Домашняя работа. Часть 4.

[illegible] Артем
ИВ Тк. 21.

8.4.12

$\int \frac{dx}{x+\sqrt[3]{x}} = [k = НОК(1;3) = 3 \Rightarrow x = t^k = t^3 \Rightarrow \sqrt[3]{x} = \sqrt[3]{t^6} = t^2;\ dx = d(t^3) = 3t^2 dt] =$

$= \int \frac{3t^2\,dt}{t^3+t^2} = 3\int \frac{t^2\,dt}{t^2(t+1)} = 3\int \frac{dt}{t+1} = [dt = d(t+1)] = 3\ln|t+1| + C = 3\ln|\sqrt[3]{x}+1| + C$

8.4.13

$\int \frac{\sqrt{x}}{1+\sqrt[4]{x^3}}\,dx = [k = НОК(2;4) = 4 \Rightarrow x = t^4 \Rightarrow \sqrt[4]{x^3} = t^3;\ dx = dt^4 = 4t^3\,dt] =$

$= \int \frac{t^2 \cdot 4t^3\,dt}{1+t^3} = \int \frac{4t^5}{1+t^3}\,dt = 4\int \frac{t^5}{1+t^3}\,dt = [u = 1+t^3,\ du = (1+t^3)'_t\,dt =$

$= 3t^2\,dt \Rightarrow \frac{1}{3}du = t^2\,dt,\ t^3 = u-1] = 4\int \frac{1}{3}\cdot\frac{(u-1)}{u}\,du = \frac{4}{3}\int du - \frac{4}{3}\int \frac{du}{u} =$

$= \frac{4}{3}u - \frac{4}{3}\ln|u| + C = \frac{4}{3}(1+t^3) - \frac{4}{3}\ln|1+t^3| + C = \frac{4}{3}\sqrt[4]{x^3} - \frac{4}{3}\ln|1+\sqrt[4]{x^3}| + C$

8.4.15

$\int \frac{\sqrt{x}\,dx}{x-\sqrt[3]{x^2}} = [k = НОК(2;3) = 6 \Rightarrow x = t^6,\ \sqrt{x} = t^3,\ \sqrt[3]{x^2} = t^4;\ dx = 6t^5\,dt] = 6\int \frac{t^3 t^5}{t^4(t^2-1)}\,dt =$

$= 6\int \frac{t^4}{t^2-1}\,dt = \left[\begin{array}{l|l} t^4+0t^3+0t^2+0t+0 & t^2-1 \\ t^4+0t^3-t^2 & t^2+1 \\ \quad -t^2+0+0 & \\ \quad t^2+0-1 & \end{array}\right] = 6\int \frac{(t^2-1)(t^2+1)+1}{t^2-1}\,dt = 6\int \frac{dt}{t^2-1} + 6\int t^2\,dt +$

$+ 6\int dt = 6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + 6\frac{t^3}{3} + 6t + C = 3\ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right| + 2\sqrt{x} + 6\sqrt[6]{x} + C$

8.4.16

$\int \frac{\sqrt{x}\,dx}{1+\sqrt{x}} = [k = 2 \Rightarrow x = t^2;\ \sqrt{x} = t;\ dx = 2t\,dt] = 2\int \frac{t\cdot t\,dt}{1+t} = 2\int \frac{t^2-1+1}{t+1}\,dt = 2\int \frac{(t-1)\cdot(t+1)\,dt}{(t+1)} +$

$+ 2\int \frac{dt}{t+1} = 2\int t\,dt - 2\int dt + 2\int \frac{dt}{t+1} = 2\frac{t^2}{2} - 2t + 2\ln|t+1| + C = x - 2\sqrt{x} + 2\ln|\sqrt{x}+1| + C$

8.4.18

$\int \frac{\sqrt{x+2}}{x}\,dx = [k = 2 \Rightarrow x+2 = t^2 \Rightarrow x = t^2-2;\ dx = d(t^2-2) = 2t\,dt] = \int \frac{t\cdot 2t}{t^2-2}\,dt = 2\int \frac{t^2}{t^2-2}\,dt =$

$= 2\int \frac{t^2-2}{t^2-2}\,dt + 2\int \frac{2}{t^2-2}\,dt = 2\int dt + 4\int \frac{dt}{t^2-2} = 2t + 4\cdot\frac{1}{2\sqrt{2}}\cdot\ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + C = 2\sqrt{x+2} +$

$+ \sqrt{2}\ln\left|\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right| + C$

8.4.20

$\int \frac{dx}{(x+1)^{\frac{3}{2}} + (x+1)^{\frac{1}{2}}} = [k = 2 \Rightarrow x+1 = t^2;\ (x+1)^{\frac{3}{2}} = t^3,\ (x+1)^{\frac{1}{2}} = t;\ dx = d(t^2-1) = 2t\,dt] = 2\int \frac{t\,dt}{t^3+t} =$

$= 2\int \frac{dt}{t^2+1} = 2\cdot\frac{1}{1}\operatorname{arctg}\frac{t}{1} + C = 2\operatorname{arctg}\sqrt{x+1} + C$

8.4.21

$$\int\frac{\sqrt{1+x}}{1+x}dx = \left[k=2 \Rightarrow x+1=t^2;\ \sqrt{1+x}=t;\ dx=(t^2-1)'_t\,dt=2t\,dt\right] = 2\int\frac{t^2+t}{t-1}dt =$$

$$=\left[t^2+t=t^2-t+2t-2+2=(t-1)(t+2)+2\right] = 2\int\frac{(t-1)(t+2)+2}{t-1}dt = 2\int t\,dt +$$

$$+4\int dt + 4\int\frac{dt}{t-1} = t^2+4t+4\ln|t-1| = x+1+4\sqrt{x+1}+4\ln|\sqrt{x+1}-1|+C$$

8.4.22

1) $\int\frac{x-1}{\sqrt{2x-1}}dx = \left[k=2 \Rightarrow 2x-1=t^2,\ \sqrt{2x-1}=t,\ x=\frac{t^2+1}{2},\ dx=d\left(\frac{t^2+1}{2}\right)=\frac{1}{2}\cdot 2t\,dt=t\,dt\right] =$

2) $=\int\frac{t^2-1}{2t}\cdot t\,dt = \frac{1}{2}\int t^2dt-\frac{1}{2}\int dt = \frac{1}{2}\cdot\frac{t^3}{3}-\frac{1}{2}t+C = \frac{\sqrt{(2x-1)^3}}{6}-\frac{\sqrt{2x-1}}{2}+C=$

3) $=\frac{(2x-1)\cdot 2\sqrt{2x-1}-3\sqrt{2x-1}}{6}+C = \frac{(x-2)\sqrt{2x-1}}{3}+C$

8.4.23

$$\int\frac{dx}{\sqrt{1-2x}-\sqrt[4]{1-2x}} = \left[k=\text{НОК}(2;4)=4 \Rightarrow 1-2x=t^4;\ \sqrt{1-2x}=t^2,\ \sqrt[4]{1-2x}=t;\ x=\frac{1-t^4}{2};\ dx=-\frac{1}{2}\cdot 4t^3dt\right]=$$

$$=-2\int\frac{t^3dt}{t^2-t} = -2\int\frac{t^2dt}{t-1} = \left[t^2=t^2-1+1=(t-1)(t+1)+1\right] = -2\int(t+1)dt-2\int\frac{dt}{t-1}=$$

$$=-2\int t\,dt-2\int dt-2\int\frac{dt}{t-1} = -2\cdot\frac{t^2}{2}-2t-2\ln|t-1|+C = -\sqrt{1-2x}-2\sqrt[4]{1-2x}-2\ln|\sqrt[4]{1-2x}-1|+C$$

8.4.24

$$\int\frac{1}{(2-x)^2}\sqrt[3]{\frac{2-x}{2+x}}\,dx = \left[3\ \text{случай};\ k=2 \Rightarrow \frac{2-x}{2+x}=t^2;\ \sqrt{\frac{2-x}{2+x}}=t;\ 2-x=2t^2+t^2x \Rightarrow 2-2t^2=t^2x+x \Rightarrow \frac{2-2t^2}{t^2+1};\ dx=d\left(\frac{2-2t^2}{t^2+1}\right)=\frac{-8t\,dt}{(t^2+1)^2}\right]\int\frac{1}{\left(2-\frac{2-2t^2}{t^2+1}\right)^2}\cdot t\cdot\frac{(-8t)}{(t^2+1)^2}dt = \int\frac{(t^2+1)^2}{16t^4}\cdot t\cdot$$

$$\cdot\frac{(-8t)}{(t^2+1)^2}dt = \int\frac{-dt}{2t^2} = -\frac{1}{2}\int t^{-2}dt = -\frac{1}{2}\frac{t^{-2+1}}{-2+1}+C = \frac{\sqrt{2+x}}{2\sqrt{2-x}}+C$$

8.4.25

$$\int\frac{dx}{\sqrt{(x-1)^3(x-2)}} = \int\frac{\sqrt{x-1}\,dx}{\sqrt{(x-1)^4}\cdot\sqrt{x-2}} = \int\frac{dx}{(x-1)^2\sqrt{\frac{x-2}{x-1}}} = \left[k=2 \Rightarrow \frac{x-2}{x-1}=t^2;\ \sqrt{\frac{x-2}{x-1}}=t;\ x-2=t^2x-t^2;\ x-t^2x=\right.$$

$$\left.=2-t^2,\ x=\frac{2-t^2}{1-t^2}=\frac{t^2-2}{t^2-1};\ dx=d\left(\frac{t^2-2}{t^2-1}\right)=\frac{2t}{(t^2-1)^2}\right] = \int\frac{1}{\left(\frac{t^2-2}{t^2-1}-1\right)^2 t}\cdot\frac{2t}{(t^2-1)^2}dt = \int\frac{(t^2-1)^2}{t}\cdot$$

$$\cdot\frac{2t}{(t^2-1)^2}dt = \int 2\,dt = 2\int dt = 2t+C = 2\sqrt{\frac{x-2}{x-1}}+C$$

8.4.27

$$\int\frac{dx}{(1-x)\sqrt{1-x^2}} = \int\frac{dx}{(1-x)\sqrt{(1-x)(1+x)}} = \int\frac{dx}{(1-x)^2\sqrt{\frac{1+x}{1-x}}} = \left[k=2 \Rightarrow \frac{1+x}{1-x}=t^2;\ \sqrt{\frac{1+x}{1-x}}=t;\ 1+x=t^2-t^2x,\right.$$

$$\left.x+t^2x=t^2-1,\ x=\frac{t^2-1}{t^2+1};\ dx=d\left(\frac{t^2-1}{t^2+1}\right)=\frac{4t}{(t^2+1)^2}\right] = \int\frac{1}{\left(1-\frac{t^2-1}{t^2+1}\right)^2 t}\cdot\frac{4t}{(t^2+1)^2}dt =$$

$\int \frac{(t^2+1)^2}{4t}\cdot\frac{4t}{(t^2+1)^2}dt = \int dt = t + C = \sqrt{\frac{1+x}{1-x}} + C$

8.4.29

$\int x^2\cdot\sqrt{1+x^2}\,dx = \int x^3\cdot(1+x^2)^{\frac{1}{2}}dx$: $m=3$, $n=2$, $p=\frac{1}{2}$, 1) $p=\frac{1}{2}\notin Z$; 2) $\frac{m+1}{n}=\frac{3+1}{2}=2\in Z \Rightarrow 1+x^2=t^2$; $x=\sqrt{t^2-1}$; $dx=\frac{t}{\sqrt{t^2-1}}dt] = \int (t^2-1)^{\frac{3}{2}}\cdot t\,\frac{t}{\sqrt{t^2-1}}dt = \int (t^2-1)\cdot t^2 dt =$

$=\int t^4 dt - \int t^2 dt = \frac{t^5}{5} - \frac{t^3}{3} + C = \frac{(1+2x^2+x^4)\sqrt{1+x^2}}{5} - \frac{(1+x^2)\sqrt{1+x^2}}{3} + C$

8.4.31

$\int \frac{dx}{\sqrt{x}(1-\sqrt{x})^2} = \int x^{-\frac{1}{2}}(1-x^{\frac{1}{2}})^{-2}dx = [m=-\frac{1}{2}; n=\frac{1}{2}; p=-2$, 1) $p=-2\in Z \Rightarrow k=HOK(2;2)=2$, $x=t^2$, $x^{\frac{1}{2}}=$
$=t$, $x^{-\frac{1}{2}}=t^{-1}$, $dx=2t\,dt] = \int \frac{2t\,dt}{t(1-t)^2} = 2\int\frac{dt}{(t-1)^2} = [dt = d(t-1)] = 2\int (t-1)^{-2}d(t-1) =$
$=2\frac{(t-1)^{-1}}{-1} + C = \frac{2}{1-\sqrt{x}} + C$

8.4.32

$\int \frac{dx}{\sqrt{x}\cdot(1-\sqrt{x})^2} = \int x^{-\frac{1}{2}}(1-x^{\frac{1}{2}})^{-2}dx = [m=-\frac{1}{2}; n=\frac{1}{2}, p=-2$; 1) $p=-2\in Z\Rightarrow k=HOK(2;2)=2$,
$x=t^2$, $x^{\frac{1}{2}}=t$, $x^{-\frac{1}{2}}=t^{-1}$, $dx=2t\,dt] = \int\frac{2t\,dt}{t(1-t)^2} = 2\int\frac{dt}{(t-1)^2} = [dt=d(t-1)] = 2\int (t-1)^{-2}d(t-1) =$
$=2\frac{(t-1)^{-1}}{-1} + C = -\frac{2}{t-1} + C = \frac{2}{1-\sqrt{x}} + C$

8.4.33

$\int\frac{dx}{x^3\sqrt[3]{2-x^3}} = \int x^{-3}(2-x^3)^{-\frac{1}{3}}dx = [m=-3, n=3, p=-\frac{1}{3}$) 1) $p=-\frac{1}{3}\notin Z$, 2) $\frac{m+1}{n}=\frac{-3+1}{3}\notin Z$) 3) $\frac{m+1}{n}+p=$
$=\frac{-3+1}{3}-\frac{1}{3}=-1\in Z\Rightarrow k=3$, $2x^{-3}-1=t^3$, $x^{-3}=\frac{t^3+1}{2}$ | $x^3=\frac{2}{t^3+1}$; $x=\sqrt[3]{\frac{2}{t^3+1}}$; $dx=-\frac{\sqrt[3]{2}\,t^2\,dt}{(t^3+1)\sqrt[3]{t^3+1}}$
$=\frac{1}{2}\int t\,dt = -\frac{1}{2}\frac{t^2}{2} + C = -\frac{(x^3-2)}{4x^2} + C$

8.4.34

$\int\sqrt{x}(1+\sqrt{x})^3dx = \int\sqrt{x}(1+3\sqrt{x}+3x+x\sqrt{x})dx = \int(x^{\frac{1}{2}}+3x+3x^{\frac{3}{2}}+x^2)dx = \int x^{\frac{1}{2}}dx +$
$+3\int x\,dx + 3\int x^{\frac{3}{2}}dx + \int x^2dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 3\frac{x^2}{2} + 3\cdot\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^3}{3} + C = \frac{2x\sqrt{x}}{3} + \frac{3x^2}{2} + \frac{6x^2\sqrt{x}}{5}$
$+\frac{x^3}{3} + C$

8.4.35

$\frac{dx}{\sqrt{1-2x-x^2}} = \int\frac{dx}{\sqrt{2-(x+1)^2}} = (p=x+1; dp=dx) = \int\frac{dp}{\sqrt{(\sqrt{2})^2-p^2}} = \arcsin\frac{p}{\sqrt{2}} + C = \arcsin\frac{(x+1)}{\sqrt{2}} + C$

8.4.38

$\int\frac{x+1}{\sqrt{2x-x^2}}dx = \int\frac{(x-1)+2}{\sqrt{2x-x^2}}dx = \int\frac{(x-1)dx}{\sqrt{2x-x^2}} + 2\int\frac{dx}{\sqrt{2x-x^2}} = [1) 2x-x^2=t$, $dt=(2-2x)dx$,

$\frac{1}{2}dt = (1-x)\,dx;\ -\frac{1}{2}dt = (x-1)dx;\ \frac{1}{2}dt = (x-1)dx;\ 2)\ y = x-1;\ 2x - x^2 = 1 - 1 + 2x - x^2 =$

$= 1-(x-1)^2 = 1-y^2;\ dx = dy) = \frac{1}{2}\int \frac{dt}{\sqrt{t}} + 2\int \frac{dy}{\sqrt{1-y^2}} = -\frac{1}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + 2\arcsin\frac{y}{1} + C =$

$= 2\arcsin(x-1) - \sqrt{2x-x^2} + C$

8.4.40

$$\int \frac{\sqrt{1-x^2}}{x}dx = \int x^{-1}(1-x^2)^{\frac{1}{2}}dx = [m=-1;\ n=2;\ p=\frac{1}{2};\ 1)\ p = \frac{1}{2} \notin Z;\ 2)\ \frac{m+1}{n} = \frac{-1+1}{2} = 0 \in Z \Rightarrow$$

$\Rightarrow k=2\ \ 1-x^2 = t^2;\ x^2 = 1-t^2,\ x = \sqrt{1-t^2},\ dx = d(\sqrt{1-t^2}) = -\frac{t\,dt}{\sqrt{1-t^2}}] = \frac{t}{\sqrt{1-t^2}}\cdot(-1)\cdot$

$\cdot\frac{t\,dt}{\sqrt{1-t^2}} = -\int\frac{t^2dt}{1-t^2} = \int\frac{t^2dt}{t^2-1} = (t^2 = (t^2-1)+1) = \int\frac{(t^2-1)+1}{t^2-1}dt = \int dt + \int\frac{dt}{t^2-1} = t +$

$+ \frac{1}{2\cdot 1}\ln\left|\frac{t-1}{t+1}\right| + C = \sqrt{1-x^2} + \frac{1}{2}\ln\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + C$

8.4.42

$$\int x\cdot\sqrt[5]{x-2}\,dx = \int x^1(x^1-2)^{\frac{1}{5}}dx = [m=1,\ n=1;\ p=\frac{1}{5},\ 1)\ p = \frac{1}{5} \notin Z;\ 2)\ \frac{m+1}{n} =$$

$= \frac{1+1}{1} = 2 \in Z \Rightarrow k=5;\ x-2 = t^5;\ x = t^5+2;\ dx = 5t^4\cdot dt] = \int(t^5+2)\cdot t\cdot 5t^4dt =$

$= 5\int t^{10}dt + 10\int t^5dt = 5\frac{t^{11}}{11} + 10\frac{t^6}{6} + C = 5\cdot(x-2)^{\frac{6}{5}}\cdot\frac{3x+5}{33} + C$

8.4.43

$$\int\frac{dx}{x^2\sqrt{x^2+1}} = \int x^{-2}(x^2+1)^{-\frac{1}{2}}dx = [m=-2;\ n=2,\ p=-\frac{1}{2},\ 1) = -\frac{1}{2} \notin Z)\ 2)\ \frac{m+1}{n} = \frac{-2+1}{2} = \frac{-1}{2} \notin Z$$

$3)\ \frac{m+1}{n} + p = \frac{-2+1}{2} - \frac{1}{2} = -1 \in Z \Rightarrow k=2 \Rightarrow x^{-2}+1 = t^2,\ x^{-2} = t^2-1)\ x^2 = \frac{1}{t^2-1};\ x = \frac{1}{\sqrt{t^2-1}} \Rightarrow dx =$

$= \frac{-t\,dt}{\sqrt{t^2-1}\,(t^2-1)} = \int\frac{t^2-1}{\frac{1}{\sqrt{t^2-1}}+1}(-1)\cdot\frac{t\,dt}{\sqrt{t^2-1}\,(t^2-1)} = -\int dt = -t + C = -\sqrt{x^{-2}+1} + C =$

$= -\sqrt{\frac{1}{x^2}+1} + C = -\frac{\sqrt{x^2+1}}{x} + C$