EE288 Data Conversions/Analog Mixed-Signal ICs
Spring 2018

Lecture 23: Oversampled ADC

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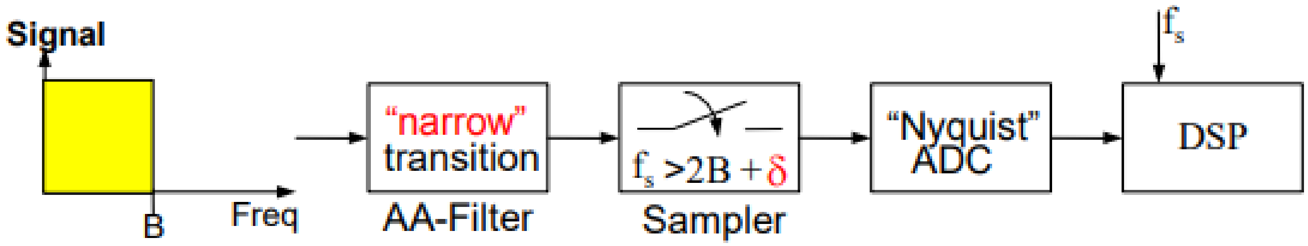
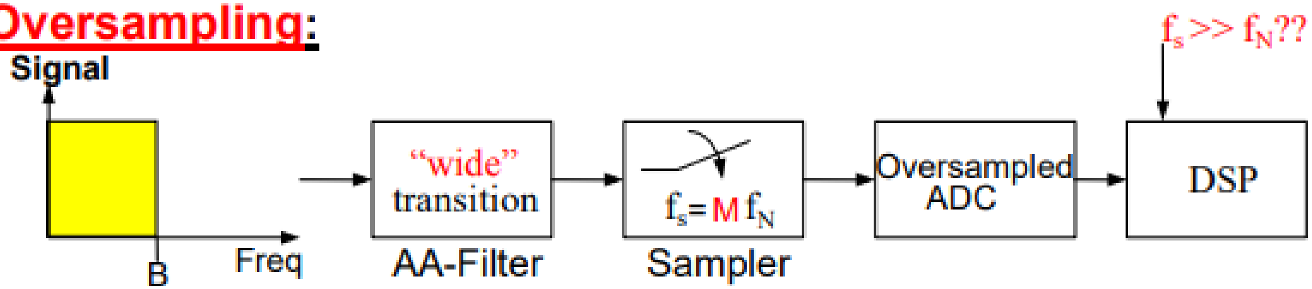
Nyquist vs. Oversampling Converters

Nyquist sampling:

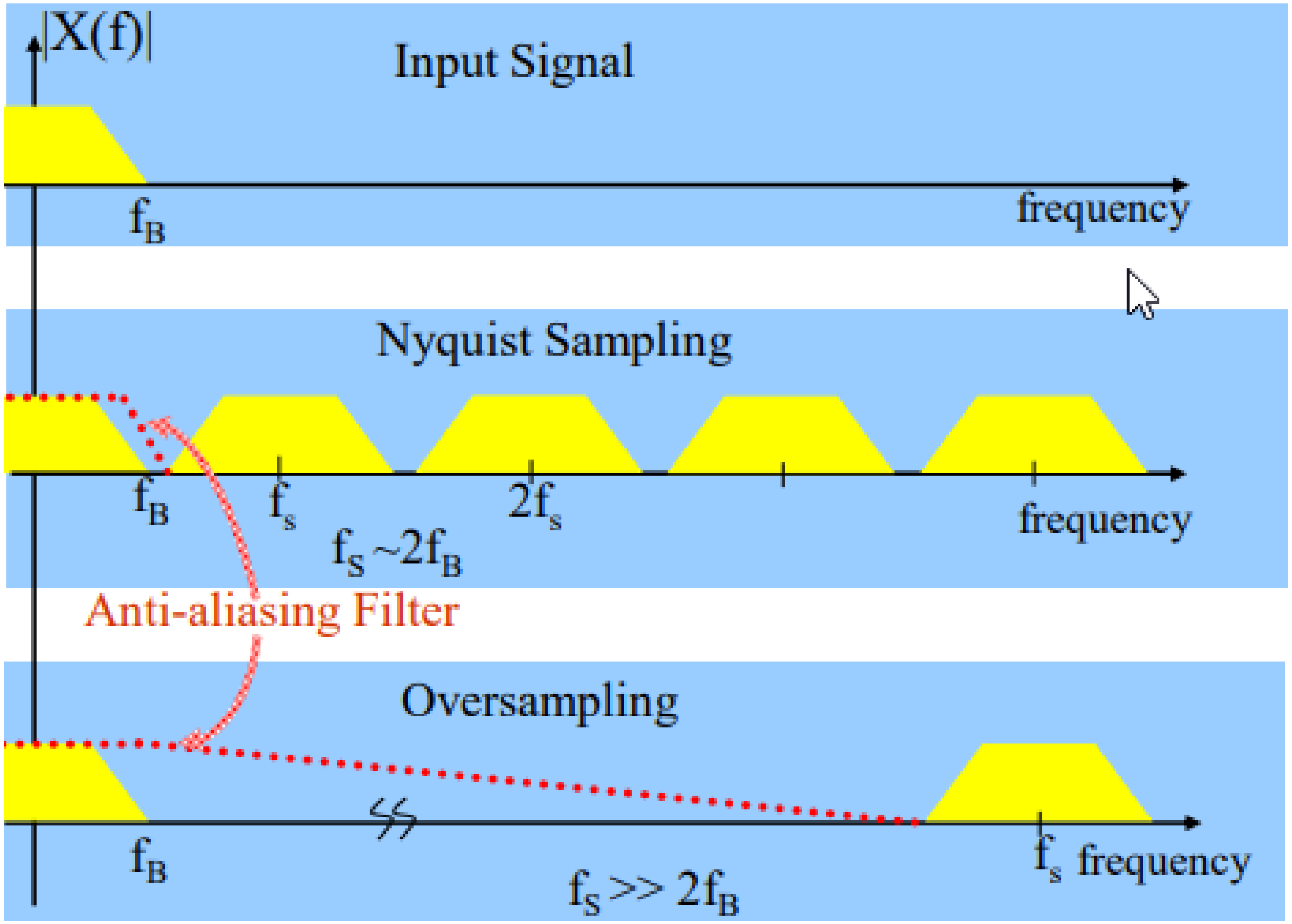


Oversampling:



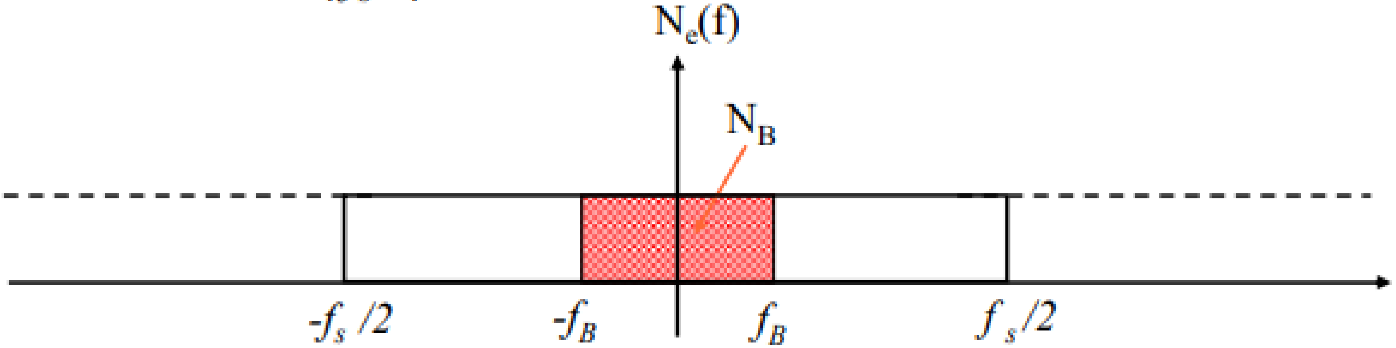
- Nyquist rate f_N ~2B
- Oversampling rate $M = f_s/f_N >> 1$

Anti-aliasing Requirement



Quantization Noise Spectrum

- For a quantizer with quantization step size Δ and sampling rate f_s :
  - Quantization noise power distributed uniformly across Nyquist bandwidth ($f_s/2$)



  - Power spectral density:

$$N_e(f) = \frac{\overline{e^2}}{f_s} = \left(\frac{\Delta^2}{12}\right)\frac{1}{f_s}$$

  - Noise is distributed over the Nyquist band $-f_s/2$ *to* $f_s/2$

Oversampled Converter Quantization Noise

$$S_B = \int_{-f_B}^{f_B} N_e(f)\,df = \int_{-f_B}^{f_B} \left(\frac{\Delta^2}{12}\right)\frac{1}{f_s}\,df$$

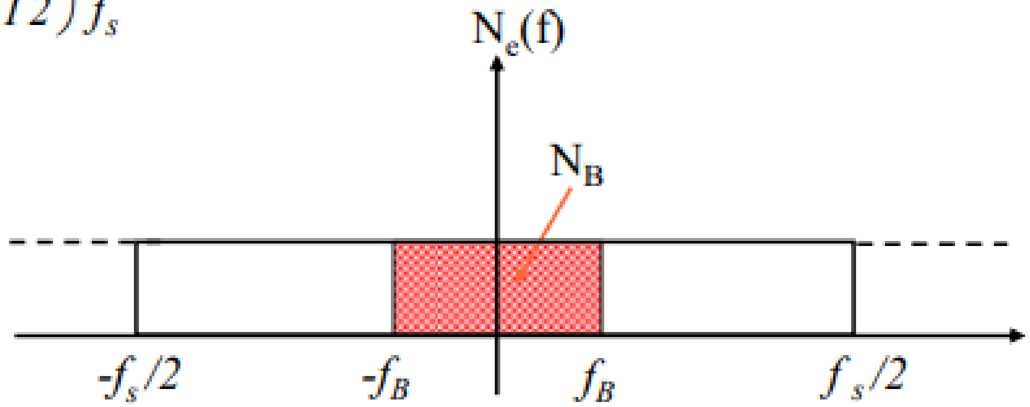
$$= \frac{\Delta^2}{12}\left(\frac{2f_B}{f_s}\right)$$

$$\text{where for } f_B = f_s/2$$

$$S_{B0} = \frac{\Delta^2}{12}$$

$$S_B = S_{B0}\left(\frac{2f_B}{f_s}\right) = \frac{S_{B0}}{M}$$

$$\text{where } M = \frac{f_s}{2f_B} = \text{oversampling ratio}$$



Oversampled Converter Quantization Noise

$$S_B = S_{B0}\left(\frac{2f_B}{f_s}\right) = \frac{S_{B0}}{M}$$

$$where\ M = \frac{f_s}{2f_B} = oversampling\ ratio$$

2X increase in M

→ 3dB reduction in S_B

→ ½ bit increase in resolution/octave oversampling

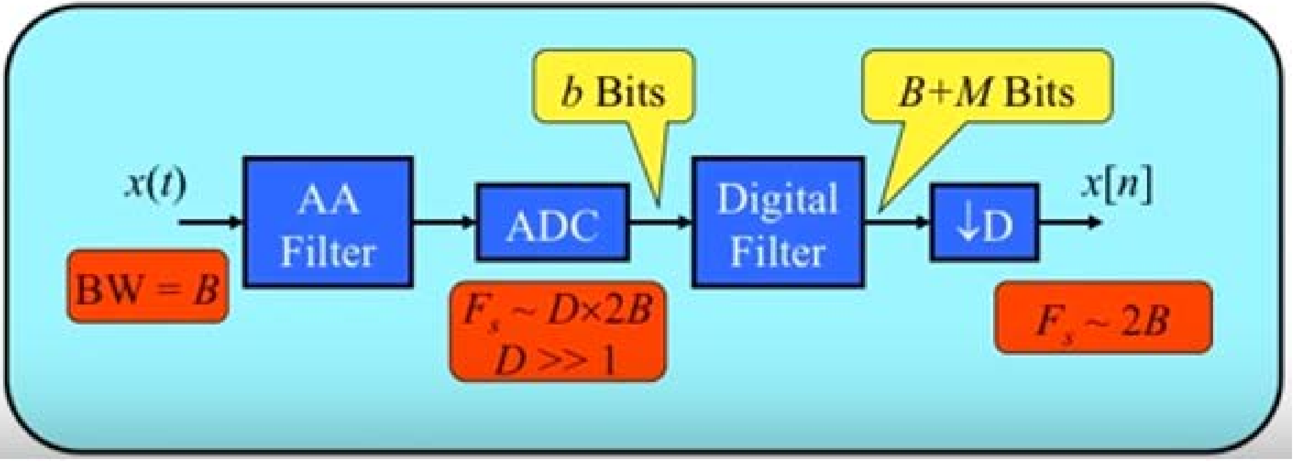
To further increase the improvement in resolution:

- Embed quantizer in a feedback loop (patented by Cutler in 1960s!)

  →Noise shaping (sigma delta modulation)

Oversampled ADC – Big Picture

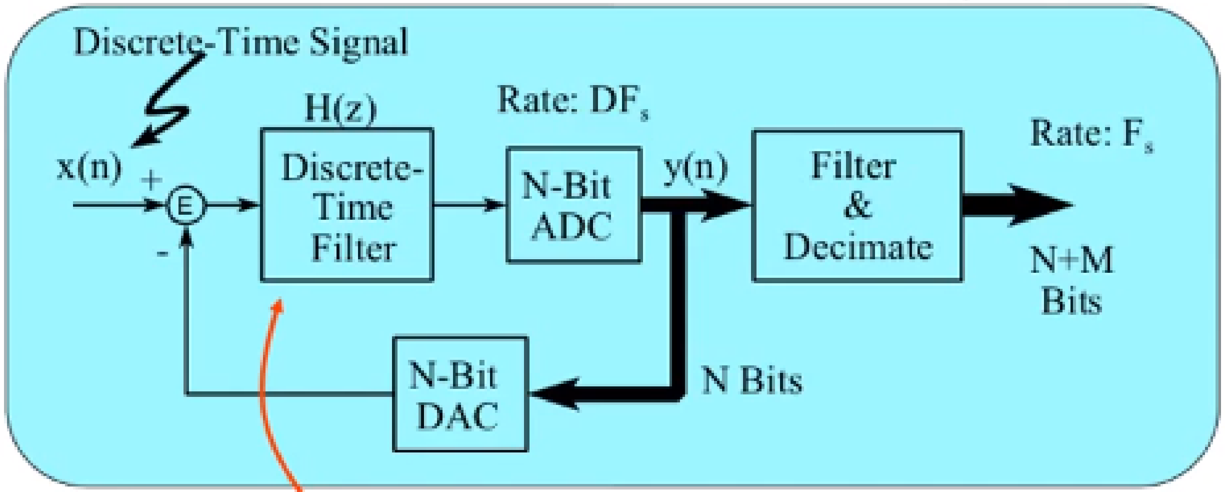
- **Oversample and Filter**:
  - Say You Have an ADC That Can Sample at a Rate of F_s.
  - Quantization Noise PSD is Uniformly Spread Over $-F_s/2$ to $F_s/2$.
  - If Signal Resides in Some Subband, Digitally Filter to that Band:
    - Signal Power Same, but Noise Power Reduced ➡ Improves the SNR
    - But… SNR is related to ENOB ➡ Increases the Effective # of Bits!
      - Analogous to averaging a bunch of integers to get a fractional value
    - Increased ENOB at the Expense of Reduced Processing BW



D = OSR

Noise Shaping ADC

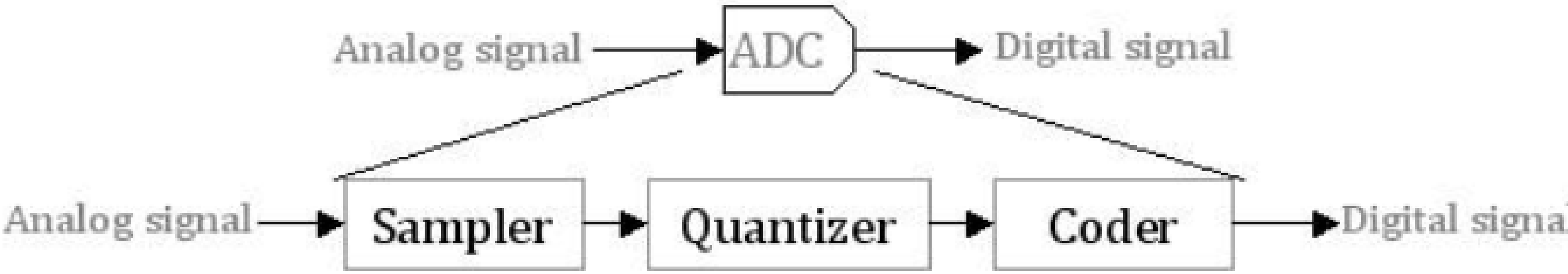
- Use VERY High Over-Sampling Rate
- Use Low-Bit ADC (sometimes even just 1 Bit)
- Use DSP Noise Shaping to Non-Uniformly Spread Noise
  - Push Most of the Quantization Noise Out of the Signal Band

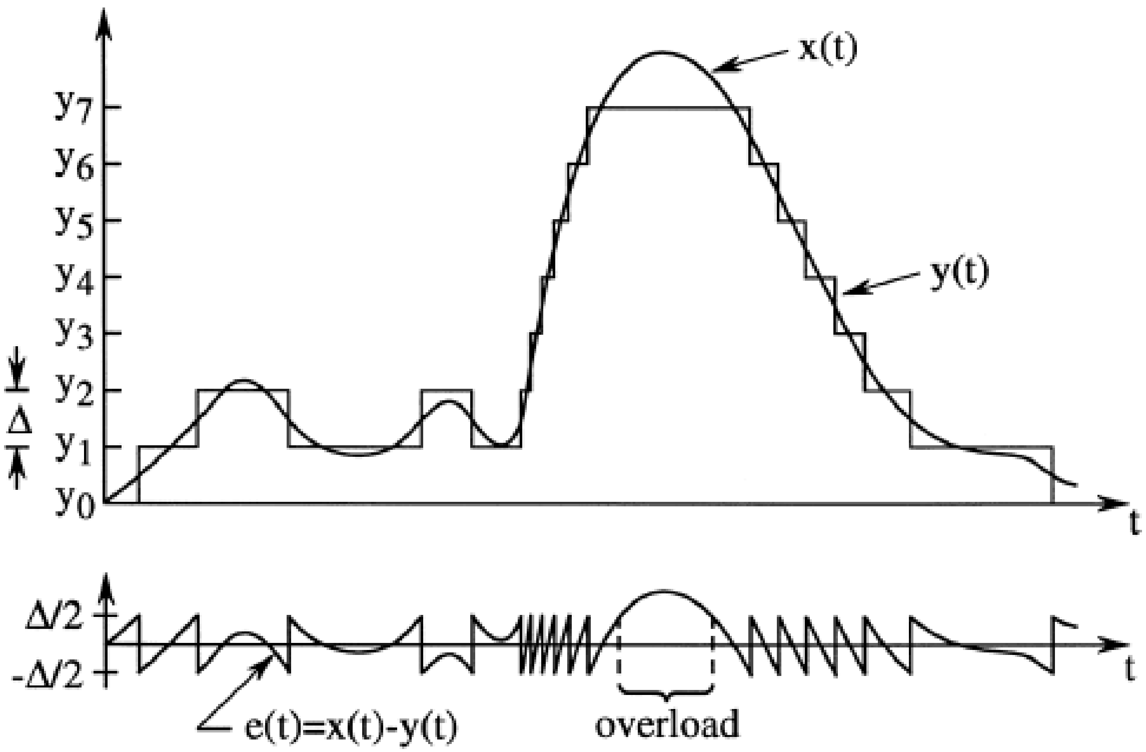


Design Filter $H(z)$ to:

- Pass Signal w/ Minimal Distortion
- Attenuate Quantization Noise in Signal Band

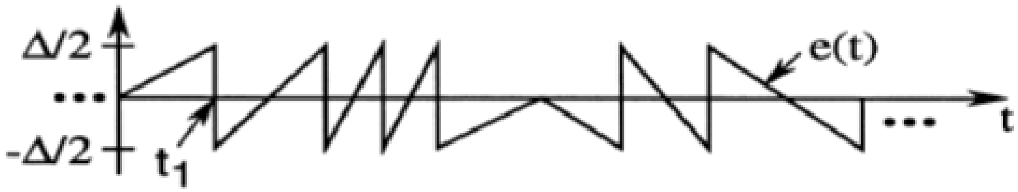
ADC Quantization





SQNR

Deterministic Sawtooth Waveform Error Model



$$e_{ms}^2 = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |e(t)|^2 dt$$

$$= \frac{1}{t_1} \int_0^{t_1} \left| \frac{\Delta/2}{t_1} t \right|^2 dt = \frac{1}{t_1^3} \frac{\Delta^2}{4} \frac{t^3}{3} \Bigg|_0^{t_1}$$

$$= \frac{\Delta^2}{12}$$

$$\text{Input FS Sinewave} = v(t) = \frac{\Delta 2^N}{2} \sin(2\pi f t).$$

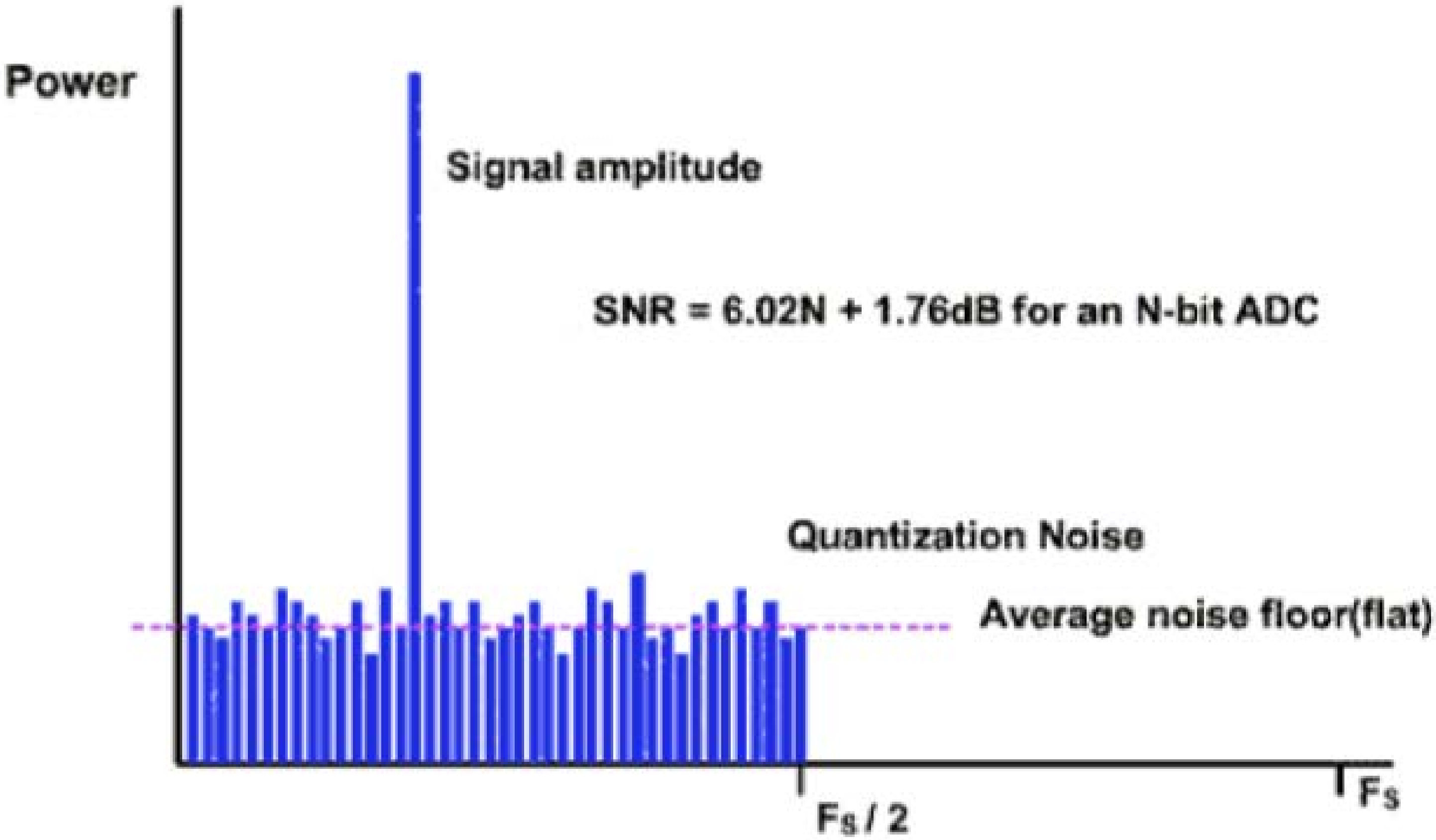
$$\text{rms value of FS input} = \frac{\Delta 2^N}{2\sqrt{2}}.$$

$$\text{SNR} = 20\log_{10} \frac{\text{rms value of FS input}}{\text{rms value of quantization noise}}$$

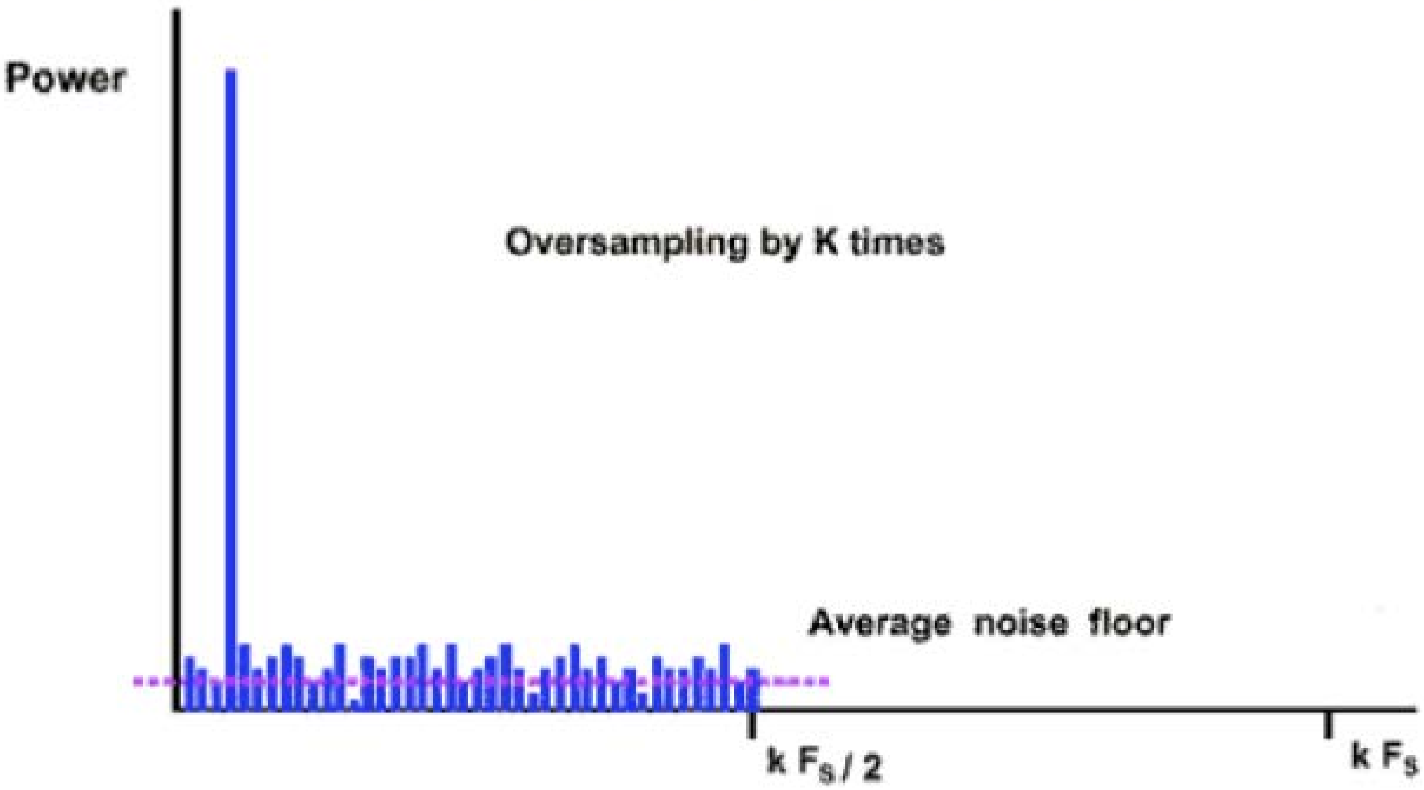
$$\text{SNR} = 20\log_{10}\left[\frac{\Delta 2^N / 2\sqrt{2}}{\Delta / \sqrt{12}}\right] = 20\log_{10} 2^N + 20\log_{10}\sqrt{\frac{3}{2}}$$

$\boxed{\text{SNR} = 6.02N + 1.76\text{dB},}$ over the dc to $f_s/2$ bandwidth.

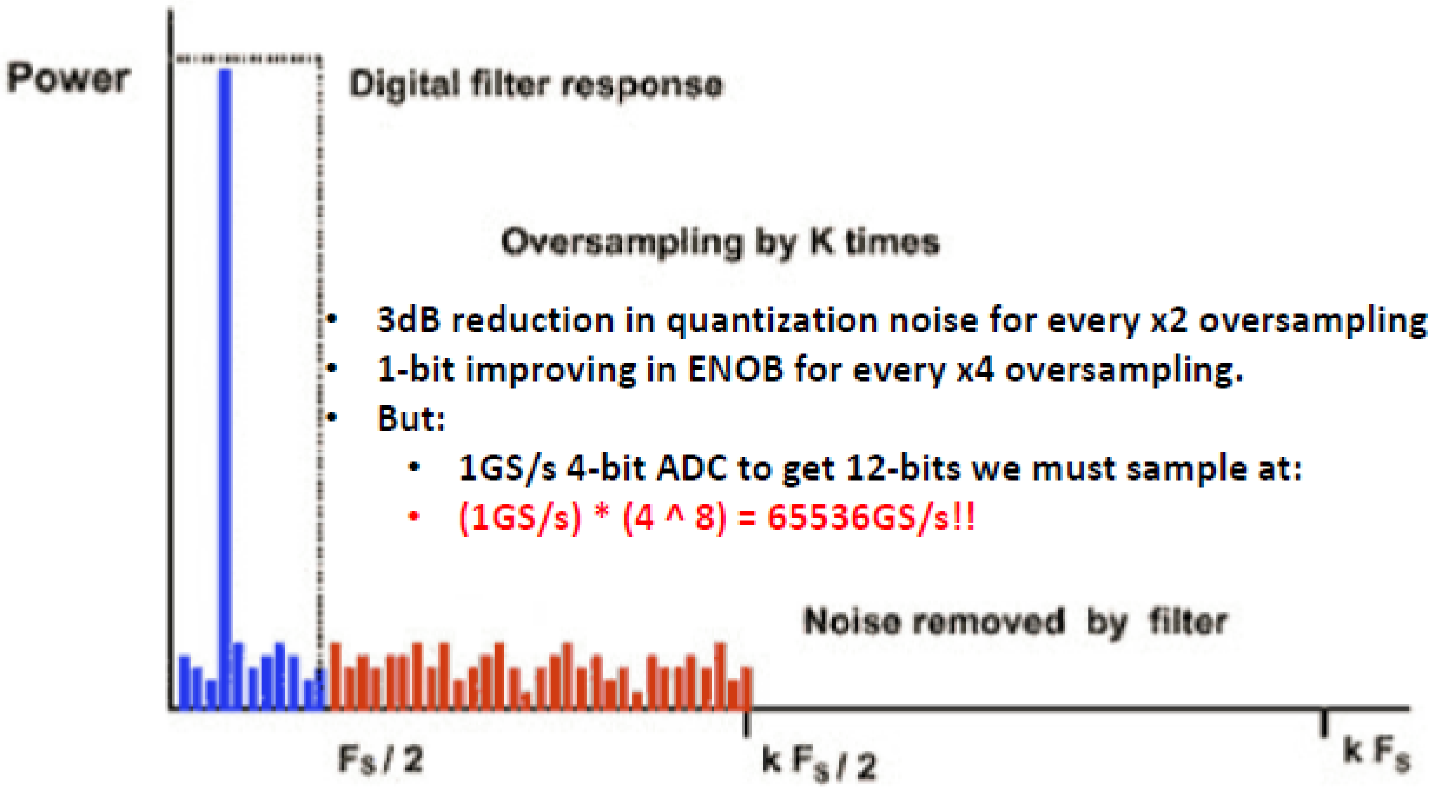
Frequency Spectrum



Frequency Spectrum with Oversampling by K times

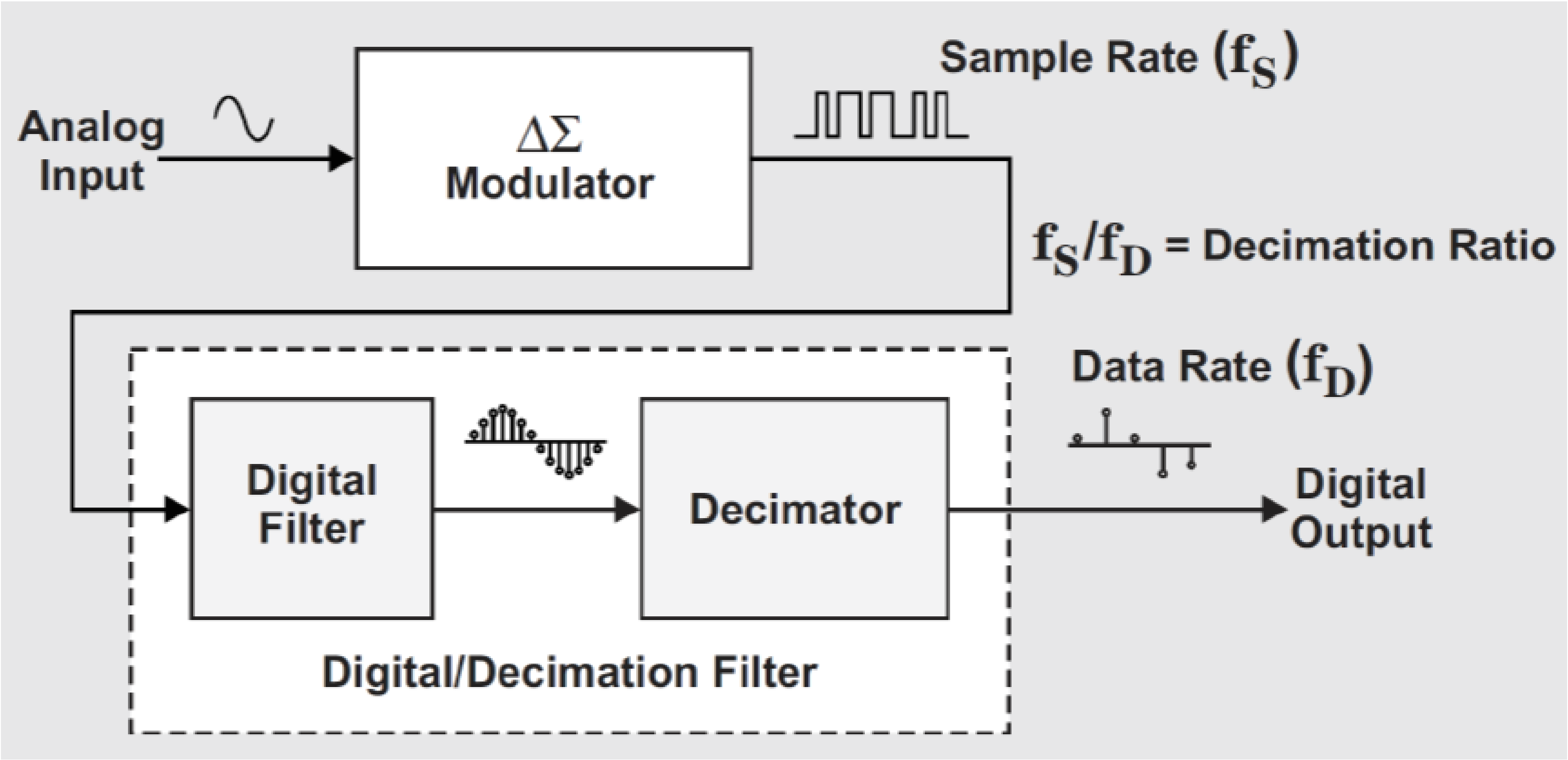


Digital Filtering

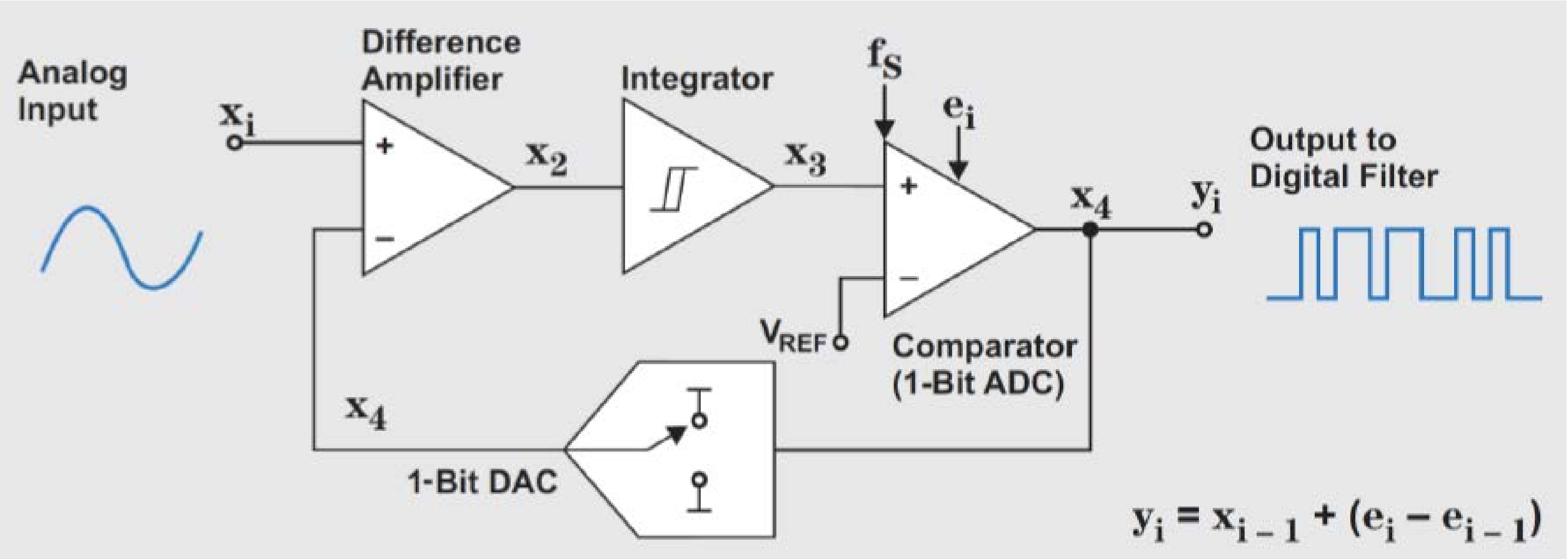


- 3dB reduction in quantization noise for every x2 oversampling
- 1-bit improving in ENOB for every x4 oversampling.
- But:
  - 1GS/s 4-bit ADC to get 12-bits we must sample at:
  - (1GS/s) * (4 ^ 8) = 65536GS/s!!

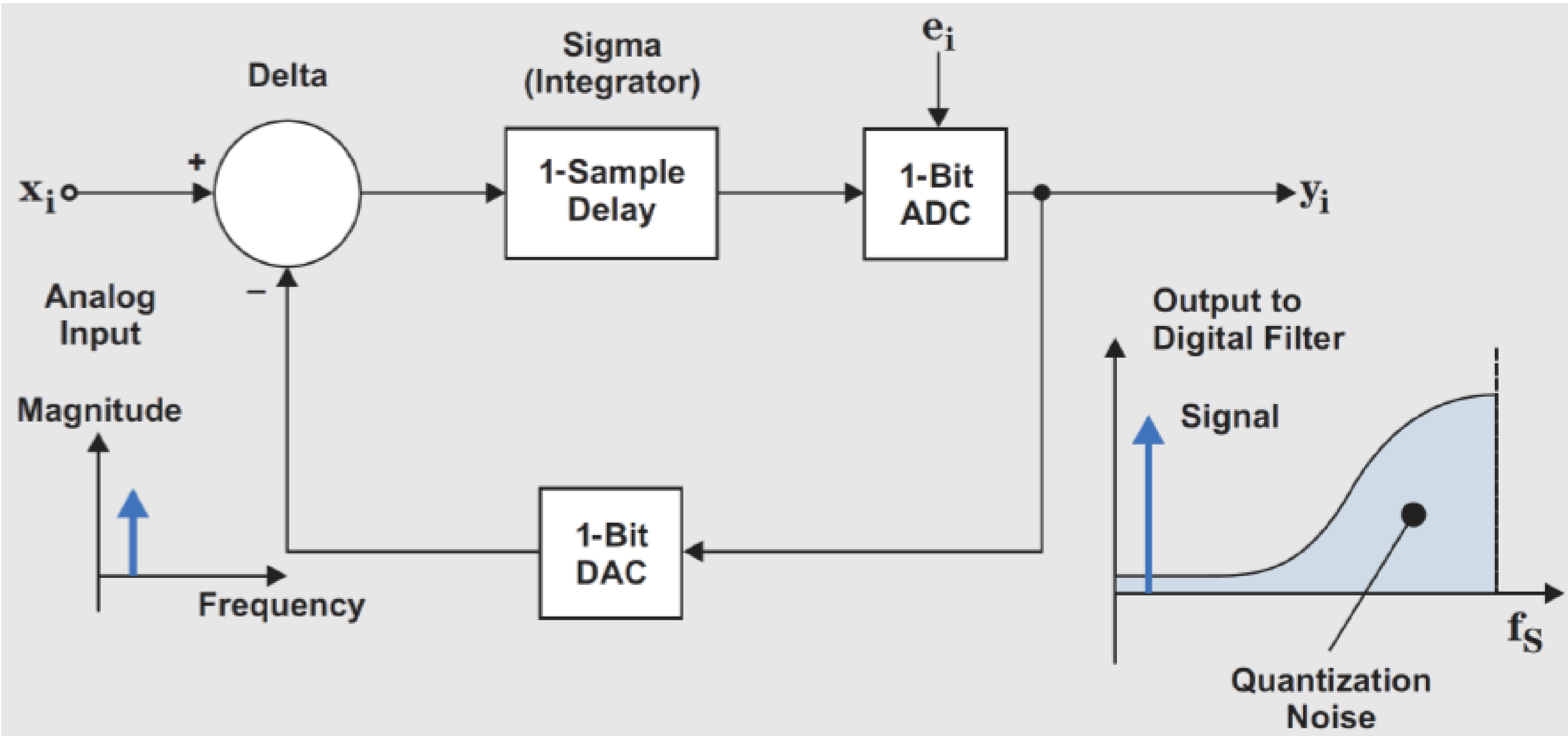
Oversampled ADC



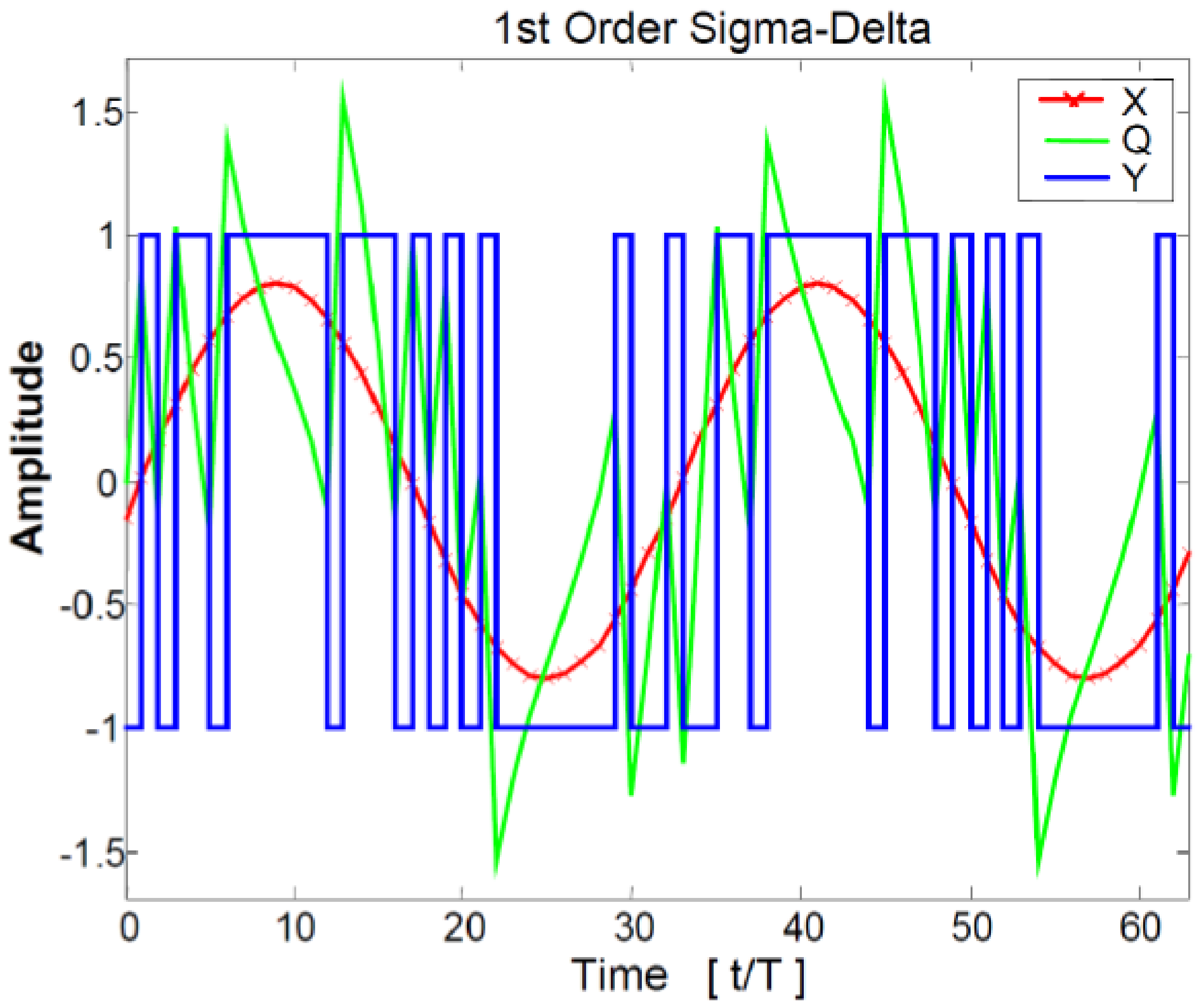
Oversampled 1-Bit ADC



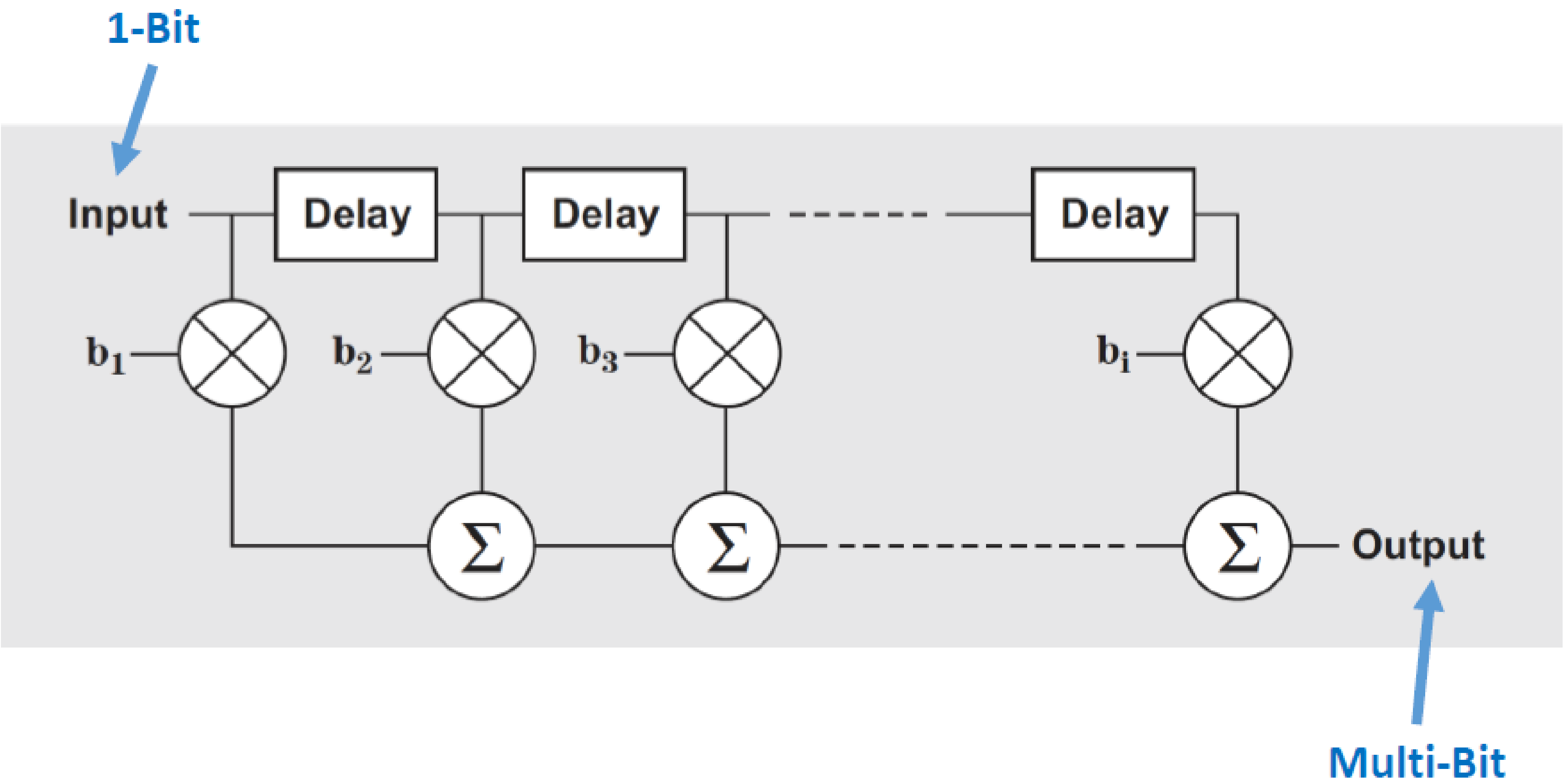
Shaped Noise at the output



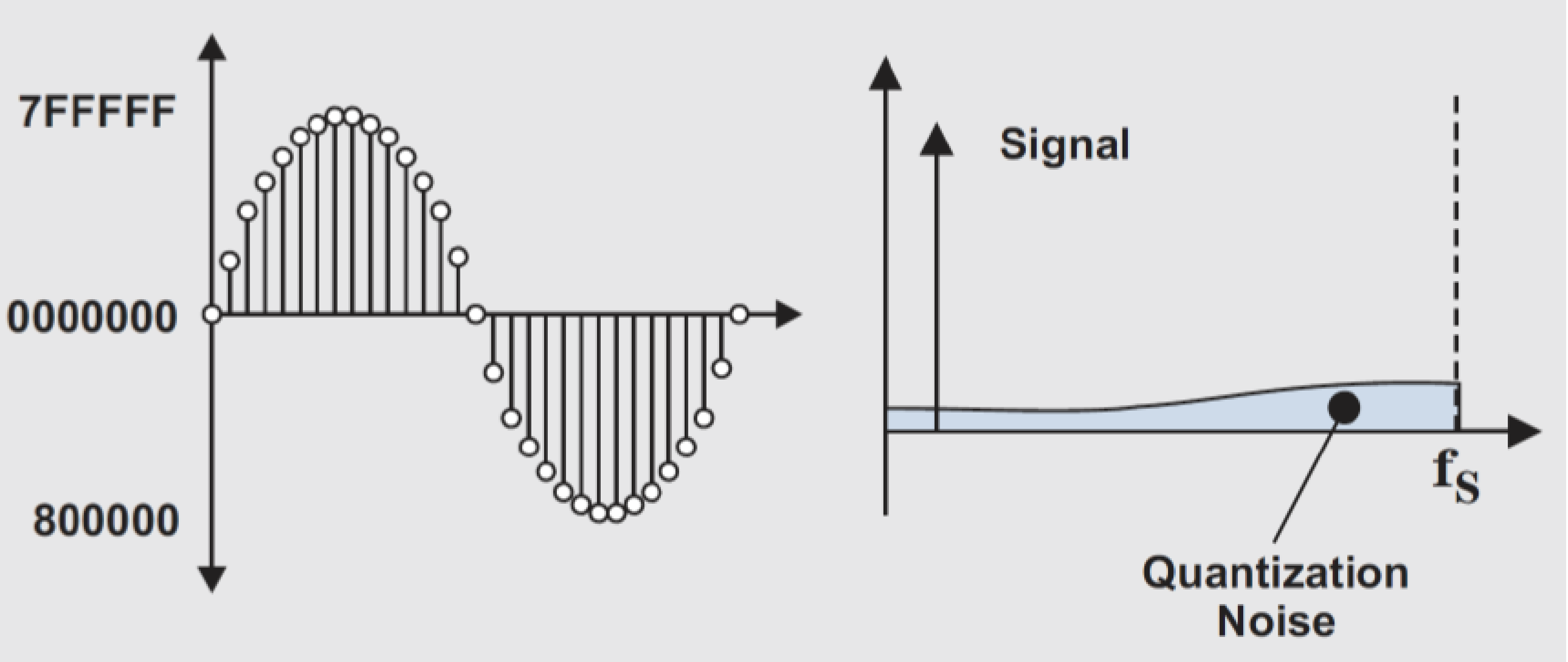
Time Domain Quantization Noise



Digital Filter



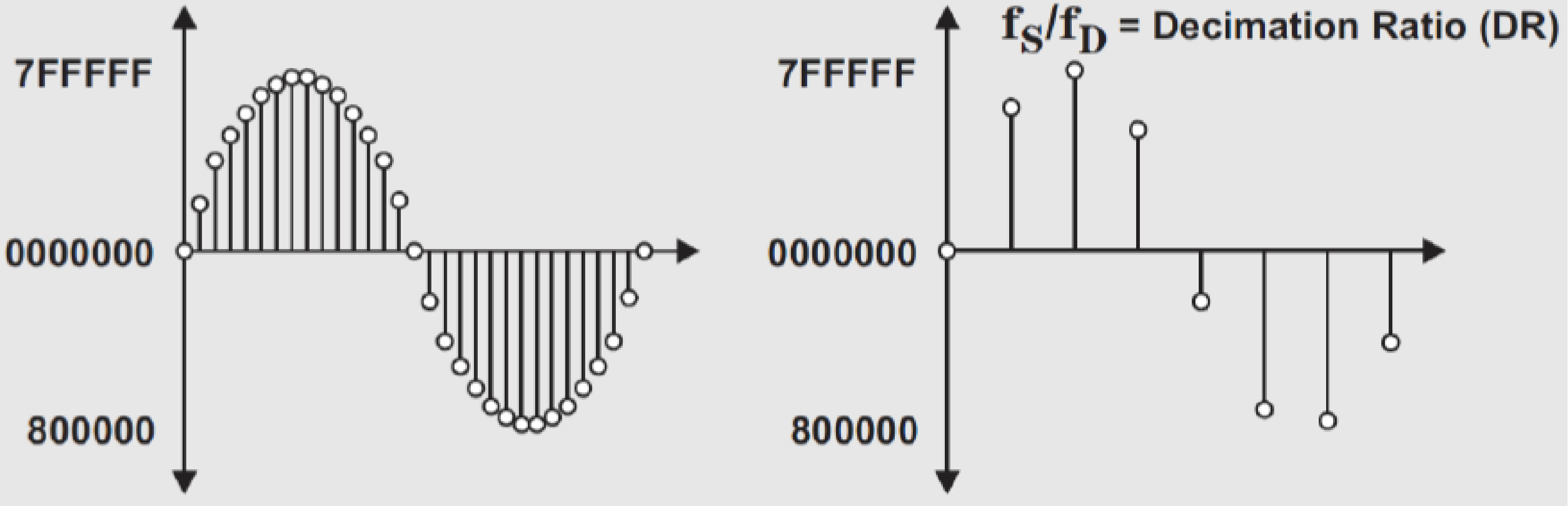
After Digital Filtering



(a) Time domain

(b) Frequency domain

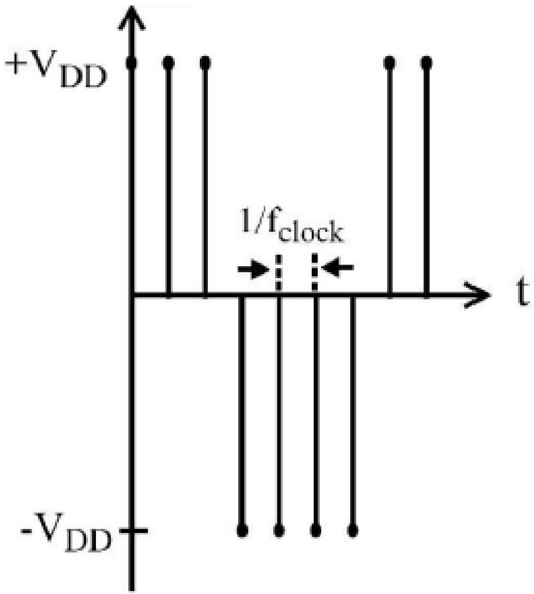
After Decimation



(a) Input sampling rate (f_S)

(b) Output-data rate (f_D)

Time Domain Signal in Oversampled ADC



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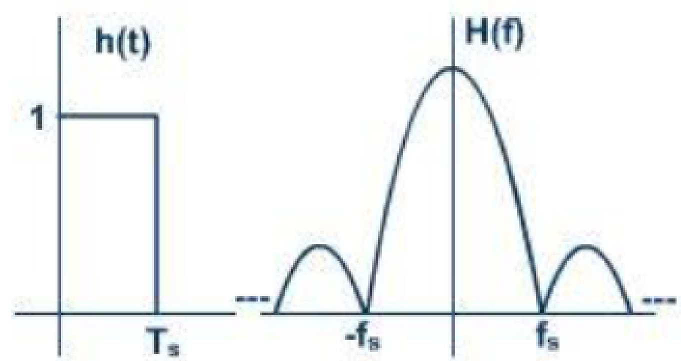

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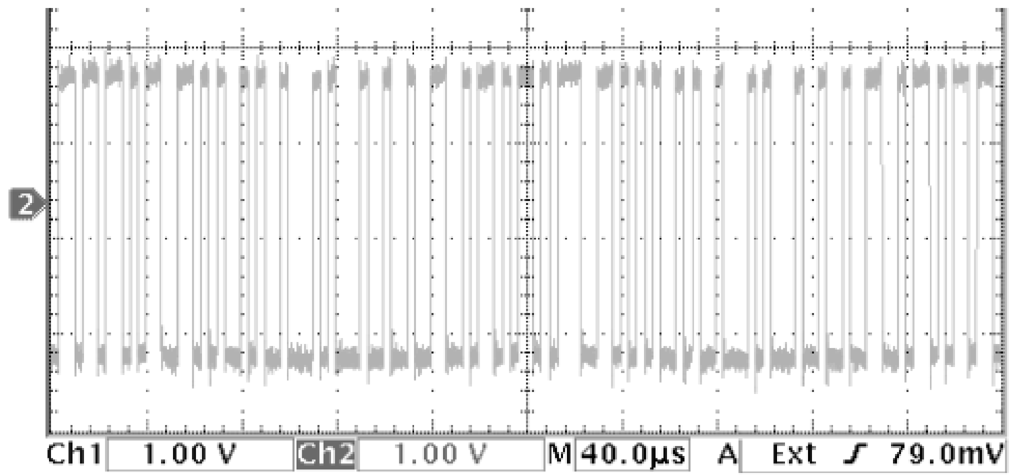
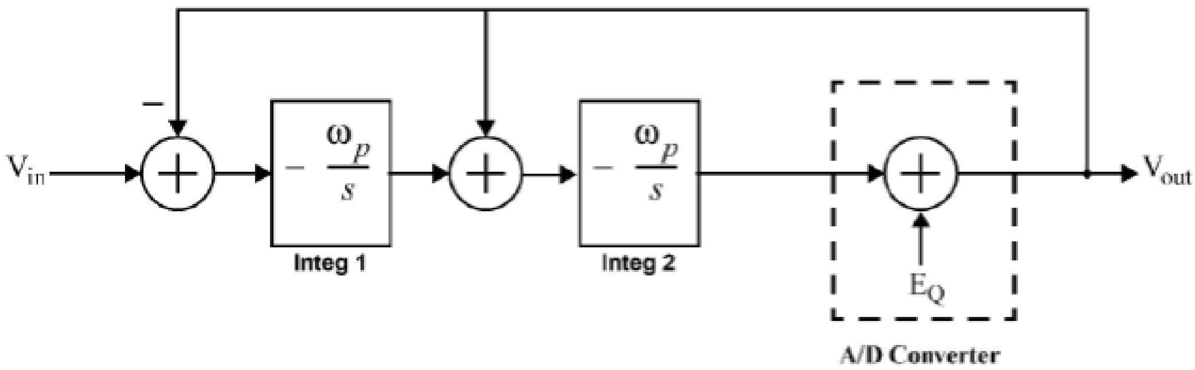
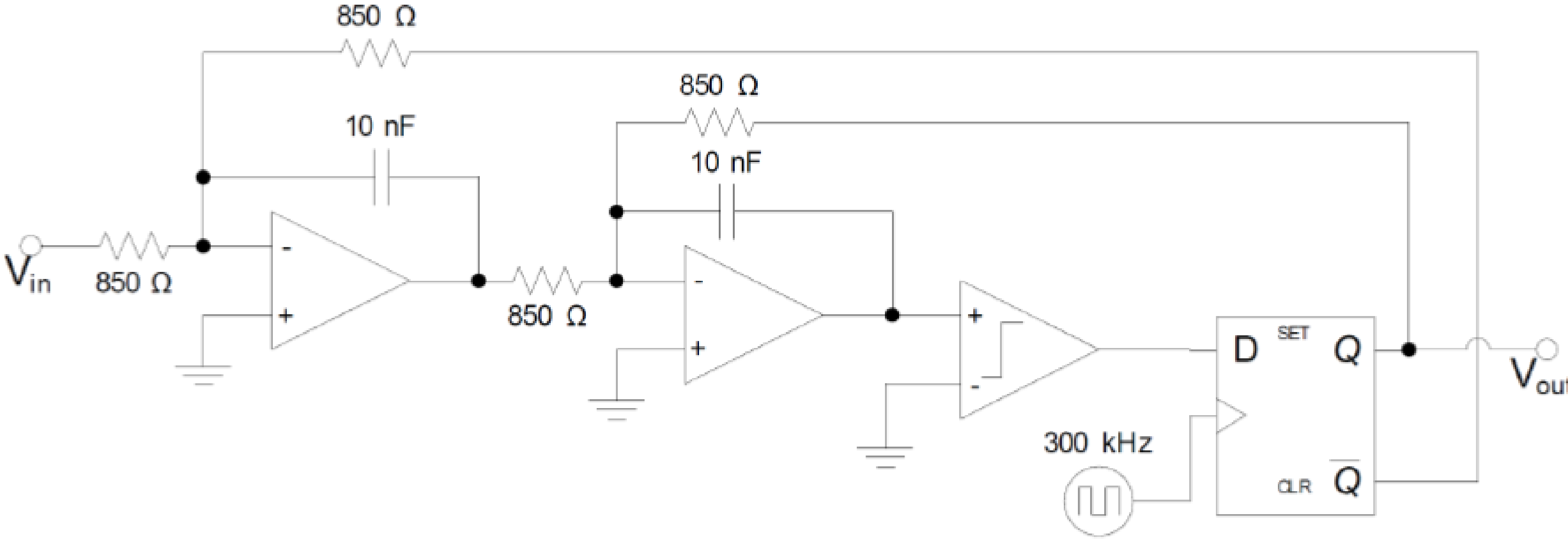
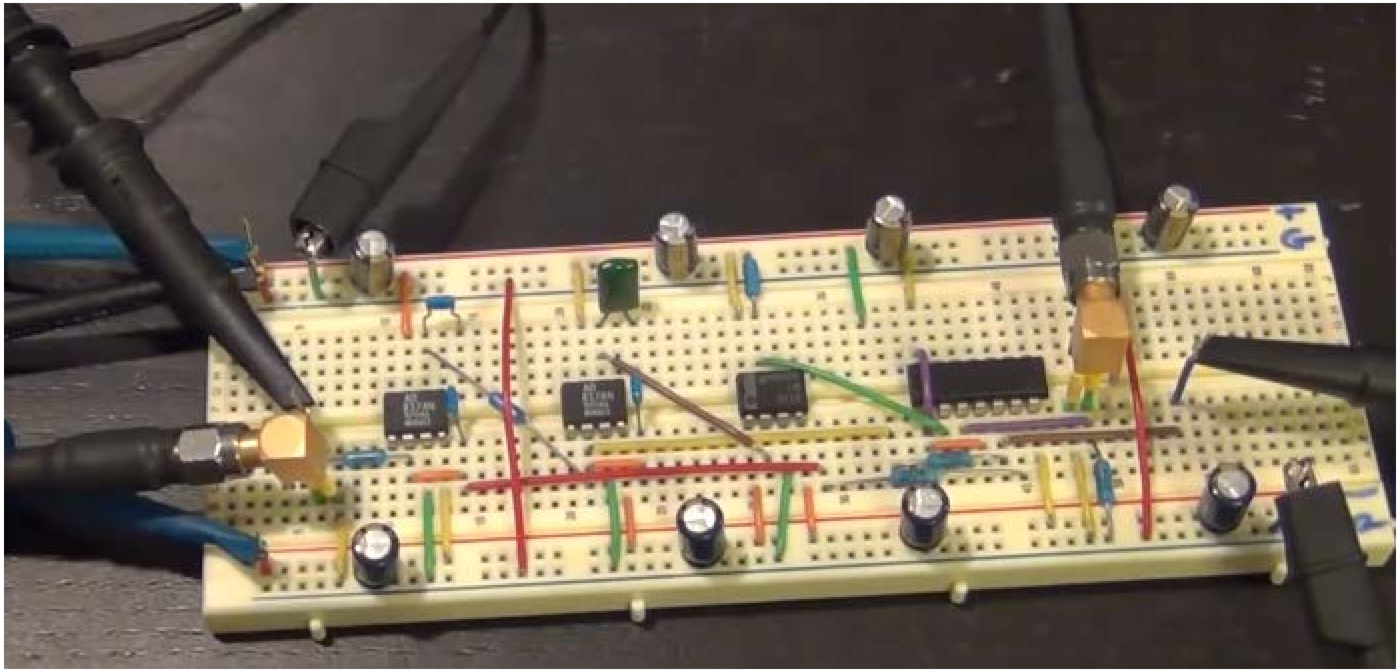


2^{nd}-Order Modulator Example

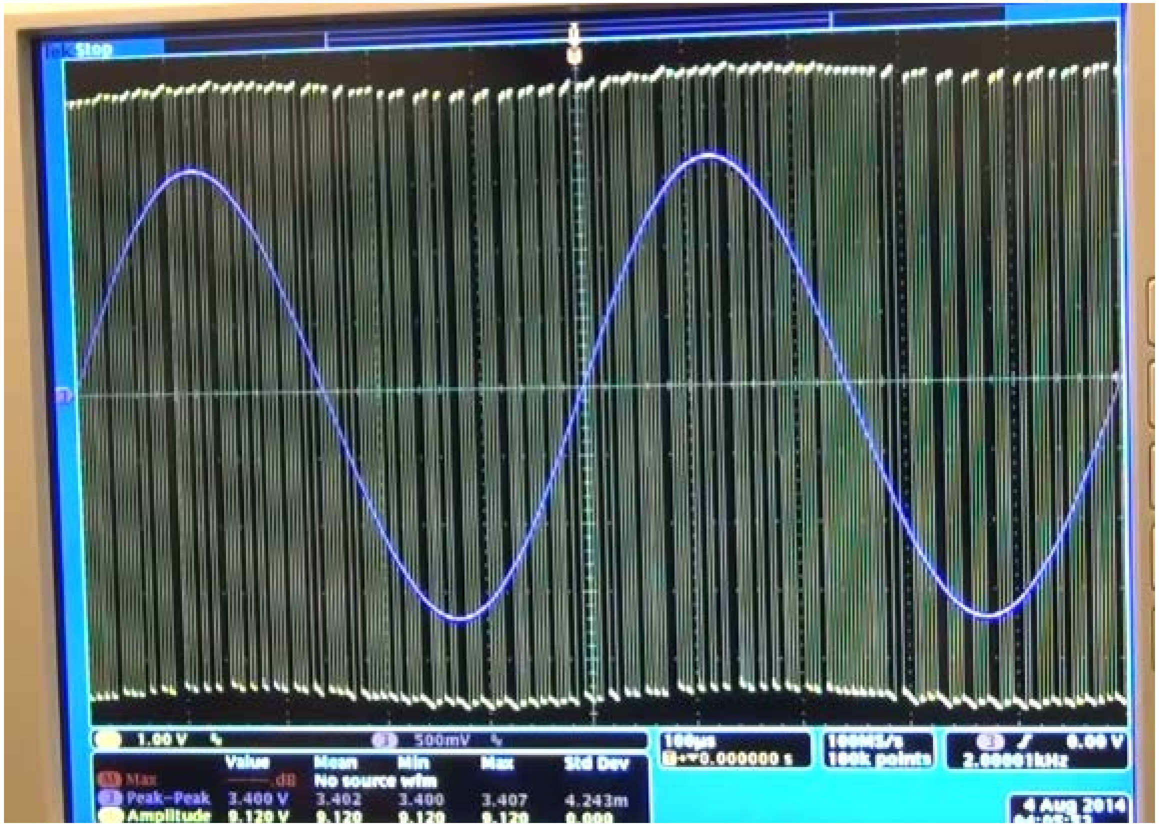




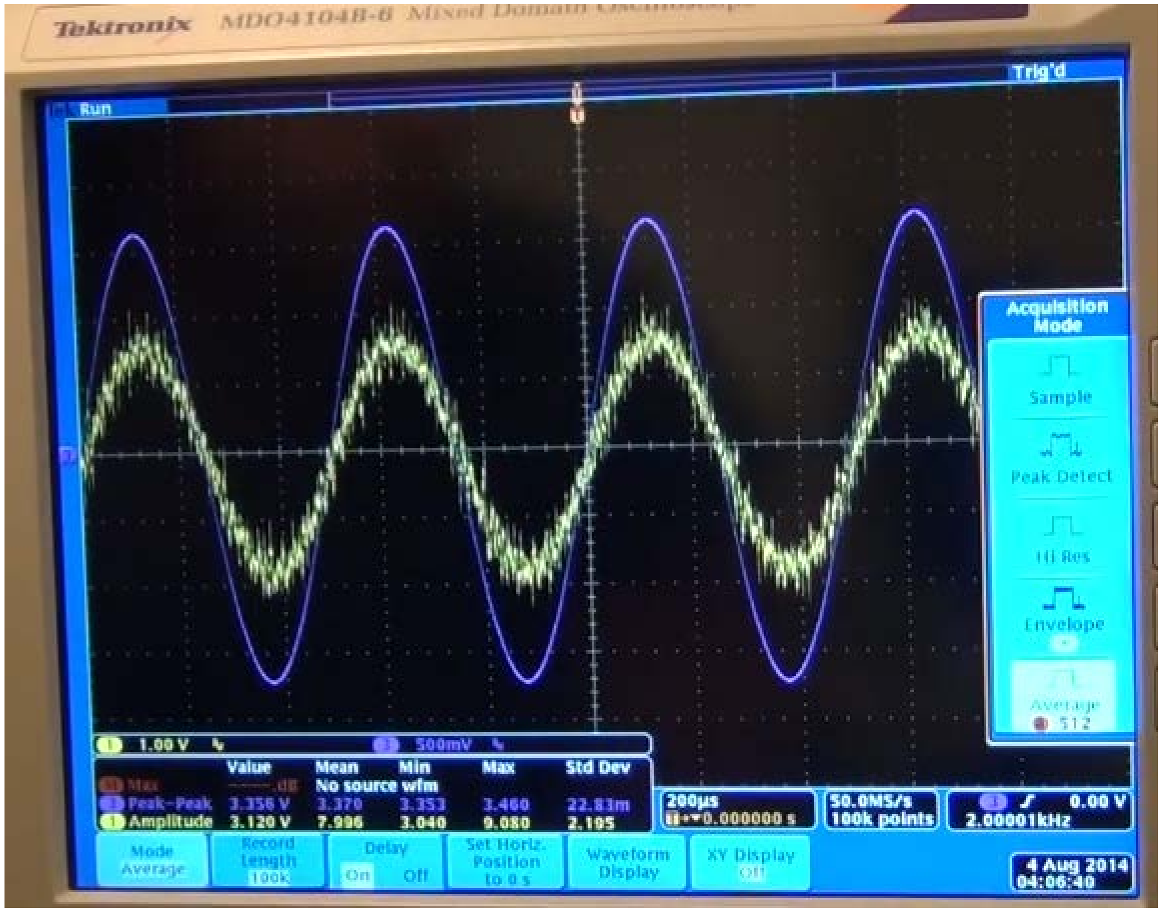
Breadboard implementation



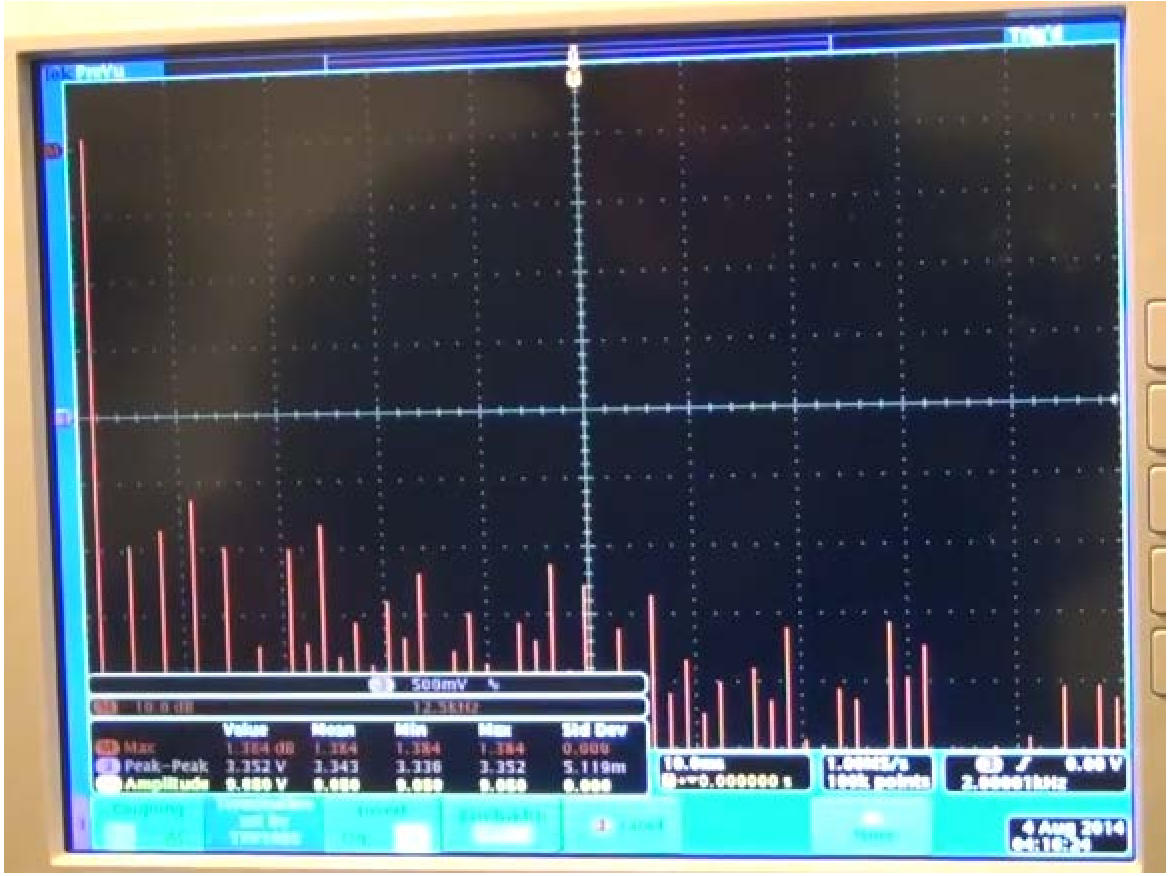
Measured Output Waveform



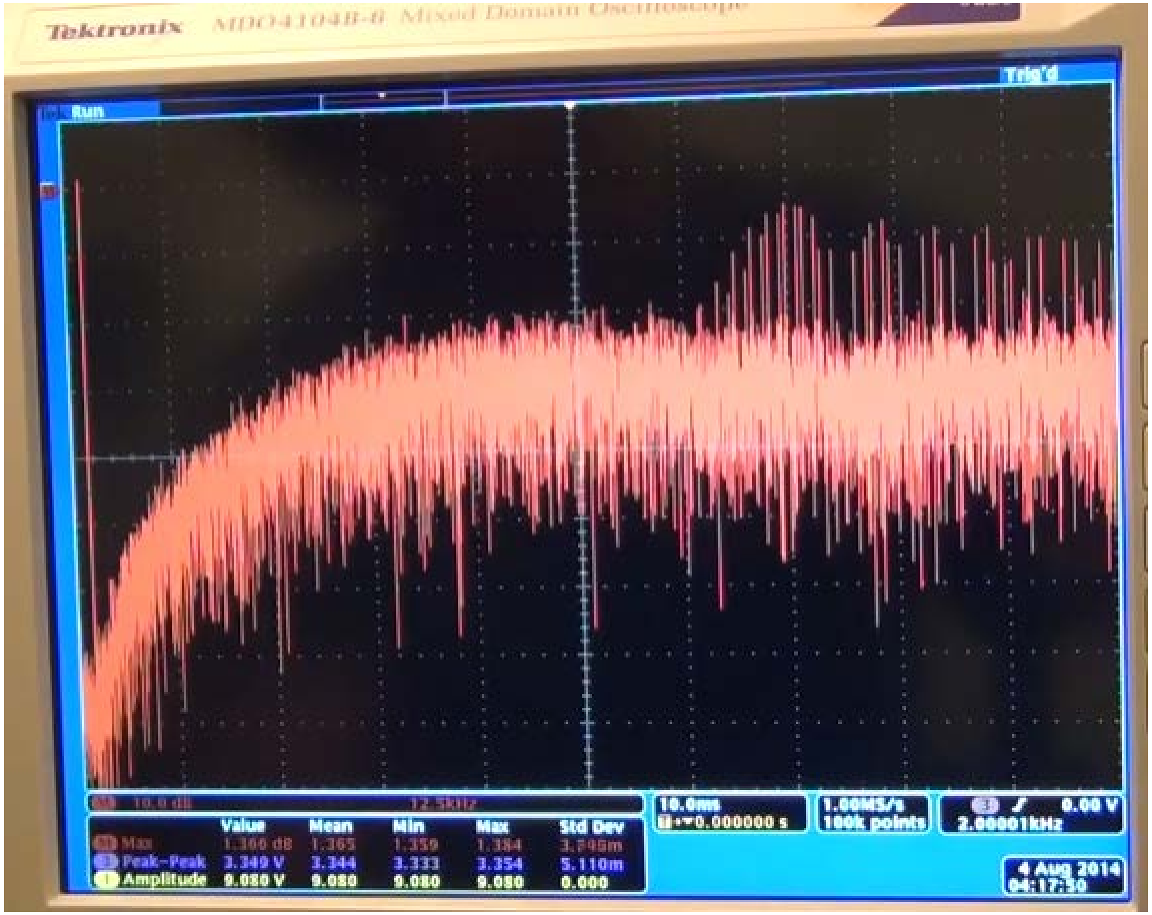
Averaging the output signal



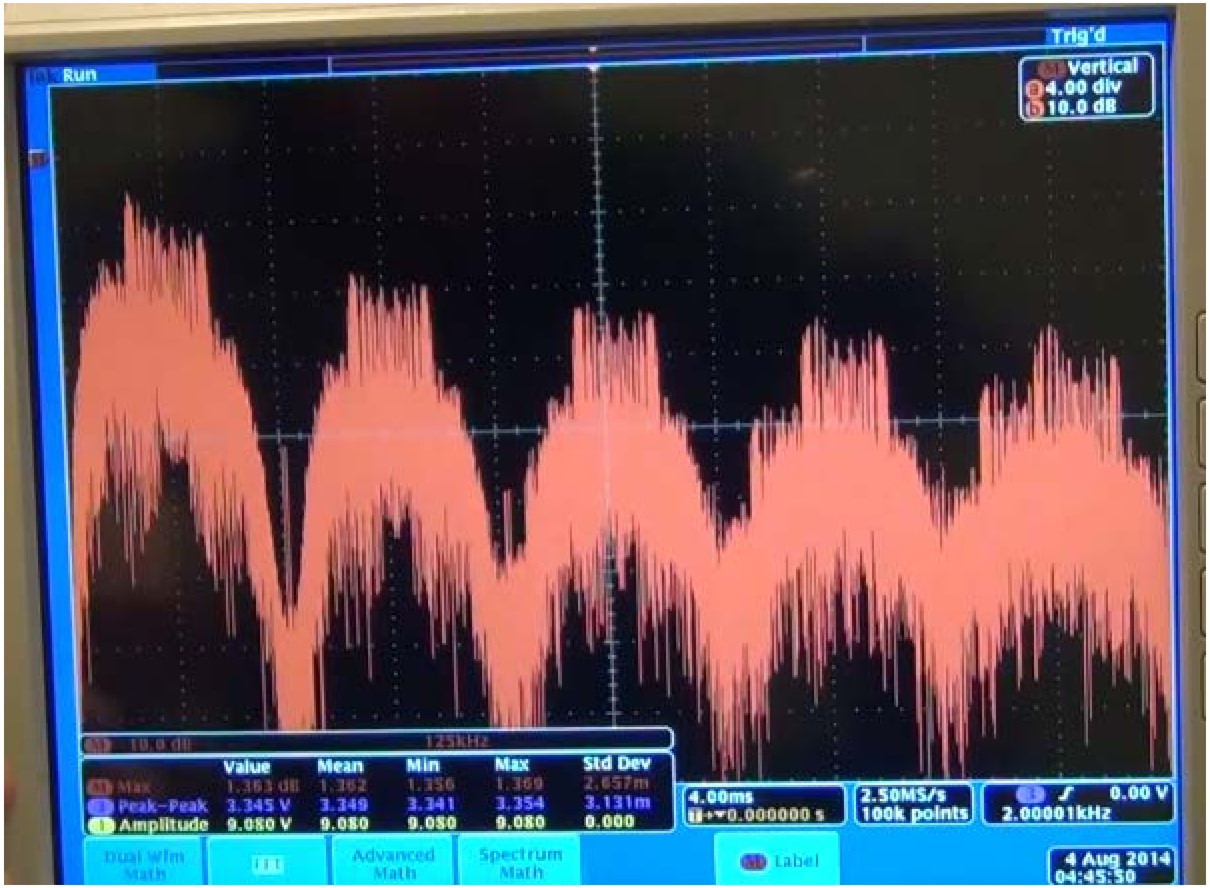
Input Signal Spectrum



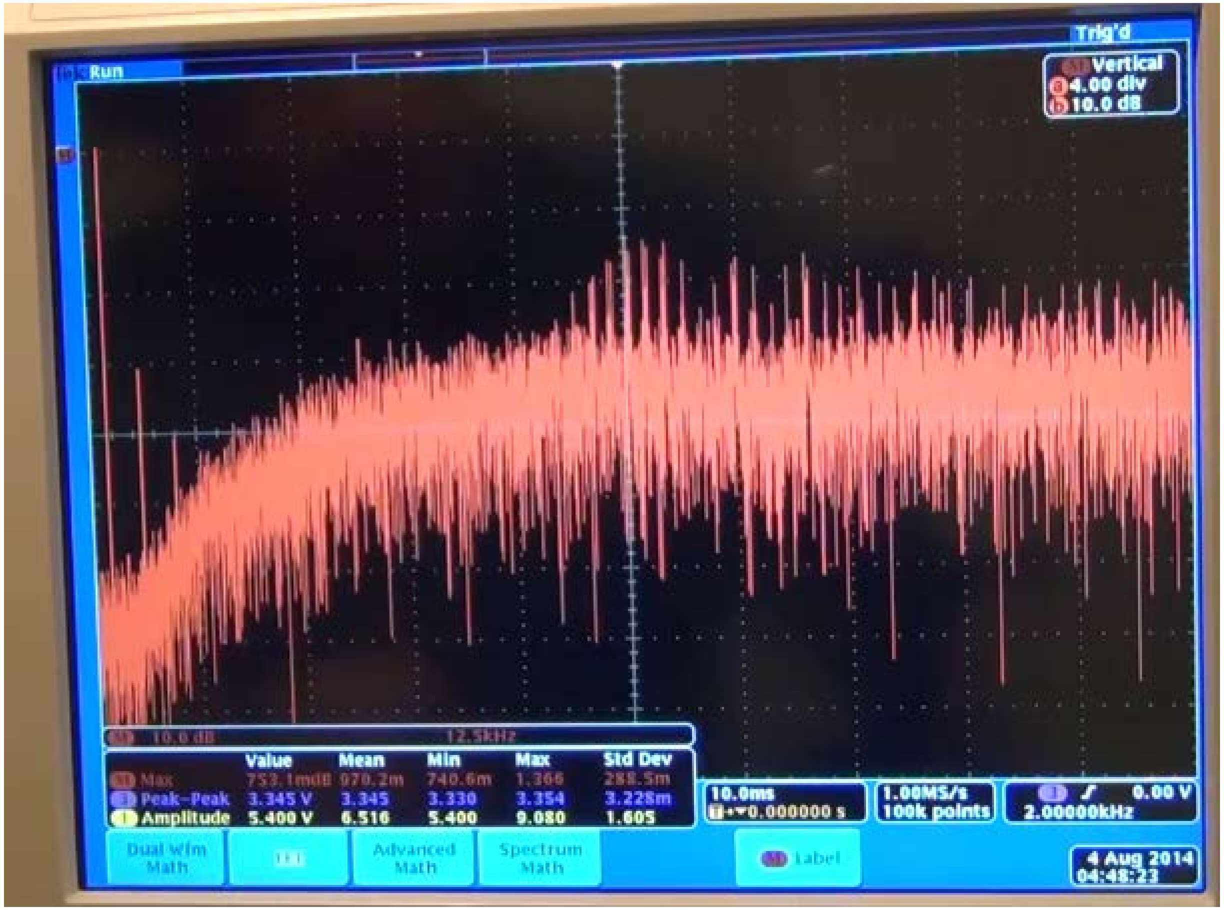
Output Signal Spectrum



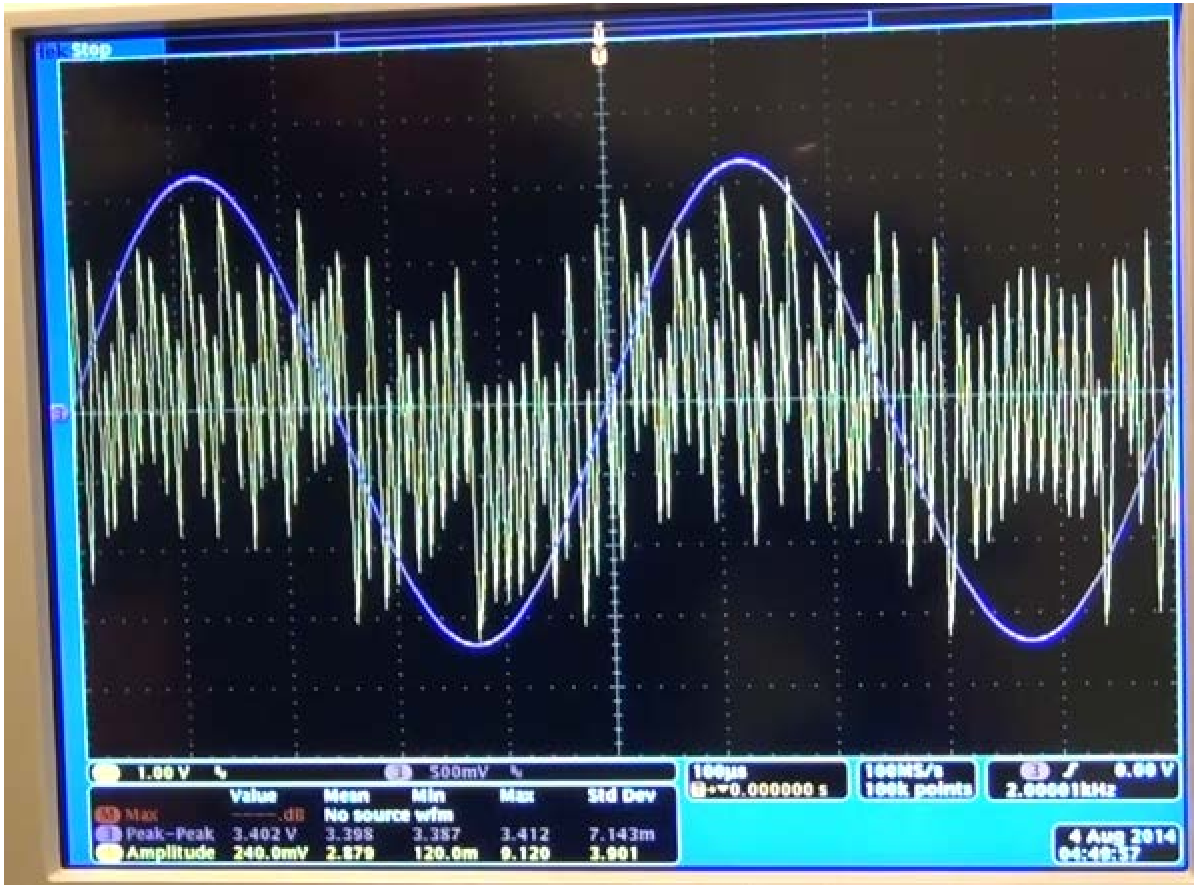
Sinc Response



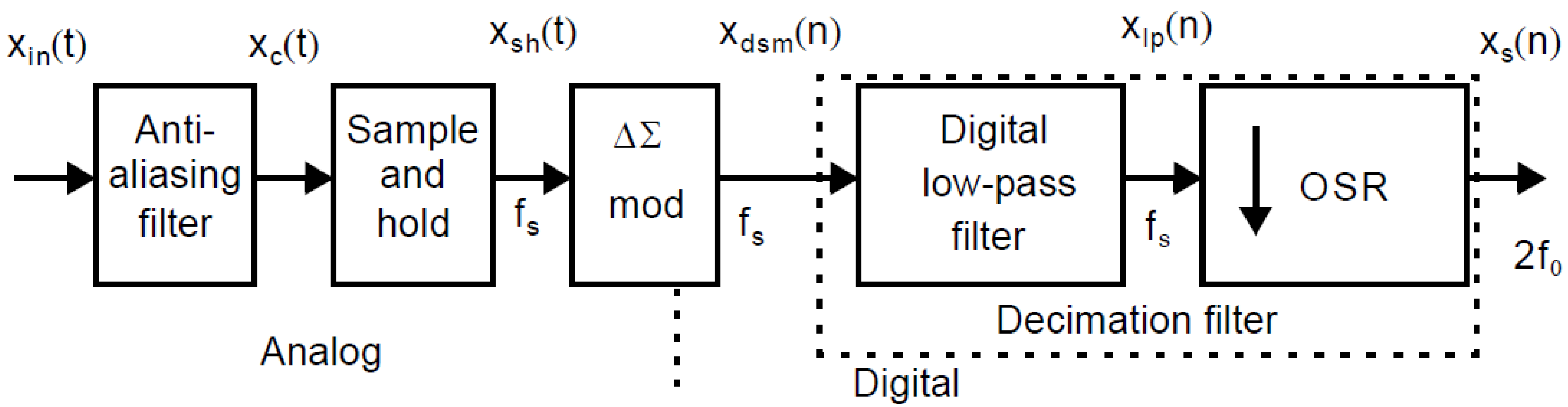
Nonlinear tones



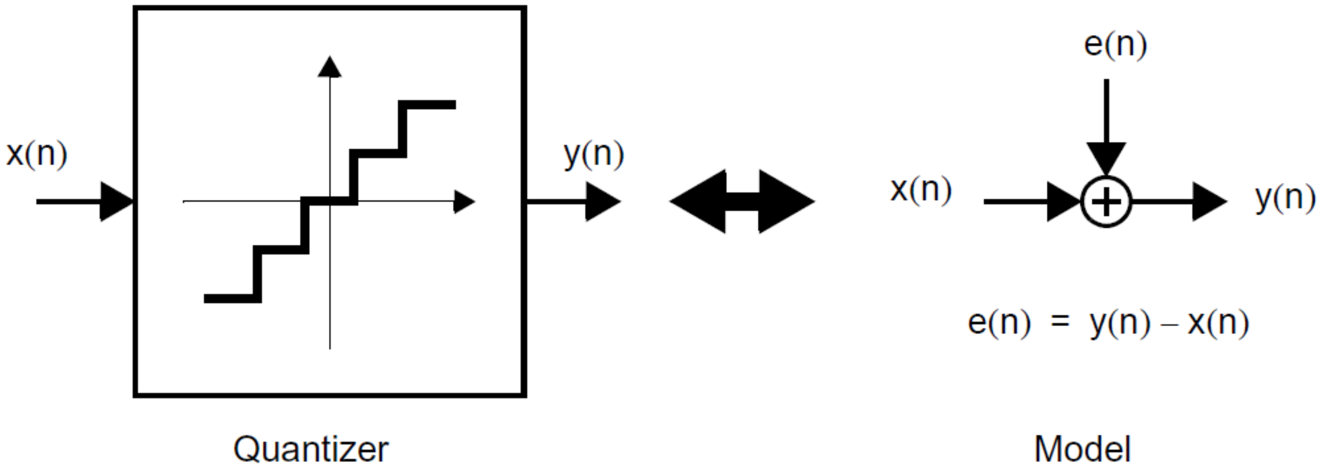
Quantized Error Signal

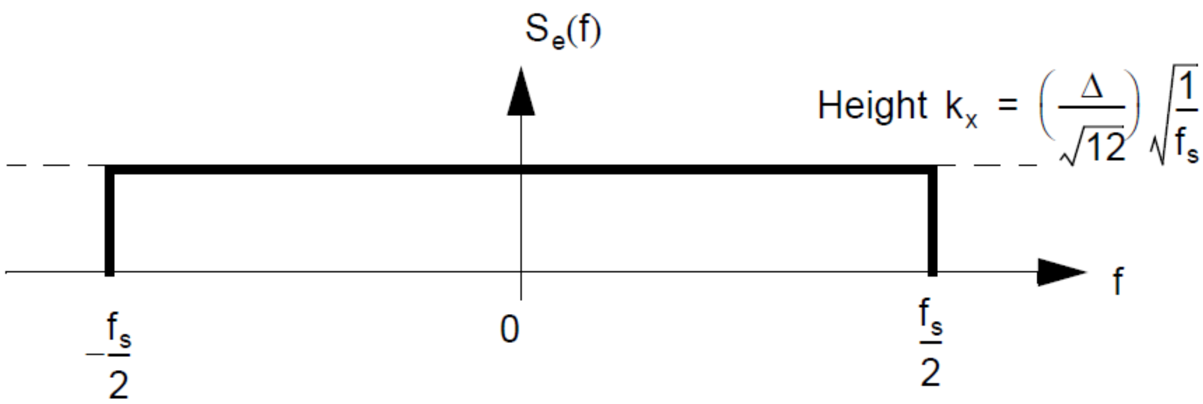


Block Diagram of Oversampled ADC

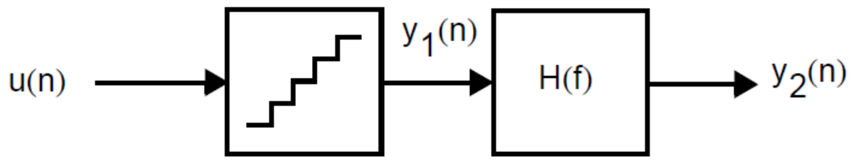


Quantization Noise Model

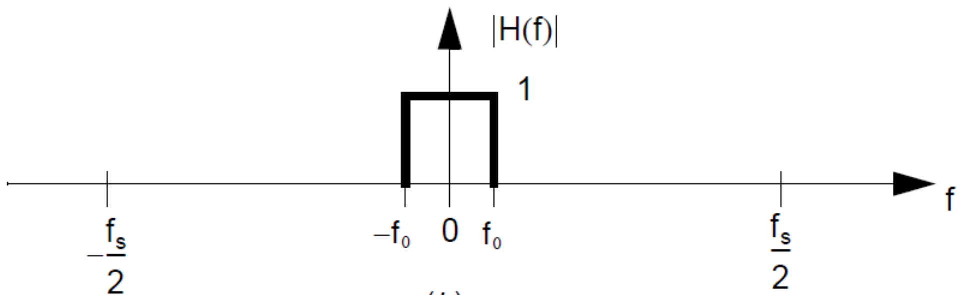




Oversampling without Noise Shaping



(*a*)



(*b*)

$$OSR \equiv \frac{f_s}{2f_0}$$

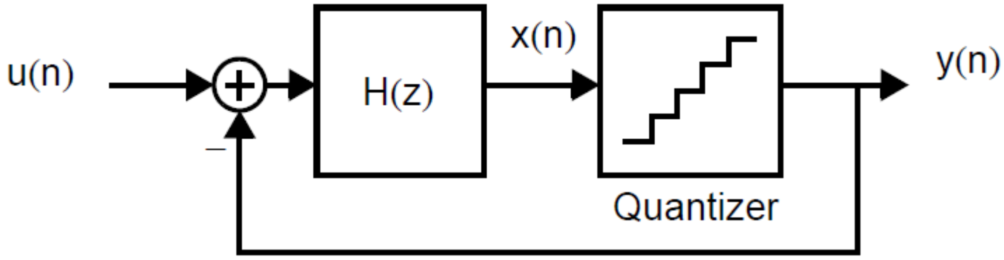
$$P_s = \left(\frac{\Delta 2^N}{2\sqrt{2}}\right)^2 = \frac{\Delta^2 2^{2N}}{8}$$

$$P_e = \int_{-f_s/2}^{f_s/2} S_e^2(f)|H(f)|^2\,df = \int_{-f_0}^{f_0} k_x^2\,df = \frac{2f_0}{f_s}\frac{\Delta^2}{12} = \frac{\Delta^2}{12}\left(\frac{1}{OSR}\right)$$

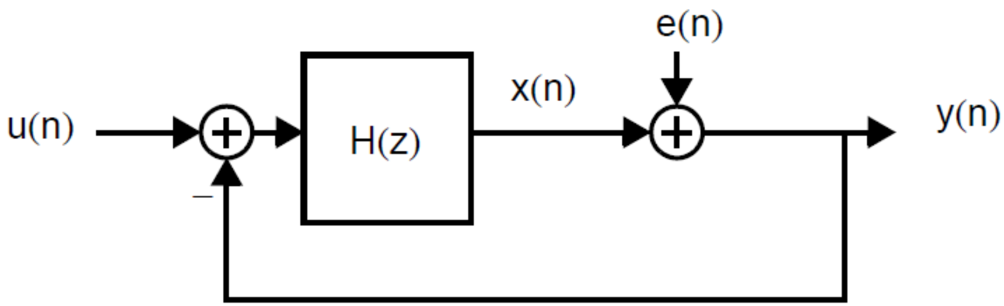
$$SQNR_{max} = 10\log\left(\frac{P_s}{P_e}\right) = 10\log\left(\frac{3}{2}2^{2N}\right) + 10\log(OSR)$$

$$= 6.02N + 1.76 + 10\log(OSR)$$

Modulator and Linear Model



(*a*)



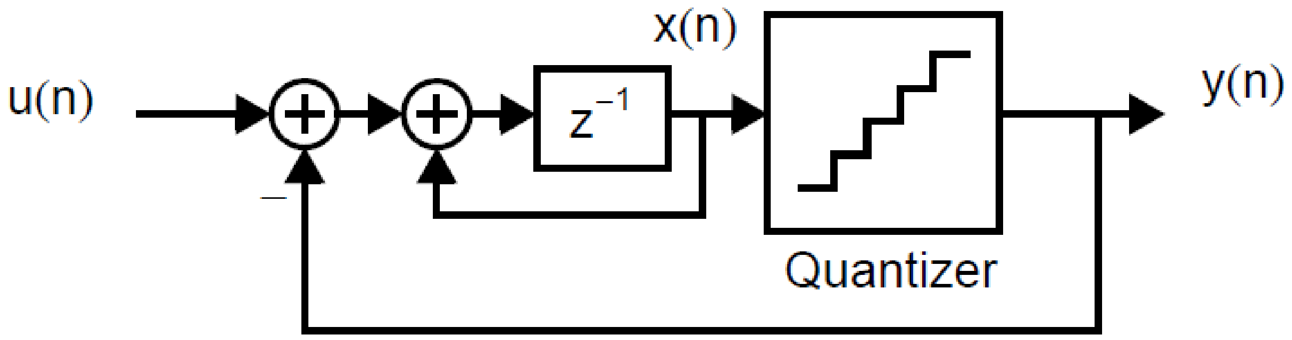
(*b*)

$$S_{TF}(z) \equiv \frac{Y(z)}{U(z)} = \frac{H(z)}{1 + H(z)}$$

$$Y(z) = S_{TF}(z)U(z) + N_{TF}(z)E(z)$$

$$N_{TF}(z) \equiv \frac{Y(z)}{E(z)} = \frac{1}{1 + H(z)}$$

First-Order Noise Shaping

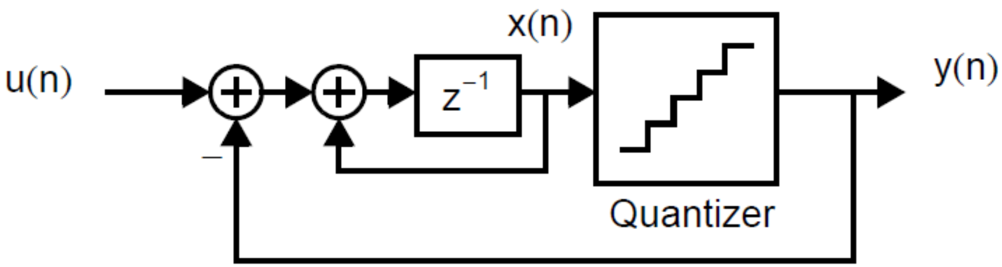


$$H(z) = \frac{1}{z-1}$$

$$S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{1/(z-1)}{1 + 1/(z-1)} = z^{-1}$$

$$N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1 + 1/(z-1)} = (1 - z^{-1})$$

First-Order Noise Shaping



$$H(z) = \frac{1}{z-1}$$

$$z = e^{j\omega T} = e^{j2\pi f/f_s}$$

$$S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{1/(z-1)}{1+1/(z-1)} = z^{-1}$$

$$N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1+1/(z-1)} = (1-z^{-1})$$

$$N_{TF}(f) = 1 - e^{-j2\pi f/f_s} = \frac{e^{j\pi f/f_s} - e^{-j\pi f/f_s}}{2j} \times 2j \times e^{-j\pi f/f_s} = \sin\left(\frac{\pi f}{f_s}\right) \times 2j \times e^{-j\pi f/f_s} \longrightarrow |N_{TF}(f)| = 2\sin\left(\frac{\pi f}{f_s}\right)$$

$$P_e = \int_{-f_0}^{f_0} S_e^2(f)|N_{TF}(f)|^2\,df = \int_{-f_0}^{f_0}\left(\frac{\Delta^2}{12}\right)\frac{1}{f_s}\left[2\sin\left(\frac{\pi f}{f_s}\right)\right]^2 df \cong \left(\frac{\Delta^2}{12}\right)\left(\frac{\pi^2}{3}\right)\left(\frac{2f_0}{f_s}\right)^3 = \frac{\Delta^2\pi^2}{36}\left(\frac{1}{OSR}\right)^3$$

$$SQNR_{max} = 10\log\left(\frac{P_s}{P_e}\right) = 10\log\left(\frac{3}{2}2^{2N}\right) + 10\log\left[\frac{3}{\pi^2}(OSR)^3\right]$$

$$= 6.02N + 1.76 - 5.17 + 30\log(OSR)$$