Numerical Differentiation

1. Forward Difference Formula:

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} - \boldsymbol{\frac{h}{2} f''(\zeta)}$$

$$\zeta \text{ lies between } x_0 \text{ and } x_0 + h$$

2. Backward Difference Formula:

$$f'(x_0) = \frac{f(x_0) - f(x_0 - h)}{h} + \boldsymbol{\frac{h}{2} f''(\zeta)}$$

$$\zeta \text{ lies between } x_0 - h \text{ and } x_0$$

3. Three Point Endpoint Formula:

$$f'(x_0) = \frac{1}{2h}\left(-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)\right) + \boldsymbol{\frac{h^2}{3} f'''(\zeta_0)}$$

$$\text{for right end point approximation replace } h \text{ by } -h$$

$$\zeta \text{ lies between } x_0 \text{ and } x_0 + 2h$$

4. Three point Midpoint Formula:

$$f'(x_0) = \frac{1}{2h}\left(f(x_0 + h) - f(x_0 - h)\right) + \boldsymbol{\frac{h^2}{6} f'''(\zeta_1)}$$

$$\zeta \text{ lies between } x_0 + h \text{ and } x_0 - h$$

5. Five point Endpoint Formula:

$$f'(x_0) = \frac{1}{12h}\left(-25f(x_0) + 48f(x_0 + h) - 36f(x_0 + 2h) + 16f(x_0 + 3h) - 3f(x_0 + 4h)\right) + \boldsymbol{\frac{h^4}{5} f^{(5)}(\zeta)}$$

$$\text{for right end point approximation replace } h \text{ by } -h$$

$$\zeta \text{ lies between } x_0 \text{ and } x_0 + 4h$$

6. Five point Midpoint Formula:

$$f'(x_0) = \frac{1}{12h}\left(f(x_0 - 2h) - 8f(x_0 - h) + 8f(x_0 + h) - f(x_0 + 2h)\right) + \boldsymbol{\frac{h^4}{30} f^{(5)}(\zeta)}$$

$$\zeta \text{ lies between } x_0 - 2h \text{ and } x_0 + 2h$$

7. Second Derivative Midpoint Formula:

$$f''(x_0) = \frac{1}{h^2}\left(f(x_0 - h) - 2f(x_0) + f(x_0 + h)\right) - \boldsymbol{\frac{h^2}{12} f^{(4)}(\zeta)}$$

$$\zeta \text{ lies between } x_0 - h \text{ and } x_0 + h$$

Problems related to Numerical Differentiation (Ex # 4.1)

1. Use the forward-difference formulas and backward-difference formulas to determine each missing entry in the following tables.

a.

x	$f(x)$	$f'(x)$
0.5	0.4794	
0.6	0.5646	
0.7	0.6442	

b.

x	$f(x)$	$f'(x)$
0.0	0.00000	
0.2	0.74140	
0.4	1.3718	

2. Use the forward-difference formulas and backward-difference formulas to determine each missing entry in the following tables.

a.

x	$f(x)$	$f'(x)$
−0.3	1.9507	
−0.2	2.0421	
−0.1	2.0601	

b.

x	$f(x)$	$f'(x)$
1.0	1.0000	
1.2	1.2625	
1.4	1.6595	

5. Use the most accurate three-point formula to determine each missing entry in the following tables.

a.

x	$f(x)$	$f'(x)$
1.1	9.025013	
1.2	11.02318	
1.3	13.46374	
1.4	16.44465	

b.

x	$f(x)$	$f'(x)$
8.1	16.94410	
8.3	17.56492	
8.5	18.19056	
8.7	18.82091	

c.

x	$f(x)$	$f'(x)$
2.9	−4.827866	
3.0	−4.240058	
3.1	−3.496909	
3.2	−2.596792	

d.

x	$f(x)$	$f'(x)$
2.0	3.6887983	
2.1	3.6905701	
2.2	3.6688192	
2.3	3.6245909	

6. Use the most accurate three-point formula to determine each missing entry in the following tables.

a.

x	$f(x)$	$f'(x)$
−0.3	−0.27652	
−0.2	−0.25074	
−0.1	−0.16134	
0	0	

b.

x	$f(x)$	$f'(x)$
7.4	−68.3193	
7.6	−71.6982	
7.8	−75.1576	
8.0	−78.6974	

c.

x	$f(x)$	$f'(x)$
1.1	1.52918	
1.2	1.64024	
1.3	1.70470	
1.4	1.71277	

d.

x	$f(x)$	$f'(x)$
−2.7	0.054797	
−2.5	0.11342	
−2.3	0.65536	
−2.1	0.98472	

18. Consider the following table of data:

x	0.2	0.4	0.6	0.8	1.0
$f(x)$	0.9798652	0.9177710	0.808038	0.6386093	0.3843735

a. Use all the appropriate formulas given in this section to approximate $f'(0.4)$ and $f''(0.4)$.

b. Use all the appropriate formulas given in this section to approximate $f'(0.6)$ and $f''(0.6)$.

25. In Exercise 10 of Section 3.4 data were given describing a car traveling on a straight road. That problem asked to predict the position and speed of the car when $t = 10$ s. Use the following times and positions to predict the speed at each time listed.

Time	0	3	5	8	10	13
Distance	0	225	383	623	742	993

26. In a circuit with impressed voltage $\mathcal{E}(t)$ and inductance L, Kirchhoff's first law gives the relationship

$$\mathcal{E}(t) = L\frac{di}{dt} + Ri,$$

where R is the resistance in the circuit and i is the current. Suppose we measure the current for several values of t and obtain:

t	1.00	1.01	1.02	1.03	1.0
i	3.10	3.12	3.14	3.18	3.24

where t is measured in seconds, i is in amperes, the inductance L is a constant 0.98 henries, and the resistance is 0.142 ohms. Approximate the voltage $\mathcal{E}(t)$ when $t = 1.00$, 1.01, 1.02, 1.03, and 1.04.