Домашняя работа Часть 5.

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8.5.19

$$\int\frac{dx}{\cos x} = \ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + C$$

8.5.20

$$\int\frac{dx}{1-\sin x} = \int\frac{dx}{1-\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}} = \left[\begin{matrix} t = tg\frac{x}{2} & t' = \frac{1}{2}(1+tg^2\frac{x}{2}) \\ dx = \frac{1}{t'}dt & \end{matrix}\right] = \int\frac{1}{1-\frac{2t}{1+t^2}}\cdot\frac{1}{\frac{1}{2}(1+t^2)}dt =$$

$$= \int\frac{1}{\frac{1+t^2-2t}{1+t^2}}\cdot\frac{1}{\frac{1}{2}(1+t^2)}dt = \int\frac{1+t^2}{1+t^2-2t}\cdot\frac{dt}{\frac{1}{2}+\frac{t^2}{2}} = \int\frac{1+t^2}{1+t^2-2t}\cdot\frac{2dt}{1+t^2} = \int\frac{2dt}{1+t^2-2t} =$$

$$= 2\int\frac{dt}{(1-t)^2} = [u=1-t,\ u'=-1\ dt=-du] = -2\int\frac{du}{u^2} = -2\int u^{-2}du = -2\cdot\frac{u^{-1}}{-1}+C =$$

$$= \frac{2}{u}+C = \frac{2}{1-t}+C = \frac{2}{1-tg\frac{x}{2}}+C$$

8.5.21

$$\int\frac{dx}{5+4\sin x} = \int\frac{dx}{5+4\left(\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}\right)} = \left[t=tg\frac{x}{2},\ t'=\frac{1}{2}(1+tg^2\frac{x}{2}),\ dx=\frac{1}{t'}dt\right] = \int\frac{dt}{5+4\left(\frac{2t}{1+t^2}\right)}\cdot$$

$$\cdot\frac{1}{\frac{1}{2}(1+t^2)} = \int\frac{1}{5+\frac{8t}{1+t^2}}\cdot\frac{1}{\frac{1}{2}(1+t^2)}dt = \int\frac{1}{\frac{5(1+t^2)+8t}{1+t^2}}\cdot\frac{1}{\frac{1}{2}(1+t^2)}dt = \int\frac{1+t^2}{5(1+t^2)+8t}\cdot$$

$$\cdot\frac{2dt}{1+t^2} = 2\int\frac{dt}{5(1+t^2)+8t} = 2\int\frac{dt}{5+5t^2+8t} = \frac{2}{5}\int\frac{dt}{t^2+\frac{8}{5}t+\frac{16}{25}+\frac{9}{25}} = \frac{2}{5}\int\frac{dt}{(t+\frac{4}{5})^2+\frac{9}{25}} =$$

$$= \left[u=t+\frac{4}{5},\ u'=1,\ dt=du\right] = \frac{2}{5}\int\frac{dt}{u^2+9/25} = \frac{2}{5}\cdot\frac{5}{3}arctg\frac{u5}{3}+C = \frac{2}{3}arctg\cdot$$

$$\cdot\frac{5tg\frac{x}{2}+4}{3}+C$$

8.5.22

$$\int\frac{2-\sin x}{2+\cos x}dx = \int\frac{2}{2+\cos x}-\frac{\sin x}{2+\cos x}dx = \left[\begin{matrix}u=2+\cos x\\ u'=-\sin x\end{matrix}\right] = 2\int\frac{1}{2+\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}}dx - \int-\frac{1}{u}du$$

$$= \left[t=tg\frac{x}{2},\ t'=\frac{1}{2}(1+tg^2\frac{x}{2})\right] = 2\int\frac{1}{2+\frac{1-t^2}{1+t^2}}\cdot\frac{1}{\frac{1}{2}(1+t^2)}dt + \ln|u| + C = 2\int\frac{1}{\frac{t^2+3}{1+t^2}}\cdot$$

$$\cdot\frac{2}{1+t^2}dt + \ln|u|+C = 2\int\frac{1+t^2}{t^2+3}\cdot\frac{2}{1+t^2}dt + \ln|2+\cos x|+C = 4\int\frac{dt}{t^2+3}+\ln|2+\cos x|$$

$$+C = 4\frac{1}{\sqrt{3}}arctg\frac{t}{\sqrt{3}}+\ln|2+\cos x|+C = \frac{4}{\sqrt{3}}arctg\frac{tg\frac{x}{2}}{\sqrt{3}}+\ln|2+\cos x|+C =$$

$$= \frac{4\sqrt{3}}{3}arctg\frac{\sqrt{3}tg\frac{x}{2}}{3}+\ln|2+\cos x|+C$$

8.5.24

a. $\int \frac{1+\sin x}{(1+\cos x)\cdot\sin x}dx = \int \frac{1+\sin x}{\sin x+\cos x\sin x}dx = \int \frac{dx}{\sin x+\cos x\sin x} + \int \frac{\sin x}{\sin x+\cos x\sin x}dx =$

$\int \frac{dx}{\sin x+\cos x\sin x} + \int \frac{dx}{1+\cos x}\left[t=tg\frac{x}{2},\ t'=\frac{1}{2}(1+t^2)\right] = \int \frac{2}{\left(\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}\cdot\frac{2t}{1+t^2}\right)(1+t^2)}dx +$

$+\int \frac{1}{1+\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}dt = \int \frac{1}{\frac{2t+2t-2t^3+2t^3}{(1+t^2)^2}}\cdot\frac{2}{1+t^2}dt + \int \frac{1+t^2}{2}\cdot\frac{2}{1+t^2}dt =$

$= \int \frac{1+t^2}{2t}dt + t + C = \frac{1}{2}\int \frac{1}{t}dt + \frac{1}{2}\int \frac{t^2}{t}dt + t + C = \frac{1}{2}\ln\left|tg\frac{x}{2}\right| + \frac{tg^2\frac{x}{2}}{4} + tg\frac{x}{2} + C$

8.5.25

$\int \frac{dx}{5\sin^2 x-3\cos^2 x+4} = \left[t=tg\,x,\ dx=\frac{dt}{1+t^2}\right] = \int \frac{dt}{\left(5\cdot\frac{t^2}{1+t^2}-3\frac{1}{1+t^2}+4\right)(1+t^2)} = \int \frac{1+t^2}{5t^2-3+4+4t^2}\cdot\frac{1}{1+t^2}dt =$

$= \int \frac{dt}{9t^2+1} = \frac{1}{9}\int \frac{dt}{t^2+\frac{1}{9}} = \frac{1}{9}\cdot\frac{3}{1}\,arctg\frac{3t}{1} + C = \frac{1}{3}\,arctg\,3tg\,x + C$

8.5.27

$\int \frac{dx}{1+3\cos^2 x} = \left[t=tg\,x,\ dx=\frac{1}{1+t^2}dt\right] = \int \frac{dt}{1+3\frac{1}{1+t^2}}\cdot\frac{1}{1+t^2} = \int \frac{dt}{\frac{1+t^2+3}{1+t^2}}\cdot\frac{1}{1+t^2} = \int \frac{1+t^2}{t^2+4}\cdot\frac{1\,dt}{1+t^2} =$

$= \int \frac{dt}{t^2+4} = \frac{1}{2}\,arctg\frac{t}{2} + C = \frac{1}{2}\,arctg\frac{tg\,x}{2} + C$

8.5.28

$\int \frac{dx}{\sin^4 x} = \left[t=tg\,x,\ dx=\frac{1}{1+t^2}dt\right] = \int \frac{dt}{\left(\frac{1}{1+t^2}\right)^2}\cdot\frac{1}{t^2+1} = \int \frac{(1+t^2)^2}{1}\cdot\frac{1}{t^2+1}dt = \int 1+t^2\,dt =$

$= \int dt + \int t^2dt = t + \frac{t^3}{3} + C = tg\,x + \frac{1}{3}tg^3x + C$

8.5.30

$\int \sin^4 x\cdot\cos^5 x\,dx = \left[t=\sin x,\ dx=\frac{1}{\sqrt{1-t^2}}dt=\frac{1}{\cos x}dt\right] = \int t^4\cdot(1-t^2)^2dt = \int t^4(1-2t^2+t^4)dt =$

$= \int t^4-2t^6+t^8\,dt = \int t^4dt - 2\int t^6dt + \int t^8dt = \frac{1}{5}t^5 - \frac{2}{7}t^7 + \frac{1}{9}t^9 + C = \frac{1}{5}\sin^5 x - \frac{2}{7}\sin^7 x + \frac{1}{9}\cdot$

$\sin^9 x + C$

8.5.31

$\int \frac{\sin 2x\,dx}{\cos^7 x} = \int \frac{2\sin x\cos x}{\cos^7 x}dx = 2\int \frac{\sin x}{\cos^6 x}dx = \left[t=\cos x,\ dx=-\frac{1}{\sin x}dt\right] = -2\int \frac{\sin x}{t^6}\cdot\frac{1}{\sin x}dt =$

$= -2\int t^{-6}dt = -2\frac{t^{-5}}{-5} + C = \frac{2}{5\cos^5 x} + C$

8.5.33

$\int \sin^6 x\,dx = \int\left(\frac{1-\cos 2x}{2}\right)^3dx = \int \frac{(1-\cos 2x)^3}{8}dx = \frac{1}{8}\int (1-3\cos 2x+3\cos^2 2x-\cos^3 2x)dx =$

$\frac{1}{8}x - \frac{3}{8}\sin 2x + \frac{3}{8}\frac{\sin^3 2x}{3} - \frac{1}{8}\frac{\sin^4 2x}{4} + C = \frac{1}{8}x - \frac{3}{8}\sin 2x + \frac{1}{8}\sin^3 2x - \frac{1}{32}\sin^4 2x + C$

8.5.34

$\int \sin^2 x \cos^4 x\,dx = \int \frac{1+\cos 2x}{2}\cdot\frac{(1-\cos 2x)^2}{4}dx = \int \frac{(1+\cos 2x)\cdot(1-\cos 2x)^2}{8}dx = [u=2x,\ u'=2,$
$dx=\frac{1}{2}du] = \frac{1}{16}\int (1+\cos u)\cdot(1-\cos u)^2 du = \frac{1}{16}\int \cos^3 u - \cos^2 u - \cos u + 1\,du = \frac{1}{16}\frac{\cos^4 u}{4} -$
$-\frac{1}{16}\frac{\cos^3 u}{3} - \frac{1}{16}\frac{\cos^2 u}{2} + \frac{u}{16} + C = \frac{\cos^4 2x}{64} - \frac{\cos^3 2x}{48} - \frac{\cos^2 2x}{32} + \frac{x}{8} + C$

8.5.36

$\int \sin x\cdot \sin 3x\,dx = \int \frac{1}{2}(\cos(x-3x) - \cos(x+3x))\,dx = \frac{1}{2}\int \cos 2x - \cos 4x\,dx = [u=2x,$
$dx=\frac{1}{2}du] = \frac{1}{2}\int \frac{\cos u}{2}du - \frac{1}{2}\int \frac{\cos t}{4}dt = \frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x + C$

8.5.37

$\int \sin\frac{x}{12}\cos\frac{x}{3}\,dx = \int \frac{1}{2}(\sin(\frac{x}{12}-\frac{4x}{12}) + \sin(\frac{x}{12}-\frac{4x}{12}))\,dx = \frac{1}{2}\int \sin(-\frac{x}{4}) + \sin(-\frac{x}{4})dx$
$= -\frac{1}{2}\int \sin\frac{x}{4}dx - \frac{1}{2}\int \sin\frac{x}{4}dx = [u=\frac{x}{4},\ u'=\frac{1}{4}\ dx=4\,du] = -\frac{1}{2}\int \sin u\cdot 4\,du - \frac{1}{2}\int \sin u\, 4\,du$
$= -2\int \sin u\,du - 2\int \sin u\,du = 2\cos u + 2\cos u + C = 4\cos\frac{x}{4} + C$

8.5.38

$\int \cos x\cdot\cos 3x\,dx = \int \frac{1}{2}(\cos 2x + \cos 4x)\,dx = \frac{1}{2}\int \cos 2x\,dx + \frac{1}{2}\int \cos 4x\,dx = [u=2x,$
$dx=\frac{1}{2}du,\ t=4x,\ dx=\frac{1}{4}dt] = \frac{1}{2}\int \cos u\cdot\frac{1}{2}du + \frac{1}{2}\int \cos t\cdot\frac{1}{4}du = \frac{1}{4}\sin u + \frac{1}{8}\sin t + C$
$= \frac{1}{4}\sin 2x + \frac{1}{8}\sin 4x + C$

~~8.5.39~~ 8.5.39

$\int \cos x\cdot\cos 3x\cdot\cos 5x\,dx = \int \frac{1}{2}(\cos(-2x) + \cos 4x)\cdot\cos 5x\,dx = \frac{1}{2}\int \cos 2x\cos 5x + \cos 4x\cdot$
$\cdot\cos 5x\,dx = \frac{1}{2}\int \frac{1}{2}(\cos 3x + \cos 7x) + \frac{1}{2}(\cos x + \cos 9x)\,dx = \frac{1}{4}\int \cos 3x + \cos 7x + \cos x + \cos 9x\,dx =$
$= \frac{1}{4}(\frac{\sin 3x}{3} + \frac{\sin 7x}{7} + \sin x + \frac{\sin 9x}{9}) + C = \frac{\sin 3x}{12} + \frac{\sin 7x}{28} + \frac{\sin x}{4} + \frac{\sin 9x}{36} + C$

8.5.41

$\int tg^4\frac{x}{2}dx = [u=\frac{x}{2},\ u'=\frac{1}{2}\ dx=2du] = 2\int tg^4 u\,du = 2\int (\frac{1}{\cos^2 u} - 1)tg^2 u\,du = 2\int \frac{tg^2 u}{\cos^2 u}du$
$-2\int tg^2 u\,du = [t=tg\,u,\ du=\frac{1}{\cos^2 x}dx] = 2\int t^2\,dt - 2\int \frac{1}{\cos^2 u} - du = 2\frac{t^3}{3} - 2\,tg\,u + 2u + C$
$= \frac{2}{3}tg^3 u - 2\,tg\frac{x}{2} + 2\cdot\frac{x}{2} + C = \frac{2}{3}tg^3\frac{x}{2} - 2\,tg\frac{x}{2} + x + C$

8.5.42.

$\int tg^7 x\,dx = \int \frac{\sin^7 x}{\cos^7 x}dx = \int \frac{(1-\cos^2 x)^3\sin x}{\cos^7 x}dx = [u=\cos x,\ dx=-\frac{1}{\sin x}du] = -\int \frac{(1-u^2)^3}{u^7}du =$
$= -\int \frac{-u^6+3u^4-3u^2+1}{u^7}du = -\int -\frac{1}{u} + \frac{3}{u^3} - \frac{3}{u^5} + \frac{1}{u^7}\,du = \ln|u| - 3\frac{u^{-2}}{-2} + 3\frac{u^{-4}}{-4} - \frac{u^{-6}}{-6} + C =$

$$= \ln|\cos x| + \frac{3}{2\cos^2 x} - \frac{3}{4\cos^4 x} + \frac{1}{6\cos^6 x} + C$$