2.4 $y[n] = 2\cos(0.3\pi n + \pi/4)$; $F_s = 4\,kHz$

a
$$S(F) = e^{-j\pi f/f_s} \cdot \frac{1}{F_s} \operatorname{sinc}\left(\frac{f}{f_s}\right) Y(\Omega)\Big|_{\Omega = \frac{2\pi f}{f_s}}$$

$$Y(\Omega) = 2\pi \sum_{k=-\infty}^{\infty} e^{j\pi/4} \delta(\omega - 0.3\pi - k2\pi) + e^{-j\pi/4} \delta(\omega + 0.3\pi - k2\pi)$$

$$Y(\Omega)\Big|_{\Omega = \frac{2\pi f}{f_s}} = 2\pi \sum_{k=-\infty}^{\infty} e^{j\pi/4} \delta\left(2\pi \frac{f}{f_s} - 0.3\pi - k2\pi\right) + e^{-j\pi/4} \delta\left(2\pi \frac{f}{f_s} + 0.3\pi - k2\pi\right)$$

$$= 2\pi \times \frac{f_s}{2\pi} \sum_{k=-\infty}^{\infty} e^{j\pi/4} \delta(f - 600 - k4000) + e^{-j\pi/4} \delta\left(2\pi \frac{f}{f_s} + 600 - k4000\right)$$

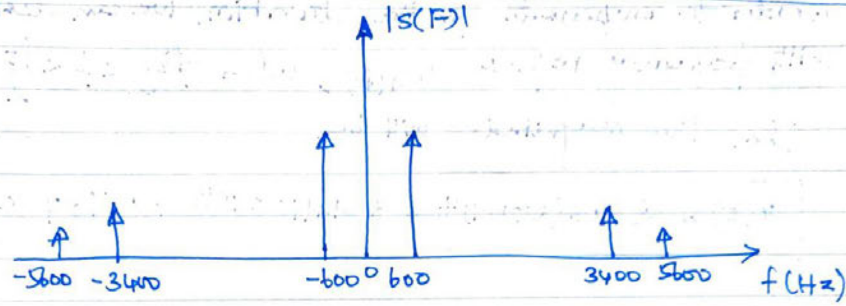
$$S(F) = f_s \sum_{k=-\infty}^{\infty} T_s e^{-j\pi\left(\frac{600 + 4000k}{4000}\right)} \operatorname{sinc}\left(\frac{600 + k4000}{4000}\right) e^{j\pi/4} \delta(f - 600 - k4000)$$
$$+ T_s e^{-j\pi\left(\frac{-600 + k4000}{4000}\right)} \operatorname{sinc}\left(\frac{-600 + k4000}{4000}\right) e^{-j\pi/4} \delta\left(\frac{2\pi f}{f_s} + 600 - k4000\right)$$

$\left(\because \text{ZOH frequency response} = T_s e^{-j\pi f/f_s} \operatorname{sinc}(f/f_s)\right)$

$$S(F) = \sum_{k=-\infty}^{\infty} (-1)^k e^{-j3\pi/20} \operatorname{sinc}\left(\frac{3}{20} + k\right) e^{j\pi/4} \delta(f - 600 \mp 4000k) +$$
$$(-1)^k e^{-j3\pi/20} \operatorname{sinc}\left(\frac{-3}{20} + k\right) e^{-j\pi/4} \delta(f + 600 - k4000)$$

from $-f_s < f < f_s \Rightarrow -4000 < f < 4000 \Rightarrow |k| \le 1$

$$\boxed{\begin{aligned} S(F) = {} & 0.17\, e^{-j2.827}\, \delta(f-3400) + 0.9634\, e^{-j0.1\pi}\, \delta(f-600) + \\ & 0.9634\, e^{j0.1\pi}\, \delta(f+600) + 0.17\, e^{j2.827}\, \delta(f+3400) \end{aligned}}$$



(b) $y(t)$ has frequencies until $f = \pm 600\,Hz$.

$$\Rightarrow y(t) = F^{-1}\left\{0.9634\, e^{-j0.1\pi}\, \delta(f-600) + 0.9634\, e^{j0.1\pi}\, \delta(f+600)\right\}$$

$$\boxed{y(t) = 2 \times 0.9634 \cos(1200\pi t - 0.1\pi)}$$

(2.5) $x(t) = 2\cos(500\pi t) - 3\sin(1000\pi t) + \cos(1500\pi t)$.

(a) $f_s = 2000\,Hz$

in absence of aliasings $Y(f) = e^{-j\pi f/f_s}\, \mathrm{sinc}\left(\frac{f}{f_s}\right) X(f)$

Max freq = 750Hz

as $f_s > 2f_{max} \Rightarrow$ no aliasing

$$\Rightarrow G(f) = e^{-j\frac{\pi f}{2000}} \frac{\sin\left(\frac{\pi f}{2000}\right)}{\frac{\pi f}{2000}}$$

$G(250) = 0.9745\, e^{-j0.392}$; $G[500] = 0.9003\, e^{-j0.785}$;

$G(750) = 0.784\, e^{-j1.178}$;

$$y(t) = 2\times 0.9745 \times \cos(500\pi t - 0.392) - 3\times 0.9003 \times \sin(1000\pi t - 0.785)$$

$$+ 0.784\cos(1500\pi t - 1.178).$$

(b) In order to compensate for the distortion, we can design a filter with frequency response $1/G(f)$, when $-f_s/2 < f < \frac{f_s}{2}$.

Then the magnitude will be.

$$\boxed{y(t) = 2\cos(500\pi t - 3\sin(1000\pi t) + 0.784\cos(1500\pi t)}$$

(2.6) (b) $f_s = 5kHz$

Incase of no aliasing.

$$G(f) = e^{\frac{-j\pi f}{5000}} \frac{\sin\left(\frac{\pi f}{5000}\right)}{\frac{\pi f}{5000}}$$

for $f = 1000$

$$G(1000) = 0.935 e^{-j0.628}$$

$$y(t) = 0.935\times\cos(2000\pi t + 0.15\pi - 0.628)$$

(d) for $f = 2500$

$$G(2500) = 0.637 e^{-j1.5708}$$

$$y(t) = 2\times 0.637 \sin(5000\pi t - 1.5708)$$

(e) $\cos(2\pi 2750 t)$ causes aliasing as $2f_{max} > f_s$

$$f_{aliased} = (5.00 - 2.75)k = 2.25kHz$$

$$x(t) = \cos(2000\pi t + 0.1\pi) - \cos(4500\pi t).$$

for $f = 1000$ & $f = 2250$

$G(1000) = 0.935\, e^{-j0.628}$ & $G(2250) = 0.699\, e^{-j0.393}$.

$y(t) = 0.935 \cos(2000\pi t + 0.1\pi - 0.628) - 0.699 \cos(4500\pi t - 1.4137)$

HW #9 Solution
EE 210
Alvin Maningding

8.13)

a) The DFS of a periodic signal is taken over a single period; say, $0 \le k < N-1$. An N-periodic signal has an N-periodic DFS representation. The plot shows one full period of the signal's magnitude response from $0 \le k \le 23$, giving $N = 24$.

b) The original analog signal was sampled at $f_s = 12$ kHz. Recall that the DFS could be considered as the result of sampling the continuous, 2π-periodic DTFT of a signal at N points. The DFS frequency index k runs from $k = 0,1,2,\cdots,N-1$, so k and Ω are related as

$$2\pi\frac{k}{N} = \Omega$$

$$= 2\pi\frac{f}{f_s}$$

Solving for analog frequency f,

$$\frac{k}{N} = \frac{f}{f_s}$$

$$f = \frac{1}{N}kf_s \qquad [\text{Hz}]$$

To determine the analog frequency content of the signal, we simply need to know the frequency indices $0 \le k \le N/2$ for which the magnitude response $|X[k]|$ is nonzero. Recall that all the information given by a DTFT, whose magnitude is even-symmetric and can be sampled to obtain the DFT/DFS, is found in $0 \le \Omega < \pi$, not $0 \le \Omega < 2\pi$. A DTFT or DFT only contains information for frequencies up to half of the sampling rate; 6 kHz, in this case.

From one period $0 \le k \le \frac{N}{2}$, or $0 \le k \le 12$, we obtain the results in Table 1.

k	***f* [kHz]**	***Ω* [rad/s]**
2	1	0.523599
4	2	1.047198
9	4.5	2.356194

Table 1: Analog and digital frequencies.

c) Digital frequencies, Ω, obtained from the above equations, are shown in Table 1.

8.22)

a) The fundamental frequency of an N-periodic signal can be calculated as

$$f_0 = \frac{f_s}{N},$$

where f_0 is in Hz and $f_s = 4$ kHz is given. As before, the magnitude plot is given for one full period from $0 \le k \le 63$, giving $N = 64$. Substituting,

$$f_0 = \frac{f_s}{N} = \frac{4000\text{ Hz}}{64} = \boxed{62.5\text{ Hz}}$$

b) The period of the signal is the reciprocal of its frequency:

$$T_0 = \frac{1}{f_0} = \frac{1}{62.5\ \text{Hz}} = \boxed{16\ \text{ms}}$$

c) The average value of the square wave is given by its DC component, found directly from the plot at $k = 0$ (note that the *y*-axis is already labelled as $|c_n| / N$):

$$\boxed{\frac{|c_0|}{N} = 41}$$

d) The 500 Hz low pass filter is known to be high-order, so we can assume that it is near-ideal. 500 Hz corresponds to

$$\begin{aligned}
f &= \frac{1}{N} k f_s \\
k &= \left\lfloor N \frac{f}{f_s} \right\rfloor \\
&= \left\lfloor 64 \cdot \frac{500\ \text{Hz}}{4000\ \text{Hz}} \right\rfloor, \\
&= \lfloor 8 \rfloor \\
k &= 8
\end{aligned}$$

where the floor function $\lfloor x \rfloor$ ensures that k remains an integer. Then, the highest frequency passed by the filter is

$$\begin{aligned}
f &= \frac{1}{N} k f_s \\
&= \frac{1}{64} (8)(4000\ \text{Hz}) \\
&= \boxed{500\ \text{Hz}}
\end{aligned}$$

The filtered signal will contain only frequencies from $0 \le k \le 8$ for which the magnitude $|c_n| / N \neq 0$. The harmonics (frequency content above the fundamental frequency) are all at odd k, as shown in Table 2.

k	***f* [Hz]**
1	62.5
3	187.5
5	312.5
7	437.5

Table 2: Frequency content after filtering.