Mechanics of Materials-Lab

Complex Engineering Problem

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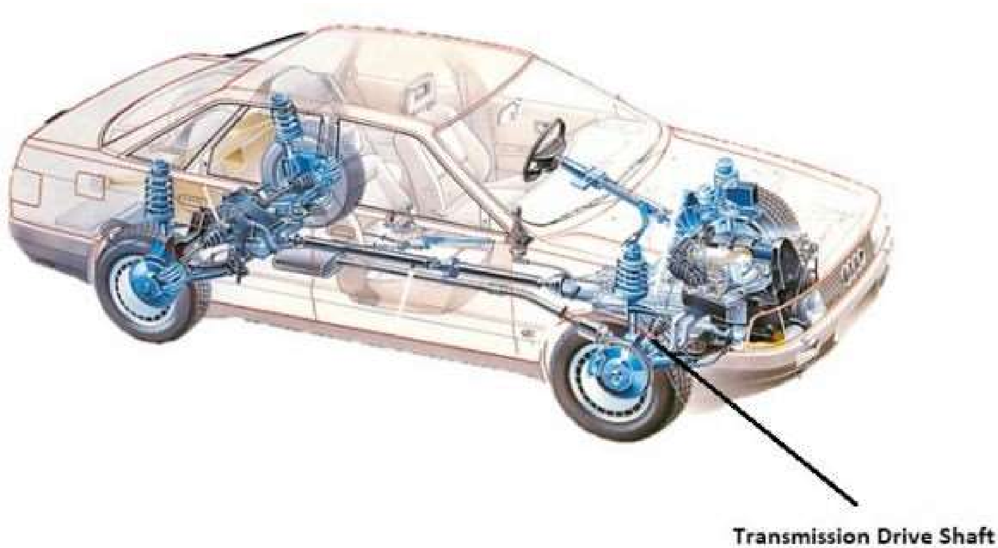
1. Objective

This report will design a hollow circular transmission shaft for a 4x2 Front Wheel Drive (FWD) car, which can transmit power between 35 and 100 kW at a rotational speed of 3000 to 7000 rpm. Two different materials will be used for designing: Titanium Alloy (Ti-6Al-4V) and Carbon Fiber Reinforced Polymer (CFRP). Shear stress, torque transmission, and safety factors will be considered for calculation in the structure of the shaft.



2. Abstract

This paper discusses the hollow transmission shaft design and the optimization of Titanium Alloy as well as CFRP. The design is considered as meeting the optimal strength, as well as weight, characteristics of the system while keeping pace with the specified operational ranges. Detailed calculations and material analysis are provided while considering safety aspects as well, and the paper concludes with a comparison of the materials to determine which one would be the best option.

3. Introduction

In an automobile industry, FWD vehicles are popular as they can fit inside the smaller body and because they are relatively inexpensive to produce. Transmission shaft is very important because it transfers power from the engine to the wheels. A hollow shaft design may contribute a reduced weight but still offers strength. The purpose of this report is to design a hollow circular shaft with requirements to meet those of a 4x2 FWD vehicle. Two advanced materials are to be compared - Titanium Alloy for its strength-to-weight ratio and CFRP for its lightness.

4. Literature Review

Advances in material science have undergone a tremendous transformation in the designing of transmission shafts. Because of superior properties and weight-saving potential, traditional materials like steel are gradually being replaced by advanced alloys and composites.

- **Hollow Shafts:** Hollow shafts have a much higher strength-to-weight ratio as compared to solid shafts and can be very effective in the automotive world.
- **Titanium Alloy (Ti-6Al-4V):** It is widely used in the aerospace as well as the automotive industries due to its excellent corrosion resistance along with high tensile strength and moderate density.
- **CFRP:** Carbon fiber composites are known for their exceptional stiffness and lightweight properties, making them suitable for high-performance applications.
- **Stress Consideration:** The design should include safety factors and polar moment of inertia to account for the torsional stress.

5. Design Requirements and Material Selection

Design Requirements:

- **Power Range:** 35–100 kW
- **Rotational Speed Range:** 3000–7000 rpm
- **Cross-Section:** Hollow circular shaft
- **Safety Factor:** 2.0

Materials Selected:

1) Titanium Alloy (Ti-6Al-4V):

- **Shear Modulus (G):** 44 GPa
- **Yield Strength (τy):** 880 MPa
- **Density:** 4.43 g/cm³

2) **CFRP**:
   - **Shear Modulus (G):** 15 GPa
   - **Yield Strength (τy):** 600 MPa
   - **Density:** 1.6 g/cm³

6. Calculations and Analysis

Given Data:

- **Power Range (P):** 35 kW to 100 Kw
- **Rotational Speed Range (N):** 3000 rpm to 7000 rpm
- **Torque (T) Formula:**

$$T = \frac{P}{\omega}, \quad \omega = \frac{2\pi N}{60}$$

Case 1: For P = 35 kW **and** N = 7000 rpm:

$$\omega = \frac{2\pi \times 7000}{60} = 733.04 \text{ rad/s}$$

$$T = \frac{35{,}000}{733.04} = 47.77 \text{ Nm}$$

Case 2: For P = 100 kW **and** N = 3000 rpm:

$$\omega = \frac{2\pi \times 3000}{60} = 314.16 \text{ rad/s}$$

$$T = \frac{100{,}000}{314.16} = 318.31 \text{ Nm}$$

Torque Range:

T = 47.77 Nm to 318.31 Nm

Material 1: Titanium Alloy (Ti-6Al-4V)

- Shear Modulus (G): 44 GPa
- Yield Strength (τy): 880 MPa
- Safety Factor (FoS): 2.0
- Ratio $k = r_i/r_o = 0.5$

Using the Torsional Shear Stress Formula:

$$\tau = \frac{T \times r_o}{J}, \; J = \frac{\pi}{2} r_o^{\,4} (1 - k^4)$$

Rearranging for r_o:

$$r_o = (\frac{2T}{\pi \, \tau \, (1 - k^4)})^{1/3}$$

For Maximum Torque (T = 318.31 Nm):

$$\tau \leq \frac{\tau y}{FoS} = \frac{880}{2} = 440 \; MPa$$

$$r_o = (\frac{2 \times 318.31 \times 10^3}{\pi \times 440 \times 10^6 \times (1 - 0.5^4)})^{1/3}$$

r_o = 19.6 mm

$r_i = k * r_o$ = 0.5 * 19.6 = 9.8 mm

Material 2: CFRP

- Shear Modulus (G): 15 GPa
- Yield Strength (τy): 600 MPa
- Safety Factor (FoS): 2.0
- Ratio $k = r_i/r_o$ = 0.5

Using the Same Formula:

$$r_o = (\frac{2T}{\pi \, \tau \, (1 - k^4)})^{1/3}$$

For Maximum Torque (T = 318.31 Nm):

$$\tau \leq \frac{\tau y}{FoS} = \frac{600}{2} = 300 \; MPa$$

$$r_o = (\frac{2 \times 318.31 \times 10^3}{\pi \times 300 \times 10^6 \times (1 - 0.5^4)})^{1/3}$$

r_o = 24.5 mm

$r_i = k * r_o$ = 0.5 * 24.5 = 12.25 mm

7. Final Results

Titanium Alloy (Ti-6Al-4V):

- Outer Radius (r_o): 19.6 mm
- Inner Radius (r_i): 9.8 mm
- Factor of Safety (FoS): 2.0

CFRP:

- Outer Radius (r_o): 24.5 mm
- Inner Radius (r_i): 12.25 mm
- Factor of Safety (FoS): 2.0

Titanium Alloy provides better strength but is heavier, while CFRP is lightweight but requires a larger diameter.

8. Conclusion

Hollow transmission shaft of a 4x2 FWD car was designed using Ti-6Al-4V Titanium Alloy and CFRP. These materials can sustain the range of torque as well as safety factor 2.0. Ti-6Al-4V is suitable for high-stress applications as it offers a compact high strength, while CFRP offers great weight saving in a product that will find application in lightweight and fuel-efficient designs. Material choice will be finally determined based on application priorities so that they ensure maximum performance and reliability.