ASSIGNMENT - 01

COURSE : Calculus & Analytical Geometry
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Roll Number : 21K-4503
SECTION - E

QUESTION : 01

a) [1, 3]

(i) [1, 3], $a=1$, $b=3$

(ii) $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

$1 \neq 3$ (D.N.E)

b) (1, 3)

(i) (1, 3), $a=1$, $b=3$

(ii) $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

$1 \neq 3$

limit D.N.E

c) $[1,2]$

(i) $(1,2)$, $a=1$, $b=2$

(ii) $\lim_{x\to 1^+} f(x) = \lim_{x\to 2^-} f(x)$

$$\boxed{1 = 1}$$

(iii) $\lim_{x\to 2^-} f(x) = f(2)$

$$1 \neq 2$$

Not Continuous

d) $(1,2)$

$(1,2)$, $a=1$, $b=2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 2^-} f(x)$

$$\boxed{3 = 3}$$

$\lim_{x\to 2^-} f(x) = f(2)$

$3 = 3$

Continuous Fn.

e) [2, 3]

$(2,3)$, $a=2$ & $b=3$

- $\lim_{x \to 2^+} f(x) = \lim_{x \to 3^-} f(x)$

$\boxed{3 = 3}$

$\lim_{x \to 3^-} f(x) = f(x)$

$\boxed{3 = 3}$

Continuous Fn.

f) (2, 3)

$(2,3)$, $a=2$, $b=3$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 3^-} f(x)$

$\boxed{3 = 3}$

$\lim_{x \to 3^-} f(x) = f(x)$

$\boxed{3 = 3}$

Continuous Fn.

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QUESTION : 2

a) $[1, 3] \rightarrow (1, 3)$, $a = 1$, $b = 3$

$$\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$$

$$3 \neq 1$$

D.N.E

b) $(1, 3) \rightarrow a = 1$, $b = 3$

$$\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$$

$$2 \neq 2$$

$$\lim_{x \to 2} f(x) \neq f(2)$$

$$2 \neq 3$$

Discountinuous Fn.

c) $[1, 2] \rightarrow (1, 2)$, $a = 1$, $b = 2$

$$\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$$

$$2 \neq 2$$

$$\lim_{x \to 2} f(x) \neq f(x)$$ Dis countinuaus FN.

$$2 \neq 3$$

d) $(1, 2) \rightarrow a = 1$, $b = 2$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$$

$$2 = 2$$

$$\lim_{x \to 2} f(x) = f(2)$$ Continuous FN.

$$\boxed{2 = 2}$$

e) $[2,3] \rightarrow (2,3)$, $a=2$, $b=3$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 3^-} f(x)$$

$$2 = 2$$

$$\lim_{x\to 2^+} f(x) = f(2)$$

$$\boxed{2 \neq 3}$$

Discountinuous Fn.

f) $(2,3) \rightarrow a=2$ & $b=3$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 3^-} f(x)$$

$$\boxed{3 = 3}$$

$$\lim_{x\to 2} f(x) = f(2)$$

$$\boxed{3 = 3}$$

Continuous Fn.

QUESTION : 3

a) $[1,3] \rightarrow (1,3)$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

$$\boxed{1 \neq 3}$$

Not Defined

b) $(1,3) \rightarrow a=1$ & $b=3$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

$$\boxed{2 = 2}$$

Continuous Fn.

Continue

c) [1, 2]

f(1) is not defined

d) (1, 2)

(i) (1, 2), a = 1, b = 2

(ii) $\lim_{x \to 1^+} f(x) = \lim_{x \to 2^-} f(x)$

$\boxed{2 = 2}$

(iii) $\lim_{x \to 2} f(x) = f(x)$ Continuous Fn

$\boxed{2 = 2}$

e) [2, 3]

f(3) is not defined

d) (2, 3)

(i) (2, 3), a = 2 & b = 3

(ii) $\lim_{x \to 2} f(x) = \lim_{x \to 3} f(x)$

$\boxed{2 = 2}$

(iii) $f(x) = \lim_{x \to 2} f(x)$ Continuous Fn.

$\boxed{2 = 2}$

QUESTION : 4

a) $[1, 3]$

~~f(3) is not defined~~

b) $(1, 3)$

~~(i) $a = 1$ & $b = 3$~~

(ii) $\lim_{x \to 1^-} f(x) = \lim_{x \to 3^+} f(x)$

$\boxed{3 = 3}$

(iii) $f(x) = \lim_{x \to 3} f(x)$

$\boxed{3 = 3}$ Continuous Fn.

c) $[1, 2]$

(i) $[1, 2]$, $a = 1$ & $b = 2$

(ii) $\lim_{x \to 1} f(x) = \lim_{x \to 2} f(x)$

$\boxed{2 = 2}$

(iii) $f(x) = \lim_{x \to 2} f(x)$

$\boxed{2 = 2}$ Continuous Fn.

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d) $(1, 2)$

(i) $(1, 2)$, $a = 1$ & $b = 2$

(ii) $\lim_{x \to 1^-} f(x) = \lim_{x \to 2} f(x)$

$$\boxed{2 = 2}$$

(iii) $\lim_{x \to 2} f(x) = f(x)$

$\boxed{2 = 2}$ Continuous Fn.

e) $[2, 3]$

$f(3)$ is not included.

f) $(2, 3)$

(i) $(2, 3)$, $a = 2$ & $b = 3$

(ii) $\lim_{x \to 2} f(x) = \lim_{x \to 3^+} f(x)$

$$\boxed{2 = 2}$$

(iii) $f(x) = \lim_{x \to 2} f(x)$

$\boxed{2 = 2}$ Continuous Fn.

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QUESTION : 5

$f(x) = \begin{cases} 1, & x \neq 4 \\ -1, & x = 4 \end{cases}$ & $g(x) = \begin{cases} 4x-10, & x \neq 4 \\ -6; & x = 4 \end{cases}$

$x = 4$
$f(x) = -1$

a) $f(x)$

$f(x)$ define at $x = 4$

$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$

$-1 \neq 1$

D.N.E

b) $g(x)$

$x = 4$

(i) $g(x)$ define at $x = 4$

(ii) $\lim_{x \to 4^-} g(x) = \lim_{x \to 4^+} g(x)$

$-6 = 4x - 10$

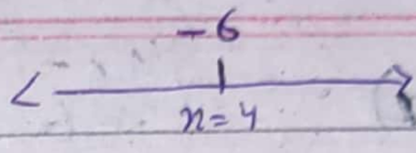
$-16 = 4x$

$\boxed{x = -4}$

(iii) $\lim_{x \to 4} f(x) = f(x)$

$\boxed{4 \neq -4}$ Discountinuous Fn.

c) $-g(x)$



(i) $-g(x)$ define at $x=4$

(ii) $\lim_{x \to 4^-} -g(x) = \lim_{x \to 4^+} -g(x)$

$$-(-6) = -(4x-10)$$

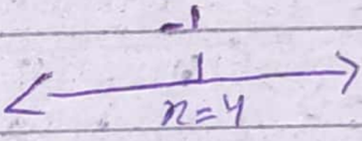
$$6 = -4x+10$$

$$\boxed{x = 4}$$

(iii) $\lim_{x \to 4} f(x) = f(x)$ Continuous Fn.

$$\boxed{4 = 4}$$

d) $|f(x)|$



$|f(x)|$ define at $x=4$

$$\lim_{x \to 4^-} |f(x)| = \lim_{x \to 4^+} |f(x)|$$

$$|-1| = |1|$$

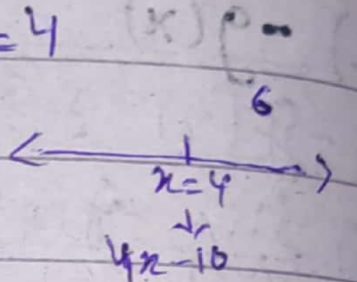
$$\boxed{1 = 1}$$

$$\lim_{x \to 4} |f(x)| = |f(x)|$$

$$\boxed{+1 = 1}$$

Continuous Fn.

e) $f(x)\,g(x) \rightarrow \begin{cases} 4x-10, & x \neq 4 \\ +6, & x = 4 \end{cases}$



(i) $fg(x)$ define at $x=4$

(ii) $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x)$

$4(4)-10 = 6$

$\boxed{6 = 6}$ limit exists.

(iii) $\lim_{x\to 4} f(x) \neq f(x)$

$\boxed{4 \neq 6}$ Discountinuous Fn.

f) $g(f(x)) \rightarrow \begin{cases} 4x-10, & x \neq 4 \\ 4x-10, & x = 4 \end{cases}$

(i) $gof(x)$ define at $x=4$

(ii) $\lim_{x\to 4^-} gof(x) = \lim_{x\to 4^+} gof(x)$

$4(4)-10 = 4(4)-10$

$\boxed{6 = 6}$

(iii) $\lim_{x\to 4} gof(x) = f(x)$ Discontinuous Fn.

$\boxed{6 \neq 4}$

$g(4) = 4(4)-10 = 16-10 = 6$

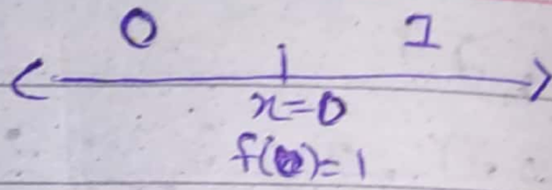
$\lim_{x\to 4} f(x) = f(x)$

$\boxed{6 = 6}$ Continuous Fn.

Q # 6 $\longrightarrow$

QUESTION : 6

a) $f(x)$

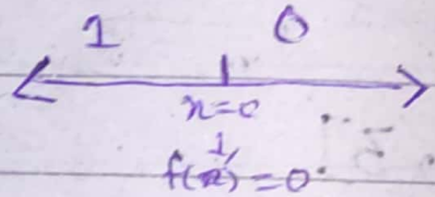


(i) $f(x)$ define at $x=0$

(ii) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\boxed{0 \neq 1}$ D.N.E

b) $g(x)$

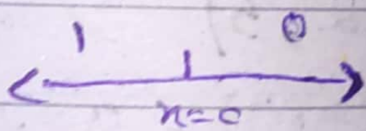


(i) $f(x)$ define at $x=0$

(ii) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\boxed{1 \neq 0}$ D.N.E

c) $f(-x)$

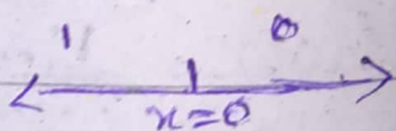


(i) $f(-x)$ define at $x=0$

(ii) $\lim_{x \to} f(x) = \lim_{x \to} f(x)$

$\boxed{1 \neq 0}$ D.N.E

d) $|g(x)|$



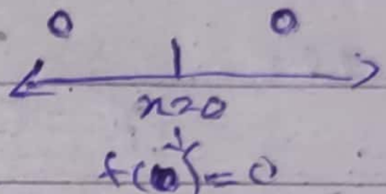
(i) $g(x)$ define at $x=0$

(ii) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\boxed{1 \neq 0}$ D.N.E

e) $f(x)\,g(x) \rightarrow \begin{cases} 0 & 0 \le x \\ 0 & x \le 0 \end{cases}$

(i) $fg(x)$ define at $x=0$



(ii) $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x)$

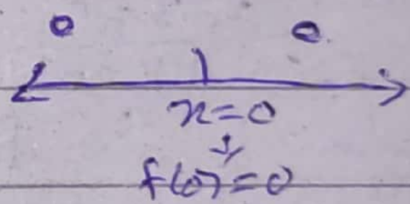
$\boxed{0 = 0}$

(iii) $\lim\limits_{x\to 0} f(x) = f(x)$

$\boxed{0 = 0}$ ∴ Continuous Fn.

f) $g(f(x)) \rightarrow \begin{cases} 0 ; & 0 \le x \\ 0 , & x < 0 \end{cases}$

(i) $gof(x)$ define at $x=0$



(ii) $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x)$

$\boxed{0 = 0}$

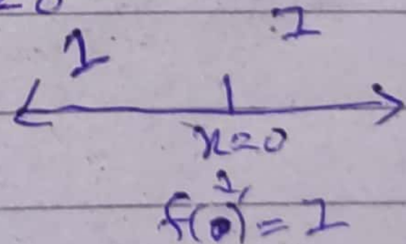
(iii) $f(x) = \lim\limits_{x\to 0} f(x)$

$\boxed{0 = 0}$

Continuous Fn.

g) $f(x)+g(x) \rightarrow \begin{cases} 1 , & 0 \le x \\ 1 ; & x < 0 \end{cases}$

(i) $f(x)+g(x)$ define at $x=0$



(ii) $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x)$

$\boxed{1 = 1}$

(iii) $\lim\limits_{x\to 1} f(x) = f(x)$

$\boxed{1 = 1}$ Continuous Fn.

QUESTION : 11 to 22

Question : 11

$f(x) = 5x^4 - 3x + 7$

$f(0) = 5(0)^4 - 3(0) + 7$

$f(0) = 7$

→ No values of x

→ Continuous Fn.

$(-\infty, \infty)$

Question : 12

$f(x) = \sqrt[3]{x-8}$

$f(0) = \sqrt[3]{0-8}$

$f(0) = \sqrt[3]{2^3}$

$f(0) = 2$

→ No values of x

→ Continuous Fn.

Question : 13

$f(x) = \frac{x+2}{x^2+4}$

Let, $f(0) = x^2 + 4$

$\boxed{f(0) = 4}$

→ No values of x.

→ Continuous Fn.

Question : 14

$f(x) = \frac{x+2}{x^2-4}$

Let, $x^2 - 4 = 0$

$x^2 = 4$

$\boxed{x = \pm 2}$

→ This Fn. is not continuous

on $x = 2$ and $x = -2$.

Question : 15

$$f(x) = \frac{x}{2x^2+x}$$

$$2x^2 + x = 0$$

$$x(2x+1) = 0$$

$\boxed{x=0}$ $\quad 2x+1=0$

$\boxed{x=-1/2}$

→ This Fn is not Continuous on $x=0$ and $x=-1/2$.

Question : 17

$$f(x) = \frac{3}{x} + \frac{x-1}{x^2-1}$$

Let,

$\boxed{x=0}$

$\boxed{x^2-1=0}$

↓

$x^2 = +1$

$\boxed{x=\pm 1}$

→ This fn is not continuous on $x=0$ and $x=\pm 1$.

Question : 16

$$f(x) = \frac{2x+1}{4x^2+4x+5}$$

$$f(0) = \frac{2x+1}{4x^2+4x+5}$$

$$f(0) = \frac{2(0)+1}{4(0)^2+4(0)+5}$$

$$f(0) = \frac{1}{5}$$

→ No values of x.

→ Continuous Fn.

Question : 18

$$f(x) = \frac{5}{x} + \frac{2x}{x+4}$$

Let, $\boxed{x=0}$ $\quad x+4=0$

$\boxed{x=-4}$

→ This is not a function on $x=0$ and $x=-4$.

Question : 19

$$f(x) = \frac{x^2 + 6x + 9}{|x| + 3}$$

$$f(0) = \frac{(0)^2 + 6(0) + 9}{|0| + 3}$$

$$f(0) = \frac{9}{3}$$

$$f(0) = 3$$

→ No values of x.

→ Continuous Fn.

Question : 20

$$f(x) = \left|4 - \frac{8}{x^4 + x}\right|$$

Let,

$$x^4 + x = 0$$

$$x(x^3 + 1) = 0$$

$$\boxed{x = 0} \quad x^3 + 1 = 0$$

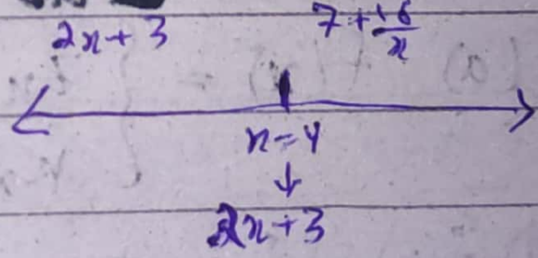
$$x^3 = -1$$

$$\boxed{x = -1}$$

→ This is not a continuous Fn on $x = 0$ & $x = -1$.

QUESTION : 21

$$f(x) = \begin{cases} 2x + 3, & x \le 4 \\ 7 + \frac{16}{x}, & x > 4 \end{cases}$$



(i) $f(x)$ define at $x = 4$

(ii) $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$

$$2(4) + 3 = 7 + 16/4$$

$$\boxed{11 = 11}$$ Limit Exists.

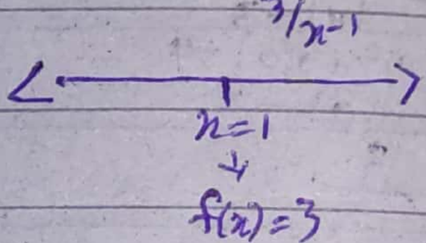
(iii) $\lim_{x \to 4} f(x) = f(x)$

$$\boxed{11 \neq 4}$$

Discontinuous Fn.

QUESTION : 22

$$f(x) = \begin{cases} \frac{3}{x-1} & x \neq 1 \\ 3 & x = 1 \end{cases}$$



(i) f(x) define at $x=1$

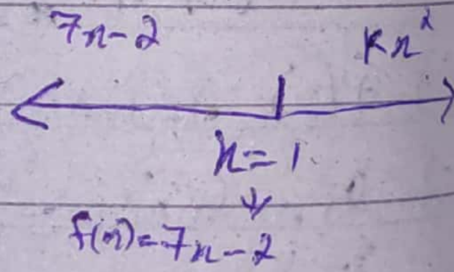
(ii) $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x)$

$3 \neq 3/2$ limit D.N.E

→ This is not a function on $x = 1$.

QUESTION : 29

(a) $f(x) = \begin{cases} 7x-2 , & x \leq 1 \\ Kx^2 , & x > 1 \end{cases}$



(i) f(x) define at $x = 1$

(ii) $\lim\limits_{x \to 1^-} f(x) = \text{limit}_{x \to 1^+} f(x)$

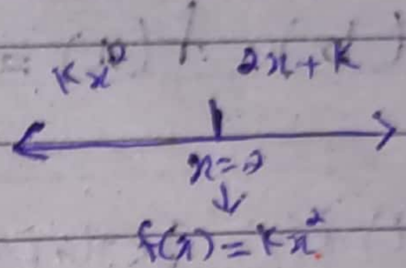
$7(1)-2 = K(1)^2$

$5 = K$

or

$\boxed{K=5}$

(b) $f(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x + k, & x > 2 \end{cases}$



(i) f(x) define at $x = 2$

(ii) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$k(2)^2 = 2(2) + k$

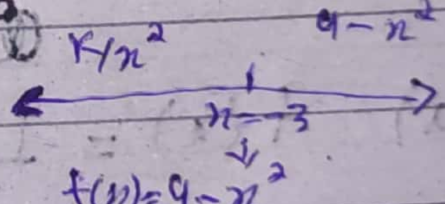
$4k = 4 + k$

$3k = 4$

$\boxed{k = 4/3}$

QUESTION : 30

(a) $f(x) = \begin{cases} 9 - x^2, & x \geq -3 \\ k/x^2, & x < -3 \end{cases}$



(i) f(x) define at $x = -3$

(ii) $\lim_{x \to -3^-} f(x) = \lim_{x \to -3^+} f(x)$

$k/(-3)^2 = 9 - (3)^2$

$k/9 = 9 - 9$

$\boxed{k = 0}$

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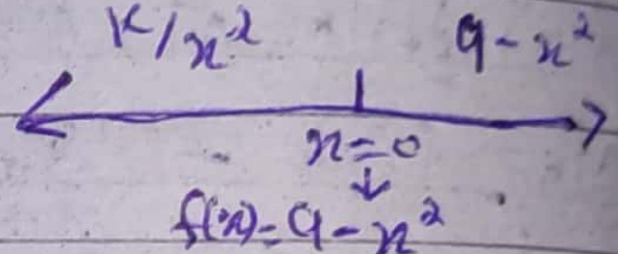
(b) $f(x) = \begin{cases} 9 - x^2, & x \geq 0 \\ K/x^2, & x < 0 \end{cases}$



(i) f(x) define at $x=0$

(ii) $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x)$

$\frac{K}{(0)^2} = 9-(0)^2$

$\boxed{0 = 9}$

D.N.E

$\rightarrow$ Function is Continuous.

QUESTION: 35

a) $f(x) = \dfrac{|x|}{x}$

Let, $\boxed{x = 0}$

(i) $f(x)$ define at $x = 0$

(ii) $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x)$

$\boxed{-1 \neq +1}$

Not removeable

b) $f(x) = \dfrac{x^2 + 3x}{x + 3}$

Let, $x + 3 = 0$

$\boxed{x = -3}$

(i) $f(x)$ define at $x = -3$

(ii) $\lim\limits_{x \to -3^-} f(x) = \lim\limits_{x \to -3^+} f(x)$

Removable

c) $f(x) = \dfrac{x - 2}{|x| - 2}$

Let, $\boxed{x = \pm 2}$

(i) $f(x)$ define at $x \neq \pm 2$

(ii) $\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to -2^+} f(x)$

$\boxed{1 \neq -2}$

Not removable

QUESTION : 36

a) $f(x) = \dfrac{x^2-4}{x^3-8}$

$f(x) = \dfrac{x^2-(2)^2}{(x)^3-(2)^3} = \dfrac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x^2-2x+4)} = \dfrac{(x+2)}{(x^2-2x+4)}$

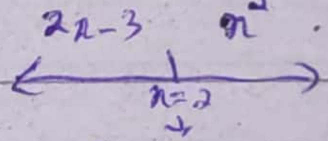
(i) $f(x)$ define at $x=2$

(ii) $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x)$

$\boxed{-2 \neq 1}$

Not removable.

b) $f(x) = \begin{cases} 2x-3 & , x \le 2 \\ x^2 & , x > 2 \end{cases}$



$f(2) = 2x-3$

(i) $f(x)$ define at $x=2$

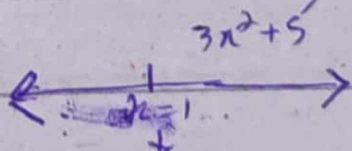
(ii) $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x)$

$2(2)-3 = (2)^2$

$\boxed{1 \neq 4}$

Not removable discontinuity on $x=2$

c) $f(x) = \begin{cases} 3x^2+5 & , x \neq 1 \\ 6, & x=1 \end{cases}$



$f(1) = 6$

(i) $f(x)$ define at $x=1$

(ii) $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x)$

$6 = 3x^2+5$

$6 \neq 8$

$\boxed{f(1) \neq 8}$

Removable discontinuity at $x=1$.

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