Домашняя работа. Часть 3 Тюков Артем.
ИВТ-21

8.3.19.

$$\int \frac{5\,dx}{x^2+\sqrt{2}} = \left[\text{1 тип: } A=5,\ a=\sqrt{2}\right] = 5\ln|x+\sqrt{2}| + C$$

8.3.20

$$\int \frac{4\,dx}{(x-0{,}5)^3} = \left[\text{2 тип: } A=4,\ a=0{,}5,\ k=3\right] = \frac{4}{1-3}\cdot\frac{1}{(x-0{,}5)^{3-1}} + C = -\frac{2}{(x-0{,}5)^2} + C$$

8.3.21

$$\int \frac{7\,dx}{(x+3)^6} = \left[\text{2 тип: } A=7;\ a=-3,\ k=6\right] = \frac{7}{1-6}\cdot\frac{1}{(x+3)^{6-1}} + C$$

8.3.24

$$\int \frac{0\,dx}{x^2+x+1} = \left[\text{3 тип: } A=0;\ B=1;\ p=1;\ q=1;\ y=x+\frac{p}{2}=x+\frac{1}{2}\Rightarrow dy=dx\right] = \int\frac{dy}{y^2+\frac{3}{4}} = \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2y}{\sqrt{3}} + C$$

8.3.25

$$\int \frac{(6x+1)\,dx}{x^2-8x+25} = \left[\text{3 тип: } A=6;\ B=1;\ p=-8,\ q=25\right] = \int\frac{3(2x-8)+(1+24)}{x^2-8x+25}\,dx = 3\int\frac{(2x-8)\,dx}{x^2-8x+25} + 25\int\frac{dx}{x^2-8x+25} =$$

$$= \left[\begin{array}{l}1)\ t=x^2-8x+25 \Rightarrow dt=(2x-8)\,dx\\ 2)\ y=x-4 \Rightarrow dy=dx\end{array}\right] = 3\int\frac{dt}{t} + 25\int\frac{dy}{y^2+9} = 3\ln|t| + 25\cdot\frac{1}{3}\operatorname{arctg}\frac{y}{3}$$

8.3.27

$$\int \frac{x+2}{x^2+3x+5}\,dx = \left[\text{3 тип: } A=1,\ B=2,\ p=3,\ q=5\right] = 0{,}5\int\frac{(2x+3)\,dx}{x^2+3x+5} + 0{,}5\int\frac{dx}{x^2+3x+5} = \left[\begin{array}{l}1)\ t=x^2+3x+5 \Rightarrow dt=(2x+3)\,dx\\ 2)\ y=x+1{,}5 \Rightarrow dx=dy\end{array}\right] =$$

$$= 0{,}5\int\frac{(2x+3)\,dt}{t(2x+3)} + 0{,}5\int\frac{dy}{y^2+2{,}75} = 0{,}5\ln|t| + 0{,}5\,\frac{1}{\sqrt{2{,}75}}\cdot\operatorname{arctg}\frac{y}{\sqrt{2{,}75}} + C$$

8.3.28

$$\int\frac{2x-1}{5x^2+2x+1}\,dx = \int\frac{(2x-1)\,dx}{5(x^2+0{,}4x+0{,}2)} = \frac{1}{5}\int\frac{2x-1}{x^2+0{,}4x+0{,}2}\,dx = \left[\text{3 тип: } A=2,\ B=-1,\ p=0{,}4,\ q=0{,}2\right] = \frac{1}{5}\int\frac{1(2x+0{,}4)+(-1-0{,}4)}{x^2+0{,}4x+0{,}2}\,dx = \frac{1}{5}\int\frac{(2x+0{,}4)\,dx}{x^2+0{,}4x+0{,}2} - \frac{7}{25}\int\frac{dx}{x^2+0{,}4x+0{,}2} = \left[\begin{array}{l}1)\ t=x^2+0{,}4x+0{,}2 \Rightarrow dt=(2x+0{,}4)\,dx\\ 2)\ y=x+0{,}2 \Rightarrow dx=dy\end{array}\right] =$$

$$= \frac{1}{5}\int\frac{(2x+0{,}4)\,dt}{t(2x+0{,}4)} - \frac{7}{25}\int\frac{dy}{y^2+0{,}16} = \frac{1}{5}\ln|t| - \frac{7}{25}\cdot\frac{1}{0{,}4}\operatorname{arctg}\frac{y}{0{,}4} + C =$$

$$= \frac{1}{5}\ln|t| - \frac{7}{25}\cdot\frac{1}{0{,}4}\operatorname{arctg}\frac{y}{0{,}4} + C = \frac{\ln|x^2+0{,}4x+0{,}2|}{5} + 0{,}7\operatorname{arctg}$$

$$\frac{x+0{,}2}{0{,}4} + C$$

8.3.30

$$\int\frac{2x+1}{(x^2+2x+5)^2}dx=[\text{4 тип}: A=2, B=1; p=2; q=5; n=2]=\int\frac{1(2x+2)+(1-2)dx}{(x^2+2x+5)^2}=$$

$$=\int\frac{(2x+2)dx}{(x^2+2x+5)^2}-\int\frac{dx}{(x^2+2x+5)^2}=\left[\begin{matrix}1) t=x^2+2x+5\Rightarrow dt=(2x+2)dx\\2) y=x+1\Rightarrow dy=dx\end{matrix}\right]=\int\frac{dt}{t^2}-\int\frac{dy}{(y^2+4)^2}=-\frac{1}{x^2+2x+5}+$$

$$+\frac{1}{2\cdot4}\cdot\frac{y}{y^2+4}+\frac{1}{4}\cdot\frac{4-3}{4-2}\cdot\int\frac{dy}{y^2+4}=-\frac{1}{x^2+2x+5}+\frac{y}{8(y^2+4)}+\frac{1}{8}\cdot\frac{1}{2}\operatorname{arctg}\frac{y}{2}+C=-\frac{1}{x^2+2x+5}+\frac{x+1}{8((x+1)^2+4)}+$$

$$+\frac{1}{16}\operatorname{arctg}\frac{x+1}{2}+C$$

8.3.31

$$\int\frac{dx}{(x^2+1)^4}=[\text{4 тип}: A=0; B=1; p=0; q=1; n=4]=\frac{1}{2\cdot3\cdot1}\cdot\frac{x}{(x^2+1)^3}+\frac{1}{1}\cdot\frac{8-3}{8-2}\int\frac{dx}{(x^2+1)^3}=\frac{1}{6(x^2+1)^3}+$$

$$+\frac{5}{6}\int\frac{dx}{(x^2+1)^3}=\frac{1}{6(x^2+1)^3}+\frac{5}{6}\left(\frac{1}{2\cdot2\cdot1}\cdot\frac{x}{(x^2+1)^2}+\frac{1}{1}\cdot\frac{6-3}{6-2}\int\frac{dx}{x^2+1}\right)=\frac{1}{6(x^2+1)^3}+\frac{5x}{24(x^2+1)^2}+\frac{15}{24}\int\frac{dx}{x^2+1}=\frac{1}{6(x^2+1)^3}+$$

$$\frac{5x}{24(x^2+1)^2}+\frac{5}{8}\operatorname{arctg}x+C$$

8.3.32

$$\int\frac{(3x+2)dx}{(x^2-3x+3)^2}=[\text{4 тип}: A=3; B=2; p=-3; q=3; n=2]=\int\frac{1{,}5(2x-3)+(2+4{,}5)dx}{(x^2-3x+3)^2}=$$

$$=1{,}5\int\frac{(2x-3)dx}{(x^2-3x+3)^2}+6{,}5\int\frac{dx}{(x^2-3x)^2}=\left[\begin{matrix}1) t=x^2-3x+3\Rightarrow dt=(2x-3)dx\\2) y=x-1{,}5\Rightarrow dx=dx\end{matrix}\right]=1{,}5\int\frac{dt}{t^2}+6{,}5\int\frac{dy}{(y^2+0{,}75)^2}$$

$$=1{,}5\ln|t|+6{,}5\left(\frac{1}{2\cdot0{,}75}\cdot\frac{y}{y^2+0{,}75}+\frac{1}{0{,}75}\cdot\frac{4-3}{4-2}\cdot\int\frac{dy}{y^2+0{,}75}\right)=1{,}5\ln|x^2-3x+3|+\frac{6{,}5}{1{,}5(x^2-3x+3)}+\frac{13}{3\sqrt{3}}$$

$$\cdot\operatorname{arctg}\frac{2(x-1{,}5)}{\sqrt{3}}+C$$

8.3.35

$$\int\frac{x\,dx}{x^2-4x-5}=\int\frac{x\,dx}{(x+1)(x-5)}=\left[\frac{x}{(x+1)\cdot(x-5)}=\frac{A}{x+1}+\frac{B}{x-5}\right.$$

$$x=(x-5)A+B(x+1)$$

$$x=Ax-5A+Bx+B$$

$$x=x(A+B)+B-5A$$

$$\begin{cases}A+B=1\\B-5A=0\end{cases}\qquad B=1-A\qquad 1-A-5A=0\qquad 6A=1\qquad A=1/6\qquad B=1-\frac{1}{6}=5/6$$

$$\Rightarrow\frac{x}{(x+1)(x-5)}=\frac{1}{6(x+1)}+\frac{5}{6(x-5)}\Big]=\frac{1}{6}\int\frac{dx}{x+1}+\frac{5}{6}\int\frac{dx}{x-5}=\left[\begin{matrix}1) t_1=x+1\Rightarrow dt_1=dx\\2) t_2=x-5\Rightarrow dt_2=dx\end{matrix}\right]=\frac{1}{6}\int\frac{dt_1}{t_1}+\frac{5}{6}\int\frac{dt_2}{t_2}$$

$$=\frac{1}{6}\ln|x+1|+\frac{5}{6}\ln|x-5|+C$$

8.3.36

$$\int \frac{2x^2-11\,dx}{x^2+x-6} = \left[\frac{-2x^2+2x-11}{2x^2+2x+12} \Big| \frac{x^2+x+6}{2} \quad \frac{2x^2-11}{x^2+x-6} = \frac{2(x^2+x+6)-2x^{-13}}{x^2+x-6} \right] =$$

$-2x-13$

$$= 2\int \frac{x^2+x+6}{x^2+x+6}dx - \int \frac{2x+23}{x^2+x+6}dx = 2x - \int \frac{2x+23}{x^2+x-6}dx = [\text{Зчит: } A=2,\ B=23,\ p=1,\ q=-6$$

$$t = x^2+x-6 \Rightarrow dt = (2x+1)dx] = 2x - \int \frac{2x+1+22}{x^2+x+6}dx = 2x - \int \frac{(2x+1)dx}{x^2+x-6} - 22\int\frac{dx}{x^2+x-6} =$$

$$= 2x - \int \frac{(2x+1)dx}{t(2x+1)} - 22\int \frac{dx}{x^2+x+6} = [y = x+0{,}5 \Rightarrow dy = dx] = 2x - \ln|t| + C - 22\int\frac{dx}{x^2+x-6} =$$

$$= 2x - \ln|x^2+x-6| - 22\,\frac{1}{2\cdot 5/2}\ln\left|\frac{y-2}{y+5/2}\right| + C = 2x - \ln|x^2+x-6| - \frac{22}{5}\ln\left|\frac{x+0{,}5-2{,}5}{x+0{,}5+2{,}5}\right|$$

$$+ C = 2x - \ln|x^2+x-6| - 4{,}4\ln\left|\frac{x-2}{x+3}\right| + C$$

8.3.37

$$\int \frac{-3x^2+x+19}{(x-4)(x-2)(x+1)} = \frac{A}{x-4} + \frac{B}{x-2} + \frac{C}{x+1}$$

$$-3x^2+x+19 = A(x-2)(x+1) + B(x-4)(x+1) + C(x-4)(x-2)$$

$$-3x^2+x+19 = Ax^2 - Ax - 2A + Bx^2 - 3Bx - 4B + Cx^2 - 6Cx + 8C$$

$$-3x^2+x+19 = x^2(A+B+C) + x(-A-3B-6C) - 2A - 4B + 8C$$

$$\begin{cases} A+B+C=-3 \\ -A-3B-6C=1 \\ -2A-4B+8C=19 \end{cases} \quad \begin{cases} A=-3-B-C \\ 3+B+C-3B-6C=1 \\ 2(3+B+C)-4B+8C=19 \end{cases}$$

$$\begin{cases} -2B-5C=1 \\ -2B+10C=13 \end{cases} \quad \begin{cases} -2B=1+5C \\ 1+5C+10C=13 \end{cases}$$

$$B = \frac{1+5C}{-2} = \frac{1+4}{-2} = -2{,}5$$

$$A = -3+2{,}5-0{,}8 = -1{,}3] = 1{,}3\int\frac{dx}{x-4} - 2{,}5\int\frac{dx}{x-2} + 0{,}8\int\frac{dx}{x+1} = \left[\begin{matrix} 1)\ t_1 = x-4 \Rightarrow dt_1 = dx \\ 2)\ t_2 = x-2 \Rightarrow dt_2 = dx \\ 3)\ t_3 = x+1 \Rightarrow dt_3 = dx \end{matrix}\right] = -1{,}3\int\frac{dt_1}{t_1} -$$

$$-2{,}5\int\frac{dt_2}{t_2} + 0{,}8\int\frac{dt_3}{t_3} = -1{,}3\ln|x-4| - 2{,}5\ln|x-2| + 0{,}8|x+1| + C$$

8.3.39

$$\int \frac{(x+2)dx}{(x-1)(x+1)^2} = \left[\frac{x^2+2}{(x-1)(x+1)^2} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}\right.$$

$$x^2+2 = A(x+1)^2 + B(x-1)(x^2+2x+1) + C(x^2-1)$$

$$x^3+2 = x^3(A+B) + x^2(3A+B+C) + x(3A-B) + A-B-C$$

$$\begin{cases} A+B=0 \\ 3A+B+C=0 \\ 3A-B=0 \\ A+B-C=2 \end{cases} \quad \begin{matrix} B=-A \\ \begin{cases} 3A-4C=1 \\ A-A-C=2 \end{cases} \end{matrix} \quad \begin{matrix} C=-2 \\ 2A+2=1 \\ 2A=-1 \end{matrix} \quad \left.\begin{matrix} A=-0{,}5 \\ B=0{,}5 \end{matrix}\right] = \int\frac{-0{,}5}{x-1}dx + \int\frac{0{,}5}{x+1}dx + \int\frac{-2}{(x+1)}dx =$$

$$= \left[\begin{matrix} 1)\ x-1=t_1 \Rightarrow dx = dt \\ 2)\ x+1 = t_2 \Rightarrow dx = dt_2 \\ 3) \end{matrix}\right] = -0{,}5\int\frac{dt_1}{t_1} + 0{,}5\int\frac{dt_2}{t_2} - 2\int\frac{dt}{t_2} = -0{,}5\ln|x-1| - 0{,}5\ln|x+1| -$$

$$-\frac{2}{x+1} + C$$

8.3.40

$$\int\frac{x+3}{(x-2)^3}dx = \left[\frac{2x+3}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{x-2} + \frac{C}{x-2} \quad 2x+3 = Ax^2 + x(-4A+B) + 4A - 2B + C\right.$$

$$\begin{cases} A=0 \\ -4A+B=2 \\ 4A-2B+C=3 \end{cases} \quad -4\cdot 0+B=2 \quad B=2 \quad 4\cdot 0-2\cdot 2+C=3 \quad C=7 \Bigg] = \int 0\,dx + 2\int \frac{dx}{(x-2)^2} + 7\int \frac{dx}{(x-2)^3} = \left[t=x-2 \Rightarrow dt=dx\right]$$

$$= 2\int \frac{dx}{t^2} + 7\int t^{-3}\,dt = -\frac{2}{t} + \frac{7t^{-2}}{-2} + C = \frac{-2}{(x-2)} - \frac{3{,}5}{(x-2)^2} + C$$

8.3.42

$$\int \frac{dx}{x^3-1} = \int \frac{dx}{(x-1)\cdot(x^2+x+1)} = \left[\frac{1}{(x-1)\cdot(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}\right.$$

$$1 = x^2(A+B) + x\cdot(A-B+C) + A-C$$

$$\begin{cases} A+B=0 \\ A-B+C \\ A-C=1 \end{cases} \quad A=-B \quad \begin{cases} -B-B+C=0 \\ -B-C=1 \end{cases}$$

$$-2B+C=0 \quad C=2B \quad -B-2B=1 \quad -3B=1 \qquad B=\tfrac{1}{3} \quad C=-\tfrac{2}{3} \quad A=-\tfrac{1}{3} \Bigg] = \frac{1}{3}\int \frac{dx}{x-1} + \int \frac{-\frac{1}{3}x-\frac{2}{3}}{x^2+x+1} = \left[3\text{ тип}: A=-\tfrac{1}{3}; B=-\tfrac{2}{3}, p=1, q=1\right]$$

$$= \frac{1}{3}\ln|x-1| + C - \frac{1}{6}\cdot\int \frac{2x+1}{x^2+x+1}dx - \frac{1}{2}\int \frac{dx}{x^2+x+1} = \begin{matrix} 1)\ t=x^2+x+1 \Rightarrow dt=(2x+1)\,dx \\ 2)\ y=x+0{,}5 \Rightarrow dx=dy \end{matrix}, \quad a=\sqrt{0{,}75} = \frac{1}{3}\ln|x-1| + C -$$

$$-\frac{1}{6}\int \frac{(2x+1)\,dt}{t(2x+1)} - \frac{1}{2}\int \frac{dy}{y^2+0{,}75} = \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| - \frac{1}{3}\operatorname{arctg}\frac{2(x+0{,}5)}{\sqrt{3}} + C$$

8.3.45

$$\int \frac{x\,dx}{(x^2-1)(x^2+1)} = \left[\frac{x}{(x^2-1)\cdot(x^2+1)} = \frac{Ax+B}{x^2-1} + \frac{Cx+D}{x^2+1} \Rightarrow x = x^3\cdot(A+C) + x^2(B+D) + x(A-C) + B-D\right.$$

$$\begin{cases} A+C=0 \\ B-D=0 \\ B+D=0 \\ A-C=1 \end{cases} \quad -2C=1 \quad C=-0{,}5 \quad A=0{,}5 \quad B=-D \quad -2D=0 \quad D=0 \Rightarrow B=0 \Bigg] = 0{,}5\int \frac{x\,dx}{x^2-1} - 0{,}5\cdot\int \frac{x\,dx}{x^2+1} = \begin{bmatrix} 1)\ t_1=x^2-1 \Rightarrow dt_1=2x\,dx \\ 2)\ t_2=x^2+1 \Rightarrow dt_2=2x\,dx \end{bmatrix} =$$

$$= 0{,}5\int \frac{x\,dt}{t_1\cdot 2x} - 0{,}5\int \frac{x\,dt_2}{t_2\,2x} = 0{,}25\ln|x^2-1| - 0{,}25\ln(x^2+1)$$

8.3.45

$$\int \frac{2x^2-3x-3}{(x^2-2x+5)(x-1)}dx = \left[\frac{2x^2-3x-3}{(x^2-2x+5)(x-1)} = \frac{Ax+B}{x^2-2x+5} + \frac{C}{x-1} \Rightarrow 2x^2-3x-3 = x^2(A+C) + x(-A+B-2C) - B+5C\right.$$

$$\begin{cases} A+C=2 \\ -A+B-2C=-3 \\ -3=-B+5C \end{cases} \quad A=2-C \quad \begin{cases} -2+C+B-2C=-3 \\ -3=-B+5C \end{cases} \quad B-C=-1 \quad B=-1+C \quad -3=1-C+5C \quad 4C=-4 \quad C=-1 \quad B=-2 \quad A=3 \Bigg] = \int \frac{3x-2}{x^2-2x+5} - \int \frac{dx}{x-1} =$$

$$\left[3\text{ тип}: A=3; B=-2, p=-2; q=5\right] = 1{,}5\int \frac{(2x-2)\,dx}{x^2-2x+5} + \int \frac{dx}{x^2-2x+5} - \int \frac{dx}{x-1} = \begin{bmatrix} 1)\ t=x^2-2x+5 \Rightarrow dt=(2x-2)\,dx \\ 2)\ y=x-1 \Rightarrow dx=dy \end{bmatrix} =$$

$$= 1{,}5\int \frac{(2x-2)\,dx}{(2x-2)\,t} + \int \frac{dy}{y^2+4} + \int \frac{dx}{x-1} = 1{,}5\ln|t| + 0{,}5\operatorname{arctg}\frac{y}{2} + \ln|x-1| + C = 1{,}5\ln|x^2-2x+5| + 0{,}5\operatorname{arctg}$$

$$\frac{x-1}{2} + \ln|x-1| + C$$

8.3.46

$$\int \frac{(x^4+x^3+x^2+x+1)\,dx}{(x^2+1)^2 x} \left[\frac{x^4+x^3+x^2+x+1}{(x^2+1)^2 x} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{E}{x} = x^4+x^3+x^2+x+1 = x^4(A+E) + x^3 B + x^2(A+C+2E) + x(B+D) + E\right.$$

$t = dx$ $\begin{cases} A+E=1 \\ E=1 \\ B=1 \\ A+C+2E=1 \\ B+D=1 \end{cases}$ $\begin{cases} A+1=1 \\ A+C+2=1 \\ 1+D=1 \end{cases}$ $A=0 \quad C=1-2-0=-1 \quad D=0$ $\Big] = \int \frac{dx}{x^2+1} + \int \frac{-x\,dx}{(x^2+1)^2} + \int \frac{dx}{(x^2+1)^2} + \int \frac{dx}{x} = [t = x^2+1 \Rightarrow dt = 2x\,dx] =$

$= \int \frac{dx}{x^2+1} - 0{,}5 \int \frac{2x\,dt}{t^2\,2x} + \int \frac{dx}{(x^2+1)^2} + \int \frac{dx}{x} = \int \frac{dx}{x^2+1} - 0{,}5 \int \frac{dt}{t^2} + \frac{1}{2} \cdot \frac{x}{x^2+1} +$

$+ \frac{1}{2} \int \frac{dx}{x^2+1} + \int \frac{dx}{x} = \operatorname{arctg} x + \frac{0{,}5}{x^2+1} + \frac{x}{2(x^2+1)} + 0{,}5 \operatorname{arctg} x + \ln|x|$

8.3.48

$\int \frac{3x+5}{x(x^2+1)^2}\,dx = \Big[\frac{3x+5}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} = 3x+5 = x^4(A+B) + Cx^3 + x^2(B+D) + x(C+E) + A \Rightarrow$

$\begin{cases} A+B=0 \\ C=0 \\ B+D=0 \\ C+E=3 \\ A=5 \end{cases}$ $\begin{matrix} A=5 \\ B=-5 \\ C=0 \\ D=5 \\ E=3 \end{matrix}$ $\Big] = 5\int \frac{dx}{x} - 5\int \frac{x}{x^2+1}\,dx + \int \frac{5x+3}{(x^2+1)} = [t = x^2+1 \Rightarrow dt = 2x\,dx] = 5\ln|x| - 5\int \frac{x\,dx}{t \cdot 2x} +$

$+ \int \frac{5x+3}{(x^2+1)^2} + C = 5\ln|x| - 2{,}5\ln|t| + \int \frac{5x+3}{(x^2+1)^2} + C = [\text{4 тип}; A=5,\ B=3,\ p=0,\ q=1,\ m=2] =$

$= 5\ln|x| - 2{,}5\ln(x^2+1) + 2{,}5 \int \frac{2x\,dx}{(x^2+1)^2} + 3\int \frac{dx}{(x^2+1)^2} = [t = x^2+1 \Rightarrow dt = 2x\,dx] = 5\ln|x| - 2{,}5\ln(x^2+1) + 2{,}5$

$\int \frac{dt}{t^2} + 3\Big[\frac{x}{2(x^2+1)} + \frac{1}{2}\int \frac{dx}{x^2+1}\Big] = 5\ln|x| - 2{,}5\ln(x^2+1) - \frac{2{,}5}{(x^2+1)^2} + \frac{1{,}5}{x^2+1} + 1{,}5 \operatorname{arctg} x + C$

8.3.50

$\int \frac{dt}{t^4-1} = \int \frac{dt}{(t^2-1)(t^2+1)} = \Big[\frac{1}{(t^2-1)(t^2+1)} = \frac{At+B}{t^2-1} + \frac{Ct+D}{t^2+1} \Rightarrow 1 = t^3(A+C) + t^2(B+D) + t(A-C) + B - D \Rightarrow$

$\begin{cases} A+C=0 \\ B+D=0 \\ A-C=0 \\ B-D \end{cases}$ $\begin{matrix} D=-B \\ -2D=-1 \\ D=0{,}5 \\ B=0{,}5 \end{matrix}$ $\begin{matrix} A=-C \\ A=C \end{matrix} \rightarrow \begin{matrix} C=-C \\ C=0 \end{matrix} \quad A=0 \Big] = \int \frac{0{,}5\,dt}{t^2-1} + \int \frac{-0{,}5\,dt}{t^2+1} = 0{,}5\int \frac{dt}{t^2-1} - 0{,}5\int \frac{dt}{t^2+1} = \frac{0{,}5}{2}$

$\ln\left|\frac{t-1}{t+1}\right| - 0{,}5 \operatorname{arctg} t + C = 0{,}25 \ln\left|\frac{t-1}{t+1}\right| - 0{,}5 \operatorname{arctg} t + C$

8.3.51

$\int \frac{e^{2x}\,dx}{e^{2x}+3e^x+2} = [e^x = t \Rightarrow dt = e^x dx] = \int \frac{t^2 \cdot dt}{t(t^2+3t+2)} = \int \frac{t\,dt}{(t+2)(t+1)} = \Big[\frac{t}{(t+2)(t+1)} = \frac{A}{t+2} + \frac{B}{t+1}$

$t = t(A+B) + A + 2B \Rightarrow \begin{cases} A+B=1 & A=1-B \\ A+2B=0 & 1+B+2B=0 \end{cases} \quad B=-1 \Rightarrow A=2 \Big] = \int \frac{2\,dt}{t+2} + \int \frac{-1\,dt}{t+1} =$

$= 2\int \frac{dt}{t+2} - \int \frac{dt}{t+1} = 2\ln|t+2| - \ln|t+1| = 2\ln|e^x+2| - \ln|e^x+1|$

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$\int \frac{\cos x\,dx}{(\sin x-1)(\sin x+2)} = [\sin x = t \Rightarrow dt = \cos x\,dx] = \int \frac{\cos x\,dt}{(t-1)\cdot(t+2)\cos x} = \int \frac{dt}{(t-1)(t+2)} = \Big[\frac{1}{(t-1)\cdot(t+2)} =$

$= \frac{A}{(t-1)} + \frac{B}{t+2} \Rightarrow 1 = t(A+B) + 2A - B$

$$\begin{cases} A=3=0 & A=-B & -3B=1 \\ 2A-B=-1 & -2B-B=1 & B=-\frac{1}{3} \Rightarrow A=\frac{1}{3} \end{cases} = \frac{1}{3}\int\frac{dt}{t-1} - \frac{1}{3}\int\frac{dt}{t+2} = \frac{1}{3}\ln|t-1| = \frac{1}{3}\ln|t-2| + C = \frac{1}{3}\ln|\sin x - 1| -$$

$$-\frac{1}{3}\ln(\sin x+2)+C$$

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$$\int\frac{\sin^4 x\,dx}{\cos x} = [\sin^2 x = a;\ da = 2\cos x\,dx] = \int\frac{a^2\,da}{2\cos^2 x} = \frac{1}{2}\int\frac{a^2\,da}{1-a} = \left[\begin{array}{l} a^2+0a+0 \;|\; -a+1 \\ a^2-a \qquad\ \ |\; -a-1 \\ \hline a+0 \\ a-1 \\ \hline 1 \end{array}\right. \Rightarrow$$

$$\Rightarrow a^2 = (1-a)(-a-1)+1] = \frac{1}{2}\int\frac{(1-a)(a-1)+1}{1-a}\,da = \frac{1}{2}\int(-a-1)\,da + \frac{1}{2}\int\frac{da}{1-a} =$$

$$= -\frac{1}{2}\int a\,da - \frac{1}{2}\int da + \frac{1}{2}\frac{da}{1-a} = -0{,}5a^2 - \frac{a}{2} + \frac{1}{2}\ln|1-a| + C = -0{,}25\sin^4 x - 0{,}5\sin^2 x +$$

$$+0{,}5\ln|\cos^2 x| + C$$