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# EE230-02 RFIC II

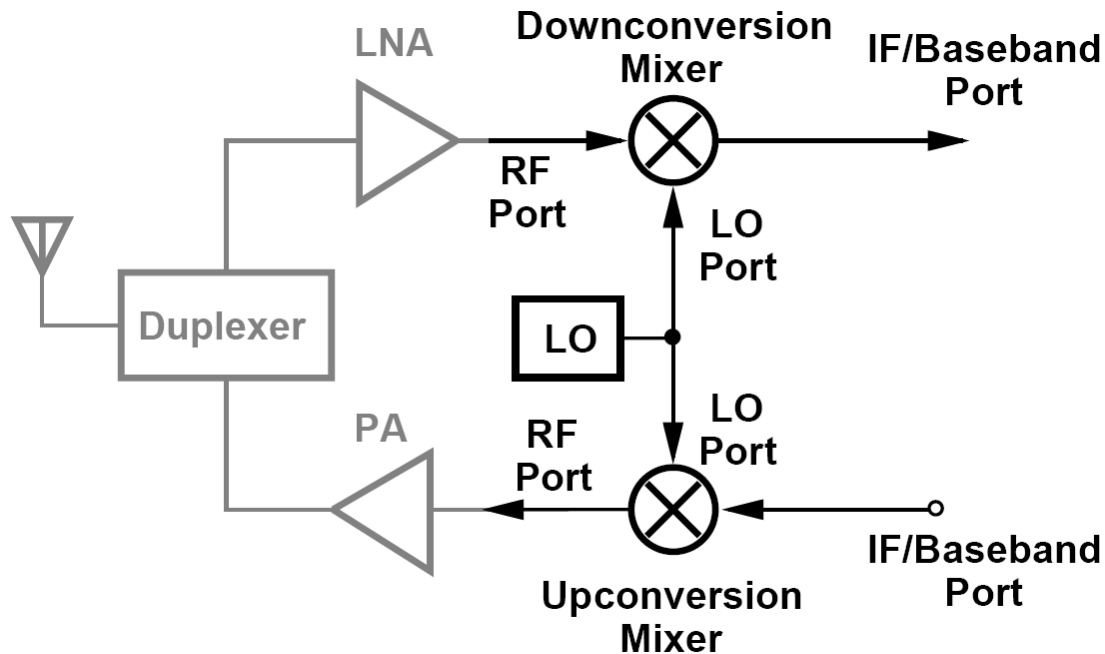
## Fall 2018

### Lecture 10: Active Mixer

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ENG-259

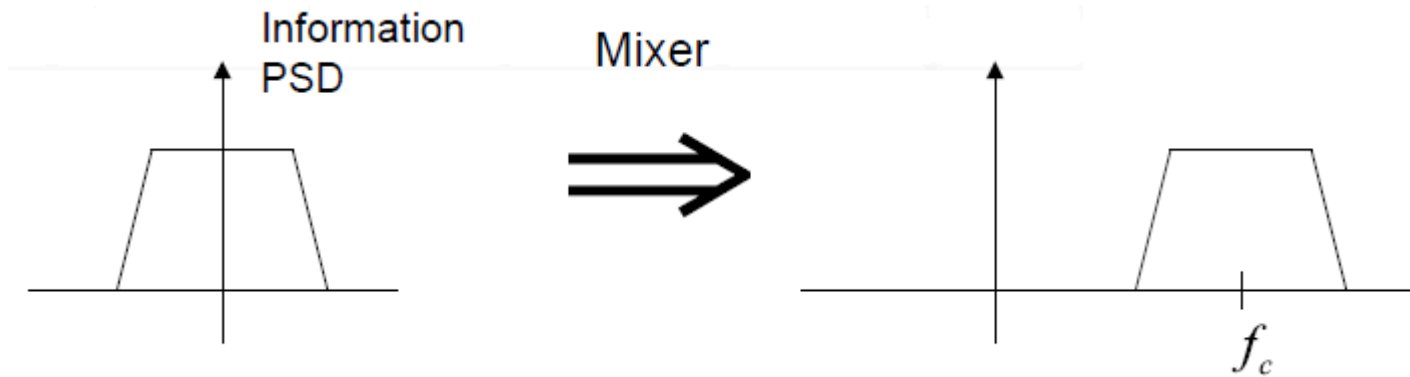
# Mixers

- Mixers perform frequency translation by multiplying two waveforms (and possibly their harmonics).
- RF or IF/Baseband port sense the signal to be frequency translated
- LO port senses the periodic waveform generated by the local oscillator



# Mixer

- The Mixer is a critical component in communication circuits.
- It translates information content to a new frequency.



# Why Use Mixer in Transmitter

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Translate information to a frequency appropriate for transmission

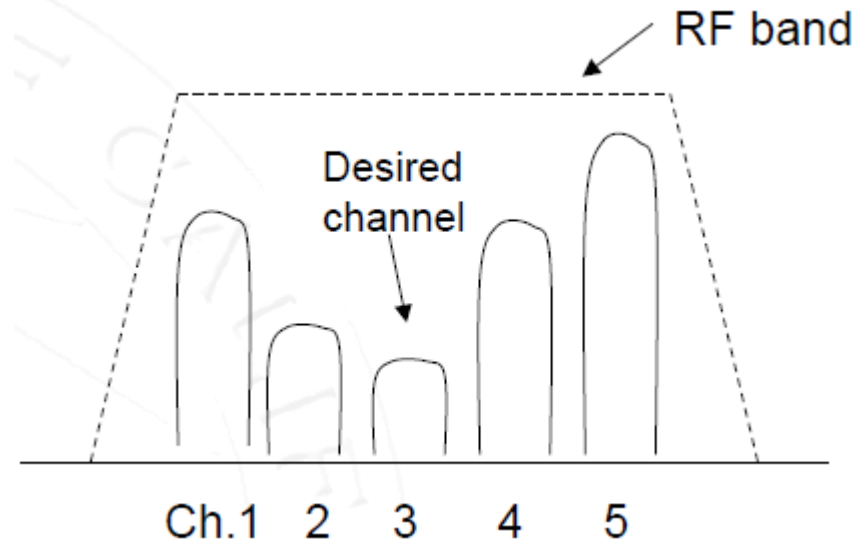
- Example: Antennas becomes smaller and more efficient at high frequencies

Spectrum sharing: Move information into separate channels in order to share spectrum and allow simultaneous use

# Why Use Mixer in Receiver

$$Q \sim \frac{\omega_o}{2\Delta\omega}$$

Bandpass filter at  $\omega_o$  requires a high-Q for narrowband signals



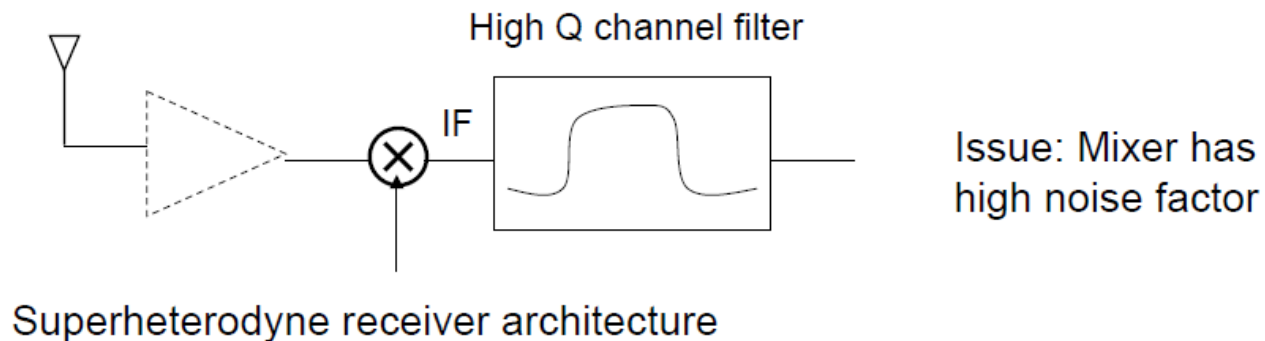
$$\Delta f \sim 200 \text{ kHz (GSM)}$$

$$f_o \sim 1\text{GHz}$$

$$Q = \frac{10^9}{2 \times 200 \times 10^6} = \frac{1000}{0.4} = 2500 \quad \leftarrow \text{High Q}$$

# Mixers in Receiver

- Filter center frequency must change to select a given channel  $\Rightarrow$  tunable filter difficult to implement
- Mixing has big advantage!  
Translate information down to a fixed (intermediate frequency) or IF.
- 1 GHz  $\Rightarrow$  10 MHz: 100x decrease in Q required
- Don't need a tunable filter

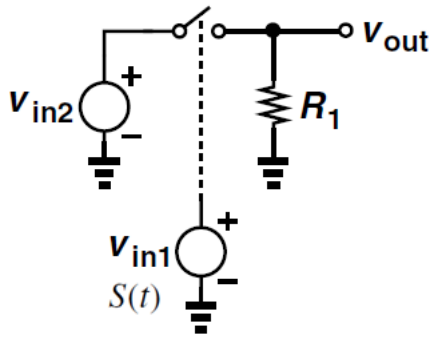


# Mixer Specifications

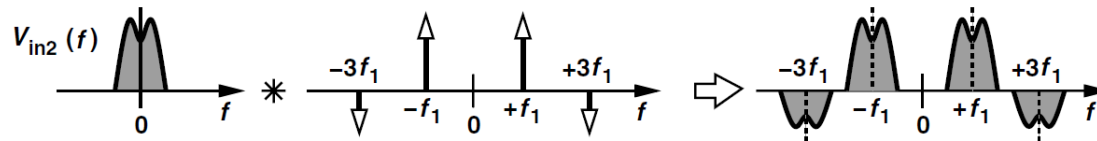
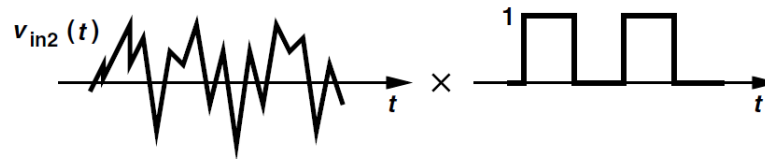
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- ❑ Conversion Gain: Ratio of voltage (power) at output frequency to input voltage (power) at input frequency
  - Down-conversion: IF power / RF power
  - Up-conversion: RF power / IF power
- ❑ Noise Figure
  - DSB versus SSB
- ❑ Linearity
- ❑ Image Rejection
- ❑ LO Feedthrough
  - Input
  - Output
- ❑ RF Feedthrough

# RF Mixer



$$v_{out}(t) = v_{in2}(t) \cdot S(t)$$



$$T_1 = 2\pi / \omega_1$$

$$V_{out}(f) = V_{in2}(f) * \sum_{n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi} \delta\left(f - \frac{n}{T_1}\right) = \sum_{n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi} V_{in2}\left(f - \frac{n}{T_1}\right)$$

**What is the amplitude when n=1?**

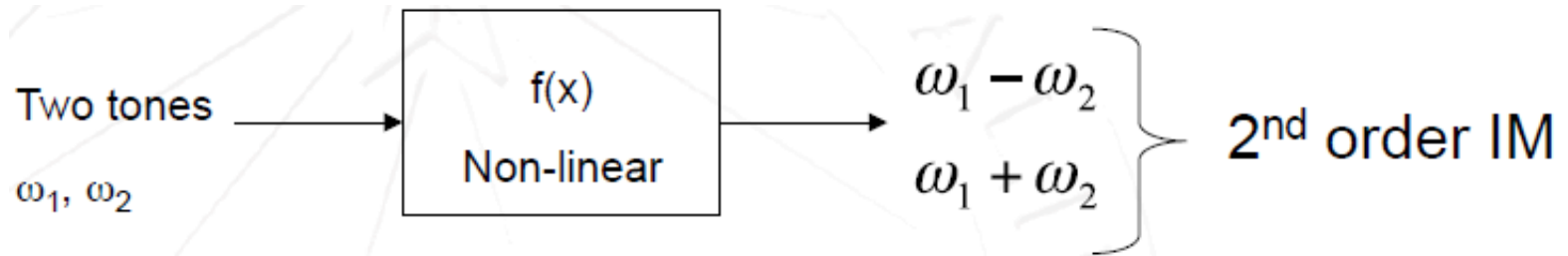


$$\frac{1}{\pi}$$

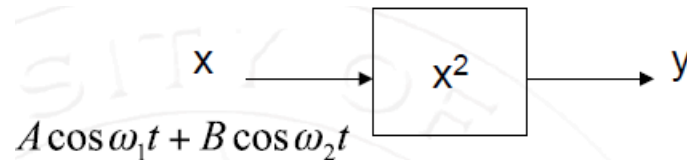


# Mixer Implementation

Any non-linear circuit acts like a mixer.



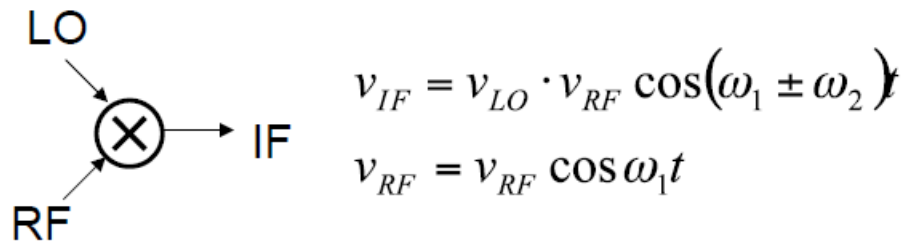
# Squarer Example



$$y = \underbrace{A^2 \cos^2 \omega_1 t + B^2 \cos^2 \omega_2 t}_{\text{DC \& second harmonic}} + \underbrace{2AB \cos \omega_1 t \cos \omega_2 t}_{\text{Desired mixing}}$$

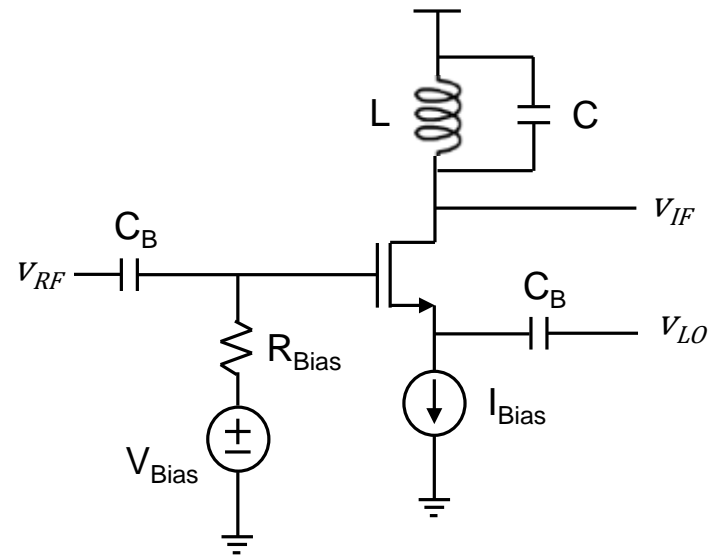
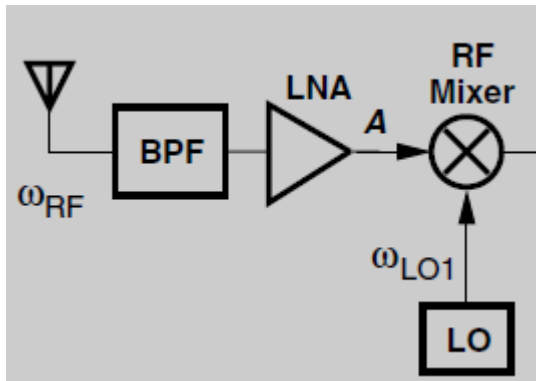
Product component:  $\frac{2AB}{2} \{ \cos(\omega_1 + \omega_2) t + \cos(\omega_1 - \omega_2) t \}$

What we would prefer:

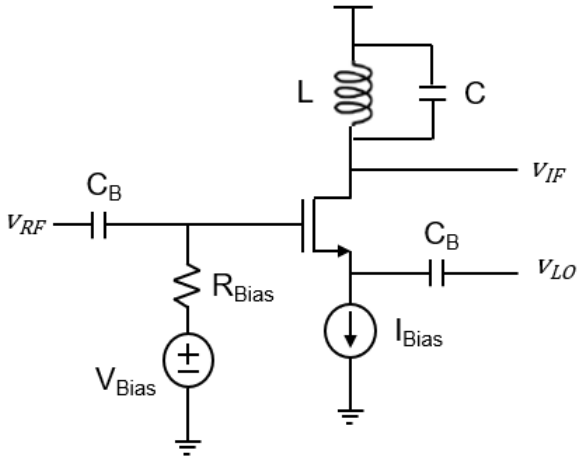


A true quadrant multiplier with good dynamic range is difficult to fabricate

# Square Law Mixer



# Square Law Mixer



$$V_{gs} = (V_{Bias} + v_{RF} \cos \omega_{RF} t) - (V_B + v_{LO} \cos \omega_{LO} t)$$

$$I_D = \frac{K' W}{2 L} (V_{gs} - V_{th})^2$$

$$= \frac{K' W}{2 L} [(V_{Bias} + v_{RF} \cos \omega_{RF} t) - (V_B + v_{LO} \cos \omega_{LO} t) - V_{th}]^2$$

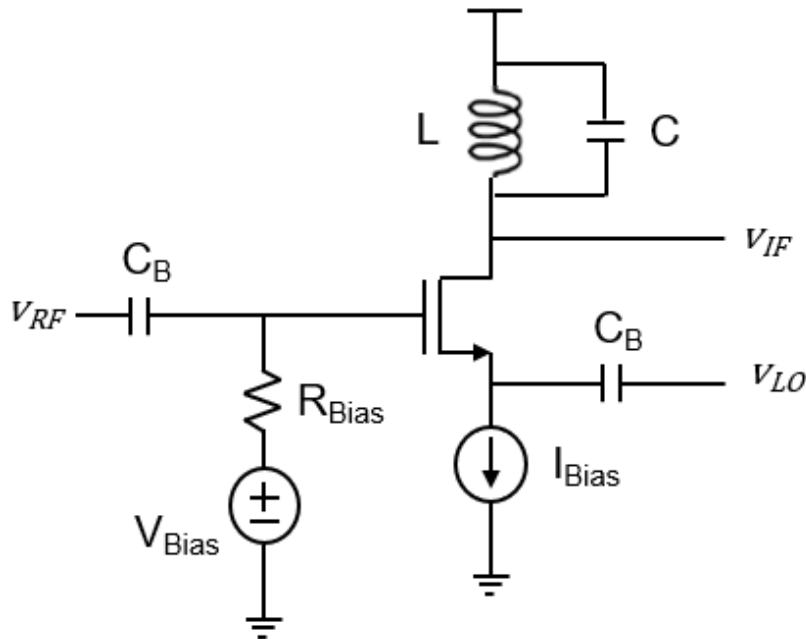
$$= \frac{K' W}{2 L} [(V_{Bias} - V_B - V_{th}) + (v_{RF} \cos \omega_{RF} t - v_{LO} \cos \omega_{LO} t)]^2$$

$$(v_{RF} \cos \omega_{RF} t - v_{LO} \cos \omega_{LO} t)^2$$

$$= (v_{RF} \cos \omega_{RF} t)^2 + (v_{LO} \cos \omega_{LO} t)^2 - 2(v_{RF} \cos \omega_{RF} t)(v_{LO} \cos \omega_{LO} t)$$

$$\frac{K' W}{2 L} (v_{RF} v_{LO}) \{ \cos(\omega_{RF} - \omega_{LO}) t - \cos(\omega_{RF} + \omega_{LO}) t \}^2$$

# Square Law Mixer



$$\frac{K' W}{2 L} (v_{RF} v_{LO}) \{ \cos(\omega_{RF} - \omega_{LO}) t \}$$

**Conversion Gain**

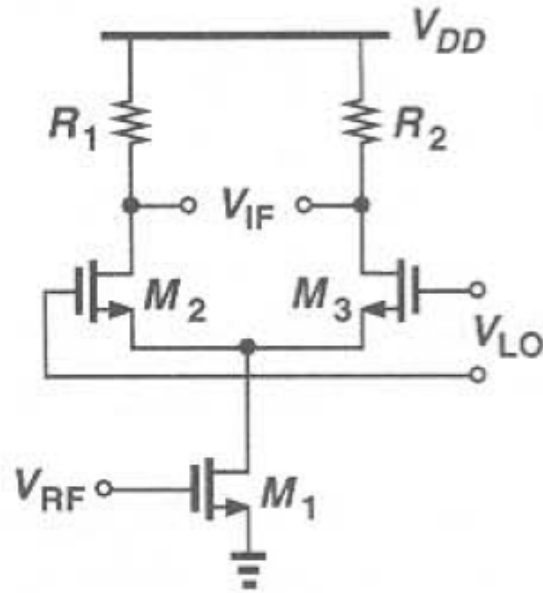
$$= \frac{\text{IF amplitude}}{\text{RF amplitude}}$$

$$= \frac{\frac{K' W}{2 L} (v_{RF} v_{LO})}{v_{RF}}$$

$$= \frac{\mu_n C_{ox} W}{2} \frac{v_{LO}}{L}$$

- Independent of Bias
- Temperature dependent
- Dependent on  $v_{LO}$

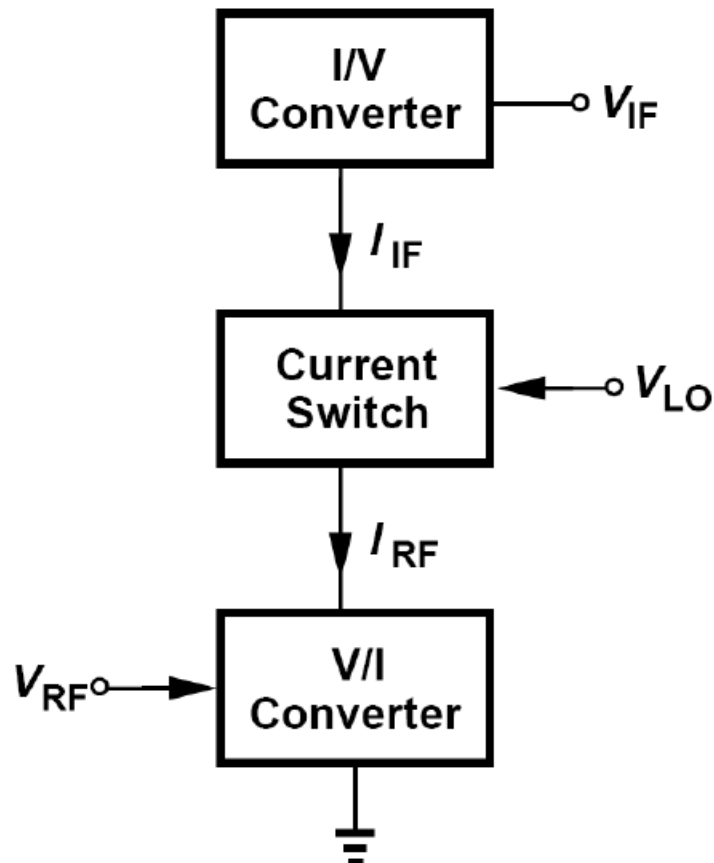
# Active Mixer



- Active mixer generally provide gain
- Reduce the noise contributed by subsequent stages
- RF input varies the drain current of  $M_1$
- $M_2$  and  $M_3$  function as a switching pair driven by the LO
- The drain current of  $M_1$  is in essence multiplied by a square wave

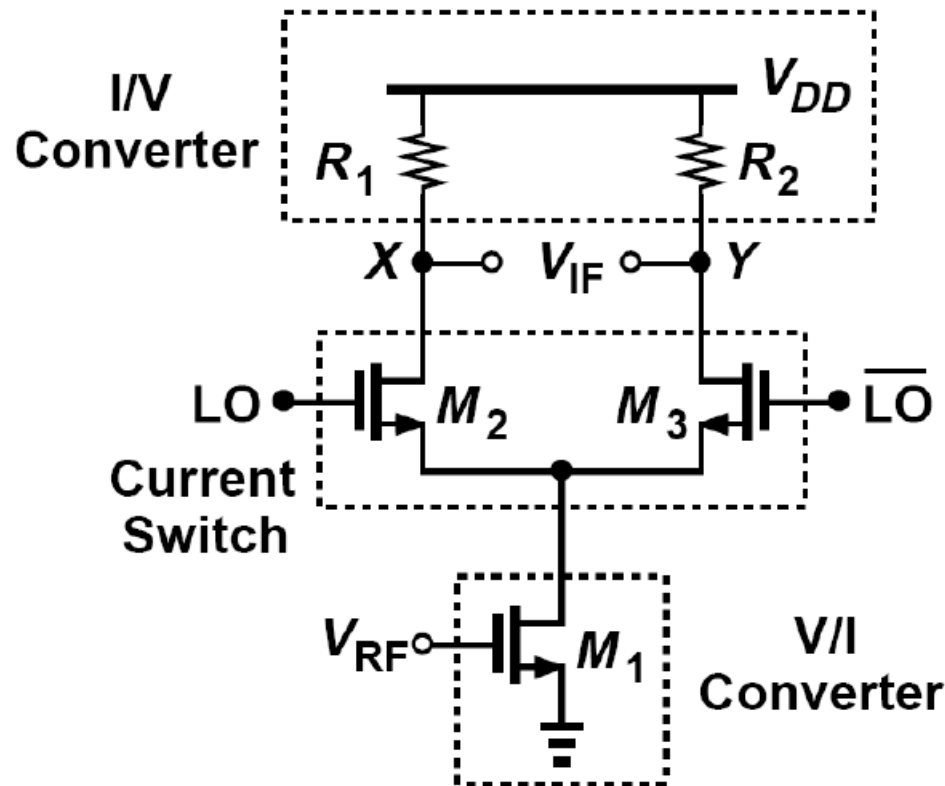
# Active Downconversion Mixers: Function

- Mixers can be realized so as to achieve conversion gain in one stage.



# Active Downconversion Mixers: Typical Realization

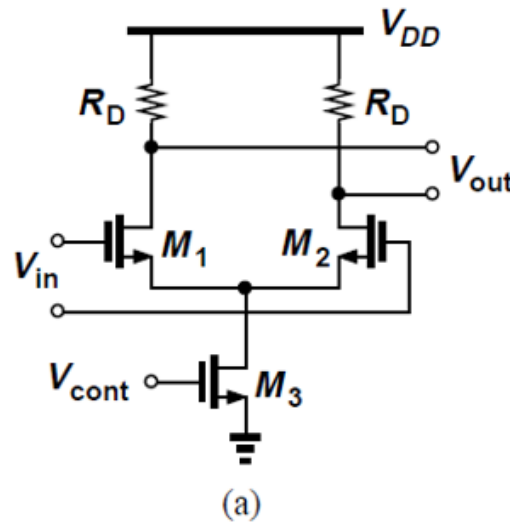
- Called active mixers, such topologies perform three functions: they convert the RF voltage to a current, steer the RF current by the LO, and convert the IF current to voltage.
- We call  $M_2$  and  $M_3$  the “switching pair.”
- The switching pair does not need rail-to-rail LO swings.





# Variable Gain Amplifier (VGA)

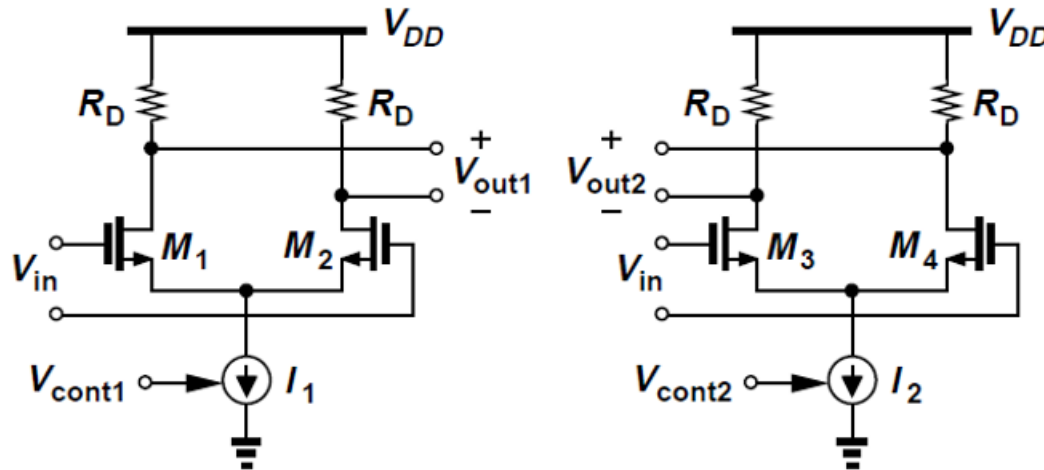
- Differential pair whose gain is controlled by a control voltage [Fig. (a)]



- In Fig.(a), the control voltage  $V_{cont}$  controls the tail current and hence the gain
- Here,  $A_v = V_{out} / V_{in}$  varies from zero (if  $I_{D3} = 0$ ) to a maximum value given by voltage headroom limitations and device dimensions
- Simple example of **Variable Gain Amplifier (VGA)**

# Variable Gain Amplifier (VGA)

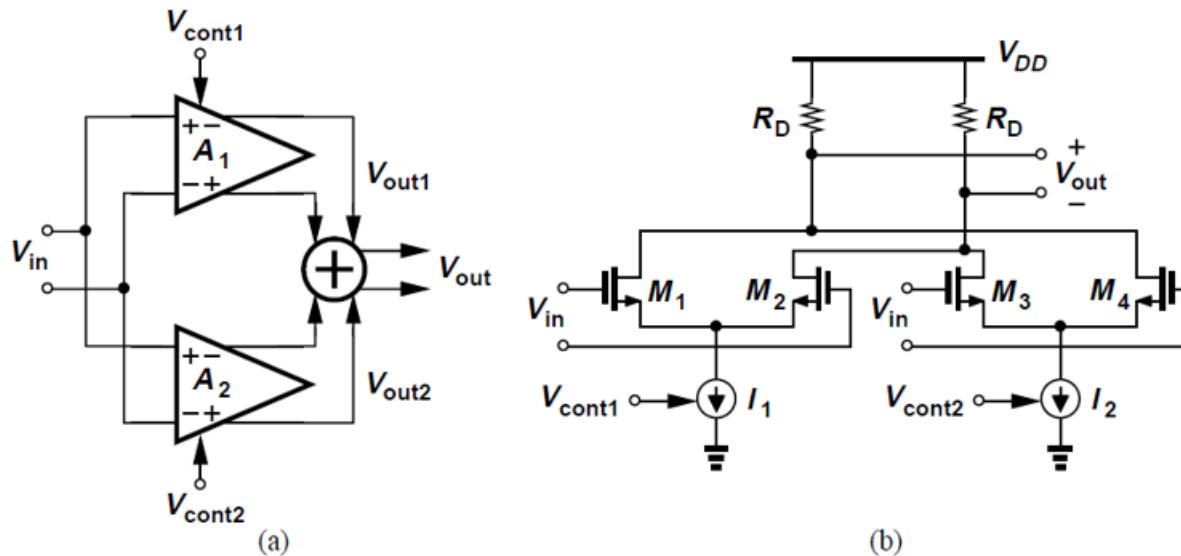
- An amplifier is sought whose gain can be continuously varied from a negative to a positive value



(b)

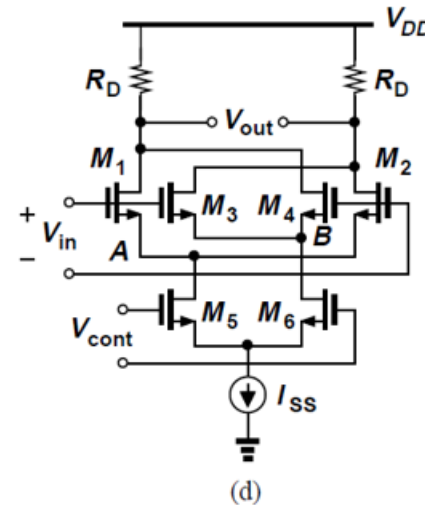
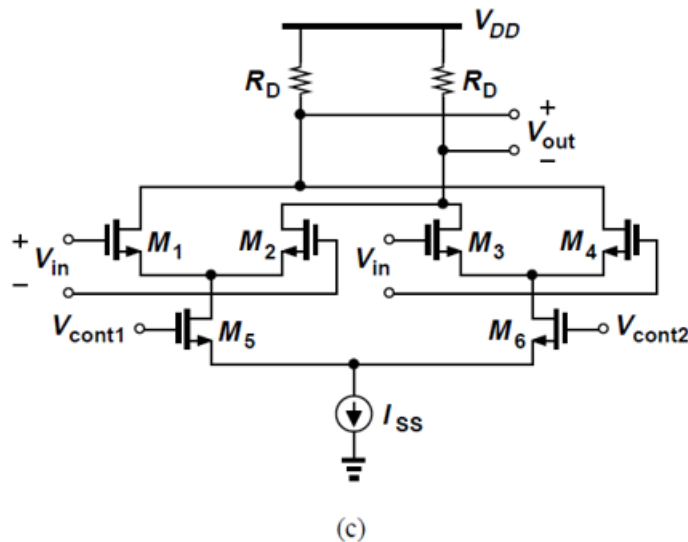
- Fig. (b) shows two differential pairs that amplify the input by opposite gains
- Here,  $V_{out1}/V_{in} = -g_m R_D$  and  $V_{out2}/V_{in} = +g_m R_D$
- If  $I_1$  and  $I_2$  vary in opposite directions, so do  $|V_{out1}/V_{in}|$  and  $|V_{out2}/V_{in}|$

# Gilbert Cell



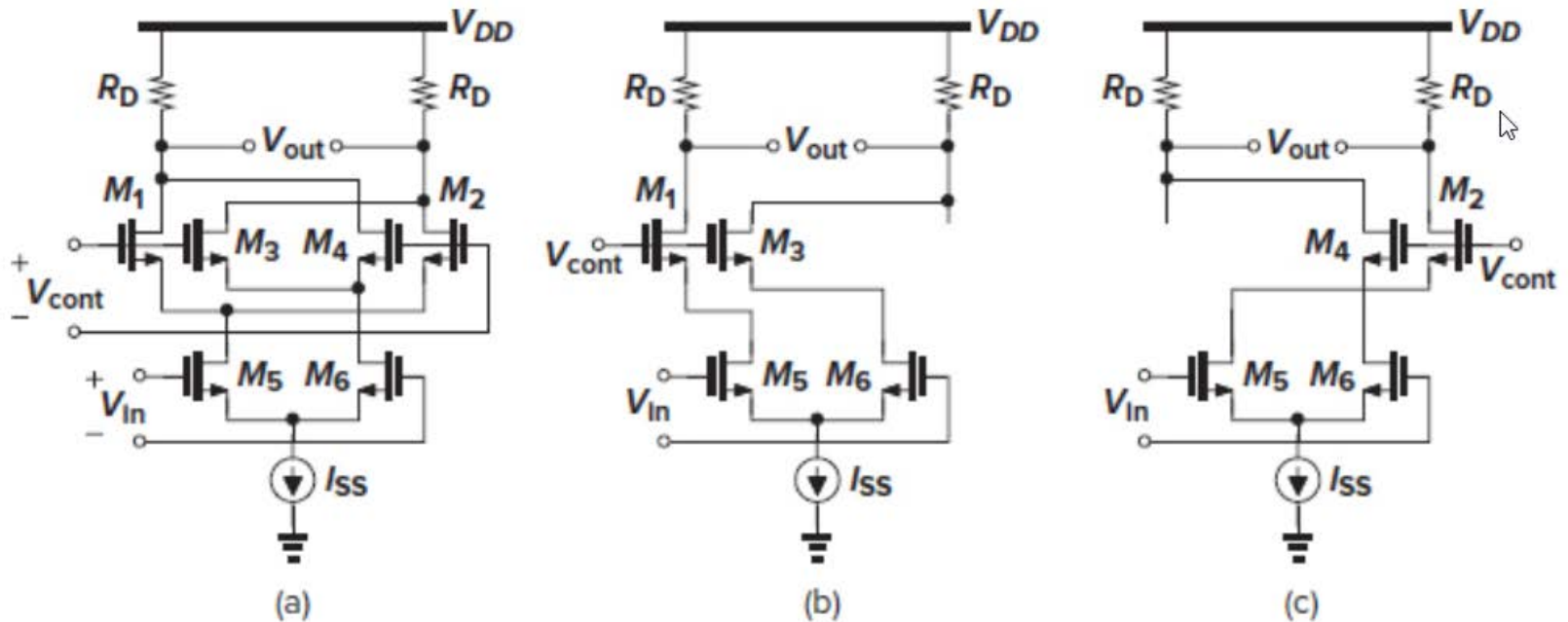
- $V_{out1}$  and  $V_{out2}$  are combined into a single output as shown in Fig. (a)
- The two voltages are summed, producing  $V_{out} = V_{out1} + V_{out2} = A_1 V_{in} + A_2 V_{in}$ , where  $A_1$  and  $A_2$  are controlled by  $V_{cont1}$  and  $V_{cont2}$  respectively
- Actual implementation shown in Fig. (b) where drain terminals are shorted to sum the currents and generate the output voltage

# Gilbert Cell

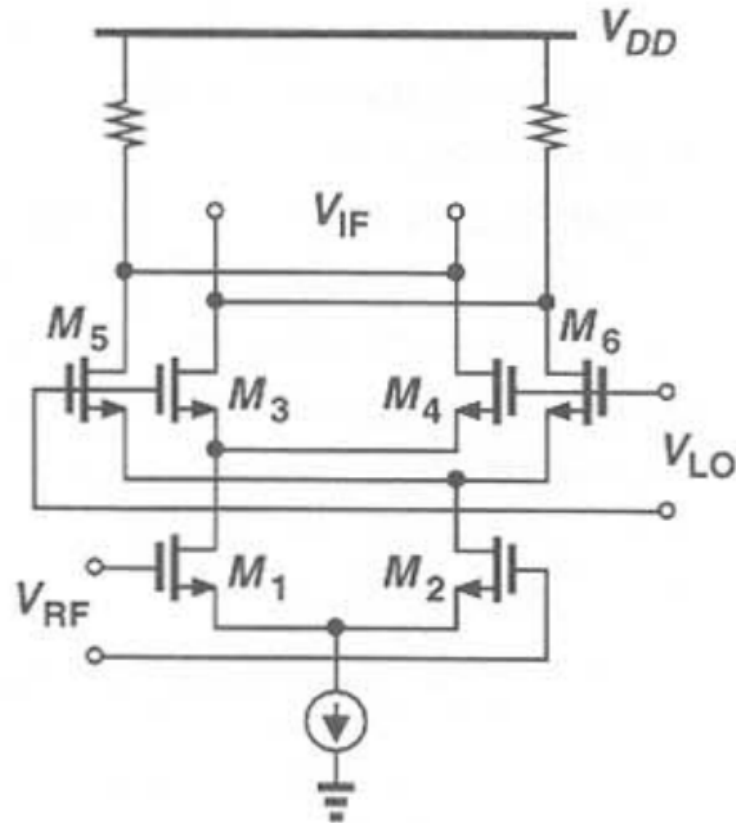


- $V_{out1}$  and  $V_{out2}$  must change  $I_1$  and  $I_2$  in opposite directions so that the amplifier gain changes monotonically
- This is done using a differential pair, as shown in Fig. (c)
- For large  $|V_{cont1} - V_{cont2}|$ , all of  $I_{SS}$  is steered to one of the top differential pairs and  $|V_{out}/V_{in}|$  is maximum
- If  $V_{cont1} = V_{cont2}$ , the gain is zero
- Simplified structure in Fig.(d), called a “Gilbert Cell”

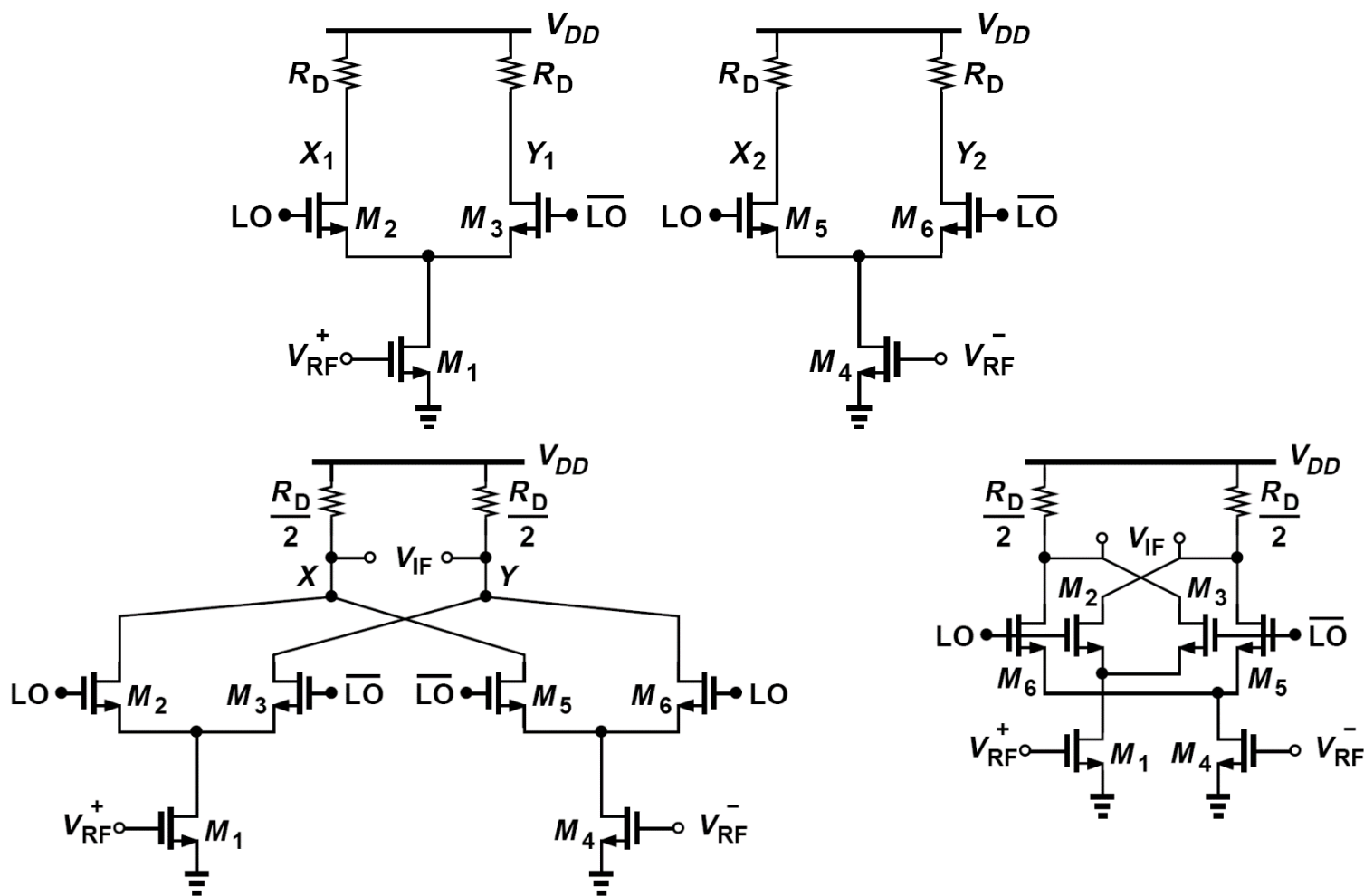
# Gilbert Cell with $V_{in}$ at Bottom Diff Pair



# Double Balanced Mixer



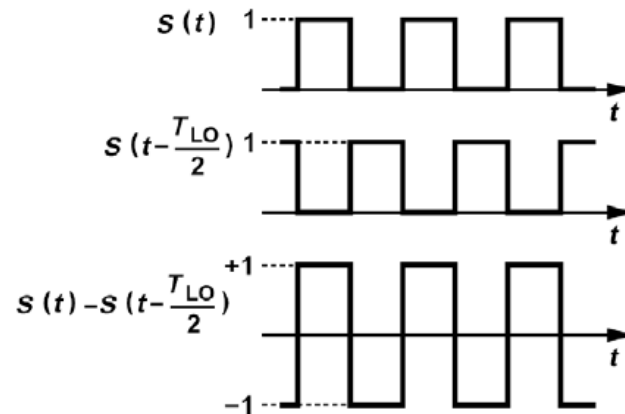
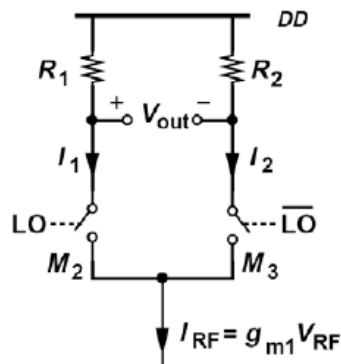
# Active Downconversion Mixers: Double-Balanced Topology



➤ One advantage of double-balanced mixers over their single-balanced counterparts stems from their rejection of amplitude noise in the LO waveform.

# Conversion Gain

With abrupt LO switching, the circuit reduces to that shown in figure below (left).



$$I_1 = I_{RF} \cdot S(t)$$

$$I_2 = I_{RF} \cdot S\left(t - \frac{T_{LO}}{2}\right)$$

We have for  $R_1 = R_2 = R_D$   $V_{out}(t) = I_{RF} R_D \left[ S\left(t - \frac{T_{LO}}{2}\right) - S(t) \right]$

The waveform exhibits a fundamental amplitude equal to  $4/\pi$ , yielding an output given by

$$V_{out}(t) = I_{RF}(t) R_D \cdot \frac{4}{\pi} \cos \omega_{LO} t + \dots$$

If  $I_{RF}(t) = g_{m1} V_{RF} \cos(\omega_{RF} t)$ , then



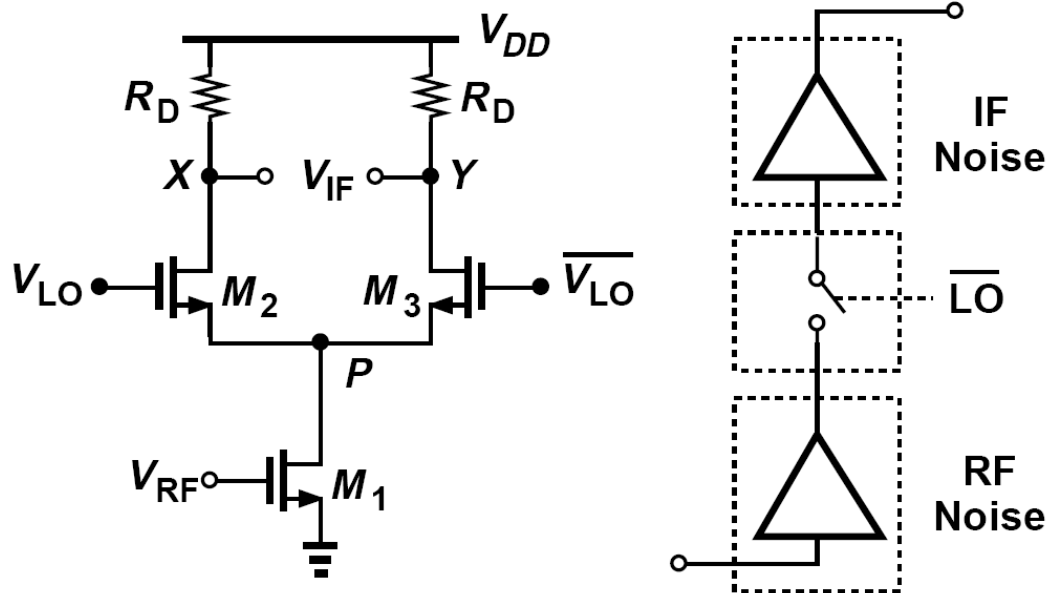
$$\frac{V_{IF,p}}{V_{RF,p}} = \frac{2}{\pi} g_{m1} R_D$$

$$V_{IF}(t) = \frac{2}{\pi} g_{m1} R_D V_{RF} \cos(\omega_{RF} - \omega_{LO}) t$$



# Noise in Active Mixers: Starting Analysis

- The noise components of interest lie in the RF range before downconversion and in the IF range after downconversion.



$$\overline{V_{n,in}^2} = \pi^2 kT \left( \frac{\gamma}{g_{m1}} + \frac{2}{g_{m1}^2 R_D} \right)$$

# Active Mixer Design Conversion Gain and NF

We know voltage conversion gain is given by:

$$\frac{V_{IF,p}}{V_{RF,p}} = \frac{2}{\pi} g_{m1} R_D$$

To compute the noise figure due to thermal noise, we first estimate the input-referred noise voltage as

$$\overline{V_{n,in}^2} = \pi^2 kT \left( \frac{\gamma}{g_{m1}} + \frac{2}{g_{m1}^2 R_D} \right)$$

We now write the single-sideband NF with respect to  $R_S = 50 \, \Omega$  as:

$$\text{NF}_{SSB} = 1 + \frac{\overline{V_{n,in}^2}}{4kT R_S}$$

# Active Mixer Design Example

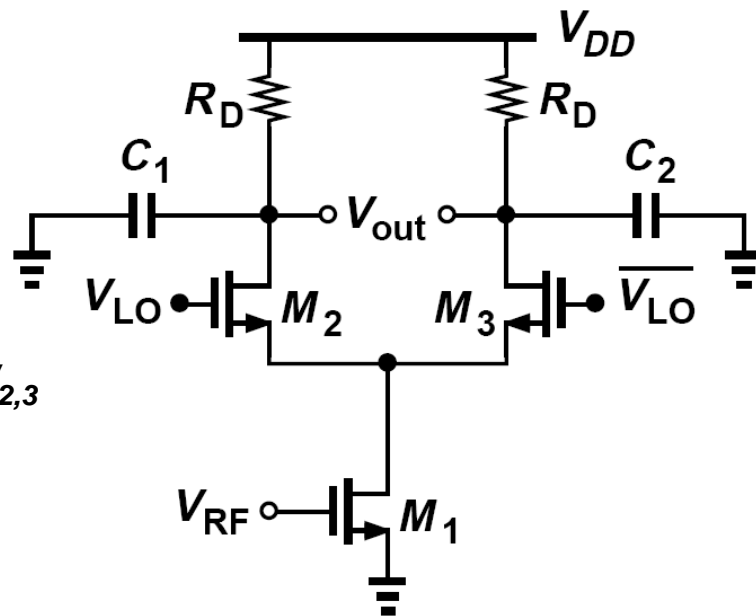
Design a 6-GHz active mixer in 65-nm technology with a bias current of 2 mA from a 1.2-V supply. Assume direct downconversion with a peak single-ended sinusoidal LO swing of 400 mV.

## *Solution:*

The design of the mixer is constrained by the limited voltage headroom. We begin by assigning an overdrive voltage of 300 mV to the input transistor,  $M_1$ , and 150 mV to the switching devices,  $M_2$  and  $M_3$  (in equilibrium).

From previous equation, we obtain a maximum allowable dc drop of about 600 mV for each load resistor,  $R_D$ . With a total bias current of 2 mA, we conservatively choose  $R_D = 500 \Omega$ .

The overdrives chosen above lead to  $W_1 = 15 \mu\text{m}$  and  $W_{2,3} = 20 \mu\text{m}$ . Capacitors  $C_1$  and  $C_2$  have a value of 2 pF to suppress the LO component at the output (which would otherwise help compress the mixer at the output).



# Active Mixer Design Example

We can now estimate the voltage conversion gain and the noise figure of the mixer.

$$\begin{aligned} A_v &= \frac{2}{\pi} g_{m1} R_D \\ &= 4.1 \text{ (= 12.3 dB)}. \end{aligned}$$

To compute the noise figure due to thermal noise, we first estimate the input-referred noise voltage as

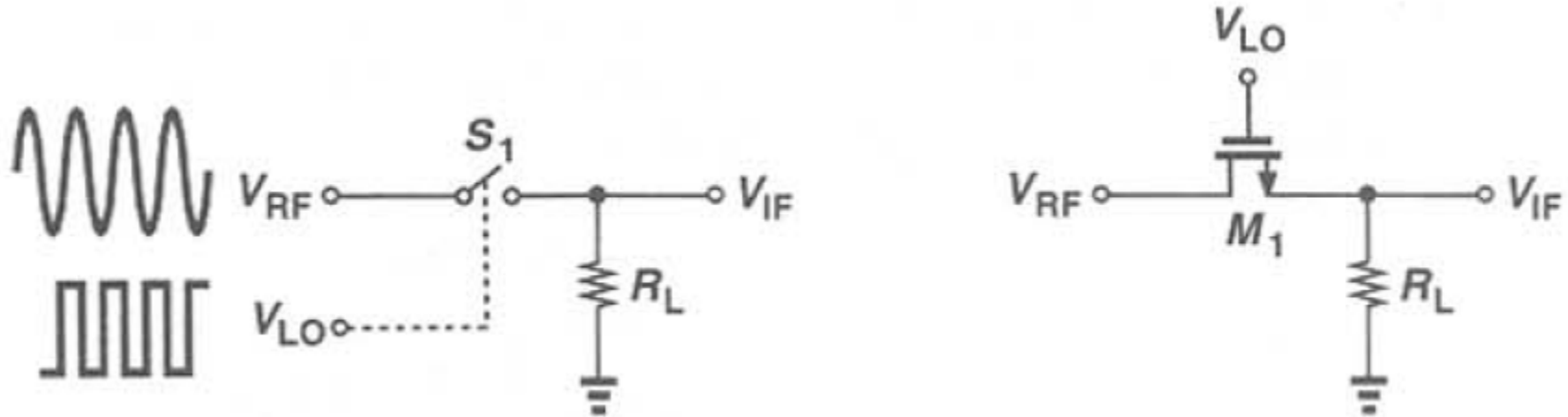
$$\begin{aligned} \overline{V_{n,in}^2} &= \pi^2 kT \left( \frac{\gamma}{g_{m1}} + \frac{2}{g_{m1}^2 R_D} \right) \\ &= 4.21 \times 10^{-18} \text{ V}^2/\text{Hz}, \end{aligned}$$

We now write the single-sideband NF with respect to  $R_S = 50 \text{ } \Omega$  as:

$$\begin{aligned} \text{NF}_{SSB} &= 1 + \frac{\overline{V_{n,in}^2}}{4kT R_S} \\ &= 6.1 \text{ (= 7.84 dB)} \end{aligned}$$

The double-sideband NF is 3 dB less.

# Passive Mixer



- **Passive mixer does not provide any gain.**
- **Passive mixer typically achieve higher linearity and speed.**
- **No power dissipation.**

# Performance Parameters: Noise and Linearity, Gain

- **Noise and Linearity:** The design of downconversion mixers entails a compromise between the noise figure and the  $IP_3$  (or  $P_{1dB}$ ).

In a receive chain, the input noise of the mixer following the LNA is divided by the LNA gain when referred to the RX input.

Similarly, the  $IP_3$  of the mixer is scaled down by the LNA gain.

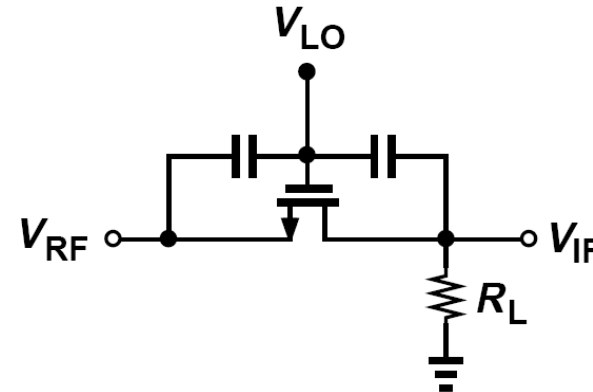
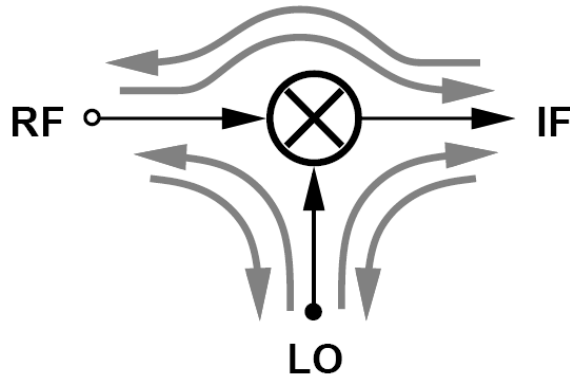
- **Gain:** mixer gain is critical in suppression of noise while retaining linearity.

Downconversion mixers must provide sufficient gain to adequately suppress the noise contributed by subsequent stages.

However, low supply voltages make it difficult to achieve a gain of more than roughly 10 dB while retaining linearity.

# Performance Parameters: Port-to-Port Feedthrough

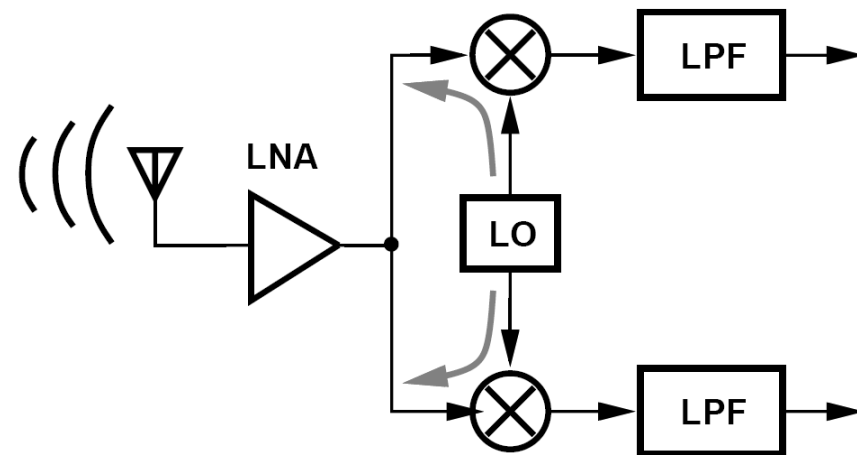
- Owing to device capacitances, mixers suffer from unwanted coupling (feedthrough) from one port to another.



In figure above, the gate-source and gate-drain capacitances create feedthrough from the LO port to the RF and IF ports.

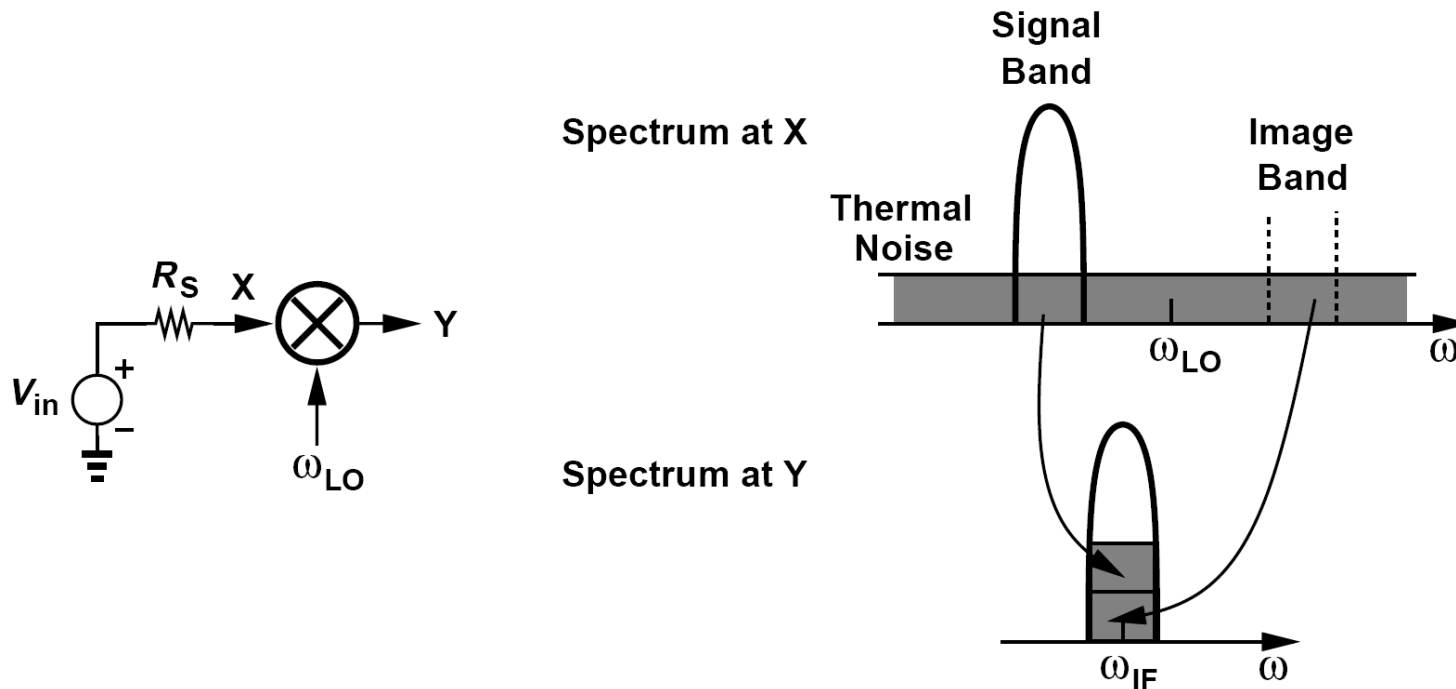
In the direct-conversion receiver:  
LO-RF feedthrough is entirely determined by the symmetry of the mixer circuit and LO waveforms.

The LO-IF feedthrough is heavily suppressed by the baseband low-pass filter(s).



# Mixer Noise Figures: SSB Noise Figure

For simplicity, let us consider a noiseless mixer with unity gain.

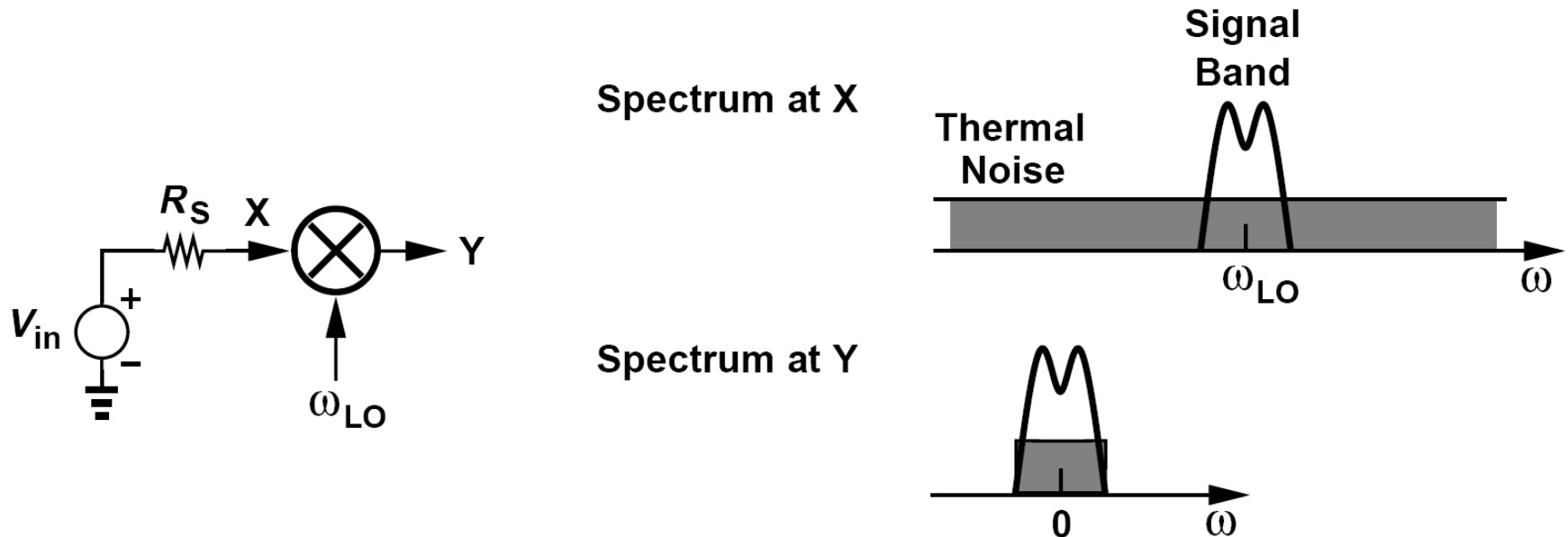


- The mixer exhibits a flat frequency response at its input from the image band to the signal band.
- The noise figure of a noiseless mixer is 3 dB. This quantity is called the “single-sideband” (SSB) noise.



# Mixer Noise Figures: DSB Noise Figure

Now, consider the direct-conversion mixer shown below.

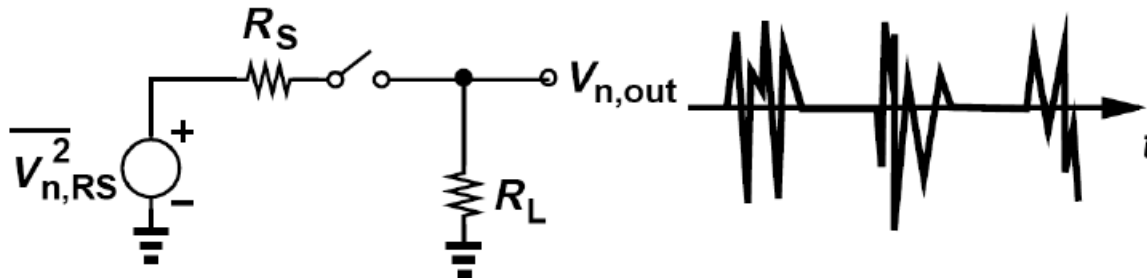


- In this case, only the noise in the signal band is translated to the baseband, thereby yielding equal input and output SNRs if the mixer is noiseless.
- The noise figure is thus equal to 0 dB. This quantity is called the “double-sideband” (DSB) noise figure

# Example of Noise Spectrum of a Simple Mixer

Consider the simple mixer shown below. Assuming  $R_L \gg R_S$  and the LO has a 50% duty cycle, determine the output noise spectrum due to  $R_S$ , i.e., assume  $R_L$  is noiseless.

*Solution:*



Since  $V_{out}$  is equal to the noise of  $R_S$  for half of the LO cycle and equal to zero for the other half, we expect the output power density to be simply equal to half of that of the input, i.e.,  $2kTR_S$ .