

CS-2009 Design and Analysis of Algorithms

Assignment Two

22L-6552

Section: 4H

Q 1) You are given two sorted arrays  $A[]$  and  $B[]$  of size " $N$ " each. Your task is to design an in-place algorithm using divide and conquer technique to determine the median of both the arrays in  $O(\log N)$ . Median is the value located at the central position of sorted data.

```

Medium(A[], B[], leftA, rightA, leftB, rightB)
{
    if (rightA - leftA + 1 <= 2 AND rightB - leftB + 1 <= 2)
    {
        return Manual Median(A, B, leftA, rightA, leftB, rightB)
    }
    else
    {
        midA = (leftA + rightA) / 2;
        midB = (leftB + rightB) / 2
        medianA = A[midA]
        medianB = B[midB]
        if (medianA == medianB)
        {
            return medianA
        }
        elseif (medianA < medianB)
        {
            return Medium(A, B, midA, rightA, leftB, midB)
        }
    }
}

```

~~scribble~~

else

{

return Median (A, B, leftA, midA, midB, rightB)

}

}

}

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Manual Median ( $A[]$ ,  $B[]$ ,  $\text{leftA}$ ,  $\text{rightA}$ ,  $\text{leftB}$ ,  $\text{rightB}$ )

{ if ( $\text{rightA} - \text{leftA} + 1 == 1$  and  $\text{rightB} - \text{leftB} + 1 == 1$ )

{ return  $(A[\text{leftA}] + B[\text{leftB}]) / 2$

{ else if ( $\text{rightA} - \text{leftA} + 1 == 1$  and  $\text{rightB} - \text{leftB} + 1 == 2$ )

{ temp [1 -- 3]  
temp [1] =  $A[\text{leftA}]$   
temp [2] =  $B[\text{leftB}]$   
temp [3] =  $B[\text{rightB}]$   
Sort (temp, 3)

return temp [2]

{ else if ( $\text{rightA} - \text{leftA} + 1 == 2$  and  $\text{rightB} - \text{leftB} + 1 == 1$ )

{ temp [1 -- 3]  
temp [1] =  $A[\text{leftA}]$   
temp [2] =  $A[\text{rightA}]$   
temp [3] =  $B[\text{leftB}]$   
Sort (temp, 3)

return temp [2]

else

{

temp[1] = 4

temp[1] = A[leftA]

temp[2] = A[rightA]

temp[3] = B[leftB]

temp[4] = B[rightB]

sort(temp, 4)

return (temp[2] + temp[3]) / 2

}

}

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Q2) Given an array of integers  $\text{Arr}[]$  of length " $N$ ". Your task is to design an algorithm using divide and conquer technique to determine the majority element in the array. A majority element is an element that appears more than half ( $N/2$ ) times in the array. Your algorithm must return the majority element if it exists in an array, otherwise -1.

$\text{findMajorityElement}(\text{Arr}[], \text{left}, \text{right})$

{ if ( $\text{left} == \text{right}$ )

{ return  $\text{Arr}[\text{left}]$

}

mid =  $(\text{left} + \text{right})/2$

left Majority =  $\text{findMajorityElement}(\text{Arr}, \text{left}, \text{mid})$

right Majority =  $\text{findMajorityElement}(\text{Arr}, \text{mid}+1, \text{right})$

if ( $\text{left Majority} == \text{right Majority}$ )

{ return left Majority

}

leftCount = countFrequency(Arr, left, right, leftMajority)

~~rightCount = countFrequency(Arr, mid + 1, right)~~

rightCount = countFrequency(Arr, left, right, rightMajority)

if (leftCount > (right - left + 1) / 2)

{  
    return leftMajority

}

else if (rightCount > (right - left + 1) / 2)

{  
    return rightMajority

}

else

{  
    return -1

}

}

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CountFrequency (Arr[], left, right, value)

{ Count = 0

for (i = left; i <= right; i++)

{

if (Arr[i] == value)

{

Count++

}

}

return Count

}

Q3) Consider an array of distinct integers  $\text{Arr}[\cdot]$  of size  $“N”$  was sorted in ascending order. The array has been rotated clockwise  $“K”$  number of times. Given such an array, your task is to find the value of  $“K”$ . Design and algorithm using divide and conquer technique to get the value of  $“K”$  in  $O(\log N)$ .

find Rotations ( $\text{Arr}[\cdot]$ , left, right)

{ if ( $\text{right} < \text{left}$ )

{ return 0

}

if ( $\text{right} == \text{left}$ )

{ return  $\text{left} - 1$

}

$\text{mid} = (\text{left} + \text{right}) / 2$

if ( $\text{mid} < \text{right}$  and  $\text{Arr}[\text{mid} + 1] < \text{Arr}[\text{mid}]$ )

{ return  $\text{mid} + 1 - 1$

}

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```
if (mid > low and Arr[mid] < Arr[mid-1])  
{  
    return mid-1  
}
```

```
if (Arr[right] > Arr[mid])  
{  
    return findRotations(Arr, left, mid-1)  
}
```

```
else  
{  
    return findRotations(Arr, mid+1, right)  
}
```

}

Q4) Assume that you are given an unsorted array of integers  $\text{Arr}[]$  of size " $N$ " and two integers " $X$ " and " $Y$ ". It is guaranteed that the values of both the integers are from the original array. Your task is to find the distance between " $X$ " and " $Y$ ". The distance between two elements of the array is the number of elements that lie between them in sorted order. Design an efficient algorithm using the divide and conquer approach to get the distance between two elements of ~~the~~ the array.

main()

{

$d = \text{DistanceXY}(\text{Arr}, \text{left}, \text{right}, X, Y)$

    return  $d - 2$  // -2 is to exclude the count of boundaries

}

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Distance XY (Arr[I], left, right, X, Y)

{ if ( left &lt; right)

{ mid = (left + right) / 2

distance = 0

distance += Distance XY (Arr, left, mid, X, Y)

distance += Distance XY (Arr, mid+1, right, X, Y)

return distance

}

else

{

if ( X &lt; Y)

{ lower Bound = X

upper Bound = Y

}

else

{

lower Bound = Y

upper Bound = X

}

~~if (Arr [left] >= lowerBound and Arr [left] <= upperBound)~~

~~if (Arr [left] >= lowerBound and Arr [left] <= upperBound)~~

if (Arr [left] >= lowerBound and Arr [left] <= upperBound)

{ return 1

}

else

{ return 0

}

}

}

Q5) Consider an unsorted array of integers  $\text{Arr}[]$  of size " $N$ " and an integer " $Y$ ". Design an algorithm using divide and conquer approach to ~~solve~~ solve this problem in  $O(N \log(N))$ !

```
findPair (Arr[], left, right, y)
{
    flag = false
```

Merge Sort (Arr, left, right)

```
for (i = left; i <= right & and flag == false; i++)
```

Search Key = Arr[i] + y ; // if b-a = y  
then b = a + y

~~Binary = Binary~~

flag = BinarySearch (Arr, left + 1, right, searchKey)

// binary search function returns the ~~index~~ of the  
// element in the array

Value if the value is found otherwise 0. <sup>index</sup>

1

return flag

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Binary Search (Arr[], left, right, searchKey)

{ while ( left <= right )

{ mid = (left + right) / 2

{ if ( Arr[mid] == searchKey )

{ return mid

{ else if ( Arr[mid] < searchKey )

{ left = mid + 1

{ else

{ right = mid - 1

{ }

return 0

}

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MergeSort (Arr[], left, right)

{ if (left &lt; right)

{ M = (left + right) / 2

MergeSort (Arr, left, M)

MergeSort (Arr, M+1, right)

Merge (Arr, left, M, right)

}

}

Merge (Arr[], left, M, right)

{ n1 = M - left + 1

n2 = right - M

A[1 - - - n1] // temporary arrays

B[1 - - - n2]

Copy (Arr, A, left, M) // copy data into

Copy (Arr, B, M+1, right) temporary arrays

$i = 1$

$j = 1$

$K = \text{left}$

while ( $i \leq n1$  and  $j \leq n2$ )

{ if ( $A[i] \leq B[j]$ )

{  $\text{Arr}[K++] = A[i++]$

}

else

{

$\text{Arr}[K++] = B[j++]$

}

}

while ( $i \leq n1$ )

{  $\text{Arr}[K++] = A[i++]$

}

while ( $j \leq n2$ )

{  $\text{Arr}[K++] = B[j++]$

}

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Delete A and B

}

Mid1 Q1) You are given a sorted array A of n integers, a search key V, and a positive integer K  $\leq n$ . Your goal is to find the K nearest values to the search key present in the array. Note that the search key may or may not be present in the array. Devise an efficient algorithm that returns the subarray (starting and ending index) containing the K nearest values.

FindSubArray (Arr[], left, right, V, K)

{ nearestCount = 0

closestIndex = BinarySearch (Arr, left, right, V)

// binary search function returns closest index or the exact index

nearestCount ++

i = closestIndex - 1

j = closestIndex + 1

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```
while (nearestCount < K and i >= left and j <= right)
{
    diff1 = AbsDiff (V, Arr[i])
    diff2 = AbsDiff (V, Arr[j])
    if (diff1 <= diff2)
    {
        i --
    }
    else
    {
        j ++
    }
    nearestCount ++
}
```

while (nearestCount < K and i >= left)

{  
    i--

    nearestCount++

}

while (nearestCount < K and j <= right)

{  
    j++

    nearestCount++

}

start = i++

end = j--

return (start, end)

}

Binary Search (Arr[], left, right, searchKey)

```
{   while (left <= right)
    {
        mid = (left + right) / 2
        if (Arr[mid] == searchKey)
        {
            return mid
        }
        else if (Arr[mid] < searchKey)
        {
            left = mid + 1
        }
        else
        {
            right = mid - 1
        }
    }
    return mid
}
```

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