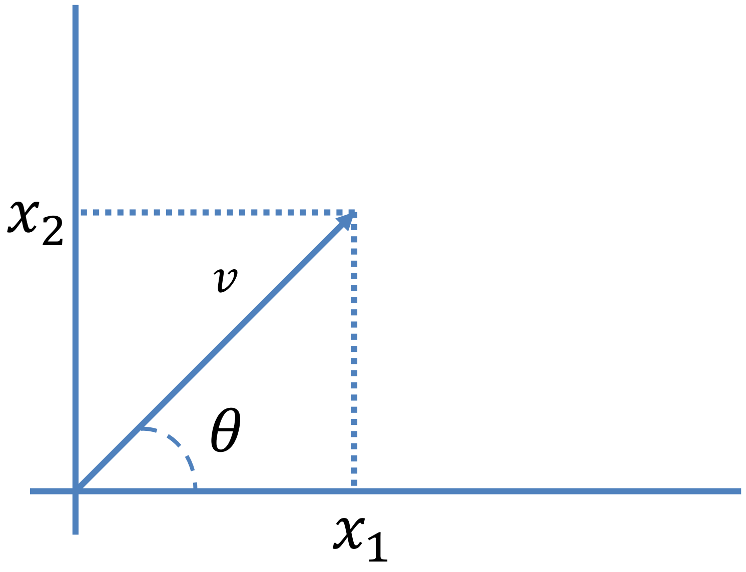
Vector review

$$v = (x_1, x_2)$$

Magnitude $$||v|| = \sqrt{x_1^2 + x_2^2}$$

Orientation: $$\theta = tan^{-1}\left(\frac{x_2}{x_1}\right)$$

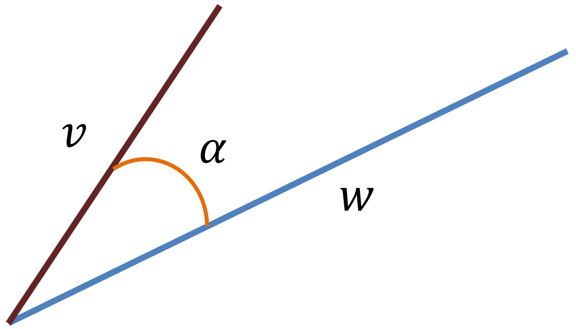


Dot product

$$v \cdot w = (x_1, x_2) \cdot (y_1, y_2) = x_1 y_1 + x_2 y_2$$

The inner product is a scalar

$$v \cdot w = (x_1, x_2) \cdot (y_1, y_2) = ||v|| \cdot ||w|| \ \cos \alpha$$



Cross product

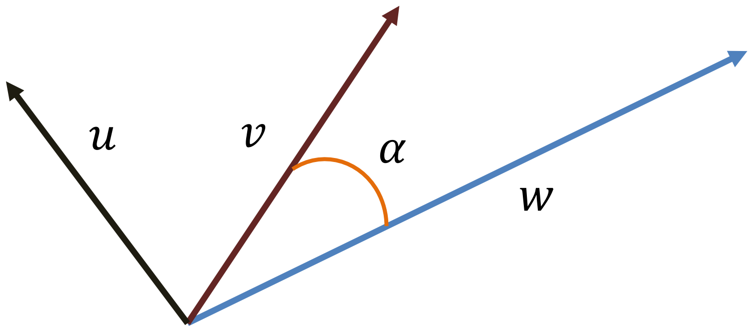
$$u = v \times w$$

Magnitude: $\|u\| = \|v \times w\| = \|v\| \|w\| \sin \alpha$

Orientation:

$$u \perp v \Rightarrow u \cdot v = (v \times w) \cdot v = 0$$

$$u \perp w \Rightarrow u \cdot w = (v \times w) \cdot w = 0$$



Matrix review

$$A_{n\times m} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & \boxed{a_{nm}} \end{bmatrix} \longleftrightarrow$$

Pixel's intensity value

Sum: $C_{n\times m} = A_{n\times m} + B_{n\times m}$ $\qquad c_{ij} = a_{ij} + b_{ij}$

Matrix multiplication

$$A_{n\times m} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix} \quad \mathbf{a}_i \qquad B_{m\times p} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1p} \\ b_{21} & b_{22} & \dots & b_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mp} \end{bmatrix}$$

$$\mathbf{b}_j$$

Product:

$$C_{n\times p} = A_{n\times \boxed{m}} B_{\boxed{m}\times p}$$

$$c_{ij} = \mathbf{a}_i \cdot \mathbf{b}_j = \sum_{k=1}^{m} a_{ik} b_{kj}$$

Matrix determinant

$$\det \begin{bmatrix} a_{11} \end{bmatrix} = a_{11}$$

$$\det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$\det \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{23}a_{32}a_{11} - a_{33}a_{12}a_{21}$$

Matrix inverse

$$AA^{-1} = A^{-1}A = I$$

$$A^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{13} & a_{14} \end{bmatrix}^{-1} = \frac{1}{\det A} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}, \det A \neq 0$$

$$If \det A = 0, A\ does\ not\ have\ an\ inverse$$

Matrix eigenvalues and Eigenvectors

$A\ eigenvalue\ \lambda\ and\ eigenvector\ u\ satisfies$

$$Au = \lambda u$$

$where\ A\ is\ a\ square\ matrix$

$Multiplying\ u\ by\ A, scales\ u\ by\ \lambda$

Rearranging the previous equation gives the system

$$Au\ - \lambda u = (A - \lambda I)u = 0$$

$which\ has\ a\ solution\ if\ and\ only\ if\ \det(A - \lambda I) = 0$

The eigenvalues are the roots of this determinant which is polynomial in λ

Substitute the resulting eigenvalues back into $Au = \lambda u$ and solve to obtain the corresponding eigenvector

Matrix eigenvalues and Eigenvectors