EE210

Midterm II

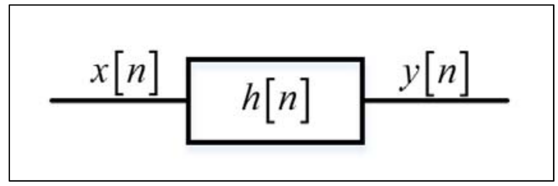
Name and ID: $\begin{cases} \text{Last:} \\ \\ \text{First:} \\ \\ \text{Student ID \#:} \\ \\ \text{Email:} \end{cases}$

5/5/2020

- Write your signature on the bottom right corner of front page
- Draw a box around your final answers otherwise you will NOT get any credits and move your solutions to the given boxes.
- Your phone must be turned off and kept in your bag.
- One (8.5x11) cheat sheet is allowed.
- Calculator is not ok.
- No cell phone for calculator
- Only pencil and eraser
- If you use pen, 20 pts will be deducted.

7.8

1. [15 pts]



A filter, $h[n]$ in the frequency domain is defined as

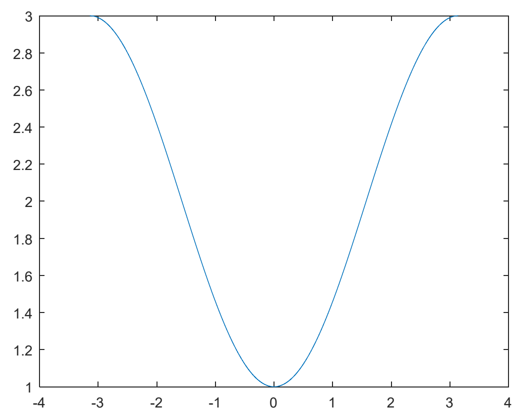
$$H(\Omega) = -0.5 + 2e^{-j\Omega} - 0.5e^{-j2\Omega} \text{ (DTFT domain)}$$

An input to the filter is defined as

$$x[n] = -3\delta[n] + 2\delta[n-1] - 2\delta[n-3] - \delta[n-4]$$

a) Plot magnitude response of $H(\Omega)$ from $-\pi$ to π

b) Find output $y[n]$

a)	 High pass filter
b)	$h[n] = -0.5\delta[n] + 2\delta[n-1] - 0.5\delta[n-2]$ $x[n] = -3\delta[n] + 2\delta[n-1] - 2\delta[n-3] - \delta[n-4]$ $y[n] = h[n] * x[n]$ $= [-0.5\ 2\ -0.5] * [-3\ 2\ 0\ -2\ -1]$ $= [1.5\quad -7\quad 5.5\quad 0\quad -3.5\quad -1\quad 0.5]$

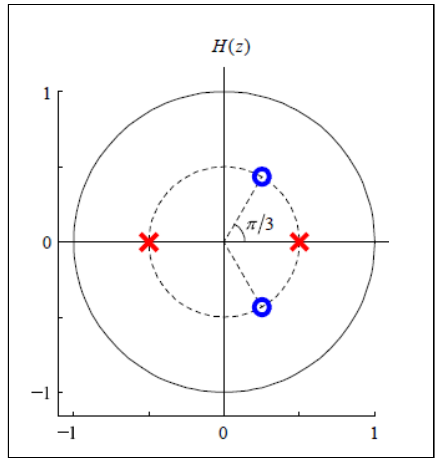
10 5

2. [15 pts]

Given a system $H(z)$, whose pole-zero plot is shown on the right, and the fact that $H(1)=1$

a) Find $H(z)$ (You need to show how you got the answer otherwise you will not get credits)

b) Find input output difference equation



prob #2.

$$H(z) = \frac{(z - K\cdot e^{-j\pi/3})(z - K\cdot e^{j3\pi/K})}{(z-K)\cdot(z+K)}$$

problem statement says that $H(1)=1$.

$$H(1) = 1 = \frac{(1 - K\cdot e^{-j\pi/3})\cdot(1 - K\, e^{j\pi/3})}{(1-K)(1+K)}$$

$$1 - K^2 = (1 - K\cdot e^{-j\pi/3})(1 - K\cdot e^{j\pi/3})$$

$$\Rightarrow 1 - K^2 = (1 - K\cdot e^{j\pi/3} - K\cdot e^{-j\pi/3} + K^2)$$

$$\Rightarrow 2K^2 = K(1/2 + \sqrt{3}/2\, j) + K(1/2 - \sqrt{3}/2\, j)$$

$$= 2K^2 = K \qquad 2K^2 - K = K(2K-1) = 0$$

$\boxed{K = 0 \text{ or } K = 1/2}$

so $\boxed{K = 1/2}$

a) $$\boxed{H(z) = \frac{(z - 1/2\cdot e^{-j\pi/3})(z - 1/2\cdot e^{j\pi/3})}{(z - 1/2)(z + 1/2)}}$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{(z^2 - \frac{1}{2}\cdot e^{-j\pi/3} - \frac{1}{2}\cdot e^{j\pi/3} + \frac{1}{4})}{z^2 - 1/4}$$

$$Y(z)(z^2 - \tfrac{1}{4}) = X(z)\{z^2 - \tfrac{1}{2}z + \tfrac{1}{4}\}$$

b) $$\boxed{y[n] = \tfrac{1}{4}y[n-2] + x[n] - \tfrac{1}{2}x[n-1] + \tfrac{1}{4}x[n-2]}$$

3. [20 pts]

It is given that

$$y[n] = \delta[n] - \left(\frac{1}{2}\right)^{n-1} \cdot u[n-1]$$

$$x[n] = \delta[n] - 3\delta[n-1] + \frac{9}{4}\delta[n-2]$$

a) Find $h[n]$

b) Find input output difference equation

$$y[n] = \delta[n] - \left(\frac{1}{2}\right)^{n-1} \cdot u[n-1]$$

$$= \delta[n] - \left[\left(\frac{1}{2}\right)^{n} \cdot u[n]\right] * \delta[n-1]$$

$$Y(z) = 1 - \frac{z}{z-1/2} \cdot z^{-1}$$

$$= 1 - \frac{1}{z-\frac{1}{2}} = \frac{z-\frac{3}{2}}{z-\frac{1}{2}}$$

$$x[n] = \delta[n] - 3\delta[n-1] + \frac{9}{4}\delta[n-2]$$

$$X(z) = 1 - 3z^{-1} + \frac{9}{4}z^{-2}$$

	$H[z]=\dfrac{Y[z]}{X[z]}$ $=\dfrac{\left(\dfrac{z-\frac{3}{2}}{z-\frac{1}{2}}\right)}{1-3z^{-1}+\frac{9}{4}z^{-2}}=\dfrac{\left(\dfrac{z-\frac{3}{2}}{z-\frac{1}{2}}\right)}{z^{-2}\left(z^2-3z+\frac{9}{4}\right)}=\dfrac{\left(\dfrac{z-\frac{3}{2}}{z-\frac{1}{2}}\right)}{z^{-2}\left(z-\frac{3}{2}\right)^2}=\dfrac{z^2}{\left(z-\frac{3}{2}\right)\left(z-\frac{1}{2}\right)}$ $\dfrac{H[z]}{z}=\dfrac{z}{\left(z-\frac{3}{2}\right)\left(z-\frac{1}{2}\right)}=\dfrac{\mathrm{A}}{\left(z-\frac{3}{2}\right)}+\dfrac{\mathrm{B}}{\left(z-\frac{1}{2}\right)}=\dfrac{\frac{3}{2}}{\left(z-\frac{3}{2}\right)}+\dfrac{-\frac{1}{2}}{\left(z-\frac{1}{2}\right)}$ $H[z]=\dfrac{\frac{3}{2}z}{\left(z-\frac{3}{2}\right)}+\dfrac{-\frac{1}{2}z}{\left(z-\frac{1}{2}\right)}$ $h[n]=\left(1.5(1.5)^n-0.5(0.5)^n\right)u[n]$

4. [20 pts]

A certain 15 point discrete Fourier series response, $X[k]$, of discrete signal, $x[n]$, has only non-zero values at $\{X[k=5]=-7.5000-j12.9904;\ X[k=11]=-7.5000+j12.9904\}$

a) Find $x[n]$

b) Find $y[n]$ which is output of the filter, if $H[k]=[4\ \ 4\ \ -4\ \ 4]$. It means, find discrete convolution of $y[n]=x[n]*h[n]$

c) Find 6 point circular convolution of $\{x[n]\ \ \&\ \ h[n]\}$

d) Find fft of $x[n]$

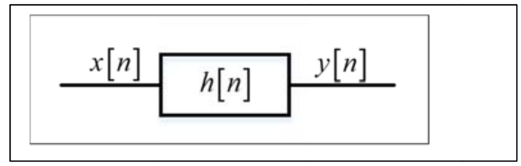
a)	$x[n]=[-1\ \ 2\ \ -1]$
b)	$y[n]=[4\quad 0\quad 4\quad -8]$
c)	$y[n]_{6\,pt}=[-4\quad 8\quad -8\quad 16\quad -20\quad 8]$
d)	$X[k]_{4\,pt\_ff}=\begin{bmatrix} 0 \\ -2j \\ -4 \\ 2j \end{bmatrix}$

```
clc; clear;

x = [-1 2 -1];
% xx = [x x x x x];
xx = [x x x x x];
N = length(xx)

XX   = fft(xx,N).'
k = (0:N-1).';
[k XX]

[k sym(XX)]

h = [2 2 -2 2];
H = fft(h,4)
X = fft([-1 2 -1 0],4)
Y = X.*H
y = ifft(X.*H,4)

%part(c)
x6 = [x 0 0 0]
h6 = [y 0 0]
y_6pt = cconv(x6,h6,6)

% part(d)
x4 = [x 0];
X_fft = fft(x4,4).'
```

5. [15 pts]

A causal linear time invariant system has system function of

$$H(z)=\frac{z+1}{(z-0.5)(z+0.25)}$$



a) Find ROC.

Time domain output signal $y[n]$ is defined as

$$y[n]=-\frac{1}{3}\left(-\frac{1}{4}\right)^n u[n]-\frac{4}{3}(2)^n u[-(n+1)]$$

b) Find the z transform $X(z)$ of an input signal $x[n]$ that can produce the above output.

---

a)	Since the statement says it is a causal system ROC is $\|z\|>0.5$

b)

$$y[n]=-\frac{1}{3}\left(-\frac{1}{4}\right)^{n}u[n]-\frac{4}{3}(2)^{n}u\left[-(n+1)\right]$$

$$Y(z)=-\frac{1}{3}\cdot\frac{z}{\left(z+\frac{1}{4}\right)}+\frac{4}{3}\cdot\frac{z}{(z-2)}=\frac{-\frac{1}{3}\cdot z(z-2)+\frac{4}{3}\cdot z\left(z+\frac{1}{4}\right)}{\left(z+\frac{1}{4}\right)(z-2)}=\frac{z(z+1)}{\left(z+\frac{1}{4}\right)(z-2)}\qquad 0.25<|z|<2$$

$$H(z)=\frac{z+1}{(z-0.5)(z+0.25)}\qquad |z|>0.5$$

$$H(z)=\frac{Y(z)}{X(z)}\Rightarrow X(z)=\frac{Y(z)}{H(z)}$$

$$X(z)=\frac{Y(z)}{H(z)}=\frac{\dfrac{z(z+1)}{\left(z+\frac{1}{4}\right)(z-2)}}{\dfrac{z+1}{(z-0.5)(z+0.25)}}=\frac{\dfrac{z\cancel{(z+1)}}{\cancel{\left(z+\frac{1}{4}\right)}(z-2)}}{\dfrac{\cancel{(z+1)}}{(z-0.5)\cancel{(z+0.25)}}}$$

$$=\frac{z(z-0.5)}{(z-2)}=\frac{z^{2}}{(z-2)}-\frac{0.5z}{(z-2)}\qquad |z|<2$$

$$x[n]=-(2)^{n+1}u[-n-2]+0.5(2)^{n}u[-n-1]$$

6. [15 pts]

An analog signal $x(t)$ shown below is sampled to give a sequence $x[n]$.

$$f_s = 100 \text{ and } N = 200 \text{ for DFT}$$
$$x(t) = 2\sin(4\pi t) + 5\cos(8\pi t)$$

Plot $X_R[k]$ and $X_I[k]$, the real and imaginary parts of the discrete Fourier transform (DFT) of $x[n]$ (plot magnitude and phase information.)

Plot $X_R[k]$ and $X_I[k]$, the real and imaginary parts of the discrete Fourier transform (DFT) of $x[n]$.

$x(t)$ contains two frequencies, $\underbrace{2\sin(4\pi t)}_{f_1=2} + \underbrace{5\cos(8\pi t)}_{f_2=4}$

The frequency property of sine and cosine is

$$\begin{aligned} F\{\sin(x)\} &= \frac{1}{2j}\left[\delta(f - f_1) - \delta(f + f_1)\right] \\ F\{\cos(x)\} &= \frac{1}{2}\left[\delta(f - f_2) + \delta(f + f_2)\right] \end{aligned} \tag{1}$$

Since continuous signal has to be sampled with pulse train, it has to be multiplied with $\frac{1}{T_s} = f_s$

The spike (frequency response) in real part is contributed by the cosine function and its amplitude is

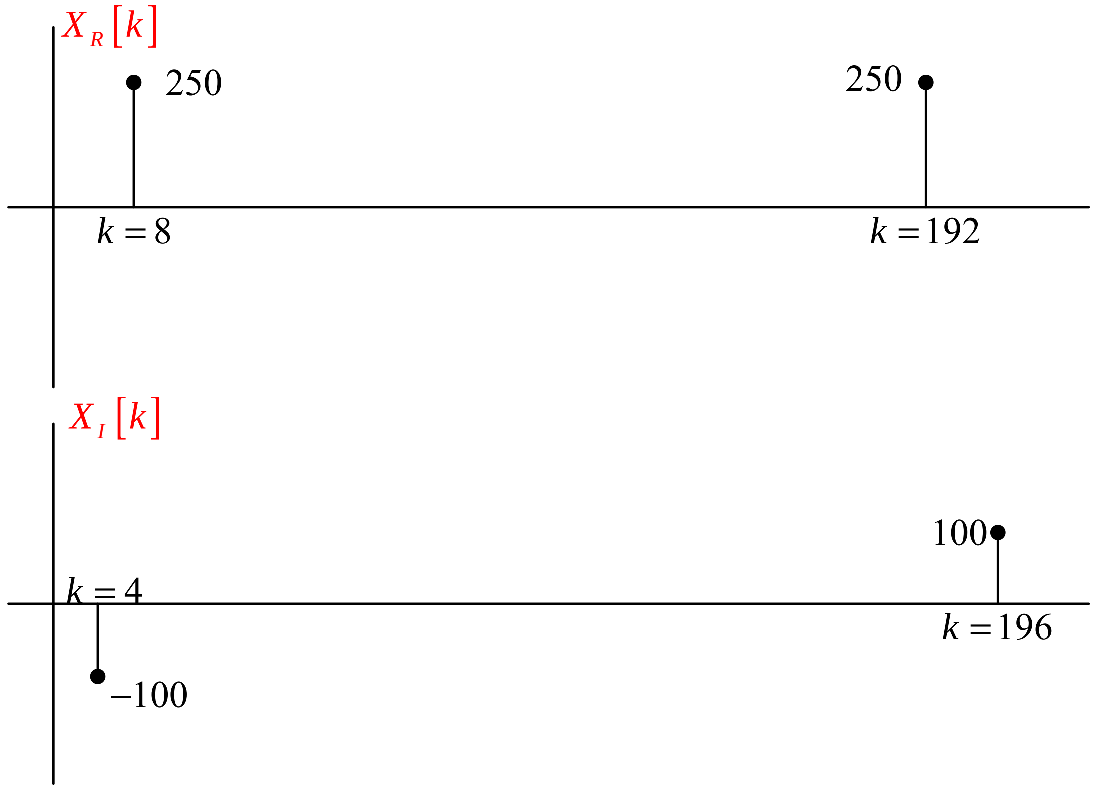
$$\frac{1}{2}(5 \cdot 100) = 250$$

So the frequency is at

$$250\left(\delta[k-8] + \delta[k-192]\right)$$

The spike for the sine function is imaginary part and its amplitude and frequency is

$$100 \text{ and } 100\left(-j\delta[k-4] + j\delta[k-196]\right)$$


$X_R[k]$
250
250
$k=8$
$k=192$
$X_I[k]$
100
$k=4$
$k=196$
−100