Practice Questions:

Question no 1: Solve the given questions by Heun's and Mid point Method.

a. $y' = te^{3t} - 2y, \quad 0 \le t \le 1, \quad y(0) = 0,$ with $h = 0.5;$ actual solution $y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}.$

b. $y' = 1 + (t - y)^2, \quad 2 \le t \le 3, \quad y(2) = 1,$ with $h = 0.5;$ actual solution $y(t) = t + \frac{1}{1-t}.$

c. $y' = 1 + y/t, \quad 1 \le t \le 2, \quad y(1) = 2,$ with $h = 0.25;$ actual solution $y(t) = t \ln t + 2t.$

d. $y' = \cos 2t + \sin 3t, \quad 0 \le t \le 1, \quad y(0) = 1,$ with $h = 0.25;$ actual solution $y(t) = \frac{1}{2}\sin 2t - \frac{1}{3}\cos 3t + \frac{4}{3}.$

Question no 2: Solve the given questions by Heun's and Mid point Method. Compare the results of both methods with actual values.

a. $y' = e^{t-y}, \quad 0 \le t \le 1, \quad y(0) = 1,$ with $h = 0.5;$ actual solution $y(t) = \ln(e^t + e - 1).$

b. $y' = \dfrac{1+t}{1+y}, \quad 1 \le t \le 2, \quad y(1) = 2,$ with $h = 0.5;$ actual solution $y(t) = \sqrt{t^2 + 2t + 6} - 1.$

c. $y' = -y + ty^{1/2}, \quad 2 \le t \le 3, \quad y(2) = 2,$ with $h = 0.25;$ actual solution $y(t) = \left(t - 2 + \sqrt{2}ee^{-t/2}\right)^2.$

d. $y' = t^{-2}(\sin 2t - 2ty), \quad 1 \le t \le 2, \quad y(1) = 2,$ with $h = 0.25;$ actual solution $y(t) = \frac{1}{2}t^{-2}(4 + \cos 2 - \cos 2t).$

Question no 3: Solve the given questions by Heun's and Mid point Method. Compare the results of both methods with actual values.

a. $y' = y/t - (y/t)^2, \quad 1 \le t \le 2, \quad y(1) = 1,$ with $h = 0.1;$ actual solution $y(t) = t/(1 + \ln t).$

b. $y' = 1 + y/t + (y/t)^2, \quad 1 \le t \le 3, \quad y(1) = 0,$ with $h = 0.2;$ actual solution $y(t) = t\tan(\ln t).$

c. $y' = -(y + 1)(y + 3), \quad 0 \le t \le 2, \quad y(0) = -2,$ with $h = 0.2;$ actual solution $y(t) = -3 + 2(1 + e^{-2t})^{-1}.$

d. $y' = -5y + 5t^2 + 2t, \quad 0 \le t \le 1, \quad y(0) = \frac{1}{3},$ with $h = 0.1;$ actual solution $y(t) = t^2 + \frac{1}{3}e^{-5t}.$

Question no 4: Solve the given questions by Heun's and Mid point Method. Compare the results of both methods with actual values.

a. $y' = \dfrac{2 - 2ty}{t^2 + 1}, \quad 0 \le t \le 1, \quad y(0) = 1, \text{ with } h = 0.1; \text{ actual solution } y(t) = \dfrac{2t + 1}{t^2 + 1}.$

b. $y' = \dfrac{y^2}{1 + t}, \quad 1 \le t \le 2, \quad y(1) = -(\ln 2)^{-1}, \text{ with } h = 0.1; \text{ actual solution } y(t) = \dfrac{-1}{\ln(t + 1)}.$

c. $y' = (y^2 + y)/t, \quad 1 \le t \le 3, \quad y(1) = -2, \text{ with } h = 0.2; \text{ actual solution } y(t) = \dfrac{2t}{1 - 2t}.$

d. $y' = -ty + 4t/y, \quad 0 \le t \le 1, \quad y(0) = 1, \text{ with } h = 0.1; \text{ actual solution } y(t) = \sqrt{4 - 3e^{-t^2}}.$

Question no 5: Repeat Question no 1 by using RK4 method.

For Solution of above questions:

Midpoint Method

https://www.emathhelp.net/calculators/differential-equations/modified-euler-method-calculator/

Heun's Method

https://www.emathhelp.net/calculators/differential-equations/improved-euler-heun-calculator/

RK-4 Method

https://www.emathhelp.net/calculators/differential-equations/fourth-order-runge-kutta-method-calculator/