Research on Digital Image Enhancement Based on Time Frequency Domain*

*Note: This article is just a simple science popularization

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Abstract—This article will introduce the relevant content of gray scale digital image enhancement in detail, including what is gray scale image, the types and sources of image noise, how to evaluate the effect of image enhancement, and the filtering methods to solve the corresponding noise. A method combining time domain and frequency domain is proposed to minimize the influence of noise. Relevant code has been opened in https://github.com/mugua1020/image-enhancement.

Index Terms—matlab, image enhancement, wave filter

I. BACKGROUNDS AND CHALLENGES

During photography, a picture is often affected by factors such as the external light source is too bright or too dark, the intermediate medium is turbid, or there is periodic noise in the picture transmission process. How to enhance the expression ability of a picture and filter the noise has become the focus of current research. The typical iamge enhancement algorithm based on wavelet proposed by Li et al., and some related technologies based on genetic algotithm. In recent years, with the continuous development of deep learning, image enhancement technology based on neural network has also received. The traditional image enhancement technology based on spatial domain and frequency domain is the basis to understande and master all this. This work has completed six common types of image filters, and will explain them one by one, and compare their performance indicators. The quantization design of frequency filter of two-dimensional iamge is often complicated, this will be explained more specifically later. Therefore, this paper completes a one-dimensional frequency filter and its quantization process as a reference.

Next, we will indroduce the concept of two-dimensional image frequency from the gray image. After that, we are going to introduce four common noises, and how to quantify the corresponding enhancement indicators and their meanings.

A. Grayscale Image

In digital photography, computer-generated imagery, and colorimetry, a grayscale image is one in which the value of each pixel is a single sample representing only an amount of light; that is, it carries only intensity information. Grayscale images, a kind of black-and-white or gray monochrome, are composed exclusively of shades of gray. The contrast ranges from black at the weakest intensity to white at the strongest. Grayscale images are distinct from one-bit bi-tonal black-and-white images, which, in the context of computer imaging, are images with only two colors: black and white (also called bilevel or binary images). Grayscale images have many shades of gray in between.

Fig. 1. Example of a Grayscale image.

Grayscale images can be the result of measuring the intensity of light at each pixel according to a particular weighted combination of frequencies (or wavelengths), and in such cases they are monochromatic proper when only a single frequency (in practice, a narrow band of frequencies) is captured. The frequencies can in principle be from anywhere in the electromagnetic spectrum (e.g. infrared, visible light, ultraviolet, etc.).

A colorimetric (or more specifically photometric) grayscale image is an image that has a defined grayscale colorspace, which maps the stored numeric sample values to the achromatic channel of a standard colorspace, which itself is based on measured properties of human vision.

If the original color image has no defined colorspace, or if the grayscale image is not intended to have the same human-perceived achromatic intensity as the color image, then there is no unique mapping from such a color image to a grayscale image.

Thank Ting Yi for her teaching.

B. Frequency of The Image

First, we need to understand the spatial domain of the image. The spatial domain of an image is a two-dimensional plane coordinate system, which has two orthogonal axes, namely, x and y axes. The amplitude of a point in the spatial domain is the gray level of the point. The gray level of a point on the image is the superposition of the gray levels of the point in the x and y directions. In this way, take x and y as abscissa and gray value as ordinate respectively to obtain two functions. Indicates the change of the gray value of the image in the x and y directions.

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})} \tag{1}$$

Secondly, we need to master Fourier series. According to the knowledge of Fourier series, a function can be written as the sum of several sine functions with different amplitudes and frequencies. Therefore, through Fourier transform, the gray-scale - x function is transformed into the amplitude - u function, and the gray-scale - y function is transformed into the amplitude - v function. The plane coordinate system with u, v as the orthogonal axis is established, so the frequency diagram is obtained.

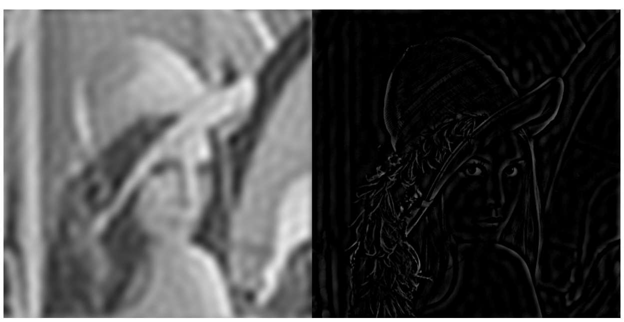

Fig. 2. Picture after frequency filter processing

The frequency of an image is an indicator of the intensity of gray level changes in the image, and it is the gradient of gray level in the plane space. As shown in Figure 2, the low frequency component represents the contour of the image, while the high frequency component represents more details of the image

C. Typical Noise

First, salt and pepper noise and random noise are common. Salt and pepper noise refers to two kinds of noise, one is salt noise, the other is pepper noise. Salt noise is white 255 in gray image, pepper noise is black 0, and it is black and white speckles in image. Random noise is similar to salt and pepper noise, but there are only white noise points. Gaussian noise is a kind of noise whose probability density function follows Gaussian distribution (i.e. normal distribution). If the amplitude distribution of a noise follows Gaussian distribution and its power spectral density is uniformly distributed, it is called Gaussian white noise.

Fig. 3. Example of periodic noise

Random noise is a kind of noise that is caused by the accumulation of a large number of fluctuation disturbances randomly generated in time, and its value cannot be predicted in a given instant. Random noise often appears in oil exploitation, exploration, image processing, signal processing, wireless communication, underwater acoustic detection, biomedical engineering, optical fiber communication and other fields, and it is often used to manually add random noise to carry out signal restoration and image restoration. Common random noises include single frequency noise, impulse noise and fluctuation noise. Single frequency noise is a continuous wave interference (such as external station signal). It can be regarded as a modulated sine wave, but its amplitude, frequency or phase cannot be predicted in advance. The main feature of this noise is that it occupies an extremely narrow frequency band, but its position on the frequency axis can be measured. Therefore, single frequency noise does not exist in all communication systems and is relatively easy to prevent. The random noise used in this paper is a simple single frequency noise.

D. Typical Indicators

At the pixel level, the evaluation criteria for image enhancement usually include the mean absolute error (MAE), mean square error (MSE), peak signal to noise ratio (PSNR) and structural similarity (SSIM). At present, the authoritative objective evaluation criteria for image enhancement (image superresolution, image denoising, image deblurring, image rain removal, image deblocking effect, etc.) are the peak signal to noise ratio (PSNR) and structural similarity (SSIM).

$$MSE = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} [X(i,j) - \tilde{X}(i,j)]^2 \tag{2}$$

$$PSNR = 10 \cdot log_{10}[\frac{(2^n-1)^2}{MSE}] \tag{3}$$

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_x^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \tag{4}$$

Structural similarity is an index used to measure the similarity of two digital images. When one of the two images is a lossless image and the other is a lossy image, the structural similarity of the two images can be regarded as a measure of the image quality of the lossy image. Compared with the traditional image quality measurement indicators, the structure similarity is more consistent with the judgment of the human eye on image quality. It is generally believed that the structure similarity index is more suitable than the mean square error (MSE) to judge the similarity of two images, because the structure similarity index considers the image brightness, contrast and structure information at the same time, which is closer to the judgment benchmark of human visual system. MSE only considers the average brightness error of the two pictures, and does not consider the structured information. Therefore, when the picture has a slight brightness change, the MSE change will be very dramatic, but the small brightness change will not cause people to judge the two pictures as completely different.

II. METHODOLOGY

For this reason, we have designed seven different filters, including mean filter, median filter, Gaussian filter and special notch filter in the spatial domain, and ideal low-pass filter, Butterworth low-pass filter and Gaussian low-pass filter in the frequency domain. Now I will introduce them one by one. After the introduction, I will propose a circuit implementation method of two-dimensional filter and a quantization example.

A. Filters in Spatial Domain

The implementation of the filter in the spatial domain is relatively simple. First, set the size of the picture. In particular, pay attention to the pre filling of the picture. This is because in the process of picture sliding convolution, the edge results of the picture need to be specially processed to ensure the size of the output picture. Secondly, we need to create corresponding convolution kernels according to the input parameters. Different filters have different convolution kernels. In the process of windowing and convolution, different operations can be completed according to different filters.

Mean filtering is also called linear filtering, and the main method used is neighborhood average method. The basic principle of linear filtering is to replace each pixel value in the original image with the average value, that is, the current pixel point (x, y) to be processed. Select a template, which is composed of several neighboring pixels. Find the average value of all pixels in the template, and then assign the average value to the current pixel point (x, y) as the gray level g (x, y) of the processed image at that point, That is, g (x, y)=Σ f (x, y)/m m is the total number of pixels including the current pixel in the template.

The main idea of the median filter is to run through the signal entry by entry, replacing each entry with the median of neighboring entries. The pattern of neighbors is called the ”window”, which slides, entry by entry, over the entire signal. For one-dimensional signals, the most obvious window is just the first few preceding and following entries, whereas for two-dimensional (or higher-dimensional) data the window must include all entries within a given radius or ellipsoidal region (i.e. the median filter is not a separable filter).

Gaussian filtering is a kind of linear smoothing filtering, which is suitable for eliminating Gaussian noise and is widely used in the noise reduction process of image processing. Generally speaking, Gaussian filtering is the process of weighting and averaging the whole image. The value of each pixel is obtained by weighting and averaging itself and other pixel values in the neighborhood. The specific operation of Gaussian filtering is to use a template (or convolution, mask) to scan every pixel in the image, and use the weighted average gray value of the pixels in the neighborhood determined by the template to replace the value of the central pixel of the template.

Notch filter refers to a filter that can rapidly attenuate the input signal at a certain frequency point to achieve the filtering effect that blocks the passage of this frequency signal. Notch filter is a kind of band stop filter, but its stop band is very narrow. As we know, periodic noise in the frequency domain is the noise caused by a certain frequency point, such as 60Hz AC in the environment. Therefore, we designed a special notch filter for periodic noise to filter this specific noise.

B. Filter in Frequency Domain

The frequency domain image enhancement processing is to transform the image from the spatial domain to the frequency domain, then perform the enhancement processing, and finally invert the transformation to the spatial domain. After the two-dimensional function of the original image is decomposed into signals of different frequencies, the high-frequency signal carries the details of the image (such as the image boundary), and the low-frequency signal contains the rough background information of the image. These signals with different frequencies can be processed to achieve the corresponding purpose of image enhancement. For example, strengthening the low-frequency signal can enhance the contrast of image details to achieve the effect of sharpening. Removing the low-frequency signal can remove the details, and only get the image with a rough background outline.

The ideal low pass filter is a relatively simple type. By multiplying the FFT result of the picture in the frequency domain by the transfer function of the low-pass filter, the result is returned to the picture after IFFT. Compared with Gaussian filter, the mathematical equation of transfer function is modified to make it more smooth. Butterworth filter is a further enhancement of the Gaussian filter.

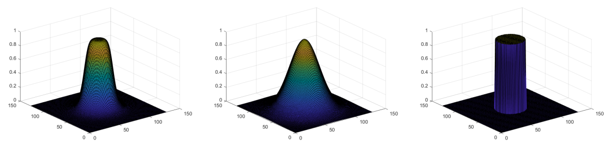

Fig. 4. Butterworth filter and Gaussian filter and ideal filter

The order N of the filter is proposed. When N is small, Butterworth filter is closer to the Gaussian filter, and when N is large, it is closer to the result of the ideal filter.

$$H(u,v) = \begin{cases} 1, D(u,v) \le D_0 \\ 0, D(u,v) > D_0 \end{cases} \tag{5}$$

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}} \tag{6}$$

$$H(u,v) = e^{-D^2(u,v)/2D_0^2} \tag{7}$$

C. Filter Quantization

For two-dimensional DFT, when the image size is relatively large, the direct calculation is too large and difficult to achieve. In fact, we often use the circuit method to achieve filtering operations in the time domain. However, it is too complicated to implement the two-dimensional filter directly on the circuit, so we often use the dimension reduction method to calculate the two-dimensional DFT.

$$\begin{aligned} F(u,v) &= \sum_{x=0}^{M-1} e^{-j2\pi ux/M} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi vy/N} \\ &= \sum_{x=0}^{M-1} F(x,v) e^{-j2\pi ux/M} \end{aligned} \tag{8}$$

$$F(x,v) = \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi vy/N} \tag{9}$$

dimensional DFT to demonstrate the quantization process of the filter. To put it simply, by designing the passband boundary frequency, stopband boundary frequency, passband ripple and stopband ripple of the filter, the order and cut-off frequency of the corresponding filter are obtained through the butord() function, and then the transfer function of the corresponding filter is obtained through the butter() function. The bilinear property of bilinear () is changed to the z domain, and the corresponding coefficients are quantized into quantizer () and quantize () to obtain the corresponding results.

III. RESULTS AND DISCUSSION

When we add periodic noise and salt pepper noise to the image, we get an image with noise, as shown in the right figure of Figure 6. Since the seven filters are too many, the typical mean filter, median filter, Gaussian filter and Gaussian low-pass filter are selected here. The rest of the filters can be run by themselves if you want to try. The corresponding MSE, PSNR and SSIM results after these filters are shown in the right figure of Figure 7. Correspondingly, if we add periodic noise and Gaussian noise to the picture, we can get a noise image as shown in the left figure of Figure 6. After four filters with the same results, the corresponding results are shown in the left figure of Figure 7.

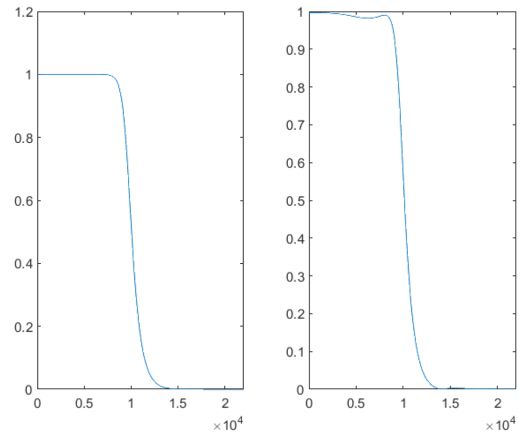

Fig. 5. Before and after quantification

Fig. 6. Pictures after adding noise

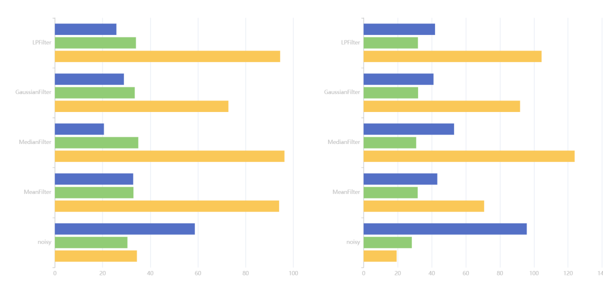

Fig. 7. Results after filtering

From the filtering results, we can know that the median filter is better for salt and pepper noise, and better for Gaussian noise. From our previous analysis of notch filter, we can know that notch filter can play a greater role in terms of periodic noise, so we can propose a combined filter. For images with periodic noise and salt pepper noise, notch filter is used to filter out the periodic noise, and median filter is used to filter out the salt pepper noise. The same method is used for images with periodic noise and Gaussian noise.

IV. CONCLUTION

To sum up, we analyzed the gray image and its principle, and also proposed the concept of image frequency domain. Then four typical noises and three typical evaluation methods are introduced. Finally, based on the above knowledge, we propose seven common filters and their structures, and select four of them, and give the results under two pictures with different noises. According to this result, we propose a more reasonable structure of the combined filter, and detailed this structure will have better output results.

V. QUESTIONNAIRE INVESTIGATION

1. I think there are a lot of teaching contents in this course. The following classes are very tense, and the teacher speaks quickly

2. To be helpful. You can have a deeper understanding of the principles and uses of these things.

3. I feel that there is a problem in the assignment of Topic 1 and 2, which may be because no one has done it before

4. Big homework is a little difficult. Due to lack of time, some of the work cannot be improved

5. It would be better if more relevant information could be provided