Домашняя работа. Часть 6 Гневиров Артем Евгеньевич. ИВТм21

9.1.27

$$\int_1^{e^2} \frac{\ln^3 x}{3x}dx = \left[u = \ln x, du = \frac{1}{x}dx\right] = \frac{1}{3}\int_1^{e^2} u^3 du = \left[\int u^n du = \frac{u^{n+1}}{n+1}\right] = \left[\frac{1}{3}\cdot\frac{u^4}{4} + C = \frac{u^4}{12} + C =\right.$$

$$\left.= \frac{\ln^4 x}{12} + C\right] = \frac{\ln^4 x}{12}\Big|_1^{e^2} = \frac{\ln^4 e^2}{12} - \frac{\ln^4 1}{12} = \frac{4}{3} - 0 = \frac{4}{3}$$

9.1.28

$$\int_{\pi}^{2\pi} \frac{x + \cos x}{2\sin x + x^2}dx = [u = 2\sin x + x^2, du = (2\cos x + 2x)dx] = \int \frac{1}{2u}du = \left[\frac{1}{2}\ln(|u|) + C =\right.$$

$$\left.= \frac{\ln(|u|)}{2} + C = \frac{\ln(|2\sin x + x^2|) + C}{2}\right] = \frac{\ln(|2\sin x + x^2|)}{2}\Big|_{\pi}^{2\pi} = \frac{\ln|2\sin 2\pi + (2\pi)^2|}{2} -$$

$$- \frac{\ln(|2\sin\pi + \pi^2|)}{2} = \frac{\ln|4\pi^2|}{2} - \ln\pi$$

9.1.30

$$\int_1^{e} \frac{\sin\ln x}{x}dx = \left[u = \ln x, du = \frac{1}{x}dx\right] = \int_1^{e} \sin u\, du = [-\cos u + C = -\cos(\ln x) + C] =$$

$$= -\cos\ln x\Big|_1^{e} = -\cos\ln e + \cos\ln 1 = 1 - \cos 1$$

9.1.31

$$\int_{-1}^{0} \frac{3^x - 2^x}{6^x}dx = \int_{-1}^{0} \frac{3^x}{6^x} - \frac{2^x}{6^x} = \int_{-1}^{0} \frac{3^x}{6^x}dx - \frac{2^x}{6^x}dx = \left[-\frac{1}{\ln(2)2^x} - \left(-\frac{1}{\ln(3)3^x}\right) + C = \frac{-\ln(3)3^x - \ln(2)2^x}{\ln 2\ln 3\, 2^x 3^x}\right.$$

$$\left.+ C = \frac{-\ln(3)3^x - \ln(2)2^x}{\ln 2\ln 3\, 2^x 3^x}\right]\Big|_{-1}^{0} = -\frac{\ln(3)3^0 - \ln(2)2^0}{\ln 2\ln 3\, 2^0 3^0} - \left(-\frac{\ln 3\, 3^{-1} - \ln 2\, 2^{-1}}{\ln 2\ln 3\, 2^{-1}3^{-1}}\right) = \frac{1}{\ln 2} - \frac{2}{\ln 3}$$

9.1.34

$$\int_0^{\pi/3} \operatorname{tg} x \ln\cos x\, dx = \left[u = \ln(\cos x), -du = \frac{\sin x}{\cos x}dx\right] = \int_{0}^{\pi/3} -u\, du = \left[-1\cdot\frac{u^2}{2} = -\frac{u^2}{2} =\right.$$

$$\left.= -\frac{\ln^2(\cos x)}{2} + C\right] = -\frac{\ln^2(\cos x)}{2}\Big|_0^{\pi/3} = -\frac{\ln^2\cos\frac{\pi}{3}}{2} - \left(-\frac{\ln^2\cos 0}{2}\right) = \frac{-\ln^2 2}{2}$$

9.1.35

$$\int_0^2 \frac{x\,dx}{\sqrt{x+2} + \sqrt{3x+2}} = \int_0^2 \frac{x\sqrt{3x+2} - x\sqrt{x+2}}{2x}dx = \frac{1}{2}\int_0^2 \sqrt{3x+2} - \sqrt{x+2}\,dx = \frac{1}{2}\left(\int_0^2\sqrt{3x+2}\,dx\right.$$

$$\left.- \int_0^2\sqrt{x+2}\,dx\right) = \left[\frac{1}{2}\cdot\left(-\left(\frac{2x\sqrt{x+2}}{3} + \frac{4\sqrt{x+2}}{3}\right) + \frac{2x\sqrt{3x+2}}{9} + \frac{4\sqrt{3x+2}}{9}\right) + C = \frac{(3x+2)^{\frac{3}{2}}}{9} +\right.$$

$$\left.+ \frac{(-3x-6)\sqrt{x+2}}{9} + C\right] = \frac{(3x+2)^{\frac{3}{2}}}{9} + \frac{(-3x-6)\sqrt{x+2}}{9}\Big|_0^{2} = \frac{(3\cdot 2+2)^{\frac{3}{2}}}{9} + \frac{(-3\cdot 2-6)\sqrt{2+2}}{9} -$$

$$- \frac{(3\cdot 0+2)^{\frac{3}{2}}}{9} + \frac{(-3\cdot 0-6)\sqrt{0+2}}{9} = \frac{5\cdot 4\sqrt{2}}{9} - \frac{8}{3}$$

9.1.66

$$\int_0^{\pi/2} \frac{5\,dx}{\cos x + 1}dx = 5\int_0^{\pi/2} \frac{1}{2\cos^2(\frac{x}{2})}dx = \left[u = \frac{x}{2}, dx = 2du\right] = 5\int_0^{\pi/2} \frac{1}{\cos^2 u}du = [5\operatorname{tg} u + C =$$

$$= 5\,tg\,u + C = 5\,tg\,\frac{x}{2} + C] = 5\,tg\,\frac{x}{2}\Big|_0^{\frac{\pi}{2}} = 5\,tg\,\frac{\frac{\pi}{2}}{2} - 5\,tg\,\frac{0}{2} = 5$$

9.1.67

$$\int_0^{\ln 4} \sqrt{e^x - 1}\,dx = [u = e^x,\ du = e^x dx] = \int_0^{\ln 4} \frac{\sqrt{u-1}}{u}\,du = [v = \sqrt{u-1},\ du = 2v\,dv] =$$

$$= \int_0^{\ln 4} \frac{2v^2}{v^2+1}\,dv = 2\int_0^{\ln 4} 1 - \frac{1}{v^2+1}\,dv = 2\left(-\int_0^{\ln 4} \frac{1}{v^2+1}\,dv + \int_0^{\ln 4} dv\right) = [2(-arctg(v) + v) + C =$$

$$= 2v - 2\,arctg\,v + C = 2\sqrt{u-1} - 2\,arctg(\sqrt{u-1}) = 2\sqrt{e^x-1} - 2\,arctg(\sqrt{e^x-1}) + C] =$$

$$= 2\sqrt{e^x-1} - 2\,arctg(\sqrt{e^x-1})\Big|_0^{\ln 4} = 2\sqrt{e^{\ln 4}-1} - 2\,arctg\sqrt{e^{\ln 4}-1} - (2\sqrt{e^0-1} - 2\,arctg\sqrt{e^0-1}) =$$

$$= 2\sqrt{3} - \frac{2\pi}{3}$$

9.1.68

$$\int_0^{\pi/2} \frac{1}{1+\sin x + \cos x}\,dx = [u = tg(\tfrac{x}{2}),\ du = \frac{1}{2\cos^2(\frac{x}{2})}dx] = \int_0^{\pi/2} \frac{1}{u+1}\,du = (\ln(|u+1|) = \ln(|tg(\tfrac{x}{2})+1|$$

$$+ C] = \ln(|tg\,\tfrac{x}{2}+1|)\Big|_0^{\frac{\pi}{2}} = \ln(|tg\,\frac{\frac{\pi}{2}}{2}+1|) - (\ln(|tg\,\tfrac{0}{2}+1|)) = \ln 2$$

9.1.70

$$\int_0^2 3x(1-x)^{17}dx = [u = 1-x,\ dx = -du] = 3\int_0^2 -(1-u)u^{17}du = -3\int u^{17} - u^{18}du = -3\left(-\int_0^2 u^{18}du + \int_0^2 u^{17}du\right) = \left(-3\left(-\left(\frac{u^{19}}{19}\right) + \frac{u^{18}}{18}\right)\right) + C = \frac{3u^{19}}{19} - \frac{u^{18}}{6} + C = \frac{3(1-x)^{19}}{19} - \frac{(1-x)^{18}}{6}\Big|_0^2 = \frac{3(1-2)^{19}}{19} -$$

$$- \frac{(1-2)^{18}}{6} - \left(\frac{3(1-0)^{19}}{19} - \frac{(1-0)^{18}}{6}\right) = -\frac{37}{114}$$

9.1.100

$$\int_{\pi/6}^{\pi/2} \frac{x\,dx}{\sin^2 x} = [u = x,\ dv = \frac{1}{\sin^2 x},\ du = dx,\ v = -ctg\,x] = -x\,ctg\,x - \int_{\pi/6}^{\pi/2} (-ctg(x))\,dx =$$

$$= -x\cdot ctg\,x + \int_{\pi/6}^{\pi/2} ctg\,x\,dx = [-x\cdot ctg\,x + \ln\sin x + C] = -x\cdot ctg\,x + \ln\sin x\Big|_{\pi/6}^{\pi/2} =$$

$$= -\frac{\pi}{2}ctg\frac{\pi}{2} + \ln\sin\frac{\pi}{2} - \left(-\frac{\pi}{6}ctg\frac{\pi}{6} + \ln\sin\frac{\pi}{6}\right) = 0 - \left(-\ln 2 - \frac{\pi}{2\sqrt{3}}\right) = \ln 2 + \frac{\pi}{2\sqrt{3}}$$

9.1.101

$$\int_0^{0.2} x e^{5x}dx = [u = x,\ dv = e^{5x}dx,\ du = dx,\ v = \tfrac{1}{5}e^{5x}] = \frac{1}{5}xe^{5x}\cdot dx - \int_0^{0.2}\frac{1}{5}e^{5x}dx =$$

$$= [\tfrac{1}{5}xe^{5x} - \tfrac{1}{25}e^{5x} + C] = \frac{(5x-1)\cdot e^{5x}}{25}\Big|_0^{0.2} = \frac{(5\cdot 0.2-1)\cdot e^{5\cdot 0.2}}{25} - \frac{(5\cdot 0-1)\cdot e^{5\cdot 0}}{25} = 0 - \left(-\frac{1}{25}\right) = 0.04$$

9.1.103

$$\int_1^{e^2} \ln^2 x\,dx = [u = \ln^2 x,\ dv = dx,\ du = 2\frac{\ln x}{x}dx,\ v = x] = x\cdot\ln^2 x - \int_1^{e^2} 2\ln x\,dx =$$

$$= [u = \ln x,\ dv = 2dx,\ du = \tfrac{1}{x}dx,\ v = 2x] = \int_1^{e^2} 2\ln x\,dx = 2x\ln x - \int_1^{e^2} 2dx = x\cdot(\ln^2 x - 2\ln x$$

$$+ 2)\Big|_1^{e^2} = e^2(\ln^2 e^2 - 2\ln e^2 + 2) - (1\cdot(\ln^2 1 - 2\ln 1 + 2)) = 2e^2 - 2$$

9.1.105

$$\int_0^1 \frac{x^3}{\sqrt{1+x^2}}\,dx = [2x\,dx = dx^2,\ t = x^2] = \int_0^1 \frac{t}{2\sqrt{t+1}}\,dt = \int_0^2 \frac{x-1}{2\sqrt{x}}\,dx = \int_0^2 \frac{1}{2}\sqrt{x}\,dx + \int_0^2 -\frac{1}{2\sqrt{x}}\,dx =$$

$$= \left[\frac{1}{3}\cdot x^{\frac{3}{2}} - \sqrt{x} + C = \frac{1}{3}(x^2+1)^{\frac{3}{2}} - \sqrt{x^2+1} + C = \frac{1}{3}(x^2+1)^{\frac{3}{2}} - \sqrt{x^2+1} + C\right] = \frac{(x^2+1)^{\frac{3}{2}}}{3} - \sqrt{x^2+1}\,\Big|_0^{2}$$

$$= \frac{(2^2+1)^{\frac{3}{2}}}{3} - \sqrt{2^2+1} - \left(\frac{(0+1)^{\frac{3}{2}}}{3} - \sqrt{0+1}\right) = \frac{2\sqrt{5}}{3} + \frac{2}{3}$$

9.1.106

$$\int_0^{\sqrt{3}} \frac{x^2}{(1+x^2)^2}\,dx = \left[u=x,\ dv=\frac{x}{(x^2+1)^2}\,dx,\ du=dx,\ v=-\frac{1}{2(x^2+1)}\right] = -\frac{x}{2x^2+2} - \int_0^{\sqrt{3}} -\frac{1}{2x^2+2}\,dx =$$

$$-\frac{x}{2x^2+2} + \int_0^{\sqrt{3}} \frac{1}{2x^2+2}\,dx = -\frac{x}{2x^2+2} + \frac{1}{2}\operatorname{arctg} x + C\,\Big|_0^{\sqrt{3}} = -\frac{\sqrt{3}}{2(\sqrt{3})^2+2} + \frac{\operatorname{arctg}\sqrt{3}}{2} - \frac{-0}{2(0)^2+2} + \frac{\operatorname{arctg}}{2}$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{8}$$

9.1.108

$$\int_0^{9} e^{\sqrt{x}}\,dx = [x=t^2,\ dx=2t\,dt] = \int_0^{9} 2x\,e^{x}\,dx = 2\int_0^{9} x e^{x}\,dx = [u=x,\ dv=e^{x}\,dx,\ du=dx,$$

$$v=e^{x}] = 2\cdot\left(x\cdot e^{x} - \int_0^{9} e^{x}\,dx\right) = [2\cdot(x-1)\cdot e^{x} + C] = (2\sqrt{x}-2)\,e^{\sqrt{x}}\Big|_0^{9} = (2\sqrt{9}-2)e^{\sqrt{9}} - (2\sqrt{0}-2)e^{\sqrt{0}} =$$

$$= 4e^{3}+2$$