

9/3. № 2. Справедлив ли
меньше матрица
↓
равна нулю

Однозначно определено:

2) $A + \Theta = \Theta + A$, где Θ - нулевая матрица same размера.

$$\square A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad \Theta = \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \end{pmatrix}$$

Наше замечание:

$$A + \Theta = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} + \begin{pmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \end{pmatrix} = \begin{pmatrix} a_{11} + 0 & a_{12} + 0 & \dots & a_{1n} + 0 \\ a_{21} + 0 & a_{22} + 0 & \dots & a_{2n} + 0 \\ \dots & \dots & \dots & \dots \\ a_{m1} + 0 & a_{m2} + 0 & \dots & a_{mn} + 0 \end{pmatrix}$$

$$F \left(\begin{array}{cccc} 0+a_{11} & 0+a_{12} \dots 0+a_{1n} \\ 0+a_{21} & 0+a_{22} \dots 0+a_{2n} \\ \vdots & \vdots \\ 0+g_m & 0+g_{m2} \dots 0+g_{mn} \end{array} \right) = \left(\begin{array}{cccc} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{array} \right) + \left(\begin{array}{cccc} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{array} \right) =$$

$$= \Theta + A = \text{nyelvezet racionális}$$

3) $A - A = \Theta$

$\square A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad \Theta = \begin{pmatrix} 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}$

Nyelvezet racionális:

$$A - A = \left(\begin{array}{cccc} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{array} \right) - \left(\begin{array}{cccc} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{array} \right) =$$

$$= \left(\begin{array}{cccc} a_{11}-a_{11} & a_{12}-a_{12} & \dots & a_{1n}-a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}-a_{m1} & a_{m2}-a_{m2} & \dots & a_{mn}-a_{mn} \end{array} \right) = \left(\begin{array}{cccc} 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{array} \right) = \text{nyelvezet racionális}$$

4) Kétszáműveletek $A + B = B + A$

$\square A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix}$

Nyelvezet racionális: $A + B = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix} =$

$$= \left(\begin{array}{cccc} a_{11}+b_{11} & a_{12}+b_{12} & \dots & a_{1n}+b_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \dots & a_{mn}+b_{mn} \end{array} \right) = \left(\begin{array}{cccc} b_{11}+a_{11} & b_{12}+a_{12} & \dots & b_{1n}+a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1}+a_{m1} & b_{m2}+a_{m2} & \dots & b_{mn}+a_{mn} \end{array} \right)$$

$$= \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} = B + A = \text{nyelvezet racionális}$$

5)

$$\text{Докажем } 2(A+B) = 2A + 2B$$

$$\text{Левая часть} = 2(A+B) = 2 \cdot [a_{11} + b_{11} \ a_{12} + b_{12} \ \dots \ a_{1n} + b_{1n}] = \\ [a_{11} + b_{11} \ a_{12} + b_{12} \ \dots \ \dots \ a_{1n} + b_{1n} \ a_{21} + b_{21} \ a_{22} + b_{22} \ \dots \ a_{2n} + b_{2n}]$$

$$= [2 \cdot a_{11} + 2 \cdot b_{11} \ 2 \cdot a_{12} + 2 \cdot b_{12} \ \dots \ 2 \cdot a_{1n} + 2 \cdot b_{1n}] = \\ [2 \cdot a_{11} + 2 \cdot b_{11} \ 2 \cdot a_{12} + 2 \cdot b_{12} \ \dots \ 2 \cdot a_{1n} + 2 \cdot b_{1n}]$$

$$= [2a_{11} \ 2a_{12} \ \dots \ 2a_{1n}] + [2b_{11} \ 2b_{12} \ \dots \ 2b_{1n}] = \\ [2a_{11} \ 2a_{12} \ \dots \ 2a_{1n}] [2b_{11} \ 2b_{12} \ \dots \ 2b_{1n}]$$

$$= 2A + 2B = \text{правая часть} \quad \square$$

$$6) (2+\mu) \cdot A = 2A + \mu A \quad A = [a_{11} \ a_{12} \ \dots \ a_{1n}] \\ [a_{21} \ a_{22} \ \dots \ a_{2n}]$$

$$\text{Левая часть} = (2+\mu) \cdot A = [(2+\mu) \cdot [a_{11} \ a_{12} \ \dots \ a_{1n}]] = \\ [2a_{11} \ 2a_{12} \ \dots \ 2a_{1n}]$$

$$= [2a_{11} \ 2a_{12} \ \dots \ 2a_{1n}] + [\mu a_{11} \ \mu a_{12} \ \dots \ \mu a_{1n}] = \\ [\mu a_{11} \ \mu a_{12} \ \dots \ \mu a_{1n}] = \\ [2a_{11} \ 2a_{12} \ \dots \ 2a_{1n}] [\mu a_{11} \ \mu a_{12} \ \dots \ \mu a_{1n}]$$

$$= 2A + \mu A = \text{правая часть} \quad \square$$

$$7) (2 \cdot M) \cdot A = 2 \cdot (M \cdot A)$$

$$\square A = [a_{11} \ a_{12} \ \dots \ a_{1n}]$$

$$[a_{21} \ a_{22} \ \dots \ a_{2n}]$$

$$\text{Левая часть} = (2 \cdot M) \cdot A = [2 \cdot M] \cdot [a_{11} \ a_{12} \ \dots \ a_{1n}] = \\ [2 \cdot M \cdot a_{11} \ 2 \cdot M \cdot a_{12} \ \dots \ 2 \cdot M \cdot a_{1n}]$$

$$= [(2 \cdot M) \cdot a_{11} \ (2 \cdot M) \cdot a_{12} \ \dots \ (2 \cdot M) \cdot a_{1n}] = \\ [(2 \cdot M) \cdot a_{11} \ (2 \cdot M) \cdot a_{12} \ \dots \ (2 \cdot M) \cdot a_{1n}]$$

$$\begin{aligned} & \left(\begin{array}{cccc} 2 \cdot (\mu \cdot a_{11}) & 2 \cdot (\mu \cdot a_{12}) & \dots & 2 \cdot (\mu \cdot a_{1n}) \\ \dots & \dots & \dots & \dots \\ 2 \cdot (\mu \cdot a_{m1}) & 2 \cdot (\mu \cdot a_{m2}) & \dots & 2 \cdot (\mu \cdot a_{mn}) \end{array} \right) = \\ & = 2 \cdot \left| \begin{array}{cccc} \mu \cdot a_{11} & \mu \cdot a_{12} & \dots & \mu \cdot a_{1n} \\ \dots & \dots & \dots & \dots \\ \mu \cdot a_{m1} & \mu \cdot a_{m2} & \dots & \mu \cdot a_{mn} \end{array} \right| = 2 \cdot |\mu \cdot A| = \\ & = \text{правильность} \end{aligned}$$

Свойства операций сложения, выражающие независимость
матриц как чисел:

1) Коммутативность $A + B = B + A$

2) Ассоциативность $A + (B + C) = (A + B) + C$

3) $A + 0 = A$

4) $A - A = 0$

5) $1 \cdot A = A$

□ $A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$

$$\begin{aligned} & \text{Найдем } 1 \cdot A = 1 \cdot \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} = \\ & = \begin{pmatrix} 1 \cdot a_{11} & 1 \cdot a_{12} & \dots & 1 \cdot a_{1n} \\ \dots & \dots & \dots & \dots \\ 1 \cdot a_{m1} & 1 \cdot a_{m2} & \dots & 1 \cdot a_{mn} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} = A - \text{правильность} \end{aligned}$$

6) Дистрибутивность: $d \cdot (A \cdot B) = d \cdot A + d \cdot B$

7) $(J + B) \cdot A = J \cdot A + B \cdot A$

8) $d \cdot (B \cdot A) = d \cdot B \cdot A$

9) $0 \cdot A = 0$

$$\square A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \ddots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

$$\text{левая разм} = 0 \cdot A = \begin{pmatrix} 0 \cdot a_{11} & 0 \cdot a_{12} & \dots & 0 \cdot a_{1n} \\ \vdots & \ddots & \ddots & \vdots \\ 0 \cdot a_{m1} & 0 \cdot a_{m2} & \dots & 0 \cdot a_{mn} \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} = 0 = \text{правая разм}$$

cb-ва операции транспонирования матриц:

$$\text{ii } (A^T)^T = A$$