Midterm Exam

March 6, 9:10-10:30am

Name: Solutions

(50 points total)

PROBLEM 1: Consider the common source amplifier shown in Fig. 1. [**12 points**]

(a) Calculate the noise factor of this circuit, assuming it is driven by a source with an output impedance of R_S. For the transistor, include C_{gs} in the small signal model. You may neglect induced gate noise, $1/f$ noise, r_o, and all other small signal capacitances. [**10**]

(b) Now assume that the amplifier is being designed to operate at the frequency ω_c, and C_{gd} is no longer small enough to be neglected. We have connected an inductor from the gate to the drain of $M1$ to increase the reverse isolation. How should the inductor be sized? [**2**]

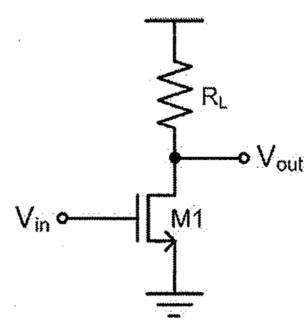
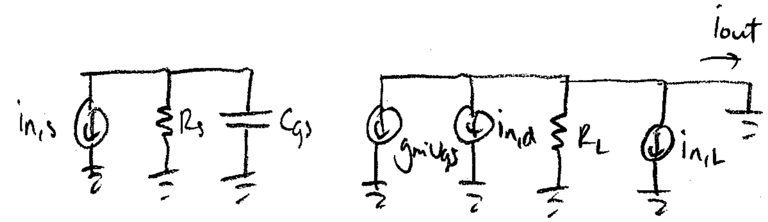


Figure 1: Common source amplifier schematic for Problem 1.

a) Draw small signal model with noise sources:



PROBLEM 1 (cont'd)

Consider noise sources one at a time:

R_S: $i_{out} = g_m \cdot i_{ns} \cdot [R_S \| C_{gs}]$ ← $= \left(\frac{1}{R_S} + sC_{gs}\right)^{-1} = \frac{R_S}{1 + sR_S \cdot C_{gs}}$

$$\overline{i_{out}^2} = g_m^2 \cdot \overline{i_{ns}^2} \cdot \frac{R_S^2}{(1 + s \cdot R_S \cdot C_{gs})^2}$$

i_d: $\overline{i_{out}^2} = \overline{i_{n,d}^2}$

R_L: $\overline{i_{out}^2} = \overline{i_{n,L}^2}$

Now sub into expression for F:

$$F = 1 + \frac{\cancel{4kT}\gamma \cdot g_{do} \cdot \cancel{\Delta f}}{\cancel{4kT \cdot \Delta f}} \cdot \frac{\cancel{R_S} \cdot (1 + s \cdot R_S \cdot C_{gs})^2}{g_m^2 \cdot R_S^{\cancel{2}}} + \frac{\cancel{4kT\Delta f}}{R_L \cdot \cancel{4kT\Delta f}} \cdot \frac{\cancel{R_S}(1 + s \cdot R_S \cdot C_{gs})^2}{R_S^{\cancel{2}} \cdot g_m^2}$$

$$\boxed{F = 1 + \frac{\gamma \cdot g_{do} \cdot (1 + \omega^2 \cdot R_S^2 \cdot C_{gs}^2)}{g_m^2 \cdot R_S} + \frac{(1 + \omega^2 \cdot R_S^2 \cdot C_{gs}^2)}{R_L \cdot R_S \cdot g_m^2}}$$

b) Choose L to resonate with C_{gd}

$$\therefore \quad \omega_c = \frac{1}{\sqrt{L \cdot C_{gd}}} \Rightarrow \boxed{L = \frac{1}{\omega_c^2 \cdot C_{gd}}}$$

PROBLEM 2: Consider the matching network shown in Fig. 2, consisting of two inductors and one capacitor. [**12 points**]

(a) If R_L is 50 Ω, choose values for C, L_1, and L_2 so that R_{IN} is 10 Ω at $\omega = 5 \times 10^9$ rad/s (hint: more than one solution is possible). [**10**]

(b) What advantages (if any) does this matching network have over a standard L-match? [**2**]

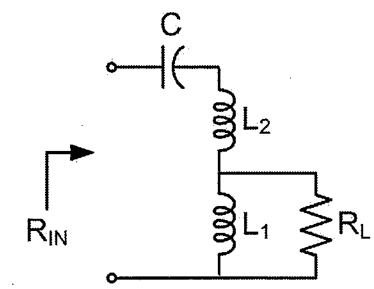
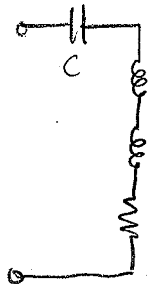


Figure 2: Matching network for Problem 2.

a) Transform to series equivalent:



$L_{1,s} = L_1 \cdot \frac{Q_1^2}{Q_1^2+1}$

$R_{L,s} = \frac{R_L}{Q_1^2+1}$

Now, C must resonate with $L_2 + L_{1,s}$ at ω_0,

so: $\omega_0 = \frac{1}{\sqrt{C(L_2 + L_{1,s})}}$ ⊛

Also, $R_{in} = R_{L,s} = 10 = \frac{50}{Q_1^2+1}$

$\therefore Q_1^2 = 4 \Rightarrow Q_1 = 2.$

Also, $Q_1 = \frac{\omega_0 \cdot L_{1,s}}{R_{L,s}}$

$\therefore 2 = \frac{5\times10^9 \cdot L_{1,s}}{10}$

$\therefore L_{1,s} = 4\times10^{-9} \Rightarrow L_1 = 4\times10^{-9}\left(\frac{2^2+1}{2^2}\right)$

$\boxed{L_1 = 5\times10^{-9} = 5\,nH}$

Now, we have some flexibility in choosing the other components. Choose $\boxed{C = 1pF}$

Now, from ⊛: $L_2 = \frac{1}{\omega_0^2 \cdot C} - L_{1,s}$

$= \frac{1}{25\times10^{18} \cdot 1\times10^{-12}} - 4\times10^{-9}$

$\boxed{L_2 = 36\times10^{-9} = 36\,nH}$

Note this yields an overall network Q of:

$Q = \frac{\omega_0(L_2+L_{1,s})}{R_{L,s}}$

$= \frac{5\times10^9(40\times10^{-9})}{10}$

$\boxed{Q = 20}$

b) So, we see that this network provides some flexibility in choosing the Q of the overall network (we can choose it to be higher than the requirement from the transformation ratio, but not lower).

PROBLEM 3: Your manager has asked you to evaluate the linearity of an RF front end. You know that for a signal input power of -30 dBm the power of the fundamental component at the output is -10 dBm, and for two tones with an input power of -30 dBm, the IM3 components have a power of -70 dBm at the output. [**12 points**]

(a) What is the gain of the system? [**1**]

(b) Using the plot in Fig. 3, calculate the IIP3 for the system. Draw in the relevant curves, and label the IP3 point on the plot. [**3**]

(c) If the bandwidth of the system is 10 MHz, the noise figure is 14 dB, what is the input referred noise floor (hint: this was contained in the formula given in class for the sensitivity)? What is the output referred noise floor? [**5**]

(d) Draw the output referred noise floor into the plot in Fig. 3, and mark the SFDR. What is the SFDR for this system? [**3**]

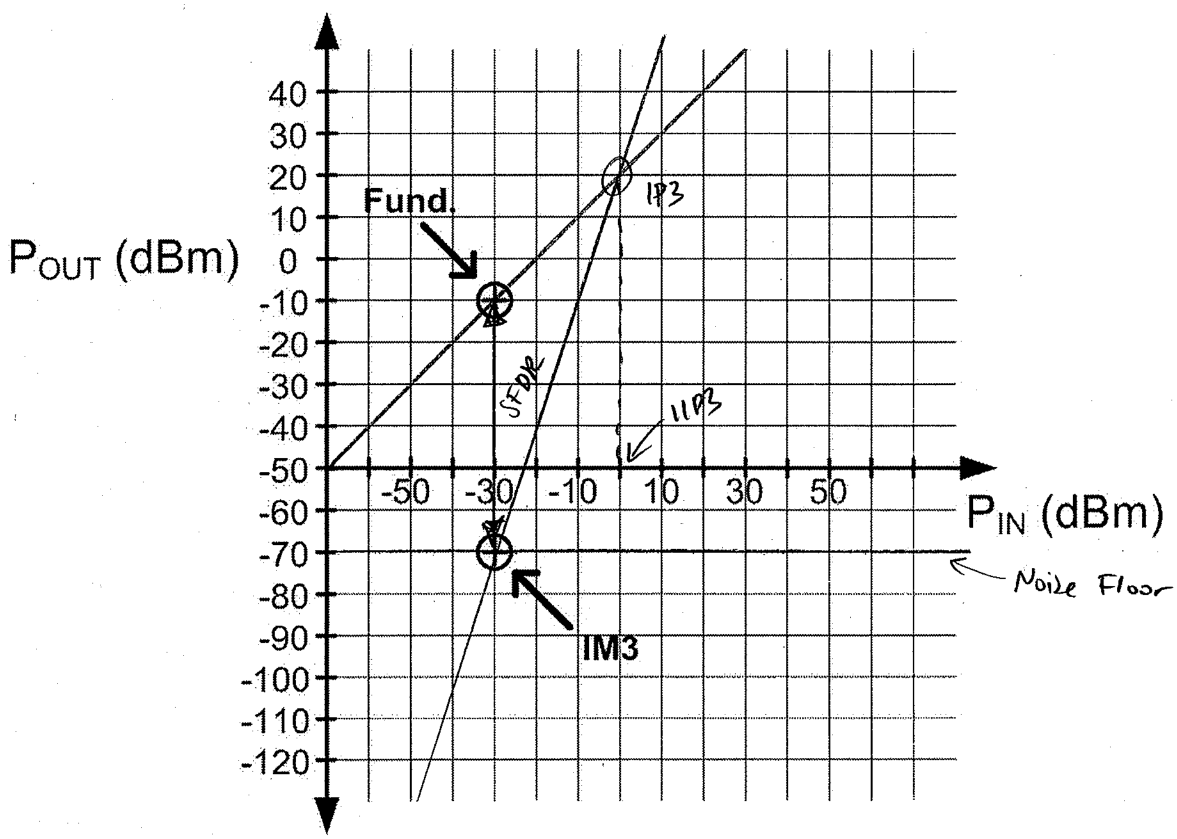


Figure 3: SFDR plot for Problem 3.

PROBLEM 3 (cont'd)

(a) gain = 20 dB

(b) IIP3 = 0 dBm

(c) I/P referred noise = -174 dBm + $10 \cdot \log B + NF$

$= -174 + 70 + 14$

$= -90$ dBm

O/P referred noise = I/P referred noise × gain

$= -90$ dBm + 20 dB

$= -70$ dBm

(d) SFDR = 60 dB

Explanation for (c):

$$F = \frac{\text{noise at O/P}}{\text{noise at O/P from I/P}} = \frac{\text{noise at O/P}}{\text{noise at I/P} \times \text{gain}}$$

$\therefore$ noise at O/P = F × gain × $\underbrace{\text{noise at I/P}}_{= kT \times BW}$

$\therefore$ O/P referred noise [dBm]

$= 10 \cdot \log (F \times \text{gain} \times kT \times BW)$

$= NF + \text{gain (in dB)} + \underbrace{10 \cdot \log kT}_{= -174\ \frac{dBm}{Hz}} + 10 \cdot \log BW$

PROBLEM 4: Shown in Fig. 4 is the down-conversion chain for an RF receiver. The incoming signal of interest is centered around $f_C = 900$ MHz, and low-side LO injection is used for down-conversion, with $f_{LO} = 850$ MHz. **[12 points]**

(a) What is the intermediate frequency (f_{IF}) for this system? **[1]**

(b) If the bandwidth of each channel is 2 MHz, what is the approximate Q required for the channel select filter? **[2]**

(c) What is the image frequency for the signal of interest? **[1]**

(d) Now assume that the specifications for each of the system blocks are as shown in Table 1. What is the noise figure of the down-conversion chain as a whole (note the distinction between the noise figure (NF, in dB) as provided in Table 1 and the noise factor also discussed in class (F, in linear units))? **[8]**

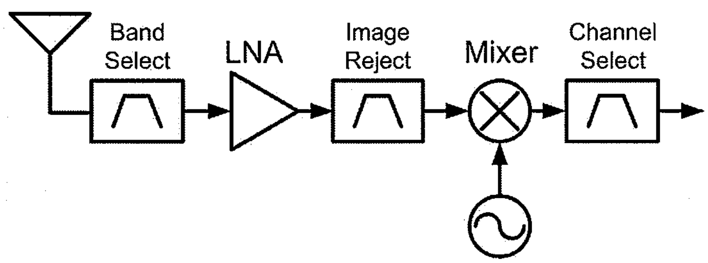


Figure 4: Down-conversion chain for Problem 4.

Component	Noise Figure (dB)	Available Power Gain (dB)
Band Select filter	0 1	-3 0.5
Low Noise Amplifier	3 2	20 100
Image Reject filter	0 1	-3 0.5
Mixer	10 10	0 1
Channel Select filter	0 1	-3 0.5

Table 1: Noise Figures and Available Power Gains for components in Fig. 4.

a) $f_{IF} = f_c - f_{LO} = 50$ MHz.

b) $Q = \frac{f_0}{\Delta f} = \frac{f_{IF}}{BW} = \frac{50 \text{ MHz}}{2 \text{ MHz}} = 25$

c) $f_{IM} = f_{LO} - f_{IF} = 800$ MHz.

PROBLEM 4 (cont'd)

d) Write linear units into Table 1.

Friis equation:

$$F_{tot} = 1 + F_1 - 1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots$$

Sub in: $F = 1 + (1-1) + \frac{(2-1)}{0.5} + \frac{(1-1)}{0.5 \cdot 100} + \frac{(10-1)}{0.5 \cdot 0.5 \cdot 100} + \frac{(1-1)}{0.5 \cdot 0.5 \cdot 100 \cdot 1}$

$$= 1 + 2 + \frac{9}{25}$$

$$= 3.36$$

$\therefore NF = 10 \cdot \log F = 5.26 \text{ dB}$

PROBLEM 5: Your boss has asked you to interview someone for a new position in the RFIC design group at your company. So far only one candidate has applied (see Fig. 5)? [**2 points**]

(a) Who is this man? [**1**]

(b) Would you trust him to design your RF front ends? [**1**]

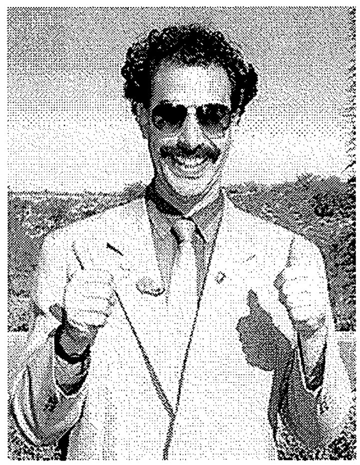


Figure 5: Job applicant for RFIC position.

a) Mr. Borat Sagdiyev

b) Yes, I bet his RF design skills are-a very nice!