National University of Computer & Emerging Sciences, Karachi

Department of Computer Science

Final Exam, SUMMER-2022

11th August, Thursday, 2022,

9:30 am – 12:30 pm

Course Code: CS-2008	Course Name: Numerical Computing
Instructor Name: Nadeem khan	
Student Roll No:	Section:

Instructions:

- Solve all the questions according to the sequence given in the question paper.
- Read each question completely before answering it. There are 8 questions and 2 pages.
- In order to receive full credit, you must show your all necessary work.
- Display answers correct up to **5** numbers of decimal places. Scientific calculator is allowed.

Time: 180 Minutes **Max. Marks: 100**

Question Number 1 CLO1 [5+5=10 Marks]

i) Evaluate the polynomial $\boldsymbol{f(x) = x^3 - 5x^2 + 6x + 0.55}$ at $\boldsymbol{x = 1.37}$ also find $\boldsymbol{f(x)}$in term of nested manner. Use 3-digit arithmetic with chopping. Evaluate the percentage relative error.

ii) Use fix point iteration method to locate roots of $\boldsymbol{y = x^3 + x^2 - 1}$ corrected up to 5 decimal places.

Question Number 2 CLO2 [5+5+10=20 Marks]

i) Approximate $\int_0^2 x^2 ln(x^2+1)dx$ using $h = 0.25$ use

a) Composite trapezoidal

b) Composite Simpsons rule

ii) Use numerical differentiation complete the given table.

x	$f(x)$	$f'(x)$
1.1	9.02501	
1.2	11.02318	
1.3	13.46374	
1.4	16.44465	

Question Number 3 CLO2 [2+4+4=10 Marks]

Solve the following initial value problem over the interval from $t = 0$ to $t = 2$ Where $y(0) = 1$.

$$\boldsymbol{y' = yt^2 - 1.1y}$$

i) Analytically

ii) Euler's method with $h = 0.5$

iii) fourth order RK method with $h = 0.5$

Question Number 4 **CLO2** **[5+5=10 Marks]**

i) Construct the divided difference table.

ii) Estimate $f(0.1)$.

x	-0.1	0	0.2	0.3
$f(x)$	5.30	2.00	3.19	1.00

Question Number 5 **CLO2** **[10+10=20 Marks]**

Solve following the system of equation using both Gauss siedel and relaxation method. Do four iteration of each. (Display five decimal places).

$$4x + 11y - z = 33$$
$$8x - 3y + 2z = 20$$
$$6x + 3y + 12z = 35$$

Question Number 6 **CLO2** **[10 Marks]**

Solve $\boldsymbol{Ax} = \boldsymbol{b}$ using Crout's method or Doolittle method of factorization.

$$\mathbf{A}=\begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{4} \\ \mathbf{8} & \mathbf{-3} & \mathbf{2} \\ \mathbf{4} & \mathbf{11} & \mathbf{-1} \end{bmatrix}, \ \mathbf{x}=\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} \mathbf{2} \\ \mathbf{0} \\ \mathbf{3} \end{bmatrix}.$$

Question Number 7 **CLO2** **[3+7=10 Marks]**

Determine whether the given matrix is
(a) singular
(b) Strictly diagonally dominant
(c) Positive definite or not
And also determine $\mathbf{LDL^t}$ Factorization of following matrix.

$$\begin{bmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{0} \\ \mathbf{-1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{0} & \mathbf{-1} & \mathbf{2} \end{bmatrix}$$

Question Number 8 **CLO2** **[5+5=10 Marks]**

i).Apply Power method to find the dominant Eigen value and corresponding Eigen vector up to Three decimal places for the following matrix.

$$\begin{bmatrix} \mathbf{4} & \mathbf{-1} & \mathbf{1} \\ \mathbf{-1} & \mathbf{3} & \mathbf{-2} \\ \mathbf{1} & \mathbf{-2} & \mathbf{3} \end{bmatrix}$$

Associated with unit Eigen vector $\boldsymbol{V} = (\mathbf{1}, \mathbf{-1}, \mathbf{1})^{\boldsymbol{t}}$.
ii).If $\Delta, \boldsymbol{\nabla}, \boldsymbol{\delta}, \boldsymbol{E}$ and μ denote forward,backward and central operators,shift operator,average operators respectively with eqaul spacing show that

a).$\boldsymbol{\delta\mu} = \frac{\Delta + \boldsymbol{\nabla}}{\mathbf{2}}$ b).$\boldsymbol{E}^{\frac{1}{2}} = \boldsymbol{\mu} + \frac{\boldsymbol{\delta}}{\mathbf{2}}$

Formula Sheet

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} - \frac{h}{2} f''(\xi).$$

$$P_n(x) = f[x_n] + f[x_n, x_{n-1}](x - x_n) + f[x_n, x_{n-1}, x_{n-2}](x - x_n)(x - x_{n-1}) + \cdots + f[x_n, \ldots, x_0](x - x_n)(x - x_{n-1}) \cdots (x - x_1).$$

$$P_n(x) = P_n(x_0 + sh) = f[x_0] + shf[x_0, x_1] + s(s-1)h^2 f[x_0, x_1, x_2] + \cdots + s(s-1)\cdots(s-n+1)h^n f[x_0, x_1, \ldots, x_n]$$

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right] - \frac{b-a}{180}h^4 f^{(4)}(\mu).$$

Three-Point Endpoint $f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)] + \frac{h^2}{3} f^{(3)}(\xi_0),$

Five-Point Endpoint $f'(x_0) = \frac{1}{12h}[-25f(x_0) + 48f(x_0+h) - 36f(x_0+2h) + 16f(x_0+3h) - 3f(x_0+4h)] + \frac{h^4}{5} f^{(5)}(\xi),$

$$\int_a^b f(x)\,dx = 2h\sum_{j=0}^{n/2} f(x_{2j}) + \frac{b-a}{6}h^2 f''(\mu).$$

$n = 4$ $\int_{x_0}^{x_4} f(x)\,dx = \frac{2h}{45}[7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4)] - \frac{8h^7}{945} f^{(6)}(\xi),$

Composite Simpson's rule $h = (b-a)/n$, and $x_j = a + jh$,

closed Newton-Cotes $n = 2$: $\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)] - \frac{h^5}{90} f^{(4)}(\xi),$

$n = 3$ $\int_{x_0}^{x_3} f(x)\,dx = \frac{3h}{8}[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)] - \frac{3h^5}{80} f^{(4)}(\xi),$

Composite Midpoint rule n be even, $h = (b-a)/(n+2)$, and $x_j = a + (j+1)h$

Euler's method is

$$w_0 = \alpha,$$
$$w_{i+1} = w_i + hf(t_i, w_i), \quad \text{for each } i = 0, 1, \ldots, N-1.$$

$$\mu f(x) = \frac{1}{2}\left[f\left(x + \frac{h}{2}\right) + f\left(x - \frac{h}{2}\right)\right]$$

$$Ef(x) = f(x+h)$$

$$\Delta f(x) = f(x+h) - f(x)$$
$$\nabla f(x) = f(x) - f(x-h)$$

$$\delta[f(x)] = f(x + h/2) - f(x - h/2)$$

Rk method

$$w_0 = \alpha,$$
$$k_1 = hf(t_i, w_i),$$
$$k_2 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_1\right),$$
$$k_3 = hf\left(t_i + \frac{h}{2}, w_i + \frac{1}{2}k_2\right),$$
$$k_4 = hf(t_{i+1}, w_i + k_3),$$
$$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$