ECSE 443 – ASSIGNMENT 3

MUHAMMAD TAHA – 260505597

# Chapter 5

## Exercises

### 5.3

1. As , the Newton iteration is given as follows, when f(x) = 0 needs to be solved.
2. Given an initial guess of 4-bit accuracy, to obtain 24-bit accuracy, the number of iterations required is . For 53-bit accuracy .

### 5.4

Let . Then, . Hence, to solve f(x) = 0 the Newton iteration is as follows:

### 5.6

1. As and , the iterative scheme is not convergent.
2. As and , the iterative scheme is locally convergent.
3. , the fixed-point iteration function given by Newton’s method is .

### 5.9

This first iteration given by Newton’s method is , where can be found via:

For

Hence s1 = 0.5 and s2 = -0.5.

Therefore, the first iteration is = x0 + s0 =

### 5.10

If

Hence,

If then

Therefore,

## Computer Problems

### 5.1

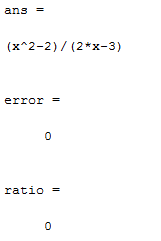
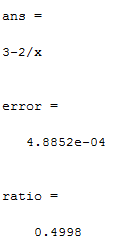
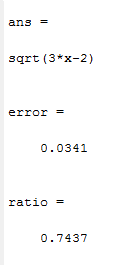
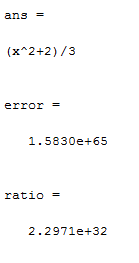
Therefore, the gradient is divergent.

Therefore, the gradient is convergent at 0.75.

Therefore, the gradient is convergent at 0.5.

Therefore, the gradient is quadratically convergent.

1. The MATLAB output for the functions above is given below:



### 5.2

The termination criteria used for each function was that the computation for each function was repeated a maximum of ten times per method and the tolerance of error was set as 10-10.

The output obtained for each function with all 3 methods is shown below.

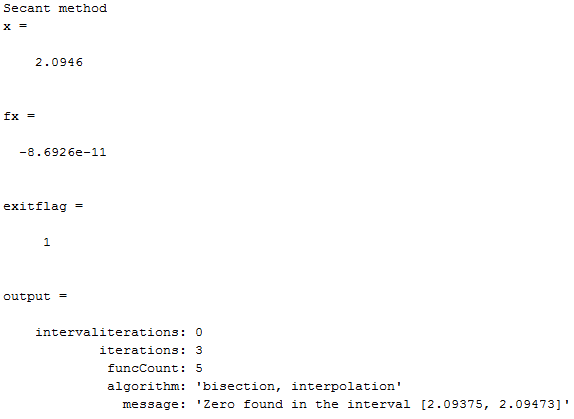
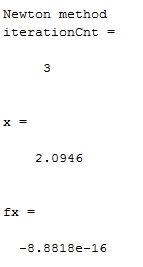
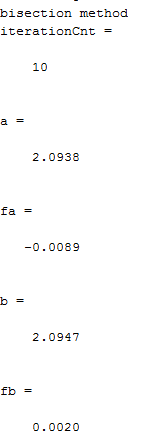
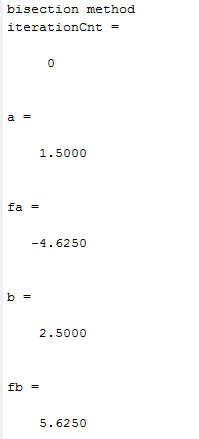


Figure : Part a

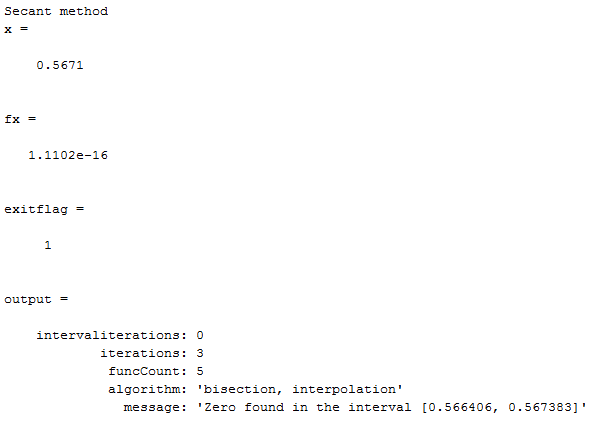
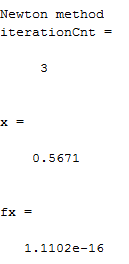
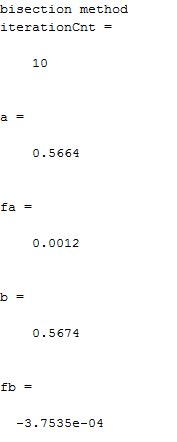
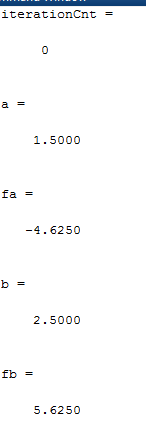


Figure : Part b

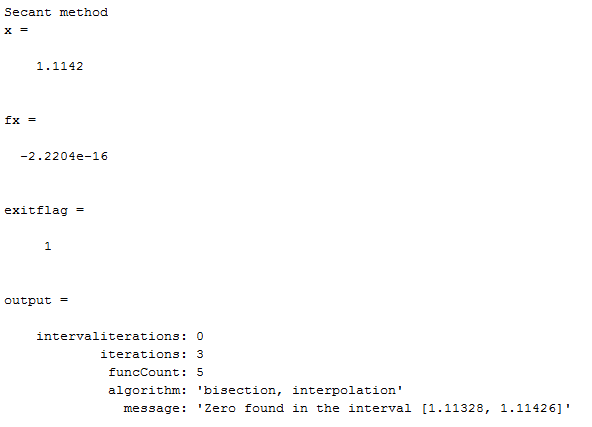
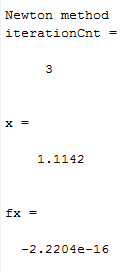
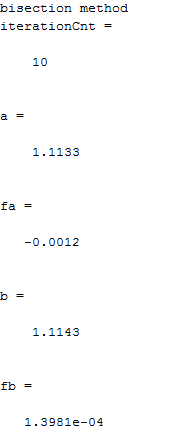


Figure : Part c

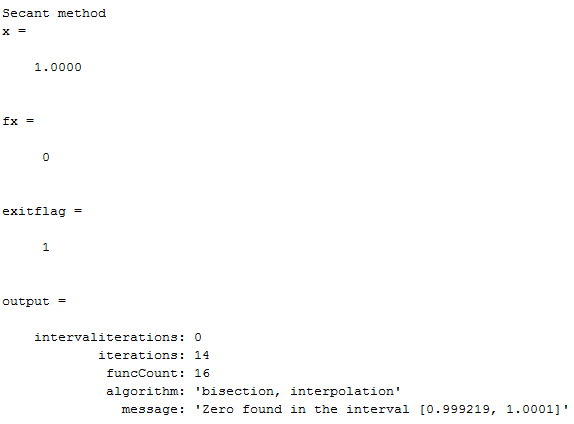
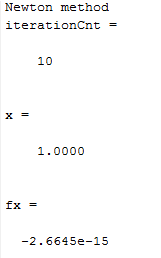
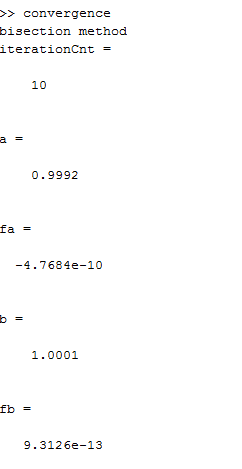
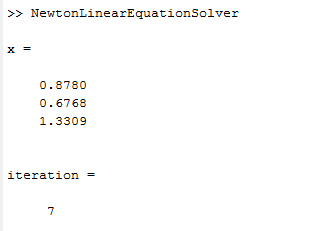
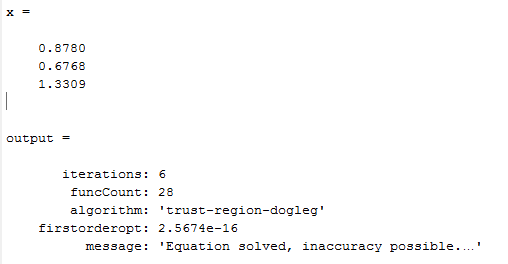


Figure : Part d

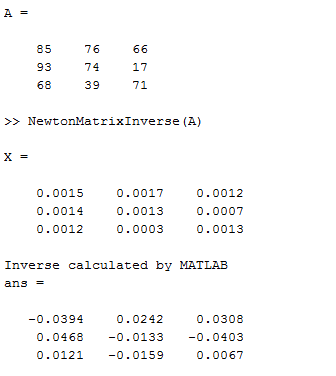
### 5.13

The following screenshots shows the result obtained by the implementation in MATLAB compared with the built-in MATLAB solver.





### 5.17



# Chapter 6

## Exercises

### 6.1

f has a minimum, critical point at as is positive definite at .

1. The matrix inverse is given by:

Hence,

Therefore,

1. The derivative is close to zero with this step as shown below

9, -2)|| = 9.2 and

1. The new iteration deviates from the solution so the chosen step is not good

|| ||= ||(1, 1)|| = 1.41

|| ||= ||)||= 2.34

### 6.2

1. When A:

The system for Newton’s method step from an arbitrary is given below

Which gives the result . Hence, we get:

Implying that . Thus, it can be observed that Newton’s method converges to the solution in one iteration from any .

1. For the steepest descent method, we need to find the minimum of α**,** where. The minimizing value for α is:

If, is an eigenvector of A, then

The following equations proves that is A’s eigenvector:

Therefore,

As ,

Showing that the steepest method converges to the solution in one iteration from the starting point.

### 6.3

If y is a non-zero vector, then

Hence, a 2x2 Hessian matrix of the Lagrange function for constrained optimization cannot be positive definite.

### 6.4

1. The following vertices are contained in the feasible region:

{(0,0), (0, 1.5), (0.857, 0.857), (1.09, 0.545), (1.2, 0)}

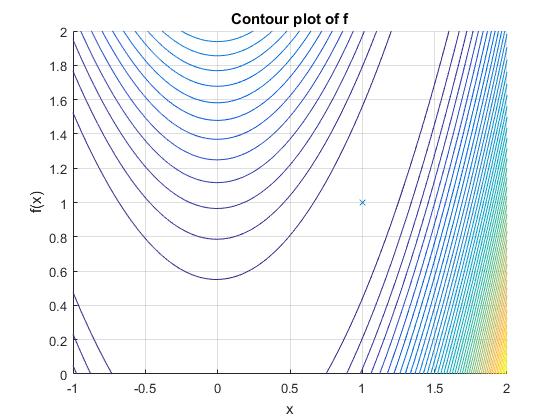
1. The values of the objective function at each vertex are: 0, -3, -4.29, -4.37 and -3.6 respectively.

The lowest cost is found at: (1.09, 0.545)

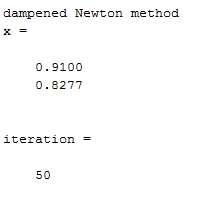
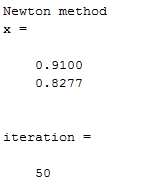
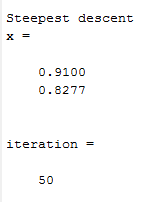
## Computer Problems

### 6.9

The plot of the path taken in plane by the solutions for each method is given below

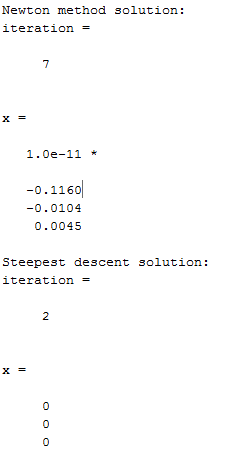


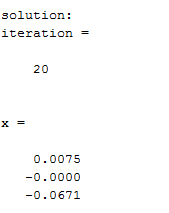
Final solutions obtained from each method after a suitable number of iterations is provided below



### 6.11

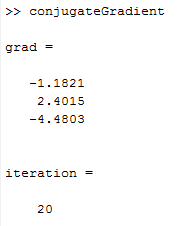
The following figures shows the solutions obtained from MATLAB



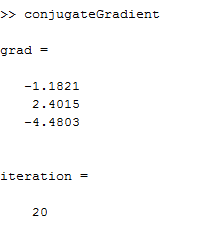


### 6.12

The solution below is obtained for the Fletcher Reeves algorithm.



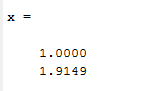
The solution below is obtained for Polak Ribiere algorithm.



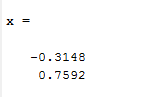
The tolerance was set as 10­-5 and the function successfully converged to the solution in n steps for an arbitrary quadratic function of n variables.

### 6.13

1. The solution obtained from MATLAB is given below



1. The solution obtained from MATLAB is given below



### 6.19 (a)

The solution obtained from MATLAB for all unknown x is shown below

