20.06.2024 г. Интегралы и ДУ Гнездов Артем Евгеньевич ИВТ 2.1.

Вариант 9

1.1. $\int \frac{2 \cdot x \cdot dx}{(19-22x^2)^4} = \left[t = 19-22x^2,\ dt = -44x,\ -\frac{1}{44} dt = x\,dx \right] = 2\int -\frac{1}{44t^4} dt =$

$= -\frac{1}{22} \cdot \left(-\frac{1}{3t^3}\right) + C = \frac{1}{66t^3} + C = \left[t = 19-22x^2 \right] = \frac{1}{66 \cdot (19-22x^2)^3} + C$

Ответ: $\frac{1}{66 \cdot (19-22x^2)^3} + C$

1.2 $\int 5 \cdot (5 \cdot x^8 + 1{,}5 \cdot x^9) \cdot \ln(x^5)\, dx = \int 25\left(5x^8 + \frac{3x^9}{2}\right) \ln(x) dx = \frac{25}{2} \int (10x^8 + 3x^9) \cdot$

$\cdot \ln x\, dx = \left[t = \ln x,\ dt = \frac{1}{x} dx,\ du = 10x^8 + 3x^9,\ u = \frac{10x^9}{9} + \frac{3x^{10}}{10} \right] = \int (3x^9 + 10x^8) \ln x\, dx =$

$= \left(\frac{3x^{10}}{10} + \frac{10x^9}{9}\right) \ln x - \int \frac{\frac{3x^{10}}{10} + \frac{10x^9}{9}}{x} dx = \left(\frac{3x^{10}}{10} + \frac{10x^9}{9}\right) \ln x - \int \frac{27x^9 + 100x^8}{90} =$

$= \left(\frac{3x^{10}}{10} + \frac{10x^9}{9}\right) \ln x - \left(\frac{3}{10}\int x^9 dx + \frac{10}{9}\int x^8 dx\right) = \left[\left(\frac{3x^{10}}{10} + \frac{10x^9}{9}\right) \ln x - \frac{3x^{10}}{100} - \frac{10x^9}{81} + C\right] =$

$= \frac{25\left(\frac{3x^{10}}{10} + \frac{10x^9}{9}\right) \ln x}{2} - \frac{3x^{10}}{8} - \frac{125x^9}{81} + C = \frac{15x^{10} \ln x}{4} + \frac{125x^9 \ln x}{9} - \frac{3x^{10}}{8} - \frac{125x^9}{81} + C$

Ответ: $\frac{15x^{10} \ln x}{4} + \frac{125x^9 \ln x}{9} - \frac{3x^{10}}{8} - \frac{125x^9}{81} + C$

1.3 $\int \frac{x - 17/5}{x^3 - 2x^2 + 4x - 8} dx = \int \frac{x - \frac{17}{5}}{(x^2+4) \cdot (x-2)} dx = \left[\frac{Ax+B}{x^2+4} + \frac{C}{x-2} = \right.$

$= \frac{(C+A)x^2 + (B-2A)x + 4C - 2B}{(x^2+4) \cdot (x-2)} \Rightarrow \begin{cases} 4C + 2B = -\frac{17}{5} \\ B - 2A = 1 \\ C + A = 0 \end{cases} \begin{cases} 4C + 2B = -\frac{17}{5} \\ B = 1 + 2A \\ C = -A \end{cases}$

$\cancel{4 \cdot 2A \cdot 2C + 2A - 2C = -\frac{17}{5}}$ $\cancel{B = 1 + 2A}$ $\cancel{C = -A}$ $\begin{cases} 4C - 2(1-2C) = -\frac{17}{5} \\ B = 1 + 2A \\ C = -A \end{cases} \begin{cases} 8C = -\frac{17}{5} + \frac{10}{5} \\ B = 1 + 2A \\ C = -A \end{cases}$

$\begin{cases} C = -\frac{7}{40} \\ A = \frac{7}{40} \\ B = \frac{27}{20} \end{cases} \begin{cases} A = \frac{7}{40} \\ B = \frac{27}{20} \\ C = -\frac{7}{40} \end{cases} \Bigg] = \int \frac{\frac{7x}{40} + \frac{27}{20}}{x^2+4} - \frac{7}{40(x-2)} dx = \int \frac{7x + 54}{40(x^2+4)} - \frac{7}{40(x-2)} dx =$

$$= \frac{1}{40}\int \frac{7x+54}{x^2+4} - \frac{7}{40}\int \frac{1}{x-2}\,dx = \left[\int \frac{7x+54}{x^2+4} = \frac{7}{2}\int \frac{2x}{x^2+4}\,dx + \int \frac{54}{x^2+4}\,dx = \right.$$

$$= \left[\begin{array}{l} 1.\ \frac{7}{2}\int \frac{2x}{x^2+4}\,dx = \left[\begin{array}{l} t=x^2+4 \\ dt=2x\,dx \end{array}\right] = \frac{7}{2}\int \frac{1}{t}\,dt = \frac{7}{2}\ln t = \frac{7}{2}\cdot\ln(x^2+4) = \frac{7\ln(x^2+4)}{2} \\ 2.\ \int \frac{54}{x^2+4}\,dx = \left[\begin{array}{l} u=\frac{x}{2},\ x=2u \\ dx=2\,du \end{array}\right] = 54\int \frac{2}{4u^2+4}\,du = \overset{27}{54}\int \frac{1}{2(u^2+1)}\,du = 27\,arctg(u) = 27\,arctg\left(\frac{x}{2}\right) \\ 3.\ \int \frac{1}{x-2}\,dx = \ln(|x-2|) \end{array}\right]$$

$$= \frac{1}{40}\cdot\left(\frac{7\ln(x^2+4)}{2} + 27\,arctg\left(\frac{x}{2}\right)\right) - \frac{7}{40}\cdot(\ln(|x-2|)) = \frac{7\ln(x^2+4)}{80} + \frac{27\,arctg\left(\frac{x}{2}\right)}{40}$$

$$- \frac{7\ln(|x-2|)}{40} + C$$

Ответ: $\frac{7\ln(x^2+4)}{80} + \frac{27\,arctg\left(\frac{x}{2}\right)}{40} - \frac{7\ln(|x-2|)}{40} + C$

(1.4) $$\int \frac{2}{9+\sqrt{10x+46}}\,dx = \left[\begin{array}{l} t=10x+46,\ dx=\frac{1}{10}\,dt \\ x=\frac{t-46}{10} \end{array}\right] = 2\int \frac{1}{10(\sqrt{t}+9)}\,dt = \left[\begin{array}{l} u=\sqrt{t}+9 \\ 2\,du=\frac{1}{\sqrt{t}}\,dt \end{array}\right] =$$

$$= \frac{1}{5}\int \frac{2u-18}{u}\,du = \frac{1}{5}\int 2 - \frac{18}{u}\,du = \frac{1}{5}\cdot\left(-18\int \frac{1}{u}\,du + 2\int 1\,du\right) = \frac{1}{5}\left(-18\ln(|u|) + 2u\right) +$$

$$+ C = \frac{2u}{5} - \frac{18\ln(|u|)}{5} + C = \frac{2\sqrt{t}}{5} - \frac{18\ln(\sqrt{t}+9)}{5} + \frac{18}{5} + C = -\frac{18\ln(\sqrt{10x+46}+9)}{5} +$$

$$+ \frac{2\sqrt{10+46}}{5} + \frac{18}{5} + C$$

Ответ: $\frac{2\sqrt{10+46}}{5} - \frac{18\ln|\sqrt{10x+46}+9|}{5} + C$

(1.5) $$\int \frac{2}{9\cos^2(x) + 17\sin^2(x)}\,dx = [\sin x = \cos x\cdot tg x] = \int \frac{2}{\cos^2 x\cdot(17 tg^2 x + 9)}\,dx =$$

$$= 2\int \frac{1}{\cos^2 x\cdot(17 tg^2 x+9}\,dx = \left[t = tg x,\ dt = \frac{1}{\cos^2 x}\,dx\right] = 2\int \frac{1}{17t^2+9}\,dt = \left[\begin{array}{l} u=\frac{\sqrt{17}t}{3} \\ t=\frac{3u}{\sqrt{17}} \\ dt=\frac{3}{\sqrt{17}}\,du \end{array}\right] =$$

$$= 2\int \frac{3}{\sqrt{17}\cdot(9u^2+9)}\,du = 2\int \frac{1}{3\sqrt{17}\cdot(u^2+1)}\,du = \frac{2}{3\sqrt{17}}\int \frac{1}{u^2+1}\,du = \frac{2}{3\sqrt{17}}\cdot arctg(u) + C =$$

$$= \frac{2\,arctg\,u}{3\sqrt{17}} + C = \frac{2\,arctg\left(\frac{\sqrt{17}t}{3}\right)}{3\sqrt{17}} + C = \frac{2\,arctg\left(\frac{17 tg x}{3}\right)}{3\sqrt{17}} + C$$

Ответ: $\frac{2\,arctg\left(\frac{17 tg x}{3}\right)}{3\sqrt{17}} + C$

1.6) $\int x\cdot\arcsin(13x)\cdot dx = \begin{bmatrix} t=\arcsin(13x) & du=x \\ dt=\frac{13}{\sqrt{1-169x^2}} & u=\frac{x^2}{2}\end{bmatrix} = \frac{x^2\arcsin(13x)}{2} -$

$- \int \frac{13x^2}{2\sqrt{1-169x^2}}dx = \Big[-\frac{13}{2}\int \frac{x^2}{\sqrt{1-169x^2}}dx \Rightarrow \Big[\frac{x^2}{\sqrt{1-169x^2}} = (Ax+B)\cdot\sqrt{1-169x^2} + C\int\frac{1}{\sqrt{1-169x^2}}dx,$

$\left((Ax+B)\cdot\sqrt{1-169x^2} + C\cdot\int\frac{1}{\sqrt{1-169x^2}}dx\right)' = A\sqrt{1-169x^2} - \frac{169x(Ax+B)}{\sqrt{1-169x^2}} + \frac{C}{\sqrt{1-169x^2}} =$

$= \frac{-338Ax^2 - 169Bx + C + A}{\sqrt{1-169x^2}} = \frac{x^2}{\sqrt{1-169x^2}}$ $\begin{cases} C+A=0 \\ -169B=0 \\ -338C=1\end{cases}$ $\begin{cases} A=-\frac{1}{338} \\ B=0 \\ C=\frac{1}{338}\end{cases}\Big] =$

$= -\frac{13}{2}\left(-\frac{x\sqrt{1-169x^2}}{338} + \frac{1}{338}\int\frac{1}{\sqrt{1-169x^2}}dx\right) = -\frac{13}{2}\left(-\frac{x\sqrt{1-169x^2}}{338} + \frac{1}{338}\int\frac{1}{13\sqrt{1-t^2}}dt\right) =$
$[t=13x,\ x=\frac{t}{13},\ dx=\frac{1}{13}dt]$

$= -\frac{13}{2}\cdot\left(-\frac{x\cdot\sqrt{1-169x^2}}{338} + \frac{1}{338}\cdot\left(\frac{\arcsin(t)}{13}\right)\right) = -\frac{13}{2}\cdot\left(-\frac{x\cdot\sqrt{1-169x^2}}{338} + \frac{1}{338}\cdot\left(\frac{\arcsin(13x)}{13}\right)\right)$

$= \frac{x\sqrt{1-169x^2}}{52} - \frac{\arcsin(13x)}{676}\Big] = \frac{x^2\arcsin(13x)}{2} + \frac{x\sqrt{1-169x^2}}{52} - \frac{\arcsin(13x)}{676} + C$

Ответ: $\frac{x^2\arcsin(13x)}{2} + \frac{x\sqrt{1-169x^2}}{52} - \frac{\arcsin(13x)}{676} + C$

1.7) $\int_{-5}^{6}\frac{1}{\sqrt{5-4x-x^2}}dx =$ [подынтегральная функция имеет разрыв в точках -5 и 1] =

$= \lim_{\delta\to 0}\int_{-5+\delta}^{1-\delta}\frac{dx}{\sqrt{5-4x-x^2}} + \lim\int_{1+\delta}^{6}\frac{dx}{\sqrt{5-4x-x^2}} = \Big[\int\frac{1dx}{\sqrt{-x^2-4x+5}} = \int\frac{1dx}{\sqrt{9-(x+2)^2}} =$

$= [t=x+2,\ x=t-2,\ dx=dt] = \int\frac{1}{\sqrt{9-t^2}}dt = [u=\frac{t}{3},\ t=3u,\ dt=3du] = \int\frac{3}{\sqrt{9-9u^2}}du =$

$= \int\frac{1}{\sqrt{1-u^2}}du = \arcsin(u)+C = \arcsin(\frac{t}{3})+C = \arcsin(\frac{x+2}{3})+C\Big] =$

$= \lim_{\delta\to 0}\arcsin\left(\frac{x+2}{3}\right)\Big|_{-5+\delta}^{1-\delta} + \lim\arcsin\left(\frac{x+2}{3}\right)\Big|_{1+\delta}^{6} = \left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) + \left(\arcsin\left(\frac{\delta}{3}\right) - \frac{\pi}{2}\right) =$

$= \arcsin\left(\frac{\delta}{3}\right) + \frac{\pi}{2}$

Ответ: $\arcsin\left(\frac{\delta}{3}\right) + \frac{\pi}{2}$

(2.1) Найти частное решение ДУ:

$y' = -2y \;;\; y(0) = 10$

$\frac{dy}{dx} = -2y \Rightarrow dy = -2y\,dx \Rightarrow \frac{1}{y}\,dy = -2\,dx\,,\; y \neq 0$

$\int \frac{1}{y}\,dy = \int -2\,dx$

$\ln|y| + C = -2x + C$

$y = e^{-2x+C}$

$y = e^{-2x} \cdot e^{C}$

$y = e^{-2x} \cdot C$

$y = \frac{1}{e^{2x}} \cdot C$

$y = \frac{C}{e^{2x}}$ – Общее решение

$10 = \frac{C}{e^{2\cdot 0}}$

$10 = \frac{C}{1}$

$10 = C \Rightarrow y = \frac{10}{e^{2x}}$ – Частное решение

Ответ: Общее решение $\frac{C}{e^{2x}}$

Частное решение $\frac{10}{e^{2x}}$

(2.2.) $x \cdot y' = 2 \cdot \sqrt{11 \cdot x^2 + y^2} + y =$

$= \left[y' = \frac{dy}{dx} \right] = \frac{x\,dy}{dx} = 2 \cdot \sqrt{11x^2 + y^2} + y$

$x\,dy = (2 \cdot \sqrt{11x^2 + y^2} + y)\,dx$ — Однор. урав. I порядка

$\left[u = \frac{y}{x}\,,\; y = ux\,,\; dy = u\,dx + x\,du \right]$

$x(u\,dx + x\,du) = (2\sqrt{u^2+11} + u)\,x\,dx$

$\cancel{ux\,dx} + x^2\,du = 2\sqrt{u^2+11}\,x\,dx + \cancel{ux\,dx}$

$x^2\,du = 2\sqrt{u^2+11}\,x\,dx \quad /: x \cdot \sqrt{u^2+11}$

$\frac{du}{\sqrt{u^2+11}} = \frac{2\,dx}{x}$

$\int \frac{1}{\sqrt{u^2+11}}\,du = \int \frac{2}{x}\,dx$

$$\int \frac{1}{u^2+\sqrt{11}^2} = 2\ln(x)+C$$

$$\ln(|\sqrt{u^2+\sqrt{11}^2}+u|) = 2\ln|x|+C$$

$$\ln(|\sqrt{u^2+11}+u|) = 2\ln|x|+C$$

$$\sqrt{u^2+11}+u = x^2\cdot e^C$$

$$\frac{y}{x}+\sqrt{\frac{y^2}{x^2}+11} = x^2\cdot C \quad /\cdot x$$

$$y+\sqrt{y^2+11x^2} = x^3\cdot C$$

$$\frac{y+\sqrt{y^2+11x^2}}{x^3} = C$$

Ответ: Общее решение: $\frac{y+\sqrt{y^2+11x^2}}{x^3} = C$