Probability: Approximate percent: ⇒ Emperical
Minimum percent: ⇒ Chebyshev

Chebyshev's Theorem:

$\boxed{\bar{x} \pm k\sigma}$ $\boxed{1 - 1/k^2}$ ⇒ For percentage

Ex: 3.57

$\bar{x} = 230$, $\sigma = 41$

For:

$\boxed{\mu \pm 2\sigma}$ ⇒ $230 \pm 2 \times 41$

$= 230 \pm 82 = (148, 312)$

$1 - 1/k^2 = 1 - 1/4 = 75\%$

For

$\mu \pm 2.5\sigma$ ⇒ $230 \pm 2.5 \times 41$

$= 23 \pm 102.5 = (127.5, 332.5)$

$1 - 1/(2.5)^2 = 84\%$

For

$\mu \pm 3\sigma$ ⇒ $230 \pm 3 \times 41$

$= 230 \pm 123 = (107, 353)$

$= 1 - 1/9 = 88.9\%$

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Empirical Formula

- $\mu \pm 1\sigma = 68\%$
- $\mu \pm 2\sigma = 95\%$
- $\mu \pm 3\sigma = 99.7\%$

Ex: 3.59

$\bar{x} = 82, \sigma = 16$

$\bar{x} \pm 1s, \bar{x} \pm 2s$ & $\bar{x} \pm 3s$

For: $\bar{x} \pm 1s$

$82 \pm 16 = (66, 98) \Rightarrow 68\%$

For $\bar{x} \pm 2\sigma$

$82 \pm 32 = (50, 114) \Rightarrow 95\%$

For $\bar{x} \pm 3\sigma$

$82 \pm 48 = (34, 130) \Rightarrow 99.7\%$

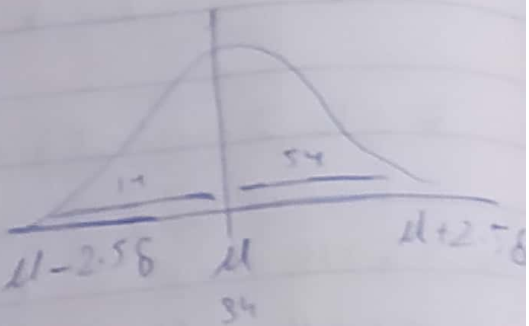
Ex-XI

$\bar{x} = 34$, $\sigma = 8$

a) [illegible] mean

i) 14 to 54

$$k = \frac{54-34}{8} = 2.5$$



$$1 - \frac{1}{2.5^2} = 84\%$$

ii) 18 to 50

$$k = \frac{50-34}{8} = 2$$

$$1 - \frac{1}{4} = 75\%$$

b)

$$1 - \frac{1}{k^2} = 89\% = 0.89$$

$$1 - 0.89 = \frac{1}{k^2}$$

$$0.11 = \frac{1}{k^2}$$

$$k^2 = \frac{1}{0.11}$$

$$k^2 = 9.09$$

$$k = 3.03$$

$34 \pm 3 \times 8$

34 ± 24

$(10, 58)$

Ex 3.63

$\bar{x} = 34$, $s = 8$

a) 10 to 58

$k = \frac{58-34}{8} = 3$

$\boxed{\mu \pm 3s}$ => 99.7%

$k = \frac{50-34}{8} = \frac{16}{8} = 2$

$\boxed{\mu \pm 2s}$ => 95%

(b) 26 to 42

$k = \frac{42-34}{8} = 1$

$\boxed{\mu \pm 1s}$ => 68%

Ex 3.63

$\bar{x} = 34$, $\delta = 8$

a) 10 to 58

$k = \frac{58-34}{8} = 3$

$\boxed{\mu \pm 3\delta} \Rightarrow 99.7\%$

$k = \frac{50-34}{8} = \frac{16}{8} = 2$

$\boxed{\mu \pm 2\delta} \Rightarrow 95\%$

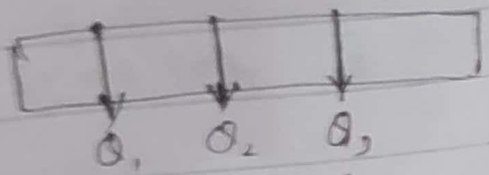
(b) 26 to 42

$k = \frac{42-34}{8} = 1$

$\boxed{\mu \pm 1\delta} \Rightarrow 68\%$

: 3.5

rtiles & Interquartile;



$IQR = (Q_3 - Q_1)$

~~Factor = 1.5~~ × IQR

Lower Inner Factor = Q_1 - Factor

~~Upper Outer Inner Factor~~ = Q_3 - Factor

Partile

~~Percentiles~~ 70th

$P_k = \text{Value}\left[\frac{k*n}{100}\right] = [\sim]$

Percentile rank $\text{Value}\left[\frac{k*n}{100}\right] = \frac{\text{No. of Values less than Value}\left[\frac{k*n}{100}\right]}{\text{Total values}} \times 100$

Ex 13.69

~~68 71 72 73~~

68 69.69 71 72 73 74 75 76 77
78 79

a) $Q_2 = 73$

$Q_1 = 69$

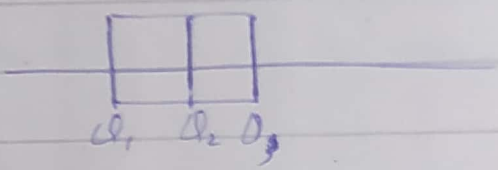
$Q_3 = 76.5$

$\left(\frac{35 \times 13}{100}\right) = 4.55$

b) $P_{(35)} = \text{Value}_{(4.55)} = \text{Value}_{(5)} = 71.$

c) Percentile Rank $= \frac{4}{13} \times 100 = 30.7\%$

Box & Whisker :-



$1.5 \times IQR = F_1$

$3 \times IQR = F_2$

Lower Inner :- $= Q_1 - F_1$

Upper Inner $= Q_3 + F_1$

Lower Outer $= Q_1 - F_2$

Upper ~~Inner~~ Outer $= Q_3 + F_2$

$Q_2 = 94$

$IQR = 197$

$f_1 = 1.5 \times IQR = 295.5$

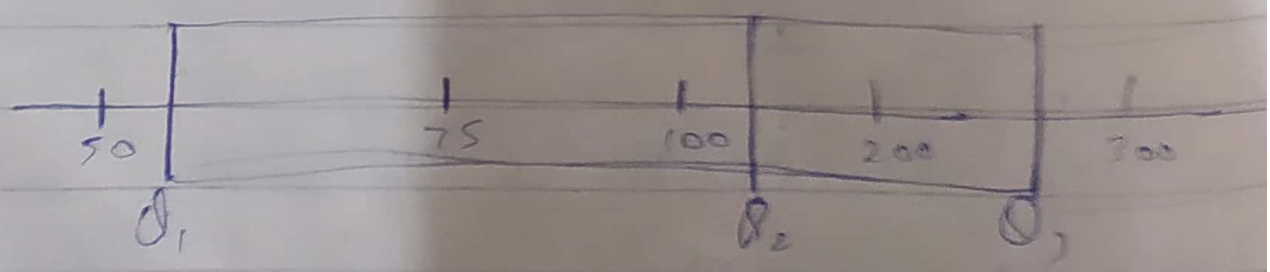
$f_2 = 3 \times IQR = 591$

Lower Inner = -209.5

Upper Inner = ~~22~~ 553.5

Lower Outer = -490

Upper Outer = 819



* 731 is mild outlier

Bayes Theorem :

$$P(\bar{B}/A) = \frac{P(A/\bar{B}) \cdot P(\bar{B})}{P(A/B)\, P(B) + P(A/\bar{B}) \cdot P(\bar{B})}$$

Ex:04

$P(Sm) = 0.1$

$P(NoSm) = 0.9$

$P(die|Sm) = 0.05$

$P(die/NonSm) = 0.005$

$P(Smo/Die) = ?$

$$P(Smo/Die) = \frac{0.05 \times 0.1}{\times 0.1 + 0.005 \times 0.9}$$

$$P(Smo/Die) = \frac{0.05 \times 0.1}{0.05 \times 0.1 + 0.005 \times 0.9}$$

$$= \frac{0.005}{0.0095}$$

$P(Smo/Die) = \cancel{0.263}\ 0.5263$

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Ex: 3

$P(B) = 0.5$

$P(\bar{B}) = 0.5$

$P(A/B) = 0.99$

$P(A/\bar{B}) = 0.05$

$$P(\bar{B}/A) = \frac{0.05 \times 0.5}{0.05 \times 0.5 + 0.99 \times 0.5}$$

$$= \frac{0.025}{0.52}$$

$P(\bar{B}/A) = 0.048076$

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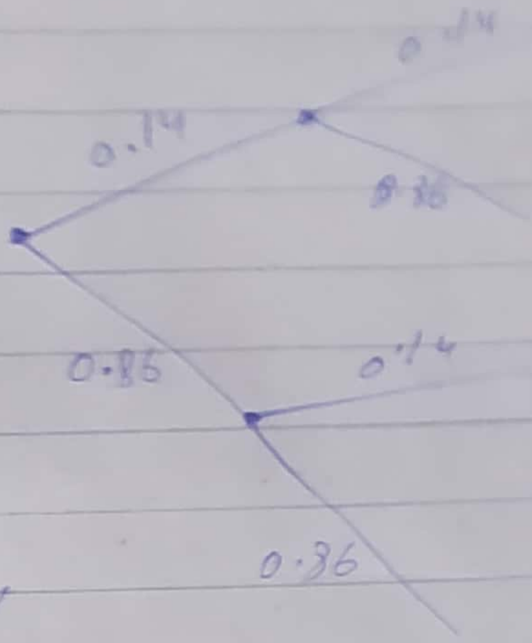
Probability for discrete random variable :



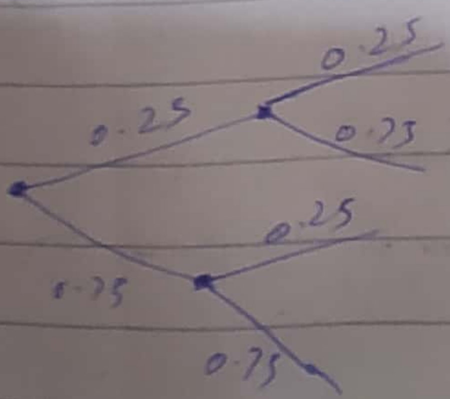
Ex: 5.4

x	P(x)
0	$(0.86)^2$
1	$(0.86)(0.14) \times 2$
2	$(0.14)^2$

~~For~~ x = 0

Ex: 5.4

Left handed = 3/12 2023



x	P(x)
0	$(0.75)^2$
1	$(0.25)(0.75)2$
2	$(0.25)^2$

Mean & Std devn

$$\mu = \Sigma x \cdot P(x)$$

$$\delta = \sqrt{\Sigma x^2 \cdot P(x) - \mu^2}$$

Ex: 5.21

x	0	1

$$x = 2.561$$

$$\delta = \sqrt{8.3076 - (2.561)^2}$$

$$= \sqrt{8.3076 - 6.5587}$$

$$\delta = \sqrt{1.74879}$$

$$\delta = 1.322$$

Joint Probability:

Ex 13.3)

$f(x,y) = cxy$ for $x = 1,2,3;\ y = 1,2,3$

$f(x,y)$		x		
		1	2	3
y	1	c	$2c$	$3c$
	2	$2c$	$4c$	$6c$
	3	$3c$	$6c$	$9c$

$$\sum\sum f(x,y) = 1$$

$$36c = 1$$

$$\boxed{c = 1/36}$$

(b) $f(x,y) = c|x-y|$ for $x = -2,0,2;\ y = -2,3$

		x		
$f(x,y)$		-2	0	2
	-2	$0c$	$2c$	$4c$
y	3	$5c$	$3c$	$1c$

$$\sum\sum f(x,y) = 1$$

$$15c = 1$$

$$\boxed{c = 1/15}$$

Ex : 3.38

$f(x,y) = \frac{x+y}{30}$, for $x = 0,1,2,3$; $y = 0,1,2$

$f(x,y)$	0	1	2	3
0	0	1/30	1/15	1/10
1	1/30	1/15	1/10	2/15
2	1/15	1/10	2/15	1/6

a) $P(X \leq 2, Y = 1) = 1/30 + 1/15 + 1/10$

$= 0.2$

b) $P(X > 2, Y \leq 1) = 1/10 + 2/15 = 0.2333$

c) $P(X > Y) = 1/30 + 1/15 + 1/10 + 1/10 + 2/15 + 1/6$

$= 0.6$

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d) $P(x+y=4) = \frac{2}{15} + \frac{2}{15} = 0.2667$

Ex: 3.39

$$\overset{x}{3 \text{ oranges}}, \overset{y}{2 \text{ apples}}, 3 \text{ bananas}$$

$$f(x,y) = \frac{{}^{3}C_{x} \cdot {}^{2}C_{y} \cdot {}^{3}C_{4-y-x}}{{}^{8}C_{4}}$$

$f(x,y)$	0	1	2	3
0	0	3/70	9/70	3/70
1	1/35	9/35	9/35	1/35
2	3/70	9/70	3/70	0

b) $P(x+y \le 2) = 0 + 3/70 + 9/70 + 1/35 + 9/35 + 3/70$

$= 0.5$

Marginal Distribution:-

$f(x,y)$	x: 2	4	$h(y)$
y: 1	0.1	0.15	0.25
3	0.2	0.3	0.5
3	0.1	0.15	0.25
$g(x)$	0.4	0.6	

Mean & Variance

Ex: 4.10

$f(x,y)$	y: 1	2	3	$g(x)$
x: 1	0.1	0.05	0.02	0.17
2	0.1	0.35	0.05	0.5
3	0.03	0.1	0.2	0.33
$h(y)$	0.23	0.5	0.27	

$$\mu_x = \sum x \cdot g(x)$$

$$\mu_x = 1\times0.17 + 2\times0.5 + 3\times0.33$$

$$\mu_x = 2.16$$

$\mu_Y = \sum y \cdot f(y)$

$= 1 \times 0.23 + 2 \times 0.5 + 3 \times 0.27$

$\mu_Y = 2.04$

Ex: 4.17

x	-3	6	9
$f(x)$	$1/6$	$1/2$	$1/3$

$\mu_{g(x)} = ?$ $g(x) = (2x+1)^2$

x	-3	6	9
$g(x)$	25	169	361

$\mu_{g(x)} = \sum g(x) f(x)$

$= 25 \times 1/6 + 169 \times 1/2 + 1/3 \times 361$

$\mu_{g(x)} = 209$

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(Ex: 4.2)

$f(x,y)$		x: 2	4	$h(y)$
	1	0.1	0.15	0.25
y	3	0.2	0.3	0.5
	5	0.1	0.15	0.25
	$g(x)$	0.4	0.6	

a) $g(x,y) = xy^2$ $\quad E[g(x,y)] = ?$

$$E(g(x,y)) = 2\times0.1 + 4\times0.15 + 18\times0.2 + 36\times0.3 + 50\times0.1 + 100\times0.15$$
$$= 35.2$$

b) μ_x, μ_y

$$\mu_x = \sum g(x)\cdot x$$
$$= 2\times0.4 + 4\times0.6$$
$$\mu_x = 3.2$$

$$\mu_y = \sum h(y)\cdot y$$
$$= 1\times0.25 + 3\times0.5 + 5\times0.25$$
$$\mu_y = 3$$

$$\boxed{\delta^2 = E(x^2) - \mu^2}$$

Ex: 4.41

x	-3	6	9
$f(x)$	$1/6$	$1/2$	$1/3$

$g(x) = (2x+1)^2$

$E(g(x)) = 25 \times 1/6 + 169 \times 1/2 + 361 \times 1/3$

$E(g(x)) = \cancel{208.5}\ 209$

$\delta^2_{g(x)} = \Sigma E(g(x)^2) - E(g(x))^2$

$\delta^2_{g(x)} = ((25)^2 \times 1/6 + (169)^2 \times 1/2 + (361)^2 \times 1/3) - (209)^2$

$\delta^2_{g(x)} = 14144$

$\delta_{g(x)} = \sqrt{14144}$

$\delta_{g(x)} = 118.9$

Ex: 4.44

$f(x,y)$		x = 0	1	2	3	$h(y)$
y	0	0	1/30	1/15	1/10	1/5
	1	1/30	1/15	1/10	2/15	1/3
	2	1/15	1/10	2/15	1/6	7/15
$g(x)$		1/10	1/3	3/10	2/5	

$\sigma_{xy} = E(XY) - \mu_x \mu_y$

$\sigma_x = ?$, $\sigma_y = ?$

$\rho_{xy} = \dfrac{\sigma_{xy}}{\sqrt{\sigma_x \sigma_y}}$

$E(x) = \sum x \cdot g(x)$

$= 1/3 + 2 \times 3/10 + 3 \times 2/5$

$E(x) = 2$

$E(x^2) = \sum x^2 g(x)$

$= 1/3 + 4 \times 3/10 + 9 \times 2/5$

$= 5$

$\sigma_x^2 = 5 - 4$

$\sigma_x = \sqrt{1}$

$\boxed{\sigma_x = 1.}$

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$\mu_Y = \sum y \cdot h(y)$

$= 1 \times 1/3 + 2 \times 7/15$

$\mu_Y = 19/15$

$E(XY) = 1/15 + 2/10 + 3 \times 2/15 + 2 \times 1/10 + 4 \times 2/15 + 6 \times 1/6$

$E(XY) = 2.4$

$\delta_{XY} = 2.4 - (2.5)^2$

$= 2.4 - 2.5$

$\delta_{XY} = -0.1$