Date March 22, 2022

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Registration no : FA20-BCS-027
Subject : Linear Algebra
Submitted To : Sir Umair Umer.

Assignment # 1.

Question # 10:

[M] The chemical reaction below can be used in industries processes such as production of arsene (AsH_3). Use exact arithematic or rational format for calculations To balance this equation.

$$MnS + As_2Cr_{10}O_{35} + H_2SO_4 \longrightarrow HMnO_4 + AsH_3 + CrS_3O_{12} + H_2O.$$

Solution:

manganese:	1		0		0			1
sulphur :	1		0		1			0
arsenic : x_1	0	$+x_2$	2	$+x_3$	0	$=$	x_4	0
chromium:	0		10		0			0
oxygen :	0		35		4			4
hydrogen :	0		0		2			1

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$$+x_5\begin{bmatrix}0\\0\\1\\0\\0\\3\end{bmatrix}+x_6\begin{bmatrix}0\\3\\0\\1\\12\\0\end{bmatrix}+x_7\begin{bmatrix}0\\0\\0\\0\\1\\2\end{bmatrix}$$

In rational format, the general solution is $x_1 = (16/327)x_7$

$x_2 = (13/327)x_7$, $x_3 = (374/327)x_7$

$x_4 = (16/327)x_7$, $x_5 = (26/327)x_7$

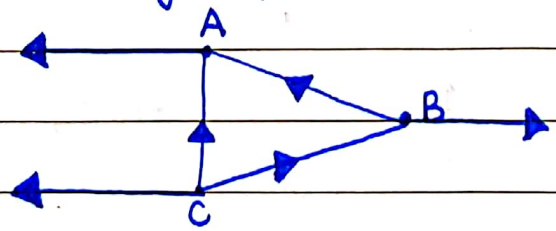
$x_6 = (130/327)x_7$, x_7 is free.

Hence; Balanced Equation is:

$$16MnS + 13As_2Cr_{10}O_{35} + 374H_2SO_4 \longrightarrow 16MnO_4 + 26AsH_3 + 130CrS_3O_{12} + 327H_2O$$

Question #11:

Find general flow pattern of the network shown in figure. Assuming that flows are all non-negative, what is the longest possible values for x_3?



Solution:

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Node	Flow in		Flow out
A	$x_1 + x_3$	=	20
B	x_2	=	$x_3 + x_4$
C	80	=	$x_1 + x_2$
Total Flows:	80	=	$x_4 + 20$

Arranging Equations:

$$x_1 + x_3 = 20$$
$$x_2 - x_3 - x_4 = 0$$
$$x_1 + x_2 = 80$$
$$x_4 = 60$$

Augmented Matrix:

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 80 \\ 0 & 0 & 0 & 1 & 60 \end{array}\right]$$

$\Rightarrow$ R3 − R1 & replace R3

$$\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 80 \\ \ominus 1 & 0 & \oplus 1 & 0 & \ominus 20 \\ \hline 0 & 1 & -1 & 0 & 60 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \end{array}\right]$$

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$R3 - R2$ & replace R3.

$$\begin{array}{ccccc} 0 & 1 & -1 & 0 & 60 \\ 0 & 1 & -1 & -1 & 0 \\ \hline 0 & 0 & 0 & +1 & 60 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & +1 & 60 \\ 0 & 0 & 0 & 1 & 60 \end{bmatrix}$$

$R3 + R2$ & replace R2.

$$\begin{array}{ccccc} 0 & 1 & -1 & 0 & 60 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 1 & 60 \end{bmatrix}$$

$R3 - R4$ & replace R4.

$$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{bmatrix} \textcircled{1} & 0 & 1 & 0 & 20 \\ 0 & \textcircled{1} & -1 & 0 & 60 \\ 0 & 0 & 0 & \textcircled{1} & 60 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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So;

$$\begin{cases} x_1 = 20 - x_3 \\ x_2 = 60 + x_3 \\ x_3 \text{ is free} \\ x_4 = 60 \end{cases}$$

Since; x_1 cannot be negative.

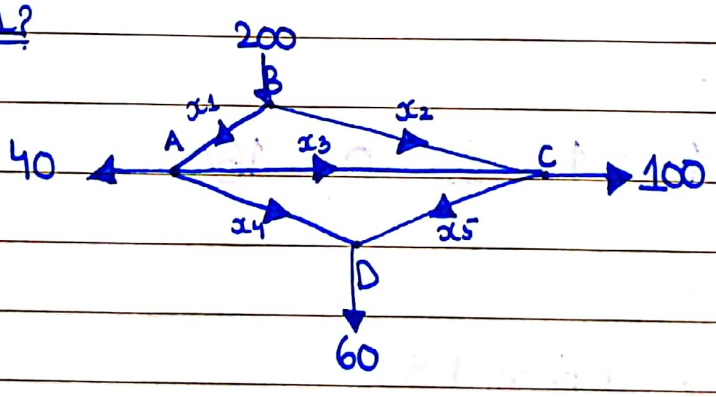
The largest value of x_3 is 20.

Question #12:

(a) Find the general Traffic pattern in subway network show in figure.

(b) Describe the general Traffic pattern when the road whose flow is x_4 is closed.

(c) When $x_4 = 0$, what is minimum value of x_1?



Intersection		Flow in		Flow out
A	:	x_1	=	$x_3 + x_4 + 40$
B	:	200	=	$x_1 + x_2$
C	:	$x_2 + x_3$	=	$x_5 + 100$
D	:	$x_4 + x_5$	=	60
Total Flow	:	200	=	$40 + 100 + 60 = 200$

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Arranging Equations:

$$x_1 + x_3 - x_4 = 40$$
$$x_1 + x_2 = 200$$
$$x_2 + x_3 - x_5 = 100$$
$$x_4 + x_5 = 60$$

Augmented Matrix:

$$\left[\begin{array}{ccccc:c} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array}\right]$$

R2 − R1 ⇒ replace R2.

$$\begin{array}{cccccc} 1 & 1 & 0 & 0 & 0 & 200 \\ 1 & 0 & -1 & -1 & 0 & 40 \\ \hline 0 & 1 & 1 & 1 & 0 & 160 \end{array}$$

$$\left[\begin{array}{ccccc:c} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array}\right]$$

R3 − R2 ⇒ replace R3.

$$\begin{array}{cccccc} 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ \hline 0 & 0 & 0 & -1 & -1 & -60 \end{array}$$

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$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

R3 + R4 & replace R4. & xing R3 by "-1".

0 0 0 0 0 0

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & +1 & +1 & +60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

R2-R3 & replace R2.

0 1 1 1 0 160

0 0 0 ±1 ±1 ±60

0 1 1 0 -1 100

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

R3+R1 & replace R1.

1 0 -1 0 1 100

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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General Solution:

$$\begin{cases} x_1 = 100 + x_3 - x_5 \\ x_2 = 100 - x_3 + x_5 \\ x_3 \text{ is free} \\ x_4 = 60 - x_5 \\ x_5 \text{ is free} \end{cases}$$

Part b:

When $x_4 = 0$,

x_5 must be 60.

Then;

$$\begin{cases} x_1 = 40 + x_3 \\ x_2 = 160 - x_3 \\ x_3 \text{ is free} \\ x_4 = 0 \\ x_5 = 60 \end{cases}$$

Part c:

Minimum value of x_1 is 40 cars/minute, because x_3 cannot be negative.

Question #13:

(a) Find the general flow pattern in network shown in figure.

(b) Assuming that flow must be in branches denoted by x_2, x_3, x_4, x_5.

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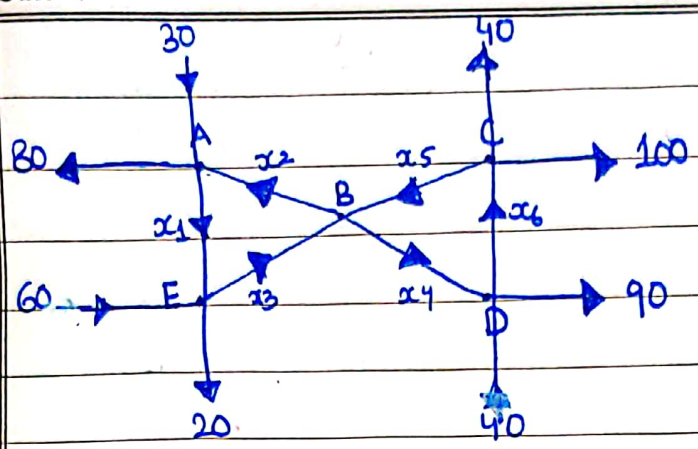

Solution:

Intersection	Flow in		Flow out.
A	$x_2 + 30$	=	$x_1 + 80$
B	$x_3 + x_5$	=	$x_2 + x_4$
C	$x_6 + 100$	=	$x_5 + 40$
D	$x_4 + 40$	=	$x_6 + 90$
E	$x_1 + 60$	=	$x_3 + 20$
Total Flow:	230	=	230

Arranged Equation:

$$x_1 - x_2 = -50$$
$$x_2 - x_3 + x_4 - x_5 = 0$$
$$x_5 - x_6 = 60$$
$$x_4 - x_6 = 50$$
$$x_1 - x_3 = -40$$

Augmented Matrix:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 1 & 0 & -1 & 0 & 0 & 0 & -40 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \textcircled{1} & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & \textcircled{1} & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & \textcircled{1} & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & \textcircled{1} & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Part a:

General Solution:

$$\begin{cases} x_1 = x_3 - 40 \\ x_2 = x_3 + 10 \\ x_3 \text{ is free} \\ x_4 = x_6 + 50 \\ x_5 = x_6 + 60 \\ x_6 \text{ is free} \end{cases}$$

Part b:

To find minimum flow x_1 cannot be negative, $x_3 > 40$. This implies that $x_2 \geq 50$. Also x_6 cannot be negative $x_4 \geq 50$, $x_5 \geq 60$

Minimum flows are $x_2 = 50$, $x_3 = 40$, $x_4 = 50$, $x_5 = 60$ (when $x_1 = 0$ & $x_6 = 0$)

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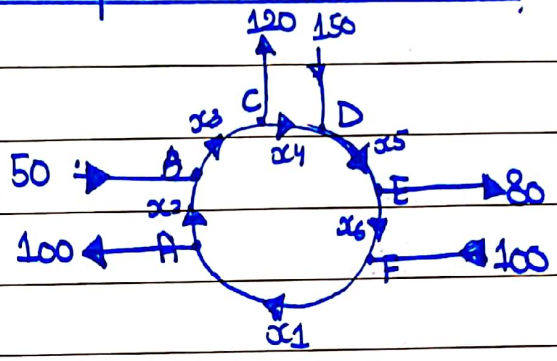
Question # 14:

Intersections in englands are often constructed as one-way "roundabouts". Assume that traffic must travel in shown directions. Find general solution and smallest possible value of x_6?



Solution

Intersection	Flow in		Flow out
A	x_1	=	$x_2 + 100$
B	$x_2 + 50$	=	x_3
C	x_3	=	$x_4 + 120$
D	$x_4 + 150$	=	x_5
E	x_5	=	$x_6 + 80$
F	$x_6 + 100$	=	x_1

Arranged Equations:

$$x_1 - x_2 = 100$$
$$x_2 - x_3 = -50$$
$$x_3 - x_4 = 120$$
$$x_4 - x_5 = -150$$
$$x_5 - x_6 = 80$$
$$x_6 = -100$$

$-x_1$

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Augmented Form.

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ -1 & 0 & 0 & 0 & 0 & 1 & -100 \end{array}\right]$$

⇒ R1+R6 & replace R6.
⇒ R1+R2 & replace R1.
⇒ R2+R3 & replace R2.
⇒ R3+R4 & replace R3
⇒ R4+R5 & replace R4
⇒ R5+R6 & replace R5.

Hence, we get

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 50 \\ 0 & 0 & 0 & 1 & 0 & 0 & -70 \\ 0 & 0 & 0 & 0 & 1 & 0 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

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General Solution:

$$\begin{cases} x_1 = 100 + x_6 \\ x_2 = x_6 \\ x_3 = 50 + x_6 \\ x_4 = -70 + x_6 \\ x_5 = 80 + x_6 \\ x_6 \text{ is free} \end{cases}$$

Since, x_4 cannot be negative.

Minimum value of x_6 is 70. Ans*

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LA Assignment
Question No 6

$$\begin{bmatrix} S \\ P \\ O \\ B \\ N \end{bmatrix} = x_1 \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -2 \\ 8 \\ -3 \\ 0 \end{bmatrix}$$

$$+x_4 \begin{bmatrix} -1 \\ 0 \\ -3 \\ 0 \\ -1 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -2 \\ 8 \\ -3 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 0 \\ -3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Augmented Matrix

$$\begin{bmatrix} 3 & 0 & -2 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 4 & 6 & 8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \end{bmatrix}$$

Exchange R_1 with R_2