EE 538B CMOS RF IC DESIGN

Midterm Examination: May 14, 2003

Student Name: Solutions

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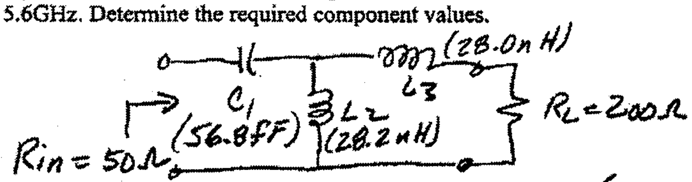
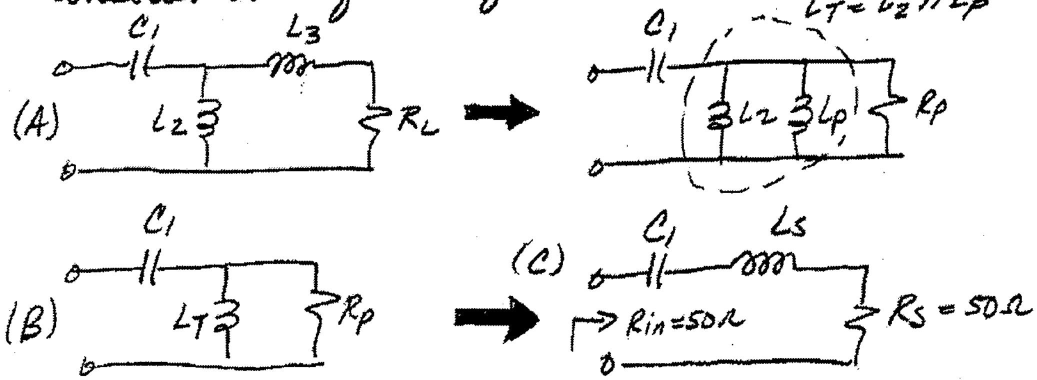
You are allowed one sheet of notes. Write legibly. Show all work. State assumptions.

Problem #	Points Possible	Points
1	25	25
2	25	25
3	25	25
4	25	25
		100

1. (25 points) With Q=10, the matching circuit below is used to convert $R_L = 200\Omega$ to $R_{in} = 50\ \Omega$ at f_o = 5.6GHz. Determine the required component values.



Consider a sequence of transformations:



The final circuit is pure series RLC circuit:

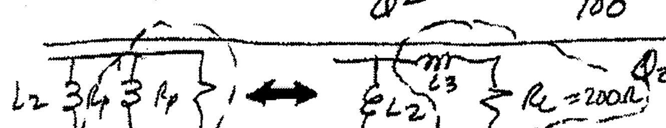
$Q = 10$; $R_S = R_{in} = 50\ \Omega$; $f_0 = 5.6\ GHz$

For (C): $Q = 10 = \frac{\omega_0 L_S}{R_S} \Rightarrow L_S = \frac{Q R_S}{\omega_0} = \boxed{14.2\ nH}$

$Q = 10 = \frac{1}{\omega_0 R_S C_1} \Rightarrow C_1 = \frac{1}{\omega_0 R_S Q} = \boxed{56.8\ fF}$

For B: $R_p = R_S (Q^2+1) = 50\ (101) = \boxed{5050\ \Omega}$

$L_T = L_S \frac{(Q^2+1)}{Q^2} = \frac{(14.2nH)(101)}{100} = \boxed{14.4\ nH}$

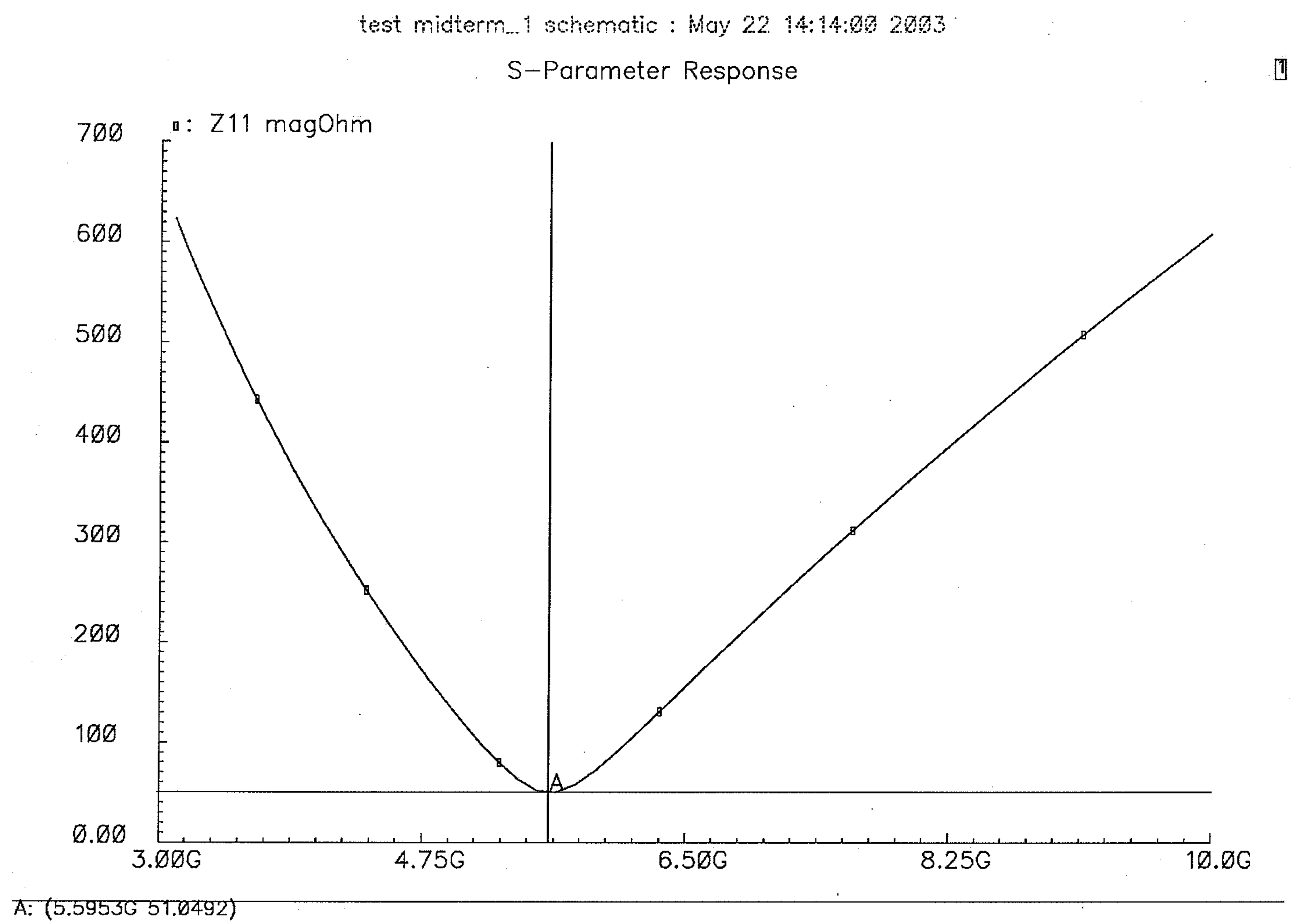


$Q_2 = \sqrt{\frac{R_p}{R_L} - 1} = 4.92$

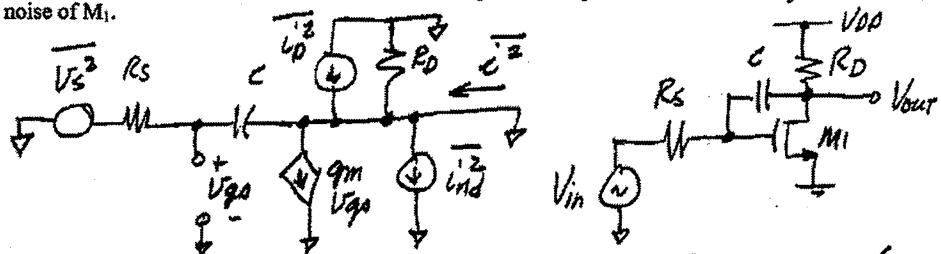
$Q_2 = \frac{\omega_0 L_3}{R_L} \Rightarrow L_3 = \frac{Q_2 R_L}{\omega_0} = \boxed{28.0\ nH}$

$Q_2 = \frac{R_p}{\omega_0 L_p} \Rightarrow L_p = \frac{R_p}{Q_2 \omega_0} = \boxed{29.2\ nH}$

$L_2 = \frac{L_T L_p}{L_p - L_T} = \boxed{28.2\ nH}$


test midterm_1 schematic : May 22 14:14:00 2003
S−Parameter Response
▫: Z11 magOhm
700
600
500
400
300
200
100
0.00
3.00G
4.75G
6.50G
8.25G
10.0G
A
A: (5.5953G 51.0492)

2. (25 points) Find the noise factor of the circuit shown with respect to the source resistance R_S. Neglect channel length modulation effects and all parasitic capacitors. Consider only drain current noise of M_1.



$\overline{v_s^2} = 4kTR_s\Delta f$; $\overline{i_{nd}^2} = 4kT\gamma g_{d0}\Delta f$; $\overline{i_D^2} = \frac{4kT}{R_D}\Delta f$

(i) Consider $\overline{i_D^2}$: $\overline{i_1^2} = \overline{i_D^2} = \frac{4kT}{R_D}\Delta f$

(ii) consider $\overline{i_{nd}^2}$: $\overline{i_2^2} = \overline{i_{nd}^2} = 4kT\gamma g_{d0}\Delta f$

(iii) consider $\overline{v_s^2}$: Note that $\overline{v_s}$ creates a noise current that flows through R_s and C to the output. $\overline{v_s}$ also creates v_{gs} via the R_s and C voltage divider and v_{gs} multiplied by g_m creates another output current noise component. These two components are completely correlated.

$$v_{gs} = \frac{1/sC}{R_s+1/sC}\overline{v_s} = \frac{1}{sR_sC+1}\overline{v_s} \Rightarrow v_{gs}^2 = \frac{1}{(\omega R_sC)^2+1}\cdot\overline{v_s^2}$$

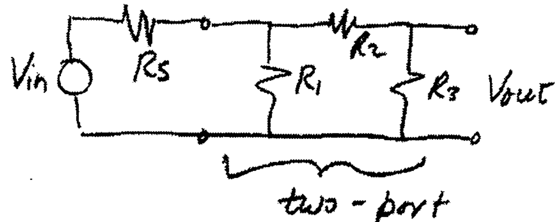
$$\overline{i_3} = g_m v_{gs} - sCv_{gs} = v_{gs}(g_m - sC)$$

$$\therefore \overline{i_3^2} = v_{gs}^2(g_m^2+(\omega C)^2) = \frac{g_m^2+(\omega C)^2}{(\omega R_sC)^2+1}\cdot\overline{v_s^2}$$

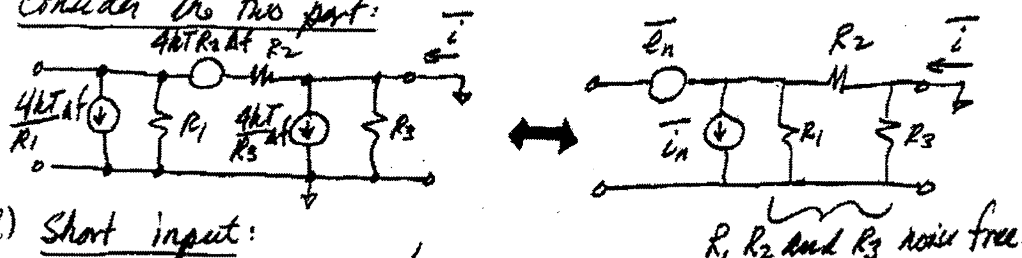
$$F = (\overline{i_1^2}+\overline{i_2^2}+\overline{i_3^2})/\overline{i_3^2}$$

$$= 1 + \frac{[(\omega R_sC)^2+1]}{[g_m^2+(\omega C)^2]}\cdot\left(\frac{1}{R_DR_s}+\frac{\gamma g_{d0}}{R_s}\right) \longleftarrow \text{ANS.}$$

3. (25 points) Determine expressions for the equivalent two-port input noise voltage and current power sources for the two-port network shown assuming it is driven from a source resistance R_S.



Consider the two port:



(a) Short input:

$$\overline{i^2} = \frac{4kT}{R_3}\Delta f + \frac{4kTR_2\Delta f}{R_2^2} = 4kT\Delta f\left(\frac{1}{R_2}+\frac{1}{R_3}\right)$$

(a) Short input:

$$\overline{i^2} = \frac{\overline{e_n^2}}{R_2^2}$$

Hence, $\overline{e_n^2} = 4kT\Delta f\, R_2^2\left(\frac{1}{R_2}+\frac{1}{R_3}\right) \longleftarrow \overline{e_n^2}$

(b) Open circuit input:

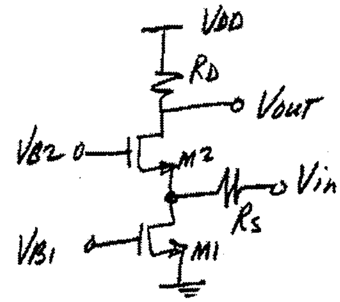
$$\overline{i^2} = \frac{4kT\Delta f}{R_1}\left(\frac{R_1}{R_1+R_2}\right)^2 + \frac{4kT\Delta f}{R_3} + \frac{4kTR_2\Delta f}{(R_1+R_2)^2} \qquad \overline{i^2} = \overline{i_n^2}\cdot\left(\frac{R_1}{R_1+R_2}\right)^2$$

$$= 4kT\Delta f\left[\frac{R_1}{(R_1+R_2)^2} + \frac{1}{R_3} + \frac{R_2}{(R_1+R_2)^2}\right]$$

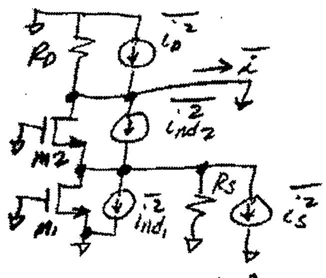
$$\therefore \overline{i_n^2} = \overline{i^2}\cdot\frac{(R_1+R_2)^2}{R_1^2} = 4kT\Delta f\left[\frac{R_1}{(R_1+R_2)^2} + \frac{1}{R_3} + \frac{R_2}{(R_1+R_2)^2}\right]\frac{(R_1+R_2)^2}{R_1^2}$$

$$= 4kT\Delta f\left[\frac{1}{R_1} + \frac{(R_1+R_2)^2}{R_1^2 R_3} + \frac{R_2}{R_1^2}\right] \longleftarrow \overline{i_n^2}$$

4. (25 points) Find an expression for the noise factor of the circuit below with respect to the source resistance R_S. Neglect channel length modulation and all parasitic capacitors; also, neglect induced gate current noise.



The noise sources are uncorrelated:

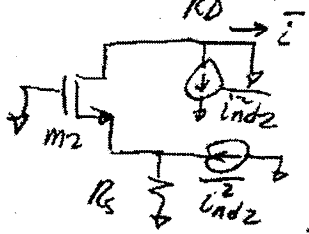


$$\overline{i_{n,R_S}^2} = \frac{4kT}{R_S}\Delta f\left(\frac{R_S}{R_S+1/g_{m2}}\right)^2$$

$$\overline{i_{n,M1}^2} = 4kT\gamma g_{d01}\Delta f\left(\frac{R_S}{R_S+1/g_{m2}}\right)^2$$

note current divider between R_S and $1/g_{m2}$.

$$\overline{i_{n,R_D}^2} = \frac{4kT}{R_D}\Delta f$$



In this model, the components of $\overline{i}$ are obviously correlated.

$$\overline{i_{n,m2}^2} = 4kT\gamma g_{d02}\Delta f\left[\frac{R_S}{R_S+1/g_{m2}}-1\right]^2$$

$$= 4kT\gamma g_{d02}\Delta f\left[\frac{1}{1+g_{m2}R_S}\right]^2$$

Finally,

$$F = \frac{\dfrac{4kT\Delta f\, g_{m2}^2 R_S}{(1+g_{m2}R_S)^2} + \dfrac{4kT\Delta f\,\gamma g_{d01} g_{m2}^2 R_S^2}{(1+g_{m2}R_S)^2} + \dfrac{4kT\Delta f\,\gamma g_{d02}}{(1+g_{m2}R_S)^2} + \dfrac{4kT\Delta f}{R_D}}{\dfrac{4kT\Delta f\, g_{m2}^2 R_S}{(1+g_{m2}R_S)^2}}$$

$$= 1 + \gamma_1 g_{d01} R_S + \frac{\gamma_2 g_{d02}}{g_{m2}^2 R_S} + \frac{(1+g_{m2}R_S)^2}{g_m^2 R_D R_S} \leftarrow F$$