Probability

The degree of belief about a statement or event is called probability of the event

$$P(A) = \frac{\text{Favourable outcomes in } A}{\text{Total Sample event}}$$

Q) Find the probability of getting a head when a coin is toss in a.

$S = \{H, T\}$

let A be the event of head

$P(\text{head}) = 1/2$

Q2) If a dice is roll once, then what is the probability of getting 1, and getting of an even number

$P(1) = 1/6$ $P(\text{even}) = 1/2$

Q) If two dice is rolled once what will be the points of getting :-

a) Sum of dot is less than
b) Same dot
c) Sum of dot is ≤5

Ans a) $6/36$

Ans b) $6/36$

Ans c) $10/36$

Axioms Of Probability:-

If S is the Sample Space of an experiment and A&B are any two events then

i) $P(A) \geq 0$

ii) $P(S) = 1$

iii) If A&B are the two disjoint mutually exclusive events then

$P(A \cup B) = P(A) + P(B)$

Addition Law Of Probability:-

If A and B are any two event then

$P(A \text{ or } B) = (A \cup B) = \cancel{P(A)+P(B)+P(}$
$P(A) + P(B) - P(A \cap B)$

* If A&B are two mutually Exclusive events i.e:-

$A \cap B = \emptyset$, $P(A \cap B) = P(\emptyset) = 0$

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

Mutually Exclusive:-

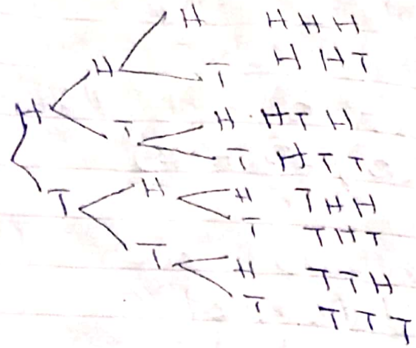
$P(A \cup B) = P(A) + P(B)$

*let B be the event that different no of appearance.

$B = S/A = S - A = A^c$
$P(B) = P(A^c) = 1 - P(A)$
$P(B) = 1 - P(A)$ Complement law

Tree diagram:-

Three Coins are tossed.



The Product Rule:-

Independent events

A & B are two independent event

Then $P(A \text{ and } B) = P(A \cap B) = P(A)P(B)$

* $P(A' \cap B) = P(A')P(B)$
* $P(A \cap B') = P(A).P(B')$
* $P(A' \cap B) = P(A').P(B)$
* If A and B are independent then they are not mutually exclusive.

* If A, B and C are independent then $P(A \cap B \cap C)' = 1 - P(A)P(B)P(C)$
* If A, B and C are independent then $P(A \cup B \cup C) = 1 - P(A' \cap B' \cap C')$

(not E) = The event E does not occur

(A & B) = The event "both A & B occurs"

(A or B) = The event "either A or B occur or both".

The Product rule (Dependent events)

$P(A \cap B) = P(A).P(B/A)$

Conditioned Probability:-

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad \text{OR} \quad P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Independent events if:-

$P(B/A) = P(B)$ or $P(A/B) = P(A)$

Q) Two Cards are drawn from Succession Of a 52 Playing Cards without replacement. What is the probability that the both Cards are Spades.

$$\frac{13}{52} \cdot \frac{12}{51}$$

Syllabus Mid 1

- Measure Central tendency
  - CV
- Variance
  - Probability (upto 56 slide)
  - Permutation and Combination

Assignment 1

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Sol. 1a) Mean = $\frac{\Sigma x}{n} = \frac{1159+1214+1190+1189+1139}{50}$

$\bar{x} = \frac{5891}{50}$

$\bar{x} = 117.82$

Median = $\frac{(n+1)}{2}^{th\ term} = \frac{25^{th}+26^{th}}{2} = 117.5$

$\frac{117+118}{2}$

Mode = 128, 97, 112, 124, 131

b) Range = 150 - 88 = 62

Variance = $\frac{1}{n}\Sigma(x-\bar{x})^2 = 255.33$

SD = $\sqrt{Var} = \sqrt{255.33} = 15.011$

Variance = 220.82
SD = 14.860

c) $\bar{y} \pm S = 117.81 \pm 15.01 = (102.8, 132.8)$

$\bar{y} \pm 2S = 117.81 \pm 2(15.01) = (87.79, 147.8)$

$\bar{y} \pm 3S = 117.81 \pm 3(15.01) = (72.8, 162.8)$

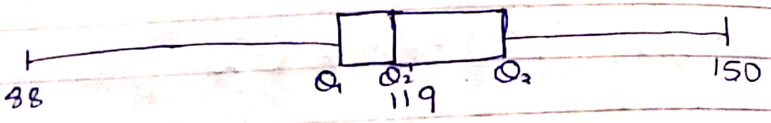
31 values lie in range $\bar{y} \pm S$
49 values lies in range $\bar{y} \pm 2S$
50 Values lies in range $\bar{y} \pm 3S$

d) $Q_1 = \frac{(n+1)}{4} = \frac{51}{4} = 12.75 \simeq 12^{th} = 108$

$Q_2 = \text{median} = 117.5$

$Q_3 = \frac{3(n+1)}{4} = 38.25 \simeq 39^{th} = 131$

Min = 88
Max = 150



e) $P_{70} = \frac{i(n+1)^{th}}{100} = \frac{70}{100} \times (51)^{th} = 35.7^{th}$

$P_{70} = 128$

Q2)

Q1).a)

x	f	fx
40	3	120
42	2	84
43	2	86
47	1	47
50	1	50
51	1	51
55	2	110
57	1	57
60	1	60
63	2	126
64	1	64
65	1	65
67	1	67
68	3	204
69	1	69
71	1	71
72	1	72
73	1	73
74	1	74
75	2	150
76	1	76
77	3	231
78	2	156
79	1	79

x	f	fx
80	1	[illegible]
82	1	[illegible]
83	1	[illegible]
86	1	[illegible]
88	1	[illegible]
95	1	[illegible]
96	1	[illegible]
97	1	[illegible]

$(x-\bar{x})^2$

2190 + 1252

$\Sigma f = 44$

$\Sigma fx = 2949$

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{2949}{44} = 67.02$$

$$\text{Mean} = \frac{n}{2} = \frac{44}{2} = \frac{22^{nd} + 23^{rd}}{2} = \frac{68+69}{2} = 68.5$$

Mod = 40, 68, ~~68~~ 77

~~Variance =~~

Range = 97 - 27 = 70

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} = 57$$

$$Q_2 = \left(\frac{n+1}{2}\right)^{th} = 71$$

$$Q_3 = 3\left(\frac{n+1}{4}\right)^{th} = 78$$

Variance = 11474.182/44

Variance = 260.776

S.D = 16.1485

Probability Distribution (PD)

Q1) A Coin is tossed three times. Find the probability of head for the following:

(i) Atleast one heads
(ii) At most two heads
(iii) All the heads

Also, draw a probability Distribution table (PD).

Sol) No of tosses = n = 3

$2^n = 2^3 = 8$ possible outcomes

S = {HHH, HHT, HTH, TH,H, TTH, THT, HTT, TTT}

let x = No of heads

PD

X	0	1	2	3	Total
P(X = No of head)	1/8	3/8	3/8	1/8	1

≥ Atleast
≤ At Most
= Exactly

(i) $P(x \geq 1) = P(x = 1, 2, 3) = 7/8$

(ii) $P(x \geq 1) = P(x=1) + P(x=2) + P(x=3)$

$$= \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$$

i) Mean $= E(x) = \sum x p(x)$

ii) Variance :

$V(x) = E(x^2) - [E(x)]^2$

$E(x) = \sum x p(x)$

$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$

$= 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$

$\boxed{E(x) = 3/2}$

$V(x) = E(x^2) - [E(x)]^2$

where

~~$E(x^2)p(x)$~~

$E(x^2) = \sum x^2 p(x)$

$E(x) = \sum x p(x)$

$E(x^2) = 0^2 \times 1/8 + 1^2 \times 3/8 + 2^2 \times 3/8 + 3^2 \times 1/8$

$= 0 + 3/8 + 12/8 + 9/8 = 3$

$E(x) = 3/2$

$V(x) = 3 - (3/2)^2 = 0.75$

Discrete Probability Distribution

i) $f(x) \geq 0$

ii) $\sum f(x) = 1$

iii) $P(X = x) = f(x)$

when these 3 condition meet a function is called Discrete Probability Distribution

Q) A shipment of 8 similar microcomputers to a retail order contains 3 that are defective. If a school make a random purchase of 2 of these computers find the probability distibution for the number of defectives.

Sol) Let $X =$ N.O of defective computers

$X = 0, 1, 2$

$f(0) = P(x=0) = \frac{\binom{3}{0}\binom{5}{2}}{\binom{8}{2}} = \frac{10}{28}$ ${}^5P_2$

$P(x=1) = \frac{\binom{3}{1}\binom{5}{1}}{\binom{8}{2}} = \frac{15}{28}$

$P(x=2) = \frac{\binom{3}{2}\binom{5}{0}}{\binom{8}{2}} = \frac{3}{28}$

Probability Distribution Table :-

X	0	1	2
$f(x)$	10/28	15/28	3/28

Q) If a car agency sells 50% of its inventory of a certain foreign car equipped with airbags, find a formula for the probability distribution of the number of cars with airbags among the next 4 cars sold by the agency.

$P(x) = 0.5$. $(2^4 = 16)$ Total Possible Outcomes

$$P(x) = \frac{\text{No of Cars}}{\text{Total Cars}}$$

$$P(X=x) = \frac{\binom{4}{x}}{16} \quad x = 0,1,2,3,4$$

X = No of Car having airbag

X = 0, 1, 2, 3, 4

Cumulative Distribution

$F(x)$ of a discrete random value X with probability distribution $f(x)$ is

$$F(x) = P(X \le x) = \sum_{t \le x} f(t), \quad -\infty < x < \infty$$

Q) Find the cumulative distribution of the random variable X in last question, using $F(x)$, verify that $f(2) = 3/8$.

Sol) Direct calculation of the probability of (last question) gives

$$f(x) = \frac{\binom{4}{x}}{16} \quad \text{for } x = 0, 1, 2, 3, 4$$

$f(0) = \frac{1}{16}$, $f(1) = \frac{1}{4}$, $f(2) = \frac{3}{8}$, $f(3) = \frac{1}{4}$

$$f(4) = \frac{1}{16}$$

Therefore

$F(0) = f(0) = \frac{1}{16}$

$F(1) = f(0) + f(1) = 5/16$

$F(2) = f(0) + f(1) + f(2) = 11/16$

$F(3) = f(0) + f(1) + f(2) + f(3) = 15/16$

$F(4) = f(0) + f(1) + f(2) + f(3) = 1$

Hence

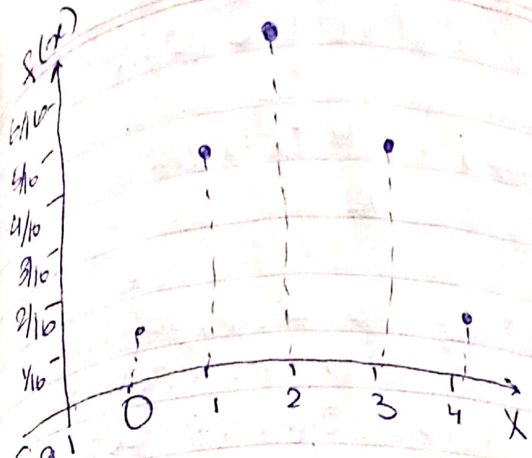
$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1/16 & \text{for } 0 \leq x \leq 1 \\ 5/16 & \text{for } 1 \leq x < 2 \\ 11/16 & \text{for } 2 \leq x < 3 \\ 15/16 & \text{for } 3 \leq x < 4 \\ 1 & \text{for } x \geq 4 \end{cases}$$

Now,

$f(2) = F(2) - F(1)$

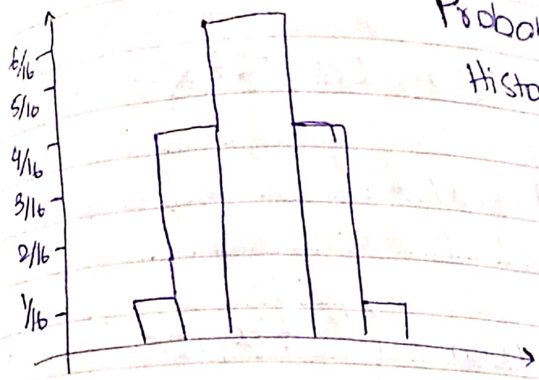
$= \frac{11}{16} - \frac{5}{16} = \frac{3}{8}$

$\boxed{f(2) = 3/8}$

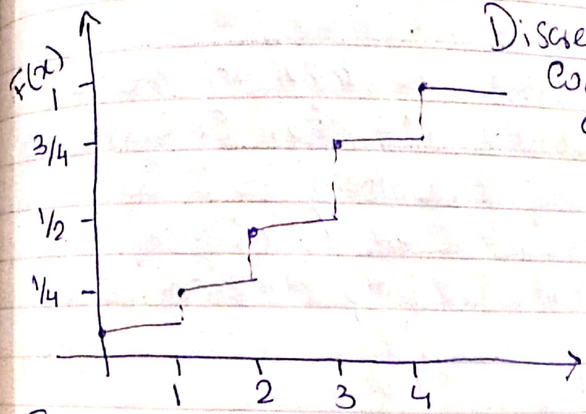


Bar chart

Fig 1



Probability Histogram Fig2



Discrete Cumulative distribution

Fig3

Continuous Probability Distributions:-

$$P(a<x<b) = \int_a^b f(x)\,dx$$

A function $f(x)$ is a probability density function (PDF) for the continuous random variable over the set of real numbers R. if

① $f(x) \geq 0$ for all $x \in R$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

③ $P(a<x<b) = \int_a^b f(x)\,dx$

Q) Suppose that the error in the reaction temperature in °C. for a controlled Laboratory experiment is a continuous random variable X having the probability density function (PDF)

$$f(x) = \begin{cases} x^2/3, \end{cases}$$

Joint Probability Distribution:-

The function $f(x,n)$ is a joint probability distribution or probability mass function (P.m.f) of a discrete random variable x and y if:-

The cumulative Distribution $F(x)$ of a continuous random variable X with Density function $f(x)$ in

$$F(x) = P(X \leq x) = \int_{-\infty}^{\infty} x f(t)\,dx$$

for $-\infty < x < \infty$

Q) For the Density Function

$$f(x) = \begin{cases} x^2/3 & -1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find $F(x)$ and use it to evaluate $P(0 < x \leq 1)$

$$f(x) = \frac{dF(x)}{dx}$$

$$F(x) = \int_{-1}^{x} f(t)\,.dt = \int_{-1}^{x} \frac{t^2}{3}\,.dt$$

$$= \left|\frac{t^3}{9}\right|_{-1}^{x}$$

$$= \frac{x^3+1}{9}$$

therefore

$$F(x) = \frac{x^3+1}{9}$$

$$F(x) = \begin{cases} 0 & x \leq -1 \\ \frac{x^3+1}{9} & -1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

0 $x \leq -1$ becz.

because probability is starting from -1 and F(x) is sum of all probabilities hence at $x \leq -1$ there is no probability hence before -1 ~~then~~ it is 0.

at $x \geq 2$
$F(x) = 1$ because from -1 to 2
so of all probabilities ~~both~~
will be 1.

For $P(0 < x \leq 1)$
upper - lower

$= F(1) - F(0)$
$= 2/9 - 1/9 = 1/9$

Joint Probability Distribution :-

The function $f(x, n)$ is a joint probability distribution or probability mas function (Pmf) of a discrete random variable X and y if

1) $f(x, y) \geq 0$ for all (x, y).

2) $\sum_x \sum_y f(x, y) = 1$

3) $P(X = x, Y = y) = f(x, y)$

For any region A in the xy-plate

$P[(x,y) \in A] = \sum_A \sum f(x, y)$

(Q) Two refills for a ball point pen are selected at random from a box that contains 3 blue refills, 2 red refills, and 3 green refills. If x is the number of blue refills and y is the number of red refills selected. Find.

a) The joint probability function f(x,y)

b) $P[(x,y) \in A)$ where A is the region $[(x,y) \; x+y \le 1]$

3 Blue	2 Red	3 Green

X = No of blue refills
Y = No of red refills

i) Joint Probability Function

$(x,y) \rightarrow (0,0), (0,1), (0,2), (1,0)$
$(1,1), (2,0)$

x \ y	0	1	2	Row Total
0	3/28	9/28	3/28	15/28
1	3/14	3/14	–	3/7
2	1/28	–	–	1/28
Total	5/14	15/28	3/28	

(X, Y) ← X blue, Y red: $(0,0), (0,1), (0,2), (1,0)$
$(1,1), (2,0)$

$$(x,y) = \frac{\binom{3}{0} \cdot \binom{2}{0}}{\binom{8}{2}} = \frac{1}{28}$$

$$f(x,y) = \frac{\binom{3}{x}\binom{2}{y}\binom{3}{2-x-y}}{\binom{8}{2}}$$

$$f(0,0) = \frac{\binom{3}{0}\binom{2}{0}\binom{3}{2-0-0}}{\binom{8}{2}} = \frac{3}{28}$$

b) $P[(x,y) \in A]$ where A is the region $[(x,y) / x+y \leq 1]$

$P[(x,y) \in A] = P(x+y \leq 1)$

$= f(0,0) + f(0,1) + f(1,0)$

$= \frac{3}{28} + \frac{3}{14} + \frac{9}{28} = 9/14$

Joint Density Function :-

For Continuous

① $f(x,y) \geq 0$, for all (x,y)

② $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dx.dy = 1$

③ $P[(x,y) \in A] = \iint f(x,y).dx.dy$

2. $f(x) = \begin{cases} \frac{20{,}003}{(x+100)^3} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$

Q) $f(x,y) = \frac{2}{5}(2x+3y)$, $0 \leq x \leq 1$, $0 \leq y \leq 1$

$f(x,y) = \begin{cases} 2/5\,(2x+3y) & 0 \leq x \leq 1,\ 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$

$\int_{y=0}^{1} \int_{x=0}^{1} \frac{2}{5}(2x+3y).\, dx\, dy$

$\frac{2}{5} \int_{y=0}^{1} \left(\int 2x . dx + 3y \int . dx\right) . dy$

$\frac{2}{5} \int_{y=0}^{1} \left[\frac{2x^2}{2} + \frac{3y}{\cancel{y}} . x\right]_0^1 . dy$

$\frac{2}{5} \int_{y=0}^{1} \{(1-0) + 3y(1-0)\} . dy$

$\frac{2}{5} \int_{y=0}^{1} (1+3y) . dy$

$2/5 \left(\int_{y=0}^{1} . dy + 3 \int_{y=0}^{1} y . dy \right)$

$= \frac{2}{5}\left[y + 3\frac{y^2}{2}\right]_0^1 = \frac{2}{5}\left[\frac{5}{2}\right] = \boxed{1}$ Ans

Def 3.10

Marginal Distribution of X alone and of Y alone are.

for discrete → $g(x) = \sum_y f(x,y)$ &

$h(y) = \sum_x f(x,y)$

for continuous → $g(x) = \int_{-\infty}^{\infty} f(x,y).dy$

and $h(y) = \int_{-\infty}^{\infty} f(x,y).dx$

$f(x,y)$

y \ x	0	1	2	Row Total
0	3/28	9/28	3/28	15/28
1	3/14	3/14	-	3/7
2	1/28	-	-	1/28
Total/Sum	5/14	15/28	3/28	1

Table 3.1

Q) Show that the columns and row of Table 3.1 gives the marginel distibution of X alone and of Y alone.

Sol) For the random variable X, we see that

$g(x) = g(0) = f(0,0) + f(0,1) + f(0,2)$

$= \frac{3}{28} + \frac{3}{14} + \frac{1}{28} = \frac{5}{14}$

$g(1) = f(1,0) + f(1,1) + f(1,2)$

$= 9/28 + \frac{3}{14} + 0 = \frac{15}{28}$

$g(2) = f(2,0) + f(2,1) + f(2,2)$

$= \frac{3}{28} + 0 + 0 = \frac{3}{28}$

In a Similar manner we Could Show that the value Of h(y) are given by the row table

x	0	1	2
$g(x)$	$5/14$	$15/28$	$3/28$

y	0	1	2
$h(y)$	$15/28$	$3/7$	$1/28$

Ex 3.17

Find $g(x)$ and $h(y)$ for the joint density Function Of Example 3.15.

$$f(x,y) = \begin{cases} 2/5(2x+3y) & 0 \le x \le 1 \quad 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,.dy$$

$$g(x) = \int_0^1 \frac{2}{5}(2x+3y)\,.dy$$

$$g(x) = \frac{2}{5}\int_0^1 (2x+3y)\,.dy$$

$$= \frac{2}{5}\int_0^1 (2x.dy + 3y.dy)$$

$$= 2/5\left[2x\int_0^1 dy + 3\int_0^1 y.\,dy\right]$$

$$= 2/5\left[2x(y)_0^1 + 3(y^2/2)_0^1\right]$$

$$g(x) = \frac{2}{5}\left[2x + 3/2\right]$$

$$\boxed{g(x) = \frac{4x+3}{5}}$$

$$h(y) = \int_{-\infty}^{\infty} f(x,y)\,.dx$$

$$h(y) = \int_{-\infty}^{\infty} \frac{2}{5}(2x+3y)\,.dx$$

$$= \frac{2}{5}\int_0^1 (2x.dx + 3y.dx)$$

$= \frac{2}{5}\left[\frac{2}{2}\left|x^2\right|_0^1 + 3y\,|x|_0^1\right]$

$= \frac{2}{5}[1 + 3y]$

$h(y) = \frac{2(1+3y)}{5}$

Def 3.11

let x and y be two r.v discrete or Continuous. The Conditional distribution of the r.v Y given that $X = x$ is

$$\boxed{f(y/x) = \frac{f(x,y)}{g(x)} \quad g(x) > 0}$$

Similarly the Conditional Distribution of the r.v X given that $Y = y$ is

$$\boxed{f(x/y) = \frac{f(x,y)}{h(y)} \quad h(y) > 0}$$

$f(x=0/y=1) = \frac{f(0,1)}{h(1)}$

Ex 3.14

Find the Conditional distribution of X given that $Y = 1$ and use it to determine $P(X=0/Y=1)$

Sol) first find $f(x/y)$, where $y = 1$, we find that $h(1) = \sum_{x=0}^{2} f(x,1) =$

$h(y) = \sum_{0}^{2} f(x,1) = f(0,1) + f(1,1) + f(2,1)$

$= 3/14 + 3/14 + 0 = 3/7$

Now $f(x/1) = \frac{f(x/1)}{h(1)} = \frac{7}{3} f(x,1)$

Then

$f(0/1) = 7/3\, f(0,1) = \left(\frac{7}{3}\right)\left(\frac{3}{14}\right) = \frac{1}{2}$

~~$f(1,1)$~~ $7/3$

$f(1,1) = 7/3\, f(1,1) = \frac{7}{3} \times \frac{3}{14} = \frac{1}{2}$

$f(2,1) = \frac{7}{3} f(2,1) = \frac{7}{3}(0) = 0$

and the Conditional distribution of X given that $Y = 1$ is

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x	0	1	2
$f(x)$	1/2	1/2	0

Therefore, if it is known that 1 of the 2 pen refills selected are red, we have a probability equal to 1/2 that the other refill is not blue.

Ex #3.19

$$f(x,y)=\begin{cases}10xy^2 & 0<x<y<1\\ 0 & \text{elsewhere}\end{cases}$$

a) Find the marginals density $g(x)$, $h(y)$ and the conditional density $f(y/x)$

b) Find $P(Y>1/2 \mid X=0.25)$

$g(x)=\int_{-\infty}^{\infty} f(x,y)\cdot dy = \int_x^1 10xy^2\, dy = 10x\int_x^1 y^2\, dy$

$= 10x\left|\frac{y^3}{3}\right|_x^1 = \frac{10x}{3}(1-x^3) \quad 0<x<1$

$h(y)=\int_{-\infty}^{\infty} f(x,y)\cdot dx = \int_0^y 10xy^2$

$= 5x^2y^2\Big|_{x=0}^{x=y} = 5y^4 \quad 0<y<1$

Now

$f(y/x) = \frac{f(x,y)}{g(x)} = \frac{10xy^2}{\frac{10x}{3}(1-x^3)} = \frac{3y^2}{1-x^3} \quad 0<x<y<1$

b) $P(Y>1/2 \mid x=0.25) =$

$= \int_{1/2}^1 f(y/x=0.25)\, dy$

$= \int_{1/2}^1 \frac{3y}{1-0.25^3}\cdot dy = 8/9$

Ex 3.20 Do it Yourself

$P(A \cap B) = P(A).P(B)$
$P(A \cap B) = P(A).P(B/A)$

Statistical Independence:-

$f(x, y) = f(x/y).h(y)$

and $\boxed{f(x, y) = g(x).h(y)}$

Q) Show that the random variable of Examples 3.14 are not statistically independent.

Sol) Let us consider the point (0,1), from table we find three probations $f(0,1)$, $g(0)$ and $h(1)$ to be

$f(0,1) = 3/14$ $\quad f(0,0) + f(0,1) + f(0,2)$

$g(0) = \sum_{y=0}^{2} f(0,y) = \frac{3}{28} + \frac{3}{14} + \frac{1}{28} = \frac{5}{14}$

$f(0,1) = 3/14$

$g(x) = g(0) = \sum_{y=0}^{2} f(0,y) = f(0,0) + f(0,1) + f(0,2)$

$= \frac{3}{28} + \frac{3}{14} + \frac{1}{28} = \frac{5}{14}$

$h(y) = h(1) = \sum_{x=0}^{2} f(x,1) = f(0,1) + f(1,1) + f(2,1)$

$= \frac{3}{14} + \frac{3}{14} + 0 = \frac{3}{7}$

$\boxed{f(x,y) \neq g(x)\,h(y)}$ and therefore X and Y are not statistically independent

$f(0,1) = g(0)\,h(1)$

$3/14 = \frac{5}{14} \cdot \frac{3}{7}$

$\boxed{\frac{3}{14} \neq \frac{15}{98}}$

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Chapter 4 Mathematical Expectation

4.1

Mean of random variable

Def:-

$\boxed{\mu = E(x) = \sum_x x f(x)}$

if x is discrete and

$\boxed{\mu = E(x) = \int_{-\infty}^{\infty} x f(x).dx}$

if x is continuous

Q) Ex 4.1

Total 7 sample to be inspected

4 good
3 defective

A Sample of 3 is taken by inspector. Find the expected value of the number of good component, in the Sample.

Sol) let x be the number of several sample i.e to be the good.

$P(X=x) = \dfrac{\binom{4}{x}\binom{3}{3-x}}{\binom{7}{3}}$

$P(X=0) = f(0) = 1/35$
$P(X=1) = f(1) = 12/35$
$P(X=2) = f(2) = 18/35$
$P(X=3) = f(3) = 4/35$

(i)	0 (ii)	(iii)	(iv)	Sum
$P(X=x)$ $f(x)$	$1/35$	$12/35$	$18/35$	$4/35$	$=1$

$V(x) = E(x^2) - [E(x)]^2$

where $S.D = \sqrt{V(x)}$

$E(x^2) = \sum_x x^2 f(x)$

&

$E(x) = \sum_x x f(x)$

Mean: $\mu = \sum x f(x)$

$\mu = 0 \times \frac{1}{35} + 1 \times \frac{12}{35} + 2 \times \frac{16}{35} + 3\left(\frac{4}{35}\right)$

$\boxed{\mu = 1.7}$

"Expected value" always means "mean"

Marginal Approach for mean:

$\mu g(x) = E[g(x)] = \sum_x g(x) f(x)$

Sol) let X be the number of sample i.e to be good, if X is discrete.

$\mu g(x) = \int_{-\infty}^{\infty} g(x) f(x) . dx$

If X is Continuous.

Expected Value:

$\mu g(x,y) = E[g(x,y)] = \sum_x \sum_y g(x,y) f(x,y)$

if X and Y are discrete

And

$\mu g(x,y) = E[g(x,y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y).f(x,y) dx dy$

if X and Y are Continuous.

Ex 4.6

$g(x,y) = xy$

$E(xy) = \sum_{x=0}^{2} \sum_{y=0}^{2} xy f(x,y)$

$= (0)(0) f(0,0) + (0)(1) f(0,1) + (0)(2) f(0,2) + (1)(0) f(1,0) + (1)(1) f(1,1) + (2)(0) f(2,0)$

$= f(1,1) = 3/14$

Ex 4.7

Q. Find $E(Y/x)$ for the Density Function

$$f(x,y) = \begin{cases} \frac{x(1+3y^2)}{4} & 0<x<2 \\ & 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$$

Sol) $E(Y/x) = \left(\frac{Y}{x}\right)\left(\frac{x(1+3y^2)}{4}\right)$

$= y\left(\frac{1+3y^2}{4}\right)$

$$E(Y/x) = \int_0^1 \int_0^2 y\left(\frac{1+3y^2}{4}\right) . dx . dy$$

$$= \int_0^1 y\frac{(1+3y^2)}{4} \int_0^2 dx \, . dy$$

$$= \int_0^1 y\left(\frac{1+3y^2}{4}\right) |x|_0^2 \, dy$$

$$= \int_0^1 (2-0)\left(y\frac{(1+3y^2)}{4}\right) . dy$$

$$E(Y|x) = \frac{2}{4}\int_0^1 (1+3y^2) y . dy$$

let $u = 1+3y^2$

$du = 6y . dy$

$\frac{du}{6} = y . dy$

$$E(Y|x) = \frac{1}{2}\int_0^1 u \frac{du}{6}$$

$$= \frac{1}{12}\int \frac{u^2}{2}$$

$$= \frac{1}{12}\left[\frac{(1+3y)^2}{2}\right]_0^1$$

$$= \frac{1}{12}\left[8 - \frac{1}{2}\right] = \frac{5}{8} \text{ Ans}$$

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Variance & Covariance :-

Variance $= \sigma^2 = E(x^2) - [E(x)]^2$

$\sigma^2 = E[(x-\mu)^2] = \sum_x (x-\mu)^2 f(x)$

if X is discrete ↑

$\sigma^2 = E(x-\mu)^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) \, d(x)$

for Continuous ↑

Ex # 4.8

X	1	2	3
f(x)	0.3	0.4	0.3

find μ

Sol) $\mu = E(x) = 1(0.3) + 2(0.4) + 3(0$

$\mu = 2.0$

$\sigma^2 = \sum_{x=0}^{2} (x-2)^2 f(x) =$

See Example 4.10

Covariance :-

$\sigma^2 g(x) = E\{[g(x) - \mu g(x)]^2\}$

$= \sum (g(x) - \mu g(x)\}^2 f(x)$

if x is discrete ↑

$\sigma^2 g(x) = E[g(x) - \mu g(x)]^2 = \int_{-\infty}^{\infty} g(x) - \mu g(x)$

$\int_{-\infty}^{\infty} [g(x) - \mu g(x)]^2 f(x) \, dx$

if x is Continuous ↑

Example 4.11

Calculate $g(x) = 2(x) + 3$

X	0	1	2	3
f(x)	1/4	1/8	1/2	1/8

X	0	1	2	3
g(x) = 2x+3	3	5	7	9
f(x)	1/4	1/8	1/2	1/8

~~$\mu_{2x+3} = E(2x+3) = \sum_{x=0}^{3} (2x+3) f(x)$~~

Sol) $\mu_{2x+3} = E(2x+3) = \sum_{x=0}^{3} (2x+3) f(x)$

$\mu_{g(x)} = 3 \times \frac{1}{4} + 5 \times \frac{1}{8} + 7 \times \frac{1}{2} + 9 \times \frac{1}{8} = 6$

$\sigma^2 = E(x-\mu)^2$

$\sigma^2_{(2x+3)} = E\{[(2x+3) - \mu_{2x+3}]^2\} = E\{[2x+3-\ldots$

$= E(4x^2 - 12x + 9)$

$= \sum_{x=0}^{3} (4x^2 - 12x + 9) f(x) = 4$

$= (4(0) - 12(0) + 9)(1/4) + (4 - 12 + 9)(1/8) + (4(2)^2 - 12(2) + 9)(1/2) + (4(3)^2 - 12(3) + 9)\ldots$

$= \boxed{4}$ Answer

See Ex 4.12

Def 4.4:-

1. $\sigma_{xy} = E(x-\mu_x)(y-\mu_y) = \sum_x \sum_y (x-\mu_x)(y-\mu_y) f(x,y)$

if x & y are discrete.

2) $\sigma_{xy} = E(x-\mu_x)(y-\mu_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x-\mu_x)(y-\mu_y) \cdot f(x,y)\,dx\,dy$

if x & y are ~~discrete~~ Continuous

Theorem 4.4

The covariance of two r.v x and y ~~with~~ with mean μ_x and μ_y respectively is given by

$\sigma_{xy} = E(xy) - \mu_x \mu_y$

See Example 4.13

See Example 4.14

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Definition 4.5 ; let X and Y be random variables with Covariance σ_{xy} and Standard deviation σ_x and σ_y respectively. The Coefficient of Covariance X and Y is

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

$$P_{xy} = \frac{Cov(x,y)}{(\text{standard dev of } x)(\text{standard dev of } y)}$$