EE288 Data Conversions/Analog Mixed-Signal ICs
Spring 2018

Lecture 4: Quantization

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Course Schedule – Subject to Change

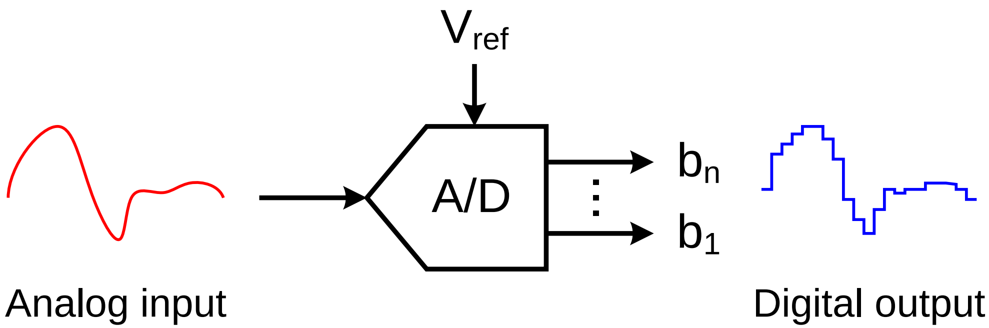
Date	Topics
24-Jan	Course introduction and ADC architectures
29-Jan	Converter basics: AAF, Sampling, Quantization, Reconstruction
31-Jan	ADC dynamic performance metrics, Spectrum analysis using FFT
5-Feb	ADC & DAC static performance metrics, INL and DNL
7-Feb	OPAMP and bias circuits review
12-Feb	SC circuits review
14-Feb	~~Sample and Hold Amplifier - Reading materials~~
19-Feb	Flash ADC and Comparator: Regenerative latch
21-Feb	Comparators: latch offset, preamp, auto-zero
26-Feb	Finish Flash ADC
28-Feb	DAC Architectures - Resistor, R-2R
5-Mar	DAC Architectures - Current steering, Segmented
7-Mar	DAC Architectures - Capacitor-based
12-Mar	SAR ADC with bottom plate sampling
14-Mar	SAR ADC with top plate sampling
19-Mar	Midterm Review
21-Mar	Midterm exam
26-Mar	Spring break
28-Mar	Spring break
2-Apr	Pipelined ADC stage - comparator, MDAC, x2 gain
4-Apr	Pipelined ADC bit sync and alignment using Full adders
9-Apr	Pipelined ADC 1.5bit vs multi-bit structures
11-Apr	Fully-differential OPAMP and Switched-capacitor CMFB
16-Apr	Single-slope ADC
18-Apr	Oversampling & Delta-Sigma ADCs
23-Apr	Second- and higher-order Delta-Sigma Modulator.
25-Apr	Hybrid ADC - Pipelined SAR
30-Apr	Hybrid ADC - Time-Interleaving
2-May	ADC testing and FoM
7-May	Project presentation 1
8-May	Project presentation 2
14-May	Final Review
20-May	Project Report Due by 6 PM

No class on Feb 14, next Wed.

Quantization & Performance Metrics

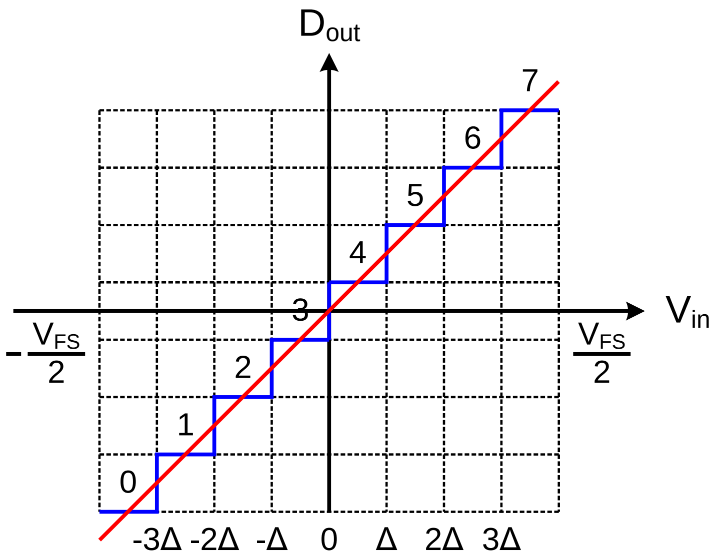
***Midterm Exam dates are approximate and subject to change with reasonable notice.**

Quantization

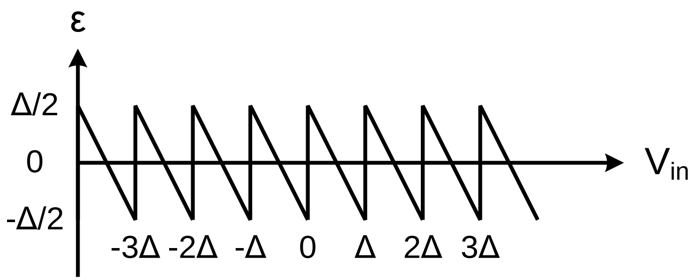


- Quantization = division + normalization + truncation
- Full-scale range (V_{FS}) is determined by V_{ref}

Quantization Error



N = 3



$$\Delta = \frac{V_{FS}}{2^N} = LSB$$

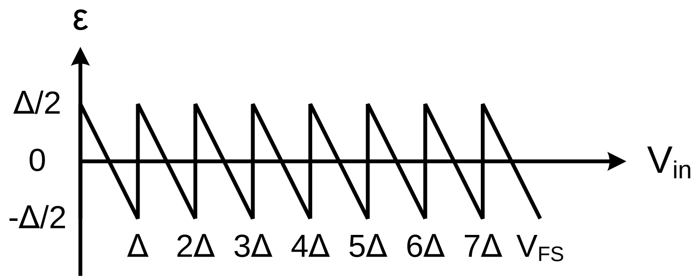
$$V_{in} \in [0, V_{FS}]$$

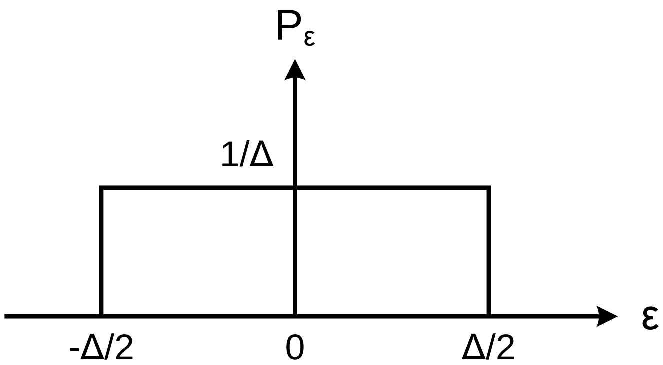
$$\varepsilon = D_{out}\Delta - V_{in} = D_{out}\left(\frac{V_{FS}}{2^N}\right) - V_{in}$$

$$-\frac{\Delta}{2} \le \varepsilon \le \frac{\Delta}{2}$$

"Random" quantization error is usually regarded as noise

Quantization Noise





Assumptions:

- N is large
- $0 \le V_{in} \le V_{FS}$ and $V_{in} >> \Delta$
- V_{in} is active
- ε is Uniformly distributed
- Spectrum of ε is white

$$\boxed{\sigma_\varepsilon^{\,2} = \int_{-\Delta/2}^{\Delta/2} \varepsilon^2 \cdot \frac{1}{\Delta} \cdot d\varepsilon = \frac{\Delta^2}{12}}$$

Ref: W. R. Bennett, “Spectra of quantized signals,” *Bell Syst. Tech. J.*, vol. 27, pp. 446-472, July 1948.

Signal-to-Quantization Noise Ratio (SQNR)

Assume V_{in} is sinusoidal with $V_{p\text{-}p} = V_{FS}$,

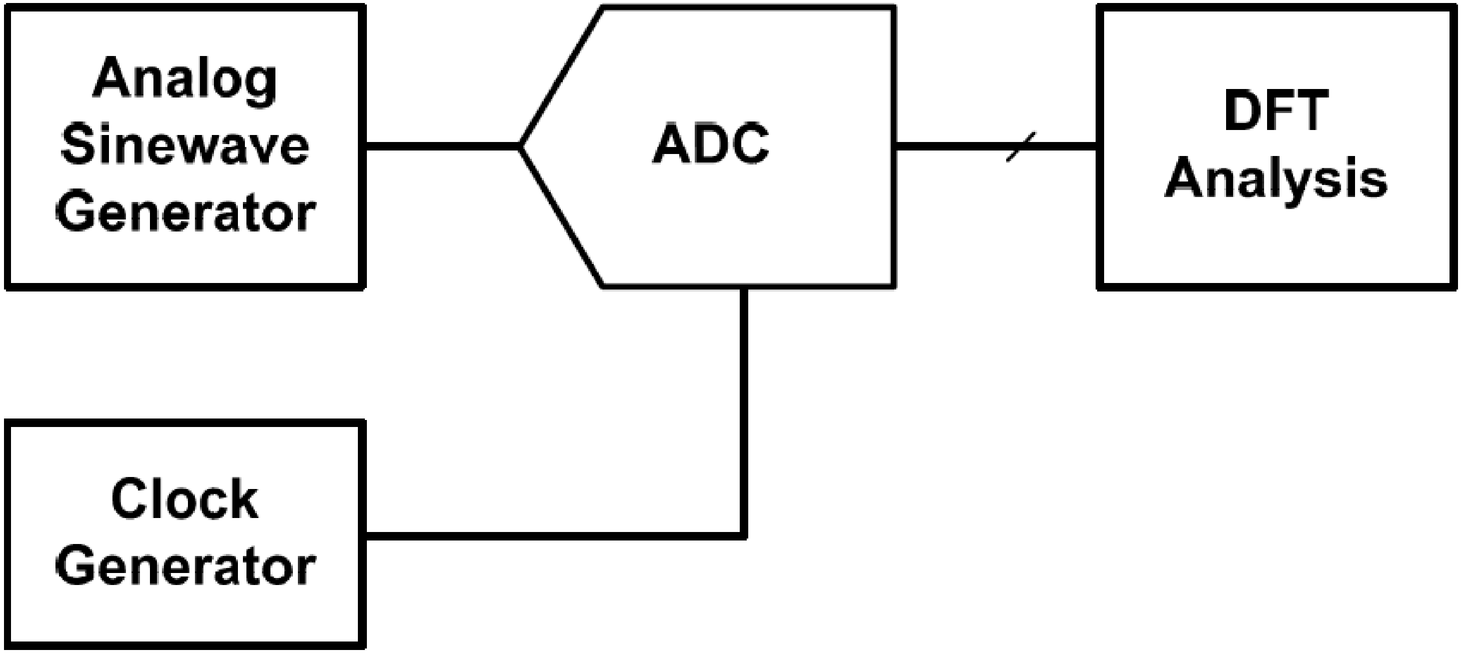
$$SQNR = \frac{V_{FS}^{2}/8}{\sigma_{\varepsilon}^{2}} = \frac{\left(2^{N}\Delta\right)^{2}/8}{\frac{\Delta^{2}}{12}} = 1.5 \times 2^{2N},$$

$$\boxed{SQNR = 6.02 \times N + 1.76\ \text{dB}}$$

N (bits)	SQNR (dB)
8	49.9
10	62.0
12	74.0
14	86.0

- SQNR depicts the theoretical performance of an ideal ADC
- In reality, ADC performance is limited by many other factors:
  - Electronic noise (thermal, 1/f, coupling/substrate, etc.)
  - Distortion (measured by THD, SFDR, IM3, etc.)

ADC Characterization



DFT

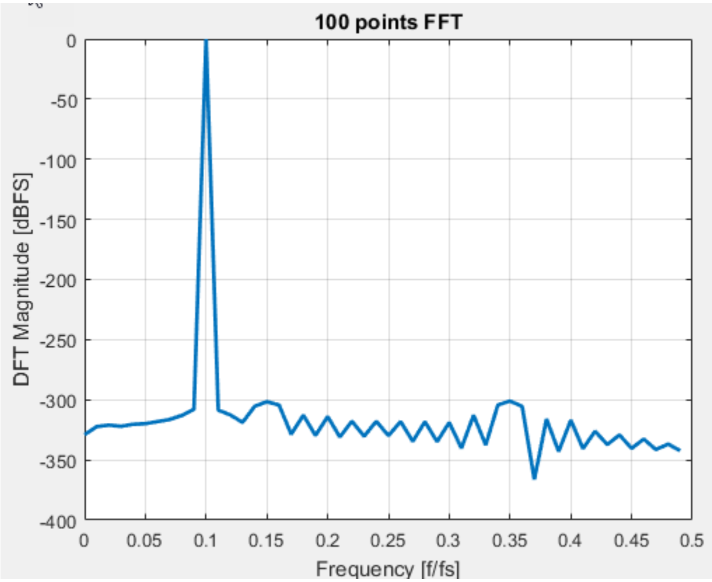
- DFT takes a block of N time domain samples (spaced $T_s = 1/f_s$) and yields a set of N frequency bins

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

- Bin k represents frequency content at $k \cdot f_s / N$ [Hz]
- DFT frequency resolution
  - Proportional to $1/(N \cdot T_s)$ in [Hz/bin]
  - $N \cdot T_s$ is total time spent gathering samples
- A DFT with $N=2^{\text{integer}}$ can be found using a computationally efficient algorithm
  - FFT = Fast Fourier Transform

FFT MATLAB Example 1

```
clear all; close all; clc;
N = 100;                        % FFT size
fs = 1000;                      % Sampling rate
fx = 100;                       % Input signal tone
FS = 1;                         % Full Scale (actually half)
t = 0:N-1;
x = FS*cos(2*pi*fx/fs*t);
s = abs(fft(x));                % FFT and take absolute
s = s(1:end/2);                 % remove redundant half of spectrum
s = 20*log10(2*s/N/FS);         % dB relative to full-scale
f = [0:N/2-1]/N;                % frequency vector
plot(f, s, 'linewidth', 2);
xlabel('Frequency [f/fs]');
ylabel('DFT Magnitude [dBFS]');
title(strcat(num2str(N),' points FFT'));
grid on;
```



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