

ZETA CONVERTER

Final Report

EE 560: POWER ELECTRONIC CONVERTERS

By

MUHAFIZ BILAL KHAN (244102111)

PRAMOD VIPPARTHI (244102112)

Under the guidance of

Dr. CHANDAN KUMAR



Department of Electronics and Electrical Engineering

Indian Institute of Technology, Guwahati

North Guwahati, Assam - 781039, India

November 2024

Team Members

- Muhafiz Bilal Khan - 244102111 (Analyzed the zeta converter in continuous, boundary and discontinuous mode of operations. Simulated the zeta converter in SIMULINK. Edited MATLAB figures in VISIO. Made \LaTeX file for report submission.)
- Pramod Vipparthi - 244102112 (Analyzed the zeta converter in continuous, boundary and discontinuous mode of operations. Simulated the zeta converter in SIMULINK. Made circuit diagrams and waveforms in VISIO.)

Contents

List of Tables	iv
List of Figures	v
1 Introduction	1
2 Zeta Converter in Continuous Conduction Mode	1
2.1 Input-Output Voltage Relationship	2
2.2 Current Relationships	3
2.3 Converter Waveforms in CCM	5
2.4 Inductor Current Ripple.....	6
2.5 Capacitor Voltage Ripple	7
3 Boundary condition between CCM and DCM in Zeta Converter	8
3.1 Converter Waveforms in Boundary Condition	8
3.2 Minimum Inductor Values For CCM.....	9
4 Zeta Converter in Discontinuous Conduction Mode	10
4.1 Converter Waveforms in DCM	10
4.2 Input-Output Voltage Relationship	11
4.3 Output Capacitor current ampere-second balance	13
5 Simulation	14
5.1 Parameters	14
5.2 Assumptions	14
5.3 Calculations	15
5.3.1 Duty ratio D	15
5.3.2 Load Resistance R	15
5.3.3 Average inductor currents I_{L_1} and I_{L_2}	16
5.3.4 Switching period T_s	16
5.3.5 Inductor values L_1 and L_2	16
5.3.6 Capacitor Values C_1 and C_2	17
5.4 SIMULINK model	17
6 Simulation Results	18
6.1 Waveforms	18
6.2 Observation.....	21

6.3	Error Analysis	22
7	Closed Loop Control of Zeta Converter	22
7.1	Simulation Parameters and Schematic for Closed Loop Control	22
7.2	Simulation Waveforms	23
7.3	Simulation Results	25
8	Future Scope	25

List of Tables

1	Parameters	14
2	Assumed Values of inductor currents ripple and output voltage ripple	15
3	Practical Values obtained from SIMULINK modelling	21
4	Percentage error in practical values	22
5	PI controller parameters.....	22
6	Practical Values obtained from SIMULINK modelling using closed loop control	25

List of Figures

1	Zeta Converter Circuit	1
2	Zeta Converter circuit when switch is on and off	2
3	Zeta Converter waveforms in CCM	5
4	Zeta Converter waveforms in Boundary Condition	8
5	Zeta Converter waveforms in DCM	11
6	Switching signal	18
7	Input Voltage	18
8	Output Voltage	18
9	Voltage across L_1	19
10	Current through L_1	19
11	Current through C_1	19
12	Voltage across L_2	20
13	Current through L_2	20
14	Current through C_2	20
15	Voltage across switch	21
16	Voltage across diode	21
17	Schematic for closed loop control of zeta converter	23
18	Output Voltage of zeta converter after closed loop control	23
19	Current through L_1 after closed loop control	24
20	Current through L_2 after closed loop control	24

1 Introduction

A Zeta converter is a type of DC-DC converter that is often used for power management in electronics. The converter is named after Zeta topology, which uses two inductors and a single capacitor as energy storage elements to transfer energy from input side to output side. It also uses another capacitor for decreasing the ripple in the output voltage.

Zeta converter is similar to a single-ended primary-inductor converter (SEPIC), with the exception that the diode in SEPIC converter is replaced with an inductor in Zeta converter. It has a different topology and operational characteristics, often resulting in smoother output for certain applications. The Zeta converter can be seen as a combination of buck-boost and Cuk converter topologies. The buck-boost topology provides the ability to step-up and step-down the voltages and Cuk converter topology provides improved voltage ripple. One of the innovations in Zeta converter was the use of coupled inductors. This provided a means to increase efficiency, reduce size and handle higher power levels.[1][2]

2 Zeta Converter in Continuous Conduction Mode

A Zeta converter steps up or steps down the unregulated dc voltage at the input side at the output end. It maintains the same polarity between the input and output.

Circuit Diagram of zeta converter is given below:

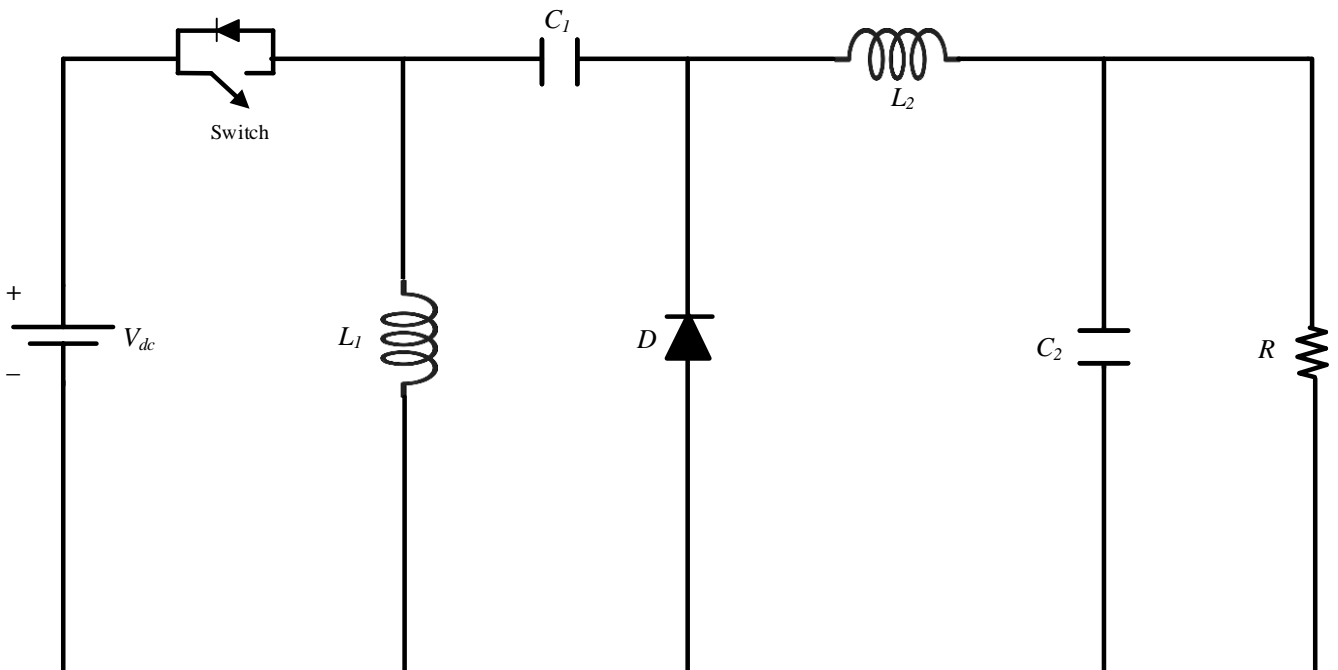


Figure 1: Zeta Converter Circuit

The figures below show the circuit during on and off condition:

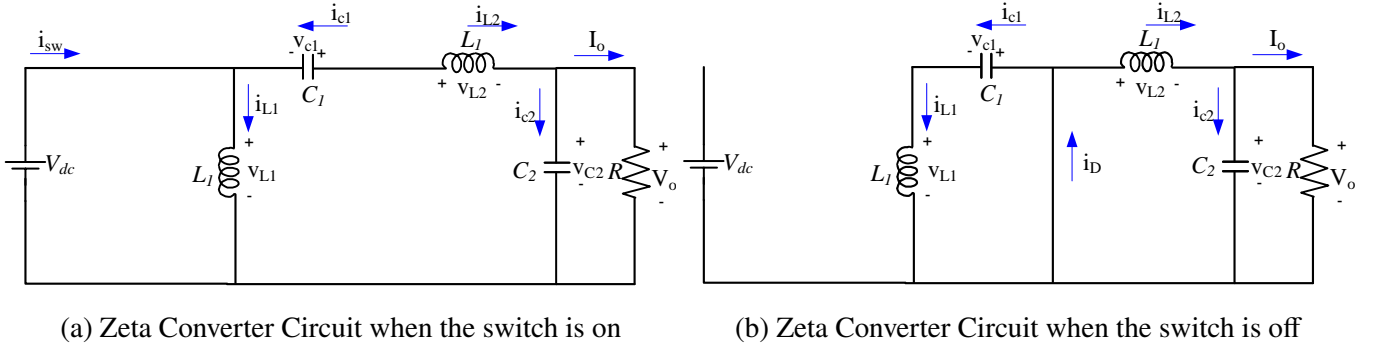


Figure 2: Zeta Converter circuit when switch is on and off

2.1 Input-Output Voltage Relationship

During continuous mode of conduction and steady state analysis, it was assumed that the switch was on during the time DT_s and off during the time interval $(1 - D)T_s$ where D was the duty ratio of switch and T_s was the switching frequency. When the switch is on, the diode will be reverse biased due to the capacitor voltage and when the switch is off, the diode will be forward biased owing to the inductor L_1 current.

Now during $T_{on} = DT_s$,

$$v_{L1} = V_{dc} \quad (1)$$

During this time, i_{L1} will be increasing owing to the fact that there is a voltage build up across it. Moreover, since the diode will be off, the capacitor C_1 will release it's energy and current through L_2 will also be increasing.

Also,

$$v_{L2} = V_{dc} + v_{c1} - V_o \quad (2)$$

During $T_{off} = (1 - D)T_s$,

$$v_{L1} = -v_{c1} \quad (3)$$

and

$$v_{L2} = -V_o \quad (4)$$

During this time both i_{L1} and i_{L2} will be decreasing owing to the fact the the voltage across both L_1 and L_2 is negative.

Applying voltage-second balance to L_1 ,

$$V_{dc}DT_s - V_{c1}(1 - D)T_s = 0$$

$$\Rightarrow V_{c1} = \frac{DV_{dc}}{1 - D} \quad (5)$$

Applying voltage-second balance to L_2 ,

$$(V_{dc} + v_{c1} - V_o)DT_s - V_o(1 - D)T_s = 0 \quad (6)$$

Substituting the value of v_{c1} from equation 5 in equation 6,

$$DV_{dc} + D \left(\frac{DV_{dc}}{1 - D} \right) - DV_o - V_o + DV_o = 0$$

$$\Rightarrow \frac{V_o}{V_{dc}} = \frac{D}{1 - D} \quad (7)$$

Thus, for $0 < D < 0.5$, the converter will step down the voltage and for $0.5 < D < 1$, the converter will step up the voltage.

2.2 Current Relationships

The capacitor currents for T_{on} and T_{off} were written as follows.

During $T_{on} = DT_s$,

$$i_{c1} = -i_{L2} \quad (8)$$

and during $T_{on} = DT_s$,

$$i_{c2} = i_{L2} - \frac{V_o}{R} \quad (9)$$

During $T_{off} = (1 - D)T_s$,

$$i_{c1} = i_{L1} \quad (10)$$

and

$$i_{c2} = i_{L2} - \frac{V_o}{R} \quad (11)$$

Applying ampere-second balance to C_2 ,

$$\left(i_{L2} - \frac{V_o}{R}\right)DT_s + \left(i_{L2} - \frac{V_o}{R}\right)(1 - D)T_s = 0$$

$$\Rightarrow I_{L2} = I_o = \frac{V_o}{R} \quad (12)$$

Applying ampere-second balance to C_2 ,

$$(-i_{L2})DT_s + i_{L1}(1 - D)T_s = 0$$

$$i_{L1} = i_{L2} \left(\frac{D}{1 - D} \right) \quad (13)$$

Taking average and substituting the value of I_{L2} from equation 12 into equation 13,

$$I_{L1} = \frac{V_o}{R} \left(\frac{D}{1 - D} \right) \quad (14)$$

$$\Rightarrow I_{L1} = I_o \left(\frac{D}{1 - D} \right) \quad (15)$$

$$\Rightarrow I_{L1} = I_{in} \quad (16)$$

Thus, upon inspecting the above relationships of average inductor currents, it was found that the average current through L_1 is the average input current I_{in} and the average current through L_2 is the average output current I_o .

2.3 Converter Waveforms in CCM

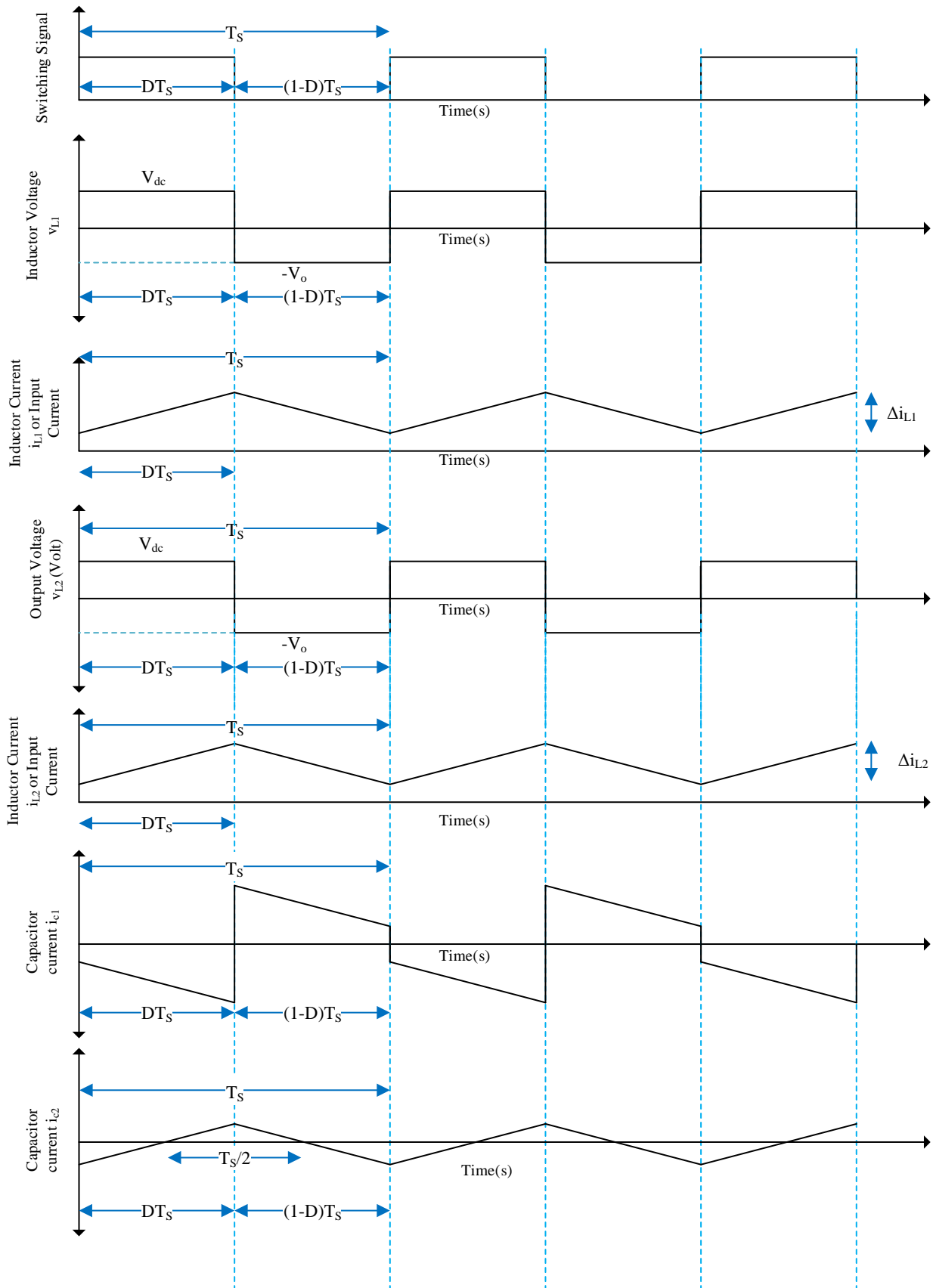


Figure 3: Zeta Converter waveforms in CCM

The waveforms in figure 3 indicate the operation of Zeta converter in continuous mode of operation. The switch is on for a duty ratio D and off for a duty ratio of $(1 - D)$. The switching signal is supplied to the switch at a time period of T_s . Now when the switch is on, the voltages across L_1 and L_2 is V_{dc} . During this time, the current through both of these inductors is rising as the voltage across them is positive as seen from the waveforms in figure 3. When the switch is off, the voltages across L_1 and L_2 is $-V_o$. During this time, the current through both of these inductors will be decreasing owing to the negative voltage across them. Moreover, the values of L_1 and L_2 are such that the current never goes to zero at or before the end of switching cycle. The mathematical analysis of the inductor voltages has already been done in section 2.1 using voltage-second balance of inductors and the equations 1 to 7 have been written according to these waveforms.

The currents through the capacitors have also been plotted. Note that the average capacitor currents are 0. The mathematical analysis of these waveforms have already been done using ampere-second balance as evident in section 2.2.

2.4 Inductor Current Ripple

For L_1 as seen from the above waveforms,

$$\Delta i_{L_1} = \left(\frac{V_{dc}}{L_1} \right) DT_s \quad (17)$$

Substituting the value of V_{dc} from equation 7 in equation 17,

$$\Delta i_{L_1} = \frac{(1 - D)V_o T_s}{L_1} \quad (18)$$

Similarly, for L_2 as seen from the above waveforms,

$$\Delta i_{L_2} = \left(\frac{V_{dc}}{L_2} \right) DT_s \quad (19)$$

Substituting the value of V_{dc} from equation 7 in equation 19,

$$\Delta i_{L_2} = \frac{(1 - D)V_o T_s}{L_2} \quad (20)$$

Equation 18 and 20 gives the relationship between inductor ripple currents, duty ratio, inductance values and output voltage.

2.5 Capacitor Voltage Ripple

From the current waveform of capacitor C_1 and equation 8, the charge stored in the capacitor can be found by the area under the curve during DT_s . Let us assume that the ripple in inductor current L_2 is very small and can be ignored and thus $i_{L_2} = I_o = \frac{V_o}{R}$. Hence,

$$\Delta Q_{c_1} = \left(\frac{1}{2}\right) (DT_s) (I_o) \quad (21)$$

Moreover, from equation 5, the average capacitor C_1 voltage is given be

$$V_{c_1} = \frac{DV_{dc}}{1-D}$$

Substituting equation 7 in the above equation,

$$\Rightarrow V_{c_1} = V_o \quad (22)$$

Writing $\Delta Q_{c_1} = C_1 \Delta V_o$ and substitution $I_o = \frac{V_o}{R}$ in equation 21,

$$C_1 \Delta V_o = \left(\frac{1}{2}\right) (DT_s) \left(\frac{V_o}{R}\right) \quad (23)$$

$$\Rightarrow \Delta V_o = \frac{DT_s V_o}{2C_1 R} \quad (24)$$

From the current waveform of capacitor C_2 , the charge stored in the capacitor can be found when the current is positive and is given by the area under the curve during that time. Hence,

$$\Delta Q_{c_2} = \left(\frac{1}{2}\right) \left(\frac{T_s}{2}\right) \left(\frac{\Delta i_{L_2}}{2}\right) \quad (25)$$

Writing $\Delta Q_{c_2} = C_2 \Delta V_o$ and substituting the value of Δi_{L_2} from equation 20 in equation 25,

$$C_2 \Delta V_o = \left(\frac{1}{8}\right) \left(\frac{T_s^2}{L_2}\right) (1-D) V_o \quad (26)$$

$$\Rightarrow \Delta V_o = \frac{(1-D) T_s^2 V_o}{8 L_2 C_2} \quad (27)$$

Equations 24 and 27 give the relationship between the output voltage ripple and output voltage. These equations are used to design capacitors C_1 and C_2 of appropriate values.

3 Boundary condition between CCM and DCM in Zeta Converter

The boundary condition is also regarded as just continuous mode of conduction. During this mode of operation, the inductor currents go to zero at the end of each switching cycle and again start from zero for the next switching cycle. So, inductor currents just touch zero and rises back up. If the inductor currents go to zero before the end of switching cycle, then we will have discontinuous mode of operation.

3.1 Converter Waveforms in Boundary Condition

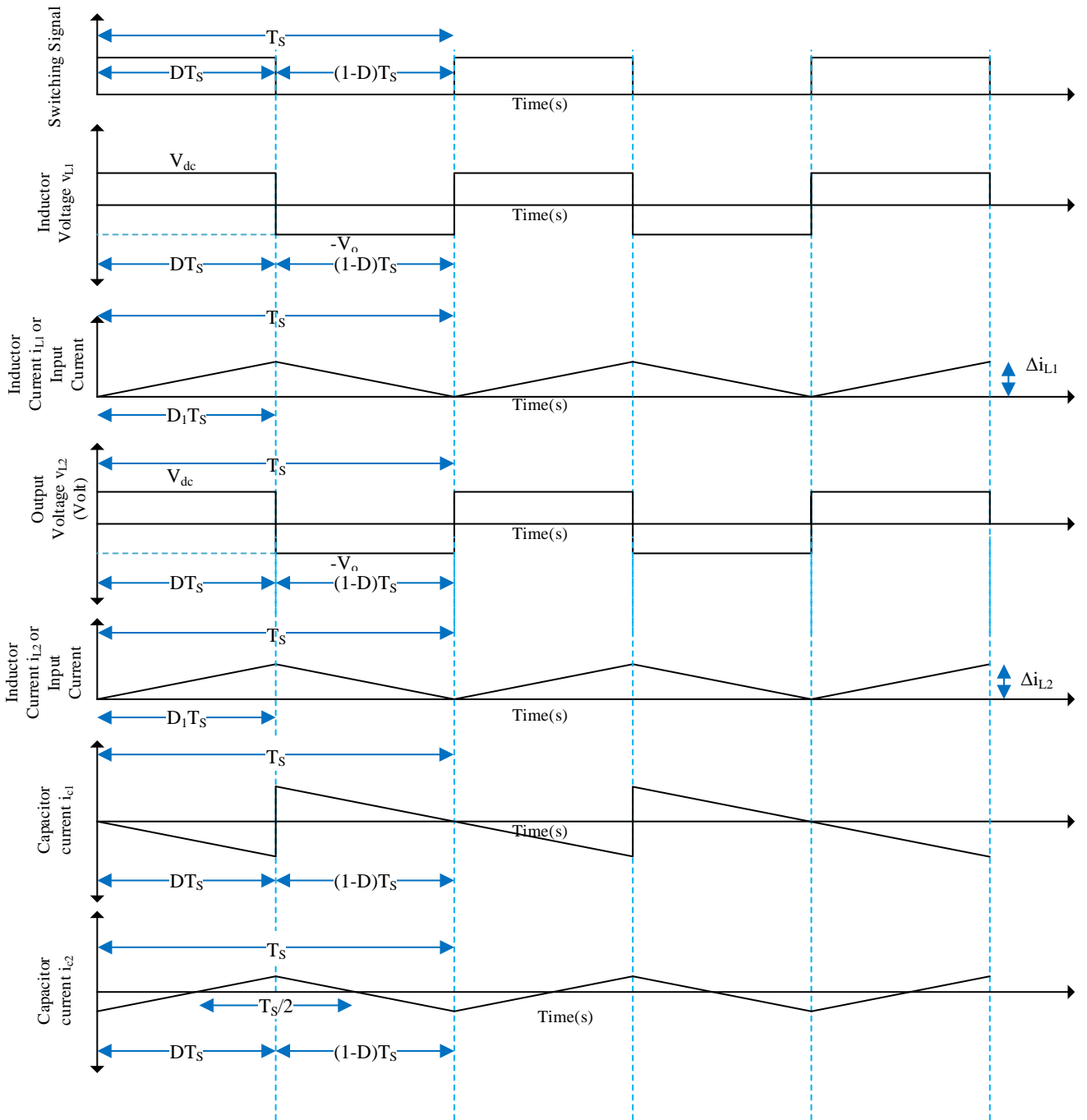


Figure 4: Zeta Converter waveforms in Boundary Condition

The waveforms in figure 4 indicate the operation of Zeta converter in boundary condition between continuous and discontinuous modes of operation. This mode is also regarded as just continuous mode of conduction. The switch is on for a duty ratio D and off for a duty ratio of $(1 - D)$. The switching signal is supplied to the switch with a time period of T_s . Now when the switch is on, the voltages across L_1 and L_2 is V_{dc} . During this time, the current through both of these inductors is rising from 0 as the voltage across them is positive as seen from the waveforms in figure 4. When the switch is off, the voltages across L_1 and L_2 is $-V_o$. During this time, the current through both of these inductors will be decreasing owing to the negative voltage across them. Moreover, the values of L_1 and L_2 are such that the current goes to zero at the end of every switching cycle and starts from 0 at the beginning of the next switching cycle. These values of L_1 and L_2 are regarded as critical values of inductors. If the inductance values falls below these critical values, the converter will operate in DCM which is explained in section 4. The critical values have been computed in equations 31 and 35. This mode of operation is continuous mode as well and hence all the equations derived for CCM will be applicable during this mode as well.

The currents through the capacitors have also been plotted. Note that the average capacitor currents are 0. The mathematical analysis of these waveforms is same as that of CCM operation. Notice that capacitor current waveforms also change with the change in operating conditions of converter.

3.2 Minimum Inductor Values For CCM

For inductor L_1 during boundary condition, average current is given by,

$$I_{LB1} = \frac{\Delta i_{L1} T_s}{2T_s} \quad (28)$$

$$\Rightarrow I_{LB1} = \frac{\Delta i_{L1}}{2} \quad (29)$$

Substituting the value of Δi_{L1} from equation 18 into equation 29,

$$I_{LB1} = \frac{(1 - D)V_o T_s}{2L_{1critical}} \quad (30)$$

where $L_{1critical}$ is the minimum inductance for current to be continuous.

Equating average inductor current through L_1 in equation 14 and equation 30,

$$\frac{V_o}{R} \left(\frac{D}{1 - D} \right) = \frac{(1 - D)V_o T_s}{2L_{1critical}}$$

Solving for L_1 ,

$$L_{1critical} = \frac{(1 - D)^2 R T_s}{2D} \quad (31)$$

This is the minimum value of L_1 for I_{L_1} to be continuous.

For inductor L_2 during boundary condition, average current is given by,

$$I_{LB_1} = \frac{\Delta i_{L_2} T_s}{2T_s} \quad (32)$$

$$\Rightarrow I_{LB_2} = \frac{\Delta i_{L_2}}{2} \quad (33)$$

Substituting the value of Δi_{L_2} from equation 20 into equation 33,

$$I_{LB_2} = \frac{(1-D)V_o T_s}{2L_{2critical}} \quad (34)$$

where $L_{2critical}$ is the minimum inductance for current to be continuous. Equating average inductor current through L_2 in equation 12 and equation 34,

$$\frac{V_o}{R} = \frac{(1-D)V_o T_s}{2L_{2critical}}$$

Solving for L_2 ,

$$L_{2critical} = \frac{(1-D)RT_s}{2} \quad (35)$$

This is the minimum value of L_2 for I_{L_2} to be continuous.

4 Zeta Converter in Discontinuous Conduction Mode

The discontinuous mode of conduction is one in which current through inductors goes to zero before the end of switching cycle. It is assumed that $D_1 T_s$ is the time during which the switch is on and the switch is off for the remaining time. Since the converter is in DCM, the inductor currents go to zero way before T_s at $(D_1 + D_2)T_s$. Afterwards the current remains zero for ΔT_s until the beginning of the next switching cycle.

4.1 Converter Waveforms in DCM

The waveforms in figure 5 indicate the operation of Zeta converter in discontinuous mode of operation. The switch is on for a duty ratio D_1 and off for a duty ratio of $(1 - D_2)$. The switching signal is supplied to the switch with a time period of T_s . Now when the switch is on, the voltages across L_1 and L_2 is V_{dc} . During this time, the current through both of these inductors is rising from 0 as the voltage across them is positive as seen from the waveforms in figure 5. When the switch is off, the inductor current goes to zero before the end of switching cycle at $(D_1 + D_2)T_s$. The voltages across L_1 and L_2 during duty $D_2 T_s$ is $-V_o$. Hence, for a period of ΔT_s , there is no current and voltage across any of the inductors as evident from

the waveforms in figure 5. Moreover, the values of L_1 and L_2 are less than the critical values of needed for the converter to operate in CCM. The mathematical analysis of the inductor voltages for DCM has been done in section 4.2 using voltage-second balance of inductors and the equations 36 to 42 have been written according to these waveforms.

The currents through the capacitors have also been plotted. In this case, the current through the capacitors is different from that of continuous mode of operation. In case of C_1 , the current goes to zero during ΔT_s . In case of C_2 , the output current flows through the capacitor in reverse direction during ΔT_s . Note that the average capacitor currents are zero. The mathematical analysis of these waveforms have already been done using ampere-second balance as evident in section 4.3.

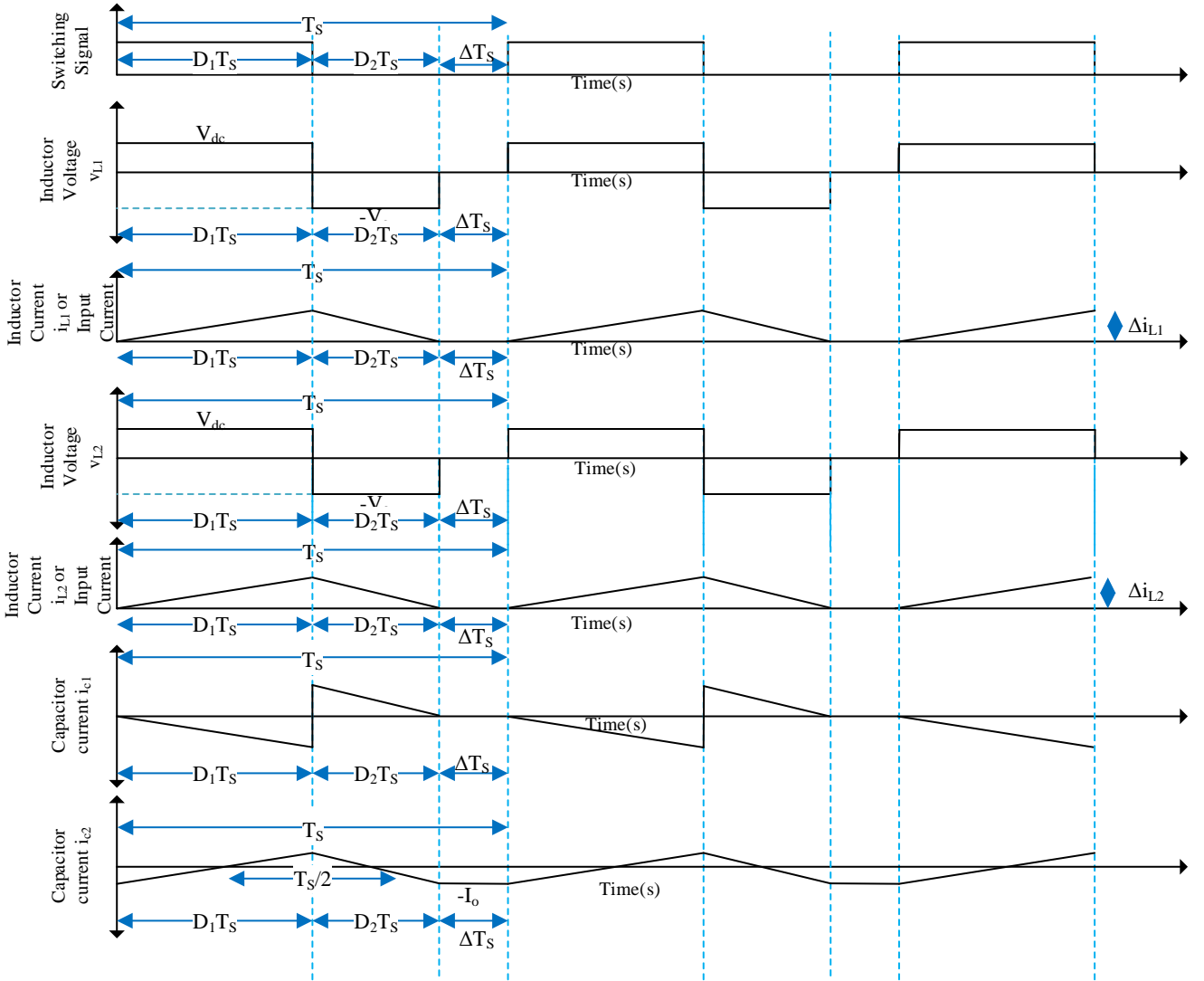


Figure 5: Zeta Converter waveforms in DCM

4.2 Input-Output Voltage Relationship

During DCM mode of conduction and steady state analysis, it was assumed that the switch Q was on during the time $D_1 T_s$ and the current went to zero at $(D_1 + D_2)$ where D_1 was the duty ratio of switch Q and T_s was the switching frequency. When the switch is on, the diode will be reverse biased due to the capacitor

voltage and when the switch is off, the diode will be forward biased owing to the inductor L_1 current.

Now during D_1T_s ,

$$v_{L_1} = V_{dc} \quad (36)$$

During this time, i_{L_1} will be increasing owing to the fact that there is a voltage build up across it. Moreover, since the diode will be off, the capacitor C_1 will release it's energy and current through L_2 will also be increasing.

Also,

$$v_{L_2} = V_{dc} + v_{c_1} - V_o \quad (37)$$

During D_2T_s ,

$$v_{L_1} = -v_{c_1} \quad (38)$$

and

$$v_{L_2} = -V_o \quad (39)$$

During this time both i_{L_1} and i_{L_2} will decrease and go to 0 way before the end of switching cycle owing to the fact the the voltage across both L_1 and L_2 is negative.

During ΔT_s , the voltage across both inductors is 0. So, there is no requirement to write that is VSB.

Applying voltage-second balance to L_1 ,

$$\begin{aligned} V_{dc}D_1T_s - V_{c_1}D_2T_s &= 0 \\ \implies V_{c_1} &= \frac{D_1V_{dc}}{D_2} \end{aligned} \quad (40)$$

Applying voltage-second balance to L_2 ,

$$(V_{dc} + v_{c_1} - V_o)D_1T_s - V_oD_2T_s = 0 \quad (41)$$

Substituting the value of V_{c1} from equation 40 in equation 41,

$$D_1 V_{dc} + D_1 \left(\frac{D_1 V_{dc}}{D_2} \right) - D_1 V_o - D_2 V_o = 0$$

$$\Rightarrow \frac{V_o}{V_{dc}} = \frac{D_1}{D_2} \quad (42)$$

This is the input-output voltage relationship when the converter operates in DCM.

The above equations were obtained by referring to [3].

4.3 Output Capacitor current ampere-second balance

During $D_1 T_s$,

$$i_{c2} = i_{L2} - \frac{V_o}{R} \quad (43)$$

During $D_2 T_s$

$$i_{c2} = i_{L2} - \frac{V_o}{R} \quad (44)$$

During ΔT_s ,

$$i_{c2} = -\frac{V_o}{R} \quad (45)$$

Applying ampere second balance to C_2 ,

$$\int_0^{T_s} i_{c2} dt = 0 \quad (46)$$

$$\Rightarrow \int_0^{D_1 T_s} \left(i_{L2} - \frac{V_o}{R} \right) dt + \int_{D_1 T_s}^{(D_1+D_2)T_s} \left(i_{L2} - \frac{V_o}{R} \right) dt + \int_{(D_1+D_2)T_s}^{T_s} \left(\frac{V_o}{R} \right) dt = 0 \quad (47)$$

$$\int_0^{(D_1+D_2)T_s} i_{L2} dt = \frac{V_o T_s}{R} \quad (48)$$

The integral signifies the area under the curve of i_{L2} from 0 to $D_1 + D_2$. Hence, equation 48 becomes

$$\left(\frac{(D_1 + D_2)T_s}{2} \right) \left(\frac{V_{dc} D_1 T_s}{L_2} \right) = \frac{V_o T_s}{R}$$

$$\Rightarrow \frac{(D_1 + D_2)D_1RT_s}{2L_2} = \frac{V_o}{V_{dc}}$$

Substituting the value of $\frac{V_o}{V_{dc}}$ from equation 42 in the above equation,

$$\Rightarrow \frac{(D_1 + D_2)D_1RT_s}{2L_2} = \frac{D_1}{D_2}$$

On further solving this equation and putting $\frac{2L_2}{RT_s} = k_2$,

$$D_2^2 + D_1D_2 - k_2 = 0 \quad (49)$$

On solving this quadratic equation and ignoring the negative root,

$$D_2 = \frac{-D_1 + \sqrt{D_1^2 + 4k_2}}{2} \quad (50)$$

Substituting this value of D_2 in equation 42,

$$\frac{V_o}{V_{dc}} = \frac{2}{\sqrt{1 + \frac{4k_2}{D_1^2}} - 1} \quad (51)$$

This equation can also be used to get the output voltage of Zeta converter in DCM.

5 Simulation

5.1 Parameters

To design the circuit in MATLAB/SIMULINK, the following parameters were taken.

Parameter	Value
Input Voltage V_{dc}	24 V
Output Voltage V_o	12 V
Power rating P_o	50 W
Switching frequency f_{sw}	100 kHz

Table 1: Parameters

5.2 Assumptions

The data given in table 1 is not enough for the design of zeta converter. We need to assume the value of certain quantities to get the values of capacitances and inductances. So, the assumptions are as follows.

Parameter	Assumed Value
Δi_{L_1}	10% of I_{L_1}
Δi_{L_2}	2.5% of I_{L_2}
ΔV_o	0.1% of V_o

Table 2: Assumed Values of inductor currents ripple and output voltage ripple

5.3 Calculations

5.3.1 Duty ratio D

Substituting the values of V_o and V_{dc} in equation 7,

$$\frac{12}{24} = \frac{D}{1-D}$$

Solving the above relation,

$$D = \frac{1}{3}$$

$$\Rightarrow D = 0.333$$

Hence the switch is on for 33.33% of the switching period.

5.3.2 Load Resistance R

From table 1,

$$P_o = 50 \text{ W}$$

Output power can be written as $P_o = \frac{V_o^2}{R}$. Hence, the above equation can be written as

$$\Rightarrow \frac{V_o^2}{R} = 50$$

$$\Rightarrow R = \frac{V_o^2}{50}$$

$$\Rightarrow R = \frac{12^2}{50}$$

$$\Rightarrow R = 2.88 \Omega$$

5.3.3 Average inductor currents I_{L_1} and I_{L_2}

Substituting the values of V_o , D and R in equation 14,

$$I_{L_1} = \frac{12}{2.88} \left(\frac{0.333}{1 - 0.333} \right)$$
$$\Rightarrow I_{L_1} = I_{in} = 2.083 \text{ A}$$

Now substituting the values of V_o and R in equation 12,

$$I_{L_2} = \frac{12}{2.88}$$
$$\Rightarrow I_{L_2} = I_o = 4.166 \text{ A}$$

5.3.4 Switching period T_s

Switching period is give as

$$T_s = \frac{1}{f_{sw}}$$
$$\Rightarrow T_s = \frac{1}{100 \times 10^3}$$
$$\Rightarrow T_s = 10^{-5} \text{ sec}$$

5.3.5 Inductor values L_1 and L_2

From table 2, $\Delta i_{L_1} = 0.1 I_{L_1}$

Substituting for Δi_{L_1} from equation 18 in the above equation,

$$\frac{(1 - D)V_o T_s}{L_1} = 0.1 I_{L_1}$$
$$\Rightarrow \frac{(1 - 0.333)(12)(10^{-5})}{L_1} = 0.1 \times 2.083$$
$$\Rightarrow L_1 = \frac{(1 - 0.333)(12)(10^{-5})}{0.20833}$$
$$\Rightarrow L_1 = 384 \mu H$$

Similarly, from table 2, $\Delta i_{L_2} = 0.025 I_{L_2}$

Substituting for Δi_{L_2} from equation 20 in the above equation,

$$\frac{(1 - D)V_o T_s}{L_2} = 0.025 I_{L_2}$$

$$\begin{aligned}
&\Rightarrow \frac{(1 - 0.333)(12)(10^{-5})}{L_2} = 0.025 \times 4.166 \\
&\Rightarrow L_2 = \frac{(1 - 0.333)(12)(10^{-5})}{0.10415} \\
&\Rightarrow L_2 = 768.122 \mu H
\end{aligned}$$

5.3.6 Capacitor Values C_1 and C_2

From table 2, $\Delta V_o = 0.001V_o$

Substituting for ΔV_o from equation 24 in the above equation,

$$\begin{aligned}
&\frac{DT_s V_o}{2C_1 R} = 0.001V_o \\
&\Rightarrow \frac{0.333 \times 10^{-5} \times 12}{2C_1 \times 2.88} = 0.001 \times 12 \\
&\Rightarrow C_1 = \frac{0.333 \times 10^{-5} \times 12}{0.06912} \\
&\Rightarrow C_1 = 13.88 mF
\end{aligned}$$

Again using $\Delta V_o = 0.001V_o$ and substituting for ΔV_o from equation 27,

$$\begin{aligned}
&\frac{(1 - D)T_s^2 V_o}{8L_2 C_2} = 0.001V_o \\
&\Rightarrow \frac{(1 - 0.333)(10^{-5})^2 \times 12}{8 \times (384 \times 10^{-6})C_2} = 0.001 \times 12 \\
&\Rightarrow C_2 = \frac{(1 - 0.333)(10^{-5})^2 \times 12}{8 \times (384 \times 10^{-6}) \times 0.012} \\
&\Rightarrow C_2 = 21.701 \mu F
\end{aligned}$$

5.4 SIMULINK model

Using the values of duty ratio, inductors, capacitors, load resistor computed in section 5.3 and using the data given in table 1, a model was simulated in the SIMULINK software. MOSFET was used as switch and its gate pulse was generated using a repeating triangular sequence having time period T_s and amplitude 1 compared with a constant block with a constant value of 0.333. This method of generating the pulses for turning on and turning off the switch is termed as pulse width modulation of PWM. A diode was also connected in the same way as depicted in figure 1. The internal resistance of MOSFET was 0.1Ω and that of diode was 0.001Ω . The SIMULINK model was formed similar to what we saw in figure 1. On the input side, a dc battery of voltage 24 V was used and connected with the MOSFET. All the current directions and voltage polarities were taken in the same way as depicted in figures 2a and 2b. Voltage and current measurement blocks were used to measure the voltage and currents through various circuit components. The output of these measurement blocks was given to scope to graphically depict the shape of waveforms.

6 Simulation Results

6.1 Waveforms

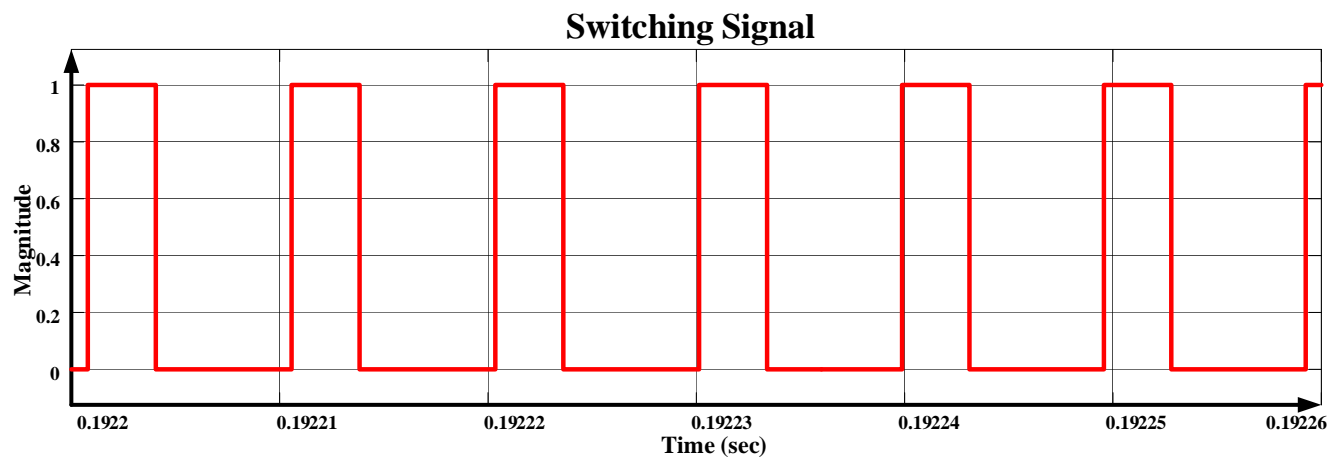


Figure 6: Switching signal

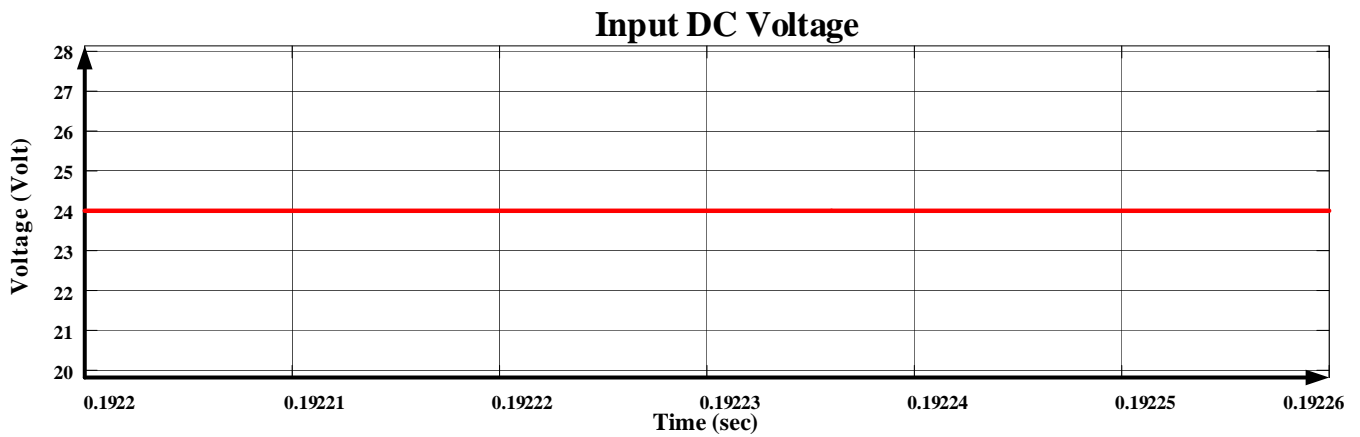


Figure 7: Input Voltage

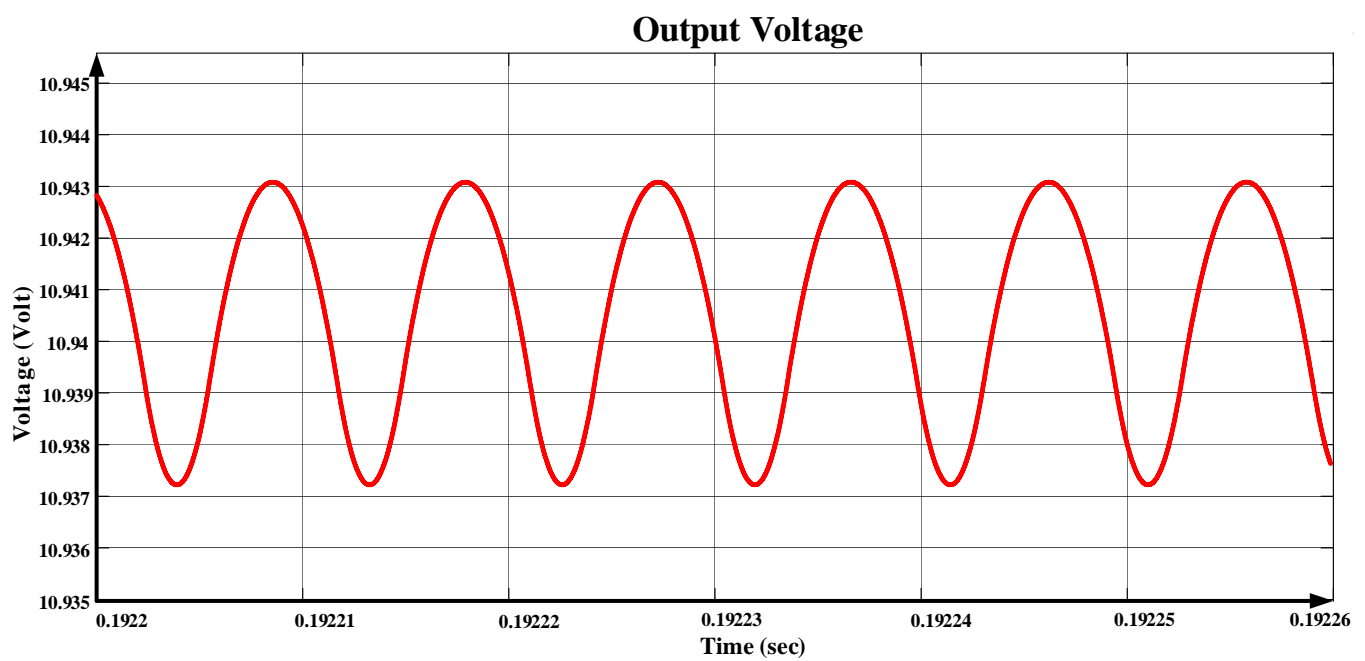


Figure 8: Output Voltage

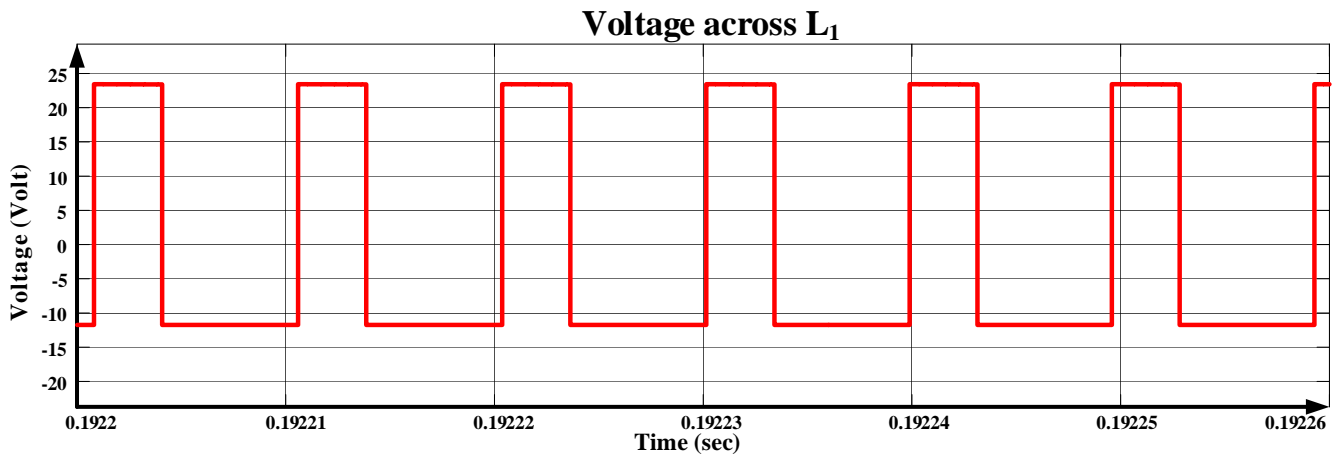


Figure 9: Voltage across L_1

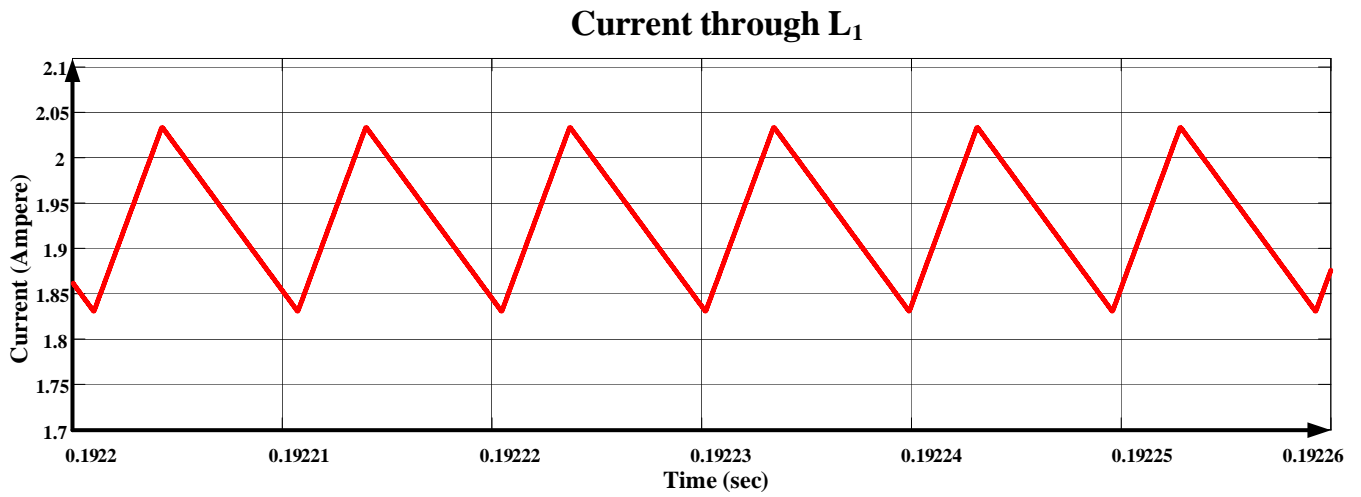


Figure 10: Current through L_1

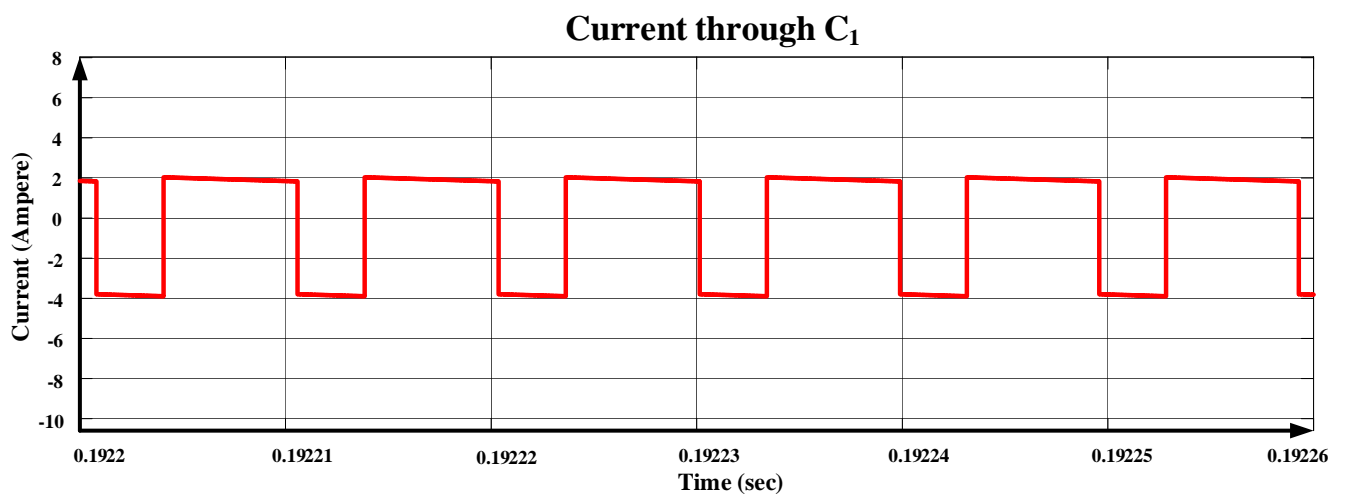


Figure 11: Current through C_1

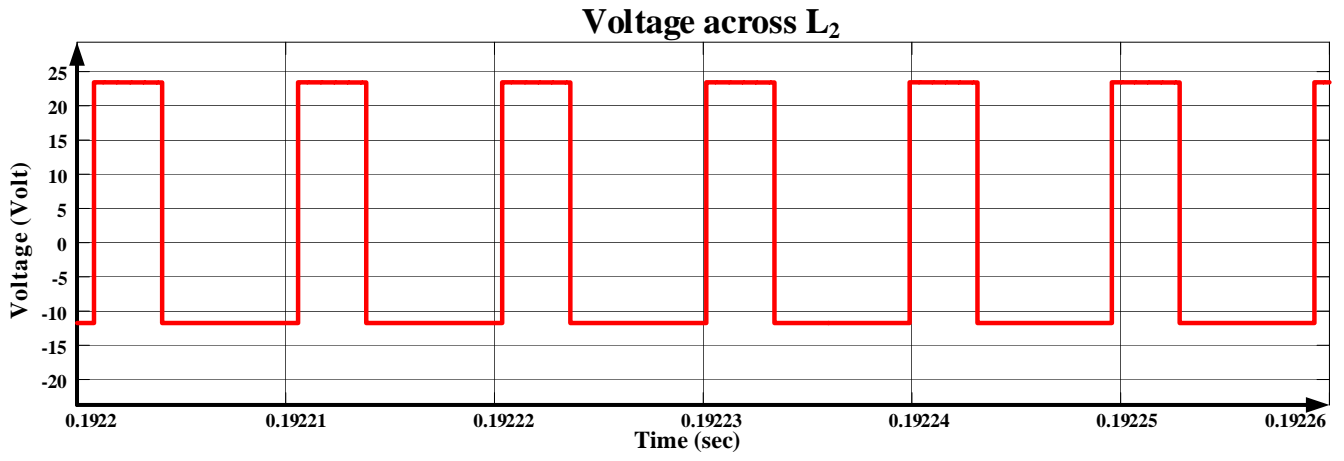


Figure 12: Voltage across L_2

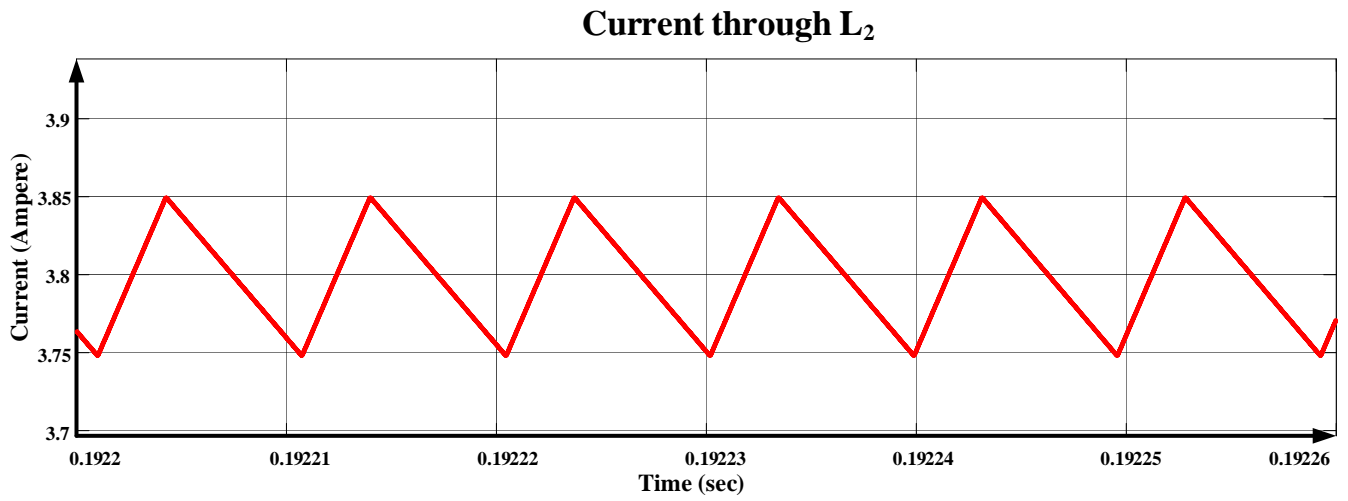


Figure 13: Current through L_2

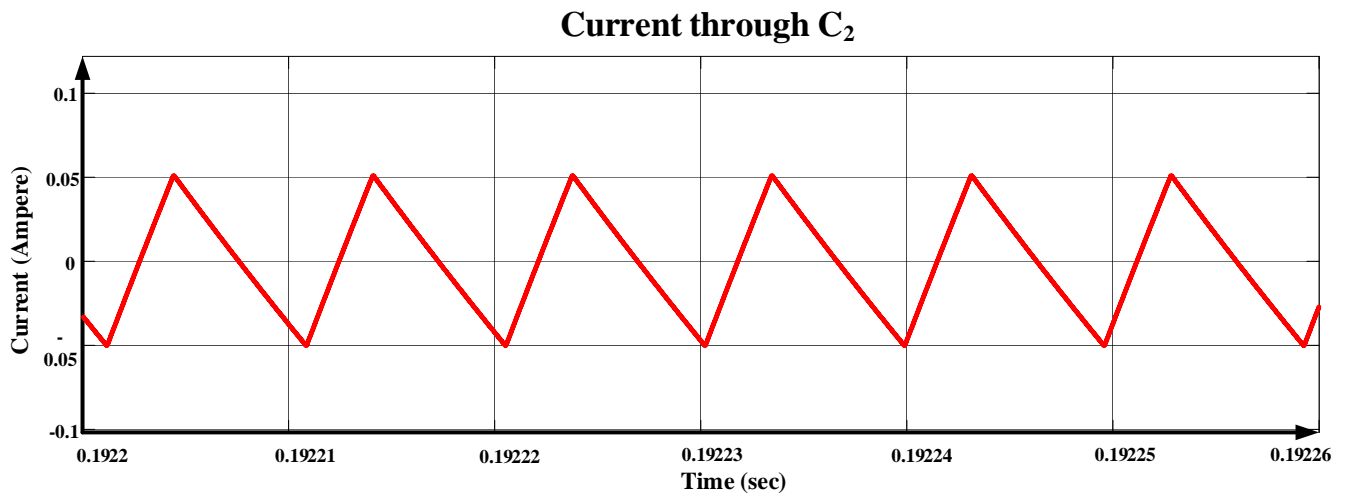


Figure 14: Current through C_2

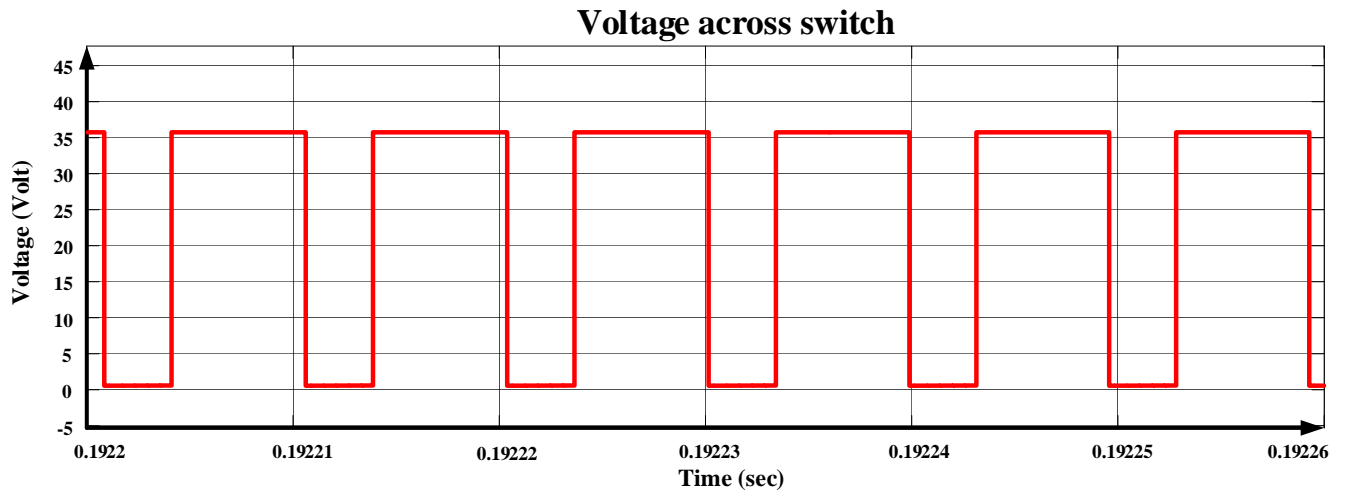


Figure 15: Voltage across switch

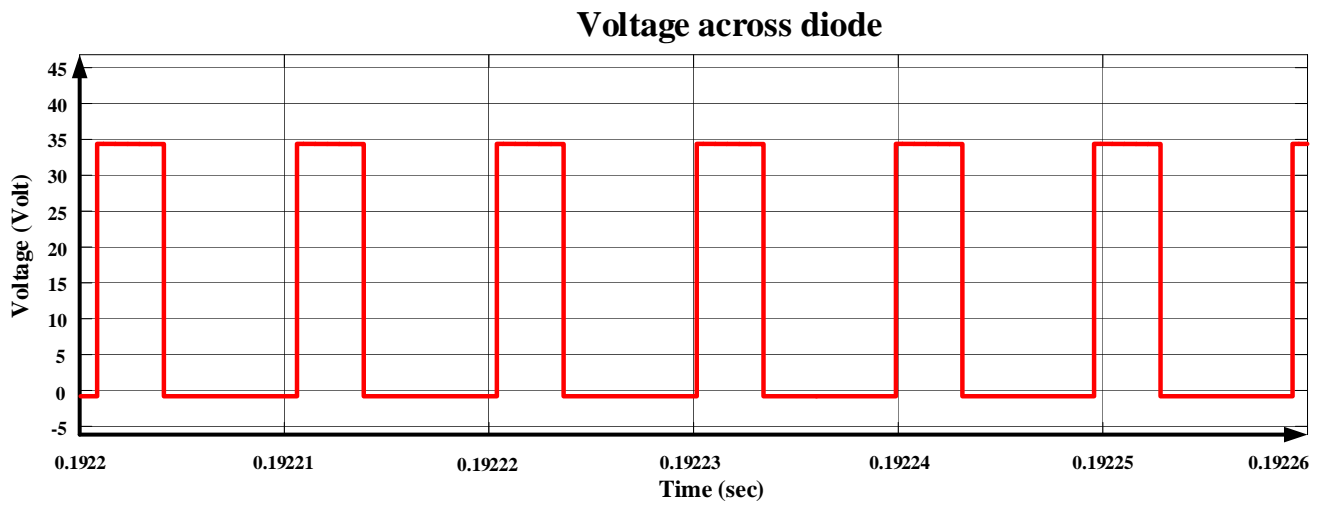


Figure 16: Voltage across diode

6.2 Observation

The following results were obtained after we ran the simulation.

Parameter	Practical Value
V_o	10.94 V
ΔV_o	0.005896 V
I_{L_1}	1.929 A
Δi_{L_1}	0.2028 A
I_{L_2}	3.8 A
Δi_{L_2}	0.1014 A

Table 3: Practical Values obtained from SIMULINK modelling

6.3 Error Analysis

The percentage error due to difference in theoretical values and practical values is given in the table below.

Parameter	Theoretical Value	Practical Value	% error
V_o	12 V	10.94 V	8.83%
ΔV_o	0.012 V	0.005896 V	50.87%
I_{L_1}	2.083 A	1.929 A	7.39%
Δi_{L_1}	0.20833 A	0.2028 A	2.65%
I_{L_2}	4.166 A	3.8 A	8.78%
Δi_{L_2}	0.10415 A	0.1014 A	2.64%

Table 4: Percentage error in practical values

7 Closed Loop Control of Zeta Converter

One way to get the output voltage as required is to employ closed loop control by using PI controller. Using PI controller has many advantages like faster settling and decrease of steady state error.

7.1 Simulation Parameters and Schematic for Closed Loop Control

The schematic shown in figure 17 shows the closed loop control of zeta converter. In the figure, the output voltage across the capacitor C_2 was sensed using a voltage sensor. The voltage sensed was compared with a reference value which in this case was 12 V. The error signal generated was fed to PI controller. The output of PI controller was compared with a repating sequence of switching period T_s using a relational operator. The output of the relational operator was given to the gate driver circuit of the MOSFET. The PI controller was tuned using the Ziegler-Nichols method[4]. The following values of proportional gain and integral gain were computed using this method.

Parameter	Value
Proportional Gain K_p	9
Integral Gain K_i	527

Table 5: PI controller parameters

All the circuit parameters computed in section 5.3 were kept the same. The only difference was the change in the method of generating the switch pulses to control the output voltage. In this case, even if the reference is changed, there will be no need to change the switching circuit.

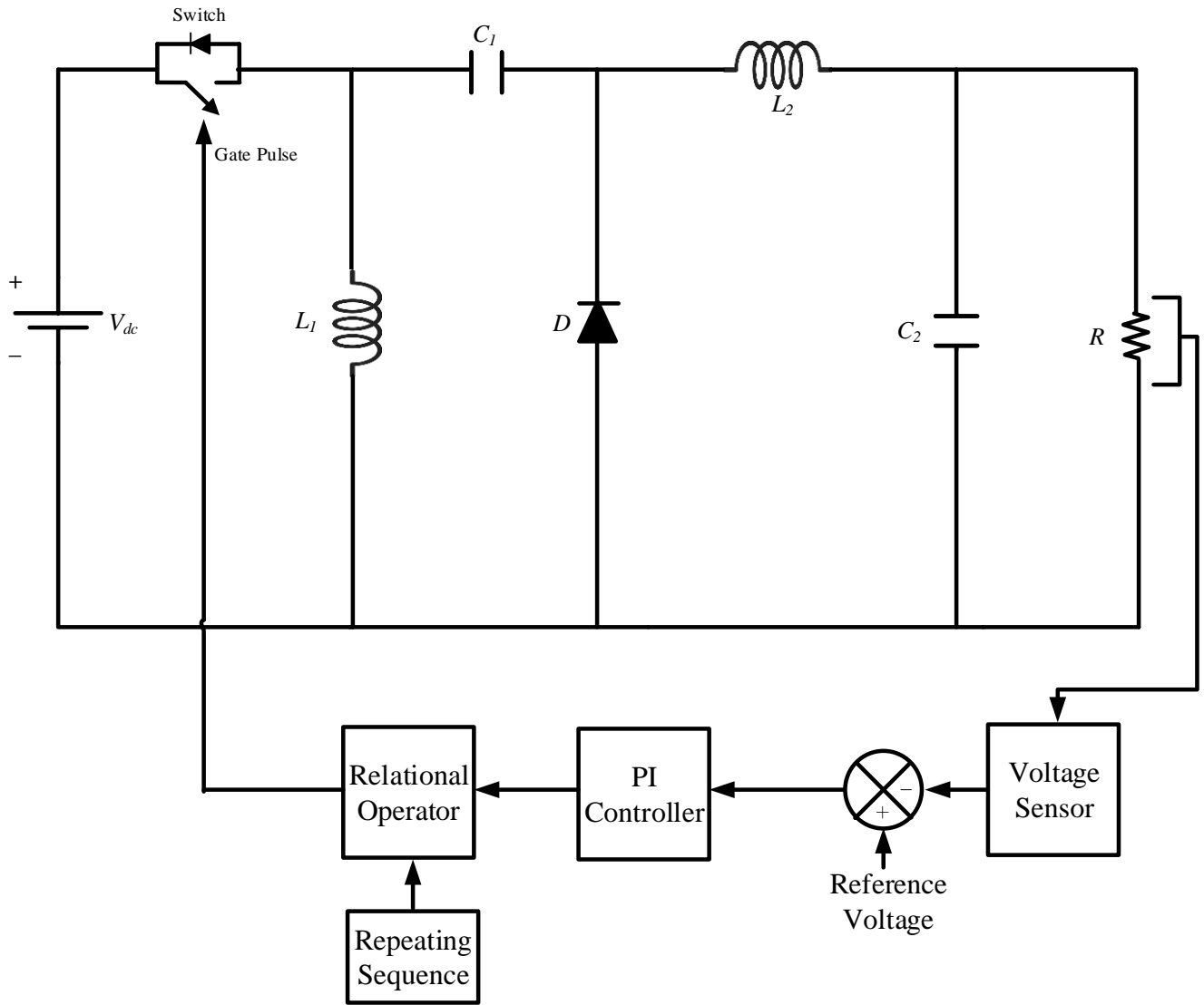


Figure 17: Schematic for closed loop control of zeta converter

7.2 Simulation Waveforms

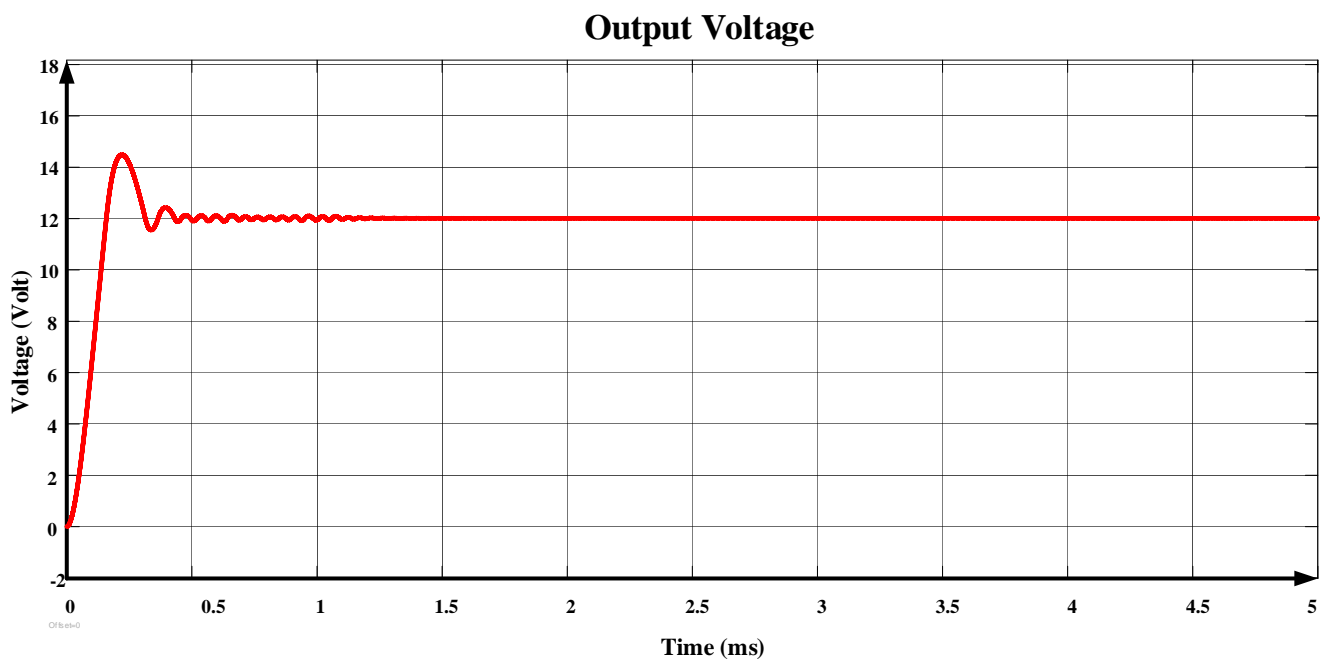


Figure 18: Output Voltage of zeta converter after closed loop control

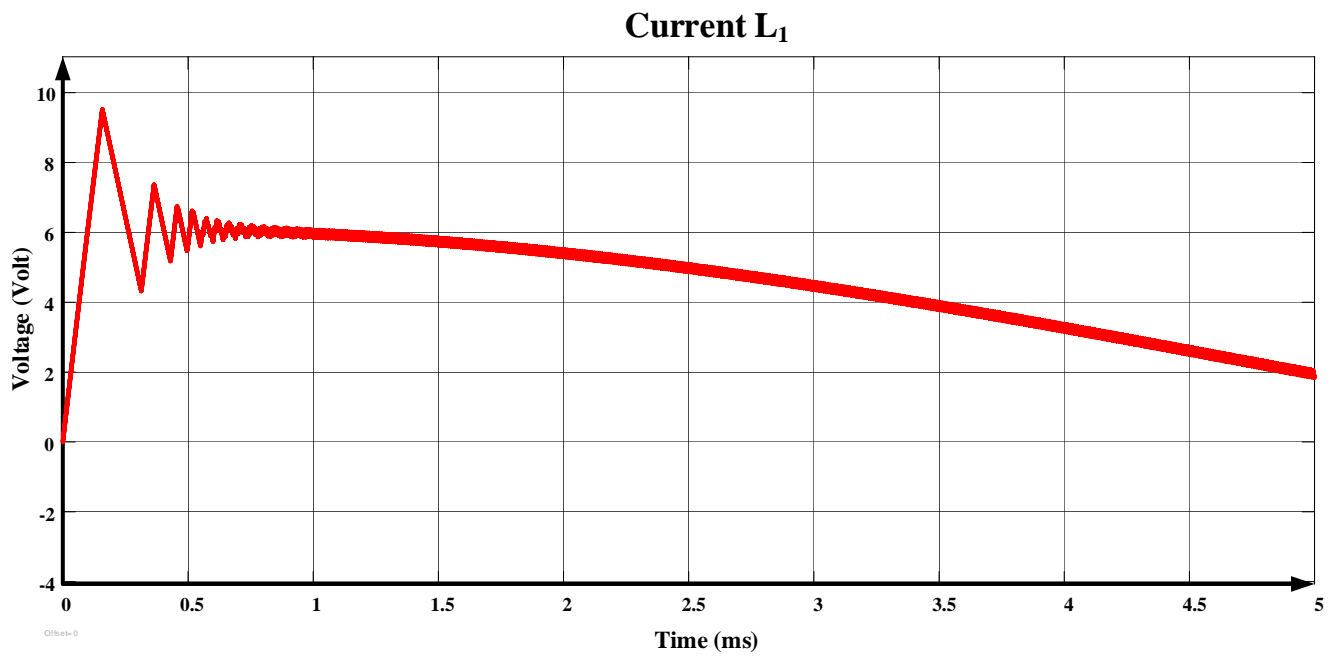


Figure 19: Current through L_1 after closed loop control

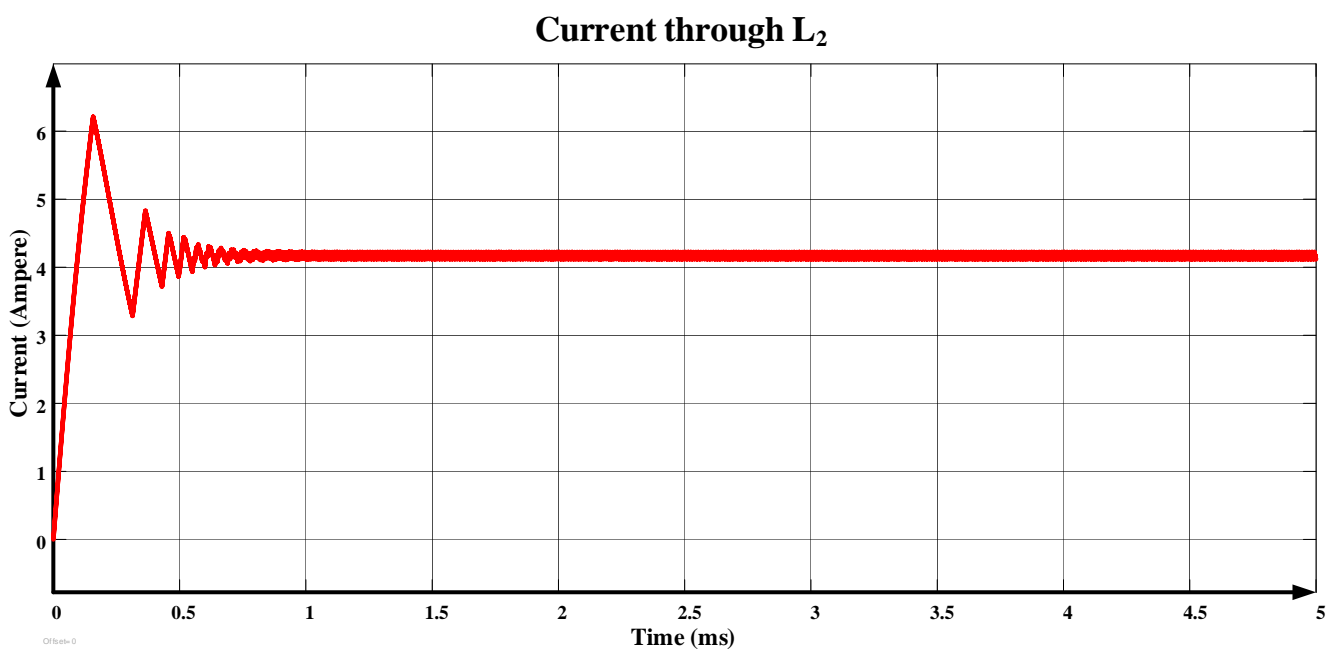


Figure 20: Current through L_2 after closed loop control

7.3 Simulation Results

The following results were obtained after running the SIMULINK model with this method.

Parameter	Practical Value
V_o	12.01 V
ΔV_o	0.006852 V
I_{L_1}	2.306 A
Δi_{L_1}	0.376 A
I_{L_2}	4.167 A
Δi_{L_2}	0.1086 A

Table 6: Practical Values obtained from SIMULINK modelling using closed loop control

8 Future Scope

The future scope of zeta converters includes several promising areas due to their versatile capabilities in power electronics. Key advancements and research directions are:

1. **Renewable Energy Systems:** Zeta converters are increasingly used for interfacing renewable energy sources like solar and wind to provide regulated output voltages. They are particularly suitable for photovoltaic (PV) systems with maximum power point tracking (MPPT) to enhance energy harvesting efficiency.
2. **Electric Vehicles (EVs):** The compact design and high efficiency of zeta converters make them ideal for electric vehicle power systems. They support voltage regulation between batteries, motors, and other electronic components, enabling better performance and energy management.
3. **Smart Grids and Energy Storage:** Zeta converters are explored in grid-connected applications for integrating energy storage systems, providing bidirectional power flow, and ensuring stability in smart grids.
4. **LED Lighting and Consumer Electronics:** Their ability to produce a continuous and regulated output voltage is crucial for LED drivers and other low-voltage DC applications.
5. **Hybrid Topologies:** Researchers are developing hybrid zeta converters with enhanced voltage gain, reduced size, and improved dynamic performance for applications requiring compact and efficient designs.[5][6]

References

- [1] N. Mohan, T. M. Undeland, and W. P. Robbins, *Power electronics: converters, applications, and design*. John wiley & sons, 2003.
- [2] R. W. Erickson and D. Maksimovic, *Fundamentals of power electronics*. Springer Science & Business Media, 2007.
- [3] A. B. Jørgensen, “Derivation, design and simulation of the zeta converter,” 2021.
- [4] V. Kumar and A. Patra, “Application of ziegler-nichols method for tuning of pid controller,” *International Journal of Electrical and Electronics Engineering*, vol. 8, no. 2, pp. 559–570, 2016.
- [5] A. B. Kancherla, N. B. Prasad, and D. R. Kishore, “Solar energy ev charging system using integrated zeta-luo converter,” *Journal of Engineering and Applied Science*, vol. 71, no. 1, p. 147, 2024.
- [6] P. L. Santosh Kumar Reddy, Y. P. Obulesu, S. Singirikonda, M. Al Harthi, M. S. Alzaidi, and S. S. Ghoneim, “A non-isolated hybrid zeta converter with a high voltage gain and reduced size of components,” *Electronics*, vol. 11, no. 3, p. 483, 2022.
- [7] M. H. Rashid, *Power electronics handbook*. Butterworth-heinemann, 2017.
- [8] M. K. Kazimierczuk, F. Corti, G. M. Lozito, and A. Reatti, “Non-isolated zeta pwm dc-dc power converter analysis for ccm including parasitics,” *IEEE Access*, 2023.