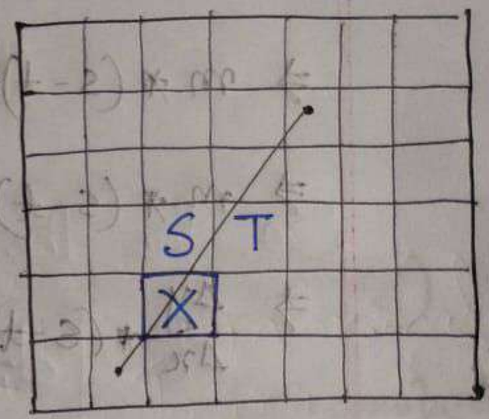
Assignment [Bresenham's Line Drawing Algorithm]

For $\boxed{m > 1}$

We can imagine the scenario like this grid →



Here, the current pixel is 'X'. The next pixel will be either 'S' or 'T'.

So if, $X \equiv (x_i, y_i)$

$S \equiv (x_i, y_{i+1})$

$T \equiv (x_{i+1}, y_{i+1})$

for any pixel, we can say that $y_i = mx + b$

For the pixel on the above row of perevious pixel,

$$y_i + 1 = mx + b \quad \text{——— (i)}$$

The distance of S & T from the actual line is:

$s = x - x_i$

$t = (x_i + 1) - x$

Distance between s & t is:

$$s - t = x - x_i - x_i - 1 + x$$

$$\Rightarrow s - t = 2x - 2x_i - 1 \quad \text{——— (ii)}$$

$$\Rightarrow s - t = 2\left(\frac{y_i + 1 - b}{m}\right) - 2x_i - 1$$

$$\Rightarrow m*(s-t) = 2y_i + 2 - 2b - 2x_i m - m$$

$$\Rightarrow m*(s-t) = 2[y_i - (mx_i + b)] + 2 - m$$

$$\Rightarrow m*(s-t) = 2[y_i - y_i] + 2 - m$$

$$\Rightarrow m*(s-t) = 2 - m$$

$$\Rightarrow \frac{\Delta y}{\Delta x}*(s-t) = 2 - \frac{\Delta y}{\Delta x}$$

$$\Rightarrow \Delta y*(s-t) = 2\Delta x - \Delta y$$

$$\therefore \boxed{d_i = 2\Delta x - \Delta y} \quad \text{(iii)}$$

Here, d_i is the decision variable for the base case.

If $d_i > 0 \Rightarrow s > t$ $\therefore$ We'll take T

If $d_i < 0 \Rightarrow s < t$ $\therefore$ We'll take S.

Now, $s - t = 2x - 2x_i - 1$ [using eqn. ii]

$$= 2\left(\frac{y_i + 1 - b}{m}\right) - 2x_i - 1 \quad \text{[using eqn. i]}$$

$$= \frac{2(y_i + 1 - b)\Delta x}{\Delta y} - 2x_i - 1$$

$$\Rightarrow \Delta y (s - t) = 2(y_i + 1 - b)\Delta x - 2x_i \Delta y - \Delta y$$

$$\Rightarrow d_i = 2y_i \Delta x - 2x_i \Delta y + (2\Delta x - 2b\Delta x - \Delta y)$$

$$\therefore d_i = 2y_i \Delta x - 2x_i \Delta y + c \quad \text{[c = constant]} \quad \text{(iv)}$$

for the next iteration,

$$d_{i+1} = 2y_{i+1}\Delta x - 2x_{i+1}\Delta y + c \quad \text{— (v)}$$

From eqn. v & iv we get,

$$d_{i+1} - d_i = 2\Delta x (y_{i+1} - y_i) - 2\Delta y (x_{i+1} - x_i)$$

$$\therefore d_{i+1} = d_i + 2\Delta x (y_{i+1} - y_i) - 2\Delta y (x_{i+1} - x_i)$$

if $d_i > 0$, we have chosen the pixel $T(x_i + 1, y_i + 1)$

$$\therefore d_{i+1} = d_i + 2\Delta x \cdot 1 - 2\Delta y \cdot 1$$

$$= d_i + 2\Delta x - 2\Delta y$$

if $d_i \leq 0$, we have chosen $S(x_i, y_i+1)$

$$\therefore d_{i+1} = d_i + 2\Delta x \cdot 1 - 2\Delta y \cdot 0$$

$$= d_i + 2\Delta x$$

So, for the first point,

$$\boxed{d_i = 2\Delta x - \Delta y}$$

For all other pixel points,

$$\boxed{\begin{aligned} d_{i+1} &= d_i + 2\Delta x - 2\Delta y \quad [\text{if } d_i > 0] \\ d_{i+1} &= d_i + 2\Delta x \quad [\text{if } d_i \leq 0] \end{aligned}}$$