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Preferential Attachment, Homophily, and the Structure of International Networks, 1816–2003

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This study examines the extent to which network formation processes in international relations parallel models that characterize the formation processes of physical, biological, and social networks. I introduce two influential models from networks sciences: Preferential Attachment (PA) models state that the probability of a new node forming a link with an existing node is a function of the latter node's centrality. Networks that form through a PA process tend to have a power-law degree distribution. The Homophily (HO) model states that nodes tend to attach to similar other nodes. Such networks evolve into a set of homogenous subgroups. An analysis of alliance and trade networks over the 1816 (1870)–2003 period reveals strong evidence that alliance networks are affected by homophily processes. Trade networks form via a preferential attachment process. The tendency of international networks to evolve according to such processes increases over time. I discuss the implications of these results.

KEYWORDS: alliances; homophily; international conflict; international networks; preferential attachment; power-law; trade

> In the days of King Amraphel of Shinar, King Arioch of Ellasar, King Chedorlaomer of Elam, and King Tidal of Goiim, these kings made war with King Bera of Sodom, King Birsha of Gomorrah, King Shinab of Admah, King Shemeber of Zeboiim, and the king of Bela (that is, Zoar). All these joined forces in the Valley of Siddim [the Dead Sea]. For twelve years they had served Chedorlaomer, but in the thirteenth year they rebelled. In the fourteenth year Chedorlaomer and the kings who were with him came and subdued the Rephaim in Ashteroth-karnaim, the Zuzim in Ham, the Emim in Shaveh-kiriathaim, and the Horites in the hill country of Seir as far as El-paran on the edge of the wilderness; then they turned back and came to En-mishpat [Kadesh], and subdued all the country of the Amalekites, and also the Amorites who lived in Hazazon-tamar. Then the king of Sodom, the king of Gomorrah, the king of Admah, the king of Zeboiim, and the king of Bela [Zohar] went out, and they joined battle in the Valley of Siddim with King Chedorlaomer of Elam, King Tidal of Goiim, King Amraphel of Shinar, and King Arioch of Ellasar, four kings against five. Now the Valley of Siddim was full of bitumen

> pits; and as the kings of Sodom and Gomorrah fled, some fell into them, and the rest fled to the hill country. So the enemy took all the goods of Sodom and Gomorrah, and all their provisions, and went their way; they also took Lot, the son of Abram's brother, who lived in Sodom, and his goods, and departed. (Genesis 14)

1. Introduction

The story of the Battle of the Vale of Siddim suggests that states (or tribes) formed strategic alliances as far back as the third millennium BC. We do not know how these war coalitions came to form, although the story suggests that the five-King coalition brought together disgruntled vassals of King Chedarlaomer—all five kings seem to have a common enemy. In modern parlance, this battle represents the outcome of a formation process leading to a polarized network structure. The two emerging alliance blocks engaged each other in one of the earliest coalition wars known to mankind.

Recent years have seen a significant growth in studies applying network analysis to international processes (Hafner-Burton et al., 2009; Maoz, 2010a: Ch. 1). These studies rely on a definition of a network as a set of units (nodes) and a rule that defines whether, how, and to what extent any two units are linked (have an edge) to each other. Most studies of international networks focused on states as the principal nodes, but differed in terms of the rules that define links. Some focused on alliances, other focused on trade, institutions, and conflicts. Consequently, mounting evidence suggests that international relations can be seen as a set of co-evolving cooperative and conflictual networks. These co-evolving networks shape central processes in international relations, such as peace and war (Maoz, 2010a: Ch. 12).

At the same time, we have only scant understanding of the processes by which these networks form. A theory of Networked International Politics (NIP) (Maoz, 2010a,b) offered a model of international network formation. Empirical tests of this model suggested a fair degree of support for many, but not all of its propositions.

The key problem with the model is that it is fairly complex. One wonders whether it could be simplified without losing a great deal of theoretical and empirical content. Furthermore, some of the ideas underlying this model parallel theories of network formation derived from physical and sociological theories. Accordingly, the present article addresses the following questions:

1. What, if any, parallels exist between processes of network formation in physical and other social settings and network formation in international relations?
2. To what extent does the historical evolution of international networks match models of physical, biological, or other social networks?

One of the key questions in networks science concerns the effect of network formation on network structure. It is commonly understood that the structure of observed networks reflects the cumulative consequences of multiple decisions about link-formation. However, most of our data consist of networks that have already evolved for some time. Such networks are an *emergent structure*, an end result of multiple choices about link formation. We do not have longitudinal data that explain how such structures evolved. Hence, the problem is to infer the micro-foundations of this structure. This is the puzzle that is implied in the Biblical story of the Battle of the Vale of Siddim.

Baseline random network models assume that each pair of nodes has a fixed probability of forming a link. This results in a network structure with a Poisson degree distribution (Erdos and Renyi, 1960).[1] But most networks—especially social ones—do not support this structure. Different nodes or different *types of nodes* have different considerations, thus yielding different probabilities of link-formation. Just what these different probabilities mean is the topic of central theories in network sciences.

Empirical tests reveal that many physical, biological, and social networks have structural characteristics that emerge from fairly simple principles of link-making. Accordingly, network scientists have developed models of network formation that connect an observed structure of a network to the processes by which it was formed. I start by introducing the two most prominent models of network formation: Preferential Attachment (PA) and Homophily (HO). The next section reviews the underlying logic of these models and shows that it converges with central paradigms of international relations. I then derive the key hypotheses from this logic with respect to the formation of international networks. The third section discusses ways in which these network formation processes can be tested empirically. Section four presents the results of the empirical analyses. Section five discusses the theoretical and empirical implications of these results.

2. Preferential Attachment (PA), Homophily (HO), and Networked International Politics (NIP) Models of Network Formation

How do international relations scholars view the processes by which networks form? There is no simple answer to this question because, until quite recently, network formation was not a topic of study in international relations research. Yet it seems that there is at least implicit treatment of this topic in various theories of international relations. Some of the central ideas in international relations theory converge quite well with interdisciplinary models of network formation. This is what we discuss next.

Preferential Attachment

The PA model asserts that the probability of a new node forming a link with an existing node is a function of the centrality of the latter; new nodes are more likely to connect to central nodes than to less central ones. This process of network

[1]A degree distribution is a distribution of the number of links of each node. The mean of the probability degree distribution is also known as *network density* (Jackson, 2008: 29).

formation induces a power-law degree distribution (Barabasi and Albert, 1999). This distribution is defined as:

$$p(K) \approx \beta K^{-\gamma} \qquad [1]$$

where the probability of a node having K links $p(K)$ equals some normalizing constant β and a power function of K with (typically but not exclusively) $2 \geq \gamma \geq 3$.

Scale-free networks with a power-law degree distribution possess a few very central nodes and a lot of peripheral ones. This results in a number of identifiable network characteristics.

- *Low density*. The density of the network—the proportion of realized edges to the number of possible edges—is generally low.
- *Low transitivity (clustering coefficient*: Watts and Strogatz, 1998). There are relatively few complete relational triangles of the type of $i \leftrightarrow j \leftrightarrow k \leftrightarrow i$ (most existing triangles are likely to be of the sort $i \leftrightarrow j \leftrightarrow k \not o i$).
- *A small number of components*. A component is a closed subset of reachable nodes (each node in the component is reachable—directly or indirectly—from all other nodes, and no node within a component is reachable from outside of the component). Because peripheral nodes are likely to have links to central nodes, and central nodes are likely to have links to each other, almost everyone is reachable from other nodes. Such networks are said to "percolate" quickly, creating one giant component (Achlioptas et al., 2009).
- *High group centralization*. Group centralization is the ratio of the difference between the most central node and all other nodes to the maximum possible difference. Because there are a lot of low-centrality nodes and a few high-centrality ones, group centralization is likely to be quite high.

A visual display of a relatively proximate scale free network is provided in Figure 1, which shows the international trade network in 1974.[2] In order to highlight the structure of this network, states (denoted by circles and labeled with COW state numbers) are grouped into three categories—Highly Central, Moderately Central, and Low Centrality states. Circle sizes reflect the degree centrality of states. We can see that central states tend to form a fairly cohesive core and marginal states tend to be in the periphery.

Does this kind of network formation process have anything to do with international relations? On the face of it, the formation of IR networks is due to strategic, economic, or cultural/ideational considerations. The centrality of existing nodes may have little to do with who forms security or economic links with whom. A more discriminating view of the literature, however, reveals an interesting overlap between PA models and theories of international networks.

[2]The evidence about the fit between this particular network and a scale-free network will be provided in the analysis section. A discussion of the trade dataset is given in the methodological appendix.

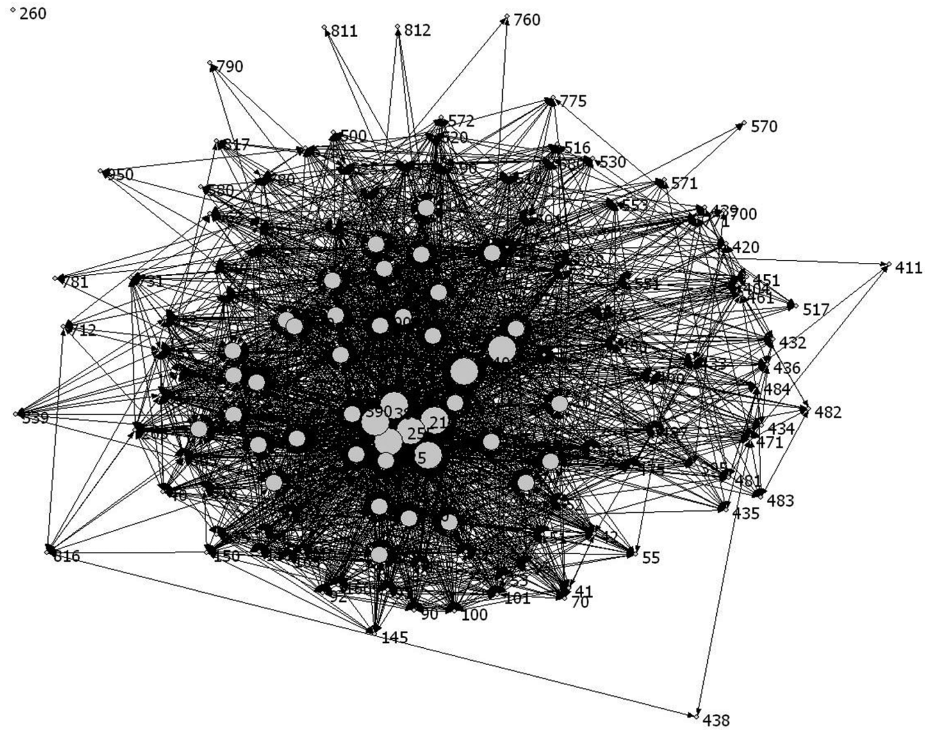

Figure 1. An Example of a Scale-Free Network—International Trade Network, 1974
Circles (with numbered labels) are COW state numbers. Arrows represent trade at level of 0.0001 of the focal (row) state's GDP.
Sizes of circles represent centrality level: Large = highly central state; Medium = moderately central state; Small = marginal state.

First, alliance theories suggest that, under certain circumstances, bandwagoning processes may drive alliance dynamics (Walt, 1988). Notions of chain-ganging (Christensen and Snyder, 1990) suggest a similar process. The idea is that states tend to join the alliance that is most likely to prevail in a conflict. In addition, free-riding is more feasible in large rather than in small alliances (Chen et al., 1996; Conybeare, 1994). This suggests clustering of states around relatively few powerful allies.

Since alliances require at least two to tango, we need to explain the conditions under which a state chooses to offer an alliance to another state and the decision of the other state to accept or reject the offer. Two kinds of considerations may drive preferential attachment in alliance networks. The first concerns the alliance choices of new states or states that are just beginning to consider forming or joining alliances. Once a nonaligned state decides that it needs allies to deal with security problems, it must decide whom to court as a would-be ally (Maoz, 2010a: 149–152). Preferential attachment or bandwagoning suggests that such states opt for a security umbrella encompassing many states. This is so for two reasons. First, a large coalition is more likely to counterbalance its enemies' capabilities. Second, allies are notoriously unreliable. A large number of allies increases the probability that some of them would help the focal state if it gets into trouble. The shortest path to a large umbrella is by approaching central states. Aligning with such states yields indirect alliances with the allies of its allies.

A state's international status as a major/minor power correlates with alliance centrality.[3] Major powers often compete with other major powers. This makes them open to offers of alliance formation. It also makes them more proactive in seeking allies among new and nonaligned states in order to increase their global influence, or to prevent minor powers from joining the opposite coalition. Both of these motivations suggest that a preferential attachment process may well account for the way in which security networks are formed.

Trade networks may be characterized by a similar logic. States prefer central trading partners because those tend to either have competitive prices and/or produce commodities that are desirable by many other states. Also, there exists a strong relationship between economy size and (a) the amount of trade that a state has, and (b) the number of trading partners (Hegre, 2009).[4] This suggests that states with large economies tend to attract and approach multiple trading partners. Since economy sizes also display power-law properties (Gabaix, 2009), it is reasonable to expect that international trade will have a similar degree distribution.

PA network formation models offer a compelling approach because they are simple and parsimonious. They require no exogenous data to model and estimate; all we need is complete network data. Second, PA applies to extremely large networks. The most common application of this model is to such networks as the internet, power grids, Hollywood actor networks, scientific citation networks, and so forth. Third, PA network formation is intuitive and simple. Consequently, it translates well to the bandwagon principle that relates to popular ideas about link formation in political science and international relations.

Yet PA models suffer from several limitations. First, they are applicable only to binary networks. When we deal with valued or signed networks, observed structures convey information that goes well beyond PA. Consequently, PA models cannot estimate the formation of valued or signed networks (unless these are binarized). Second, PA fails to explain the presence of the number of isolates. Yet, quite a few networks—such as those analyzed herein—contain a fair proportion of isolates. This requires some—quite heroic—assumptions regarding the modeling of isolates in preferential attachment models. Third, the origins of the network cannot be modeled. PA models are—strictly speaking—models of

[3]The average degree centrality (standard deviation) of minor, regional, and major powers, respectively is 0.078 (0.1), 0.097 (0.11), and 0.154 (0.13). $F(2, 13016) = 170.02$, $p < .0001$. Data sources are given below. (See also Maoz, 2010a: 230–234.)

[4]A model using the trade dataset (discussed below) shows that per capita GDP has a strong effect on both the magnitude of a state's trade and the number of trading partners.

network evolution, not of network formation. They describe how a given network with a given type of degree distribution evolves once new nodes join the network. It does not tell us what determines the decision of the first two nodes to form a link.

Both the advantages and limitations of PA models carry over to the international realm. Thinking of alliance formation or institutional network formation as a bandwagoning process is both simple and intuitive. However, if we want to model the level of commitment entailed in an alliance or the level of trade between two states, a PA process does not really translate into a valued degree distribution. Likewise, this model does not tell us how to account for multiple nonaligned states or for states that have almost no trade with other states.

Despite these limitations, we can state some hypotheses that relate PA to international network formation.

> *H1*. International networks form according to the principle of preferential attachment. This implies:
> *H1a*. New links are a function of the network centrality of dyad members.
> *H1b*. The degree distribution of international networks approximates a power-law distribution.
> *H1c*. The structural characteristics of international networks match those of scale-free networks: low density, few components, and high group centralization.

Homophily

The Homophily principle asserts that nodes sharing one or more attributes in common are more likely to form links than nodes having different attributes. The structure of the network is a function of the distribution of these attributes. The simplest generalization is that the network is divided into groups, whose size and composition is a function of the distribution of these attributes. Groups tend to be fairly homogenous with respect to members' attributes, and intergroup differences are attribute-related (McPherson et al., 2001; McPherson and Smith-Lovin, 1987; Currarini et al., 2009). Most HO studies focus on a single attribute. Consequently, a simple measure of homophily in a network follows the following logic.[5] Suppose a group of N people are distributed over a given attribute. Denote the number of people with attribute value i who have links with people of attribute value j by n_{ij}. Table 1 describes the probability distribution summarizing the links of nodes with other nodes across the values of this attribute (where $a_{ij} = \frac{n_{ij}}{N}$).

A simple homophily coefficient for this table would then be

$$HC = \frac{\sum_k a_{ii} - \sum_k r_i c_i}{1 - \sum_k r_i c_i} \qquad [2]$$

[5]I use the formulation of Newman (2003), although this is a general measure (e.g. Currarini et al., 2009; McPherson and Smith-Lovin, 1987; Zeng and Xie, 2008; Weare et al., 2009).

Table 1. Probability Distribution of a Network Links in Terms of a Specific Attribute

Value of Attribute a	*1*	2	…	*k*	*Row Sum (r)*
1	a_{11}	a_{12}	…	a_{1k}	$\Sigma_j a_{1j} = r_1$
2	a_{21}	a_{22}	…	a_{2k}	$\Sigma_j a_{2j} = r_2$
…	…	…	…	…	…
k	a_{k1}	a_{k2}	…	a_{kk}	$\Sigma_j a_{kj} = r_k$
Column Sum (c)	$\Sigma_i a_{i1} = c_1$	$\Sigma_i a_{i2} = c_2$	…	$\Sigma_i a_{ik} = c_k$	$\Sigma_i \Sigma_j a_{ij}$

The homophily index varies from some negative number that is smaller than –1 to +1.[6] Positive values indicate that, given the distribution of the population over the attribute, homophily is higher than expected by chance. Negative values indicate *heterophily* (or what Newman calls disassortative mixing), that is, a tendency of nodes to have links to nodes that are different from themselves. In order to examine the statistical significance of the HO coefficient, we need to estimate the variance of this distribution. This is given by:

$$\sigma^2_{HO} = \frac{\sum_k r_i c_i + [\sum_k r_i c_i]^2 - \sum_k r_i^2 c_i - \sum_k r_i c_i^2}{N(1 - \sum_k r_i c_i)} \qquad [3]$$

And the T-statistic of this index is given by

$$T = HC \Big/ \sqrt{\sigma^2_{HO}} \qquad [4]$$

The principle of homophily indexes is intuitive. It is simply the extent to which nodes are connected to similar nodes as a proportion of random links (considering the distribution of properties in the population). Yet, in most real-life situations we need a more complex definition of homophily. Some attributes are continuous rather than discrete. Moreover, "sameness" may be defined across multiple attributes rather than a single one, as seen in Table 1. For example, in a society, people may identify themselves along several dimensions (e.g. race, sex, education, age). Patterns of friendship may mix these dimensions. Concomitantly, some groups may show greater or different conception of homophily than other groups. Homophily in such cases is present, but it is group specific rather than general. There is evidence that large groups tend to show greater degrees of homophily than small groups friendship networks (Currarini et al., 2009). More complex measures of homophily have indeed been developed,[7] and we will discuss them below.

[6]The reason for this odd range is that heterophily is a function of some sort of random distribution of links. Since the denominator is determined by the size of "randomness" in a network with a given set of marginal distributions, HO can be –1 iff the trace of the matrix is zero and $\Sigma_k r_i c_i = 0.5$. This can never happen. The maximum value of the trace in a $k \times k$ matrix with a random distribution is a function of k but is always smaller than 0.5.

[7]For example, Axelrod, 1997; Newman, 2002; Kuperman, 2006; Zeng and Xie, 2008.

What is the impact of homophily on network structure? As homophily increases, the network exhibits local convergence but global polarization (Axelrod, 1997; Ruyu and Kuperman, 2007; Centola et al., 2007). The evolution of networks under more or less deterministic assumptions of homophily, but also assuming that interaction affects the commonality of traits, suggests that networks evolve into groups that are culturally homogenous (that is, share a high level of within-group homophily), with reduced cross-group interaction (Centola et al., 2007). A number of identifiable network characteristics follow.

- The number of components is a function of the number and distribution of the traits across the population. Specifically, when homophily is defined by a single trait, the number of components will be correlated with the number of categories of this trait. When homophily is defined along multiple traits, the number of components will decline because the distribution becomes more complex.
- The number of nodes within the largest (giant) component typically declines with the multidimensionality of the trait distribution and the number of categories within traits.
- Network density decreases with the number of categories within a given trait and with the number of traits that define homophily.
- Transitivity is relatively high.
- Group centralization declines with the number of categories and the number of traits.

Here too, a picture is useful to illustrate the emergent properties of a network in which there is some tendency of homophily. Figure 2 displays the alliance network of the year 2002, with regime type as the homophily attribute. States are defined as democracies, anocracies, or autocracies.

Clearly, some alliances are dominated by democracies (e.g. NATO) while others are almost exclusively made up of anocracies and/or autocracies (e.g. the Arab League, OAU). The relatively small number of democracies accentuates the disproportionate tendency of democracies to form alliances with other democracies. On the other hand, the alliances between non-democratic states are not surprising given that the opportunity to form such alliances is high. That is why the actual proportion of alliances between non-democratic states or between mixed (democratic–non-democratic) regime types is not different than what one can expect by chance alone. This is also the reason why the homophily index in this year is quite low.

How does homophily apply to international network formation? The idea that states tend to form cooperative links with states that are "similar" has strong roots in the theory and empirics of international relations. The question, of course, is what we mean by "sameness" in international relations. Different theories have different conceptions of "sameness" or "similarity", but the general idea that similarities attract is common to all three major paradigms. Let us spell out some of these meanings.

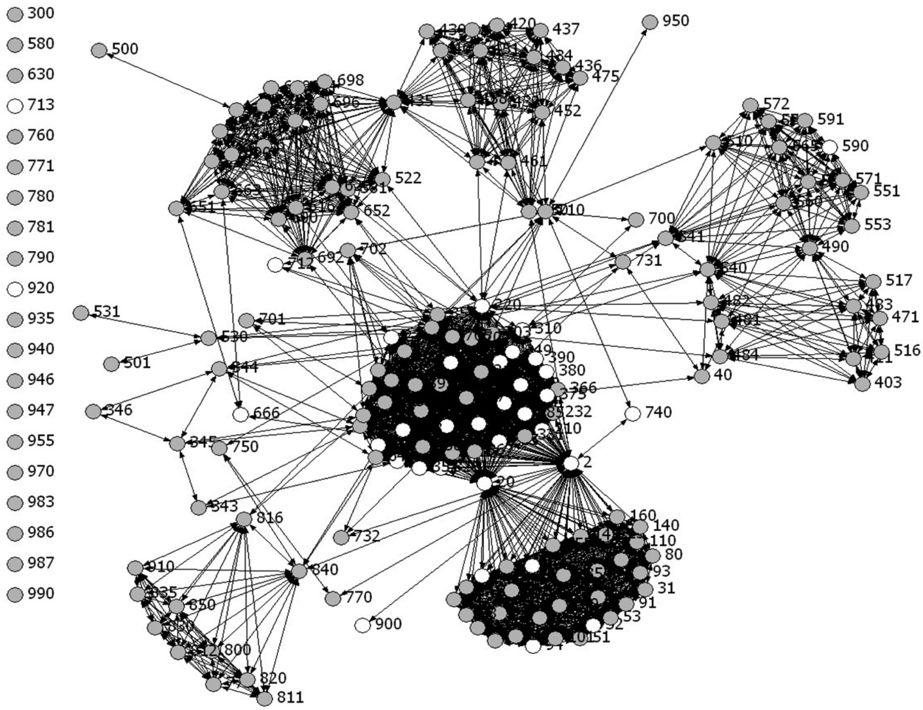

Figure 2. An Example of Homophily in IR Network—Alliance Network, 2002
Homophily Coefficient, HC = 0.086, SD = 0.029; T = 2.99, $p < 0.01$.
Labels = COW state numbers. White circle = democracy; gray circle = non-democracy.

In realism, the concept of similarity or affinity reads “common interests”. Mearsheimer (1994/5: 13) articulates what realists mean by common interests: “balance-of-power logic causes states to form alliances and cooperate against common enemies”. The only common interest that matters to realists is a common threat (Walt, 1988). This implies that the “enemy of my enemy is my ally” homophily process tends to result in highly polarized international systems (Lee et al., 1994; Maoz et al., 2007; Saperstein, 2004).

The liberal concept of sameness is specific to democratic norms and/or institutions. Democracies trust each other more because their political system is built on sustainable contractual arrangements. This makes democracies more likely to trust other democracies. The same does not apply to non-democratic political systems. These are both more suspicious and more likely to exploit each other (as well as democratic states), hence making for less reliable and more risky allies (Maoz, 2010a: 159–162). This extends to trade relations as well. Empirical analyses provide considerable evidence for the democratic homophily principle in alliance politics (Siverson and Emmons, 1991; Maoz, 2000) and trade relations (Aidt and Gassebner, 2010; Long, 2008; Mansfield et al., 2000).

Constructivists assert that states are driven by identity factors and shared ideas. Homophily, in this context, implies that states that have shared identities and ideas are likely to form links with each other. One way of interpreting identity-based sameness concerns cultural similarity between states. The notion that that common identities attract closely parallels models of cultural revolution (Axelrod, 1997; Centola et al., 2007).

Thus, all three major paradigms emphasize some form of homophily in processes of international network formation. This suggests several hypotheses.

> *H2:* International networks are formed by attachment of states to other states that they consider the same as or similar to themselves. The general rule is that states use similar criteria to assess similarity. However, different types of states define "sameness" differently. Specifically,
>
> *H2a:* All states use three criteria to determine "sameness". These criteria are: democracy, shared enemies, and cultural similarity. However, the order of importance of these criteria varies by regime types. Specifically,
>
> *H2b:* Democracies' order of preference is (1) joint democracy, (2) shared enemies, (3) cultural similarity.
>
> *H2c:* Non-democracies' order of preference is (2) shared enemies, (2) cultural similarity, (3) democracy.
>
> *H2d:* Given that HO processes drive network formation, cohesive groups within networks are likely to display high degrees of homophily as defined above.
>
> *H2e:* The emergent implications of these patterns of network formation are: low density, a relatively large number of components, high transitivity, and low group centralization.

These hypotheses combine the ideas of the three paradigms. It is possible to test each of the paradigms separately, but the idea here is not to examine which paradigm provides the best homophily model. Rather, it is to examine a broader conception of sameness that recognizes that networking choices in IR are complicated and embody strategic calculations (shared enemies), political affinities (joint democracy), and cultural links (cultural similarity). This model also recognizes the contextual nature of sameness. Specifically, non-democratic states are more "realist" and "cultural" than democratic states (Maoz, 2010a: Ch. 5).

The homophily model of network formation has a number of useful properties. First, as in the case of PA models, HO models of network formation are conceptually simple and intuitive. Given some attribute or set of attributes we consider to define "sameness", we can spell out a fairly simple model of network formation. Second, we can work back from data on a general structure of the network to the process that formed this structure. Third, unlike the PA model, the HO model can be applied to both binary and valued networks; it can account for linked nodes and for isolates; and it can explain both the initial links and the evolution of the network.

HO models have a several problems. First, they require exogenous data on nodal attributes. Network data are not sufficient to establish HO processes. In most real-life situations we need information about multiple attributes. Second, homophily models can become rather complex. Third, general models of homophily cannot be pre-specified across different networks. Each network (or each family of networks) may be driven by different types of attributes that determine network formation processes. For example, the meaning of "sameness" might be different for alliance choices of states than for trade partner choices or institutional affiliation choices. Accordingly, the definition of how many attributes go into the definition of "sameness", in what order, and at what level of importance is very much an open question. The answer to this question depends on the context (type of network, theoretical considerations) and the researcher's expectations. The case of a single model that fits everything (as in the PA model) does not apply here.

3. Testing Models of Network Formation in International Relations

(a) Testing PA Processes and Power-Law Distributions

A test of a preferential attachment model requires us to compare the actual degree distribution of a given international network to a power-law distribution. It must be said, however, that this test is somewhat unfair. This model is more adequate in large networks than in small ones (Barabasi and Albert, 1999). International networks with states as nodes are comparatively small. Nevertheless, since we can treat each year as reflecting a network that, in principle, can assume a different structure from those of previous years, it is possible to offer two sets of tests of this principle.

First, the simplest test is a test of the process of network formation. New links are considered more likely to form the more central one of the nodes. Thus, we can examine whether the number of new attachments is a function of the centrality of the nodes. Specifically, the model to be tested is given by:

$$p\left[\left(s_{ij}\right)_{t1}=1 \mid \sum_{j} s_{ij\,t0}=0\right]=f(c_{jt0}, X_{t0}) \qquad [5]$$

where c_{jt0} is the degree centrality score of node j in the previous year. In words, this means that the probability of observing a "new" link between two nodes i and j is a function of j's degree centrality in the previous year. (Note, the X on the right-hand side of Equation [5] is a vector of control variables.)

The second, and more conventional test compares the degree distribution of international networks to a power-law distribution. The problem of this comparison is threefold. First, we need to determine the value of γ that provides the best fit of the actual degree distribution to a power-law distribution. Second, we need to decide which part of the actual degree distribution to fit to the power-law distribution. Third, we need to account for isolates in the actual degree distribution, or rather to modify the actual degree distribution to focus only on valid links and ignore the isolates.

We can confront these problems in the following manner. We start by varying the value of gamma between 0.5 and 4 via an iterative algorithm. This algorithm determines which value induces a distribution that provides the best fit to the degree distribution of actual networks. The argument about the fit between higher degree nodes and a power-law distribution is not justified in the case of international networks. This is so primarily because most of these networks are large

Table 2. Adjusted Probability Distribution of Connected Nodes

Original Degree	*Relative Frequency*	*Connected Probability*
0	0.200	NA
1	0.250	0.313
2	0.110	0.138
3	0.050	0.063
….	–	
$N-1$	–	

enough to provide a sense of the entire range of the distribution. We also need to make significant modifications (as will be shown below) to deal with the fairly large number of isolates in empirically observed networks. This suggests that the empirical distributions are already transformed to improve the probability of fit with a power-law distribution.

In order to allow for a connected network, we need to transform the degree distribution of empirically observed networks such that they will only reflect those nodes that have at least one link to another node in the network. This is done by resetting the distribution such that the probability of a nodal degree reflects only connected nodes. Let the probability of a nodal degree K ($0 \leq K \leq N-1$) be $P(K)$. Denote the adjusted probability that reflects only connected nodes as $P'(K'|K' > 0)$. The revised probability distribution is given by:

$$P'(K') = P(K|K > 0)/(1 - P(K|K = 0) \qquad [6]$$

This is exemplified in Table 2.

We can now compare the adjusted degree distribution to a power-law distribution. This comparison must be based on two facts. First, these distributions are discrete with respect to nodal degrees. Second, the expected (power-law) distribution is strongly left-skewed. This implies that a simple chi-square comparison provides a straightforward test of the extent to which these two distributions match.

(b) Testing Homophily

There are a number of ways of testing the extent to which empirical networks fit the HO model of network formation. Instead of going through some alternatives, I discuss an approach that seems to best meet the requirements of a more complex approach to homophily. I label this type of homophily as "contingent homophily". The strategy of measuring homophily in a contingent fashion consists of establishing the probability of a link between two nodes in the network on the basis of an *a priori* model of homophily. Such a model is established in the following manner.

1. State the dimensions of homophily (common traits) that are considered relevant for matching pairs of nodes. Some of these dimensions may be

categorical (e.g. race in a friendship network), and some may be continuous (e.g. income).
2. For each trait define the range it may take.
3. Assign to each node a value on each of the traits.
4. Determine a nodal type on a single category.
5. For each nodal type, specify a vector of weights (W) that defines the relative importance of the various traits for this specific nodal type. (Let w_{it} be the weight assigned by node type i to trait t. The constraints on W are $0 \leq w_{it} \leq 1$, $\Sigma_t w_{it} = 1 \; \forall \; i \in N$.)
6. The expected probability of a link between two nodes is defined as:

$$P(x_{ij}) = \sum_{t=1}^{k} \frac{(1 - |v_{it} - v_{jt}|)w_{it}}{\text{rnge}(t)} \qquad [7]$$

where v_{it}, v_{jt} are, respectively, the values of node i and j on trait t and *rnge(t)* is defined as the range of values on trait t. (For continuous traits, this is *rnge(t)* = *max(t)* − *min(t)*. For ordinal traits, this is defined as *rnge(t)* = *max(t)* − *min(t)* + 1. For nominal traits this is simply 1.) This index takes into account (a) multiple traits, (b) traits that are defined on different measurement scales, (c) traits with different ranges, (d) the idea that different "types" of nodes assign different weights to certain traits.

Testing homophily on a dyadic level requires matching the expected probability of a link to the observed presence or absence of a link. The expectation is that the higher the expected probability of a link between two nodes, the more likely are they to be connected in the real world. Comparing the estimates of alliance links based on the homophily index to one generated by a baseline model can give us a sense whether a HO network formation improves on a baseline prediction of who forms an alliance with whom.

There are a number of ways to test HO on emergent network properties. These can be done at the individual node level, by measuring the number of links for each individual node that reflect homophily as a proportion of the total number of links of that node. We can examine the extent to which subsets of nodes that form substantive groups (cliques, blocks, communities, etc.) are populated by homophilous nodes (Currarini et al., 2009). I focus here on the general structure of the entire network. At this level, we can estimate the Homophily Coefficient (HC) and test its statistical significance, using the test shown in Equation [4] above. This provides us with an assessment of the prevalence of homophily in the network as a whole.

We now turn to the results of the empirical analyses of network formation in international networks. The analysis focuses on two types of international networks: security (formal alliance) networks and international trade networks. The appendix outlines the research methodology and design.

4. Results

We start by examining the extent to which states actually follow PA and HO principles when forming alliances or forging new trade or institutional links. Table 3 provides the results of this analysis. Table 4 displays the marginal change in the probability of a link as a function of a specified change in the values of each independent variable, controlling for the other independent variables. This allows us to assess the relative contribution of each variable to the model.

The leftmost columns in each section of the tables are the baseline model for the PA and the homophily index. As expected, the baseline model seems quite meaningful: all the components that go into the homophily index have a statistically significant impact on the probability of alliance and trade links. These effects are considerable and are all in the expected direction, judging from the increase/decrease in the probability of links as a function of the change in the value of the independent variables. Joint democracy, the presence of common enemies, and cultural similarity contribute significantly to the probability of states forming and maintaining alliance or trade links. The overall fit of the alliance model is moderate-to-high and so is the improvement in fit over the modal category (PIF = 0.69). With the alliance onset dependent variable, the overall fit of the baseline model drops significantly, although the independent variables maintain their significance.

The results of the baseline trade model are quite similar. All of the independent variables have a significant effect on the probability of trade between states. Here, however, the fit of the baseline model is moderate. Substituting the independent variables with the centrality and homophily indices suggests that both preferential attachment and homophily considerations have a significant effect on alliance and trade choices. Running the preferential attachment and the homophily models separately maintains these results.[8] This provides dyadic evidence for both the PA and HO network formation models.

I now turn to a distributional analysis of the PA model. Figure 3 shows the fit between various power-law distributions and the average annual degree distribution of the alliance and trade network. The difference between the two parts of this figure is quite striking: the alliance degree distribution does not seem to fit well any specific power-law distribution. On the other hand, the trade degree distribution seems to provide a good fit to a power-law distribution with γ =1, and clearly falls between the range of $0.75 \leq \gamma \leq 1.25$. In order to provide a more reliable test of this observation, I use a chi-square test on the difference between the actual degree distribution or alliance/trade networks and the best fitting power-law distribution. The results of the annual chi-square levels are displayed in Figure 4.

[8]The overall level of the model's fit is maintained in the separate runs of the preferential attachment and homophily indices. This is so because half of the variance is accounted for by the binary time-series cross-sectional controls (non-event years and cubic splines). A simple logit model without these controls suggests a marginal drop in the model fit when the PA and HO are run separately.

Table 3. Probability of Alliance and Trade Links, Binary Time-Series Cross-Sectional Logit of All Dyads 1816(1870)–2003

Independent/Control Variable	*Alliance Years*		*Alliance Onset*		*Trade Year*		*Trade Onset*	
	Baseline	*PA/HO*	*Baseline*	*PA/HO*	*Baseline*	*PA/HO*	*Baseline*	*PA/HO*
Joint Democracy	0.589**		0.721**		0.839**		0.994**	
	(0.026)		(0.062)		(0.021)		(0.022)	
Enemy of My Enemy	1.165**		2.132**		0.361**		0.477**	
	(0.057)		(0.06)		(0.033)		(0.037)	
Cultural Similarity	0.84**		0.407**		0.505**		0.468**	
	(0.014)		(0.04)		(0.014)		(0.016)	
State A Major Power	0.693**		0.076		0.521**		0.378**	
	(0.023)		(0.056)		(0.023)		(0.025)	
State B Major Power	0.967**		0.107		2.203**		2.045**	
	(0.025)		(0.061)		(0.014)		(0.015)	
Log Distance	−0.782**	−0.565**	−0.512**	−0.453**	−0.505**	−0.341**	−0.3**	−0.249**
	(0.009)	(0.011)	(0.009)	(0.01)	(0.006)	(0.007)	(0.005)	(0.006)
Degree Centrality State A		4.24**		1.051**		−1.19**		−1.296**
		(0.144)		(0.233)		(0.069)		(0.076)
Degree Centrality State B		7.062**		1.063**		3.162**		3.191**
		(0.209)		(0.256)		(0.054)		(0.058)
Homophily Index		0.239**		0.448**		0.253**		0.277**
		(0.009)		(0.022)		(0.007)		(0.008)
No. Years w/o Link+	−1.775**	−1.89**	−0.286**	−0.322**	−1.124**	−1.694**	−0.806**	−1.216**
	(0.021)	(0.028)	(0.011)	(0.015)	(0.009)	(0.015)	(0.007)	(0.012)
Constant	6.341**	4.268**	0.257**	−0.447**	3.019**	3.001**	0.534**	1.331**
	(0.079)	(0.093)	(0.096)	(0.108)	(0.054)	(0.057)	(0.048)	(0.05)

(continued)

Table 3. (Continued)

Independent/Control Variable	*Alliance Years*		*Alliance Onset*		*Trade Year*		*Trade Onset*	
	Baseline	*PA/HO*	*Baseline*	*PA/HO*	*Baseline*	*PA/HO*	*Baseline*	*PA/HO*
Model Statistics								
N	674,692	653,632	634,181	597,253	1,280,896	1,242,161	1,319,211	1,278,407
chi-square	45,264.73	56,167.41	9,351.93	7,155.15	77,882.00	79,211.11	55,917.37	54,624.83
R-Squared	0.686	0.796	0.129	0.111	0.565	0.613	0.436	0.466
PIF[++]	0.687	0.854	0.001	0.001	0.384	0.579	0.166	0.184

Numbers in parentheses are robust standard errors.

[+] Cubic spline parameters are omitted to conserve space.

[++] PIF: Percent Improvement in Fit: $PIF = \frac{CP|Model-MC}{1-MC}$ where *CP|Model* is the percentage of cases predicted correctly given the model, and *MC* is the percentage of cases in the modal category.

$^{*}p < 0.05$; $^{**}p < 0.01$.

Table 4. Percentage Change in the Probability of the Dependent Variable as a Result of Change in the Value of the Independent Variable

	Alliance Years		*Alliance Onset*		*Trade Year*		*Trade Onset*	
	Baseline	*PA/HO*	*Baseline*	*PA/HO*	*Baseline*	*PA/HO*	*Baseline*	*PA/HO*
Baseline prob. of link (all ind. vars. at mean).	9.10		0.56		3.80		2.10	
Joint Democracy*	79.85		104.86		130.61		168.81	
Enemy of My Enemy*	218.29		721.83		43.23		60.89	
Cultural Similarity*	130.79		49.94		65.48		59.47	
State A Major Power*	99.48		7.83		68.02		45.71	
State B Major Power*	161.65		11.28		786.75		659.37	
Log Distance**	−79.48	−80.75	−87.20	−97.86	−81.20	−73.81	−76.45	−76.61
Degree Centrality State A**		37.86		20.06		−10.84		−16.07
Degree Centrality State B**		71.69		20.30		35.82		54.08
Homophily Index**		21.61		143.44		30.96		51.94

*Change in the value of the variable from 0 to 1.
**Change in the value defined as movement from the value of the 25th percentile to the value of the 75th percentile.

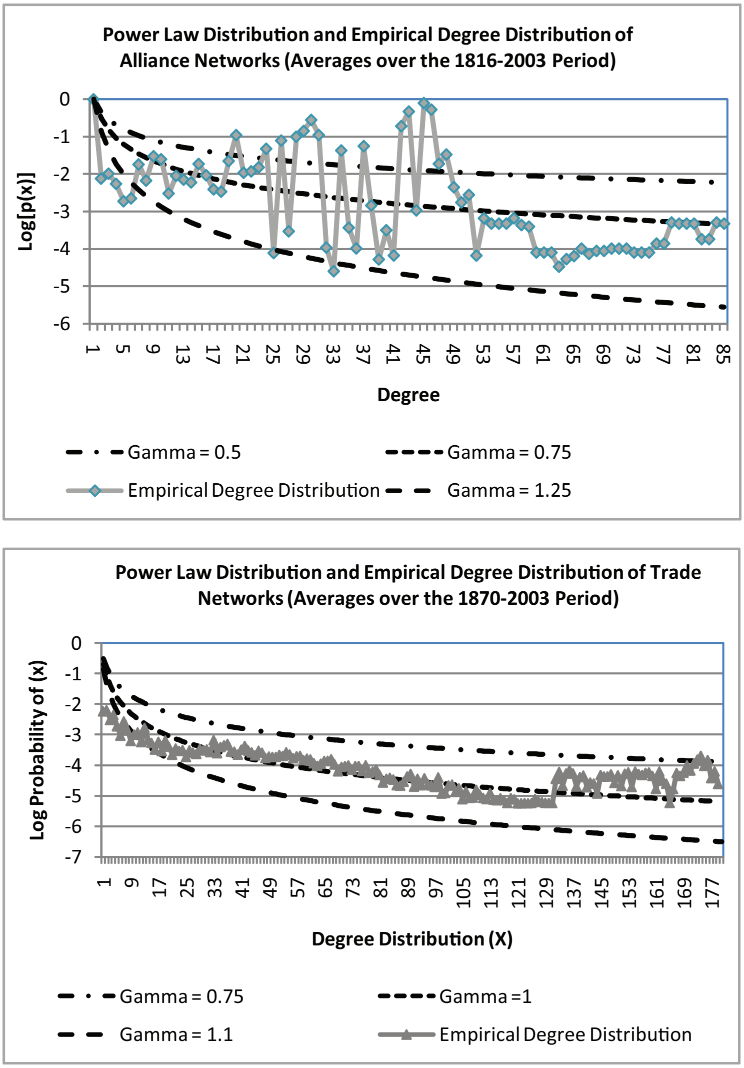

Figure 3. Power Law and Empirical Degree Distributions of Alliances and Trade Networks, 1816(1870)–2003

This figure tells a simple story. There is strong evidence that trade networks were formed according to the preferential attachment model. The fit between the degree distribution of trade networks and the power-law distribution is exceptionally strong. The maximum chi-square score of the difference between the trade degree distribution and the power-law distribution is observed in 1920 and assumes the value of 2.75 (which with 38 degrees of freedom has a probability of 1 of fitting to the actual power-law distribution). On the other hand, the fit between the alliance degree distribution and the power-law distribution is almost never statistically significant. The only exceptions are the years 1816–1820 for alliances,

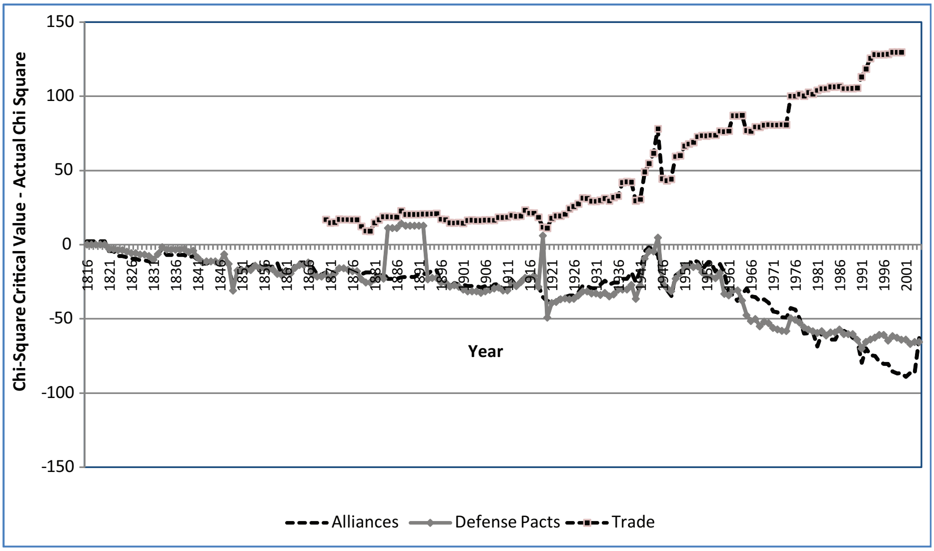

Figure 4. Chi Square Figures of Degree of Fit between Actual Networks and Power-Law Degree Distribution by Year

The lines reflect the difference between the actual chi-square value and the critical value of chi-square at $p < 0.05$ (for alliances and defense pacts) and $p < 0.001$ for trade. Positive values mean that the probability of a fit between the degree distribution of the actual network and the best-fitting power-law distribution is greater than 95% (for alliances) and greater than 99.9% (for trade). Negative values indicate bad (insignificant) fit. (Degrees of freedom are adjusted for each year, $df_t = k-1 = max(d_t)-1$.)

1884–1892, 1919, and 1945 for defense pacts. The fit in later years makes sense due to crowding at the end of the two World Wars. Yet there is no apparent reason for the PA process that marked alliance networks in the 1884–1892 period.

This aspect of the test confirms the PA process of trade formation that we have seen at the dyadic level, but disconfirms the PA process of alliances and defense pacts. This suggests that while states' choice of allies may be affected by the centrality of candidates, the degree of preferential attachment as a guiding principle is not as strong as we would expect. I will come back to this point below. Interestingly, the goodness of fit between the degree distribution of trade networks and a power-law distribution increases over time, despite the increased degrees of freedom. In contrast, the fit between the degree distribution of alliance networks and a power-law distribution worsens with time. This suggests that PA considerations have become increasingly prominent in the formation of trade links over time. In contrast, such considerations became less prominent in alliance formation behavior as time progressed.[9]

[9]In fact, the "poorness" of the fit between alliance and power-law degree distribution is even more pronounced considering the fact that, on average throughout the 1816–2004 period, about 47.3% of the states were isolates. These, of course, are not even included in the PA model.

I now turn to additional tests of the HO hypothesis. As the preceding analyses show, the effects of HO processes are considerably stronger for alliance networks than for trade networks. This point is accentuated once we consider the distributional network implications of such patterns. The results of the HC estimates are provided in Figure 5.

This figure illustrates clearly that homophily had a much stronger effect on the formation of alliance networks than on trade networks. The average HC for alliances (0.165) was more than twice that of the trade HC (0.073). The trend is also interesting: while we see a constant decline in homophily for alliances during the 19th and first half of the 20th century, the HC spikes during World War II and remains fairly high afterwards. Trade homophily shows no secular trend. The significance of both the alliance and trade homophily coefficients also increases after the mid 1940s.

We end this section by showing a number of structural consequences of these network formation processes. This is done in Figure 6, which confirms the kind of network characteristics that are consistent with PA and HO processes. The number of components is far lower for trade networks than for alliance networks, and the group centralization of trade networks is significantly higher than for alliance networks. In contrast, transitivity is significantly higher for alliance networks and lower for trade networks. These patterns corroborate the key results of this study: there is evidence that both PA and HO processes operate on international networks. Yet, each of these models is consistent with one type of network. Trade networks form according to a PA logic: new trade links between states are a function of the trade centrality of each. Alliance networks are influenced by HO processes. States tend to form alliance links with "similar" states.

5. Conclusion

This is the first study showing that the way states form cooperative relations seems to parallel the way people join social networks on the internet, the patterns of citations of scholarly works, or the ways children form friendships. However, different international networks are marked by different logics of link formation. There is fairly strong evidence suggesting that trade network formation is based on preferential attachment. Likewise, there is fairly significant evidence suggesting that alliance networks are formed by homophily considerations.

This evidence spans levels of analysis. These patterns are observed at the dyadic trade and alliance levels. However, the differences in patterns of network formation are most pronounced at the network level. It is at this level that we can most clearly observe the consequences of dyadic patterns. And it is at this level that we can best associate trade networks with PA processes and alliance networks with HO processes. And it is this result that highlights the tremendous value of network analysis for international relations. Specifically, this approach allows us to identify the linkages between micro-level processes, such as the factors that affect the choices of individual units, and macro-level consequences—discernible structures in international politics. The fact that international systems are not

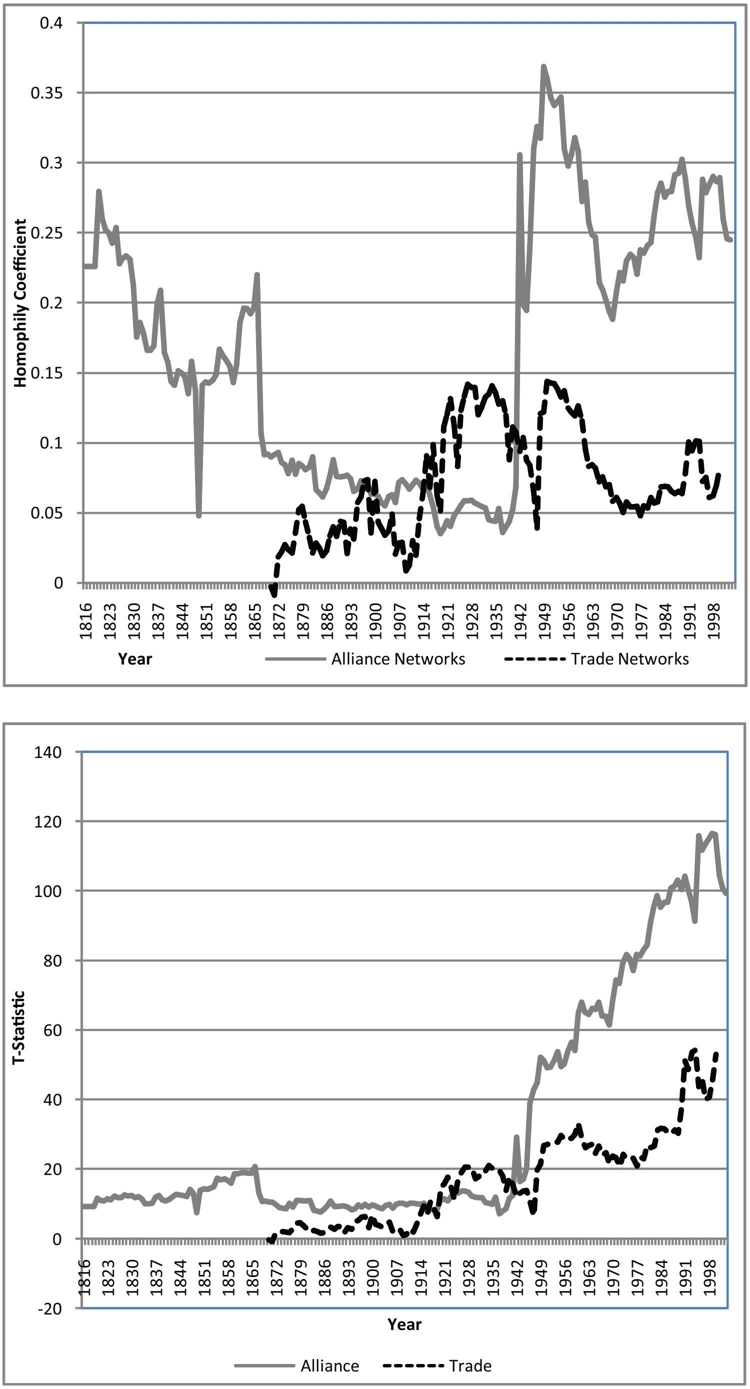

Figure 5. Homophily in Alliance and Trade Networks over Time

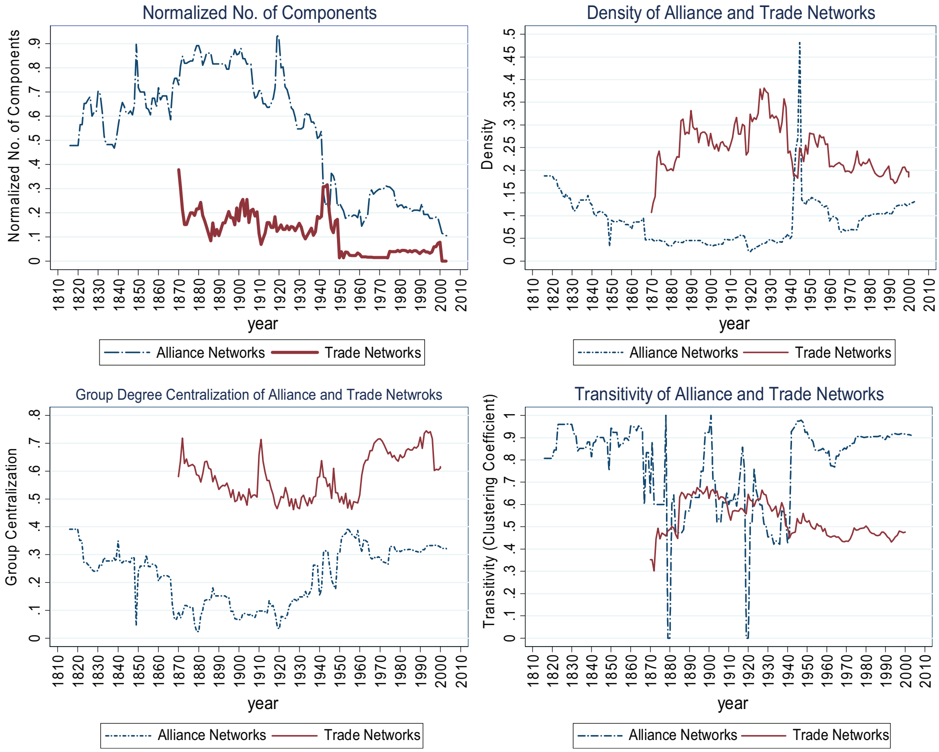

Figure 6. Network Characteristics: Alliance and Trade Networks, 1816–2003

mere aggregations of units is something that most students of international relations are familiar with. However, how international structures emerge is something that is far less obvious and far less understood. The present study shows how network analysis can effectively establish this link.

The analysis here offers a number of insights into the study of international structures. First, it shows a consistent relationship between dynamic processes of network formation and network evolution and the emergent international structures that result from them. The results are not entirely new: we knew that regime homophily affects both alliance and trade choices. What we did not know was the kind of structures that emerge from these patterns. Second, the results of the fit between (a) alliances and HO network formation processes and (b) trade and PA processes suggest that international structures can be due to fairly simple rules of interaction. Third, these rules of interaction correspond to various paradigms in international relations, as well as to more integrative models of network evolution (Maoz, 2010a).

This study opens the door to further investigations. Homophily in trade networks may imply different things than in strategic cooperation networks. Trade-related sameness may have to do with the size of economies, the structure of economies, and domestic regime. It would be interesting to see whether an economic conception of homophily may provide a better fit between the HO model and trade networks. In addition, with data on more networks (institutional, diplomatic, substate), it is now possible to test whether such simple network formation models apply to other international networks. The current study offers both a theoretical framework and a methodology for such analyses.

Methodological Appendix

1. Data

This study uses several datasets.

a. *Alliance Treaties and Obligations Provisions (ATOP)*. This dataset consists of all formal alliance treaties between states and covers the 1816–2004 period (Leeds, 2005).
b. *International Trade dataset*. This dataset covers values of trade between states over the period of 1870–2003 (Gleditsch, 2002; Barbieri et al., 2008).
c. *Regime dataset*. The POLITY IV dataset provides information on regime characteristics of states over the 1800–2004 period (Marshall and Jaggers, 2004).
d. *The Dyadic MID dataset* (Maoz, 2005) is used to generate data on "the enemy of my enemy" principle (Maoz et al., 2007).
e. *The Cultural (Religious/Linguistic) dataset* was collected by Phil Schaffer at the University of Michigan. It contains data on the religious and linguistic groups within states over the 1820–1990 period. The religion dataset is currently being updated and cleaned. The linguistic dataset is preserved as is. It was used in a number of previous studies (Henderson, 1998, 2004; Henderson and Tucker, 2001; Maoz, 2010a).
f. *The Minimum Distance dataset* (Gleditsch and Ward, 2001) specifies the distance between state capitals.

2. Units of Analysis and Empirical Domain

The spatial-temporal domain covers all states/dyads over the period of 1870 to 2003. In some cases (where we use only alliance data but not trade), the period analyzed is 1816–2003. The analyses herein focus on two units of analysis: the dyad-year unit and the network-year unit.

3. Measurement

Dependent Variables

Alliance Links. Alliance links are defined in several ways. First, a dyad-year is assigned a score of 1 if the dyad members had a formal alliance of any type. Second, a defense/offense variable is assigned a score of 1 if dyad members had a formal offensive or defense pact for that year and 0 otherwise. Third, a similar coding was applied to alliance onset. However, here a dyad was assigned a score of 1 for the first year it had an alliance, 0 for every year prior to that, and a missing value for each year the alliance was active (right censoring). Finally, a commitment level was defined such that zero commitment = no alliance, low commitment = consultation, non-aggression, and neutrality pact, and high commitment = defense or offense pact.

Trade links. A trade matrix R is created for each year. An entry r_{ij} receives a score of 1 if the trade (imports + exports) between states *i* and *j* exceeded one-hundredth of one percent of *i*'s GDP, and 0 otherwise. This means that, in contrast to the alliance network, the trade network is asymmetric ($r_{ij} \neq r_{ij}$). Trade onset is assigned a value of 1 for the first year trade links existed between states and missing value for each subsequent year. For the HC analysis, trade levels were also broken down into three levels: no trade, low trade = less than 0.1% of the state's GDP, and high trade = 0.1% or more of the state's GDP.

Note that if an alliance or trade links were disrupted for one or more years in a dyad, the years of disruption were again assigned a zero. The first year that these links were resumed was assigned a score of 1 and all subsequent years wherein links were maintained were assigned missing values and were excluded from the analysis of onset.

Degree Centrality. This is simply the number of alliance/trade links of the state with other states. The normalized degree centrality is the degree centrality divided by the number of states for that year minus 1.

Degree Distribution. The frequency distribution of degree centrality. Power-law degree distributions are extracted using Equation [1] above with variable gamma scores. For more convenient display, the actual and power-law distributions are log-transformed.

Independent/Control Variables

Joint Democracy. I use two versions of this variable. The first version—joint democracy—is used in the baseline models discussed below. The second version—joint regime type—is used in the construction of the homophily coefficient. A dyad is assigned a score of 1 if both states were democracies (following the cutoffs of Maoz, 1998: 97–98), 0 if one state was a democracy and the other not, and –1 if neither was a democracy.

Enemy of My Enemy. Using the Dyadic MID dataset, a conflict network was set for each year such that an entry mid_{ij} was assigned a score of –1 if dyad members had at least one MID during that year and 0 otherwise. The MID matrix was then squared. The MID^2 matrix is now a valued matrix with diagonal entries mid^2_{ii} denoting the number of dyadic MIDs that a state was engaged in and mid^2_{ij} denoting the number of common enemies shared by states *i* and *j*. This matrix is binarized, so that a dyad receives a score of 1 if it had at least one common enemy and 0 otherwise (Maoz et al., 2007).

Cultural Similarity. Following Maoz (2010a: 205–206), the cultural similarity of a dyad is a function of the degree of linguistic and religious similarity of their populations. Cultural similarity of a dyad is assigned a value of 1 if the cultural similarity between dyad members was higher than the mean cultural similarity in the system that year and 0 otherwise.

Log Distance. The number of kilometers between states' capitals (Gleditsch and Ward, 2001). Log distance is used due to large variation in the distance data.

Homophily Index. The homophily index uses the joint democracy, enemy of my enemy, and cultural similarity indices. The calculation of the dyadic index relies on Equation [7] above and is defined as follows:

$$HO_{ij} = \left\{ \left(1 - \left|dem_i - dem_j\right|\right) w_{demi} + mid^2_{ij} w_{cei} + cs_{ij} w_{csi} \right\} \qquad [8]$$

where *dem* is the democracy score of the state ($i \neq j$), mid^2_{ij} is the binarized *ij* entry of the MID^2 matrix, *cs* is the cultural similarity value of the dyad, and the w_{xxi} scores are the weights assigned by state *i* to the trait. The *W* vector is defined differently for democratic and non-democratic states based on Maoz (2010a), as follows.

Trait	*Weight-Democracy*	*Weight-Non-Democracy*
Democracy	0.5	0.1
Enemy of My Enemy	0.3	0.6
Cultural Similarity	0.2	0.3

The weights of these traits were changed a number of times while retaining the relative ranking of traits, and this had very little effects on the results. The homophily index varies from 0 to 1 and is directional ($HO_{ij} \neq HO_{ji}$). In order to estimate the HC, I partition the HO index to low (zero homophily), medium (lower two-thirds), and high (upper third) homophily. The HC, the variance, and the T-statistic are computed as in Equations [2]–[4] above.

4. Methods

I use several estimation methods. First, I use logit models (with non-alliance years and cubic splines; Beck et al., 1998) to test the probability of alliance links as a function of degree centrality or as a function of homophily. These analyses start out with baseline models that use the independent variables found by Maoz (2010a) and others to affect the probability of alliance and trade links. Models not shown in the present article use the centrality and homophily indices separately. The results are similar to those reported in Tables 2 and 3.

Second, I compare the degree distribution of real-world alliance or trade networks with a power-law network. This is done for each year by calculating a chi-square statistic for the year. This statistic is calculated as:

$$\chi^2 = \sum_{j=1}^{k} \frac{\left(d_{aj} - d_{plj}\right)^2}{d_{plj}} \qquad [9]$$

where $j = 1, 2, \ldots, k$ indexes degrees, d_{aj} is the frequency of nodes with degree j in the alliance (or trade) network, and d_{plj} is the expected frequency of nodes with degree j in a power-law distribution with gamma selected for best fit (as seen in Figure 3). Note that in examining real world networks, we ignore isolates and adjust probabilities according to Table 1 above. The degrees of freedom for each year vary and are $k-1$.

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