Discrete Structures (CS 1005)

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Assignment Number : 1 (One)

Q1) For each of these arguments, explain which rules of inference are used for each step.

Q1) Part 1) "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."

Let $J(x)$: x knows JAVA

Let $C(x)$: x is in this class

Let $P(x)$: x can get a high paying job

Premise: $C(\text{Doug})$

Premise: $J(\text{Doug})$

Premise: $\forall x(J(x) \rightarrow P(x))$

Conclusion: $\exists x(C(x) \land P(x))$

The steps are as following:

1. $C(\text{Doug})$ Premise

2. $J(Doug)$ Premise

3. $\forall x(J(x) \rightarrow P(x))$ Premise

4. $J(Doug) \rightarrow P(Doug)$ Universal instantiation from 3

5. $P(Doug)$ Modus Ponens from 4 and 2

6. $C(Doug) \wedge P(Doug)$ Conjunction from 1 and 5

7. $\exists x(C(x) \wedge P(x))$ Existential generalization from 6

$\Rightarrow$ 7 is the ~~req~~ required Conclusion.

Answer

Q1) Part 2) "Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution."

Let $P(x)$: x cares about ocean pollution

Let $C(x)$: x is in this class

Let $W(x)$: x enjoys whale watching

Premise: $\forall x\,(W(x) \rightarrow P(x))$

Premise: $\exists x\,(W(x) \wedge C(x))$

Conclusion: $\exists x\,(C(x) \wedge P(x))$

The steps are as following:

1. $\exists x\,(W(x) \wedge C(x))$ Premise
2. $\forall x\,(W(x) \rightarrow P(x))$ Premise
3. $W(a) \wedge C(a)$ Existential instantiation from 1
4. $W(a)$ Simplification from 3
5. $W(a) \rightarrow P(a)$ Universal instantiation from 2
6. $P(a)$ Modus Ponens from 4 and 5
7. $C(a)$ Simplification from 3
8. $C(a) \wedge P(a)$ Conjunction from 6 and 7

⑨ $\exists x(C(x) \wedge P(x))$ Existential generalization from ⑧

$\Rightarrow$ ⑨ is the required conclusion. Answer

Q1) Part 3) "Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke, a student in this class, can use a word processing program."

Let $C(x)$: x is in this class

Let $P(x)$: x owns a personal computer

Let $W(x)$: x can use a word processing program

Premise: $\forall x(C(x) \rightarrow P(x))$

Premise: $\forall x(P(x) \rightarrow W(x))$

Premise: $C(\text{Zeke})$

Conclusion: $W(\text{Zeke})$

The steps are as following:

1. $\forall x\,(C(x) \rightarrow P(x))$ Premise

2. $\forall x\,(P(x) \rightarrow W(x))$ Premise

3. $C(\text{Zeke})$ Premise

4. $C(\text{Zeke}) \rightarrow P(\text{Zeke})$ Universal instantiation from 1

5. $P(\text{Zeke})$ Modus ponens from 3 and 4

6. $P(\text{Zeke}) \rightarrow W(\text{Zeke})$ Universal instantiation from 2

7. $W(\text{Zeke})$ Modus ponens from 5 and 6

$\Rightarrow$ 7 is the required Conclusion. Answer.

Q 1) Part 4) "Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean."

Let $J(x)$: x lives in New Jersey

Let $O(x)$: x lives within 50 miles of the ocean

Let $S(x)$: x has seen the Ocean

Premise: $\forall x\,(J(x) \rightarrow O(x))$

Premise: $\exists x\,(J(x) \wedge \neg S(x))$

Conclusion: $\exists x\,(O(x) \wedge \neg S(x))$

The steps are as following:

① $\forall x\,(J(x) \rightarrow O(x))$ Premise

② $\exists x\,(J(x) \wedge \neg S(x))$ Premise

③ $J(a) \wedge \neg S(a)$ Existential instantiation from ②

4 $J(a)$ Simplification from 3

5 $J(a) \longrightarrow O(a)$ Universal instantiation from 1

6 $O(a)$ Modus Ponens from 4 and 5

7 $\neg S(a)$ Simplification from 3

8 $O(a) \wedge \neg S(a)$ Conjunction from 6 and 7

9 $\exists x(O(x) \wedge \neg S(x))$ Existential generalization from 8

$\Rightarrow$ 9 is the required conclusion.] Answer

Q2) You are given as input a (propositional) function "bool P(int x, int y)" and two arrays, int D1[n] containing elements of the domain of the first variable 'x' and int D2[m] containing elements of the domain of the second variable 'y'. Write a code fragment to determine the truth value of each of the following (nested) quantified statements:

Q2) Part 1) $\forall x \forall y P(x, y)$

```
bool xyHitEveryTime = true;

For(int i = 0 ; i < n ; i++)
{
    For(int j = 0 ; j < m ; j++)
    {
        if ( P(D1[i], D2[j]) == false)
        {
            xyHitEveryTime = false;
            j = m ;
            i = n ;

        }

    }
}

Cout << "The truth value is: " << xyHitEveryTime;
```

Q2) Part 2) $\forall x \exists y \; P(x, y)$

```
bool yHitOnce;
for(int i=0; i<n; i++)
{
    yHitOnce = false;
    for(int j=0; j<m; j++)
    {
        if( P(D1[i], D2[j]) == true)
        {
            yHitOnce = true;
            j = m;
        }
    }
    if( yHitOnce == false)
    {
        break;
    }
}

cout << "Truth value is: " << yHitOnce;
```

Q2) Part 3) $\exists x \forall y P(x, y)$

```
bool xHitOnce;
For(int i=0; i<n; i++)
{
      xHitOnce = true;
      For(int j=0; j<m; j++)
      {
            if( P(D1[i], D2[j]) == false)
            {
                  xHitOnce = false
                  j=m;
            }
      }
      if (xHitOnce == true)
      {
            break;
      }
}
Cout << "Truth value is: " << xHitOnce;
```

Q2) Part 4) $\exists x \exists y P(x, y)$

```
bool xyHitOnce = false;
for(int i= 0 ; i<n ; i++)
{
      for(int j=0; j<m; j++)
      {
            if( P(D1[i], D2[j])==true)
            {
                  xyHitOnce = true;

                  j = m;
                  i = n;
            }
      }
}

Cout<< "Truth Value is:"<< xyHitOnce;
```