Гневков Артем Евгеньевич, ИВТб. 2.1.

Домашнее Задание

Блок I

№ 1.3.17.

$$A=\begin{pmatrix} 1 & -3 & 1 & 14 & 22 \\ -2 & 1 & 3 & 3 & -9 \\ -4 & -3 & 11 & -19 & 17 \end{pmatrix} \begin{matrix} \\ \\ \mathrm{III}+4\mathrm{I} \end{matrix} \sim \begin{pmatrix} 1 & -3 & 1 & 14 & 22 \\ -2 & 1 & 3 & 3 & -9 \\ 0 & -15 & 15 & -75 & 105 \end{pmatrix} \begin{matrix} \mathrm{II}+2\mathrm{I} \\ \mathrm{III}-3\mathrm{II} \\ \end{matrix} \sim$$

$$\sim \begin{pmatrix} 1 & -3 & 1 & -14 & 22 \\ 0 & -5 & 5 & -25 & 35 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow r(A)=2$$

№ 1.3.18

$$A=\begin{pmatrix} 1 & 2 & 4 & -3 \\ 3 & 5 & 6 & -4 \\ 3 & 8 & 2 & -19 \end{pmatrix} \begin{matrix} \mathrm{II}-3\mathrm{I} \\ \mathrm{III}-3\mathrm{I} \\ \end{matrix} \sim \begin{pmatrix} 1 & 2 & 4 & 3 \\ 0 & -1 & -6 & 5 \\ 0 & 2 & -10 & -10 \end{pmatrix} \mathrm{III}+2\mathrm{II} \sim$$

$$\sim \begin{pmatrix} 1 & 2 & 4 & -3 \\ 0 & -1 & -6 & 5 \\ 0 & 0 & -22 & 0 \end{pmatrix} \Rightarrow r(A) = 3$$

№1.3.19.

$$A = \begin{pmatrix} 3 & -1 & 3 & 2 & 5 \\ 5 & -3 & 2 & 3 & 4 \\ 1 & -3 & -5 & 0 & -7 \\ 7 & -5 & 1 & 4 & 1 \end{pmatrix} \begin{matrix} 3\cdot\mathrm{III} - \mathrm{I} \\ 3\cdot\mathrm{II} - 5\mathrm{I} \\ 3\cdot\mathrm{IV} - 7\mathrm{I} \end{matrix} \sim \begin{pmatrix} 3 & -1 & 3 & 2 & 5 \\ 0 & -4 & -9 & -1 & -13 \\ 0 & -8 & -18 & -2 & -26 \\ 0 & -8 & -18 & -2 & -32 \end{pmatrix} \begin{matrix} \mathrm{IV} - \mathrm{III} \\ \mathrm{III} - 2\mathrm{II} \end{matrix} \sim$$

$$\sim \begin{pmatrix} 3 & -1 & 3 & 2 & 5 \\ 0 & -4 & -9 & -1 & -13 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 \end{pmatrix} \Rightarrow r(A) = 3$$

№1.3.20.

$$A = \begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 71 & 141 & -72 \end{pmatrix} \begin{matrix} 24\cdot\mathrm{II} - 49\mathrm{I} \\ 24\cdot\mathrm{III} - 73\mathrm{I} \\ 24\cdot\mathrm{IV} - 47\mathrm{I} \end{matrix} \sim$$

$$\begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 0 & 29 & -12 & 0 & -58 \\ 0 & 29 & -276 & 0 & -58 \\ 0 & -29 & 12 & 0 & 58 \end{pmatrix} \mathrm{IV} + \mathrm{II} \sim \begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 0 & 29 & -12 & 0 & -58 \\ 0 & 29 & -276 & 0 & -58 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow$$

1.3.21.

$$A=\begin{pmatrix}4 & 3 & -5 & 2 & 3\\ 8 & 6 & -7 & 4 & 2\\ 4 & 3 & -8 & 2 & 7\\ 4 & 3 & 1 & 2 & -5\\ 8 & 6 & -1 & 4 & -6\end{pmatrix}\begin{matrix}II-2I\\ III-I\\ IV-I\\ V-2I\end{matrix} \sim \begin{pmatrix}4 & 3 & -5 & 2 & 3\\ 0 & 0 & 3 & 0 & -4\\ 0 & 0 & -3 & 0 & 4\\ 0 & 0 & 6 & 0 & -8\\ 0 & 0 & 9 & 0 & -12\end{pmatrix}\begin{matrix}III+II\\ IV-2II\\ V-3II\end{matrix} \sim$$

$$\sim\begin{pmatrix}4 & 3 & -5 & 2 & 3\\ 0 & 0 & 3 & 0 & -4\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{pmatrix} \Rightarrow r(A)=2$$

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1.3.22.

$$A=\begin{pmatrix}17 & -28 & 45 & 11 & 39\\ 29 & -37 & 61 & 13 & 50\\ 25 & -7 & 32 & -18 & -11\\ 31 & 12 & 19 & -43 & -55\\ 42 & 13 & 29 & -55 & -68\end{pmatrix}\begin{matrix}17II-29I\\ 17III-25I\\ 17IV-31I\\ 17V-42I\end{matrix} \sim \begin{pmatrix}17 & -28 & 45 & 11 & 39\\ 0 & 43 & -43 & -43 & -86\\ 0 & 581 & -581 & -581 & -1162\\ 0 & 1072 & -1072 & -1072 & \\ 0 & 1397 & -1397 & -1397 & -2794\end{pmatrix}$$

$$\begin{matrix}III-\frac{581}{43}\cdot II\\ IV-\frac{1072}{43}\cdot II\\ V-\frac{1397}{43}\cdot II\end{matrix} \sim \begin{pmatrix}17 & -28 & 45 & 11 & 39\\ 0 & 43 & -43 & -43 & -86\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{pmatrix} \Rightarrow r(A)=2$$

№ 1.3.23

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$$

1) $M_1 = |a_{11}| = 3 \neq 0 \Rightarrow r(A) \geq 1$

2) $M_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 3 & -1 \\ 4 & -3 \end{vmatrix} = 3 \cdot (-3) - 4 \cdot (-1) = -5 \neq 0 \Rightarrow$

$\Rightarrow r(A) \geq 2$

3) $M_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{vmatrix} = 3 \cdot (-3) \cdot 0 + (-1) \cdot 3 \cdot 1 +$

$+ 4 \cdot 3 \cdot 2 - 1 \cdot (-3) \cdot 2 - (-1) \cdot 4 \cdot 0 - 3 \cdot 3 \cdot 3 = 0 \Rightarrow 2 \leq r(A) < 3 \Rightarrow \underline{r(A) = 2}$

Базисный минор – M_2 $\begin{vmatrix} 3 & -1 \\ 4 & -3 \end{vmatrix}$ ∎

№ 1.3.24

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

1) $M_1 = |a_{11}| = 3 \neq 0 \Rightarrow r(A) \geq 1$

2) $M_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 3 & -1 \\ 4 & -3 \end{vmatrix} = -5 \neq 0 \Rightarrow r(A) \geq 2$

3) $M_3 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 2 \end{vmatrix} = 3 \cdot (-3) \cdot 2 + (-1) \cdot 3 \cdot 1 + 4 \cdot 3 \cdot 2 -$

$- 1 \cdot (-3) \cdot 2 - 4 \cdot (-1) \cdot 2 - 3 \cdot 3 \cdot 3 = -10 \neq 0 \Rightarrow r(A) \geq 3 \Rightarrow \underline{r(A) = 3}$

Базисный минор – $M_3 = \begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 2 \end{vmatrix}$

№1.3.25

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$A = \begin{pmatrix} 2 & -1 & 5 & 6 \\ 1 & 1 & 3 & 5 \\ 1 & -5 & 1 & -3 \end{pmatrix}$

1) $M_1 = |a_{11}| = 2 \neq 0 \Rightarrow r(A) \geq 1$

2) $M_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 \cdot 1 - 1 \cdot (-1) = 3 \neq 0 \Rightarrow r(A) \geq 2$

3) $M_3^1 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 2 & -1 & 5 \\ 1 & 1 & 3 \\ 1 & -5 & 1 \end{vmatrix} = 2 \cdot 1 \cdot 1 + (-1) \cdot 3 \cdot 1 + (1 \cdot (-5) \cdot$
$\cdot 5 - 1 \cdot 1 \cdot 5 - (1 \cdot 1 \cdot (-1)) - (-5) \cdot 3 \cdot 2 = 0$

$M_3^2 = \begin{vmatrix} a_{11} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \end{vmatrix} = \begin{vmatrix} 2 & -1 & 6 \\ 1 & 1 & 5 \\ 1 & -5 & -3 \end{vmatrix} = 2 \cdot 1 \cdot (-3) + (-1) \cdot 5 \cdot 1 +$
$+ (1 \cdot (-5)) \cdot 6 - 1 \cdot 1 \cdot 6 - 1 \cdot (-1) \cdot (-3) - (-5) \cdot 5 \cdot 2) = 0 \Rightarrow r(A) < 3$

$2 \leq r(A) < 3 \Rightarrow r(A) = 2$

Базисный минор = $M_2 = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$

№ 1.3.26.

$$A = \begin{pmatrix} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -1 & 1 & -3 \\ 1 & 3 & 0 & -3 & 1 \\ 0 & -7 & -3 & 1 & -3 \end{pmatrix}$$

1) $M_1 = |a_{11}| = 1 \neq 0 \Rightarrow r(A) \geq 1$

2) $M_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1 \cdot 1 - 2 \cdot 0 = 1 \neq 0 \Rightarrow r(A) \geq 2$

3) $M_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{vmatrix} = 1 \cdot 1 \cdot 0 + (-2) \cdot (-1) \cdot 1 +$

$+ 0 \cdot 3 \cdot 3 - 1 \cdot 1 \cdot 3 - 0 \cdot (-2) \cdot 0 - (-1) \cdot 3 \cdot 1 = 2 \neq 0 \Rightarrow r(A) \geq 3$

4) $M_4^1 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = \begin{vmatrix} 1 & -2 & 3 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & -3 \\ 0 & -7 & 3 & 1 \end{vmatrix} =$

$= 1 \cdot \begin{vmatrix} 1 & -1 & 1 \\ 3 & 0 & -3 \\ -7 & 3 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} -2 & 3 & -4 \\ 1 & -1 & 1 \\ -7 & 3 & 1 \end{vmatrix} = 0$

$M_4^2 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{45} \end{vmatrix} = \begin{vmatrix} 1 & -2 & 3 & 4 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 1 \\ 0 & -7 & 3 & 3 \end{vmatrix} =$

$= 1 \cdot \begin{vmatrix} 1 & -1 & -3 \\ 3 & 0 & 1 \\ -7 & 3 & 3 \end{vmatrix} + 1 \cdot \begin{vmatrix} -2 & 3 & 4 \\ 1 & -1 & -3 \\ -7 & 3 & 3 \end{vmatrix} = 0 \Rightarrow r(A) < 4$

$3 \le r(A) < 4 \Rightarrow r(A) = 3$

Базисный минор: $M_3 \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{vmatrix}$

№1.3.27

$$A = \begin{pmatrix} 1 & -2 & 1 & -1 & 1 \\ 2 & 1 & -1 & 2 & -3 \\ 3 & -2 & -1 & 1 & -2 \\ 2 & -5 & 1 & -2 & 2 \end{pmatrix}$$

1) $M_1 = |a_{11}| = 1 \neq 0 \Rightarrow r(A) \geq 1$

2) $M_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = 1 \cdot 1 - 2 \cdot (-2) = 5 \neq 0 \Rightarrow r(A) \geq 2$

3) $M_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ 3 & -2 & -1 \end{vmatrix} = 1 \cdot 1 \cdot (-1) + (-2) \cdot (-1) \cdot 3 + 2 \cdot (-2) \cdot$

$\cdot 1 - 3 \cdot 1 \cdot 1 - 2 \cdot (-2) \cdot (-1) - (-2) \cdot (-1) \cdot 1 = -8 \neq 0 \Rightarrow r(A) \geq 3$

4) $M_4^1 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 & -1 \\ 2 & 1 & -1 & 2 \\ 3 & -2 & -1 & 1 \\ 2 & -5 & 1 & -2 \end{vmatrix} =$

$= 1 \overset{-4}{\begin{vmatrix} 1 & -1 & 2 \\ -2 & -1 & 1 \\ -5 & 1 & -2 \end{vmatrix}} - (-2) \overset{4}{\begin{vmatrix} 2 & -1 & 2 \\ 3 & -1 & 1 \\ 2 & 1 & -2 \end{vmatrix}} + 1 \overset{4}{\begin{vmatrix} 2 & 1 & 2 \\ 3 & -2 & 1 \\ 2 & -5 & -2 \end{vmatrix}} - 1 \overset{8}{\begin{vmatrix} 2 & 1 & -1 \\ 3 & -2 & -1 \\ 2 & -5 & 1 \end{vmatrix}} = 0$

$M_4^2 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{45} \end{vmatrix} = \begin{vmatrix} 1 & -2 & 1 & 1 \\ 2 & 1 & -1 & -3 \\ 3 & -2 & -1 & -2 \\ 2 & -5 & 1 & 2 \end{vmatrix} = 1 \overset{-18}{\begin{vmatrix} 2 & 1 & -3 \\ 3 & -2 & -2 \\ 2 & -5 & 2 \end{vmatrix}} + 1 \overset{-5}{\begin{vmatrix} 1 & -2 & 1 \\ 3 & -2 & -2 \\ 2 & -5 & 2 \end{vmatrix}}$

$- 1 \overset{-5}{\begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -3 \\ 2 & -5 & 2 \end{vmatrix}} - 1 \overset{-18}{\begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -3 \\ 3 & -2 & -2 \end{vmatrix}} = 0 \Rightarrow r(A) < 4$

$3 \le r(A) < 4 \Rightarrow \underline{r(A) = 3}$

Базисный минор: $M_3 \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ 3 & -2 & -1 \end{vmatrix}$

Д №1.3.28

$$A = \begin{pmatrix} 2 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -2 \\ 1 & 3 & 0 & 3 & 1 \\ 0 & -7 & 3 & 1 & -3 \end{pmatrix} \quad \begin{pmatrix} 2 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -2 \\ 3 & 3 & -3 & -3 & 4 \\ 4 & 5 & -5 & -5 & 7 \end{pmatrix}$$

1) $M_1 = |a_{11}| = 2 \ne 0 \Rightarrow r(A) \ge 1$

2) $M_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = 2 \cdot (-1) - 1 \cdot 1 = -3 \ne 0 \Rightarrow r(A) \ge 2$

3) $M_3^1 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 3 & -3 \end{vmatrix} = 2 \cdot (-1) \cdot (-3) + 1 \cdot 1 \cdot 3 + 1 \cdot 3 \cdot (-1) -$

$- 3 \cdot (-1) \cdot (-1) - 1 \cdot 1 \cdot (-3) - 3 \cdot 1 \cdot 2 = 0$

$M_3^2 = \begin{vmatrix} a_{11} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{24} \\ a_{41} & a_{42} & a_{44} \end{vmatrix} = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 4 & 5 & -5 \end{vmatrix} = 2 \cdot (-1) \cdot (-5) + 1 \cdot 1 \cdot 4 + 1 \cdot 5 \cdot (-1) -$

$- 4 \cdot (-1) \cdot (-1) - 1 \cdot 1 \cdot (-5) - 5 \cdot 1 \cdot 2 = 0$

$M_3^3 = \begin{vmatrix} a_{11} & a_{12} & a_{15} \\ a_{21} & a_{22} & a_{25} \\ a_{31} & a_{32} & a_{35} \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 3 & 3 & 4 \end{vmatrix} = 2 \cdot (-1) \cdot 4 + 1 \cdot (-2) \cdot 3 + 1 \cdot 3 \cdot 1 -$

$- 3 \cdot (-1) \cdot 1 - 1 \cdot 1 \cdot 4 - 3 \cdot (-2) \cdot 2 = 0$

$$M_3^4 = \begin{vmatrix} a_{11} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \end{vmatrix} = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 3 & -3 \end{vmatrix} = 2\cdot(-1)\cdot(-3) + 1\cdot1\cdot3 + 1\cdot3\cdot(-1) -$$

$$- 3\cdot(-1)\cdot(-1) - 1\cdot1\cdot(-3) - 3\cdot1\cdot2 = 0$$

$$M_3^5 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{41} & a_{42} & a_{43} \end{vmatrix} = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 4 & 5 & 5 \end{vmatrix} = 2\cdot(-1)\cdot(-5) + 1\cdot1\cdot4 + 1\cdot5\cdot(-1) -$$

$$- 4\cdot(-1)\cdot(-1) - 1\cdot1\cdot(-5) - 5\cdot1\cdot2 = 0$$

$$M_3^6 = \begin{vmatrix} a_{11} & a_{12} & a_{15} \\ a_{21} & a_{22} & a_{25} \\ a_{41} & a_{42} & a_{45} \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 4 & 5 & 7 \end{vmatrix} = 2\cdot(-1)\cdot7 + 1\cdot5\cdot1 + 1\cdot(-2)\cdot4 -$$

$$- 4\cdot1\cdot(-1) - 1\cdot1\cdot7 - 5\cdot(-2)\cdot2 = 0 \Rightarrow r(A) < 3$$

$$2 \le r(A) < 3 \Rightarrow \underline{r(A) = 2}$$

Базисный минор $= M_2 = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$

№ 1.3.29.

$$A = \begin{pmatrix} 1 & -3 & 2 & 0 \\ 2 & -3 & -1 & 3 \\ 3 & -6 & -1 & \lambda \\ 1 & -2 & 0 & 1 \end{pmatrix} \begin{matrix} II - 2I \\ III - 3I \\ IV - I \end{matrix} \sim \begin{pmatrix} 1 & -3 & 2 & 0 \\ 0 & 3 & -5 & 3 \\ 0 & 3 & -7 & \lambda \\ 0 & 1 & -2 & 1 \end{pmatrix} \begin{matrix} III - II \\ 3\cdot IV - II \end{matrix} \sim$$

$$\sim \begin{pmatrix} 1 & -3 & 2 & 0 \\ 0 & 3 & -5 & 3 \\ 0 & 0 & -2 & \lambda-3 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{matrix} III - 2IV \\ III \Leftrightarrow IV \end{matrix} = \begin{pmatrix} 1 & -3 & 2 & 0 \\ 0 & 3 & -5 & 3 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & \lambda-3 \end{pmatrix}$$

$\lambda - 3 = 0$

$\lambda = 3 \Rightarrow$ При $\lambda = 3$ 4 строка нулевая $\Rightarrow$

При $\lambda = 3$, $r(A) = 3$

При $\lambda \neq 3$, $r(A) = 4$

№ 1.3.30

№1.3.30

$$A=\begin{pmatrix}3&1&1&4\\2&4&10&1\\1&7&17&3\\2&2&4&3\end{pmatrix} \begin{matrix}I\Leftrightarrow III\\ II\Leftrightarrow IV\end{matrix} \sim \begin{pmatrix}1&7&17&3\\2&2&4&3\\3&1&1&4\\2&4&10&1\end{pmatrix} \begin{matrix}II-2I\\III-3I\\IV-\lambda I\end{matrix} \sim$$

$$\sim\begin{pmatrix}1&7&17&3\\0&-12&-30&-3\\0&-20&-50&-5\\0&4-7\lambda&10-17\lambda&1-3\lambda\end{pmatrix} \frac{3\cdot III}{5}-II \sim \begin{pmatrix}1&7&17&3\\0&-12&-30&-3\\0&0&0&0\\0&4-7\lambda&10-17\lambda&1-3\lambda\end{pmatrix} \xrightarrow{12IV-(4-7\lambda)\cdot II} \sim$$

$$\sim\begin{pmatrix}1&7&17&3\\0&-12&-30&-3\\0&0&0&0\\0&0&6\lambda&-15\lambda\end{pmatrix} III\Leftrightarrow IV \sim \begin{pmatrix}1&7&17&3\\0&-12&-30&-3\\0&0&6\lambda&-15\lambda\\0&0&0&0\end{pmatrix}$$

$$\begin{cases}6\lambda=0\\-15\lambda=0\end{cases} \Rightarrow \lambda=0 \Rightarrow 3 \text{ строка нулевая и } r(A)=2$$

При $\lambda=0$, $r(A)=2$; при $\lambda\neq0$, $r(A)=3$.

№1.3.31

$\square$

$$A=\begin{pmatrix}\lambda&1&1&1\\1&\lambda&1&\lambda\\1&1&\lambda&\lambda^2\end{pmatrix} I\Leftrightarrow III \sim \cancel{\begin{pmatrix}1&\lambda&1&\lambda\\\lambda&1&1&1\\1&1&\lambda&\lambda^2\end{pmatrix}} \begin{pmatrix}1&1&\lambda&\lambda^2\\1&\lambda&1&\lambda\\\lambda&1&1&1\end{pmatrix} \begin{matrix}II-I\\III-\lambda I\end{matrix} \sim$$

$$\sim\begin{pmatrix}1&1&\lambda&\lambda^2\\0&\lambda-1&1-\lambda&\lambda-\lambda^2\\0&1-\lambda&1-\lambda^2&1-\lambda^3\end{pmatrix} III+II \sim \begin{pmatrix}1&1&\lambda&\lambda^2\\0&\lambda-1&1-\lambda&\lambda-\lambda^2\\0&0&(1-\lambda)\cdot(2+\lambda)&(1-\lambda)\cdot(1+\lambda)^2\end{pmatrix}$$

III строка будет нулевая при $\begin{cases}(1-\lambda)\cdot(2+\lambda)=0\\(1-\lambda)\cdot(1+\lambda)^2=0\end{cases}$

$(1-\lambda)\cdot(2+\lambda)=0$

$1-\lambda=0$ или $2+\lambda=0$

$\underline{\lambda=1}$ $\qquad \lambda=-2$

$(1-\lambda)\cdot(1+\lambda)^2=0$

$1-\lambda=0$ или $(1+\lambda)^2=0$

$\underline{\lambda=1}$

$\lambda^2+2\lambda+1=0$

$\lambda=-1$

При $\lambda=1$, 3 строка нулевая и $r(A)=2$

2) II строка будет нулевая при: $\begin{cases}\lambda-1=0\\1-\lambda=0\\\lambda-\lambda^2=0\end{cases} \Rightarrow \begin{cases}\lambda=1\\\lambda=+1\\\left[\begin{array}{l}\lambda=1\\\lambda=0\end{array}\right.\end{cases} \Rightarrow$

II строка будет нулевой при ~~любом значении~~ значении $\lambda=1$

При $\lambda=1$, $r(A)=1$

При $\lambda\neq1$, $r(A)=3$