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QUESTION 21:

By hypothesis, there exists P and Q that $P^{-1}GP = A$ and $Q^{-1}CB = A$.

$\rightarrow P^{-1}BP = Q^{-1}CQ$

$QP^{-1}BPQ^{-1} = QQ^{-1}CQQ^{-1}$

$C = QP^{-1}BPQ^{-1} = (PQ^{-1})^{-1}B(PQ^{-1})$

This shows that B is similar to C.

QUESTION 22:

If A is diagonizable, then $A = PDP^{-1}$ for some P.
If B is similar to A, then $B = QAQ^{-1}$ for some Q.

$\rightarrow B = Q(PDP^{-1})Q^{-1}$
$= (QP)P(P^{-1}Q^{-1}) = (QP)D(QP)^{-1}$

$\therefore$ B is diagonilazble.

QUESTION 23:

If $Ax = \lambda x$, then $P^{-1}Ax = \lambda P^{-1}x$
if $B = P^{-1}AP$, then $B(P^{-1}x) = P^{-1}AP(P^{-1}x) = P^{-1}Ax = \lambda P^{-1}x$
$P^{-1}x \neq 0$ because $x \neq 0$ and P^{-1} is invertable.

Hence (1) shows that $P^{-1}x$ is an eigen vector of B.

QUESTION 24:

If $A = PBP^{-1}$, then rank A = rank $P(BP^{-1})$ = rank BP^{-1} rank BP^{-1} = rank B, since P^{-1} is invertible $\therefore$ rank A = rank B.

QUESTION 25:

If $A = PBP^{-1}$ then $tr(A) = tr((PB)P^{-1}) = tr(P^{-1}PB)$
$= tr(IB) \Rightarrow tr(B)$

if B is diagonal, then diagonal entries of B must be eigenvalues of A.

QUESTION 26:

If $A = PDP^{-1}$ for some P, then $trA = tr[(PP)P^{-1}] = trD$.
since eigen values of A are on the main diagonal of D trD is the sum of eigen values of A.

QUESTION 27:

for each j, $I(b_j) = b_j$ since standard cordinate vector of any vector in $\mathbb{R}^n$ is itself, $\rightarrow$ matrix for I relative to B and E is $(b_1, b_2\ b_3 \dots b_n)$.
This matrix is precisely change of cordinates matrix.

QUESTION 28:

For each j, $I(b_j) = b_j$ and $[I(b_j)]_C = [b_j]_C$.

A matrix for I relative to B and C is $[[b_1]_C [b_2]_C \dots [b_n]_C]$.

This is called change of cordinates matrix from B to C.

QUESTION 29:

If $B = \{b_1, b_2, b_3 \dots b_n\}$, then B-cordinate vector of B_j is e_j. Thus, $[I(b_j)]_B = [B_j]_B = e_j$ and,
$[I]_B = [[I(b_1)]_B \dots [I(b_n)]_B] =$
$[e_1, e_2, \dots e_n] = I$

THANKS