Домашняя работа. Часть I. Грелков Артем
ИВПт 21

8.1.29

$$\int \frac{dx}{x^2\sqrt{x}} = \int x^{-2\frac{1}{2}} dx = \frac{x^{-1\frac{1}{2}}}{-1\frac{1}{2}} + C = -\frac{32}{3\sqrt{x^3}}$$

8.1.30

$$\int \frac{dx}{x^2+3} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + C$$

8.1.31

$$\int \frac{1}{5^x} dx = \frac{\left(\frac{1}{5}\right)^x}{\ln\frac{1}{5}} + C$$

8.1.32

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin \frac{x}{2} + C$$

8.1.33

$$\int \frac{dx}{\sqrt{x^2-1}} = \ln| x + \sqrt{x^2-1} + C$$

8.1.34

$$\int \frac{dx}{x^2-25} = \frac{1}{10} \ln\left|\frac{x-5}{x+5}\right| + C$$

8.1.35

$$\int \left(x+\frac{2}{x}\right)^2 dx = \int \left(x^2+4+\frac{4}{x^2}\right) dx = \int x^2 dx + \int 4\, dx + 4\int x^{-2} dx = \frac{x^3}{3} + 4x + 4\frac{x^{-1}}{1} + C =$$

$$= \frac{x^3}{3} + 4x - \frac{4}{x} + C$$

8.1.36

$$\int \frac{dx}{4x^2+1} = \frac{1}{4}\int \frac{dx}{4x^2+1} = \frac{1}{4} \cdot \operatorname{arctg} \frac{x}{3} + C$$

8.1.37

$$\int \left(7^x - \frac{8}{x} + 4\cos x\right) dx = \int 7^x dx - 8\int \frac{dx}{x} + 4\int \cos x\, dx = \frac{7^x}{\ln 7} - 8\ln|x| + 4\sin x + C$$

8.1.38

$$\int \left(\frac{\sqrt{3}}{\cos^2 x} - \sqrt[3]{x} - \frac{2}{x^4}\right) dx = \sqrt{3}\int \frac{1}{\cos^2 x} dx - \int x^{\frac{1}{3}} dx - 2\int x^{-4} dx = \sqrt{3}\, \operatorname{tg} x - \frac{3\sqrt[3]{x^4}}{4} - \frac{2}{3x^3} + C$$

8.1.39

$$\int \frac{\sqrt{x} - 3\sqrt[5]{x^2} + 1}{\sqrt[4]{x}} dx = \int \frac{x^{\frac{1}{2}} - 3x^{\frac{2}{5}} + 1}{x^{\frac{1}{4}}} dx = \int x^{\frac{1}{2}-\frac{1}{4}} dx - 3\int x^{\frac{2}{5}-\frac{1}{4}} dx + \int x^{-\frac{1}{4}} dx =$$

$$= \frac{4\sqrt[4]{x^5}}{5} - 60\sqrt[20]{x^{33}} + \frac{4\sqrt[4]{x^3}}{3} + C$$

8.1.40

$\int (10{,}8x^{-81} + 0{,}2(0{,}5)^{x}) \cdot dx = \int 0{,}7x^{-81} dx + (\frac{0{,}2}{0{,}5})^{x} dx = 0{,}7\int x^{-81} dx + 0{,}2\int 0{,}5^{x} dx = \frac{0{,}7x^{-80}}{-80} +$

$\frac{0{,}2(0{,}5)^{x}}{\ln(0{,}5)} = \frac{7}{9x^{80}} + \frac{0{,}2(0{,}5)}{\ln(0{,}5)}$

8.1.41

$\int (5\,sh\,x - 7\,ch\,x + 1) dx = 5\int sh\,x\,dx - 7\int ch\,x\,dx + \int dx = 5\,ch\,x - 7\,ch\,x + x + C$

8.1.42

1) $\int (x^2 - 1)(\sqrt{x} + 4) dx = \int (x^2\sqrt{x} - \sqrt{x} + 4x^2 - 4) dx = \int x^{5/2} dx - \int x^{1/2} dx + 4\int x^2 dx - \int 4 dx =$

2) $= \frac{x^{7/2}}{\frac{7}{2}} - \frac{x^{3/2}}{3/2} + \frac{4x^3}{3} - 4x + C = \frac{2}{7}\sqrt{x^7} - \frac{2}{3}\sqrt{x^3} + 4x + C$

8.1.43

$\int \frac{7 - \sqrt{x^2 + \pi}}{\sqrt{x^2 + \pi}} dx = \int \frac{7}{\sqrt{x^2 + \pi}} dx + \int \frac{\sqrt{x^2 + \pi}}{\sqrt{x^2 + \pi}} = 7\int \frac{dx}{\sqrt{x^2 + 5}} + \int dx = 7\ln \cdot |x + \sqrt{x^2 + \pi}| + x + C$

8.1.44

$\int (\frac{\sqrt{x} - 5}{x})^3 dx = \int \frac{(\sqrt{x} - 5)^3}{x^3} dx = \int \frac{x^{3/2} - 15x + 75x^{1/2} - 125}{x^3} dx = \int (x^{-3/2} - 15x^{-2} + 75x^{-5/2} - 125x^{-3}) dx =$

$= \frac{x^{-1/2}}{-\frac{1}{2}} - \frac{15x^{-1}}{-1} + \frac{75x^{-3/2}}{-3/2} - \frac{125x^{-2}}{-2} + C = -\frac{2}{\sqrt{x}} + \frac{15}{x} - \frac{50}{\sqrt{x^3}} + \frac{125}{2x^2} + C$

8.1.45

$\int \sin 7x\,dx = -\frac{1}{7}\cos 7x + C$

8.1.46

$\int \sqrt[5]{2x - 8}\,dx = \int (2x - 8)^{1/5} dx = \frac{1}{2}\,\frac{(2x - 8)^{6/5}}{6/5} = \frac{5 \cdot \sqrt[5]{(2x - 8)^6}}{12} + C$

8.1.47

1) $\int (1 - 4x)^{2001} dx = -\frac{1}{4}\,\frac{(1 - 4x)^{2002}}{2002} + C = -\frac{(1 - 4x)^{2002}}{8008} + C$

8.1.48

$\int \frac{dx}{9x + 7} = \frac{1}{9x + 7} = \frac{1}{9}\ln|9x + 7| + C = \frac{\ln|9x + 7|}{9} + C$

8.1.49

$\int \frac{dx}{(6x + 11)^4} = \frac{1}{6}\,\frac{(6x + 11)^{-3}}{-3} = -\frac{1}{18(6x + 11)^3}$

8.1.50

$\int \frac{dx}{25x^2 + 1} = \frac{1}{25}\,arctg\,5x + C$

8.1.51

$$\int 3^{2-11x}dx = \frac{-1}{11}\cdot\frac{3^{2-11x}}{11\ln 3}+C$$

8.1.52

$$\int\frac{dx}{\sqrt{4x^2-1}} = \frac{1}{2}\ln\left|2x+\sqrt{4x^2-1}\right|+C$$

8.1.53

$$\int \sin^2 3x\,dx = \int(1-\cos 6x)\,dx = \int dx-\int\cos 6x\,dx = x-\frac{1}{6}\sin 6x+C$$

8.1.54

$$\int \cos^2 8x\,dx = \int(\cos 16x+1)\,dx = \int\cos 16x\,dx+\int dx = \frac{1}{16}\sin 16x+C$$

8.1.55

$$\int tg^2 x\,dx = \int\left(\frac{1}{\cos^2 x}-1\right)dx = \int\frac{dx}{\cos^2 x}-\int dx = tg\,x-x+C$$

8.1.56

$$\int\frac{4x+1}{x-5}dx = \int\frac{4(x-5)+21}{x-5}dx = \int\frac{4(x-5)}{x+5}dx+\int\frac{21}{x-5}dx = 4\int dx+21\int\frac{dx}{x-5} =$$

$$= 4x+21\ln(x-5)+C$$

8.1.57

$$\int(3tg\,x-2ctg\,x)^2dx = \int(9tg^2x-12tg\,x\cdot ctg\,x+4ctg^2x)\,dx = \int 9tg^2x\,dx-\int 12dx+$$

$$+\int 4ctg^2x\,dx = 9\int\left(\frac{1}{\cos^2 x}-1\right)dx-\int 12\,dx+4\int\left(\frac{1}{\sin^2 x}-1\right)dx = 9\int\frac{dx}{\cos^2x}-9\int dx-12\int dx+$$

$$+4\int\frac{dx}{\sin^2 x}-4\int dx = 9tg\,x-4ctg\,x-25x+C$$

8.1.60

$$\int\frac{\sin 2x}{\cos x}dx = \int\frac{2\sin x\cos x}{\cos x}dx = 2\int\sin x = -2\cos x+C$$