Applied Physics Assignment-1

Vectors

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Question : 01

A displacement vector in the xy plane is 7.3m long and directed at angle of 30° in Fig 1. Determine
a) x component b) Y component of vector.

Solution:

Data:

$\vec{\gamma} = 7.3\,m$

$\gamma_x = ?$

$\gamma_y = ?$

Formula:

$\gamma_x = |\vec{\gamma}| \cos\theta$

$\gamma_y = |\vec{\gamma}| \sin\theta$

Calculation:

$\gamma_x = |\vec{\gamma}| \cos\theta$

$\gamma_x = (7.3)\cos 30°$

$\boxed{\gamma_x = 6.32\,m}$

$\gamma_y = |\vec{\gamma}| \sin\theta$

$\gamma_y = (7.3)\sin 30°$

$\boxed{\gamma_y = 3.65\,m}$

QUESTION : 02

The two vectors a & b (Fig-2) have equal mag of 10m and the angles are $\theta_1 = 30$ & $\theta_2 = 105$. Find (a) x & y components of their vector sum γ (b) Mag of γ and (c) Angle γ makes with the positive direction of x-axis.

Solution :

Data:

$$\vec{a} = \vec{b} = 10\,m$$
$$\theta_1 = 30 \text{ \& } \theta_2 = 105$$

Formula :

(i) $a_x = |\vec{a}| \cos\theta$.
(ii) $a_y = |\vec{a}| \sin\theta$
(iii) $\gamma = \sqrt{a^2 + b^2}$
(iv) $\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$

Calculation :

Finding x & y Component

$\vec{a}_x = |\vec{a}| \cos\theta$
$\vec{a}_x = |10| \cos 30^\circ$
$\boxed{\vec{a}_x = 8.6602\,m}$

$\vec{a}_y = |\vec{a}| \sin\theta$
$\vec{a}_y = |10| \sin 30^\circ$
$\boxed{\vec{a}_y = 5\,m}$

$\vec{b}_x = |\vec{b}| \cos\theta$
$\vec{b}_x = |10| \cos 105^\circ$
$\boxed{\vec{b}_x = -7.0711\,m}$

$\vec{b}_y = |\vec{b}| \sin\theta$
$\vec{b}_y = |10| \sin 105^\circ$
$\boxed{\vec{b}_y = 7.071\,m}$

For x/y component of vector Sum;

$$\gamma_x = a_x + b_x$$
$$\gamma_x = (8.6602) + (-7.0711)$$
$$\boxed{\gamma_x = 1.5891\,m}$$

$$\gamma_y = a_y + b_y$$
$$\gamma_y = (5) + (7.071)$$
$$\boxed{\gamma_y = 12.071\,m}$$

For Magnitude of γ;

$$\gamma = \sqrt{\gamma_x^2 + \gamma_y^2}$$

$$\gamma = \sqrt{(1.58)^2 + (12.07)^2}$$

$$\boxed{\gamma = 12.1752 \text{ m}}$$

Finding Angle with positive x-axis;

$$\theta = \tan^{-1}\left(\frac{\gamma_y}{\gamma_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{12.0711}{1.5891}\right)$$

$$\boxed{\theta = 82.50^\circ}$$

QUESTION: 03

For vector in Fig-3 with a=4, b=3.8 c=5, what are (a) the mag and direction of $a \times b$. (b) the mag & direction of $a \times c$ and (c) the mag & direction of $b \times c$?

Solution:

Data:

$$a = 4 \quad , \quad b = 3.8 \quad c = 5$$

Formulas:

(i) $\theta = \tan^{-1}\left(\frac{P}{B}\right)$

(ii) $|a \times b| = ab \sin\theta$

For α;

$$\alpha = \cos^{-1}\left(\frac{a \cdot b}{ab}\right)$$

For β;

$$\beta = \cos^{-1}\left(\frac{b \cdot c}{bc}\right)$$

Angle b/w a & b:

$$\theta_1 = 90^\circ$$

Angle b/w b & c:

$$\theta_2 = \tan^{-1}\left(\frac{\text{Perp}}{\text{Base}}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 36.86^\circ$$

Angle b/w a & c:

$$\theta_1 + \theta_2 + \theta_3 = 180^\circ$$

$$\theta_3 = 180^\circ - 90^\circ - 36.86^\circ$$

$$\theta_3 = 53.13^\circ$$

Calculation:

Magnitude of $a \times b$;

$|a \times b| = ab \sin\theta_1 = (4)(3)\sin 90^\circ$

$\boxed{|a \times b = 12 \text{ units}}$

Magnitude of $b \times c$;

$|b \times c| = bc \sin\theta_2 = (3)(5)\sin 36.86^\circ$

$\boxed{|b \times c| = 9 \text{ units}}$

Magnitude of $a \times c$;

$|a \times c| = ac \sin\theta_3 = (4)(5)\sin 53.13^\circ$

$\boxed{|a \times c| = 16 \text{ units}}$

Direction of $\vec{a} \times \vec{b}$

$\alpha = \cos^{-1}\left(\frac{a.b}{ab}\right)$

$\alpha = 90^\circ$

Direction of $\vec{b} \times \vec{c}$

$\beta = \cos^{-1}\left(\frac{b.c}{bc}\right)$

$\beta = 36.86^\circ$

Direction of $\vec{a} \times \vec{c}$

$\gamma = \cos^{-1}\left(\frac{a.c}{ac}\right)$

$\boxed{\gamma = 53.13}$

QUESTION : 04

By Considering the above problem (2), Find the (a) $\underline{a.b}$ (b) $\underline{b \times c}$ (c) angle b/w a & b.

Solution :

Data: $\vec{\gamma} = 10$ m

Formula:

(i) $a.b = ab\cos\theta$

Angles $\vec{a}$ and $\vec{b}$ make with x-axis are $\theta_1 = 30^\circ$ & $\theta_2 = 135^\circ$

Calculation :

Finding $a.b$;

$a.b = ab\cos\theta$ — eq (i)

Angle b/w a and b is $= \theta_3$

$$\theta_3 = 180^\circ + 30^\circ - 135^\circ$$

$$\boxed{\theta_3 = 75^\circ}$$

eq (i) $\Rightarrow$ $\vec{a} \cdot \vec{b} = ab \cos 75^\circ$

$$= (10)(10) \cos 75^\circ$$

$$\boxed{\vec{a} \cdot \vec{b} = 25.88 \text{ m}}$$

Finding $\vec{a} \times \vec{b}$:

$$\vec{a} \times \vec{b} = ab \sin \theta$$

$$= (10)(10) \sin 75^\circ$$

$$\boxed{\vec{a} \times \vec{b} = 96.59 \text{ m}}$$

QUESTION : 05

The x component of vector A is 25m and y component is 40m (a) Mag of A? (b) Angle b/w direction of & positive direction of x?

Solution:

Data: $A_x = 25$ m

$A_y = 40$ m

Formula:

(i) $|A| = \sqrt{a^2 + b^2}$

(ii) $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

Calculation :

Finding Magnitude of A;

$$|A| = \sqrt{A_x^2 + A_y^2}$$

$$|A| = \sqrt{(25)^2 + (40)^2}$$

$$\boxed{|A| = 47.169}$$

For Angle;

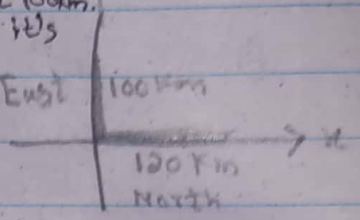
$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{40}{25}\right)$$

$$\boxed{\theta = 58^\circ}$$

QUESTION : 06

A ship sets out to sail to a point 120Km due north, An unexpected storm blows the ship to a point 100km. (a) How far? (b) in what direction it sail to reach its original destination.



Solution :

Data:

$A_x = 120$ Km

$A_y = 100$ Km

Formula :

(i) $|D| = \sqrt{a^2 + b^2}$

(ii) $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

Calculation :

For Distance ;

$$D = \sqrt{A_x^2 + A_y^2} = \sqrt{(120)^2 + (100)^2}$$

$$\boxed{D = 156.2 \text{ Km}}$$

For Angle ;

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{100}{120}\right)$$

$$\boxed{\theta = 50.194^\circ}$$

QUESTION : 07

Three vectors a, b & c, have a mag of 50m and lie in an xy plane. Their direction is relative to a point 100 Km due east of starting point (a) How far (b) 30°, 195° & 315°. what are the mag & angle of a+b+c & a-b+c. Find fourth vector d such that (a+b) - (c+d) = 0?

Formula :

(i) $A_x = a\cos\theta$

(ii) $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

Solution :

Data:

$$a = b = c = 50 \text{ m}$$

$\theta_1 = 30^\circ$, $\theta_2 = 195^\circ$ and $\theta_3 = 315^\circ$

Calculation :

$A_x = a\cos\theta_1$ $\quad$ $B_x = b\cos\theta_2$ $\quad$ $C_x = c\cos\theta_3$

$\boxed{A_x = 25.93 \text{ m}}$ $\quad$ $\boxed{B_x = -28.97 \text{m}}$ $\quad$ $\boxed{C_x = 21.21 \text{ m}}$

$A_y = a\sin\theta_1$ $\quad$ $B_y = b\sin\theta_2$ $\quad$ $C_y = c\sin\theta_3$

$\boxed{A_y = 15 \text{ m}}$ $\quad$ $\boxed{B_y = -7.76 \text{m}}$ $\quad$ $\boxed{C_y = -21.21 \text{ m}}$

Let,

$$a + b + c = F$$

So,

$$F_x = a_x + b_x + c_x \qquad F_y = a_y + b_y + c_y$$

$$\boxed{F_x = 18.21\ m} \qquad \boxed{F_y = -13.97\ m}$$

$$\boxed{F = 22.96\ m}$$

$$a_x = 43.30\ m \qquad b_x = -48.29\ m \qquad c_x = 35.35\ m$$

$$a_y = 25\ m \qquad b_y = -12.94 \qquad c_y = -35.35\ m$$

$$F_x = 30.36\ m$$

$$F_y = -23.29\ m$$

$$\boxed{F = 38.26\ m}$$

For angle;

$$\theta_F = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{23.29}{30.36}\right)$$

$$\boxed{\theta_F = 37.49^\circ}$$

For $a - b + c$;

Let,

$$a - b + c = G_1$$

$$G_x = a_x + b_x + c_x = (43.30) + (-48.29) + (35.35)$$

$$\boxed{G_x = 126.95\ m}$$

$$G_y = a_y + b_y + c_y = (25) + (-12.94) + (-35.35)$$

$$\boxed{G_y = 2.58\ m}$$

$$\boxed{G_1 = 126.97\ m}$$

$$\boxed{\theta_{G_1} = 1.166^\circ}$$

For Magnitude & Angle of fourth vector d such that
$(a + b) - (c + d) = 0$

$(a_x + b_x) - (c_x + d_y) \Rightarrow 13.30 - 48.29 - (35.35 + d_x) = 0$
$(-4.995) - (35.35) + d_x \Rightarrow -40.35 - d_x \Rightarrow \boxed{-d_x = 40.35}$

$$\boxed{d_x = -40.35 \text{ m}}$$

$(a_y + b_y) - (c_y + d_y) = 0$
$(25 - 12.94) - (-35.35 + d_y) \Rightarrow 47.4143 - d_y = 0$

$$\boxed{d_y = -47.4143 \text{ m}}$$

$$\boxed{d = 62.2596 \text{ m}}$$

$$\boxed{\theta = 49.60^\circ}$$

QUESTION : 08

Solution :

Formula :

(i) $\cos\theta = \frac{A_x}{|A||x|}$

Data:

$$\vec{A} = 2i - 3j + 5k$$

Calculation :

For θ_x ;

$$\cos\theta_x = \frac{\vec{A} \cdot \vec{x}}{|A||x|} = \frac{2}{6.164}$$

$$\theta_x = \cos^{-1}\left(\frac{2}{6.164}\right)$$

$$\boxed{\theta_x = 35.79^\circ}$$

For θ_y ;

$$\cos\theta_y = \frac{\vec{A} \cdot \vec{y}}{|A||y|} = \frac{-3}{6.164}$$

$$\theta_y = \cos^{-1}\left(\frac{-3}{6.164}\right)$$

$$\boxed{\theta_y = 116.07^\circ}$$

For θ_z :

$$\cos\theta_z = \frac{\vec{A}\cdot\vec{z}}{|A||z|} = \frac{5}{6.164}$$

$$\theta_z = \cos^{-1}\left(\frac{5}{6.164}\right)$$

$$\boxed{\theta_z = 35.79^\circ}$$

QUESTION : 09

Solution :

Formula :

(i) $|a| = \sqrt{a^2+b^2+c^2}$

(ii) $a.b = \frac{a.b}{ab}$

(iii) $\cos\theta = \frac{\vec{a}.\vec{b}}{ab}$

Data :

$$a = 5i + 4j - 6k$$
$$b = -2i + 2j + 3k$$
$$c = 4i + 3j + 2k$$
$$\gamma = a + b + c$$

Calculation :

Finding Angle b/w a & b

$$|a| = \sqrt{(5)^2+(4)^2+(-6)^2}$$
$$|a| = \sqrt{25+16+36}$$
$$\boxed{|a| = 8.775}$$

$$|b| = \sqrt{(-2)^2+(2)^2+(3)^2}$$
$$|b| = \sqrt{4+4+9}$$
$$\boxed{|b| = 4.1231}$$

$$a.b = \frac{\vec{a}.\vec{b}}{ab} \Rightarrow (i)$$

$$a \cdot b = (5i + 4i - 6K) \cdot (-2i + 2j + 3K)$$

$$a.b = -10 + 8 - 18$$

$$\boxed{a.b = -20}$$

eq (i) $\Rightarrow$ $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{ab}$

$$\cos\theta = \frac{-20}{(8.775)(4.1231)}$$

$$\theta = \cos^{-1}(-0.55)$$

$$\boxed{\theta = 123.5593^\circ}$$

$$\gamma = a + b + c$$

$$\gamma = (5 - 2 + 4)i + (4 + 2 + 3)j + (-6 + 3 + 2)K$$

$$\gamma = 7i + 9j - K$$

$$|\gamma| = \sqrt{(7)^2 + (9)^2 - (1)^2}$$

$$|\gamma| = \sqrt{49 + 81 + 1}$$

$$\boxed{|\gamma| = 11.4455}$$

Angle with Z-axis

$$\cos\theta = \frac{\vec{\gamma} \cdot \vec{Z}}{\gamma Z} \Rightarrow \frac{-1}{11.4455}$$

$$\boxed{\theta = 95^\circ}$$

QUESTION : 10

Solution :

Data :

Formula :

(i) $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$

$\vec{A} = 6$ units

$\vec{B} = 7$ units

$\vec{A} \cdot \vec{B} = 14$

Calculation :

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\cos\theta = \frac{14}{(6)(7)}$$

$$\cos\theta = 0.333$$

$$\theta = \cos^{-1}(0.333)$$

$$\boxed{\theta = 70.52^{\circ}}$$

X ——— X