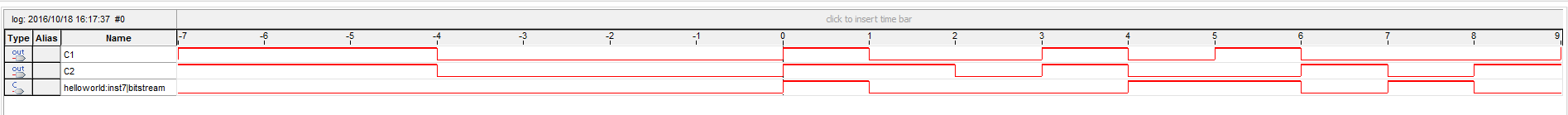
## Question 1

1. Refer to **figure x** in appendix for the block diagram of rate-half encoder. The “hello world” block is the bit stream generator as seen in **figure y** in appendix, which was implemented using the provided code from Verilog documents. The results (c1 and c2) from the half-rate encoder, obtained via signal-tap analysis is shown in **figure x** below.
2. Refer to **figure x** in appendix for the filter\_demo block diagram, which implements an echo system. The “2048\_32bit\_regs” uses the provided 32-bit register Verilog file to create a delay for the input sound. The 32-bit register was cascaded for a total of 2048 times to create a noticeable delay. Adding this delayed sound to the original creates the echo.

## Question 2

a1) Refer to **Figure x** in Appendix to find the script for the decoder of (8,4,4) Hamming code with comments explaining it. The script finds the shortest distance because each codeword in the codebook and received signal. The distance between one codeword and the received vector is found by:

This distance is calculated for all sixteen codewords in (8,4,4) Hamming code and codeword with minimum distance is the one matched.

Using the decoder with given received vector = [0.54, -0.12, 1.32, 0.41, 0.63, 1.25, 0.37, -0.02]. The minimum distance was matched to the following codeword:

0 0 1 1 1 1 0 0

a2) The number of floating operations can be found by analysing the additions, multiplications, subtractions to find codeword. Knowing that we find distance between one codeword and received vector using:

This includes 8 subtractions, 8 multiplications and 7 additions which gives a total 23 flops to compute distance between one codeword and received vector. This is done for all 16 codewords, which results in number of flops:

16 × 23 = 368 flops

b1) The trellis for the (8,4,4) Hamming code with 4 states at most per stage is shown in **Figure** :

0 /01

11 /11

1/10

1 /10

0 /01

11 /11

00 /00

00 /00

x x 0 0

x x 1 1

x x 0 1

x x 1 0

0 /01

1 /11

1 /10

1 /10

0 /01

1 /11

0 /00

0 /00

x 0 0 0

x 0 1 1

x 1 0 1

x 1 1 0

10 /10

01 /01

11 /11

0 0 0 0

0 0 0 0

0 0 1 1

1 1 0 1

1 1 1 0

00 /00

- /11

- /00

- /01

- /10

0 0 0 0

m1m2/ m1m2

m3/ m3p1

m4/ m4p2

-/ p3p4

b2) The appropriate edge weights for the individual trellis edges is calculated using this **equation**:

d(e) = |ri – c­i(e)|2 + |ri+1 – c­i+1(e)|2

With received vector = [0.54, -0.12, 1.32, 0.41, 0.63, 1.25, 0.37, -0.02], the edge weights calculated using previous equation are shown in trellis in **Figure**.

0.2306

0.4106

1.9106

1.9106

0.2306

0.4106

1.7306

1.7306

x x 0 0

x x 1 1

x x 0 1

x x 1 0

1.8793

0.2939

0.4993

0.4993

1.8793

0.2939

2.1393

2.1393

x 0 0 0

x 0 1 1

x 1 0 1

x 1 1 0

0.226

1.546

1.466

0 0 0 0

0 0 0 0

0 0 1 1

1 1 0 1

1 1 1 0

0.306

1.4373

0.1373

1.1773

0.3973

0 0 0 0

m1m2/ m1m2

m3/ m3p1

m4/ m4p2

-/ p3p4

This shows the edges at each stage, then the survivors of each stage will be searched, by ADD-COMPARE-STORE step.

The survivors of stage one are:

0.226

1.546

1.466

0.306

The survivors of stage two are:

0.7253

2.0453

1.7599

0.5999

The survivors of stage 3 are:

0.9559

2.2759

2.1705

1.0105

The survivor paths of stage 4 and the least distance path is shown in red:

1.3532

3.4532

3.6078

1.1478

This path is the one corresponding to the 0 0 1 1 1 1 0 0 codeword.

b3) The total number of flops using Viterbi decoder is reduced to 23 flops. This is achieved by symmetry of the trellis. The equation used to calculate the edge distances, can be expanded as:

d(e) = |ri – c­i(e)|2 + |ri+1 – c­i+1(e)|2 = ri2 + ri+12 ± 2ri  ± 2ri+1 + 2

At each stages, ri+12, ri2, and 2 are common among all paths. This reduces calculating the edge distances to:

d(e) = ± 2ri  ± 2ri+1

Taking out the common factor of 2, reduces the calculation further to:

d(e) = ± ri  ± ri+1

The edge distances at stage one will be r1 + r2, -r1 - r2, -r1 + r2, and r1 - r2. The negation of r1 + r2 gives -r1 - r2 = –( r1 + r2), and negation of -r1 + r2 gives r1 - r2 = –(-r1 + r2). Negation is not considered a floating point operation in two’s complement representation, sign bit is changed to negate result. This reduces calculating the edge distance at stage one to 2 flops. This symmetry property for calculating edge distances extends to stages 2, 3, and 4 giving a total of 8 flops for this step.

For the second step of ADD-COMPARE-STORE, the number of flops for this stage is 15 flops. This can be achieved a to be completed…

c1) The Matlab code for Viterbi decoder for (8,4,4) Hamming Code is shown in **Figures 1 ,2 3 ,4 5** in Appendix.

c2) The Gaussian noise at each SNR level is calculated by multiplying square root of the variance, standard deviation by randn(1, 8). Power of received signal is 1, so variance is calculated using:

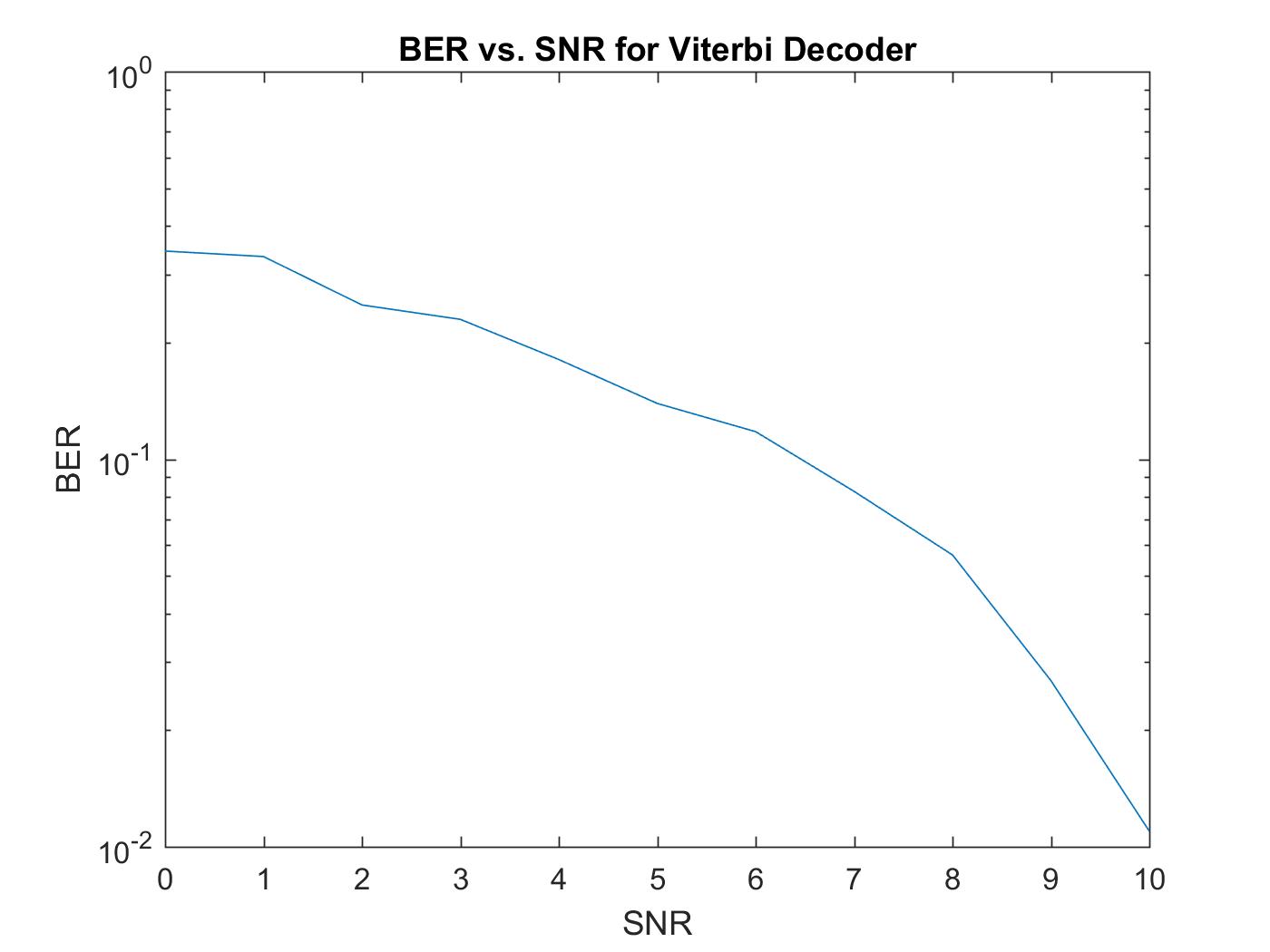
σ2 =

The noise will be equal to:

Noise = σ\*randn(1,8)

This noise is added to each codeword, then passed to Viterbi decoder to retrieve original codeword. The number of bit errors in message bits is taken to give BER over 10,000 packets or when 100 codeword errors are reached.

The BER vs. SNR plot for Viterbi decoder performance with additive Gaussian noise is shown in **Figure.**



## Question 3

1. Plot below shows the discrete plot of impulse response (h[n]).



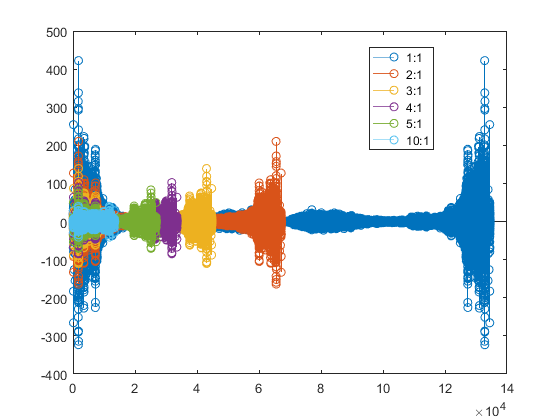
Frequency is 16000 Hz, so the delay = 149/16000 = 0.0093 seconds. Speed of sound = 340 m/s, so distance between microphone and the person is d = 340\*0.0093 = 3.1662 m.

This result was confirmed with MATLAB as well, by looping over the impulse vector and finding the index where the first non-zero value occurs.

1. The sound is still audible after convolution, although there some noise could be also be heard. The background noise seems to have degraded the sound quality of the original audio. The code for this convolution can be seen in **figure** in appendix.
2. An delayed version of the sound can be heard along with the original, therefore creating the impression of an echo. The code to create this echo sound is in **figure** in appendix.
3. The sound is played backwards by reversing the original sound vector. This is achieved by using the flipud function in MATLAB. Playing the backwards signal resulted in speech which could not be understood. The code for this reversal and playback can be seen in **figure** in appendix.

|  |  |
| --- | --- |
| Frequency | Comments |
| 13000 | The sound seems slower and the voice is now thick, low-pitched voice |
| 14500 | The sound is quite like the original but still slow, quite thick, low-pitched voice |
| 17000 | The sound is quite similar but is a faster, higher pitched version of the original |
| 18500 | The sound is much faster and higher pitched version of the original, and seems much higher than the previous frequency |
| 20000 | At this frequency, the sound is way high pitched and much faster than the original |

1. The plot below shows the Fast Fourier Transform (FFT) plot, comparing the original signal, with the sub-sampling signals. The MATLAB code for aliasing can be seen in **fig** in appendix.

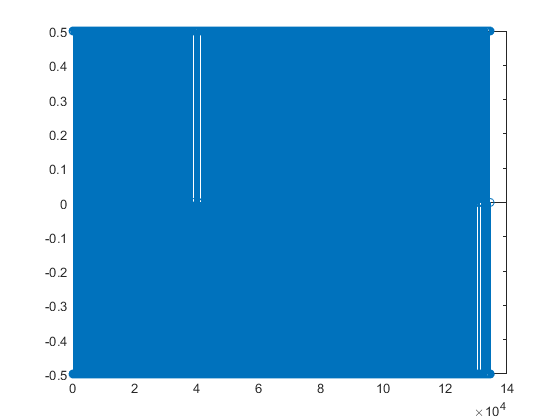


|  |  |
| --- | --- |
| Sub-Sample ratio | Ratio |
| 2:1 | The sound is like the original with some minor losses in the audio |
| 3:1 | This sound is also like the original, but the signal seems more lossy now |
| 4:1 | This sound is more lossy than before, with the sound almost resembling a muffled voice |
| 5:1 | This sound is more muffled than before with the audio almost not hearable in low volume |
| 10:1 | This sound has the most losses, having the greatest muffled audio. Only a little bit of the audio could be understood |

1. The following results were obtained after quantization. The MATLAB code used to achieve quantization can be seen in **figures** in the appendix:

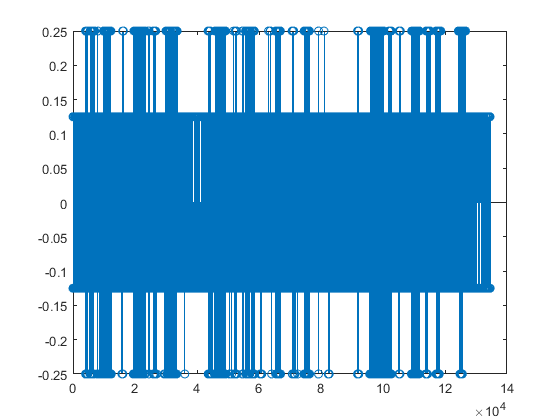
**1-bit quantization:**

The graph below shows the discrete plot of the 1-bit quantized signal. In this case, significant noise could be heard along with the audio clip, the noise occurring due to quantization



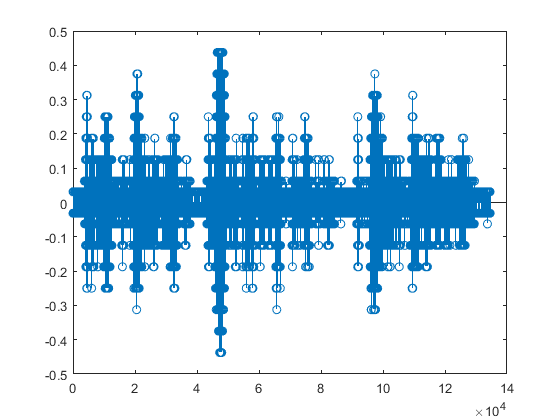
**2-bit quantization:**

The graph below shows the plot for the 2-bit quantized signal. In this case, the signal could be heard along with background noise, severely hampering the quality. However, the sound quality was noticeably better compared with 1-bit quantization.



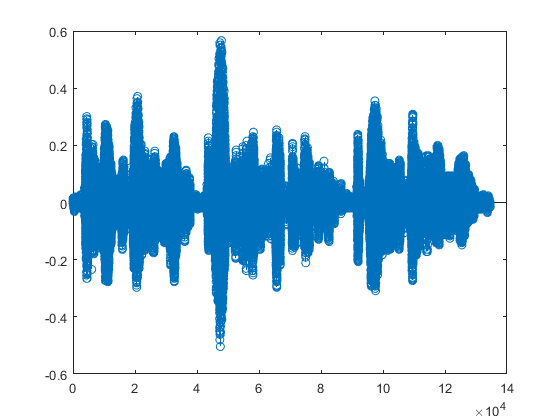
**4-bit quantization:**

The graph for 4-bit quantization is as shown below. In this case, the original signal could be heard along with noise, however the quality was much better compared with 1-bit and 2-bit quantization with the noise intensity seemed lower than 1-bit and 2-bit quantization noise.



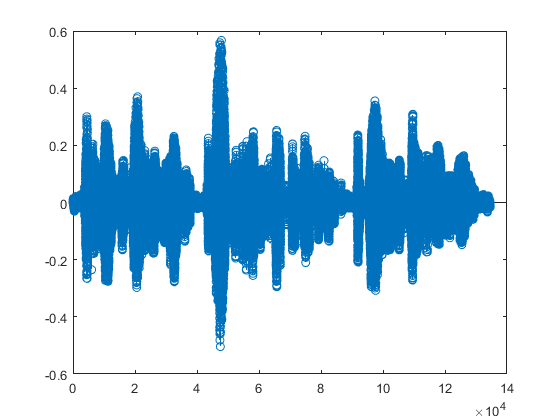
**8-bit quantization:**

In this case the sound was very clear, almost close to the original. Some low and minor noise could be heard with the signal at very specific points and not throughout the audio playback, as was the case with 4-bit quantization. The discrete plot is shown below, which is almost the same as the discrete plot of the original signal.



**16-bit quantization:**

In this case the sound quality was the same as the original and no noise could be heard. The plot is shown below, which is now closer to the original and better in shape compared to the plot of 4-bit quantization.



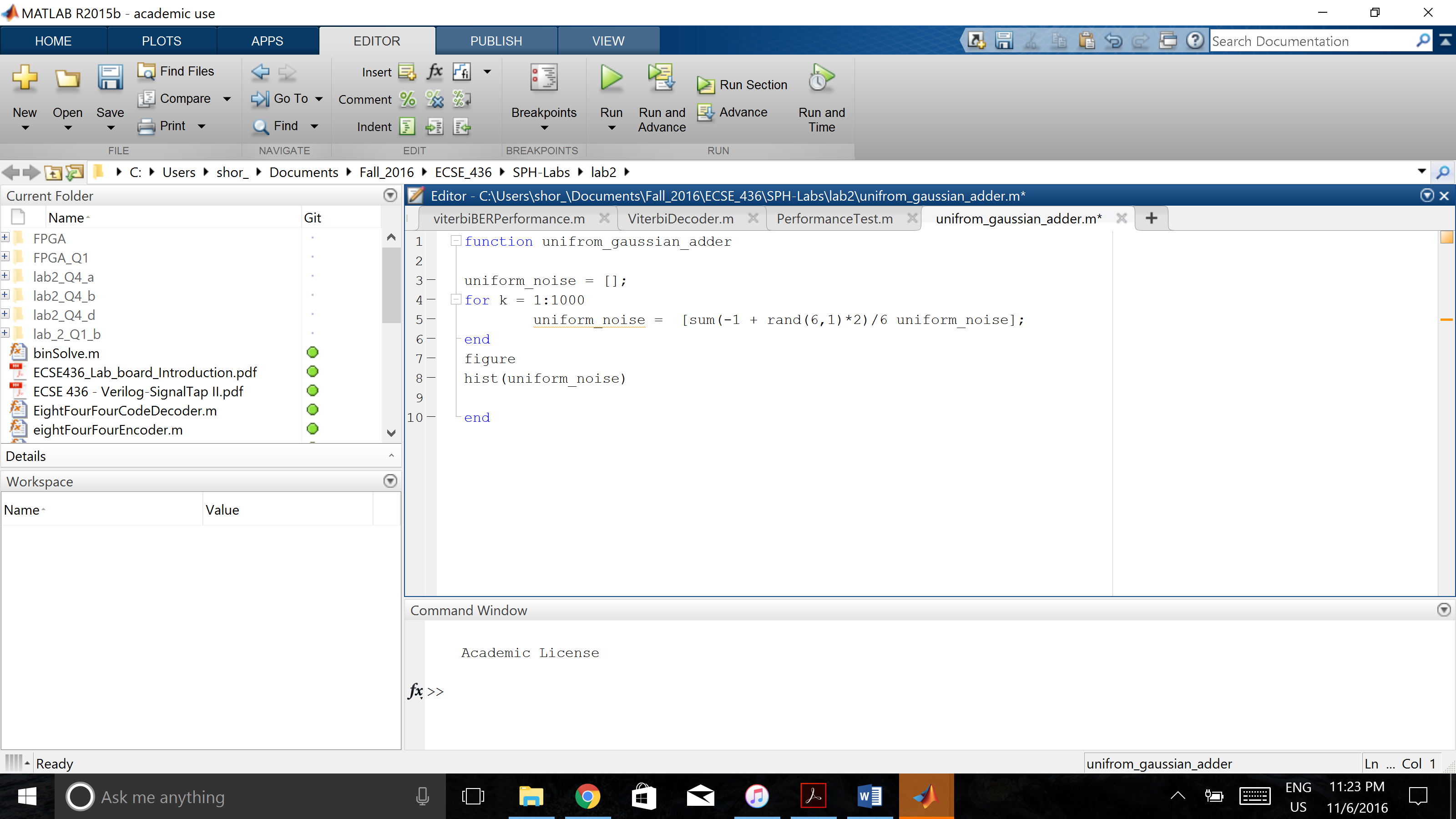
## Question 4

e) In MATLAB, the rand function can be used to generate uniformly distributed numbers such as:

a + rand(n, 1) \* (b – a)

where a is the minimum, b is the maximum and n is the number of random number generated in vector.

In our case, where the interval is [-1 1] and we are adding 6 uniform generators, a is -1, b is 1 and n is 6, Matlab script to show the noise generated is in fact close to being Gaussian distributed is shown in **Fig ..**:



The plot of this script in **Fig 1** shows that the noise generated with six uniform number generators has a Gaussian distribution.



f) The mean of noise in part (d) is 0 while the variance is 0.2.

g) The noise power was adjusted by masking most significant bits in noise. Estimating the each masked bit corresponds to 6dB and that SNR of 0dB is such that the most significant 4 bits of noise are shifted to the right. The different SNR values were obtained as shown in **Table**:

|  |  |
| --- | --- |
| **SNR /dB** | **Right Shift in Added Gaussian Noise** |
| -10 | Shifted two bits |
| 0 | Shifted four bits |
| 10 | Shifted six bits |
| 20 | Shifted seven bits |
| 30 | Shifted nine bits |
| 40 | Shifted eleven bits |

## Question 5

1. Here is an example of 4 equations with 4 unknowns where the real and mod2 solutions differ

Solving the above four equations yields the following results in base-10: a = -1, b = -1, c = 2 and d = 0. Solving the equations in mod-2 gives the following: a = 1, b = 1, c = 0 and d = 0.

1. The following example shows 4 linear equations with 4 unknowns where the real and mod-2 solutions differ but only contain 0 and 1:

Solving the above four equations yields the following results in base-10: a = 0, b = 0, c = 0 and d = 1. Solving the equations in mod-2 gives the following: a = 1, b = 1, c = 1 and d = 0.

This difference occurs when A matrix in base-10 is invertible but in mod-2 it’s not

.

1. The above examples prove that it is necessary to implement our own function to solve any given system of linear equations in binary. The approach of solving the equation in real(base-10) domain and then converting the answer to binary does not always work, as the above example has proven. This is a necessary step during the implementation of erasure decoder for binary error control.
2. Refer to **figures** for the implementation on the function “binSolve” which uses Gaussian elimination to solve 4 linear equations with 4 unknowns in mod-2.

## Question 6

1. A system is said to be causal if its output depends only on present and past input values, and not future inputs. For discrete LTI systems, all outputs are defined via the convolutional theorem:

For non-zero h[n] when n < 0, the summation would expand as such:

It can be noticed from the last two terms of the summation that when h[n] is negative the output depends on future values of x (x[n+1], x[n+2] and so on). This goes against the definition of causality and hence the LTI system is not causal if impulse response is non-zero for n < 0.

1. For BIBO stability, any bounded input, i.e. |x[n]| < ∞, should give a bounded output, i.e. |y[n]| < ∞. For LTI systems, a system is not BIBO stable if |h[n]| is not stable. This can be proved via the convolutional theorem, where assume that summation of |x[n]| = A, a finite constant:

= ∞

Therefore, the system is not stable.

1. For a system to be invertible, there should exist an impulse response such that when the output signal of the system is multiplied (or convoluted) with the inverse, the original input is recovered:

h[n]

h`[n]

x[n]

y[n]

x[n]

The following equation shows the output for the echo system used in Question 1. The echo is generated by delayed response of the input signal, modelled by the term x[n – A], where A is a real, positive constant.

Converting this equation discrete time Fourier transform:

The invertible system is defined by:

= 1/

Hence the system is only invertible when the term 1 + e­-jwA is non-zero, thus making H`(ejw) real. Due to this condition, the system is not strictly invertible.

1. In this case, the impulse response of the system will be 1 for certain frequencies only. Assume that the impulse response is given as follows:

The z-transform for this impulse response is the following:

//complete

1. The convolution theorem states the following for LTI systems:

It is also known that an LTI system has h[n] as its output when the input is . This can be stated as:

Due to time-invariance we can say:

Due to linearity:

Again, due to linearity:

Hence, the above states and proves the convolution theorem:

1. The Fourier transform for any signal x[n] is defined by the following equality:

The convolutional property states the following:

Therefore, Y(ejw) is given as such:

It can be observed that the two summations above correspond to the definition of discrete Fourier transform. Thus:

1. Aliasing refers to the sub-sampling of a signal. With the aid of aliasing, a continuous signal can be sampled at different frequencies to arrive at its close approximation in discrete time. A sample taken at every other interval will be a closer approximation to a sample taken at every 3rd interval. The plots below show the effect of aliasing by sampling a sine waveform from continuous time domain to discrete time domain. In **Fig2** the sample is taken at every 0.1 seconds. In **Fig3** the sample is taken at every 0.3 seconds. In both figures, some points from the original waveform are missing due to aliasing.

****

Figure 1: Sine plot in continuous time domain

****

Figure 2: Sine function in discrete time domain, sample taken at every 0.1 seconds

****

Figure 3: Sine function in discrete time domain, sample taken at every 0.3 seconds

## Appendix

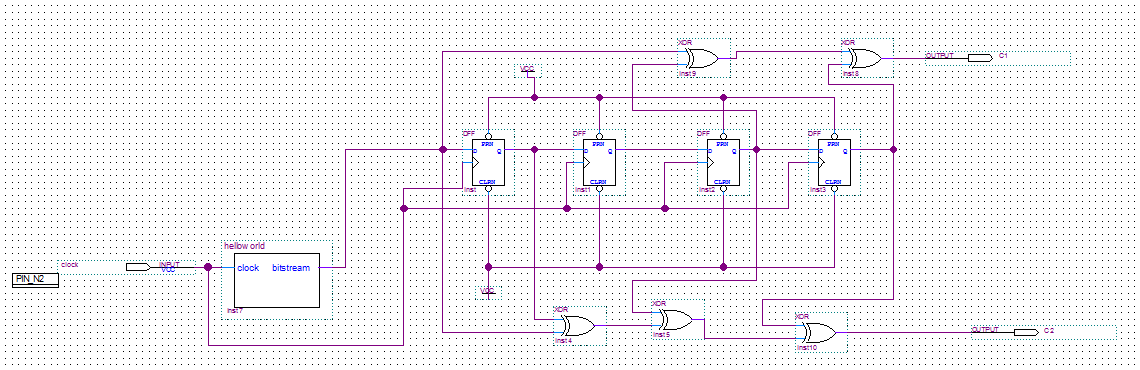


Figure 4: rate-half encoder

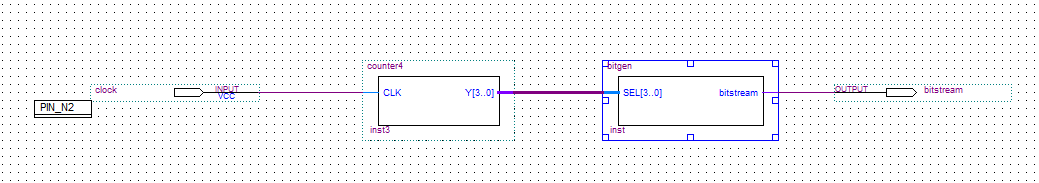


Figure 5: bit stream generator for rate half encoder

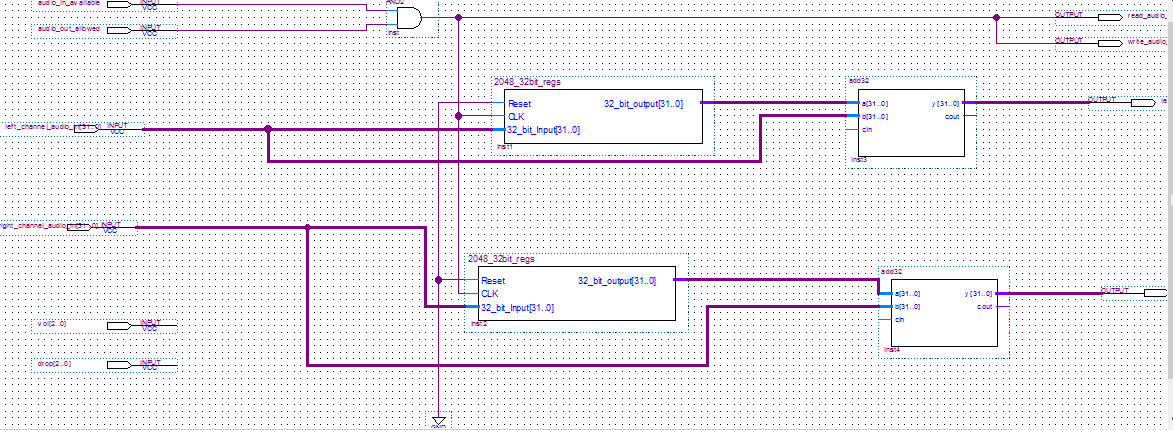
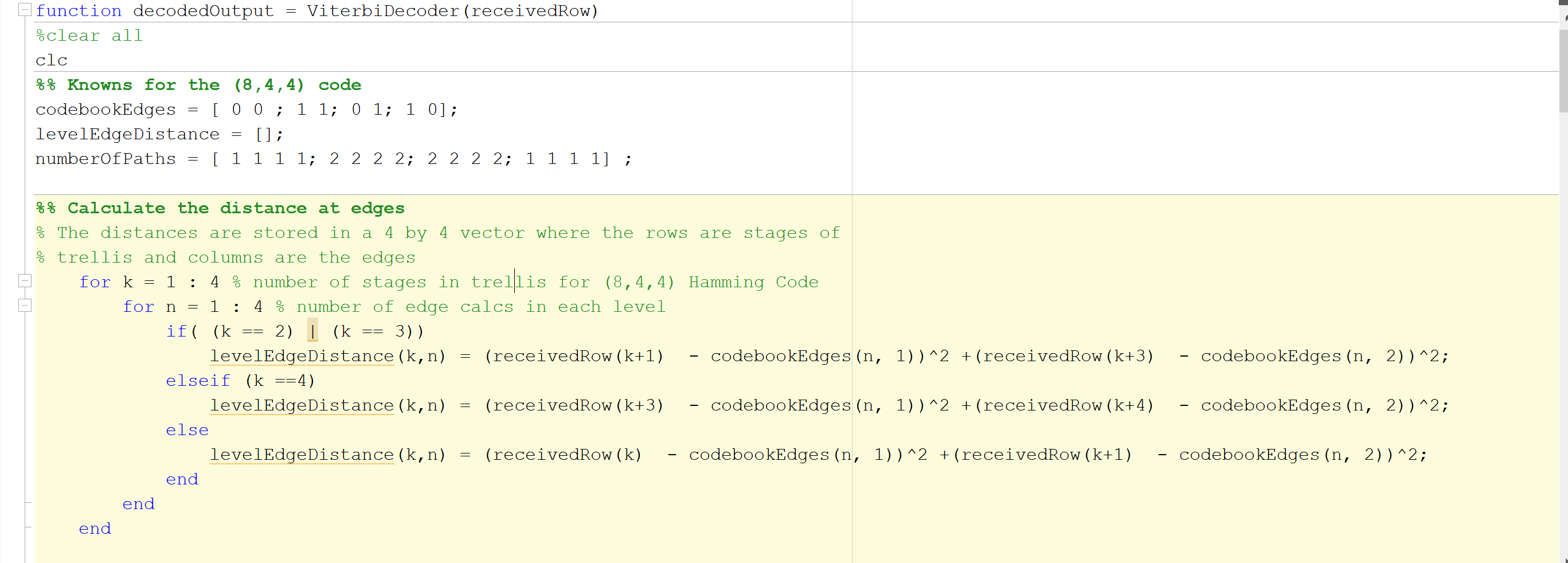
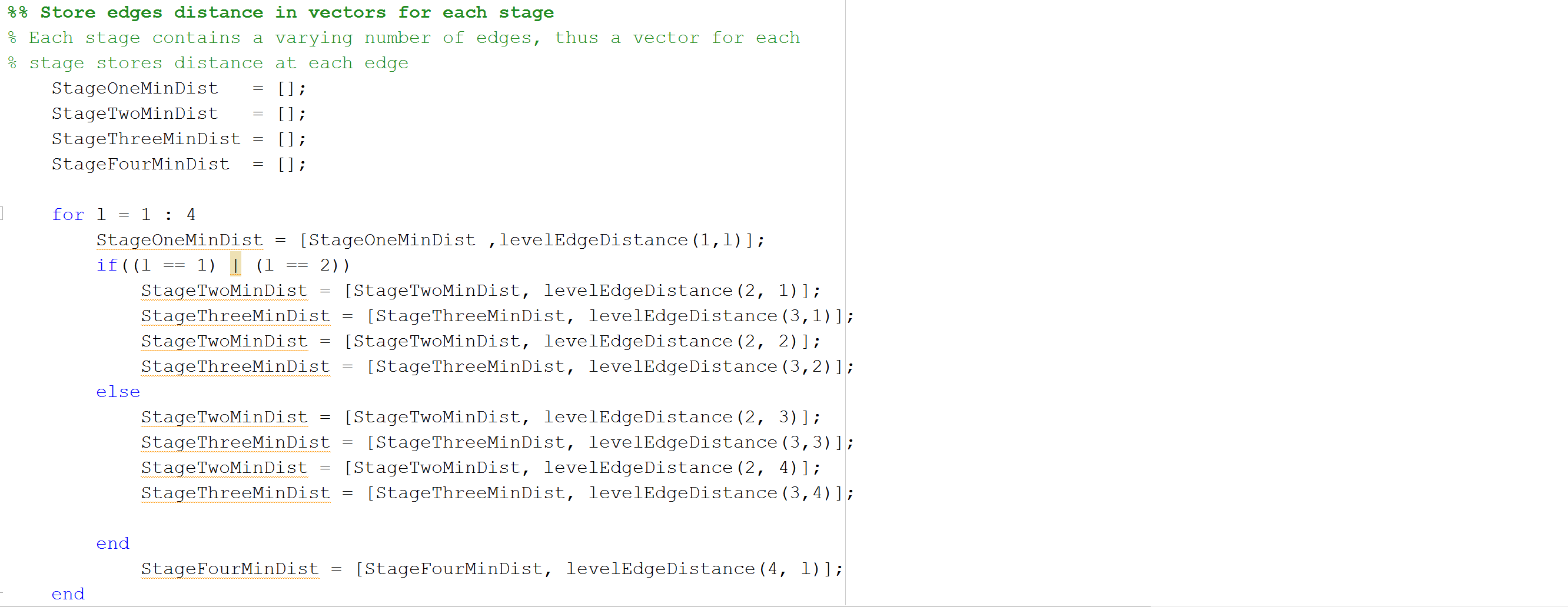
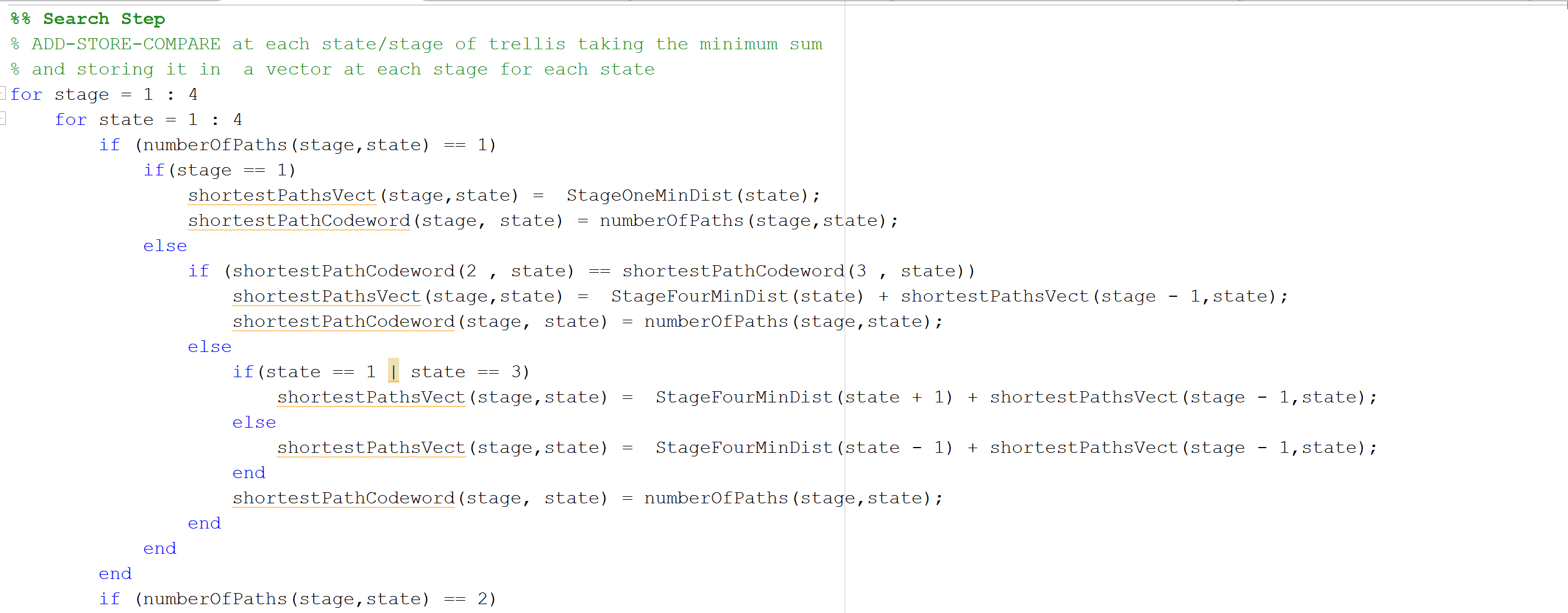
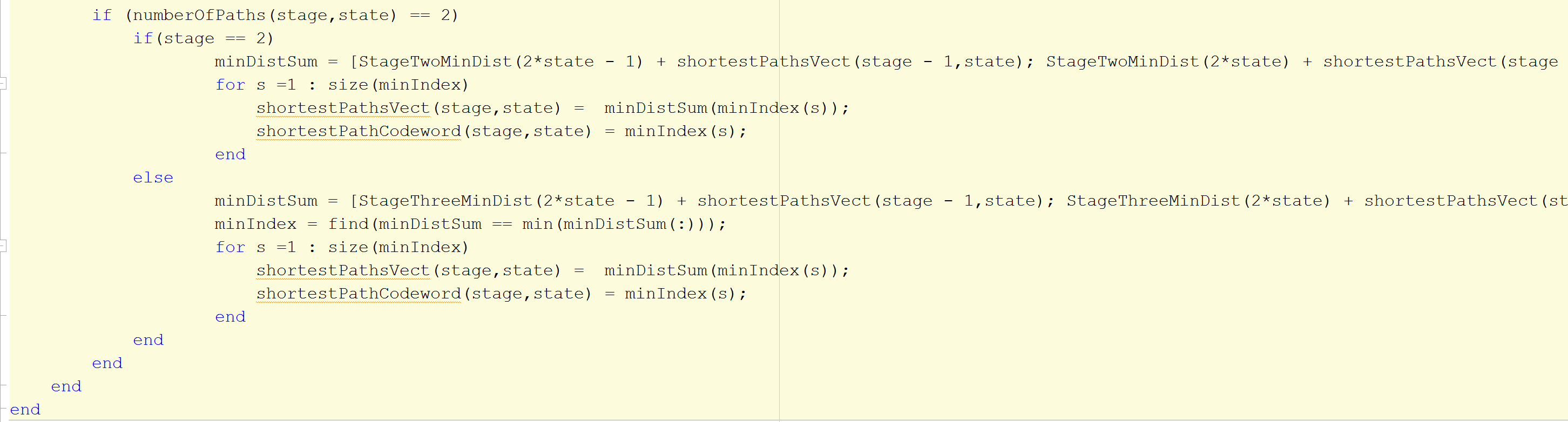


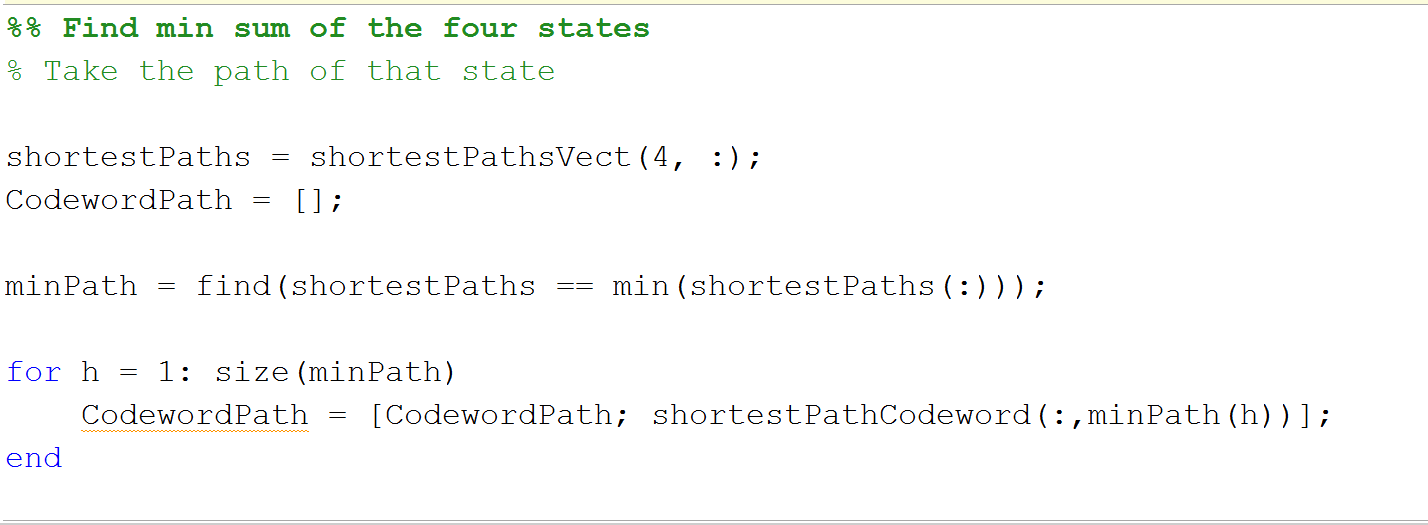
Figure 6: filter\_demo for echo system











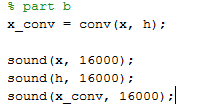


Figure 7: Convolution of impulse with audio

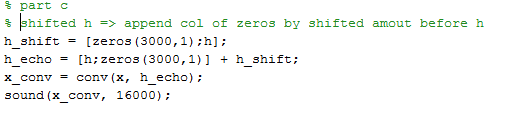


Figure 8: echo sound generator

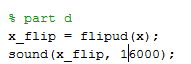


Figure 9: Playing the sound in reverse

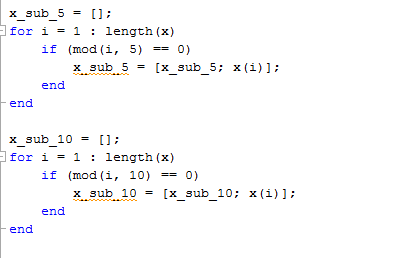
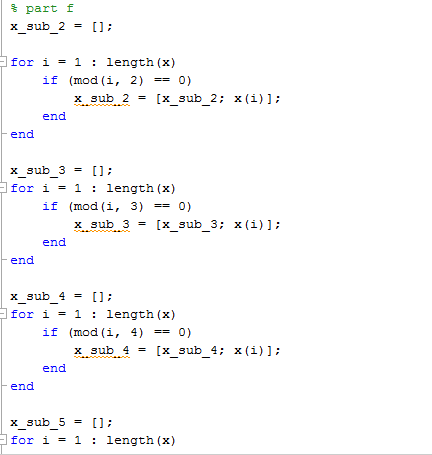


Figure 10: Aliasing implementation

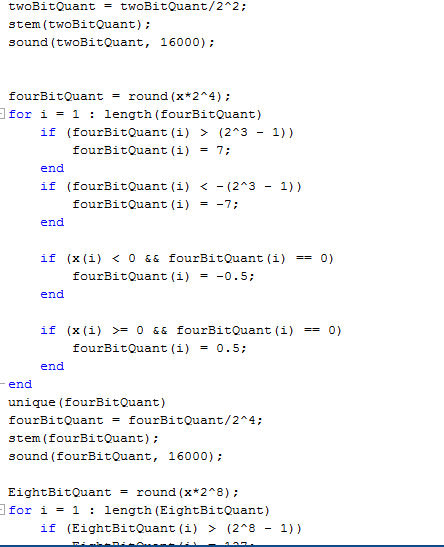
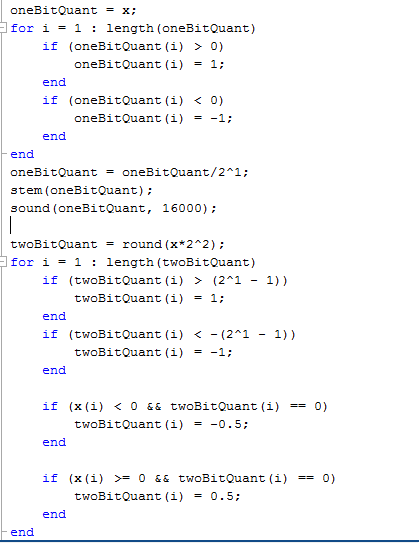


Figure 11: 1-bit and 2-bit quantization

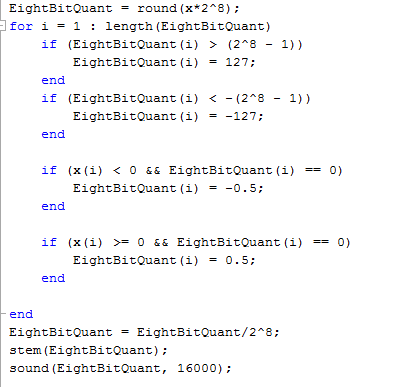
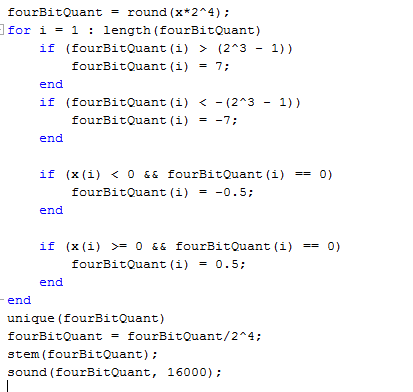


Figure 12: 4-bit and 8-bit quantization

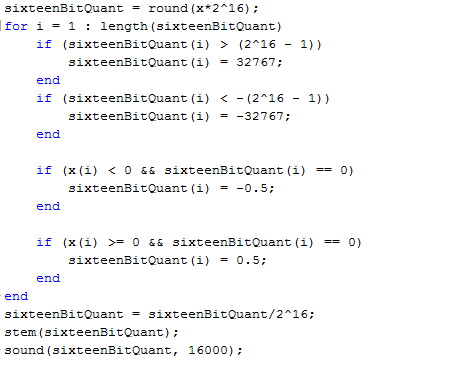


Figure 13: 16-bit quantization

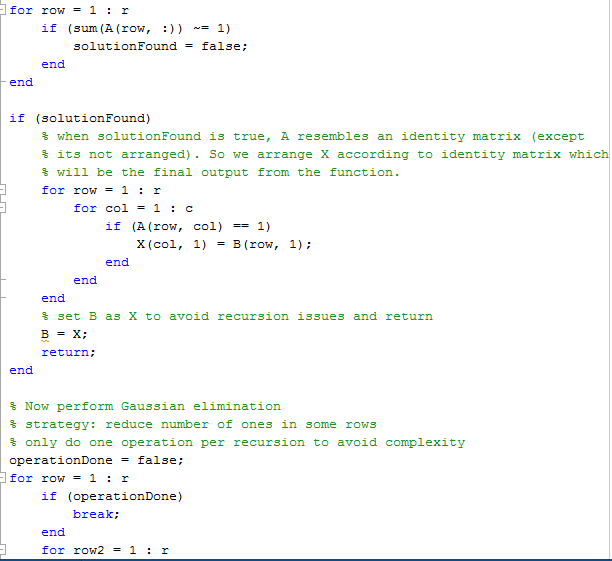
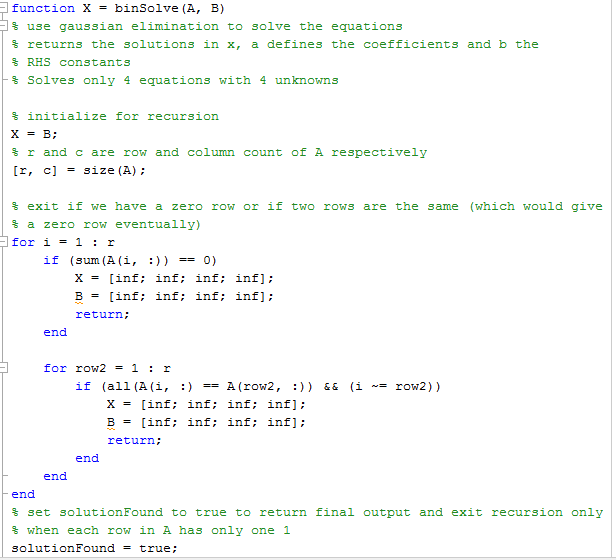


Figure 14: Mod-2 Gaussian elimination solver

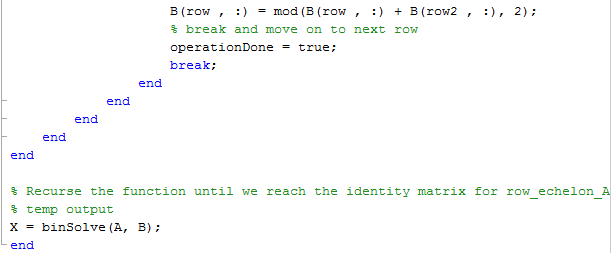
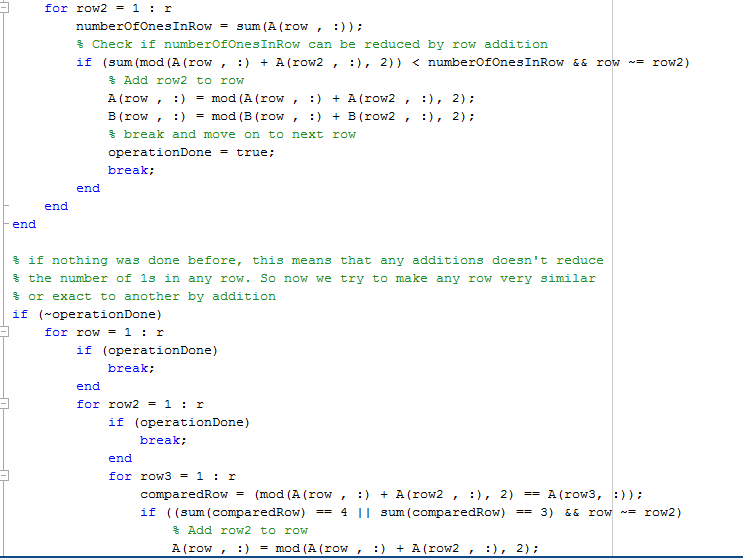


Figure 15: Mod-2 Gaussian elimination solver 2