6.2 Applications of the Normal Distributions

- The standard normal distribution curve can be used to solve a wide variety of practical problems. The only requirement is that the variable be normally or approximately normally distributed.
- For all the problems presented in this chapter, you can assume that the variable is normally or approximately normally distributed.

Applications of the Normal Distributions

- To solve problems by using the standard normal distribution, transform the original variable to a standard normal distribution variable by using the *z* value formula.

- This formula transforms the values of the variable into standard units or *z* values. Once the variable is transformed, then the Procedure Table (Sec. 6.1) and Table E in Appendix C can be used to solve problems.

Example 6-6: Summer Spending

A survey found that women spend on average $146.21 on beauty products during the summer months.

Assume the standard deviation is $29.44.

Find the percentage of women who spend less than $160.00. Assume the variable is normally distributed.

Example 6-6: Summer Spending

Step 1: Draw the normal distribution curve.

Example 6-6: Summer Spending

Step 2: Find the *z* value corresponding to \$160.00.

$$z = \frac{X - \mu}{\sigma} = \frac{160.00 - 146.21}{29.44} = 0.47$$

Step 3: Find the area to the left of *z* = 0.47.

Table E gives us an area of .6808.
68% of women spend less than \$160.

Example 6-7a: Newspaper Recycling

Each month, an American household generates an average of 28 pounds of newspaper for garbage or recycling. Assume the standard deviation is 2 pounds. If a household is selected at random, find the probability of its generating between 27 and 31 pounds per month. Assume the variable is approximately normally distributed.

Step 1: Draw the normal distribution curve.

Example 6-7a: Newspaper Recycling

Step 2: Find *z* values corresponding to 27 and 31.

$$z = \frac{27-28}{2} = -0.5 \qquad z = \frac{31-28}{2} = 1.5$$

Step 3: Find the area between *z* = -0.5 and *z* = 1.5.

Table E gives us an area of 0.9332 – 0.3085 = 0.6247. The probability is 62%.

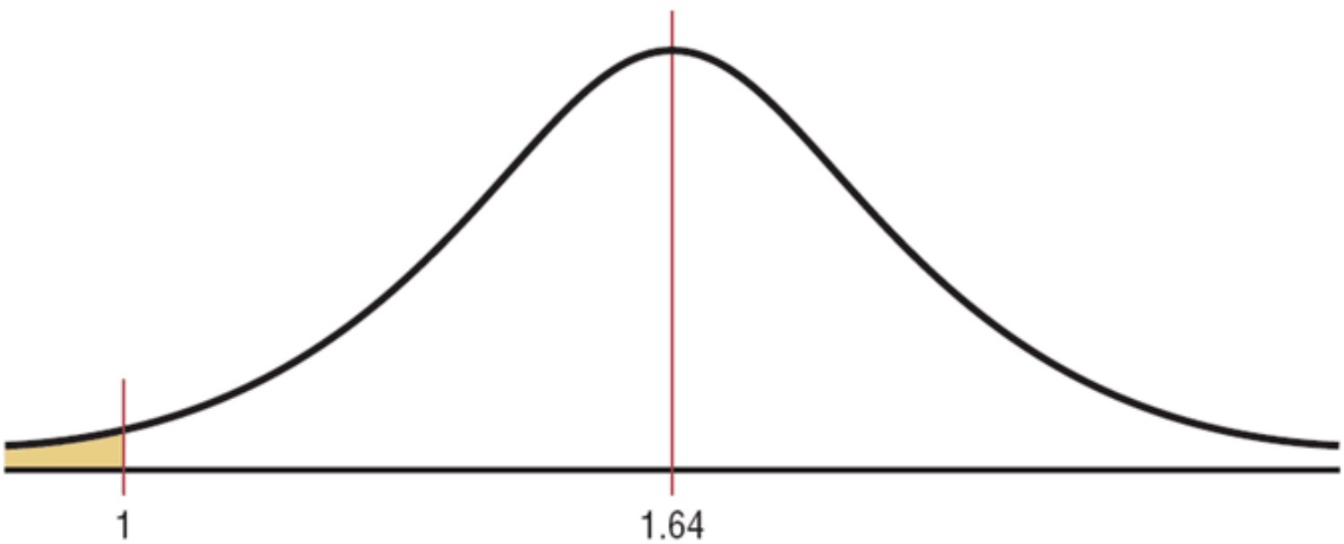
Example 6-8: Coffee Consumption

Americans consume an average of 1.64 cups of coffee per day. Assume the variable is approximately normally distributed with a standard deviation of 0.24 cup.

If 500 individuals are selected, approximately how many will drink less than 1 cup of coffee per day?

Example 6-8: Coffee Consumption

Step 1: Draw the normal distribution curve.



Example 6-8: Coffee Consumption

Step 2: Find the *z* value for 1.

$$z = \frac{1-1.64}{0.24} = -2.67$$

Step 3: Find the area to the left of $z = -2.67$. It is 0.0038.

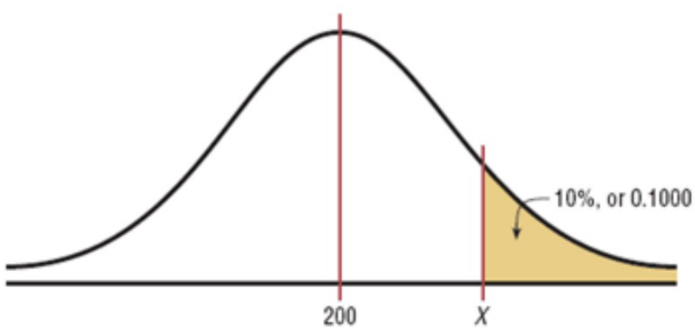
Step 4: To find how many people drank less than 1 cup of coffee, multiply the sample size 500 by 0.0038 to get 1.9.

Since we are asking about people, round the answer to 2 people. Hence, approximately 2 people will drink less than 1 cup of coffee a day.

Example 6-9: Police Academy

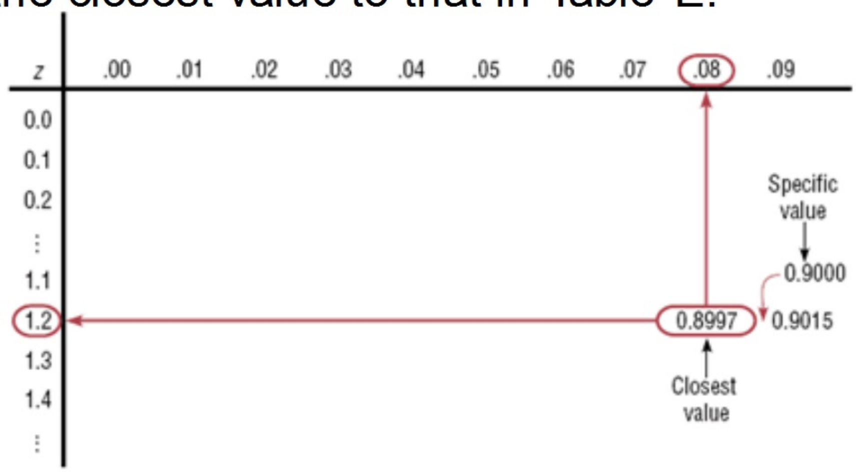
To qualify for a police academy, candidates must score in the top 10% on a general abilities test. The test has a mean of 200 and a standard deviation of 20. Find the lowest possible score to qualify. Assume the test scores are normally distributed.

Step 1: Draw the normal distribution curve.



Example 6-8: Police Academy

Step 2: Subtract 1 – 0.1000 to find area to the left, 0.9000. Look for the closest value to that in Table E.



Step 3: Find X.

$$X = \mu + z\sigma = 200 + 1.28(20) = 225.60$$

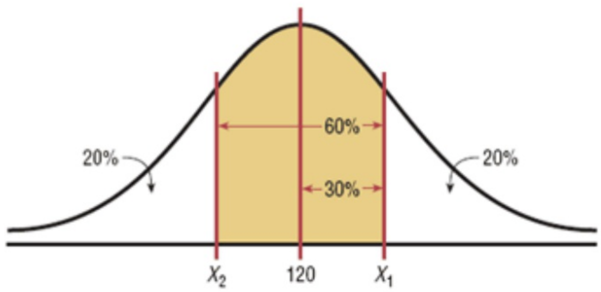
The cutoff, the lowest possible score to qualify, is 226.

Example 6-10: Systolic Blood Pressure

For a medical study, a researcher wishes to select people in the middle 60% of the population based on blood pressure. If the mean systolic blood pressure is 120 and the standard deviation is 8, find the upper and lower readings that would qualify people to participate in the study.

Step 1: Draw the normal distribution curve.

Example 6-10: Systolic Blood Pressure



Area to the left of the positive z: 0.5000 + 0.3000 = 0.8000.

Using Table E, z = 0.84.

$$X = 120 + 0.84(8) = 126.72$$

Area to the left of the negative z: 0.5000 – 0.3000 = 0.2000.

Using Table E, z = –0.84.

$$X = 120 - 0.84(8) = 113.28$$

The middle 60% of readings are between 113 and 127.

Normal Distributions

- A normally shaped or bell-shaped distribution is only one of many shapes that a distribution can assume; however, it is very important since many statistical methods require that the distribution of values (shown in subsequent chapters) be normally or approximately normally shaped.
- There are a number of ways statisticians check for normality. We will focus on three of them.

Checking for Normality

- Histogram
- Pearson's Index PI of Skewness
- Outliers
- Other Tests
    - Normal Quantile Plot
    - Chi-Square Goodness-of-Fit Test
    - Kolmogorov-Smikirov Test
    - Lilliefors Test

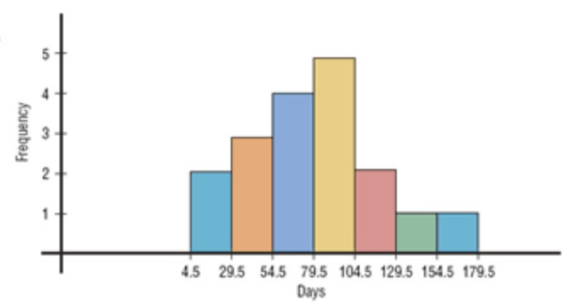
Example 6-11: Technology Inventories

A survey of 18 high-technology firms showed the number of days' inventory they had on hand. Determine if the data are approximately normally distributed.

5	29	34	44	45	63	68	74	74
81	88	91	97	98	113	118	151	158

Method 1: Construct a Histogram.

Class	Frequency
5–29	2
30–54	3
55–79	4
80–104	5
105–129	2
130–154	1
155–179	1



The histogram is approximately bell-shaped.

Example 6-11: Technology Inventories

Method 2: Check for Skewness.

$$\bar{X} = 79.5, MD = 77.5, s = 40.5$$

$$\text{PI} = \frac{3(\bar{X} - MD)}{s} = \frac{3(79.5 - 77.5)}{40.5} = 0.148$$

The PI is not greater than 1 or less than –1, so it can be concluded that the distribution is not significantly skewed.

Method 3: Check for Outliers.

Five-Number Summary: 5 - 45 - 77.5 - 98 - 158

Q1 – 1.5(IQR) = 45 – 1.5(53) = –34.5

Q3 + 1.5(IQR) = 98 + 1.5(53) = 177.5

No data below –34.5 or above 177.5, so no outliers.

Example 6-11: Technology Inventories

A survey of 18 high-technology firms showed the number of days' inventory they had on hand. Determine if the data are approximately normally distributed.

5	29	34	44	45	63	68	74	74
81	88	91	97	98	113	118	151	158

Conclusion:

- The histogram is approximately bell-shaped.
- The data are not significantly skewed.
- There are no outliers.

Thus, it can be concluded that the distribution is approximately normally distributed.