Домашняя работа. Часть 2. Гнелнов Артем
ИВТ.2.1

8.2.33

$$\int \cos(6x+1)\,dx$$

$$t=6x+1 \Rightarrow dt=6\,dx \Rightarrow dx=\frac{1}{6}\,dt$$

$$\int \cos(6x+1)\,dx=\int \cos t\,\frac{1}{6}\,dt=\frac{1}{6}\int \cos\,dt=\frac{1}{6}\sin+C=\frac{1}{6}\sin(6x+1)+C$$

8.2.34

$$\int\frac{dx}{\sqrt[3]{(5x-2)^4}}=\left[t=5x-2 \Rightarrow dt=5\,dx \Rightarrow dx=\frac{1}{5}\,dt\right]=\frac{1}{5}\int\frac{1}{t^{4/3}}\,dt=\frac{1}{5}\;\frac{t}{5/3+1}+C=-\frac{3}{5\sqrt[3]{t}}+C=$$

$$=\frac{-3}{5\sqrt[3]{5x-2}}+C$$

8.2.35

$$\int\frac{\sqrt{tg\,x}\,dx}{\cos^2 x}=\left[t=tg\,x \Rightarrow \frac{dt}{dx}=\frac{1}{\cos^2 x} \Rightarrow dx=\cos^2 x\cdot dt\right]=\int\frac{\sqrt{t}\cos^2 x\,dt}{\cos^2 x}=\sqrt{t^2}\,dt=\frac{2\sqrt{t^3}}{3}+C=$$

$$=\frac{2\sqrt{tg^3 x}}{3}+C$$

8.2.36

$$\int\frac{e^x\,dx}{e^{2x}+9}=\left[t=\frac{e^x}{3} \Rightarrow \frac{dt}{dx}=\frac{e^x}{3} \Rightarrow dx=3e^{-x}\,dt\right]=\int\frac{3\,dt}{9t^2+9}=\frac{1}{3}\int\frac{dt}{t^2+1}=\frac{arctg\,t}{3}+C=\frac{arctg\,\frac{e^x}{3}}{3}+C$$

8.2.37

$$\int\frac{x^5\,dx}{\sqrt{x^6+7}}=\left[t=x^6+7 \Rightarrow \frac{dt}{dx}=6x^5 \Rightarrow dx=\frac{1}{6x}\,dt\right]=\int\frac{1}{6}\int\frac{1}{\sqrt{t}}\,dt=\frac{\sqrt{t}}{3}+C=\frac{\sqrt{x^6+7}}{3}+C$$

8.2.39

$$\int\frac{(2x+3)\,dx}{(x^2+3x-1)^4}=\left[t=x^2+3x-1 \Rightarrow \frac{dt}{dx}=2x+3 \Rightarrow dx=\frac{dt}{2x+3}\right]=\int\frac{(2x+3)\,dx}{t^4\cdot(2x+3)}=\int\frac{dt}{t^4}=-\frac{1}{3t^3}+C=\frac{1}{3(x^2+3x-1)}+C$$

8.2.40

1) $$\int \cos^{11}2x\cdot\sin 2x\,dx=\left[t=\cos 2x \Rightarrow \frac{dt}{dx}=-2\sin 2x \Rightarrow dx=\frac{dt}{-2\sin 2x}\right]=\int\frac{t^{11}\sin 2x\,dt}{-2\sin 2x}=\int\frac{t^{11}\,dt}{-2}=-\frac{1}{2}\int t^{11}\,dt=$$

2) $$=-\frac{t^{12}}{24}+C=-\frac{\cos^{12}2x}{24}+C$$

8.2.42

$$\int\frac{e^{\frac{1}{x}}}{x^2}\,dx=\left[t=e^{\frac{1}{x}} \Rightarrow \frac{dt}{dx}=\frac{e^{\frac{1}{x}}}{x^2} \Rightarrow dx=\frac{x^2\,dt}{-e^{1/x}}\right]=\int\frac{t\,x^2\,dt}{-e^{1/x}x^2}=\int\frac{t\,dx}{-e^{1/x}}$$

8.2.43

$$\int\frac{\ln 5x\,dx}{x}=\left[t=\ln 5x \Rightarrow \frac{dt}{dx}=\frac{1}{x} \Rightarrow dx=x\,dt\right]=\int\frac{t\,x\,dx}{x}=\int t\,dx=\frac{t^2}{2}+C=\frac{\ln^2|5x|}{2}+C$$

8.2.45

$$\int 4x\sqrt[3]{x^2+8}\,dx = \left[t=x^2+8 \Rightarrow \frac{dt}{dx}=2x \Rightarrow dx=\frac{dt}{2x}\right] = \int 4x\sqrt[3]{t}\,\frac{dt}{2x} = \int 2\sqrt[3]{t}\,dt =$$
$$= \frac{1}{2}\int t^{\frac{1}{3}}\,dx = \frac{3\sqrt[3]{t^4}}{8}+C = \frac{3\sqrt[3]{(x^2+8)^4}}{8}+C$$

8.2.46

$$\int \frac{\cos x\,dx}{\sin^2 x} = \left[t=\sin x \Rightarrow \frac{dt}{dx}=\cos x \Rightarrow dx=\frac{dt}{\cos x}\right] = \int \frac{\cos x\cdot dx}{t^2\cdot\cos x} = \int \frac{dt}{t^2} = \frac{t^{-1}}{-1}+C = -\frac{1}{\sin x}+C$$

8.2.48

$$\int \frac{x\,dx}{x^4+1} = \left[t=x^2 \Rightarrow \frac{dt}{dx}=2x \Rightarrow dx=\frac{dt}{2x}\right] = \int \frac{\sqrt{t}\,dt}{(t^2+1)\,2\sqrt{t}} = \frac{1}{2}\int \frac{dt}{t^2+1} = \frac{1}{2}\cdot \operatorname{arctg} t + C = \frac{\operatorname{arctg} x^2}{2}+C$$

8.2.49

$$\int e^{-x^3}\cdot x^2\,dx = \left[t=e^{-x^3} \Rightarrow \frac{dt}{dx}=-3x^2e^{-x^3} \Rightarrow dx=\frac{dt}{-3x^2e^{-x^3}}\right] = \int \frac{t\,x^2\cdot dt}{-3x^2\cdot t} = -\frac{1}{3}\int dt = -\frac{1}{3}t+C = \frac{e^{-x^3}}{-3}+C$$

8.2.51

$$\int (8\cos\tfrac{x}{3}-5)^2\sin\tfrac{x}{3}\,dx = \left[t=\cos\tfrac{x}{3}-5 \Rightarrow \frac{dt}{dx}=-\frac{8}{3}\sin\tfrac{x}{3} \Rightarrow dx=\frac{dt}{-\frac{8}{3}\cdot\sin\frac{x}{3}}\right] = \int t^2\sin\tfrac{x}{3}\,\frac{dt}{-\frac{8}{3}\sin\frac{x}{3}} = \int -\frac{3}{8}t^2\,dt =$$
$$= -\frac{3}{8}\int t^2\,dt = -\frac{3}{8}\,\frac{t^3}{3}+C = -\frac{t^3}{8}+C = -\frac{(8\cos\frac{x}{3}-5)^3}{8}+C$$

8.2.52

$$\int \frac{(3x^2-2x+7)\,dx}{\sqrt{x^3-x^2+7x-2}} = \left[t=x^3-x^2+7x-2 \Rightarrow \frac{dt}{dx}=3x^2-2x+7 \Rightarrow dx=\frac{dx}{3x^2-2x+7}\right] = \int \frac{(3x^2-2x+7)\,dt}{\sqrt{t}\,(3x^2-2x+7)} = \int \frac{dt}{\sqrt{t}} =$$
$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}}+C = 2\sqrt{t}+C = 2\sqrt{x^3-x^2+7x-2}+C$$

8.2.54

$$\int (x-2)\cdot\sqrt{x+4}\,dx = \left[t=x+4 \Rightarrow \frac{dt}{dx}=1 \Rightarrow dx=dt\right] = \int (t+2)\cdot\sqrt{t}\,dt = \int (t^{\frac{3}{2}}+2t)^{\frac{1}{2}}dt = \int t^{\frac{3}{2}}dt+2\int t^{\frac{1}{2}}dt =$$
$$= \frac{2t^{\frac{5}{2}}}{5}+\frac{4}{3}t^{\frac{3}{2}}+C = \frac{2\sqrt{(x+4)^5}}{5}+\frac{4\sqrt{(x+4)^3}}{3}+C$$

8.2.55

$$\int \frac{3\sqrt{x}-2\cos\frac{1}{x^2}}{x^3}\,dx = \int \frac{3\sqrt{x}}{x^3}\,dx - \int \frac{2\cos\frac{1}{x^2}}{x^3}\,dx = 3\int \frac{1}{x^{\frac{5}{2}}} - 2\int \frac{\cos\frac{1}{x^2}}{x^3}\,dx = 3\left(-\frac{2}{3x^{\frac{3}{2}}}\right)+C-2\int \frac{\cos t\,(-x^3)\,dt}{x^3\cdot 2} =$$
$$\left[t=\frac{1}{x^2} \Rightarrow \frac{dt}{dx}=-\frac{2}{x^3} \Rightarrow dx=-\frac{x^3}{2}dt\right] = -\frac{2}{\sqrt{x^3}}+C+\int \cos t\,dt = -\frac{2}{\sqrt{x^3}}+\sin\frac{1}{x^2}+C$$

8.2.57

$$\int \frac{dx}{e^x+e^{-x}} = \int \frac{dx}{e^x}+\int \frac{dx}{e^{-x}} = \int e^{-x}dx+\int e^x dx = \left[t=-x \Rightarrow \frac{dt}{dx}=-1 \Rightarrow dx=-dt\right] = \int -e^t dt+e^x+C = -\int e^t dt+e^x+C =$$
$$= -e^t+e^x+C = -e^{-x}+e^x+C$$

8.2.58

$$\int \frac{x+8}{x^2+3}dx = \int \frac{x\,dx}{x^2+3} + \int \frac{8\,dx}{x^2+3}$$

1) $\int \frac{x\,dx}{x^2+3} = \left[t = x^2+3 \Rightarrow dx = \frac{dt}{2x}\right] = \int \frac{x\,dt}{t\cdot 2x} = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln|x^2+3| + C$

2) $\int \frac{8\,dx}{x^2+3} = 8\int \frac{dx}{x^2+3} = \left[p = \frac{x}{\sqrt{3}} \Rightarrow dx = \sqrt{3}\,dp\right] = 8\int \frac{\sqrt{3}\,dp}{3p^2+3} = 8\int \frac{dp}{p^2+1} = \frac{8}{\sqrt{3}}\operatorname{arctg} p + C = \frac{8}{\sqrt{3}}\operatorname{arctg}\left(\frac{x}{\sqrt{3}}\right) \Rightarrow$

$\Rightarrow \int \frac{x\,dx}{x^2+3} + \int \frac{8\,dx}{x^2+C} = \frac{1}{2}\ln|x^2+3| + \frac{8}{\sqrt{3}}\operatorname{arctg}\left(\frac{x}{\sqrt{3}}\right) + C$

8.2.60

1) $\int \frac{1-6x}{(x+1)\cdot(x-1)}dx = \int \frac{1-6x}{x^2-1}dx = \int \frac{dx}{x^2-1} - 6\int \frac{x\,dx}{x^2-1} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C - 6\int \frac{x\,dx}{x^2-1} = \left[t = x^2-1 \Rightarrow \frac{dt}{dx} = 2x\right.$

2) $\left.\Rightarrow dx = \frac{dt}{2x}\right] = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C - 6\int \frac{x\,dt}{t\cdot 2x} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - 3\ln|t| + C = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - 3\ln|x^2-1| + C$

8.2.61

$\int (\cos^2 x - \sin^2 x)\sqrt[3]{1+\sin 2x}\,dx = \cos 2x\sqrt[3]{1+\sin 2x}\,dx = \left[t = \sin 2x + 1 = \frac{dt}{dx} = 2\cos 2x \Rightarrow dx = \frac{dt}{2\cos 2x}\right]$

$= \int \frac{\cos 2x\cdot\sqrt[3]{t}\,dt}{2\cos 2x} = \frac{1}{2}\int \sqrt[3]{t}\,dt = \frac{1}{2}\cdot\frac{3t^{4/3}}{4} + C = \frac{3\sqrt[3]{(\sin 2x+1)^4}}{8} + C$

~~8.63~~ 8.2.63

$\int \sqrt{16-x^2}\,dx = \left[x = 4\sin t \Rightarrow \frac{dx}{dt} = 4\cos t \Rightarrow dx = 4dt\cdot\cos t\right] = \int \sqrt{16-16\sin^2 t}\cdot 4dt\cos t = 4\int \sqrt{16(1-\sin^2 t)}\cdot$

$\cdot dt\cdot\cos t = 4\int 4\cos t\cdot\cos t\,dt = 16\int \cos^2 t\,dt = 16\int \frac{\cos 2t+1}{2}dt = 8\int \cos 2t\,dt + 8\int dt =$

$= \left[u = 2t \Rightarrow dt = \frac{du}{2}\right] = 8\int \cos u\cdot\frac{du}{2} + 8t + C = 4\int \cos u\,du + 8t + C = 4\sin u + 8\arcsin\frac{x}{4} + C$

$= 4\sin 2\left(\arcsin\frac{x}{4}\right) + 8\arcsin\frac{x}{4} + C = 2x\sqrt{1-\frac{x^2}{16}} + 8\arcsin\frac{x}{4} + C$

8.2.64

$\int \frac{dx}{1+\sqrt{x}} = \left[x = (t-1)^2 \Rightarrow dx = (2t-2)dt\right] = \int \frac{(2t-2)dt}{1+t-1} = 2\int \frac{(t-1)dt}{t} = 2\int \frac{t}{t}dt - 2\int \frac{1}{t}dt =$

$= 2t - 2\ln + C = 2(\sqrt{x}+1) - 2\ln(\sqrt{x}+1) + C$

8.2.66

$\int \frac{dx}{(x+1)\sqrt{x}} = \left[x = t^2 \Rightarrow dx = 2\sqrt{x}\,dt\right] = \int \frac{2\sqrt{x}\,dt}{(t^2+1)\sqrt{x}} = 2\int \frac{dt}{t^2+1} = 2\operatorname{arctg} t + C = 2\operatorname{arctg}\sqrt{x} + C$

8.2.67

$\int \frac{x\,dx}{\sqrt{1-x}} = \left[x = 1-t \Rightarrow dx = -dt\right] = \int \frac{-(1-t)\,dt}{\sqrt{1-1+t}} = -\int \frac{(1-t)dt}{\sqrt{t}} = -\left(\int \frac{dt}{\sqrt{t}} - \int \frac{t\,dt}{\sqrt{t}}\right) = -\int \frac{dt}{\sqrt{t}} + \int \frac{t\,dt}{\sqrt{t}} =$

$= -2\sqrt{t} + \frac{2t^{3/2}}{3} + C = -2\sqrt{1-x} + \frac{2(1-x)^{3/2}}{3} + C = \frac{-6\sqrt{1-x} + 2(1-x)\sqrt{1-x}}{3} + C = \frac{\sqrt{1-x}(-6+2-2x)}{3} + C =$

$= \frac{-2\sqrt{1-x}^{3}\cdot(x+2)}{3} + C$

8.2.69

$$\int x\ln x\,dx = \left[\int uv'dx = uv - \int vu'dx;\ u=\ln x,\ v'=x \Rightarrow u'=\frac{1}{x},\ v=\frac{x^2}{2}\right] = \frac{\ln x\cdot x^2}{2} -$$
$$-\int\frac{x^2}{2}\cdot\frac{1}{x}\,dx = \frac{\ln x}{2}\cdot x^2 - \frac{1}{2}\int x\,dx = \frac{\ln x\cdot x^2}{2} - \frac{x^2}{4} + C = \frac{2\ln x\cdot x^2 - x^2}{4} + C = \frac{x^2(2\ln x - 1)}{4} + C$$

8.2.70

$$\int(2x+3)\cos x\,dx = [u=2x+3,\ v'=\cos x \Rightarrow u'=2;\ v=\sin x] = (2x+3)\sin x - \int 2\sin x\,dx =$$
$$= (2x+3)\sin x + 2\cos x + C$$

8.2.72

$$\int\frac{x\cos x\,dx}{\sin^3 x} = \left[u=x,\ v'=\frac{\cos x}{\sin^3 x} \Rightarrow u'=1,\ v=-\frac{1}{2\sin^2 x}\right] = -\frac{x}{2\sin^2 x} - \int\frac{-dx}{2\sin^2 x} = -\frac{x}{2\sin^2 x} = -\frac{x}{2\sin^2 x} -$$
$$-\frac{\operatorname{ctg} x}{2} + C$$

8.2.73

$$\int x^2\ln x\,dx = \left[u=\ln x,\ v'=x^2 \Rightarrow u'=\frac{1}{x};\ v=\frac{x^3}{3}\right] = \frac{x^3\ln x}{3} - \int\frac{x^3}{3x}\,dx = \frac{x^3\ln x}{3} - \frac{1}{3}\int x^2dx$$
$$= \frac{x^3\ln x}{3} - \frac{x^3}{9} + C = \frac{x^3(3\ln x - 1)}{9} + C$$

8.2.75

$$\int x^3e^x\,dx = [u=x^3,\ v'=e^x \Rightarrow u'=3x^2,\ v=e^x] = x^3e^x - \int 3x^2e^x = x^3e^x - 3\int x^2e^x\,dx = [u=x^2,\ v'=e^x \Rightarrow u'=2x,$$
$$v=e^x] = x^3e^x - 3\left(x^2e^x - \int 2xe^x\,dx\right) = [u=x,\ v'=e^x \Rightarrow 1,\ u=e^x] = x^3e^x - 3\left(x^2e^x - 2(xe^x -\right.$$
$$\left.-\int e^x\,dx)\right) = x^3e^x - 3(x^2e^x - xe^x + 2e^x) + C = e^x(x^3 - 3x^2 + 6x - 6) + C$$

8.2.76

$$\int\frac{\arccos x\,dx}{\sqrt{1+x}} = \int\arccos x\cdot(1+x)^{-\frac{1}{2}}\,dx = \left[u=\arccos x,\ v'=(1+x)^{-\frac{1}{2}} \Rightarrow u'=\frac{1}{-\sqrt{1-x^2}};\ v=2\sqrt{x+1}\right] =$$
$$= \arccos x\cdot 2\sqrt{x+1} - \int\frac{2\sqrt{x+1}\,dx}{-\sqrt{1-x^2}} = 2\sqrt{x+1}\cdot\arccos x + 2\int\frac{dx}{\sqrt{1-x}} = [t=1-x \Rightarrow dx=-dt] = 2\sqrt{x+1}\cdot$$
$$\cdot\arccos x + 2\int\frac{-dt}{\sqrt{t}} = 2\sqrt{x+1}\arccos x - 4\sqrt{t} + C = 2\sqrt{x+1}\arccos x - 4\sqrt{1-x} + C$$

8.2.81

$$\int e^{\sqrt{x}}\,dx = [t=\sqrt{x} \Rightarrow dx = 2\sqrt{x}\,dt] = \int e^t\,2t\,dt = 2\int e^t\cdot t\,dt = [u=t,\ v=e^t \Rightarrow u'=1,\ v=e^t] =$$
$$= 2\left(t\cdot e^t - \int e^t\,dt\right) = 2te^t - 2e^t + C = 2e^t(t-1) + C = 2e^{\sqrt{x}}(\sqrt{x}-1) + C$$

8.2.82

$$\int\frac{x\,dx}{\cos^2 x} = \left[u=x,\ v'=\frac{1}{\cos^2 x} \Rightarrow u'=1,\ v=\operatorname{tg} x\right] = x\operatorname{tg} x - \int\operatorname{tg} x\,dx = x\operatorname{tg} x + \ln|\cos x| + C$$

8.2.84

$$\int\ln(x+\sqrt{x^2+1})\,dx = \left[u=\ln(x+\sqrt{x^2+1});\ v'=1 \Rightarrow u'=\frac{\frac{x}{\sqrt{x^2+1}}+1}{\sqrt{x^2+1}+x};\ v=x\right] = x\ln(x+\sqrt{x^2+1}) - \int\frac{x\left(\frac{x}{\sqrt{x^2+1}}+1\right)}{\sqrt{x^2+1}+x} =$$

$= x\ln|x+\sqrt{x^2+1}| - \int \frac{x}{\sqrt{x^2+1}}dx = [t=x^2+1 \Rightarrow dx=\frac{dt}{2x}] = x\ln|x+\sqrt{x^2+1}| - \int\frac{x\,dt}{\sqrt{t}\,2x} = x\ln(x+\sqrt{x^2+1}) -$

$-\frac{1}{2}\int\frac{dt}{\sqrt{x}} = x\ln|x+\sqrt{x^2+1}| - \frac{1}{2}2\sqrt{t}+C = x\ln(x+\sqrt{x^2+1}) - \sqrt{x^2+1}+C$

8.2.85

$\int \sin 2x \cdot \ln\sin x\,dx = [u=\ln\sin x,\ v'=\sin 2x \Rightarrow u'=\frac{\cos x}{\sin x},\ v=\sin^2 x] = \sin^2 x\ln\sin -$

$-\int\frac{\cos x\sin^2 x}{\sin x}dx = \sin^2 x\ln\sin x - \int\cos x\sin x\,dx = [u=\cos x,\ v'=\sin x \Rightarrow u'=-\sin x,\ v=-\cos x]$

$= \sin^2 x\ln\sin x - \int\cos x\sin x\,dx = \sin^2 x\ln\sin x + \cos^2 x + \int\sin x\cos x\,dx.$

1) $2\int\sin x\cos x\,dx = -\cos^2 x$

2) $\int\sin x\cos x\,dx = -\frac{\cos 2x}{2} \Rightarrow \int\sin 2x\ln\sin x\,dx = \sin^2 x\ln\sin + \frac{\cos^2 x}{2}+C$

8.2.88

$\int\arcsin^2 x\,dx = [t=\arcsin x \Rightarrow dx=\sqrt{1-x^2}dt] = \int t^2\sqrt{1-x^2}dt = \int t^2\cos t\,dt = [u=t^2,\ v'=\cos t \Rightarrow$

$\Rightarrow u'=2t,\ v=\sin t] = t^2\sin t - \int 2t\sin t\,dt = t^2\sin t - 2\int t\sin t\,dt = [u=t,\ v'=\sin t \Rightarrow u'=1,\ v=-\cos t] =$

$= t^2\sin t - 2(-\cos t - \int -\cos t\,dt) = t^2\sin t + 2t\cos t - 2\int\cos t\,dt = t^2\cdot\sin t + 2t\cos t - 2\sin t + C =$

$= x\arcsin^2 x + 2\arcsin x\cdot\cos(\arcsin x) - 2x + C = x\arcsin^2 x - 2\sqrt{1-x^2}\arcsin x - 2x + C$

8.2.89

$\int\frac{\cos 3\sqrt{x}}{\sqrt{x}}dx = [t=\sqrt{x} \Rightarrow dx = 2\sqrt{x}\,dt] = \int\frac{\cos 2t\cdot dt}{t} = 2\int\cos t\,dt = 2\sin t + C = 2\sin\sqrt{x}+C$

8.2.91

$\int\frac{dx}{\sin x} = \int 1\sin^{-1}x\,dx = [u=\sin^{-1}x,\ v'=1 \Rightarrow u'=\frac{-1}{\sin^2 x},\ v=x] = \frac{x}{\sin x} - \int\frac{-x\,dx}{\sin^2 x} = \frac{x}{\sin x} + \int\frac{x\,dx}{\sin^2 x} =$

$= [u=x,\ v'=\frac{1}{\sin^2 x} \Rightarrow u'=1,\ v=-\operatorname{ctg}x] = \frac{x}{\sin x} + (-x\operatorname{ctg}x + \int\operatorname{ctg}x\,dx) = \frac{x}{\sin x} - x\operatorname{ctg}x + \ln|\sin x| + C$

8.2.92

$\int\frac{\ln^2 x}{x\sqrt{3-\ln x}}dx = [t=3-\ln x \Rightarrow -x\,dt,\ \ln^2 x=(3-t)^2] = \int\frac{-(3-t)^2 x\,dt}{x\sqrt{t}} = -\int\frac{(t-3)^2dt}{\sqrt{t}} = -\int\frac{t^2-6t+9}{\sqrt{t}} =$

$= \int t^{3/2}dt + 6\int\sqrt{t}\,dt + 9\int\frac{dt}{\sqrt{t}} = -\frac{2t^{5/2}}{5} + \frac{12t^{3/2}}{3} - 18\sqrt{t} + C = \frac{2(3-\ln x)^{5/2}}{5} + 4(3-\ln x)^{3/2} -$

$-18\sqrt{3-\ln x}+C$

8.2.94

$\int\frac{3x+5\sin(\frac{1}{e^x})}{e^x}dx = \int e^{-x}(3x+5\sin(e^{-x}))dx = \int e^{-x}3x\,dx + \int e^{-x}5\sin(e^{-x})dx =$

$= 3\int xe^{-x}dx + 5\int e^{-x}\sin(e^{-x})$

1) $\int xe^{-x}dx = [u=x,\ v'=e^{-x} \Rightarrow u'=1;\ v=-e^{-x}] = -xe^{-x} - \int -e^{-x}dx = [t=-x \Rightarrow dx=-dt] =$

$-xe^{-x} - \int e^{t} dt = xe^{-x} - e^{x} + C = -xe^{-x} - e^{-x} + C = e^{-x}(x-1) + C$

2) $\int e^{-x} \sin(e^{-x}) dx = \left[t = e^{-x} \Rightarrow dx = \frac{dt}{-e^{x}} \right] = \int \frac{t \sin t \, dt}{-t} = -\int \sin t \, dt = \cos t =$

$= \cos t \; e^{-x} + C$

$\Rightarrow \int \frac{3x + 5\sin\left(\frac{1}{e^{x}}\right)}{e^{x}} dx = 3e^{-x} \cdot (-x-1) + 5\cos(e^{-x}) + C$