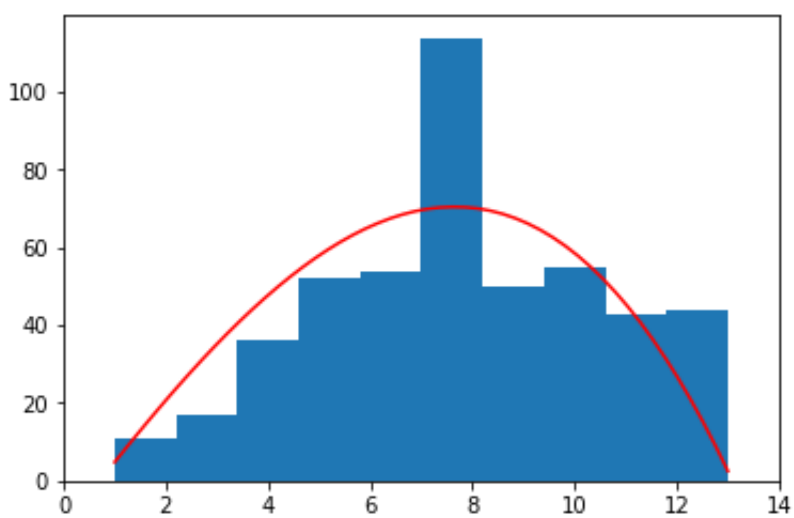
In [234]:
```
results.showSpreading(tag)
```



In [235]:
```
print(nx.info(graph12))
```

```
Name:
Type: Graph
Number of nodes: 476
Number of edges: 1831
Average degree:   7.6933
```

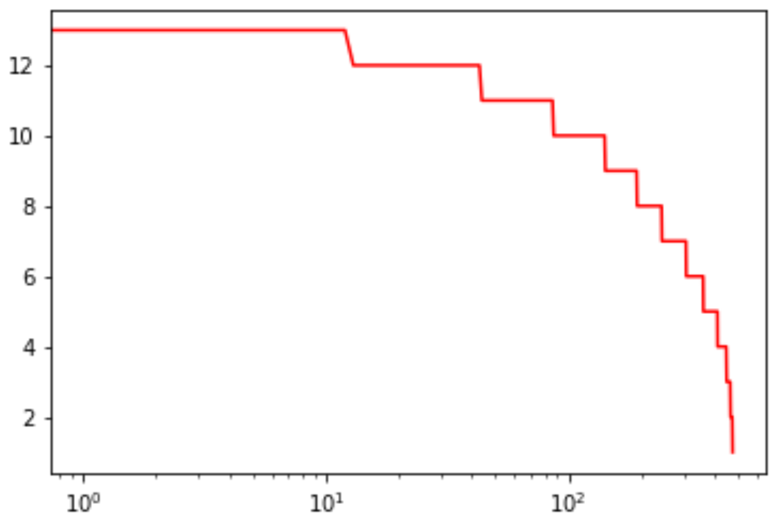
In [236]:
```
showDegreeDistributionWithFit(graph12)
```



In [237]:
```
graphDegreeTopN(graph12)
```

```
N     Node      Degree
---- -------- --------
   1      710       13
   2      713       13
   3      893       13
   4      931       13
   5      960       13
   6      973       13
   7      982       13
   8      985       13
   9      986       13
  10      987       13
```

In [238]:
```
showDegreeDistributionLogScale(graph12)
```



In [ ]:

Results Analysis

In [311]:
```
scenarios = {
    1:  "static-er-spreading-only",
    2:  "static-ba-spreading-only",
    3:  "dynamic-er-additive-only",
    4:  "dynamic-ba-additive-only",
    5:  "dynamic-er-additive-with-random-detractive",
    6:  "dynamic-ba-additive-with-random-detractive",
    7:  "dynamic-er-additive-with-targeted-detractive",
    8:  "dynamic-ba-additive-with-targeted-detractive",
    9:  "dynamic-er-additive-with-invasive-detractive",
    10: "dynamic-ba-additive-with-invasive-detractive",
    11: "dynamic-er-additive-with-extremely-invasive-detractive",
    12: "dynamic-ba-additive-with-extremely-invasive-detractive"
}
```

In [333]:
```
labels = {
    0: "spreading",
    1: "additive",
    2: "random",
    3: "targeted",
    4: "invasive",
    5: "extreme",
}
```

In [312]:
```
randomGraphs = [1, 3, 5, 7, 9, 11]
scalefreeGraphs = [2, 4, 6, 8, 10, 12]
```

Fix The Data

adjust the size of the shorter results DataFrames so we can plot them all together

we append the final row 'n' times to grow the DataFrames to the desired size

scenario 1: "static-er-spreading-only"

In [304]:
```python
# before
results.data["static-er-spreading-only"]
```

Out[304]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_t
0	0	1	1	NaN	NaN	NaN	I
1	1	5	6	NaN	NaN	NaN	I
2	2	22	28	NaN	NaN	NaN	I
3	3	89	117	NaN	NaN	NaN	I
4	4	234	351	NaN	NaN	NaN	I
5	5	136	487	NaN	NaN	NaN	I
6	6	8	495	NaN	NaN	NaN	I
7	7	1	496	NaN	NaN	NaN	I
8	8	0	496	NaN	NaN	NaN	I

In [305]:
```python
# append
for j in range(9, 11):
    dfa = pd.DataFrame(columns=results.data["static-er-spreading-only"].c
    dfa.time = [j]
    dfa.spread_count = results.data["static-er-spreading-only"][-1:].spre
    dfa.spread_total = results.data["static-er-spreading-only"][-1:].spre
    results.data["static-er-spreading-only"].loc[j] = dfa.loc[0]
```

In [306]:

```
# after
results.data["static-er-spreading-only"]
```

Out[306]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	NaN	NaN	NaN	
1	1	5	6	NaN	NaN	NaN	
2	2	22	28	NaN	NaN	NaN	
3	3	89	117	NaN	NaN	NaN	
4	4	234	351	NaN	NaN	NaN	
5	5	136	487	NaN	NaN	NaN	
6	6	8	495	NaN	NaN	NaN	
7	7	1	496	NaN	NaN	NaN	
8	8	0	496	NaN	NaN	NaN	
9	9	0	496	NaN	NaN	NaN	
10	10	0	496	NaN	NaN	NaN	

scenario 2: "static-ba-spreading-only"

In [308]:

```
# before
results.data["static-ba-spreading-only"]
```

Out[308]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_t
0	0	1	1	NaN	NaN	NaN	I
1	1	7	8	NaN	NaN	NaN	I
2	2	155	163	NaN	NaN	NaN	I
3	3	334	497	NaN	NaN	NaN	I
4	4	3	500	NaN	NaN	NaN	I
5	5	0	500	NaN	NaN	NaN	I

In [309]:

```
# append
for j in range(6, 11):
    dfa = pd.DataFrame(columns=results.data["static-ba-spreading-only"].c
    dfa.time = [j]
    dfa.spread_count = results.data["static-ba-spreading-only"][-1:].spre
    dfa.spread_total = results.data["static-ba-spreading-only"][-1:].spre
    results.data["static-ba-spreading-only"].loc[j] = dfa.loc[0]
```

In [310]:
```
# after
results.data["static-er-spreading-only"]
```

Out[310]:

	time	spread_count	spread_total	growth_count	growth_total	destruction_count	destruction_
0	0	1	1	NaN	NaN	NaN	
1	1	5	6	NaN	NaN	NaN	
2	2	22	28	NaN	NaN	NaN	
3	3	89	117	NaN	NaN	NaN	
4	4	234	351	NaN	NaN	NaN	
5	5	136	487	NaN	NaN	NaN	
6	6	8	495	NaN	NaN	NaN	
7	7	1	496	NaN	NaN	NaN	
8	8	0	496	NaN	NaN	NaN	
9	9	0	496	NaN	NaN	NaN	
10	10	0	496	NaN	NaN	NaN	

Plot Metrics

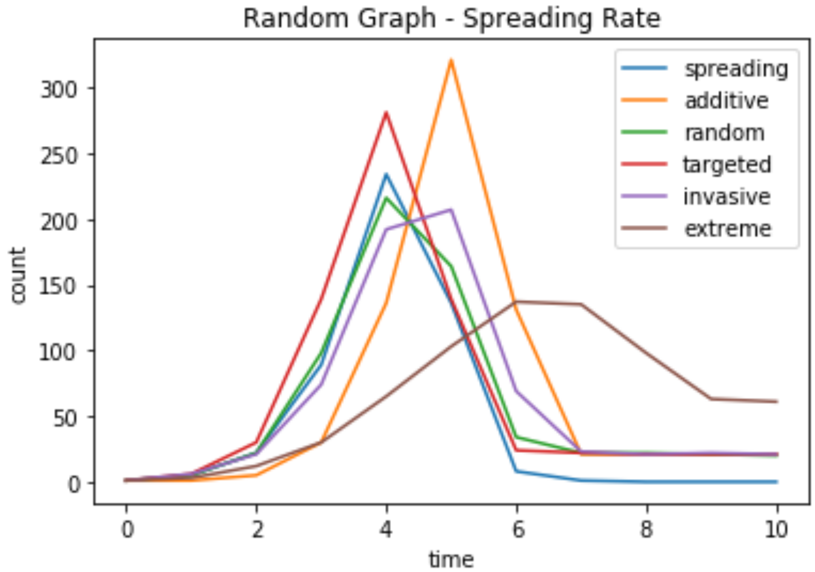
now plot values for each scenario, grouped by graph type, and metric

In [358]:
```
def plotSeries(title, x, data):
    for i in range(len(data)):
        plt.plot(x, data[i], label=labels[i])
    plt.xlabel('time')
    plt.ylabel('count')
    plt.title(title)
    plt.legend()
    plt.show()
```

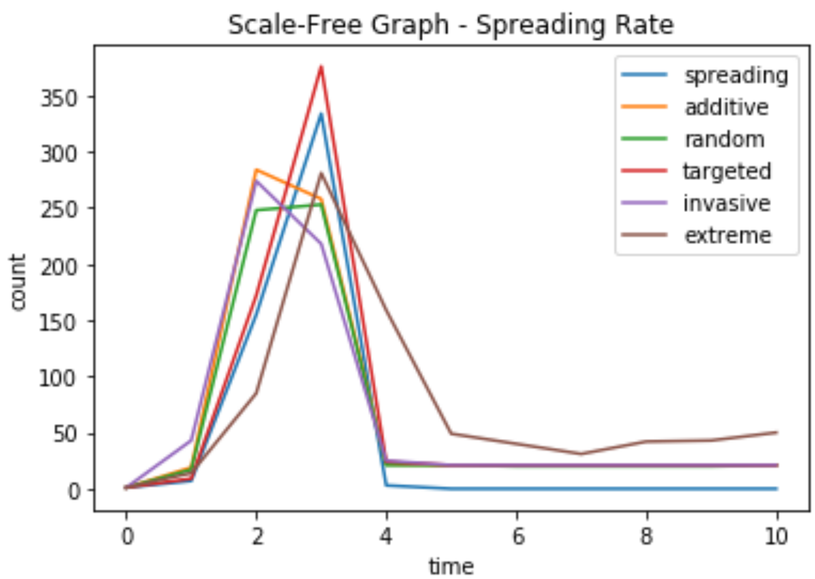
In [359]:
```
def plotScenarios(title, scenarioIndexes, column):
    time = []
    data = []
    for i in scenarioIndexes:
        # get the scenario 'tag'
        scenario = scenarios[i]
        # get the vector of time just once
        if len(time) == 0:
            time = results.data[scenario].time.values
        # append the data for this scenario into the data array
        data.append(results.data[scenario][column].values)
    plotSeries(title, time, data)
```

Spreading Rate

In [360]:
```
plotScenarios("Random Graph - Spreading Rate", randomGraphs, 'spread_coun
```

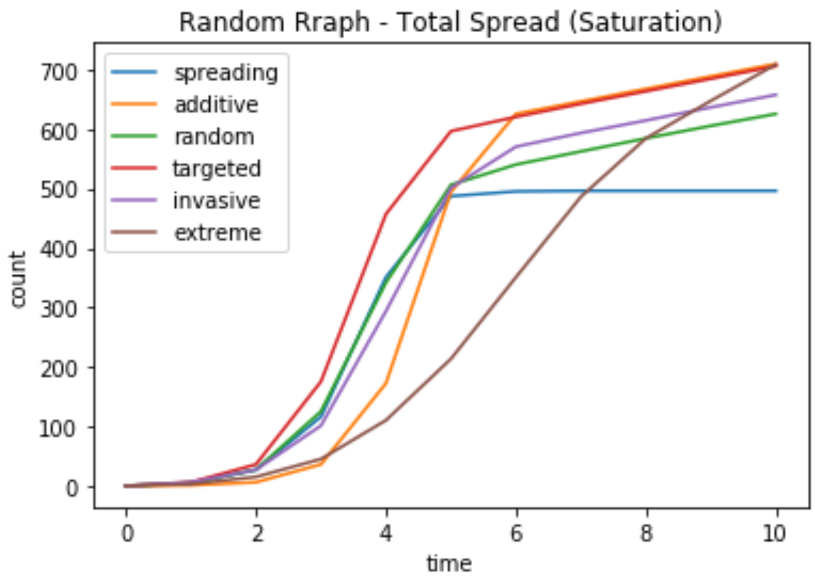


In [361]:
```
plotScenarios("Scale-Free Graph - Spreading Rate", scalefreeGraphs, 'spre
```

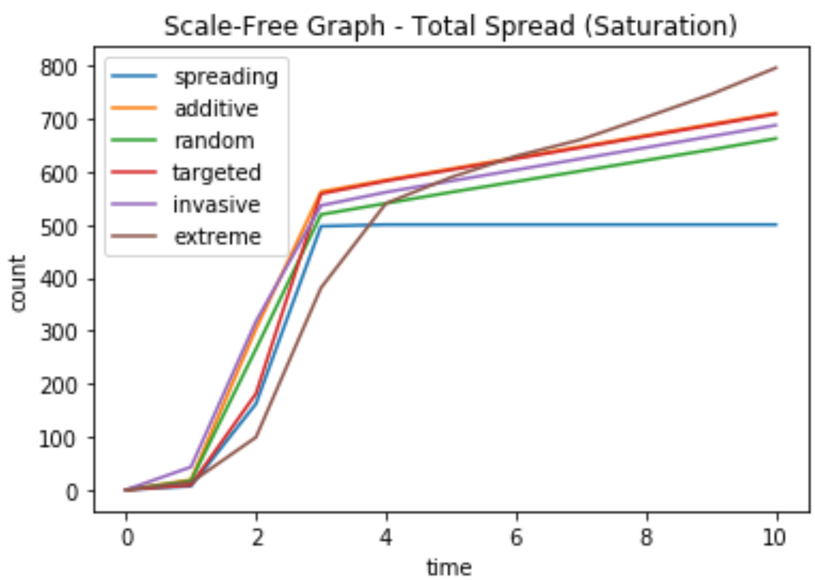


Total Spread (Saturation)

In [362]:
```
plotScenarios("Random Rraph - Total Spread (Saturation)", randomGraphs, '
```



In [363]:
```
plotScenarios("Scale-Free Graph - Total Spread (Saturation)", scalefreeGr
```



In [ ]:

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