Date:

# Binomial Experiment

Bernoulli trial

Segment repeat (trails)

* n repeated trialls
* probablity of trails each remains constant
* Each trail has two out comes P (success) Q (fail)

outcomes.

$P+q=1$

* trails are independent.

Ex: Tossing of coin 10 times:

- $n=10$ repeated toss
- two outcomes : head and tail $P \rightarrow h$ $q \rightarrow t$ $0.5 + 0.5 = 1$
- probablity remains same,
  everytime 0.5 remains
- independent trails

∴ hence these are binomial experiment

Ex: 75% instagram users.

- 5 students repeated
- Yes/no (2)
- Yes $\rightarrow p \rightarrow 0.75$ No $\rightarrow q \rightarrow 0.25$
  ↳ for each
- independent

∴ hence binomial experiment.

Ex 9 out of 12 users, 3 selected

- 3 trails
- 2 outcomes
- 1. $9/12$ then $8/11$ then $7/10$ so not constant
- dependent

∴ not binomial experiment

n and p } binomial parameters

Date: 14-3-23

**b.p.d.f.** exactly x success in n trails

$$\boxed{B(x) = {}^nC_x \, p^x q^{n-x}}$$

p → prob of success
x → no. of success
n → trials
q → Prob. of fail

Ex 1

n = 10, x = 3 (if 3 heads), p = 0.5, q = 0.5

$B(3) = {}^{10}C_3 (0.5)^3 (0.5)^7$
$= 0.11719$

Ex 2

n = 5, x = 3, p = 0.75, q = 0.25

$B(3) = {}^5C_3 (0.75)^3 (0.25)^2$
$= 0.26367$

if exactly so use B(x) formula.

if at most x so $B(x) = \sum_{x=0}^{m} {}^nC_x \, p^x q^{n-x}$

if at least x so $B(x) = \sum_{x=m}^{n} {}^nC_x \, p^x q^{n-x}$ or $1 - \sum_{x=0}^{m-1}$

Ex

n = 3

p = 0.33

q = 1 − p.

| x | P(X=x) = f(x) |
|---|---|
| 0 | 0.301 |
| 1 | 0.4441 |
| 2 | 0.21889 |
| 3 | 0.03594 |

$P(X=x) = {}^nC_x \, p^x q^{n-x}$

At x = 0

$P(X=0) = {}^3C_0 (0.33)^0 (1-0.33)^3$

At x = 1

$P(X=1) = {}^3C_1 (0.33)^1 (0.67)^2$

At x = 2

$P(X=2) = {}^3C_2 (0.33)^2 (0.67)^1$

At x = 3

$= {}^3C_3 (0.33)^3 (0.67)^0$

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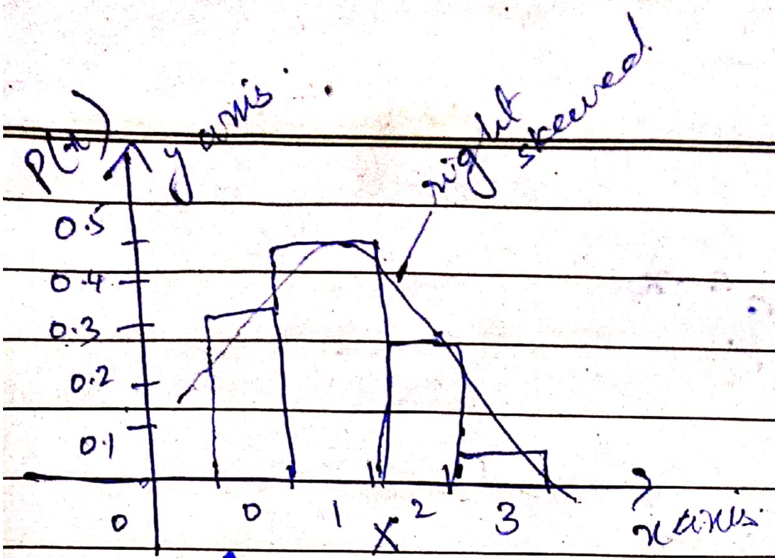



$\mu = 0.99$
$\sigma = 0.814$
average people who take interest.

$$\text{Mean} = \mu = np$$

$$\sigma = \sqrt{npq}$$

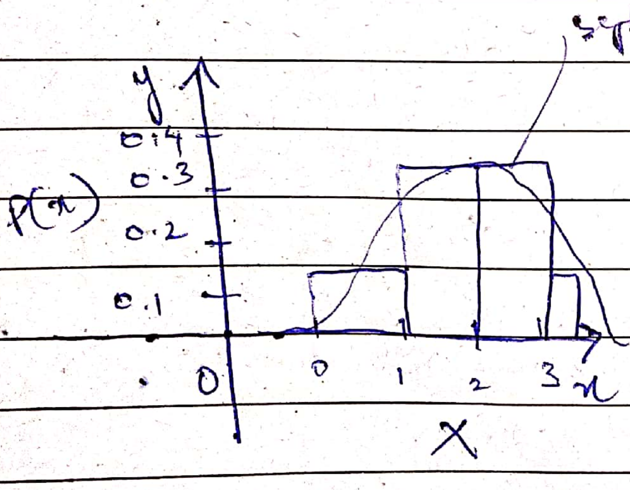
**Ex**

$n = 3$ (coin toss)

$p = 0.5 \quad q = 0.5$

| $x$ | $p(x = X) = f(x)$ |
|---|---|
| 0 | 0.125 |
| 1 | 0.375 |
| 2 | 0.375 |
| 3 | 0.125 |

$P(X=0) = {}^3C_0 \, p^0 q^3$
$= {}^3C_0 (0.5)^0 (0.5)^3$
$P(X=1) = {}^3C_1 (0.5)^1 (0.5)^2$
$P(X=2) = {}^3C_2 (0.5)^2 (0.5)^1$
$P(X=3) = {}^3C_3 (0.5)^3 (0.5)^0$




$\mu = 1.5$
$\sigma = 0.866.$

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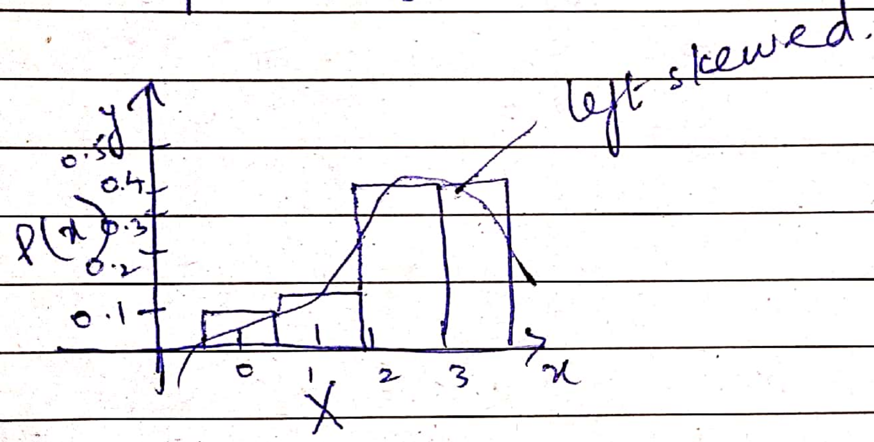
**Ex** $n = 3$, $p = 0.75$, $q = 0.25$

$P(X=0) = {}^3C_0 (0.75)^0 (0.25)^3$
$P(X=1) = {}^3C_1 (0.75)^1 (0.25)^2$
$P(X=2) = {}^3C_2 (0.75)^2 (0.25)^1$
$P(X=3) = {}^3C_3 (0.75)^3 (0.25)^0$

| $x$ | $f(x) = P(X=x)$ |
|---|---|
| 0 | 0.01563 |
| 1 | 0.14063 |
| 2 | 0.42188 |
| 3 | 0.42188 |

$\mu = 2.25$
$\sigma = 0.75$




If success $< 0.5$
so right skewed
$p = 0.5$
so symmetric
$p > 0.5$
so left skewed.

⇒ If unstable probability:-
→ trails not independent.

## Hypergeometric distribution:-

$N \rightarrow$ size of population
$r \rightarrow$ no. of success in pop.
$N - r \rightarrow$ no. of failures pop
$n - x \rightarrow$ no. of failures in sample
sample size $\leftarrow n \rightarrow$ no. of trials samples
$x \rightarrow$ no of success in sample.

$$P(x) = \frac{{}^rC_x \; {}^{N-r}C_{n-x}}{{}^NC_n}$$

(sample size)

} first 2 fulfill
3rd not constant

1st two fulfill,

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Ship ment

$P(3) = ?$

$N = 25$

$r = 20$

$N-r = 5$

$n = 4$ (sample size). selection

$x = 3$

$n-x = 1$

$$P(x) = \frac{{}^{r}C_x \cdot {}^{N-r}C_{n-x}}{{}^{N}C_n} = \frac{{}^{20}C_3 \ {}^{5}C_1}{{}^{25}C_4} = 0.45$$

**Ex**

12 heads

7 females 5 males

a) select 3 so check if all are females?
find probability. that all three are females.

so

$N = 12$ $x = 3$

$n = 3$

$r = 7$ $$P(3) = \frac{{}^{7}C_3 \ {}^{12-7}C_{3-3}}{{}^{12}C_3} = 0.15909$$

b) Find prob. that atmost one is F atmost.

$$P(x \leq 1) = \underbrace{P(0)}_{sum1} + \underbrace{P(1)}_{sum2} = 0.36345$$

for sum1 $N=12, r=7, n=3, x=0$

$$\therefore P(0) = \frac{{}^{7}C_0 \ {}^{5}C_3}{{}^{12}C_3} = 0.045$$

for sum2. $N=12, r=7, n=3, x=1$

$$P(1) = {}^{7}C_1 \ {}^{12-7}C_{3-1} / {}^{12}C_3 = 0.318$$

repeat two things in times.

Date:

# Poison Probablity Distribution:-

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$\lambda \rightarrow$ poison parameter

no specified time

Conditions:

1. random variable is discrete.
2. The occurrence are random and indepedent
3. The average no. of occurrence in some interval is given.

$\lambda \rightarrow$

**Ex** 9.5 calls per week  2 weeks (multiply by 2).

Find the probablity that exactly 6 maker in a week.

hence, $\lambda = 9.5$
$x = 6$

$$P(x) = \frac{(9.5)^6 e^{-9.5}}{6!} = 0.07642$$

**Ex** 3 times per month, calculate 12 times per year.

$\lambda = 3 \times 12 = 36$
$x = 12$

$$P(12) = \frac{36^{12} e^{-36}}{12!} = 2.2 \times 10^{-6}$$

$$P(x \leq 12) = \sum_{x=0}^{12} P(x).$$

**Ex** find the probablity of $x$ occurrences in any event in some interval. (general).

$$\boxed{\text{Mean} = \lambda \qquad \sigma = \sqrt{\lambda}}$$

can be -ve values.

compare means.

$x = 6$ so $\bar{x} = \lambda$

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**Def:-** for continuous random variable $X$, $f(x)$ is p.d.f, if.

1. $f(x) \geq 0, \quad x \in \mathbb{R}$

2. $\int_{-\infty}^{\infty} f(x).dx = 1$

3. $P(a < X < b) = \int_a^b f(x).dx. \rightarrow$ not see usually

**Remarks:-**

1. $P(X = x) = 0$
2. $P(a \leq X < b) = P(X = a) + P(a < X < b)$
$= P(a < X < b).$

**Ex:-** $f(x) = \begin{cases} \frac{x^2}{3} & , -1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$

a) is $f(x)$ in p.d.f?
b) $P(0 \leq x < 1)$.

a) 1. Since for all $x \in \mathbb{R}$, $f(x) \geq 0$

2. $\int_{-\infty}^{\infty} f(x).dx. = \int_{-\infty}^{-1} 0\, dx + \int_{-1}^{2} \frac{x^2}{3}.dx + \int_{2}^{\infty} 0\, dx$

$= 0 + \frac{x^3}{9}\Big|_{-1}^{2} + 0$

$= \frac{1}{9}\{8 - (-1)^3\} = \frac{9}{9} = 1.$

b). $P(0 \leq x < 1) = \int_0^1 \frac{x^2}{3}.dx$

$= \frac{x^3}{9}\Big|_0^1 = 1/9$

Date: 21-03-23

c.d.f

Def $F(x) = P(X \leq x)$

$$\boxed{= \int_{-\infty}^{+x} f(t) \cdot dt}$$

$-\infty < x < +\infty$

dummy $\to t$

here, $\int_{-\infty}^{x} f(t)\,dt$ so $\int_{-1}^{2} \frac{t^2}{3}\,dt = \frac{x^3}{9}\Big|_{-1}^{2} = m$

c.d.f $F(x) = \begin{cases} 0 & x < -1 \\ m & -1 < x < 2 \\ 1 & x \geq 2 \end{cases}$

Joint Probablity Function:-

$X = x$ $\quad F(x, y)$ :-

$Y = y$

i) $f(x,y) \geq 0$ for all $(x,y) \in \mathbb{R}^3$

ii) $\sum_x \sum_y f(x,y) = 1$ {for discrete countable}

iii) $P(X=x, Y=y) = f(x,y)$

Ex

BOX: 3B, 2R, 3G

Select 2 pens.

$X \to B$

$Y \to R$

$Z \to G$

$f(x,y) = \frac{{}^3C_x \, {}^2C_y \, {}^3C_3}{{}^8C_2}$

$z = 2-x-y$

$$f(x,y) = \frac{{}^3C_x \, {}^2C_y \, {}^3C_{2-x-y}}{{}^8C_2}$$

$x = 0, 1, 2$

$y = 0, 1, 2$

So

| $y$ \ $x$ | 0 | 1 | 2 | |
|---|---|---|---|---|
| 0 | 0.1 | 0.2 | 0.035 | h(0) |
| 1 | 0.3 | 0.2 | 0 | h(1) |
| 2 | 0.1 | 0 | 0 | h(2) |
| | g(0) | g(1) | g(2) | |

marginal probablity

in correct entries.

Date:

$h(y) = \sum_{x} f(x, y)$ $\qquad$ $g(x) = \sum f(x, y)$.

if Marginal distribution of $X$ so,

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $g(x)$ | $g(0)$ | $g(1)$ | $g(2)$ |

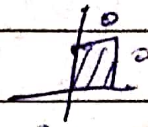
**for continuous:-**

1. $f(x,y) \geq 0$ for all $(x, y) \in \mathbb{R}^2$
2. $\int_{-\infty}^{+\infty} \int_{-\infty}^{\infty} f(x,y) \cdot dy \cdot dx$
3. $P[(x, y) \in A] = \iint_A f(x,y) \cdot dy \cdot dx$

$A \to$ region. $\qquad$ $x, y$ is not interval but a region.

**Ex**

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y), & 0 \leq x \leq 1 \\ & 0 \leq y \leq 1 \\ 0 & , \text{otherwise} \end{cases}$$



i) Since for all $(x, y) \in \mathbb{R}^2$, $f(x, y) \geq 0$

ii) Now,

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \cdot dy \cdot dx.$$

$$= \int_{x=0}^{1} \int_{y=0}^{1} \frac{2}{5}(2x + 3y) \cdot dy \cdot dx$$

$$= \frac{2}{5} \int_{x=0}^{1} \left\{ 2xy + \frac{3y^2}{2} \right\} \Big|_0^1 \cdot dx$$

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$$= \frac{2}{5}\int_0^1 \left(2x + \frac{3}{2}\right) dx$$

$$= \frac{2}{5}\left[\frac{2x^2}{2} + \frac{3}{2}x\right]_0^1$$

$$= \frac{2}{5}\left\{1 + \frac{3}{2}\right\}$$

$$= 1$$

Let $A = \{(x,y) \mid 0 < x < \frac{1}{2}, \frac{1}{2} < y < \frac{1}{2}\}$

find $P[(x,y) \in A]$

$$= \int_0^{1/2}\int_{\frac{1}{4}}^{1/2} f(x,y)\cdot dy\, dx$$

$$= \int_0^{1/2} \left\{\frac{2}{5}\left(2xy + \frac{3y^2}{2}\right)\right\}_{y=\frac{1}{4}}^{1/2} \cdot dx$$

$$= \frac{2}{5}\int_0^{1/2} \left(x + \frac{3}{8} - \frac{x}{2} - \frac{3}{32}\right)\cdot dx$$

$$= \frac{2}{5}\left[\frac{x^2}{2} + \frac{3x}{8} - \frac{x^2}{4} - \frac{3x}{32}\right]_0^{1/2}$$

$$= \frac{2}{5}\left\{\frac{1}{8} + \frac{3}{8\times 2} - \frac{1}{16} - \cancel{\frac{3}{32}} - \cancel{\frac{3}{8\times 0}} + \cancel{\frac{3}{32}}\right\} =$$

$$= \frac{2}{5}\left(\frac{1}{8} - \frac{1}{16}\right) = \frac{\cancel{2}}{5}\left(\frac{1}{\cancel{16}_8}\right) = \frac{1}{40}$$

$$= \frac{2}{5}\left\{\frac{1}{8} + \frac{3}{16} - \frac{1}{16} - \frac{3}{64}\right\} = \frac{13}{160}$$

Date:

The expected value of X if p.d.f in f(x).

$\mu = E(X) = \sum_{x} x f(x)$, if X is discrete.

$\mu = E(X) = \int_{-\infty}^{\infty} x f(x) \cdot dx$ if X is continuous.

Ex

7

4G 3D

Select 3

$f(x) = \frac{{}^{4}C_{x} \, {}^{3}C_{3-x}}{{}^{7}C_{3}}$, $x = 0, 1, 2, 3$

$\mu = \sum_{x=0}^{3} x f(x) = 0 \cdot f(0) + 1 \cdot f(1) + 2 \cdot f(2) + 3 f(3)$

Ex.- $f(x) = \begin{cases} \frac{20,000}{x^3} & , x > 100 \\ 0 & , \text{else} \end{cases}$

$\mu = \int_{-\infty}^{\infty} x f(x) \cdot dx$

$\mu = \int_{-\infty}^{100} x \, f(x) \cdot dx + \int_{100}^{+\infty} x \cdot f(x) \cdot dx$

$\mu = \int_{-\infty}^{100} \cancel{x \cdot 20,} (0) \cdot dx + \int_{100}^{+\infty} \cancel{x} \cdot \frac{20,000}{x^{\cancel{3}2}} dx$

0

$\mu = 0 + \int_{100}^{+\infty} \frac{x^{-1}}{-1} 20,000 \cdot dx$

$\mu = -20000 \left[ \frac{1}{\infty} - \frac{1}{100} \right]$

$\boxed{\mu = 200}$

Date:

**Def** X is a random variable :- $g(x)$

$$\mu_{g(x)} = E(g(x)) = \sum_x g(x) \cdot f(x)$$

$$\mu_{g(x)} = E[g(x)] = \int_{-\infty}^{+\infty} g(x) \cdot f(x) \cdot dx.$$

**Ex**

| $x$ | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|
| $f(x)$ | 1/12 | 1/12 | 1/4 | 1/4 | 1/6 | 1/6 |

$g(x) \to$ dependent on $f(x)$.

$g(x) = 2x - 1$

$$\mu_{g(x)} = E[g(x)] = \sum_{x=4}^{9} g(x) \cdot f(x).$$

$$= g(4) \cdot f(4) + g(5) \cdot f(5) + g(6) \cdot f(6) + g(7) \cdot f(7) + g(8) \cdot f(8) + g(9) \cdot f(9).$$

**Joint:-**

$$\mu_{g(x,y)} = E[(g(x,y))] = \sum_x \sum_y g(x,y) f(x,y).$$

$$\mu_{g(x,y)} = E[g(x,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) \cdot f(x,y) \cdot dx.$$

$f(x,y)$.

| Y \ X | 0 | 1 | 2 | |
|---|---|---|---|---|
| 0 | 3/28 | 9/28 | 3/28 | h(0) |
| 1 | 3/14 | 3/14 | 0 | h(1) |
| 2 | 1/28 | 0 | 0 | h(2) |
| | g(0) | g(1) | g(2) | |

$g(x,y) = X \cdot Y.$

$\frac{1}{28} + \frac{15}{28} + \frac{6}{14}$ Marginal helps to transfer in single classified

Date:

$\mu_{g(x,y)} = E[g(X,Y)]$ · $\boxed{g(x,y) = X \cdot Y}$

$\Rightarrow \sum_x \sum_y g(x,y) \cdot f(x,y)$

$= g(0,0) f(0,0) + g(0,1) \cdot f(0,1) + g(0,2) f(0,2) + \cdots + g(2,2) \cdot f(2,2)$

$= g(1,1) \cdot f(1,1)$

$= \frac{3}{14}.$

$\mu_x = E(x) = \sum_x x g''(x)$   g prime for marginal distribution.

$\mu_y = E(Y) = \sum_y y h(y)$

Ex $f(x,y)$ (1,2) not (2,1) use from up.

**Variance** Let X be a random variable with p.d.f. $f(x)$

variance $\Rightarrow \sigma^2 = E(X-\mu)^2$

$= \sum_x (x-\mu)^2 f(x)$. discrete

$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x) \cdot dx$   X is continuous

$\sum (x-\mu)^2 \cdot f(x) \Rightarrow \boxed{\sigma^2 = E(X^2) - \mu^2}$

$(x^2 - 2x\mu + \mu^2) f(x)$.

$= \sum \{ f(x) \cdot x^2 - 2 f(x) \cdot x \cdot \mu + f(x) \cdot \mu^2 \}$

$= \sum f(x) \cdot x^2 - 2 \sum f(x) \cdot x \mu + \sum f(x) \cdot \mu^2$

$= E(x^2) - 2\mu^2 + \mu^2 (1) = E(X^2) - \mu^2$

Cases of covariance, when zero?

$y = y_c + y_p$ = Sin + Cos x

$y(0) = 1$ $y'(0) = \cos(0) + 0$

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$y'(0) = 1$

A | B

| $x$ | $f(x)$ |
|---|---|
| 1 | 0.3 |
| 2 | 0.4 |
| 3 | 0.3 |

| $x$ | $f(x)$ |
|---|---|
| 0 | 0.2 |
| 1 | 0.1 |
| 2 | 0.3 |
| 3 | 0.3 |
| 4 | 0.1 |

$\mu_A = 0.2$ $\mu_B = 0.2$ $\mu = \sum x \cdot f(x)$.

$\sigma^2_A$ $\sigma^2_B$.

$\sigma^2 = E(X^2) - \mu^2$

$E(X^2) = \sum x^2 \cdot f(x)$ so

$\sigma^2_A = 0.6$ $\sigma^2_B = 1.6$

This shows, mean is same but variance can't be same.

Covariance

Joint variable :-

$\sigma_{xy} = E[(X - \mu_x)(Y - \mu_y)]$ $\mu_x \Rightarrow$ marginal

$= \sum\sum (x - \mu_x)(y - \mu_y) f(x, y)$

marginal density function

| $x$ | $g(x)$ or | $h(y)$ |
|---|---|---|
| | | |

$= \sum\sum xy\, f(xy) - \mu_x \sum\sum y\, f(xy) - \mu_y \sum\sum x\, f(x, y) + \mu_x \cdot \mu_y \sum f(x, y)$.

$= E(XY) - \mu_x \cdot \mu_y - \mu_y \mu_x + \mu_x \mu_y$

$$\boxed{\sigma_{x \cdot y} = E(X \cdot Y) - \mu_x \cdot \mu_y}$$

$$\boxed{\rho = \frac{\sigma_{x \cdot y}}{\sigma_x \sigma_y}}$$ correlation coefficient

$\sigma^2 = SD.$

Page [ ] Victory

| $f(x,y)$ | $x=0$ | $1$ | $2$ | |
|---|---|---|---|---|
| $y=0$ | $3/28$ | $9/28$ | $3/28$ | $15/28$ |
| $1$ | $3/14$ | $3/14$ | $0$ | $6/14$ |
| $2$ | $1/28$ | $0$ | $0$ | $1/28$ |
| | $g(0)=\frac{10}{28}$ | $g(1)=\frac{15}{28}$ | $3/28$ | |

Q. find $\sigma_{xy}$?

Using, $\sigma_{xy} = E(XY) - \mu_x \mu_y$

$E(XY) = \frac{3}{14}$ $\sum\sum xy\, f(x,y) =$

$\mu_x = \Sigma x \cdot g(x) = 1 \cdot g(1) + 2g(2) = \frac{15}{28} + \frac{6}{28} = \frac{21}{28} = \frac{3}{4}$

$\mu_y = \Sigma y\, h(y) = 1(h(1)) + 2(g(2))$

$= \frac{\cancel{15}}{28} + \cancel{2} = \frac{6}{14} + \frac{1}{14} = \frac{7}{14} = \frac{1}{2}$

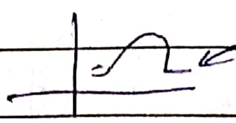
$\sigma_{x.y} = \frac{3}{14} - \frac{1}{2} \cdot \frac{21}{28}$

$\sigma_{xy} = \frac{12 - 21}{56} = -\frac{9}{56}$

## 4. Normal probablity distribution :-

→ bell shaped curve of continious function

1. Area under curve is 1.0
2. Curve is symmetric about the mean.



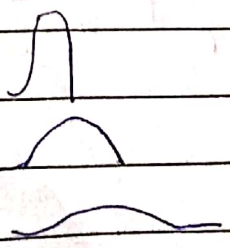
not touch

$-\infty$, to $+\infty$

$\sigma = 5$

$\sigma = 10$

$\sigma = 16 =$



} with same mean

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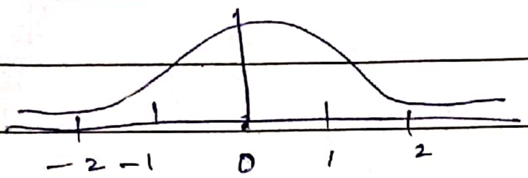
$\mu = 0$ , $\sigma = 1$ is called the standard normal distribution.




use $x \Rightarrow z$ (bottom values represented by $z$)

$z = 1.95$

go to 1.9 and see 0.05

Examples:-

1. .... left of $z = a$.
2. ... from $z = a$ to $z = b$ $\rightarrow$ $T(b) - T(a)$.
3. ...... right of $z = a$. $\rightarrow$ $1 - T(z = a)$

can be for probability.

if $P(1.19 \leq x < 2.12)$

so zero of this side

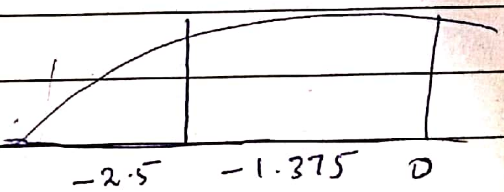
## Converting to a standard normal distribution:-

$$\boxed{z = \frac{x - \mu}{\sigma}}$$

$z < 0.50$.

$\frac{30-50}{8} = \frac{-\cancel{20}^{\cancel{10}\,5}}{\cancel{8}\,\cancel{4}\,2} = \frac{-5}{2}$

$= \frac{39-50}{8} = \frac{-11}{8} = -1.375$




$-2.5 \leq z \leq -1.375$ $\mu = 0$

$= P(a \leq Z \leq b) =$

$= P(a < z < b) + \cancel{P(a)}^{0} + \cancel{P(b)}^{0}$

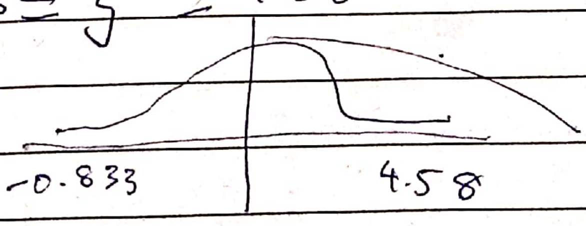
$= T(b) - T(a)$

(2) $\frac{70-80}{12}$

$= -\frac{\cancel{10}\,5}{\cancel{12}\,6} = -\frac{5}{6}$

$\frac{135-80}{12} = \frac{55}{12}$

$-0.833 \leq z \leq 4.58$



$\rho = 0$, no relation (linear). not
can be other.

Date:

|  |  | $x$ |  |  |
|---|---|---|---|---|
|  |  | 0 | 1 |  |
| $y$ | 0 | 0.5 | 0.2 | $h(0)$ |
|  | 1 | 0.2 | 0.1 | $h(1)$ |
|  |  | $g(0)$ | $g(1)$ |  |

find $\rho_{xy}$ so,

$\rho_{xy} = \dfrac{\sigma_{xy}}{\sigma_x \cdot \sigma_y}$ — (i)

$h(0) = 0.7$ $\quad h(1) = 0.3$

$g(0) = 0.7$ $\quad g(1) = 0.3$

| $x$ | 0 | 1 |
|---|---|---|
| $g(x)$ | 0.7 | 0.3 |

| $y$ | 0 | 1 |
|---|---|---|
| $h(y)$ | 0.7 | 0.3 |

$\mu_x = \Sigma x\, g(x)$

$\boxed{\mu_x = 0.3}$

$\mu_y = \Sigma y\, h(y)$

$\boxed{\mu_y = 0.3}$

$\sigma_x^2 = E(X^2) - \mu_x^2$

$= \Sigma (x^2 \cdot g(x)) - 0.3^2$

$= 0.3 - 0.3^2$

$\sigma_x^2 = 0.21$

$\boxed{\sigma_x = 0.458}$

$\sigma_y^2 = E(Y^2) - \mu_y^2$

$= 0.3 - 0.3^2$

$\sigma_y^2 = 0.21$

$\boxed{\sigma_y = 0.458}$

Now, $\sigma_{xy} = E(X \cdot Y) - \mu_x \cdot \mu_y$

$= \Sigma\Sigma\, xy \cdot f(x, y) - \mu_x \mu_y$

$= 0.1 - 0.3^2 = +0.01 = 0.01$

$\rho_{xy} = \dfrac{0.01}{0.458^2}$

$\boxed{\rho_{xy} = 0.05}$

6,7,8

Date:

| $x$ | 0 | 1 |
|---|---|---|
| $f(x)$ | 0.7 | 0.3 |

$$E(z) = \sum z \cdot f(x)$$

general form

$$\sigma^2_{g(x)} = E(g(x)^2) - \mu^2_{g(x)}$$

$$\mu_{g(x)} = \sum g(x) \cdot f(x).$$

$$f(x,y) = \frac{x(1+3y^2)}{4} \qquad 0 < x < 2, \quad 0 < y < 1$$

$$0 \qquad \text{otherwise.}$$

find $E(Y/X) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{y}{x} f(x,y) \cdot dx \cdot dy.$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{y}{x} \frac{x(1+3y^2)}{4} \cdot dx \cdot dy.$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{y(1+3y^2)}{4} \cdot dx \cdot dy.$$

limit of piece

$$= \int_0^1 \int_0^2 \frac{y(1+3y^2)}{4} \cdot dx \cdot dy.$$

$$= \int_0^1 \frac{y(1+3y^2)}{4} [x]_0^2 \cdot dy$$

$$= \frac{1}{\cancel{4}_2}\int_0^1 y(1+3y^2)(\cancel{2}) \cdot dy.$$

$$= \frac{1}{2}\int_0^1 (y+3y^3) \cdot dy$$

$$= \frac{1}{2}\left[ \left.\frac{y^2}{2}\right|_0^1 + \frac{3}{4}\left. y^4 \right|_0^1 \right]$$

$$= \frac{1}{2}\left[\frac{1}{2} + \frac{3}{4}\right] = \frac{5}{8}$$