# Problem 1

Assume you a call and a put option with the following

- Current Stock Price $165
- Current Date 03/03/2023
- Options Expiration Date 03/17/2023
- Risk Free Rate of 4.25%
- Continuously Compounding Coupon of 0.53%

Calculate the time to maturity using calendar days (not trading days).

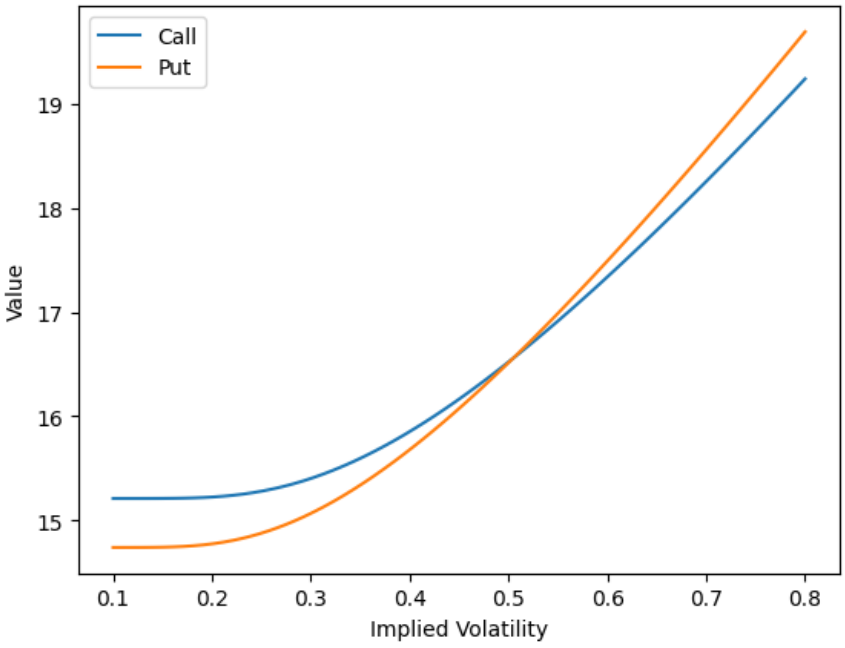
For a range of implied volatilities between 10% and 80%, plot the value of the call and the put.

Discuss these graphs. How does the supply and demand affect the implied volatility?

## Analysis

Firstly, I calculated the time to maturity. Since the current date is 03/03/2023, expiration date is 03/17/2023, and 365 days in 2023, the time to maturity is 0.0384.

Then I change the implied volatility between 10% and 80%, set strike price for call option is 150 and for put option is 180, and plotted the graph: value of call and put versus different implied volatility.



It can be seen that as implied volatility increase, the value of call and put option will increase as well. From the perspective of supply and demand, this is because an increase in implied volatility indicates an increase in the uncertainty or riskiness of the underlying asset. This, in turn, increases the potential payoff of the option and makes it more valuable to investors, who are willing to pay a higher price for the option. Additionally, an increase in implied volatility often leads to an increase in demand for options as investors seek to hedge against potential losses or speculate on potential gains. This increased demand for options can also drive up their value.

When demand for an asset in the market exceeds its supply, the price increases and implied volatility also increases. This is because buyers are willing to pay higher prices to acquire the asset, which increases the market's expectations of future price fluctuations and leads to an increase in implied volatility. Conversely, when supply exceeds demand, the price drops and implied volatility decreases.

# Problem 2

Use the options found in AAPL_Options.csv

- Current AAPL price is 151.03
- Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

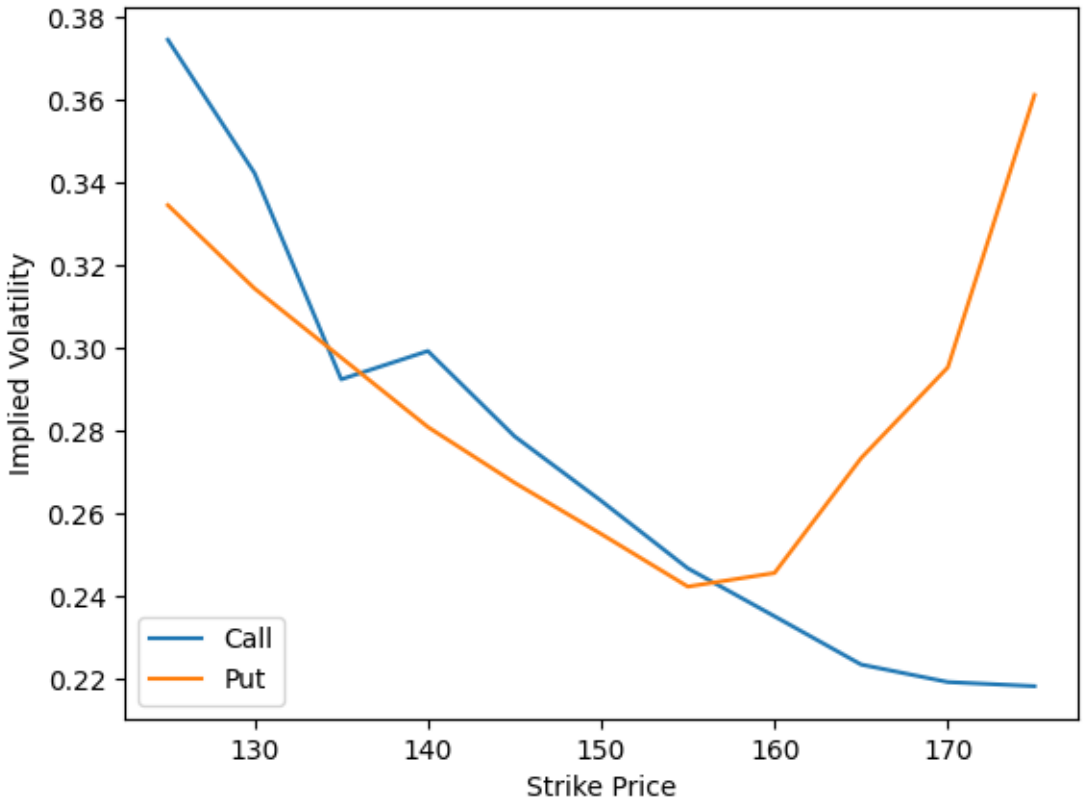
Calculate the implied volatility for each option.

Plot the implied volatility vs the strike price for Puts and Calls. Discuss the shape of these graphs. What market dynamics could make these graphs?

There are bonus points available on this question based on your discussion. Take some time to research if needed.

## Analysis

I used the method that is like the root finder in Julia to find implied volatilities and plotted the implied volatility vs the strike price for Puts and Calls.




It can be clearly seen how the implied volatility changes as strike price changes. The shape of this curve can be affected by many market dynamics, such as Market sentiment, Economic data and events, Market liquidity, Options demand and supply. For call option, as strike price increase, implied volatility decreases. It is usually due to an increase in market risk aversion. In such a scenario, investors tend to prefer low-risk options, such as call options with low strike prices, over high-risk options, such as call options with high strike prices. As a result, the demand for call options with low strike prices increases, causing their prices to rise, while the demand for call options with high strike prices decreases, causing their prices to fall, which in turn leads to a monotonically decreasing implied volatility. Additionally, a decrease in the market's expectation of future volatility may also contribute to a monotonically decreasing implied volatility of call options.

While for put option, as strike price increase, implied volatility decreases first and increases then. The pattern of put is called Volatility Smile. When the market's expectation of future volatility increases, so does the implied volatility. In some cases, as the strike price increases, the market's expectation of future volatility also increases, leading to an increase in implied volatility with the strike price.

There are several factors in the market that can cause Volatility Smile:

Changes in risk aversion: The degree of risk aversion among investors can affect the prices and implied volatility of options. In times of high market risk, investors are more

likely to purchase higher strike call options and lower strike put options, which can result in higher implied volatility for these options and the phenomenon of volatility smile.

Market expectations of future volatility: Market expectations of future volatility can also impact implied volatility. When investors expect higher market volatility in the future, they are more likely to purchase higher strike call options and lower strike put options, which can result in higher implied volatility for these options and the phenomenon of volatility smile.

Impact of asymmetric information: Some investors in the market may have more or more accurate information than other investors. This information can lead to changes in the demand or supply of certain options in the market, thereby affecting the implied volatility of these options. Especially in times of high market risk, investors are more likely to be concerned about uncertain information, increasing the demand for lower strike put options and resulting in higher implied volatility for these options and the phenomenon of volatility smile.

# Problem 3

Use the portfolios found in problem3.csv

- Current AAPL price is 151.03
- Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

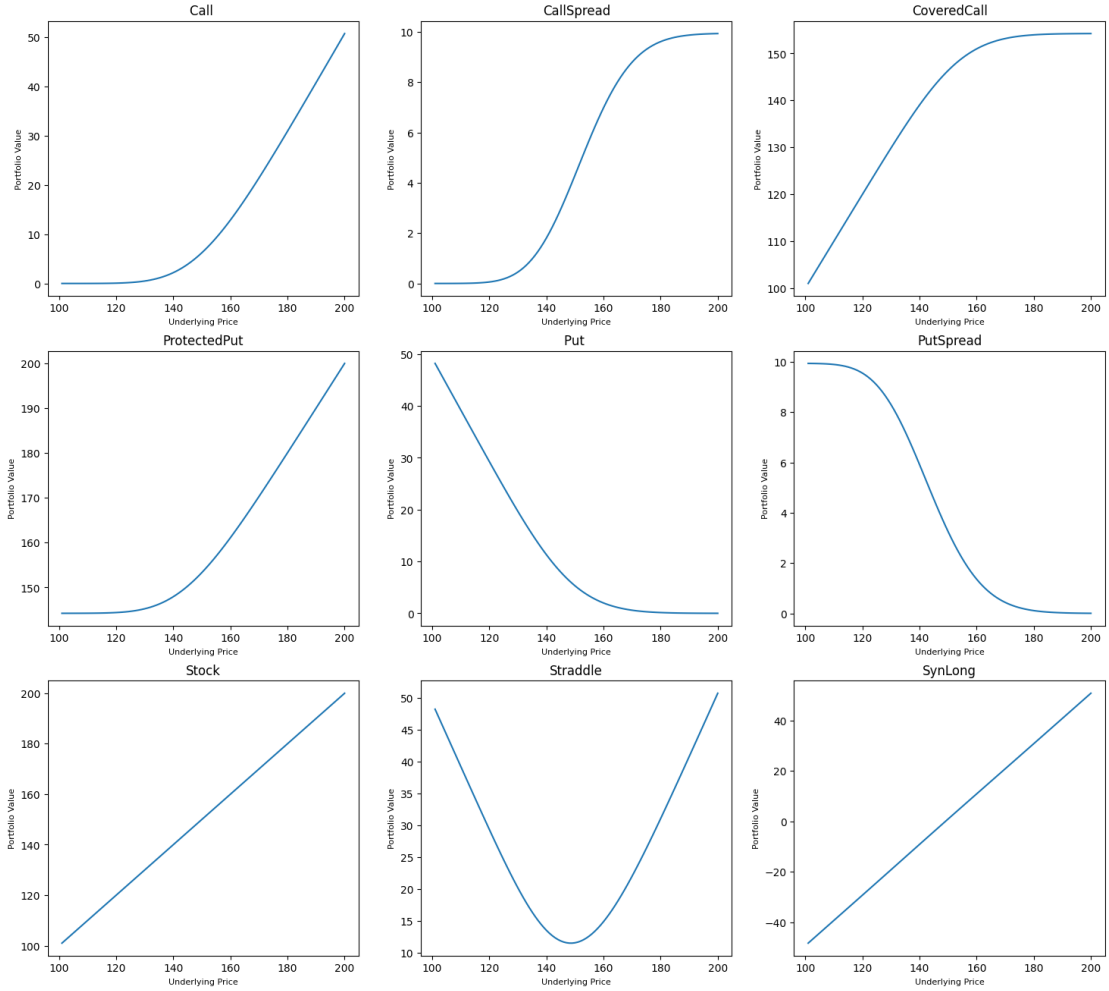
For each of the portfolios, graph the portfolio value over a range of underlying values. Plot the portfolio values and discuss the shapes. Bonus points available for tying these graphs to other topics discussed in the lecture.

Using DailyPrices.csv. Calculate the log returns of AAPL. Demean the series so there is 0 mean. Fit an AR(1) model to AAPL returns. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES. Discuss.

Hints:
- you will need to calculate the implied volatility - might not be the same as #2
- you need to take into account the change in dates for option valuations. You are simulating forward in time and options valuations are a function of time
- Calculate the PL from the current portfolio value using Current Dat

# Analysis

Since current AAPL price is 151.03, I chose the range of 101 – 200 for underlying prices and plotted portfolio value versus underlying price for each portfolio.




Call: The value of a call option increases slowly at first, then linearly as the price of the underlying asset increases.

CallSpread: The value of a call spread increases slowly at first, then linearly, and then slowly again as the price of the underlying asset increases.

CoveredCall: The value of a covered call option increases linearly at first, and then slowly as the price of the underlying asset increases.

Put: The value of a put option decreases linearly at first, and then slowly as the price of the underlying asset increases.

ProtectedPut: The value of a protected put option increases slowly at first, and then linearly as the price of the underlying asset increases.

PutSpread: The value of a put spread decreases slowly at first, then linearly, and then slowly again as the price of the underlying asset increases.

Stock: The value of a stock is positively linear with the price of the underlying asset.

Straddle: The value of a straddle option decreases linearly at first, then increases as the price of the underlying asset increases.

Synlong: The value of a synthetic long position can be negative and is positively linear with the price of the underlying asset.

After simulating returns 10 days ahead:

| Portfolio | Mean | VaR | ES |
|---|---|---|---|
| Call | 1.438734 | 6.748684 | 6.102715 |
| CallSpread | 0.094138 | 4.539950 | 3.963676 |
| CoveredCall | -1.097706 | 27.895374 | 15.950549 |
| ProtectedPut | 1.388067 | 9.517721 | 8.378670 |
| Put | 1.116069 | 4.834178 | 4.442612 |
| PutSpread | 0.302948 | 2.997564 | 2.710433 |
| Stock | 0.380893 | 31.937077 | 19.767668 |
| Straddle | 2.554803 | 0.148441 | 0.144970 |
| SynLong | 0.322664 | 32.226231 | 19.954103 |

For Call and Put: These are basic options that make money as the stock price goes up or down. The risks are moderate.

For CallSpread and PutSpread: These are options strategies that add a limit to the basic Call and Put options, lowering the risk by sacrificing potential returns.

For CoveredCall and ProtectedPut: These strategies involve a mix of stocks and options, and have similar patterns to Call and Put options. The risks are also between those of stocks and options.

For SynLong and Stock: SynLong has the same straightforward pattern as Stock, but with double returns. Both have high risks.

For Straddle: This is a special portfolio that can make money as long as the stock price moves, regardless of the direction. The risk is low compared to the other portfolios.