

$$\begin{aligned}
 1) \textcircled{a} & -3(0+2E) \\
 & = -3 \left[ \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} + 2 \begin{bmatrix} 6 & 1 & 3 \\ -1 & 2 & 2 \\ 4 & 1 & 3 \end{bmatrix} \right] \\
 & = -3 \left[ \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 12 & 2 & 6 \\ -2 & 4 & 4 \\ 8 & 2 & 6 \end{bmatrix} \right] \\
 & = -3 \begin{bmatrix} 13 & 7 & 8 \\ -3 & 4 & 5 \\ 11 & 4 & 10 \end{bmatrix} = \begin{bmatrix} -39 & -21 & -24 \\ 9 & -12 & -15 \\ -33 & -12 & -30 \end{bmatrix}
 \end{aligned}$$

$$\textcircled{b} A (2DC)$$

$$\begin{aligned}
 & = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 11 & 1 \end{bmatrix} \begin{bmatrix} 1 & 15 & 3 \\ 6 & 2 & 10 \end{bmatrix} \\
 & = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 11 & 1 \end{bmatrix} \begin{bmatrix} 2 & 30 & 9 \\ 12 & 4 & 20 \end{bmatrix} \\
 & = \begin{bmatrix} 6+0 & 20+0 & 27+0 \\ -2+24 & -30+8 & -9+40 \\ 22+12 & 330+4 & 99+20 \end{bmatrix} \\
 & = \begin{bmatrix} 6 & 20 & 27 \\ -2 & -22 & 31 \\ 34 & 334 & 119 \end{bmatrix}
 \end{aligned}$$

$$\textcircled{c} (C^T B) A^T$$

$$\begin{aligned}
 & = \left[ \begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \right] \begin{bmatrix} 3 & -1 & 11 \\ 0 & 2 & 1 \end{bmatrix} \\
 & = \begin{bmatrix} 4 & 5 \\ 16 & -2 \\ 8 & 8 \end{bmatrix} \begin{bmatrix} 3 & -1 & 11 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 12 & 6 & -39 \\ 48 & -20 & 174 \\ 24 & 8 & 96 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{d} (D - 2B)^T \\
 & = \left[ \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 12 & 2 & 6 \\ -2 & 4 & 4 \\ 8 & 2 & 6 \end{bmatrix} \right]^T \\
 & = \begin{bmatrix} -11 & 3 & -4 \\ 1 & -4 & -3 \\ -5 & 0 & -2 \end{bmatrix}^T \\
 & = \begin{bmatrix} -11 & 1 & -5 \\ 3 & -4 & 0 \\ -4 & 3 & -2 \end{bmatrix}
 \end{aligned}$$

$$\textcircled{e} D^T$$

$$= \begin{bmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{bmatrix}$$

$$\textcircled{f} AB$$

$$\begin{aligned}
 & = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 11 & 1 \end{bmatrix} \begin{bmatrix} 9 & -1 \\ 0 & 2 \end{bmatrix} \\
 & = \begin{bmatrix} 12 & -3 \\ -9 & 5 \\ 99 & -9 \end{bmatrix}
 \end{aligned}$$

Jawab: a)  $3 \times 2$  c)  $2 \times 3$  e)  $3 \times 3$   
b)  $2 \times 2$  d)  $3 \times 3$

3) Tentukanlah invers dari matriks kuadrat berikut

a)  $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 1 \\ 2 & 1 & 9 \end{bmatrix}$

b)  $B = \begin{bmatrix} 2 & -4 & 0 & 0 \\ 1 & 2 & 12 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -1 & -4 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{\det A} \text{Adj } A$

Adj A:

$\det A = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 1 \\ 2 & 1 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 1 & 4 \\ 2 & 1 \end{vmatrix}$

$\det A = (1 \cdot 4 \cdot 9) + (2 \cdot 1 \cdot 2) + (3 \cdot 1 \cdot 1) - (2 \cdot 1 \cdot 9) - (3 \cdot 4 \cdot 2)$   
 $= 36 + 4 + 3 - 18 - 24$

$A^{-1} = \frac{1}{0} \cdot \text{Adj } A$

Tidak terdefinisi;

$B = \begin{bmatrix} 2 & -4 & 0 & 0 \\ 1 & 2 & 12 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -1 & -4 & 5 \end{bmatrix}$

$B^{-1} = \frac{1}{\det B} \text{Adj } B$

$\begin{bmatrix} 2 & -4 & 0 & 0 \\ 1 & 2 & 12 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -1 & -4 & 5 \end{bmatrix} \begin{vmatrix} 2 & -4 \\ 1 & 2 \\ 0 & 0 \\ 0 & -1 \end{vmatrix}$

$(2, 2, 2, 5) = 0 + 0 - 0 - 0 - 0$

$\det B = 40$

$\frac{1}{\det B} \cdot \text{Adj } B$

$\text{Adj } B = (\text{cof } B^T)$

$= \begin{bmatrix} 2 & 1 & 0 & 0 \\ -4 & 2 & 0 & -1 \\ 0 & 12 & 2 & -4 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

$\frac{1}{\det B} \cdot \text{Adj } B = \frac{1}{40}$

$\begin{bmatrix} 2 & 1 & 0 & 0 \\ -4 & 2 & 0 & -1 \\ 0 & 12 & 2 & -4 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

$$\begin{bmatrix} \frac{2}{40} & \frac{1}{40} & 0 & 0 \\ \frac{4}{40} & \frac{2}{40} & 0 & \frac{1}{40} \\ \frac{0}{40} & \frac{12}{40} & \frac{2}{40} & \frac{4}{40} \\ \frac{0}{40} & 0 & 0 & \frac{9}{40} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{20} & \frac{1}{40} & 0 & 0 \\ \frac{-1}{10} & \frac{1}{10} & 0 & \frac{-1}{40} \\ 0 & \frac{3}{10} & \frac{1}{20} & \frac{-1}{10} \\ 0 & 0 & 0 & \frac{1}{8} \end{bmatrix}$$

Vektor

$$\textcircled{1} \quad u = (-3, 1, 2)$$

$$v = (4, 0, -8)$$

$$w = (6, -1, -4)$$

$$\begin{aligned} a) \quad v - w &= (4-6, (0+1), (-8-4)) \\ &= (-2, 1, -12) \end{aligned}$$

$$\begin{aligned} b) \quad 6u + 2v &= 6(-3, 1, 2) + 2(4, 0, -8) \\ &= (-18, 6, 12) + (8, 0, -16) \\ u + v &= (-18, 6, 12) + (8, 0, -16) \\ &= (-10, 6, -4) \end{aligned}$$

$$\begin{aligned} c) \quad -3(v - 8w) &= -3((v) + (-8w)) \\ &= -3((4, 0, -8) + (-8w)) \\ &= -3((4, 0, -8) + (-48, 8, 24)) \\ &= (-12, 0, -24) + (-48, 8, 24) \\ &= (-12 - (-48), (0, 8), (-24 - 24)) \\ &= (36, -8, -48) \end{aligned}$$

$$\begin{aligned} d) \quad (2u - 7w) - (8v + u) &= ((-6, 24) - (-28, 0, -56)) - ((32, 0, -64) + (-3, 1, 2)) \\ &= (22, 2, 60) - (31, 1, -62) \\ &= (22-31, 2-1, 60+62) \\ &= (-9, 1, 122) \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad a) \quad \|v\| &= \sqrt{2^2 + 2^2 + 2^2} \\ &= \sqrt{4 + 4 + 4} \\ &= \sqrt{12} \end{aligned}$$

$$\begin{aligned} b) \quad \|v\| &= \sqrt{(-2)^2 + (3)^2 + (3)^2 + (-1)^2} \\ &= \sqrt{4 + 9 + 9 + 1} \\ &= \sqrt{23} \end{aligned}$$

$$\textcircled{4} \quad v = (2, 3, 0, 6)$$

$$\|kv\| = 4$$

$$\begin{aligned} \|kv\| &= \sqrt{(2k)^2 + (3k)^2 + 0^2 + 6k^2} \\ &= \sqrt{4k^2 + 9k^2 + 36k^2} \\ &= \sqrt{49k^2} \\ &= 7\|k\| = 4 \\ &= \|k\| = \pm \frac{4}{7} \end{aligned}$$

$$\textcircled{5} \quad u = (2, 0, 6)$$

$$v = (1, 3, 1)$$

$$w = (5, 1, 1)$$

$$u \cdot (v \times w)$$

$$\bullet \quad v \times w$$

$$= \begin{vmatrix} 1 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 1 & 1 \\ 5 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 1 \end{vmatrix}$$

$$\bullet \quad (1(3 \cdot 1) - (1 \cdot 1)) - (1(1 \cdot 1) - 5 \cdot 1) + (1(1 \cdot 1) - 3 \cdot 5)$$

$$= (3 - 1) - (1 - 5) + (1 - 15)$$

$$= (2, -6, -9)$$

$$\begin{aligned} & \bullet \quad u \cdot (2, -6, -9) \\ & \quad (2, 0, 6) \cdot (2, -6, -9) \\ & = (2 \cdot 2) + (0 \cdot -6) + (6 \cdot -9) \\ & = 4 + 0 + (-54) \\ & = -50 \end{aligned}$$