Random variable (r.v)

A r.v is a function that associates a real n.o with each element in the sample space

Q) Let X= no. of Heads in toss of 3 coins

$2^3 = 8$

x		P(x)	F(x)
0	← TTT	$1/8$	$1/8$
1	← HTT, THT, TTH	$3/8$	$4/8$
2	← HHT, THH, HTH	$3/8$	$7/8$
3	← HHH	$1/8$	$8/8$

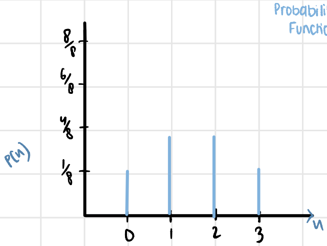
Cumulative Distribution function (CDF)

$F(X) = P(X \leq n)$

$$F(X) = \begin{cases} 0 & n < 0 & P(n<0) \\ 1/8 & 0 \leq n < 1/8 & P(n=0) \\ 4/8 & 1/8 \leq n < 4/8 & P(n=0)+P(n=1) \\ 7/8 & 4/8 \leq n < 7/8 & P(n=0)+P(n=1)+P(n=2) \\ 1 & 7/8 \leq n < 1 & P(n=0)+P(n=1)+P(n=2)+P(n=3) \end{cases}$$

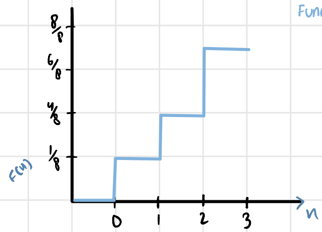
Line Graph → PMF → uses P(n)

Probability Mass Function



Line Graph → CDF → uses F(n)

Cumulative Function



Q) Let X= no. of Heads in toss of 4 coins

$2^4 = 16$

x		P(x)	F(x)
0	← TTTT	$1/16$	$1/16$
1	← HTTT, THTT, TTHT, TTTH	$4/16$	$5/16$
2	← HHTT, TTHH, HTHT, THTH	$4/16$	$9/16$
3	← THHH, HTHH, HHTH, HHHT	$4/16$	$13/16$
4	← HHHH	$1/16$	$14/16$

Q) find $P(n=2) = f(n=2)$ using CDF

$f(2) = F(2) - F(1)$

$= P(n \le 2) - P(n \le 1)$

$= 11/16 - 5/16$

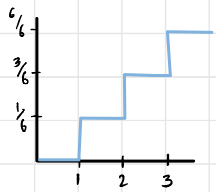
$= 3/8$

Q) PMF $f(n) = n/6$, $n = 1,2,3$

1. find CDF and sketch its graph
2. $P[1.5 < n \le 4.5]$

n	f(n)	F(n)
1	$1/6$	$1/6$
2	$2/6$	$3/6$
3	$3/6$	$6/6$

$$F(n) = \begin{cases} 0 & n < 1 \\ 1/6 & 1 \le n \le 2 \\ 3/6 & 2 \le n \le 3 \\ 6/6 & n \ge 3 \end{cases}$$



Q1) A fair coin is tossed until H appears for the first time. Find

a) PMF = $\left(\frac{1}{2}\right)^n$; $n = 1,2,3,\dots$

b) CDF = $\sum_{n=1}^{n=n} \left(\frac{1}{2}\right)^n$

c) $F(4) = \sum_{n=1}^{n=4} \left(\frac{1}{2}\right)^n$

Q2) CDF : $F(n) = 1 - \left(\frac{1}{2}\right)^{n+1}$ for $n = 0,1,2\dots$

a) $P(3) = F(1) - F(2)$

b) $P[7 \le n < 10] = F(9) - F(6)$

c) PMF = $F(n) - F(n-1)$

$= 1 - \left(\frac{1}{2}\right)^{n+1} - \left[1 - \left(\frac{1}{2}\right)^{n-1+1}\right]$

$= \left(-\frac{1}{2}\right)^{n+1} + \left(\frac{1}{2}\right)^n$

$= \left(\frac{1}{2}\right)^n \left(\frac{1-1}{2}\right)$

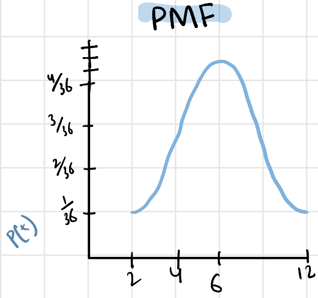
$= \frac{1}{2^{n+1}}$

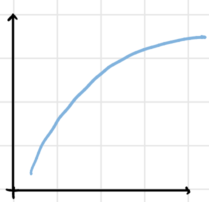
Q) if two dice are rolled once, find the PMF of the sum of points on dice, CDF and their graph

let $t = x + y$

z	2	3	4	5	6	7	8	9	.....	12
$f(t) = P(t)$	$\frac{1}{36}$	$\frac{2}{36}$	$3/36$	$4/36$	$5/36$	$6/36$	$7/36$	$8/36$	.....	
$F(t)$	$\frac{1}{36}$	$3/36$	$6/36$	.....						$36/36$

$$F(t) = \begin{cases} 0 & z < 2 \\ 1/36 & 2 \le z \le 3 \\ 3/36 & 3 \le z \le 4 \\ \vdots & \\ 36/36 & z \ge 12 \end{cases}$$





Q3) A coin is biased so that head across 3 times of tails.

If the coin is tossed 3 times, find the prob dist. for the no of heads $P[1 \le n \le 3]$

H	T
$\frac{3}{4}$	$\frac{1}{4}$

0 { TTT $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times 1$

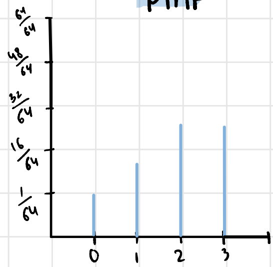
1 { HTT $\frac{3}{4} \times \frac{1}{4} \times \frac{1}{4} \times 3$
THT
TTH

2 { HHT $\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} \times 3$
HTH
THH

3 { HHH $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times 1$

X	P(n)
0	$\frac{1}{64}$
1	$\frac{9}{64}$
2	$\frac{27}{64}$
3	$\frac{27}{64}$

PMF



4) Determine the value of c so that the f(n) becomes PMF

$f(n) = c(n^2+4)$ for $n = 0,1,2,3$

$$\sum_{n=0}^{3} c(n^2+4) = 1 \qquad c = \frac{1}{30}$$

* Expected Value

let suppose a coin tossed 2 times

X = Head calculate

E(X) = Mass of a rov X

X	P(x)	XP(x)	$x^2P(x)$
0	$\frac{1}{4}$	0	0
1	$\frac{2}{4}$	$\frac{2}{4}$	$\frac{3}{4}$
2	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{4}{4}$
Sum	1	E(x) = 1	$E(x^2) = \frac{3}{2}$

$E(x) = \sum (x) P(x) \longrightarrow$ Expected value

$\sigma^2 x = V(x) = E(x^2) - [E(x)]^2 \longrightarrow$ Variance

$3B, 2R, 3G \Longrightarrow 8 \rightarrow 2$ w/o replacement

$X = B$ & $Y = R$

JOINT PROB MASS FUNCTION

$x \backslash y$	0	1	2	Total (Marginal dist $g(x)$)
0	$\frac{{}^3C_0\,{}^2C_0\,{}^3C_2}{{}^8C_2}$	$\frac{{}^3C_0\,{}^2C_1\,{}^3C_1}{{}^8C_2}$	$\frac{{}^3C_0\,{}^2C_2\,{}^3C_0}{{}^8C_2}$	$\frac{10}{28}$
1	$\frac{{}^3C_1\,{}^2C_0\,{}^3C_1}{{}^8C_2}$	$\frac{{}^3C_1\,{}^2C_1\,{}^3C_0}{{}^8C_2}$	—	$\frac{15}{28}$
2	$\frac{{}^3C_2\,{}^2C_0\,{}^3C_0}{{}^8C_2}$	—	—	$\frac{1}{28}$
$h(y)$ Total marginal dist of y	$\frac{13}{28}$	$\frac{12}{28}$	$\frac{1}{28}$	1

MATHEMATICAL EXPECTATION

Mean: $E(x) = \Sigma x f(x)$

$E[\ ] = \Sigma[\ ] f(x)$

x	0	1	2	
$g(x)$	$\frac{10}{28}$	$\frac{15}{28}$	$\frac{3}{28}$	1
$x\,g(x)$	0	$\frac{15}{28}$	$\frac{6}{28}$	$\frac{21}{28} = E(x)$
$x^2 g(x)$	0	$\frac{15}{28}$	$\frac{12}{28}$	$\frac{27}{28} = E(x^2)$

$E(x^4)$

$E(\sqrt{x})$

$E[3x] = 3E(x)$

$V(x) = \sigma_x^2 = E(x^2) - [E(x)]^2$

$= \frac{27}{28} - \left(\frac{21}{28}\right)^2$

a) $P[X \leq 2, Y = 1] = f(0,1) + f(1,1) + f(2,1)$
$= \frac{6}{28} + \frac{6}{28} + \frac{3}{28}$
$= \frac{12}{28}$

f) $P[Y=1 / X=0] = \frac{f(y=1, x=0)}{g(x=0)}$
$= \frac{6/28}{10/28}$

b) $P[X \geq 2, Y \leq 1] = f(2,1) + f(2,0)$

g) $E[XY] = \Sigma\Sigma\, xy f(x,y) = \frac{6}{28}$

c) $P[X > Y] = f(1,0) + f(2,0)$

h) Covariance $(x,y) = E(XY) - E(X).E(Y)$
$= \frac{6}{28} - \left(\frac{21}{28}\right)\left(\frac{14}{28}\right)$
$= -\frac{9}{56}$

d) $P[X+Y=4] = 0$

Conditional Probability

e) $P[X=0 / Y=1] = \frac{f(x=0, y=1)}{h(y=1)}$
$= \frac{6/28}{12/28} = \frac{6}{12}$

$-1 \leq$ Corelation $(X,Y) = \frac{\text{Covariance}(x,y)}{\sqrt{V(X)}\sqrt{V(Y)}} \leq +1$
$= \frac{-9/56}{\sqrt{\frac{45}{112}}\sqrt{\frac{9}{28}}}$
$= -0.441$ moderate -ve relationship

$\pm 1 \longrightarrow$ strong

$\pm 0.5 \longrightarrow$ moderate

$0 \longrightarrow$ negligible

JPMF

$f(x,y) = \frac{x+y}{30}$ for $x = 0,1,2,3$

$y = 0,1,2$

Calculated

a) $P[X \le 2, Y = 1] = 6/30$

b) $P[X > 2, Y \le 1] = 7/30$

c) $P[X > Y] = 18/30$

d) $P[X + Y = 4] = 8/30$

f) " covariance (X,Y)

g) " correlation (X,Y)

e) Calculate $E(X)$, $E(X^2)$, $V(X)$, $V(Y)$,

$E(X) = 60/30 = 2$, $E(X^2) = \frac{150}{30} = 5$, $V(X) = 5 - (2)^2 = 1$, $V(Y) = 66/30 - \left(38/30\right)^2 = 134/225$

marginal Dist X

X \ Y	0	1	2	g(x)	xg(x)	$x^2 g(x)$
0	0	1/30	2/30	3/30		
1	1/30	2/30	3/30	6/30		
2	2/30	3/30	4/30	9/30		
3	3/30	4/30	5/30	12/30		
h(y)	6/10	10/30	14/30	1	E(x)= 2	E(x²)= 5
yh(y)				E(y):		
y²h(y)				E(y²):		

$E(XY) = \sum\sum xy\, f(x,y) = 2.4$

00	0
01	$1/30 \times 0$
02	$2/30 \times 0$
10	$1/30 \times 0$
11	$2/30 \times 1$
12	$3/30 \times 2$
20	$2/30 \times 0$
21	$3/30 \times 2$
22	$4/30 \times 4$
30	$3/30 \times 0$
31	$4/30 \times 3$
32	$5/30 \times 6$

$V(X) = E(X^2) - [E(X)]^2 = 1$

$V(Y) = E(Y^2) - [E(Y)]^2 = 0.59$

$E(XY) = \sum\sum xy\, f(x,y) = 2.4$

Covariance $(X,Y) = E(XY) - E(X).E(Y) = 2.4 - 2\left(38/30\right) \rightarrow -2/15 \rightarrow -0.133$

Corelation $(X,Y) = \dfrac{\text{Covariance }(X,Y)}{\sqrt{V(X)}\ \sqrt{V(Y)}} = -0.17277$

↳ weak relation

↳ inverse relation as -ve sign