Probability & Statistics Assignment #2

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K21-3309 , BCS-4F

Q1)

Net Profit	Probability
-5,000	0.2
10,000	0.5
30,000	0.3

$$\mu = E(x) = \sum_x x f(x)$$

$$= -5000 \times 0.2 + 10000 \times 0.5 + 30000 \times 0.3$$

$$= 13,000$$

Q2) $P(x) = \frac{5-x}{10}$, $X = 1, 2, 3, 4$

X	P(X)
1	0.4
2	0.3
3	0.2
4	0.1

$$\mu = E(x) = \sum_x x f(x)$$

$$= 0.4 + 0.6 + 0.6 + 0.4 = 2$$

$$\sigma^2 = E(x^2) - \mu^2$$

$$= \sum_x x^2 f(x) - 2^2$$

$$= 0.4 + 1.2 + 1.8 + 1.6 - 4$$

$$= 1$$

Q3)

X	P(X)
20	0.2
21	0.4
22	0.2
23	0.1
24	0.1

$$\mu = E(x) = \sum_x x f(x)$$

$$= 4 + 8.4 + 4.4 + 2.3 + 2.4 = 21.5$$

$$\sigma^2 = \sum_x x^2 f(x) - \mu^2$$

$$= 80 + 176.4 + 96.8 + 52.9 + 57.6 - 21.5^2$$

$$= 1.45$$

$$\sigma = \sqrt{1.45} = 1.2042$$

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Q4)

X	P(X)
0	0.5
10	0.25
20	0.15
30	0.1

i) $\mu = E(x) = \sum_x x f(x) = 0 + 2.5 + 3 + 3 = 8.5$

ii) The expected value represents the average payoffs per project

iii) $\sigma^2 = E(x^2) - \mu^2 = 0 + 25 + 60 + 90 - (8.5)^2$

$= 102.75$

iv) $P(X \geq 20) = 0.15 + 0.1 = 0.25$

Q5)i) $g(x) = \int_0^\infty xe^{-x(1+y)}\,dy = -e^{-x(1+y)}\Big]_0^\infty = e^{-x}$

ii) $h(y) = \int_0^\infty xe^{-x(1+y)}\,dx = \frac{xe^{-x(1+y)}}{-(1+y)}\Big]_0^\infty - \int_0^\infty \frac{e^{-x(1+y)}}{-(1+y)}\,dx$

$= \frac{xe^{-x(1+y)}}{-(1+y)}\Big]_0^\infty - \frac{e^{-x(1+y)}}{(1+y)^2}\Big]_0^\infty$

$= -e^{-x(1+y)}\frac{(x+xy+1)}{(1+y)^2}\Big]_0^\infty$

$= \lim_{x\to\infty}\left[\frac{-e^{-x(1+y)}(x+xy+1)}{(1+y)^2}\right] + \frac{e^0(0+0+1)}{(1+y)^2}$

Using L'hôpital Rule

$= \lim_{x\to\infty}\left[\frac{-y-1}{x(y+1)^3 e^{x(y+1)}}\right] + \frac{1}{(1+y)^2} = \frac{1}{(1+y)^2}$

$h(y) = \frac{1}{(1+y)^2}$

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iii) $P(X>1 \text{ or } Y>1) = 1 - P(X \leq 1 \text{ and } Y \leq 1)$

$$P(X \leq 1 \text{ and } Y \leq 1) = \int_0^1 \int_0^1 xe^{-x(1+y)}\, dy\, dx$$

$$= \int_0^1 \left[-e^{-x(1+y)}\right]_0^1 dx = \int_0^1 \left(-e^{-x(1+1)} + e^{-x(1+0)}\right) dx$$

$$= \int_0^1 \left(-e^{-2x} + e^{-x}\right) dx = \left[\frac{e^{-2x}}{2} - e^{-x}\right]_0^1$$

$$= \left[\frac{1}{2e^{2x}} - \frac{1}{e^x}\right]_0^1 = \left[\frac{1-2e^x}{2e^{2x}}\right]_0^1$$

$$= \frac{1-2e}{2e^2} - \left[\frac{1-2}{2}\right]$$

$$= \frac{1-2e}{2e^2} + \frac{1}{2}$$

$$= \frac{1-2e+e^2}{2e^2} = 0.199788$$

$P(X>1 \text{ or } Y>1) = 1 - 0.199788$

$= 0.800212$

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Q6)

$f(x,y)$	x				
y	0	1	2	3	$h(y)$
0	0.15	0.30	0.05	0	0.50
1	0.05	0.15	0.05	0.05	0.30
2	0	0.05	0.10	0.05	0.20
$g(x)$	0.20	0.50	0.20	0.10	1.00

i) $P(Y>X) = f(0,1) + f(0,2) + f(1,2)$

$= 0.05 + 0 + 0.05 = 0.10$

ii) $\sigma_{xy} = E[(X-\mu_x)(Y-\mu_y)] = \sum\sum (x-\mu_x)(y-\mu_y) f(x,y)$

$= E(xy) - \mu_x \mu_y$

$\mu_x = E(x) = \sum x g(x) = 0 + 0.50 + 0.40 + 0.30 = 1.20$

$\mu_y = E(y) = \sum y h(y) = 0 + 0.30 + 0.40 = 0.70$

$\Rightarrow \sigma_{xy} = E(xy) - 1.2 \times 0.7$

$E(xy) = \sum\sum xy f(x,y)$

$= 1\cdot1\cdot f(1,1) + 1\cdot2\cdot f(1,2) + 2\cdot1\cdot f(2,1) + 2\cdot2\cdot f(2,2) + 3\cdot1\cdot f(3,1) + 3\cdot2\cdot f(3,2)$

$= 0.15 + 2\times0.05 + 2\times0.05 + 4\times0.1 + 3\times0.05 + 6\times0.05$

$= 1.2$

$\Rightarrow \sigma_{xy} = 1.2 - 1.2\times0.7 = 0.36$

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Q7)

X	P(X)
0	${}^5C_0 (0.7)^0 (0.3)^5 = 0.0024$
1	${}^5C_1 (0.7)^1 (0.3)^4 = 0.02835$
2	${}^5C_2 (0.7)^2 (0.3)^3 = 0.1323$
3	${}^5C_3 (0.7)^3 (0.3)^2 = 0.3087$

Q8) $\mu = np = 2$, $\sigma^2 = npq = 1$

$\Rightarrow q = \frac{1}{2}$, $p = \frac{1}{2}$, $n = 4$

$\Rightarrow P(X=2) = {}^4C_2 (0.5)^2 (0.5)^2 = 0.375$

Q9) $p = 0.2$, $n = 7$

i) $P(X=4) = {}^7C_4 (0.2)^4 (0.8)^3 = 0.02867$

ii) $P(X<\mu) = P(X<1.4) = P(X=0) + P(X=1)$

$= {}^7C_0 (0.2)^0 (0.8)^7 + {}^7C_1 (0.2)^1 (0.8)^6 = 0.5767$

iii) $P(X \geq 5) = P(X=5) + P(X=6) + P(X=7)$

$= {}^7C_5 (0.2)^5 (0.8)^2 + {}^7C_6 (0.2)^6 (0.8) + {}^7C_7 (0.2)^7 (0.8)^0 = 0.00467$

Q11) $n = 7$, $p = 0.95$

i) $P(X=2) = {}^7C_2 (0.95)^2 (0.05)^5 = 0.00000592$

ii) $P(X \geq 5) = P(X=5) + P(X=6) + P(X=7) = 0.99624$

iii) $P(X=5) = {}^7C_5 (0.95)^5 (0.05)^2 = 0.04062$

Q12) $n = 10$, $p = 0.08$

i) $P(X=3) = {}^{10}C_3 (0.08)^3 (0.92)^7 = 0.03427$

ii) $\mu = E(X) = np = 0.8$

iii) $\sigma^2 = V(X) = npq = 0.736$

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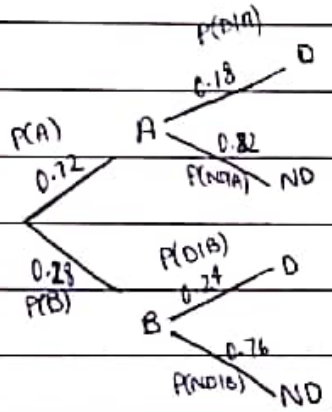
Q13)



i) $P(A \cap D) = P(A)\,P(D|A) = 0.72 \times 0.18 = 0.1296$

ii) $P(D) = P(A \cap D) + P(B \cap D) = P(A)P(D|A) + P(B)P(D|B) = 0.72 \times 0.18 + 0.28 \times 0.24$

$= 0.1296 + 0.0672 = 0.1968$

iii) $P(A|D) = \dfrac{P(D|A)\,P(A)}{P(D|A)P(A) + P(D|B)P(B)} = \dfrac{P(A \cap D)}{P(D)} = \dfrac{0.1296}{0.1968} = 0.6585$