Course Code:MT1004	Course Name: Linear Algebra
Instructor Names: Ms. Amber Shaikh & Ms. Alishba Tariq	
Student Roll No:	Section No:

Instructions:
- Return the question paper.
- Read each question completely before answering it. **There are 05 Questions and 02 pages.**
- In case of any ambiguity, you may make assumption. But your assumption should not contradict any statement in the question paper.
- Graphical Calculator is not allowed.

Time: 180 minutes   **Max Marks:50**

Question 01: CLO 2   **[5+5=10]**

a. Determine whether each of the following set is a basis for P_2. Also Find the coordinates of $p(t) = 6 - 5t + 2t^2$ relative to that basis.

$$\{1, -1 + t, 1 - 2t + t^2\}$$

b. Find the **QR-Decomposition** of the matrix **A**

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

Question 02: CLO 2   **[3+5+2=10]**

Let

$$A = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 3 \\ 0 & 3 & 3 \end{bmatrix}$$

a. Find the characteristics equation of A.
b. Find the eigenvalues of A and their corresponding eigenvectors.
c. Is A diagonalizable? Explain.

a. Find A^{10}, where

$$A = \begin{bmatrix} 4 & 3 & 0 & 0 \\ 3 & -4 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

b. If A is 5×8 matrix and $\mathbf{dim}(Null\ A) = 6$, Find:
  i. $\dim(row\ A) =$
  ii. $\dim(Col\ A) =$
  iii.   Rank $\mathbf{A} =$

a. Find the ranks of the following real matrix for all possible values of a.

$$\begin{bmatrix} a & 1 & 2 \\ 1 & 1 & 1 \\ -1 & 1 & 1-a \end{bmatrix} \quad \text{Where } a \text{ is a real no.}$$

b. Determine the basis for the solution space of the system, and find the dimension of the solution space.

$$2x_1 + 5x_2 + x_3 = 0$$
$$x_1 + 3x_2 + 2x_3 = 0$$
$$3x_1 + 4x_2 - 9x_3 = 0$$

a. Show that the following matrix is orthogonal matrix.

$$\begin{bmatrix} \dfrac{1}{3} & \dfrac{2}{3} & \dfrac{2}{3} \\ \dfrac{2}{3} & \dfrac{1}{3} & \dfrac{-2}{3} \\ \dfrac{2}{3} & \dfrac{-2}{3} & \dfrac{1}{3} \end{bmatrix}$$

b. Find a symmetric 3×3 matrix whose eigenvalues are $8, -1\ and -1$ with corresponding Eigen vectors $(2,1,2), (-1,0,1)\ and\ (-1,2,0)$.

c. Use the change of variables to eliminate the cross product term in the following quadratics form

$$Q(x) = 4x_1^2 + 4x_2^2 + 14x_1x_2$$