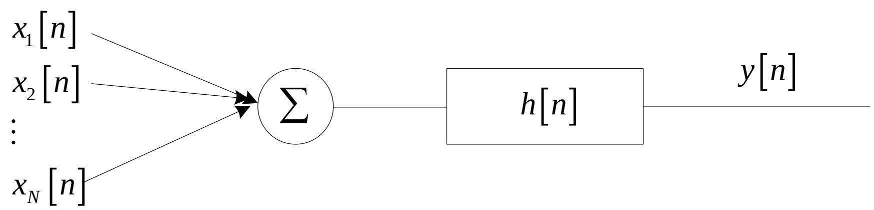
Lec08

Superposition



$$y[n] = h[n] * x[n]$$

$$x[n] = x_1[n] + x_2[n] + \ldots x_N[n] \tag{1}$$

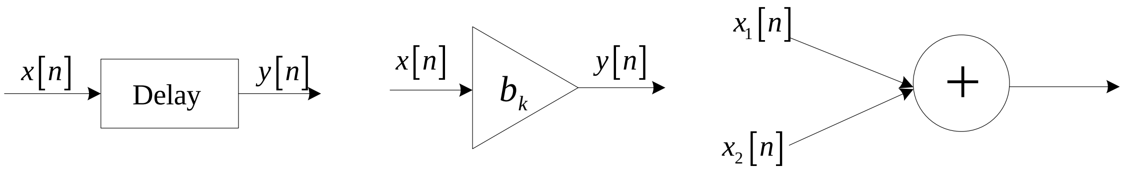
$$y[n] = h[n] * \left(x_1[n] + x_2[n] + \ldots x_N[n] \right)$$

$$\begin{aligned} y[n] &\leftrightarrow Y(f) \\ h[n] &\leftrightarrow H(f) \end{aligned} \tag{2}$$

$$\begin{aligned} Y(f) &= H(f)X(f) \\ &= H(f)\left(X_1(f) + X_2(f) + \ldots + X_N(f) \right) \\ &= H(f)X_1(f) + H(f)X_2(f) + \ldots + H(f)X_N(f) \end{aligned} \tag{3}$$

$$y[n] = h[n] * x_1[n] + h[n] * x_2[n] + \ldots + h[n] * x_N[n]$$

Difference equation diagrams



Difference equation diagrams

$$\begin{aligned} & a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] + \ldots + a_N y[n-N] \\ & = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \ldots + b_M x[n-M] \end{aligned} \tag{4}$$

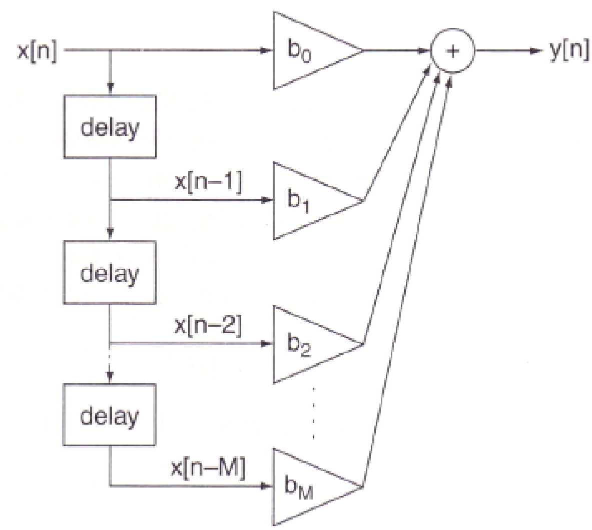
$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k] \tag{5}$$

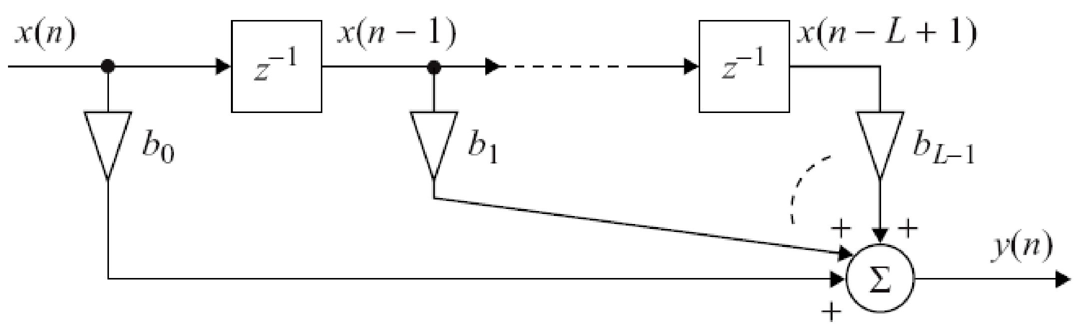
Non-recursive difference equations

Past and present inputs are required for the calculation of each new output.

$$y[n] = \sum_{k=0}^{M} b[k]\,x[n-k] \qquad (6)$$
$$= b_0 x[n] + b_1 x[n\text{-}1] + b_2 x[n\text{-}2] + \ldots + b_M x[n\text{-}M]$$

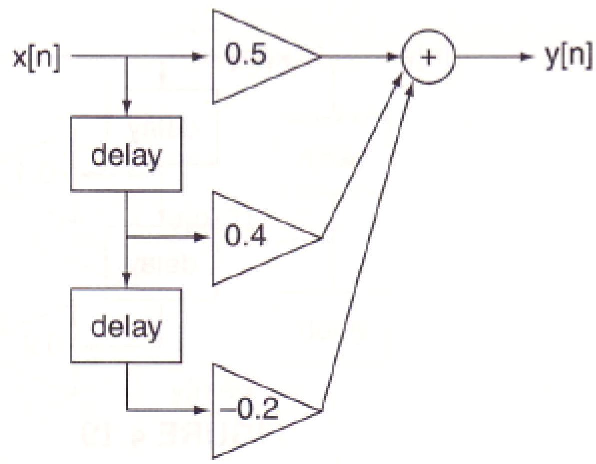
FIGURE 4.15
Nonrecursive difference equation diagram.



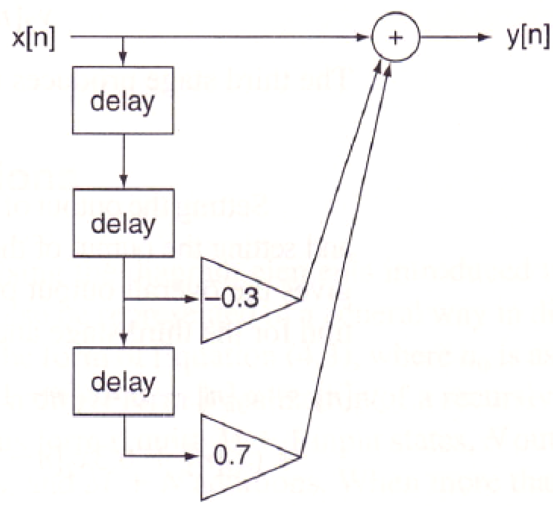


Ex) Draw a diagram for the difference equation

$$y[n] = 0.5x[n] + 0.4x[n-1] - 0.2x[n-2]$$



Ex)



Write the difference equation that corresponds to the diagram in the figure shown above.

$$y[n] = x[n] - 0.3x[n-2] + 0.7x[n-3]$$

Recursive difference equations

$$y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b[k] x[n-k]$$

$$= -a_1 y[n-1] - a_2 y[n-2] - \ldots - a_N y[n-N] \qquad (7)$$

$$+ b_0 x[n] + b_1 x[n\text{-}1] + b_2 x[n\text{-}2] + \ldots + b_M x[n\text{-}M]$$

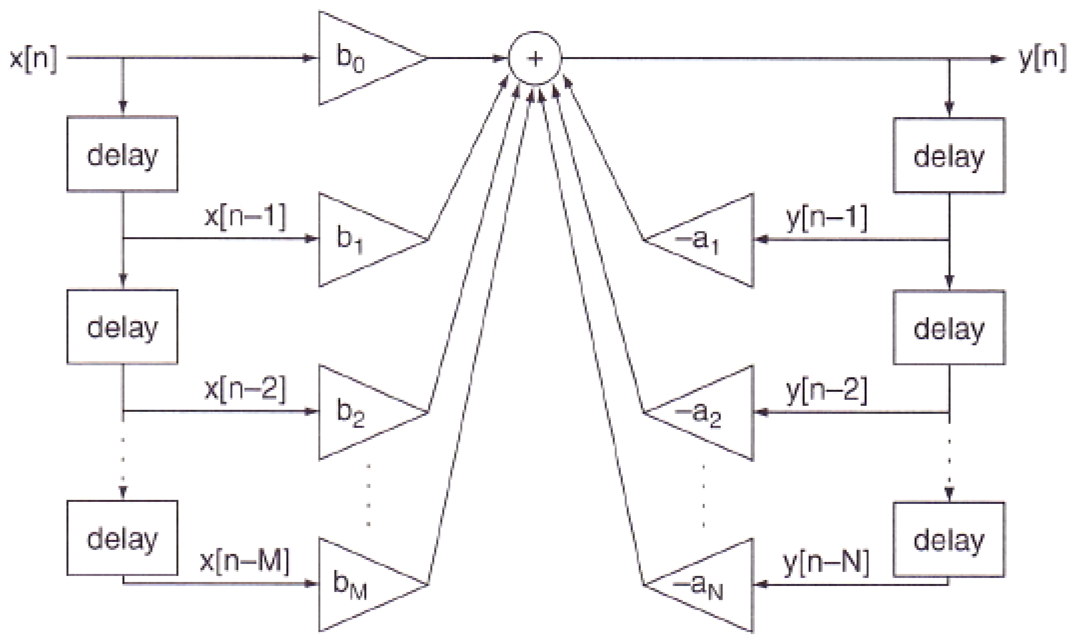
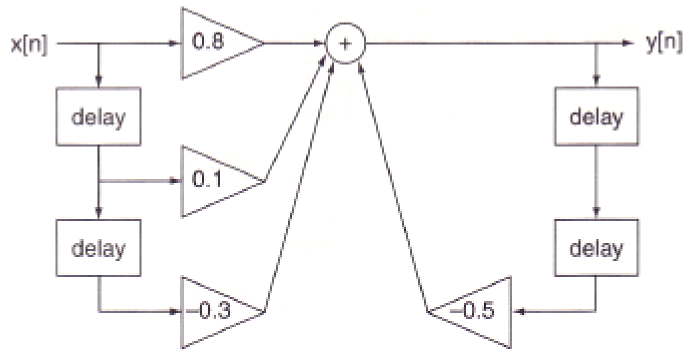


FIGURE 4.20
Recursive difference equation diagram, direct form 1 realization.

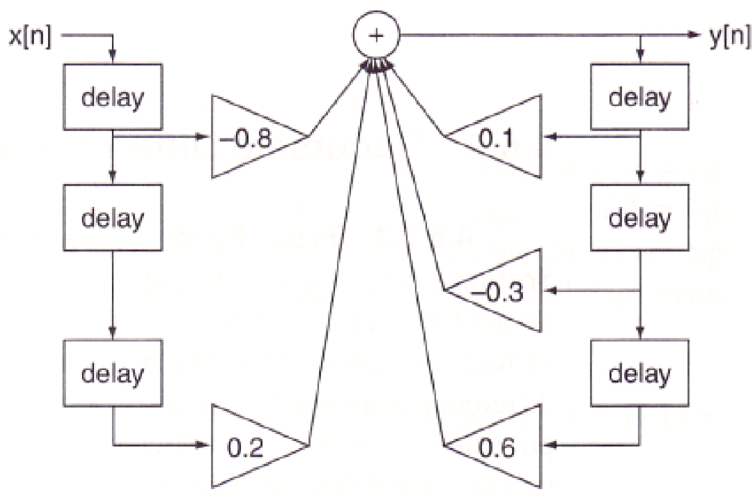
Ex) Draw a direct form 1 difference equation diagram to describe the recursive digital filter

$$y[n]+0.5y[n-2]=0.8x[n]+0.1x[n]-0.3x[n-2]$$

$$y[n]=-0.5y[n-2]+0.8x[n]+0.1x[n]-0.3x[n-2]$$

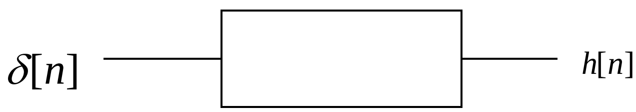


Ex) Write the difference equation that corresponds to the diagram shown

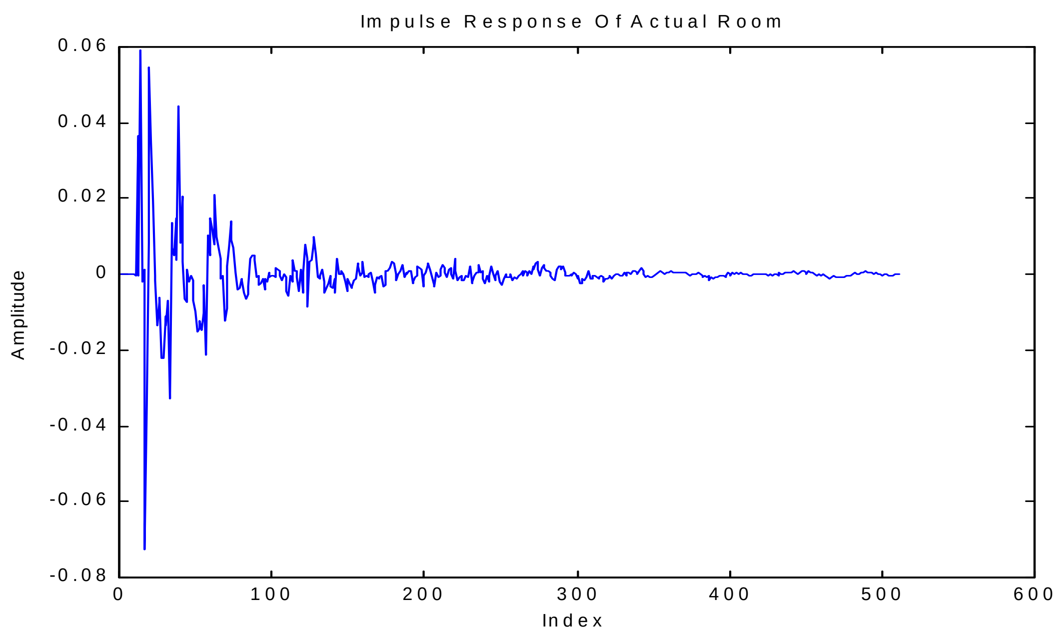


$$y[n]=0.1y[n-1]-0.3y[n-2]+0.6y[n-3]-0.8x[n-1]+0.2x[n-3]$$

The impulse response



The impulse response for a filter is the response of the filter to an impulse. When the input to a filter is a unit impulse function, the output from the filter is unit impulse response.



If the impulse response for a linear filter is known, then the output for any other input can easily calculated.

EXAMPLE 4.11

Find the first six samples of the impulse response for the difference equation

$$y[n] - 0.4y[n-1] = x[n] - x[n-1]$$

First, replace $x[n]$ with $\delta[n]$, and $y[n]$ with $h[n]$ to give:

$$h[n] - 0.4h[n-1] = \delta[n] - \delta[n-1]$$

or

$$h[n] = 0.4h[n-1] + \delta[n] - \delta[n-1]$$

Starting with $n = 0$:

$$h[0] = 0.4h[-1] + \delta[0] - \delta[-1]$$

The values for the impulse function $\delta[n]$ are known: At $n = 0$, it has the value one, and at all other values of n it has the value zero. The filter can be assumed to be causal, which means that the impulse response is zero before $n = 0$. Therefore,

$$h[0] = 0.4(0.0) + 1.0 - 0.0 = 1.0$$

Notice that $\delta[-1] = 0$ because zero is the value of the function $\delta[n]$ when $n = -1$, not a consequence of causality.

The subsequent impulse response samples are:

$$h[1] = 0.4h[0] + \delta[1] - \delta[0] = 0.4(1.0) + 0.0 - 1.0 = -0.6$$

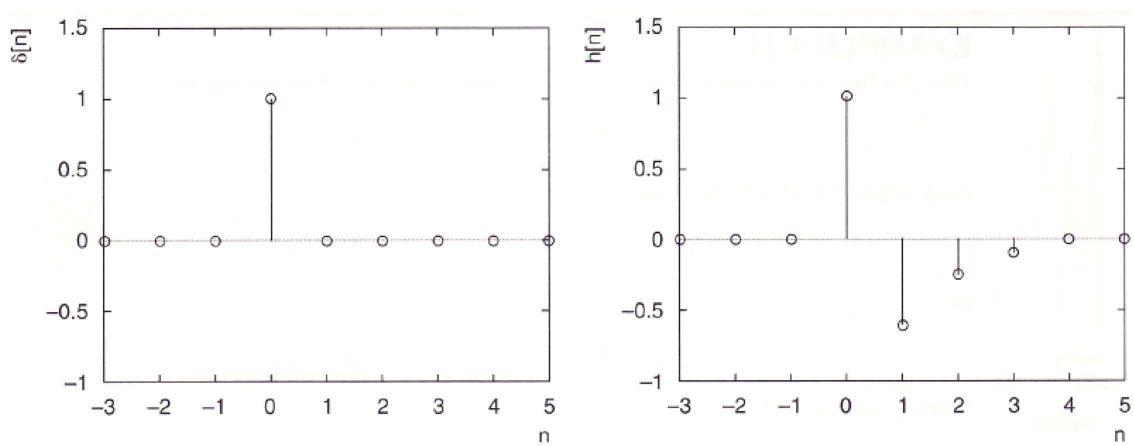
$$h[2] = 0.4h[1] + \delta[2] - \delta[1] = 0.4(-0.6) + 0.0 - 0.0 = -0.24$$

$$h[3] = 0.4h[2] + \delta[3] - \delta[2] = 0.4(-0.24) + 0.0 - 0.0 = -0.096$$

$$h[4] = 0.4h[3] + \delta[4] - \delta[3] = 0.4(-0.096) + 0.0 - 0.0 = -0.0384$$

$$h[5] = 0.4h[4] + \delta[5] - \delta[4] = 0.4(-0.0384) + 0.0 - 0.0 = -0.01536$$

The impulse function and impulse response are shown in Figure 4.28(a) and (b).



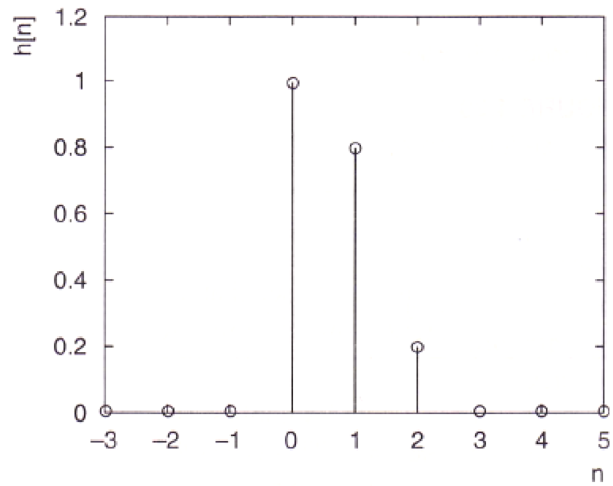
(a) Impulse Function

(b) Impulse Response

EXAMPLE 4.13

Write the difference equation for the filter whose impulse response is shown in Figure 4.30.

FIGURE 4.30
Impulse response for Example 4.13.



The impulse response can be written as a sum of impulse functions.

$$h[n] = \delta[n] + 0.8\delta[n-1] + 0.2\delta[n-2]$$

so the difference equation has the parallel structure

$$y[n] = x[n] + 0.8x[n-1] + 0.2x[n-2]$$

Since the number of nonzero samples in the impulse response is finite, the difference equation has a finite impulse response (FIR) characteristic.