

Assignment # 01

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Reg No : FA20-BCS-069

Subject : Linear Algebra

Section : BCS-4A

Questions : 6, 7, 8, 9.

Submitted to
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Date: _____

LA Assignment Question No 6

$$\begin{bmatrix} S \\ P \\ D \\ B \\ N \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \\ 8 \\ -3 \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} -1 \\ 0 \\ -3 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \\ 8 \\ -3 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ -3 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Augmented Matrix

$$\begin{bmatrix} 3 & 0 & -2 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 4 & 6 & 8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \end{bmatrix}$$

Exchange R_1 with R_2

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$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 6 & 8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \end{array} \right]$$

Add R_1 with R_2

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 6 & 8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \end{array} \right]$$

Add $-3R_1$ to R_2

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 4 & 6 & 8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \end{array} \right]$$

Exchange R_2 with R_4

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 6 & 0 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 2 & 0 & 1 & 0 \end{array} \right]$$

Add $-6R_2$ to R_3

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 18 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 0 & 6 & -1 & 0 \end{array} \right]$$

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Add $-2R_2$ to R_1 .

$$\begin{bmatrix} 1 & 0 & -2 & 0 & = & 0 \\ 0 & 1 & -3 & 0 & = & 0 \\ 0 & 0 & 18 & -3 & = & 0 \\ 0 & 0 & 6 & -1 & = & 0 \\ 0 & 0 & 6 & -1 & = & 0 \end{bmatrix}$$

Scaling R_3 by $1/18$

$$\begin{bmatrix} 1 & 0 & -2 & 0 & = & 0 \\ 0 & 1 & -3 & 0 & = & 0 \\ 0 & 0 & 1 & -1/6 & = & 0 \\ 0 & 0 & 6 & -1 & = & 0 \\ 0 & 0 & 6 & -1 & = & 0 \end{bmatrix}$$

Add $-1R_3$ to R_4

$$\begin{bmatrix} 1 & 0 & -2 & 0 & = & 1 \\ 0 & 1 & -3 & 0 & = & 1 \\ 0 & 0 & 1 & -1/6 & = & 1 \\ 0 & 0 & 0 & 0 & = & 1 \\ 0 & 0 & 6 & 1 & = & 1 \end{bmatrix}$$

Add $-6R_3$ to R_5 .

$$\begin{bmatrix} 1 & 0 & -2 & 0 & = & 0 \\ 0 & 1 & -3 & 0 & = & 0 \\ 0 & 0 & 1 & -1/6 & = & 0 \\ 0 & 0 & 0 & 0 & = & 0 \\ 0 & 0 & 0 & 0 & = & 0 \end{bmatrix}$$

Add $3R_3$ to R_2 ; Add $2R_3$ to R_1

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & = & 0 \\ 0 & 1 & 0 & -1/2 & = & 0 \\ 0 & 0 & 1 & -1/6 & = & 0 \\ 0 & 0 & 0 & 0 & = & 0 \\ 0 & 0 & 0 & 0 & = & 0 \end{bmatrix}$$

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General Solution

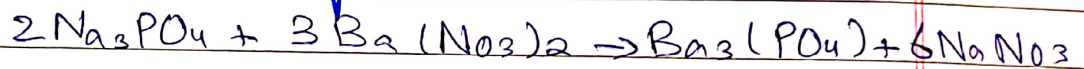
$$\Rightarrow x_1 = \frac{1}{3} x_4 \rightarrow x_1 = 2$$

$$x_2 = 2x_4 \rightarrow x_2 = 3$$

$$x_3 = \frac{2}{3} x_4 \rightarrow x_3 = 1$$

x_4 is a free variable
Taking $x_4 = 3$

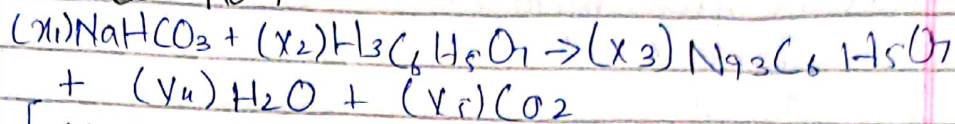
Balanced equation:-



Question 7

Solution:-

Question No 7



$$\begin{bmatrix} \text{Na} \\ \text{H} \\ \text{C} \\ \text{O} \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 8 \\ 6 \\ 7 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -5 \\ -6 \\ -7 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -2 \\ 0 \\ -1 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ -1 \\ -2 \end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 1 & 8 & -5 & -2 & 0 & 0 \\ 1 & 6 & -6 & 0 & -1 & 0 \\ 3 & 7 & -7 & -1 & -2 & 0 \end{bmatrix}$$

$$R_2 = R_2 - R_1; R_3 = R_3 - R_1; R_4 = R_4 - 3R_1$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 6 & -3 & 0 & -1 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 \end{bmatrix}$$

→ add $-6R_2$ to R_3 , $-7R_2$ to R_4 ; Scaling R_2 by $2/3$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 15/4 & 3/4 & -2 & 0 \end{bmatrix}$$

Add $-15/4 R_3$ to R_4

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 9/2 & -9/2 & 0 \end{bmatrix}$$

→ General Solution

$$x_1 = 13x_3$$

$$x_2 = \frac{1}{4}x_3 + \frac{1}{4}x_4$$

$$x_3 = \frac{1}{3}x_4 + \frac{2}{3}x_5$$

$$\frac{9}{2}x_4 = \frac{9}{2}x_3$$

x_5 is free Variable

Take $x_5 = 3$

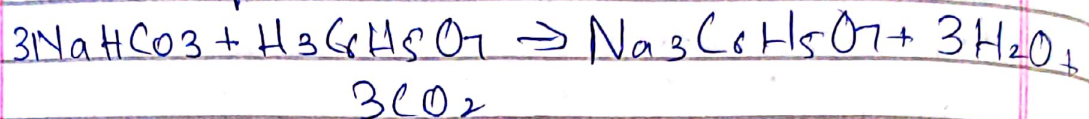
$$x_1 = x_5 = 3$$

$$x_2 = 1$$

$$x_3 = 1$$

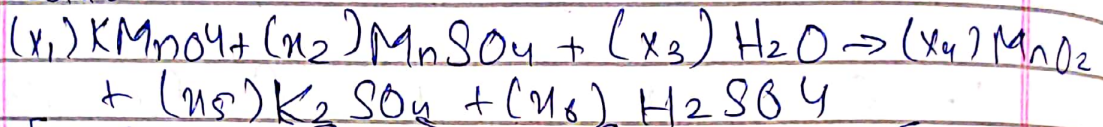
$$x_4 = 3$$

Balanced equation



Question No 8

Solution



$$\begin{bmatrix} \text{K} \\ \text{M} \\ \text{O} \\ \text{n} \\ \text{H} \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}$$

$$+ x_4 \begin{bmatrix} 0 \\ -1 \\ -2 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -2 \\ 0 \\ 4 \\ -1 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 0 \\ 0 \\ -4 \\ -1 \\ -2 \end{bmatrix}$$

Augmented Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 & =0 \\ 1 & 1 & 0 & -1 & 0 & 0 & 0 & =0 \\ 4 & 4 & 1 & -2 & -4 & -4 & -4 & =0 \\ 0 & 1 & 0 & 0 & -1 & -1 & -1 & =0 \\ 0 & 0 & 2 & 0 & 0 & 0 & -2 & =0 \end{bmatrix}$$

Adding $-1R_1$ to R_2 ; $-4R_1$ to R_3

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 4 & 1 & -2 & 4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{bmatrix}$$

Adding $-4R_2$ to R_3 ; $-1R_2$ to R_4

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{bmatrix}$$

Adding $-2R_3$ to R_5 ; R_4 to R_2

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & -4 & 8 & 6 & 0 \end{bmatrix}$$

Adding $-2R_4$ to R_5 ; $4R_4$ to R_3

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & -4 & 2 & 0 \end{bmatrix}$$

Scaling R_5 by $-1/4$; Adding $3R_5$ to R_4

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & -2.5 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.5 & 0 \end{bmatrix}$$

Adding $-2R_2$ to R_3 ; R_5 to R_2 , $2R_2$ to R_1

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1.0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1.5 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1.0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -2.5 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 & 0 \end{bmatrix}$$

4.5

$$n_1 = n_6$$

$$n_2 = (1.5)n_6$$

$$n_3 = n_6$$

$$n_4 = (2.5)n_6$$

$$n_5 = (0.5)n_6$$

n_6 is free variable;

$$\text{Take } n_6 = 2$$

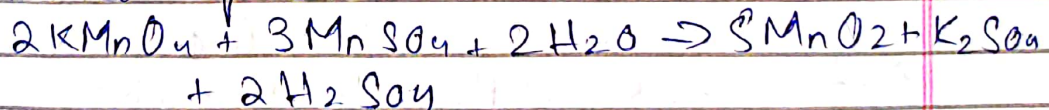
$$n_1 = n_3 = 2 \quad - \text{ i}$$

$$n_2 = 3 \quad - \text{ ii}$$

$$n_4 = 5 \quad - \text{ iii}$$

$$n_5 = 1 \quad - \text{ iv}$$

Balanced equation



Date: _____

Question 9

Solution

$$\begin{array}{c} \text{Pb} \\ \text{N} \\ \text{Cr} \\ \text{Mn} \\ 0 \end{array} = x_1 \begin{bmatrix} 1 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 8 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ -3 \end{bmatrix}$$

$$+ x_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ -2 \end{bmatrix} + x_6 \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

Augmented matrix

$$\left[\begin{array}{cccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{array} \right]$$

$$-6R_1 + R_2 \rightarrow R_2$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{array} \right]$$

Replace R_2 with R_3

$$\left[\begin{array}{cccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{array} \right]$$

Date: _____

$$-2R_2 + R_4 \rightarrow R_4$$

$$-3R_2 + R_5 \rightarrow R_5$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & = 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & = 0 \\ 0 & 0 & 18 & 0 & 0 & -1 & = 0 \\ 0 & 0 & 0 & 4 & -1 & 0 & = 0 \\ 0 & 0 & -4 & 13 & -2 & -1 & = 0 \end{bmatrix}$$

Scaling R_3 by $1/18$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & = 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & = 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & = 0 \\ 0 & 0 & 0 & 4 & -1 & 0 & = 0 \\ 0 & 0 & 0 & 13 & -2 & -1/9 & = 0 \end{bmatrix}$$

$-13R_4 + R_5$; Scaling R_4 by $1/4$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & = 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & = 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & = 0 \\ 0 & 0 & 0 & 1 & -1/4 & 0 & = 0 \\ 0 & 0 & 0 & 0 & 1 & -11/36 & = 0 \end{bmatrix}$$

$1/4 R_5 + R_4 \rightarrow R_4$; $2R_4 + R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & = 0 \\ 0 & 1 & 0 & 0 & 0 & -22/45 & = 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & = 0 \\ 0 & 0 & 0 & 1 & 0 & -11/36 & = 0 \\ 0 & 0 & 0 & 0 & 1 & -11/36 & = 0 \end{bmatrix}$$

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$$3R_3 + R_1 \rightarrow R_1$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -9/6 & = 0 \\ 0 & 1 & 0 & 0 & 0 & -22/48 & = 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & = 0 \\ 0 & 0 & 0 & 1 & 0 & -11/48 & = 0 \\ 0 & 0 & 0 & 0 & 1 & -44/48 & = 0 \end{array} \right]$$

General Solution

$$x_1 = \frac{6}{6} (n_6)$$

$$x_2 = \frac{22}{48} (n_6)$$

$$x_3 = \frac{1}{18} (n_6)$$

$$x_4 = \frac{11}{48} (n_6)$$

$$x_5 = \frac{44}{48} (n_6)$$

n_6 is free

$$n_1 = 18$$

$$n_2 = 44$$

$$n_3 = 5$$

$$n_4 = 22$$

$$n_5 = 88$$

$$n_6 = 90$$

Balanced eq

