1a)

1b)

2a) First prove that dmin <= min wt(c) where wt(c) equals the number of non-zero symbols for a codeword (c) in codebook, where c is non-zero.

We know that c = m x G, where m = message, c = codeword and G = generator matrix.

2b)

2c) The Hamming bound theorem states that any (n, k, dmin) binary code must satisfy the following formula:

2k ((n 0) + (n 1) + (n 2) + … + (n (dmin – 1)/2)) <= 2n

For (24, 12, 9) binary code, the formula is as follows:

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Where 2k  corresponds to the total number of spheres, which is multiplied by all possible points in each sphere (for example, (n 1) corresponds to all points which differ from 0 codeword by 1 symbol). 2n corresponds to all possible points.

2d) We know that a codebook for any binary code is formed via b \* G, where b = all possible binary vectors for the given code, and G = generator matrix. In order for each codeword in the codebook to be decodable, each codeword has to differ by at most dmin – 1. Hence, the binary code can always correct up to dmin – 1 erasures.

Consider two codewords, c1 and c2 in the figure below. Each circle around that codeword indicates all possible symbols which differ from the codeword by d, where d is the difference between the codeword and the symbol. C1 and c2 differ by dmin.

dmin

d = 2

d = 1

d = 2

d = 1

X – received vector

For X to be decoded as c1, it has to fall within the sphere of c1 (as shown above). Hence the maximum radius of a sphere is (dmin - 1 )/2, so the code can always correct (dmin - 1 )/2 binary errors in the codeword.

2e) The table below lists all codewords for (6,3,3) code

|  |
| --- |
| Codewords |
| 000000 |
| 100110 |
| 010101 |
| 110011 |
| 001011 |
| 101101 |
| 011110 |
| 111000 |

We can see from above that the minimum distance between any two codewords in the table above is 3. So dmin = 3.

2f) There are 16 codeword pairs which differ by dmin distance (3) for this code (we used matlab a script for this). Whats the spectrum of code??

2g) The codewords for (8,4,4) is given in the table below

|  |
| --- |
| Codewords |
| 0000 0000 |
| 1000 1110 |
| 0100 1101 |
| 1100 0011 |
| 0010 1011 |
| 1010 0101 |
| 0110 0110 |
| 1110 1000 |
| 0001 0111 |
| 1001 1001 |
| 01011010 |
| 1101 0100 |
| 0011 1100 |
| 1011 0010 |
| 0111 0001 |
| 1111 1111 |

There are 112 codeword pairs which differ by dmin distance (4) for this code (we used matlab a script for this). Whats the spectrum of code??

2h)

3d) The performance of the decoders were measured with a range of probability of erasures, starting from Pe = 0.1 and ending at Pe = 0.8, with Pe incrementing with 0.05. Random messages were generated 100 times for each probability. The bit-error-rate (BER) was recorded against each probability, taking the average BER for all 100 messages. The results were plotted in the graphs shown below.





3e) **Time complexity for Exhaustive decoder:**

The exhaustive decoder is essentially divided into two steps. The first step is to form the codebook for the given code. This is achieved by m\*G, where m consists of all possible binary messages for the give code, the size of m being (2^k x k). G is the generator matrix for the code.

For (6,3,3) code generating codebook takes about 0.0002 seconds. So we can assume that the operational time increments linearly and the for (10000, 5000) code it takes approximately 0.0002\* 5000 = 1 seconds.

The next step is to loop over all the vectors from the received matrix, then loop over all the codewords in the codebook and compute difference between all the codewords in the codebook, and the vector under consideration from the received matrix. The codeword with the minimum distance is also found in the outer loop. The dimensions of codebook are (2^k x n) and the received vector is (k x n). Therefore, the complexity will be O(k)\*O(2^k), which is almost equivalent to O(2^k). For (6,3,3) code, considering the worst case scenario (where the entire received matrix contains erasures) the execution time takes about 0.001 seconds. So the time for (10000, 5000) will almost take 0.001\*25000-3, which is almost 101500 seconds.

So the total complexity for (10000, 5000) is almost O(2^5000) or about 101500 seconds.

**Time complexity for Gaussian decoder:**

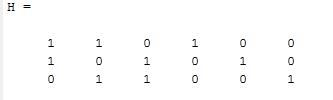
The Gaussian decoder begins by looping over the received matrix and replacing all 0.5 values with e-symbols, denoting erasure unknowns. Next, k equations are formed, which the solved by Matlab’s “linsolve” utility. The “linsolve” utility has about O(n) time complexity. For (6,3,3) code the “linsolve” takes about 0.07 seconds to solve for each row. Each row in this code has 6 columns. So for (10000, 5000) the time taken would be about 0.07\*10000 = 700 seconds.

The outer loop loops over all the rows of the received matrix, which is k. This is an O(k) operation. For (6,3,3) this takes about 0.4 seconds.

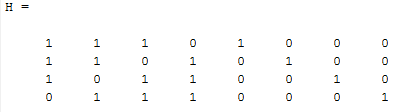
The overall time complexity for this algorithm is O(n) + O(k) which is approximately 700 + (0.4\*500) = 900 seconds for (10000, 5000) code.

3f) The parity check matrices, H, are formed by making the parity symbols for an identity matrix, such that, H = (P | I).

For (6,3,3) code the H matrix is given below:



For (8,4,4) code the H matrix is given below:

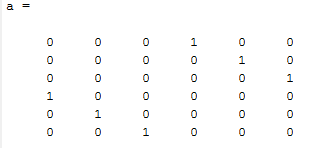


3g) It can be observed that H = (P | I) while G = (I | P). So in order to get H from G for any system, we need to swap the P and I sections from G matrix. This is achieved by multiplying G with another matrix, say A, to get H. So we have:

H = G \* A

For any system (n, k, dmin), matrix A is an n\*n matrix. It consists of 2 identity matrices of k\*k dimensions, and the other entries of matrix A are padded with 0’s to make it n\*n. The first identity matrix starts at k+1 column and goes up to n columns for the first k rows. The next identity matrix starts at 0th column and goes up to n-k columns for the remaining n-k rows.

For (6,3,3) code the A matrix looks like:



3h) The graph below shows the performance of syndrome decoder, with the same test as performed 3(d).

