Chap: 4 Probability ::

- The set of all possible outcomes of a statistical experiment is called the sample space denoted by S.

- Each outcome in a sample space is called an element or member or a sample point.
- An event is a subset of a sample space.
- Two events A and B are mutually exclusive or disjoint if $A \cap B = \emptyset$

- A permutation is an arrangement of all or part of a set of objects.

- The number of permutation of n objects is $n!$.
- The number of permutations on n distinct objects taken r at a time is $^nP_r = \dfrac{n!}{(n-r)!}$
- The number of permutations of n objects arranged is $(n-1)!$.
- The number of distinct permutations of n things of which n_1 is of one kind, n_2 is of second kind, ... n_k of k^n kind is: $\dfrac{n!}{n_1!\, n_2! \cdots n_k!}$
- The number of combinations of n distinct objects taken r at a time is

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = {}^nC_r$$

- Addition Rule:
  $$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

- If A & B are mutually exclusive:
  $$P(A\cup B) = P(A) + P(B)$$
- Conditional Probability: $P(A|B) = \dfrac{P(A\cap B)}{P(B)}$; $P(B) > 0$

- Two events A & B are independent if and only if
  $P(B|A) = P(B)$ or $P(A\cap B) = P(A) \times P(B)$

- If an experiment the events A and B can both occur then; $P(A\cap B) = P(A) \cdot P(B|A)$ or
  $P(B\cap A) = P(B) \cdot P(A|B)$

<u>Baye's Rule:</u>
- Suppose that 36 of those employed and 12 of those unemployed are members of the Rotary Club. We wish to find the probability of the event A, that the individual selected is a member of the Rotary club.

<u>E. in</u> Event E is the event person is employed;

- $P(A) = P(E\cap A) + P(E'\cap A)$
  $= \cancel{P(E) \cdot P(E|A)}$ $P(E) \cdot P(A|E) + P(E') \cdot P(A|E')$

- Rule of Elimination or Theorem of Total Probability:

- If the events $B_1, B_2, \ldots B_K$ constitute a partition of the sample space S such that $P(B_i) \neq 0$ for $i = 1, 2, \ldots K$ then for any event A of S.

$$P(A) = \sum_{i=1}^{K} P(B_i \cap A) = \sum_{i=1}^{K} P(B_i) \cdot P(A|B_i)$$

Baye's Rule:-

If the events $B_1, B_2, \ldots, B_K$ constitute a partion of the sample space S such that $P(B_i) \neq 0$; for $i = 1, 2, 3 \ldots K$ then for any event A in S such that $P(A) \neq 0$

$$P(B_r|A) = \frac{P(B_r \cap A)}{P(A)} \quad ; \text{for } r = 1, 2, \ldots K$$

$$= \frac{P(B_r) \cdot P(A|B_r)}{\sum_{i=1}^{K} P(B_i) \cdot P(A|B_i)}$$

Probability and Stats

Chap: Probability Distribution ::

1) Discrete Random Variable or Probability Mass function:

i) $f(x) \geq 0$; $\Sigma f(x) = 1$; $P(X=x) = f(x)$

ii) $0 \leq P(x) \leq 1$

A random variable is a variable whose value is determined by the outcome of a random sample.

A discrete random variable assumes countable values.

The probability distribution of a discrete random variable lists all possible values that the random variable can assume and their corresponding probabilities.

Mean or Expected value ($E(X)$ or μ) $= \Sigma x \cdot P(x)$

Standard deviation $(\sigma) = \sqrt{\Sigma x^2 \cdot P(X) - \mu^2}$

II) Continuous Random Variable or Probability density function::

i) $f(x) \geq 0$; $\int_{-\infty}^{\infty} f(x) = 1$; $P(a < x < b) = \int_{a}^{b} f(x) \cdot dx$

- A random variable that can assume any value contained in one or more intervals.
- Mean or Expected ($E(X)$ or μ) $= \int_{-\infty}^{\infty} x \cdot f(x)\, dx$
- S.d. $(\sigma) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$;

Q) 3) If $f(x,y) = \begin{cases} K(x^2+y^2) & ; 30 \le x \le 50 ;\ 30 \le y \le 50 \\ 0 & ; \text{elsewhere} \end{cases}$

i) find K if $f(x,y)$ is in pdf
ii) find $P(30 \le x < 40)$ & $P(40 \le y \le 50)$

Q1) Two ~~balls~~ pen are randomly from a bag that contain 3 blue pen & 2 red pen and 3 green pen. If X is the no. of blue pen selected; and y is the no. of red pen selected; find,

i) joint propability $f(x,y)$;
ii) $P(X,Y) \in A$; when A is the region $\{(x,y) \mid x+y \le 1\}$

Q2) From a sack of fruit's contains 3 oranges, 2 Apples, 3 banana and a random sample of 4 pieces is selected. X is the number of oranges; Y is the number of apples selected finds.

i) joint propability $f(x,y)$;
ii) $P(x+y \le 2)$

Q2)

i)

Y \ X	0	1	2	3	4
0	0	$\frac{3}{70}$	$\frac{7}{70}$	$3/70$	0
1	0	$10/70$	$\frac{10}{70}$	$2/70$	
2		$\frac{2}{70}$	$\frac{3}{70}$	$10/70$	0
3	0	0	0	0	0
4	0	0	0	0	0

$$\frac{{}^3C_x \cdot {}^2C_y \cdot {}^3C_{4-x-y}}{{}^6C_4}$$

4) $P(x+y<2) = P(0,1) + P(0,2) + P(1,1)$

$= \frac{3}{7}$

Q3) i) $\int_{30}^{50}\int_{30}^{50} K(x^2+y^2)\, dx\, dy = 1$

$= \int_{30}^{50} \frac{Kx^3}{3} + y^2x \Big|_{30}^{50} dy = 1$

$\int_{30}^{50} \left[\frac{K}{3}(50)^3 + 50y^2\right] - \left(\frac{30^3K}{3} + 30y^2\right) dy = 1$

$1 = \int_{30}^{50} \frac{125000K - 27000K}{3} + 20y^2\, dy =$

$1 = \frac{98000Ky}{3} + \frac{20y^3}{3}\Big|_{30}^{50}$

$1 = \frac{196000}{3}K + 1960000$

$K =$

Q3) i) $\int_{30}^{50}\int_{30}^{50} K(x^2+y^2)\, dy \cdot dx$

$= \int_{30}^{50} \frac{Kx^3}{3} + Ky^2x \Big|_{30}^{50} dy$

$= \int_{30}^{50} \frac{196000K}{3} + 20Ky^2\, dy$

$= \frac{196000Ky}{3} + \frac{20}{3}Ky^3 \Big|_{30}^{50} = 1$

$= \frac{196000(20K)}{3} + \frac{20K(50^3 - 30^3)}{3} = 1$

$\Rightarrow \boxed{K = \frac{1}{1960000}}$

$$f(x,y) = \begin{cases} \frac{1}{1960000}(x^2+y^2) & ; 30 \le x \le 50 ; 30 \le y \le 50 \\ 0 & ; \text{elsewhere} \end{cases}$$

ii) $P(30 \le x \le 40) + P(40 \le y \le 50) = \int_{40}^{50}\int_{30}^{40} \frac{1}{1960000}(x^2+y^2)\,dx\,dy$

$$= \frac{1}{1960000}\int_{40}^{50} \frac{x^3}{3} + y^2x \Big|_{30}^{40} dy$$

$$= \frac{1}{196000}\left[\frac{37000y}{3} + \frac{10y^3}{3}\right]_{40}^{50}$$

$$= \cancel{\frac{1}{196000}} = \boxed{\frac{1}{6}} \text{ Ans}$$

Q1) i)

X\Y	0	1	2
0	3/28	6/28	4/28
1	9/28	6/14	-
2	3/28	-	-

$$P(X=x, Y=y) = \frac{{}^3C_x \cdot {}^2C_y \cdot {}^3C_{2-x-y}}{{}^8C_2}$$

ii) $P(X,Y) \in A = P(0,0) + P(0,1) + P(1,0)$

$$= \frac{18}{28}$$

$$= \boxed{\frac{9}{14}} \text{ Ans}$$

Probability & Stats

Topic:: Commulative distribution function (Discrete):

$\cdot F(x) = P(X \leq x) = \sum_{t=0}^{m} f(t)$ for $-\infty < x < \infty$

Cummulative distribution function (continous):

$\cdot F(x) = \int_{-\infty}^{x} f(t)\,dt = P(X \leq x)$; $-\infty < x < \infty$

$\cdot P(a < x < b) = F(b) - F(a)$

$f(x) = \dfrac{dF(x)}{dx}$

Q) Find cummulative function:

i)

x	0	1	2	3	4
$f(x)$	1/16	4/16	6/16	4/16	1/16

$F(0) = 1/16$

$F(1) = F(0) + F(1) = 5/16$

$$F(x) = \begin{cases} 0 & ; x \leq 0 \\ 5/16 & ; 1 \leq x \leq 2 \\ 1/16 & ; 0 \leq x < 1 \\ 11/16 & ; 2 \leq x < 3 \\ 15/16 & ; 3 \leq x < 4 \\ 1 & ; x \geq 4 \end{cases}$$

ii) $f(x) = \begin{cases} x^2/3 & -1 < x < 2 \\ 0 & ; \text{elsewhere} \end{cases}$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

$$= \int_{-1}^{x} \frac{t^3}{3}\,dt = \frac{x^3+1}{9}$$

$$F(x) = \begin{cases} 0 & ; x < -1 \\ \frac{x^3+1}{9} & ; -1 < x < 2 \\ 1 & ; x \geq 2 \end{cases}$$

Q) find

a) $P(T=5)$; b) $P(T>3)$, c) $P(1.4 < T < 6)$; d) $P(T \leq 5 \text{ of } T \geq 2)$

$$F(T) = \begin{cases} 0 & ; t < 1 \\ 1/4 & ; 1 \leq t < 3 \\ 1/2 & ; 3 \leq t < 5 \\ 3/4 & ; 5 \leq t < 7 \\ 1 & t \geq 7 \end{cases}$$

a) $P(T=5) = P(T \leq 5) - P(T<5)$

$\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

b) $P(T>3) = 1 - F(3)$

c) $P(1.4 < T < 6) = F(6) - F(1.4)$

$3/4 - 1/4$

$= \frac{1}{2}$

d) $\frac{P(2 \leq T \leq 5)}{P(T \geq 2)} = \frac{\frac{1}{2}}{3/4}$

$= \frac{2}{3}$

Q) find $F(x)$

x	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

$$F(x) = \begin{cases} 0 & ; x < 0 \\ 0.78 & ; 0 \le x < 1 \\ 0.94 & 0 \le x \le 2 \\ 0.99 & 0 \le x \le 3 \\ 1 & 0 \le x < 4 \\ 1 & ; x \ge 4 \end{cases}$$

Q) $F(x)$; $f(x) = \begin{cases} 3x^{-4} & x > 1 \\ 0 & ; \text{elsewhere} \end{cases}$

$$F(x) = \int_1^x 3t^{-4}\, dt$$

$$F(t) = -t^{-3}$$

$$F(t) = \begin{cases} 0 & ; x \le 1 \\ 1 - x^{-3} & ; x > 1 \end{cases}$$