Mechanical waves:Objectives

- At the end of this lesson, you will be capable of
    - At the end of the course you will be capable of Explain the difference between a mechanical wave and an electromagnetic wave
    - Derive a wave equation
    - Recall the properties of the wave: frequency, wave number, wavelength, angular speed, velocity
    - State the huygen's principle
    - Explain the interference and its application
    - Explain the interference patterns for the double-slit Young experiment
    - Explain diffraction phenomena and its applications

Introduction: Mechanical waves

- A wave is a propagation of disturbances or energy produced at a given point and propagate from one point to another without transport of matter.
- **Mechanical waves** are those which can only propagate in medium but fail to propagate in vacuum. They are mainly produced by repeatedly displacement of a section of a medium. so they are due to motion of particles
- here we are focusing on those waves propagating in deformable medium (or elastic medium).
- The disturbance (deformation) produced travels through the medium, from one section to the closer one, putting it in the same vibration but at successively later time. We say that there is **vibration propagation** or simply **a progressive wave**.

Introduction: Electromagnetic waves

- Electromagnetic wave is born when electric charge oscillate in a given section of a conducting material. It is made of propagating coupled Electric and magnetic field
- Electromagnetic waves propagate in maedium and also in vacuum. Electromagnetic wave don't require medium to propagate

- We first focus on **a pulse** traveling through a medium. Consider a pulse created by flicking the end of a string. As the end of the string moves up and down repeatedly, a pulse travels along the string and a traveling wave (**progressive wave**) is then created.

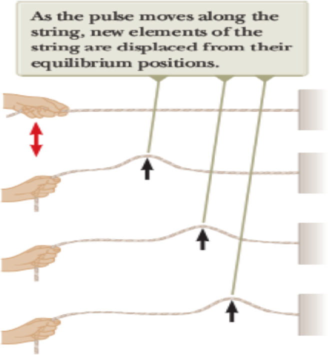
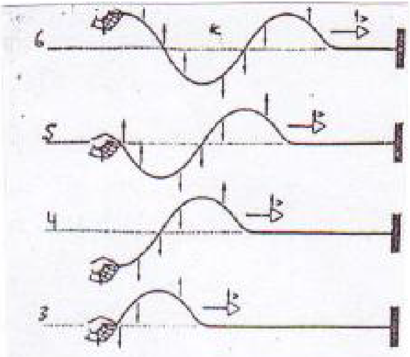


Figure: *Propagation of a disturbance along a stretched string*

Mechanical waves: types of waves

- According to direction of particles' oscillations and the wave propagation direction, mechanical waves can be **transverse** or **longitudinal**.
  A wave is **transverse** when on its passage, the particles of the propagation medium oscillate perpendicular to the wave direction of propagation. One example of transverse wave is the propagation of an upward shake along a stretched rope or string .



Mechanical waves: types of waves

- A wave is **longitudinal** when on its passage, the particles of the propagation medium oscillate in the same direction as the wave direction of propagation. An example is the propagation of a compression–extension along a stretched spring.

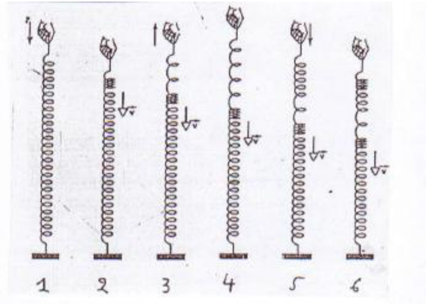


Figure: *Longitudinal wave along a stretched spring*

Mechanical waves: types of waves in space dimensions

- According to the number of space dimensions in which the wave propagates, waves can be:

  One–dimension waves: waves along a stretched rope or spring
  Two–dimension waves: concentric circular waves (crest and valleys) on a liquid surface or on a drum membrane
  Three–dimension waves: sound waves, seismic waves in solids, light waves, electromagnetic waves used in Telecommunication.

Mechanical waves: Note

- When a wave propagates in a medium, the particles of that medium oscillate at the same place, without moving in the wave's direction.
- When a wave propagates in a medium, there is no transport of matter, but only **a transport of energy**.
- The propagation of **mechanical waves** always needs an **elastic material medium**; contrarily to electromagnetic waves that can propagate through empty space (such as radio or television waves, telephone waves, light waves, X rays and γ rays).
- The propagation velocity $\vec{v}$ of a wave is called **the wave speed or the wave celerity**, and it only depends on the properties of the propagation medium.
- It is the elasticity of the propagation medium that produces in each point, a restoring elastic force $\vec{F} = -k\vec{x}$ that tend to restore each medium's section to its equilibrium position.

Consider two photos of the same rope, along which a harmonic wave is traveling. The photos are taken at two different times, separated by a time duration t.

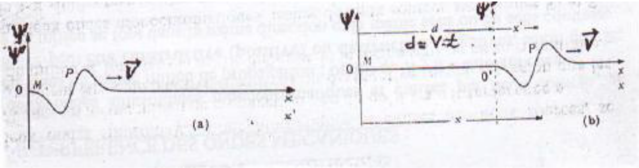


Figure: *Photos of a same harmonic wave along a stretched rope, taken at different times*

Let at time $t_0 = 0$, the physical quantity associated to the wave propagation be described by $\Psi(x, 0) = \Psi(x)$ in the reference frame O. Imagine also another reference frame O' that has the same motion as the wave, but that occupies the position O at time $t_0 = 0$ (e.g.: a deformation of a vibrating rope, a compression of a spring).

Mechanical waves:One–Dimension Wave Function

At any later time t, that physical quantity is characterized by $\Psi(x') = \Psi(x - d)$. And since the two quantities describe a same wave, but delayed in time by t; the one–dimension wave function for a wave traveling in the positive direction of $\overrightarrow{OX}$ is:

$$\Psi(x, t) = \Psi(x') = \Psi(x - d) = \Psi(x - vt) \tag{1}$$

By analogy, the one–dimension wave function for a wave traveling in the negative direction of $\overrightarrow{OX}$ is:

$$\Psi(x, t) = \Psi(x') = \Psi(x + d) = \Psi(x + vt) \tag{2}$$

And since a straight line has two opposite directions, the general one–dimension wave function for a wave traveling along $\overrightarrow{OX}$ axis becomes:

$$\Psi(x, t) = \Psi(x') = \Psi(x \mp vt) \tag{3}$$

For a wave traveling in the positive direction, we use the negative $(-)$ sign, but for a wave traveling in the negative direction we use the positive $(+)$ sign.At a fixed time t (like in the case of taking a snapshot of the pulse), the wave function becomes $\Psi(x)$; it is called **the waveform** and it defines a curve representing the geometric shape of the pulse at that time.

Analysis Model: Traveling Wave

In this section, we introduce an important function $\Psi(x)$ whose shape is sinusoidal. The wave represented by such a curve is called **a sinusoidal wave** or **a harmonic wave**.

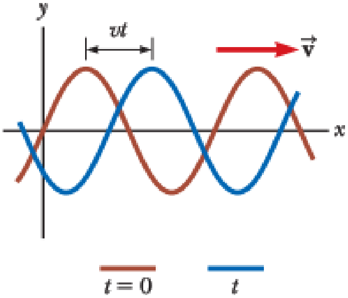


Figure: *One–dimension harmonic wave traveling to the right at a wave velocity v*

Analysis Model: Traveling Wave

A **harmonic** or **sinusoidal** wave is defined as a wave that results from the propagation of a *simple harmonic motion* in a given medium. A harmonic wave is characterized by the following physical quantities:

- The **wave amplitude**: the maximum value that can take the physical quantity associated to the wave.
- The wave period T or **temporal period**: the time duration of one complete vibration; it is expressed in seconds (1 s).
- The wavelength λ or **spatial period**: the distance covered by the wave during one period of time T; it is expressed in meters (1 m).
- The wave frequency ν: the number of vibrations or cycles per second; it is expressed in hertz (1 Hz).

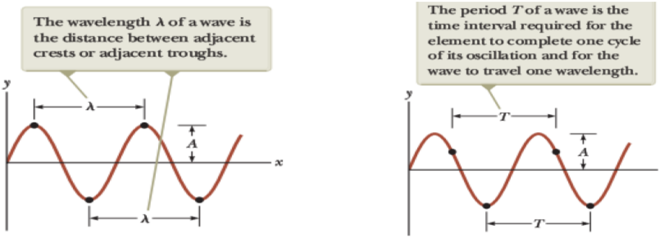


Figure: *Wavelength and period of a harmonic wave*

In general, the wave function of a harmonic wave traveling along $\overrightarrow{OX}$ axis is a sine or cosine function of x and time t given by the physical quantity:

$$\Psi(x,t) = \Psi(x \mp vt) = \Psi_m \sin\left[k\left(x \mp vt\right)\right] \; or \; \Psi(x,t) = \Psi(x \mp vt) = \Psi_m \cos\left[k \tag{4}$$

where Ψ_m is the **harmonic wave amplitude**: the maximum value that can take the physical quantity $\Psi(x,t)$ associated with the wave.

Analysis Model: Traveling Wave

Since the **sine or cosine** function is a periodic function of period 2π, the harmonic wave function $\Psi(x,t)$ that propagates, takes again the same algebraic value and the same variation, when the wave has traveled a distance equal to the wavelength λ ; this occurs after a time that equals to one period T.

$$\Psi(x,t) = \Psi[(x+\lambda),t] = \Psi[x,(t+T)]$$

The harmonic wave function for a one-dimension harmonic wave traveling along $\overrightarrow{OX}$ axis can then be given by one of the following expressions:

$$\begin{aligned}\Psi(x,t) &= \Psi_m \sin\left[k(x+\lambda) \mp kvt\right] = \Psi_m \sin\left[k(x \mp vt) + 2\pi\right] \\ &= \Psi_m \sin\left[kx \mp kv(t+T)\right] = \Psi_m(kx \mp wt)\end{aligned} \tag{5}$$

The wave number k on aa distance equals to 2π meters is defined by:

$$\lambda k = 2\pi \Leftrightarrow k = \frac{2\pi}{\lambda} \tag{6}$$

Putting 6 into 5, the harmonic wave function for a one-dimension harmonic wave travelling along $\overrightarrow{OX}$ axis can also be given by:

$$\begin{aligned}\Psi(x,t) &= \Psi \sin\left(\frac{2\pi}{\lambda}x \mp \frac{2\pi}{\lambda}vt\right)\\ \Rightarrow \Psi(x,t) &= \Psi_m \sin\left[2\pi\left(\frac{x}{\lambda} \mp \frac{t}{T}\right)\right] = \Psi_m \sin(kx \mp \omega t)\end{aligned} \tag{7}$$

or by:

$$\begin{aligned}\Psi(x,t) &= \Psi \cos\left(\frac{2\pi}{\lambda}x \mp \frac{2\pi}{\lambda}vt\right)\\ \Rightarrow \Psi(x,t) &= \Psi_m \cos\left[2\pi\left(\frac{x}{\lambda} \mp \frac{t}{T}\right)\right] = \Psi_m \cos(kx \mp \omega t)\end{aligned} \tag{8}$$

Analysis Model: Traveling Wave

The angular frequency ω of harmonic waves and expressed in radians per second (1 rad/s) becomes:

$$\omega = \frac{2\pi}{\lambda}v = kv = \frac{2\pi}{T} = 2\pi f \tag{9}$$

The relationship between the wavelength λ, the period T and the frequency f of a harmonic wave is given by:

$$\lambda = vT = \frac{v}{f} \tag{10}$$

Speed of Waves on Strings

Consider a string of mass per unit of length (linear mass density μ), stretched by a tension force $\vec{T}$ and a transverse wave that travels along the string at celerity $\vec{v}$.

The different points of the string reached by the wave front, oscillate in the $\overrightarrow{OX}$ direction with a velocity of vibration $\vec{v} = \vec{v}_y$; but during the same time the wave travels in the $\overrightarrow{OX}$ direction at a speed $\vec{v}_x = \vec{v}$

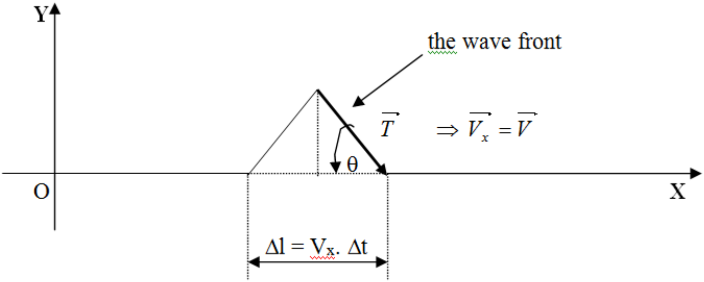


Figure: *Wave front propagation and wave celerity*

Speed of Waves on Strings

The motion of an element mass Δm of the stretched string, reached by the wave front travelling and having an element length Δl, is composed by **two perpendicular motions**: A **uniformly varying rectilinear motion** along the $\overrightarrow{OY}$ axis, with acceleration $\vec{a}_y$:

$$a_y = \frac{T_y}{\Delta m} = \frac{T \sin\theta}{\Delta m} \quad \textit{and} \quad v_y = a_y \Delta t \Rightarrow v_y = \frac{T \sin\theta}{\Delta m} \times \Delta t \qquad (11)$$

A **uniformly varying rectilinear motion** along the $\overrightarrow{OX}$ axis, such as:

$$\Delta l = v_x \Delta t \Leftrightarrow \Delta t = \frac{\Delta l}{v_x} = \frac{\Delta l}{v} \tag{12}$$

Putting 12 in 11, where $v_x = v$, the wave celerity along the string, we get:

$$v_y = \frac{T \sin\theta}{\Delta m} \times \frac{\Delta l}{v} \quad with \quad v_y = v \tan\theta \Rightarrow v^2 = v_x^2 = T \times \frac{\Delta l}{v} \times \cos\theta$$

Speed of Waves on Strings

The wave celerity v along a stretched string or rope is given by the following general expression:

$$v = \sqrt{\frac{T}{\mu}\cos\theta} \tag{13}$$

However, the angle θ is generally very small; then $\cos\theta \approx 1$ The wave celerity v along a stretched rope or string simply becomes:

$$v = \sqrt{\frac{T}{\mu}} \tag{14}$$

Note that **the mechanical wave celerity always depends on elastic and inertial properties of the medium of propagation**.

Reflection and Transmission: Refraction of the Wave Pulse

When a travelling wave pulse reaches a rigid obstacle, one part is **transmitted** and another part is **reflected**. The main part moves back from the obstacle in opposite direction and becomes reflected, but a small part is absorbed. The reflected wave pulse is **inverted**.

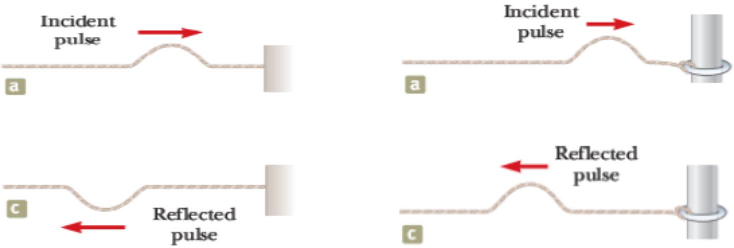
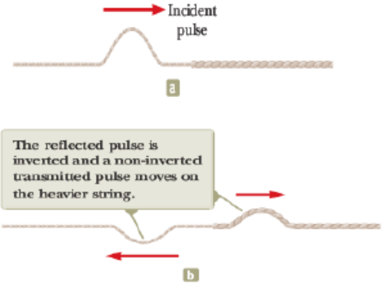
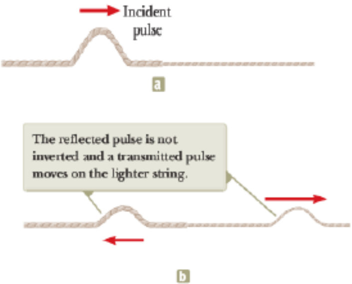


Figure: *Reflection of a wave pulse by non deformable and deformable obstacles*

Reflection and Transmission: Refraction of the Wave Pulse

At the boundary of two media with different inertia, an incident wave pulse sets up:
Weak refracted wave pulse, vibrating in the same direction as the incident wave pulse; and if the second medium's inertia is the highest: $v_2 < v_1$.
A weak refracted wave pulse and a strong reflected wave pulse, both vibrating in the same direction as the incident wave pulse; and if the second medium's inertia is the smallest: $v_2 > v_1$.





Energy Transport by Waves

The wave motion always corresponds to a transport of energy and not matter of the medium of propagation. Consider the simplest case of a harmonic wave. During the wave propagation, it is energy used to put in oscillation the wave source at the beginning that propagates through the medium at a constant wave celerity v.

According to the expression of the total mechanical energy of a harmonic oscillator (Simple harmonic motion); the total mechanical energy E, carried on by a harmonic wave is also:

$$E = \frac{1}{2} m\omega^2 \Psi_m^2 = \frac{1}{2} m\omega^2 A^2 \tag{15}$$

The **density of energy** ε in J/m^3 carried on by a harmonic wave equals to the total energy per unit volume V_o:

$$\varepsilon = \frac{E}{V_o} = \frac{1}{2}\frac{m}{V_o}\omega^2 \Psi_m^2 = \frac{1}{2}\rho\omega^2 \Psi_m^2 = \frac{1}{2}\rho\omega^2 A^2 \tag{16}$$

where ρ is the mass density of the propagation medium.

Energy Transport by Waves

The power P (in$W$) carried on by the harmonic wave becomes:

$$P = \frac{E}{t} = \frac{\varepsilon V_o}{t} = \varepsilon v S = \frac{1}{2}\rho\omega^2 v S \Psi_m^2 = \frac{1}{2}(\rho\omega^2 v S)A^2 \quad (17)$$

where:

$V_o = Sd = Svt$,

v –is the wave celerity,

ε – is the density energy,

V_o – is the volume of the propagation medium,

S – is the wave section of penetration through the propagation medium.

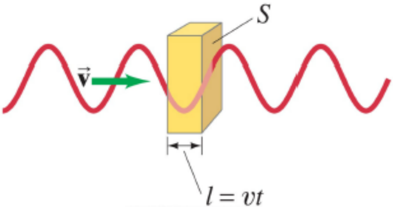


Figure: *Wave intensity is related to amplitude and frequency*

The **wave intensity** I in W/m^2 is the energy carried on by a wave per unit time and per unit cross-section area (power carried on per unit of cross–section area):

$$I = \frac{T}{tS} = \varepsilon v = \left(\frac{1}{2}\rho\omega^2 v\right)\Psi_m^2 = \left(\frac{1}{2}\rho\omega^2 v\right)A^2 \tag{18}$$

Note that **for a harmonic wave, the wave intensity I is always proportional to the square of the wave amplitude**

Exercises

1. Give one example of a transverse wave and one example of a longitudinal wave, being careful to note the relative directions of the disturbance and wave propagation in each.
2. A sinusoidal transverse wave has a wavelength of 2.80 m. It takes 0.10 s for a portion of the string at a position x to move from a maximum position of $y = 0.03$ m to the equilibrium position $y = 0$. What are the period, frequency, and wave speed of the wave? What is the difference between propagation speed and the frequency of a mechanical wave? Does one or both affect wavelength? If so, how?
3. If the tension in a string were increased by a factor of four, by what factor would the wave speed of a wave on the string increase?
4. Storms in the South Pacific can create waves that travel all the way to the California coast, 12,000 km away. How long does it take them to travel this distance if they travel at 15.0 m/s?
5. Waves on a swimming pool propagate at 0.75 m/s. You splash the water at one end of the pool and observe the wave go to the opposite end, reflect, and return in 30.00 s. How far away is the other end of the pool?

6. Wind gusts create ripples on the ocean that have a wavelength of 5.00 cm and propagate at 2.00 m/s. What is their frequency?
7. Radio waves transmitted through empty space at the speed of light (v = c = 3.00 x 108 m/s) by the Voyager spacecraft have a wavelength of 0.120 m. What is their frequency?
8. Ultrasound of intensity 1.50 x 102 W/m2 is produced by the rectangular head of a medical imaging device measuring 3.00 cm by 5.00 cm. What is its power output?
9. Given a sinusoidal wave represented by $y = 0.2sin(kx - \omega t)$, where $k = 4rad/s$, and $\omega = 8rad/s$, determine the amplitude, wavelength, frequency, and speed of this length.
10. A harmonic wave travelling along a string has the form $y = 0.25\sin(3x - 40t)$, where x is in meters and t in seconds.
    (a) Find the amplitude, wave number, angular frequency, and speed of this wave.
    (b) Find the wavelength, period, and frequency of this wave