$$p(x) = \left(\frac{(x-8.3)(x-8.6)}{0.1}\right)(16.94410) +$$
$$\left(\frac{(x-8.1)(x-8.6)}{-0.06}\right)(17.56492) +$$
$$\left(\frac{(x-8.1)(x-8.3)}{0.15}\right)(18.50515)$$

$$p(x) = -3.38882 + 17.56492 + 3.70103$$

$$p(x) = 17.877$$

Theorem:- (3.1) (Error Bond)

Suppose $x_0, x_1 \ldots x_n$ are distinct number in the interval $[a\ b]$ Then for each x in $[a\ b]$, a number $\xi(x)$ b/w x_0, x_1, x_n and hence in $(a\ b)$ exist with

$$f(x) = P(x) + \frac{f^{n+1}((\xi))}{(n+1)!}(x-x_0)(x-x_1)\ldots(x-x_n)$$

I-

Q3) Use Theorem to find error bound for the approximation $f(x) = \cos x$, $x_0 = 0$, $x_1 = 0.6$, $x_2 = 0.9$

Construct polynomial of degree at most one and at most two to approximate $f(0.45)$

Ans) one degree polynomial

$P(x) = -0.29110x + 1$

Error Bond :-

$n = 1$

By Error bond

$$\frac{f''((\xi))}{2!}(x-x_0)(x-x_1)$$

$$= \left|\frac{-\cos(\xi)}{2!}\right| \cdot |(x-0)(x-0.6)|$$

$= \; 0 < \xi < 0.6$

$$\left|\frac{-\cos(0)}{2}\right| = |-0.5| \leq 0.5$$

put lower bound value in ξ

let $g(x) = x(x-0.6)$

$g(x) = x^2 - 0.6x$

$g(x) = x^2 - 0.6x$

$g'(x) = 2x - 0.6$

$g'(x) = 0$

$0 = 2x - 0.6$

$x = 0.3$

$\cancel{g(0.3) = (0.3)^2 - 0.6(0.3)}$

$g(0.3) = 0.3(0.3 - 0.6)$

$= (0.3)(-0.3)$

$= |-0.09| \leq 0.09$

Error Bond $= (0.5)(0.09)$

$= 0.045$

$f(0.45) = P(0.45) + 0.045$

$= 0.86900 + 0.045$

$\boxed{f(0.45) = 0.91400}$

Divided Diff Table:-

Constructing DD Table:-

x	y	1st D	2nd D	3rd Diff
x_0	y_0	$D_0 = \frac{y_1-y_0}{x_1-x_0}$	$D_0 = \frac{D_1-D_0}{x_2-x_0}$	$= \frac{D_1-D_0}{x_3-x_0}$
x_1	y_1	$D_1 = \frac{y_2-y_1}{x_2-x_1}$	$D_1 = \frac{D_2-D_1}{x_3-x_1}$	
x_2	y_2	$D_2 = \frac{y_3-y_2}{x_3-x_2}$		
x_3	y_3			

D_0 1st D $= f(x_0, x_1)$

D_0 2nd D $= f(x_0, x_1, x_2)$

D_0 ~~8th~~ n^{th} D $= f(x_0, x_1, \ldots, x_n)$

Polynomial DD:-

$$P_n(x) = f(x_0) + \sum_{k=1}^{n} f(x_0, x_1, x_k)[(x-x_0)\ldots(x-x_{k-1})]$$

One Degree:-

$$P_1(x) = f(x_0) + (x-x_0) f(x_0, x_1)$$

Two Degree:-

$$P_2(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2).$$

Three Degree:-

$$P_3(x) = \text{''} + \text{''} + \text{''} + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3).$$

Q1) Construct DD Table

x	0	1	2	3	4
y	0	1	8	27	64

x	y	1st D	2nd D	3rd D	
0	0	$D_0 = 1$	$D_0 = 3$	$D_0 = 1$	$D_0 = 0$
1	1	$D_1 = 7$	$D_1 = \cancel{1.5/2}\ 6$	$D_1 = 1$	
2	8	$D_2 = 19$	$D_2 = 9$		
3	27	$D_3 = 37$			
4	64				

Q1) Construct interpolating Polynomial Of degree one, Two and three, find P(8.4)

x	y	1st D	2nd	3rd
8.1	16.9446	$D_0 = 3.1041$	$D_0 = 0.06$	
8.3	17.56492	$D_1 = 3.1341$		$= -2.0833 \times 10^{-3}$
8.6	18.50515	$D_2 = 3.1576$	$D_1 = 0.05875$	
8.7	18.82091			

One Degree :-

$$P_1(x) = f(x_0) + (x - x_0) f(x_0, x_1)$$

$$P_1(x) = 16.94410 + (x - 8.1)(3.1041)$$

$$P_1(8.4) = 17.87533$$

Two Degree :-

$$P_2(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2)$$

$$P_2(x) = \overset{16.94410}{\cancel{17.56492}} + (x - 8.1)(\overset{3.1041}{\cancel{16.9446}}) + (x - 8.1)(x - 8.3)(0.06)$$

$$P(8.4) = \cancel{18.49795}\ 17.87713$$

"for equal space (diff in x)"

Forward Difference Interpolation

x	y	Δf_0	$\Delta^2 f_0$	$\Delta^3 f_0$	$\Delta^4 f_0$
x_0 1	0				
		1.0986			
3	1.098		-0.587		
		0.511		0.4129	
5	1.609		-0.1741		-0.3244
		0.3369		0.0885	
7	1.9459		-0.0856		
		0.2513			
x_n 9	2.1972				

forward difference interpolation polynomial

$$f(x) = f(x_0) + S \Delta f_0 + \frac{S(S-1)}{2!} \Delta^2 f_0 + \frac{S(S-1)(S-2)}{3!} \Delta^3 f_0 + \frac{S(S-1)(S-2)(S-3)}{4!} \Delta^4 f_0$$

where

$$S = \frac{x - x_0}{h}$$

h → diff in x values i.e 2

Δ → Forward Operator

Same table will be drawn for forward, Central and Backward interpolation.

... 25 by Forward diff

x	y	Δf_0	$\Delta^2 f_0$	$\Delta^3 f_0$	$\Delta^4 f_0$
10	0.1736				
		0.1687			
20	0.3423		-0.011		
		0.1577		-0.0039	
30	0.5		-0.0149		
		0.1428		-0.0107	-0.0068
40	0.6428		-0.0256		
		0.1172			
50	0.760				

$x_0 = 10$

$h \to 10 \to$ difference in x

$$S = \frac{x-10}{10}$$

$$f(x) = 0.1736 + \left(\frac{x-10}{10}\right)(0.1687) + \frac{\left(\frac{x-10}{10}\right)\left(\frac{x-20}{10}\right)(-0.011)}{2!}$$

$$+ \frac{\left(\frac{x-10}{10}\right)\left(\frac{x-20}{10}\right)\left(\frac{x-30}{10}\right)(-0.0039)}{6} +$$

$$\frac{\left(\frac{x-10}{10}\right)\left(\frac{x-20}{10}\right)\left(\frac{x-30}{10}\right)\left(\frac{x-40}{10}\right)(-0.0068)}{24}$$

$= 0.1736 + 0.10122 + (-0.004125) - 0.00024375 - 0.0001593375$

$= 0.42665 - 0.004125 + 0.00024375 - 0.0001593$

$= 0.4226$

X

Backword Difference Interpolation:-

x	y	∇f_n	$\nabla^2 f_n$	$\nabla^3 f_n$	$\nabla^4 f_n$
x_0 1	0				
		1.098			
3	1.098		-0.587		
		0.511		0.4129	
5	1.609		-0.1741		-0.3244
		0.3369		0.085	
7	1.9459		-0.0856		
		0.2513			
x_n 9	2.1972				

$$f(x) = f(x_n) + S\,\nabla f_n + \frac{S(S+1)}{2!}\nabla^2 f_n + \frac{S(S+1)(S+2)\nabla^3 f_n}{3!} + \frac{S(S+1)(S+2)(S+3)\nabla^4 f_n}{4!}$$

where $S = \dfrac{x - x_n}{h}$

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Find 4.12

(Q2)

	x	y	∇f_n	$\nabla^2 f_n$	$\nabla^3 f_n$	$\nabla^4 f_n$
x_0	0	1				
			1			
	1	2		1		
			2			
	2	4		2	1	
			4			
	3	8		4	2	1
			8			
	4	16		8	4	2
			16			
x_n	5	32				

$$S = \frac{x - x_n}{h} = x - \cancel{8}2 = \cancel{-27.88}\ -0.88$$

~~$f(x) = f(x_n) + S\nabla f$~~

$$\cancel{f(x) = 32 + (-27.88)(16) + \frac{(-27.88)(-26.88)8}{2} + \frac{(-27.88)(-26.88)(-25.88)(4)}{6} + \frac{(-27.88)(-26.88)(-25.88)(-24.88)(2)}{24}}$$

$$\cancel{f(x) = 32 - 446.08 + 2997.6576 + 107.649 + 84.8835}$$

$=$

$$f(x) = 32 + (-0.88)(16) + \frac{(-0.88)(0.12)8}{2} + \frac{(-0.88)(0.12)(1.12)4}{6} + \frac{(-0.88)(0.12)(1.12)(2.12)(2)}{24}$$

$$f(x) = 32 - 14.08 - 0.4224 + 0.0788 - 0.0208$$

$$f(x) = 17.398$$

Central Diff Interpolation
Or
Stirling formula

$$P(x) = f_0 + P \cdot \mu \delta f_0 + \frac{P^2}{2!} \delta^2 f_0 + \frac{P(P^2-1)}{3!} \mu \delta^3 f_0 + \frac{P^2(P^2-1)}{4!} (\delta^4) f_0$$

where $P = \dfrac{(x - x_0)}{h}$

Q1)

x	y	δf_0	$\delta^2 f_0$	$\delta^3 f_0$	$\delta^4 f_0$
0	1				
0.2	1.2214	0.2214			
x_0 0.4	1.49182	0.27042	0.04908 0.05988	0.01086	
0.6	1.82212	0.3303	0.07342		0.00238
0.8	2.22554	0.40342		0.01324	
		μ		μ	

μ means difference

$x = 0.43$

$x_0 = 0.4$

$h = 0.2 - 0$

$f_0 = 1.49182$

$P = \dfrac{x - 0.4}{0.2}$

$P = 0.15$

$$p(x) = 1.49182 + (0.15)(0.30036) + \frac{(0.15)^2}{2}(0.05998) + \frac{(0.15)(0.15^2-1)(0.01206)}{6} + \frac{(0.15)^2(0.15^2-1)(0.00236)}{24}$$

$$p(x) = 1.49182 + 0.045054 + 0.000674775 + (-0.0002947 1625) + \frac{(0.0225)(-0.9775)}{24}(0.00236$$

$$p(x) = 1.49182 + 0.045054 + 0.000674775 - 0.00029471625 - 0.0000021627185$$

$$\boxed{p(x) = 1.53725}$$

x	$f(x)$	δf_0	$\delta^2 f_0$	$\delta^3 f_0$	$\delta^4 f_0$
-1	1				
0	4	3			
1	11	7	4		
2	16	5	-2	-6	
	13	-3	-8	-6	0

$x = 2.2$
$x_0 = 0$
$h = 1$
$f_0 = 11$

$\boxed{p = 2.2}$

$$P(x) = 11 + 2.2(6) + \frac{(2.2)^2}{2}(-2) + \frac{(2.2)(2.2^2-1)(-6)}{6}$$

$$P(x) = 11 + 13.2 + (-4.84) + (-8.448)$$

$$\boxed{P(x) = 10.912}$$

Numerical Diffrentiation

- Forward Diff
- Backward Diff

1. Forward Diff

1st Derivative

$$f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h} \quad -(1)$$

$$f''(x_0) = \frac{f'(x_0+h) - f'(x_0)}{h} \quad -(2)$$

$$f'(x_0+h) = \frac{f(x_0+2h) - f(x_0+h)}{h} \quad -(3)$$

from (1) & (3)

$$(2)\ f''(x_0) = \frac{\frac{f(x_0+2h) - f(x_0+h)}{h} - \frac{f(x_0+h) - f(x_0)}{h}}{h}$$

$$\boxed{f''(x_0) = \frac{f(x_0+2h) - 2f(x_0+h) + f(x_0)}{h^2}} \quad (4)$$

$$f'''(x_0) = \frac{f'(x_0+2h) - 2f'(x_0+h) + f'(x_0)}{h^2} \quad -(5)$$

$$f'(x_0+2h) = \frac{f(x_0+3h) - f(x_0+2h)}{h} \quad -(6)$$

$$f'''(x_0) = \frac{\frac{f(x_0+3h) - f(x_0+2h)}{h} - 2\left[\frac{f(x_0+2h) - f(\ldots)}{h}\right] + \frac{f(x_0+h) - f(x_0)}{h}}{h^2}$$

$$f'''(x_0) = \frac{f(x_0+3h) - 3f(x_0+2h) + 3f(x_0+h) - f(\ldots)}{h^3}$$

2.) Backward Diff

$$f'(x_0) = \frac{f(x_0) - f(x_0-h)}{h}$$

$$f'(x_0-h) = \frac{f(x_0-h) - f(x_0-2h)}{h}$$

$$f''(x_0 \ldots) = \frac{f'(x_0) - f'(x_0-h)}{h}$$

Make derivates equation by making them like forward diff

Q1) Use the forward Diff formula to approximate the derivative of $f(x) = \ln x$, at $x_0 = 1.8$ using $h = 0.1$, $h = 0.05$, and $h = 0.01$ and determine bounds for the approximation error.

Sol) $f'(x_0) = \frac{f(x_0+h) - f(x_0)}{h}$

$x_0 = 1.8$, $h = 0.1$, $f(x) = \ln x$

$f'(1.8) = \frac{f(1.8+0.1) - f(1.8)}{0.1}$

~~$f'(1.8)$~~ $f(1.8) = \ln(1.8) = 0.5887$

$f(1.9) = \ln(1.9) = 0.6418$

$f'(1.8) = \frac{0.6418 - 0.5887}{0.1} = 0.541$

For bounds error:-

$\left|\frac{h f''(\xi)}{2\xi^2}\right| = \frac{|h|}{2\xi^2}$; $1.8 < \xi < 1.9$ → Take smallest value

$= \frac{|0.1|}{2(1.8)^2} = 0.0154$

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h	f'(1.8)	Bounds Error
0.1	0.541	0.0154
0.05	0.548	0.007716
0.01	0.554	0.001543

For Backward

h	f'(1.8)	Bounds Error
0.1	0.57158	0.01730
0.05	0.56341	0.008163
0.01	0.5571	0.0015605

Q2) values for $f(x) = xe^x$ are given in table, Use all three point and five point formulae to approximate $f'(2.0)$.

x	$f(x)$
1.8	10.889365
1.9	12.703199
2.0	14.778112
2.1	17.148957
2.2	19.855030

Three Point Endpoint formulae:-

$$f'(x_0) = \frac{1}{2h}\left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\right]$$

Three point mid point :-

$$f'(x_0) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right]$$

five Point Midpoint :-

$$f'(x_0) = \frac{1}{12h}\left[f(x_0-2h) - 8f(x_0-h) + 8f(x_0+h) - f(x_0+2h)\right]$$

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five point Endpoint formulae:-

$$f'(x_0) = \frac{1}{12h}\left[-25f(x_0) + 48f(x_0+h) - 36f(x_0+2h) + 16f(x_0+3h) - 3f(x_0+4h)\right]$$

Sol) $h = 1.9 - 1.8 = 0.1$

$x_0 = 2.0$

Three point Endpoint formulae:-

$$f'(x_0) = \frac{1}{2h}\left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\right]$$

$$= \frac{1}{2(0.1)}\left[-3(14.7781122) + 4(17.148757) - (19.855030)\right]$$

$$= 22.02$$

Three Point Midpoint:-

$$f'(x_0) = \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right]$$

$$f'(x_0) = \frac{1}{0.2}(17.148757 - 12.703199)$$

$$= \boxed{22.22775}$$

five Point Midpoint:-

$$f'(x_0) = \frac{1}{12h}\left[f(x_0-2h) - 8f(x_0-h) + 8f(x_0+h) - f(x_0+2h)\right]$$

$$= \frac{1}{1.2}\left[10.889 - 8(12.703) + 8(17.148) - 19.855\right]$$

$$\boxed{f'(x_0) = 122.16}$$

Ex 4.1

Q1) Use the forward and backward differentiation formulae to determine each missing entry

x	$f(x)$	$f'(x)$
0.5	0.4794	10.44
0.6	0.5646	12.088
0.7	0.6442	0.796

Forward:

$f'(x_0) = f(x_0+h) + f(x_0) / h$

Backward:

$f'(x_0) =$

$$f(x_0) = \frac{f(x_0) - f(x_0-h)}{h}$$

Ex 4.1

Q5) Use the most accurate three point formula to determine each missing entry in following Table.

a)

x	$f(x)$	$f'(x)$
1.1	9.025013	17.769705
1.2	11.02318	22.19
1.3	13.46374	27.107
1.4	16.44465	32.510

For first and last value use endpoint formula.
For all mid values use ~~endpoint~~ midpoint formula.

replace h by '-h' in last entry

Q4) The distance 'x' of a runner from fixed point is measured (in meters) is given table.

Time (t)	0.2	0.4	0.6	0.8	1.0
Distance (x)	0.9798652	0.917710	0.80803	0.[illegible]609	0.[illegible]

a) Approximate the runner velocity at $t = 0.2s$ and $t = 1.0s$

b) Approximation the runner velocity and acceleration at time $t = 0.6$.

Second Derivative

$$f''(x_0) = \frac{1}{h^2}\left[f(x_0-h) - 2f(x_0) + f(x_0+h)\right]$$

	$f'(x)$
Ans a) 0.2	-0.195
1.0	-1.54

b) for 0.6 velocity use 5 point midpoint
for 0.6 accelleration use

Numerical Integration

Ex#4.3

- Trapezoidal Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right] ; h = b-a$$

- Simpson's Rule

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right], h = \frac{b-a}{2}$$

Q1) Compute the Trapezoidal and Simpson's rule approximation

$\int_0^2 f(x)\,dx$ when $f(x)$

a) x^2

By Trapezoidal rule

$h = b-a$ $\quad$ $h = 2-0$, $h = 2$

$$= \frac{h}{2}\left[f(x_0) + f(x_1)\right]$$

$x_j = a + jh$

$f(x) = x^2$

$h = 2$

$x_0 = 0 + 0(2) = 0$

$x_1 = 0 + 1(2) = 2$

$\boxed{a = 0 \quad b = 2}$

$$\int_0^2 x^2\,dx = \frac{2}{2}\left[f(0) + f(2)\right]$$

$$= \left[0 + (2)^2\right] = \boxed{4}$$

By Simpson's rule :-

$$h = \frac{b-a}{2}$$

$$\int fx \cdot dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_0)\right]$$

$$h = \frac{2-0}{2} = 1$$

$$= \frac{1}{3}\left[f(0) + 4f(1) + f(2)\right]$$

$$= \frac{1}{3}\left[0^2 + 4(1)^2 + (2)^2\right]$$

$$= \frac{1}{3}\left[4+4\right] = \boxed{8/3}$$

Closed Newton Cotes Formula:-

$$h = \frac{b-a}{n}$$

n=1

$$\int f(x) \cdot dx = \frac{h}{2}\left[f(x_0) + f(x_1)\right]$$

n=2

$$\int f(x) \cdot dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

n=3

$$= \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

n=4

$$= \frac{2h}{45}\left[7f(x_0) + 32f(x_1) + 12f(x_2) + 32f(x_3) + 7f(x_4)\right]$$

Open Cotes Newton Formula:-

$$\boxed{h = \frac{b-a}{n+2}}$$

$$\boxed{x_j = a + (j+1)h}$$

n=0

$$\int f(x)\, dx = 2h f(x_0)$$

n=1

$$\int f(x) \cdot dx = \frac{3h}{2}\left[f(x_0) + f(x_1)\right]$$

n=2

$$\int f(x) \cdot dx = \frac{4h}{3}\left[2f(x_0) - f(x_1) + 2f(x_2)\right]$$

n=3

$$\int f(x) \cdot dx = \frac{5h}{4}\left[11f(x_0) + f(x_1) + f(x_2) + 11f(x_3)\right]$$

Q1) Compare the result of the closed program and open Newton Cotes formula.

$$\int_0^{\pi/4} \sin x \, dx \simeq 0.2929$$

Closed formula

$n=1$ $h=\pi/4$ $x_j = a + jh$

$\frac{h}{2}[f(x_0) + f(x_1)]$

$\frac{\pi}{8}[\cancel{0} + \cancel{0.707107}]$

0.27777

$\frac{\pi}{8}\left[0 + \frac{\sqrt{2}}{2}\right] = \frac{\sqrt{2}\pi}{16} = \boxed{0.2776}$

$n=2$

~~$\pi/12 [0 + \ldots 2\sqrt{2} + \pi/6]$~~

$h = \pi/4/2 = \pi/8$

$\pi/12 \left[0 + 0.3826 + \frac{1}{\sqrt{2}}\right] = 0.2929$

$n=3$

$h = \pi/4/3 = \pi/12$ $a + jh$

$\frac{3}{8}\left(\frac{\pi}{12}\right)\left[0 + 3\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) + 3\left(\frac{1}{2}\right) + \frac{\sqrt{2}}{2}\right]$ $\sin(\pi/12)$ $\sin(\pi/6)$

$\boxed{0.2929}$

$n=4$

$h = \pi/16$

$\frac{2}{45}\left[\frac{\pi}{16}\right][7(0) + 32(0.1951) + 12(0.3826) + 32(0.5556) + 7(\sqrt{2}/2)]$

$\frac{2}{45}\left[\frac{\pi}{16}\right][0 + 6.2432 + 4.5912 + 17.7792 + 4.94975]$

$= \boxed{0.2929}$

Open Cotes Formula :-

$\underline{n=0}$ $\quad h = \pi/8$

$2\left(\frac{\pi}{8}\right)$

$\boxed{0.30055}$

$\underline{n=1}$ $\quad h = \pi/12$

$\frac{3}{2}\left(\frac{\pi}{12}\right)\left[\frac{\sqrt{6}-\sqrt{2}}{4} + \frac{1}{2}\right]$

$\boxed{0.29798}$

$\underline{n=2}$

0.2928

$n=3$

0.2927

$h = \frac{b-a}{n}$ $\quad x_j = a + jh$

Composite Trapezoidal Rule :-

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right] - \frac{b-a}{12}h^2 f''(\mu)$$

Composite Simpsons Rule :-

$$\frac{h}{3}\left[f(a) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right] - \frac{b-a}{180}h^4 f^{(iv)}(\mu)$$

Composite Midpoint Rule

$h = \frac{b-a}{n+2}$

$$= 2h\sum_{j=0}^{n/2} f(x_{2j}) + \frac{b-a}{6}h^2 f''(\mu)$$

$x_j = a + (j+1)h$

(1) $\int_1^2 x\ln x\,dx \quad n=4$

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Composite Trapezoidal Rule:-

$$\int_1^2 x\ln x\,dx = \frac{1}{8}\left[0 + 2(0 + 2\ln 2 + 3\ln 3) + 4\ln 4\right] - \left(\frac{1}{12}\right)\left(\frac{1}{4}\right)^2 f''(\mu)$$

$$= \frac{1}{8}\left[2(4.68213) + 2\ln 2\right]$$

$$= \frac{1}{8}\left[6.0684\right] + \frac{1}{192} f''(\mu)$$

$$\int_1^2 x\ln x\,dx = \frac{1}{8}\left[0 + 2(0.2789 + 0.608 + 0.979) + 2\ln(2)\right]$$

$$= \boxed{0.63}$$

Simpson's Rule:-

$$= \frac{1}{12}\left[f(1) + 2f(1) + f(1.25) + f(1.75) + f(2)\right]$$

$$= \frac{1}{2}\left[0 + 0 + 0.3465 + 0.34657 + 0.2789 + 2\ln 2\right]$$

$=$

Simpsons Rule

$$= \frac{1}{12}\left[f(1) + 2f(5/4) + 4f(0.25) + 4f(0.5) + f(2)\right]$$

$$= \frac{1}{12}\left[0 + 0.557858 + 0.34657 + 0.34657 + 1.38629\right]$$

$$= \frac{1}{12}\left[0 + 0.55786 - 2.77856 + 1.38629\right]$$

$=$

By Midpoint

$$h = \frac{b-a}{n+2} = \frac{2-1}{4+2} = \frac{1}{6}$$

$$x_j = a + (j+1)h$$

$$x_0 = 1 + \frac{1}{6} = 7/6$$

$$x_2 = 1 + \frac{3}{6} = 9/6$$

$$x_4 = 1 + 5/6 = 11/6$$

$$= 2h\sum_{j=0}^{2} f(x_{2j})$$

$$= 2\left(\frac{1}{6}\right)\left[f(x_0) + f(x_2) + f(x_4)\right]$$

Q8) Approximate $\int_0^2 x^2 e^{-x^2}.dx$ using $h = 0.25$

a) Composite trapezoidal

b) Composite Simpsons

c) " Midpoint

Ans a) $n=8$

$$\frac{0.25}{2}[0 + 2(f(0.25) + f(0.5) + f(0.75) + f(1) + f(1.25) + (1.50))]$$

$$\frac{0.25}{2}[0.23485 + 0.3894 + 0.42733 + 0.36788 + 0.2620 + 0.1581 + 0.03663]$$

$$\frac{0.25}{2}[0.0587 + 0.1947 + 0.3205 + 0.3679 + 0.37757 + 0.237 + 0.03863]$$

$\boxed{0.394}$

Complete this using Simpson and Midpoint

Q2) Determine h and n, an approximate error of less than 0.0002

$\int_0^{\pi} x \cdot \sin x . dx$

(a) Composite trap

b) Composite Simpsons.

a) By Composite trap

$$= \frac{b-a}{12} h^2 \; f''(\mu)$$

$a=0$, $b=\pi$, $f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$$\left|\frac{b-a}{12}h^2\right|\left|-\sin(x)\right|$$ Max value of $\sin x$ b/w $[0\ \pi]$

$|-\sin(\pi/2)| = 1$

$$\left|\frac{(\pi-0)h^2}{12}\right| < 0.00002$$

$$\frac{\pi h^2}{12} < 0.00002$$

$$h = \frac{b-a}{n}$$

$$h = \frac{\pi}{n}$$

$$\frac{\pi}{12}\left(\frac{\pi}{x}\right)^2 < 0.00002$$

$n = 360$

$$h = \frac{\pi}{360}$$

$h = 8.72 \times 10^{-3}$

Differential Equation:-

- Eulers Method

$$w_{i+1} = w_i + hf(t_i, w_i).$$

- Mid Point method

$$w_{i+1} = w_i + hf\left[t_i + \frac{h}{2}, w_i + \frac{h}{2}f(t_i, w_i)\right].$$

Ex 5.2

Q1) Use Euler's method to approximate the sol for each of the following initial value Problem.

a) $y' = te^{3t} - 2y$

$0 \leq t \leq 1$

$y(0) = 0$

$h = 0.5$

Sol)

$f(t, w) = y' = te^{3t} - 2y$

$w = y$

$y(0) = 0$

$\cancel{y(0)}\ y_0 = w_0 = 0$

$t_0 = 0$

1st iteration $i=0$

By Euler's method

$w_1 = w_0 + hf(t_0, w_0)$

$= 0 + (0.5)f(0,0)$

$= (0.5)(0 \times e^{3\times0} - 2\times0)$

$= 0$

$\boxed{w_1 = 0, \ t_1 = 0.5}$

2nd iteration:-

$w_2 = w_1 + hf(t_1, w_1)$

$w_2 = 0 + (0.5)f(0.5, 0)$

$w_2 = 0 + 0.5(0.5 \times e^{0.5\times3} - 2(0))$

$\boxed{w_2 = 1.12 \quad t_2 = 1}$

d) $y' = \cos 2t + \sin 3t$

$0 \le t \le 1$

$y(0) = 1$

$h = 0.25$

1st iteration

$w_1 = w_0 + hf(t_0, w_0)$

$w_1 = 1 + 0.25(\cos 2(0) + \sin 3(0))$

$\boxed{w_1 = 1.25}$

2nd iteration

$w_2 = 1.25 + 0.25(\cos 2(0.25) + \sin(3\times0.25))$

$\boxed{w_2 = 1.64}$

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