Lec06

Most common elemental digital signal is

- Impulse
- Step
- Exponential
- Sinusoidal

A digital signal x is designated as $x[n]$

- n: an integer referring to the number of the sample
- The sequence $x[n]$ is the collection of all the samples.

Ex 2]

- $x[n-1]$: Sequence shifted to the right by one sample.
- $x[n+1]$: Sequence shifted to the left by one sample.
- $x[kn]$: Decimation

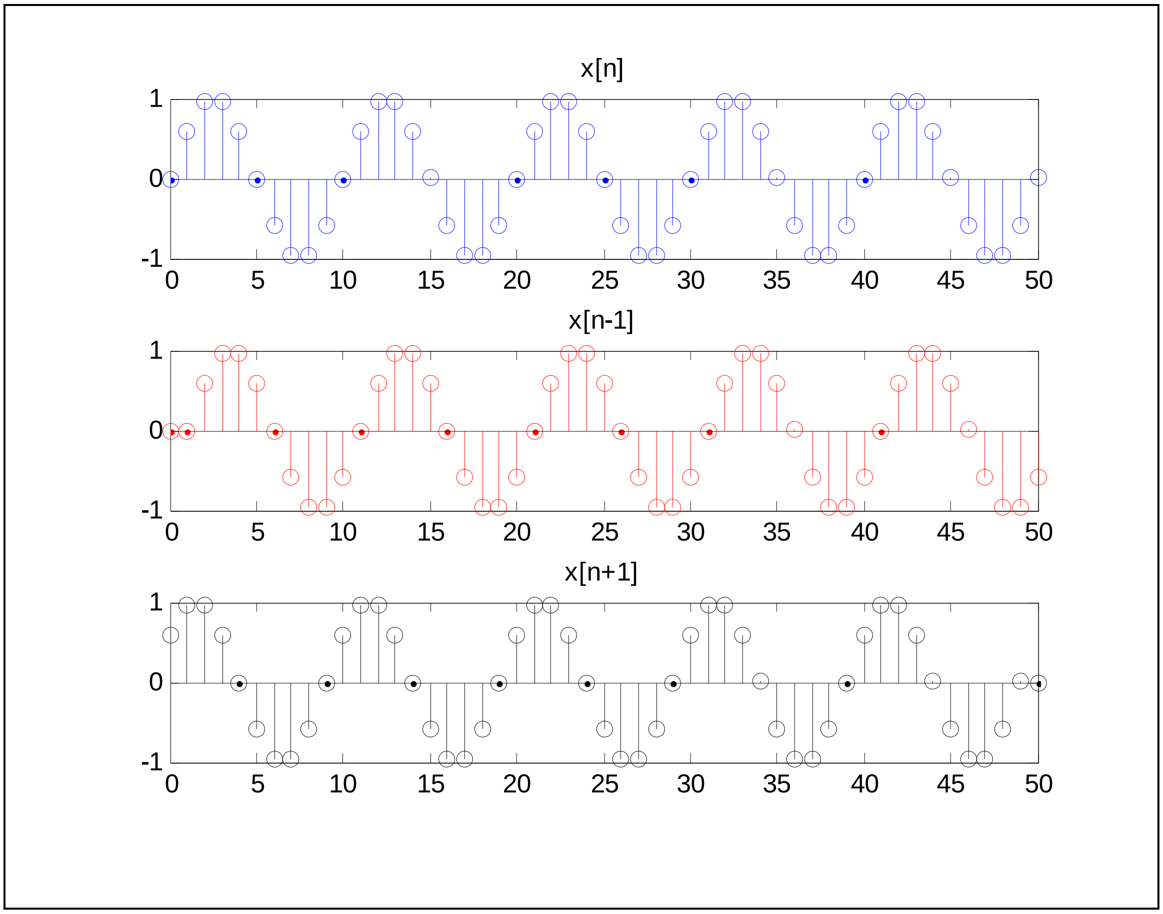


Figure 1: $x[n]$, $x[n-1]$ and $x[n+1]$

Ex 3]

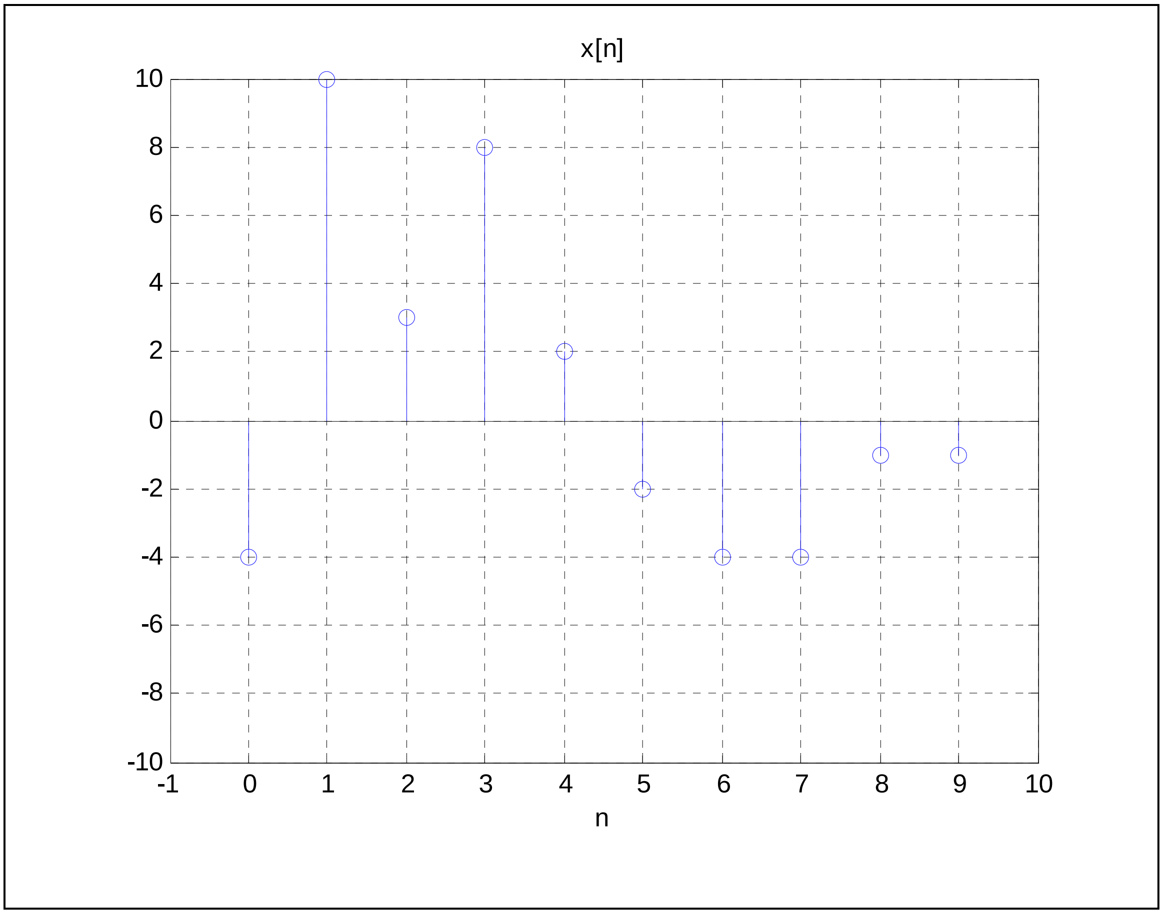


Figure 2: Signal $x[n]$

Find the following based on the above plot.

$x[0]$
$x[5]$
$x[n-1]$
$x[n-2]$
$x[2n]$

Impulse function

$$x[n]=\delta[n]+0.5\delta[n-1]+0.2\delta[n-2]$$

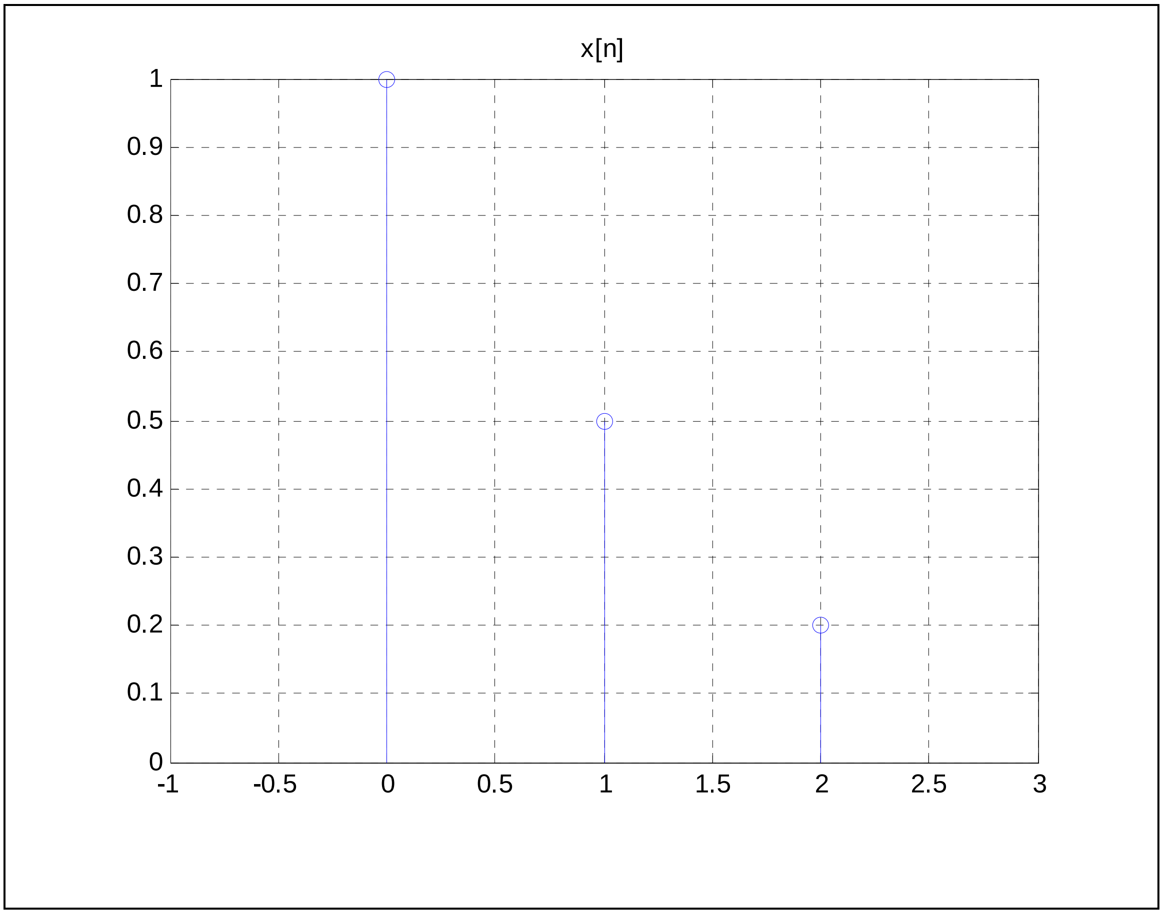


Figure 3: $x[n]=\delta[n]+0.5\delta[n-1]+0.2\delta[n-2]$

What is $x[n-1]$?

$$x[n-1]=\delta[n-1]+0.5\delta[n-2]+0.2\delta[n-3]$$

Step function

Draw the signal

$$x[n] = 3u[n]$$
$$x[n] = u[-n]$$

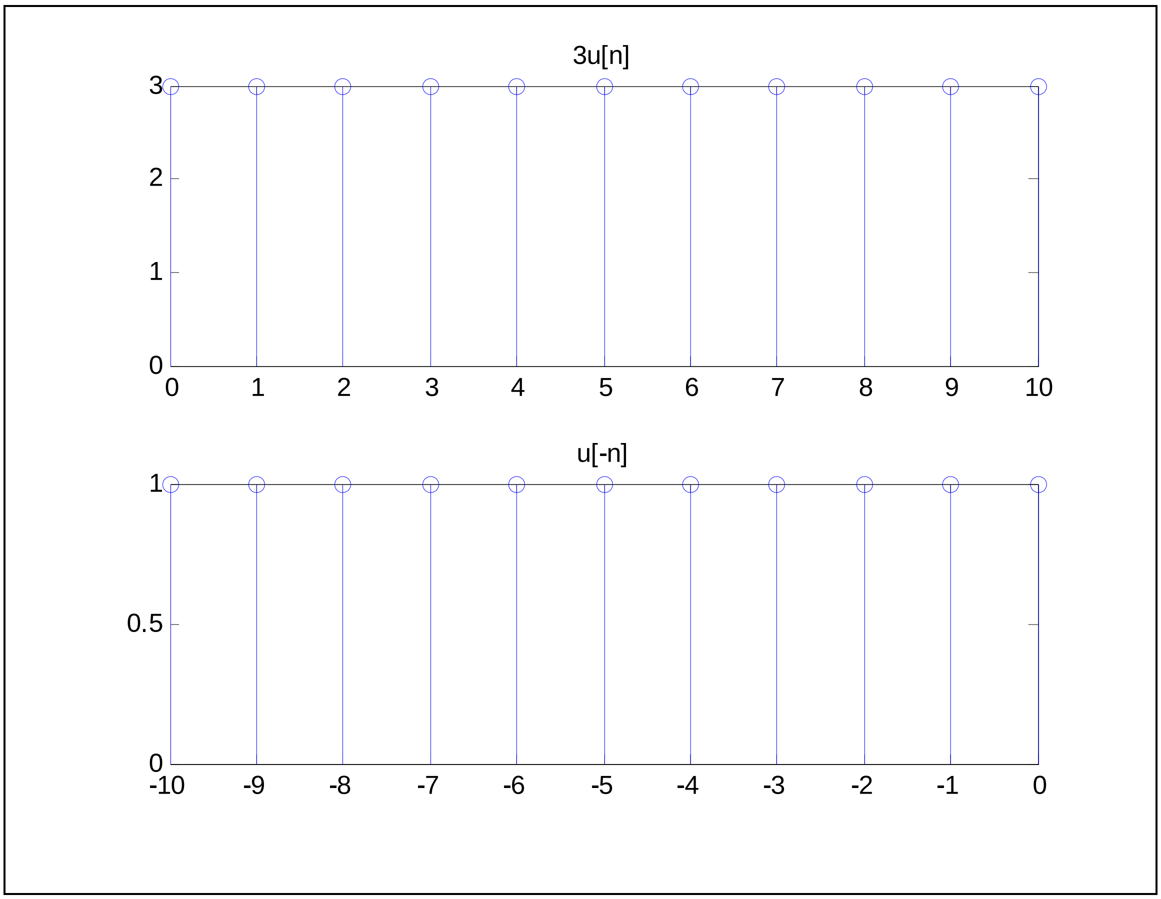


Figure 4: $u[n]$ and $u[-n]$

$$x[n] = u[n] - u[n-3]$$

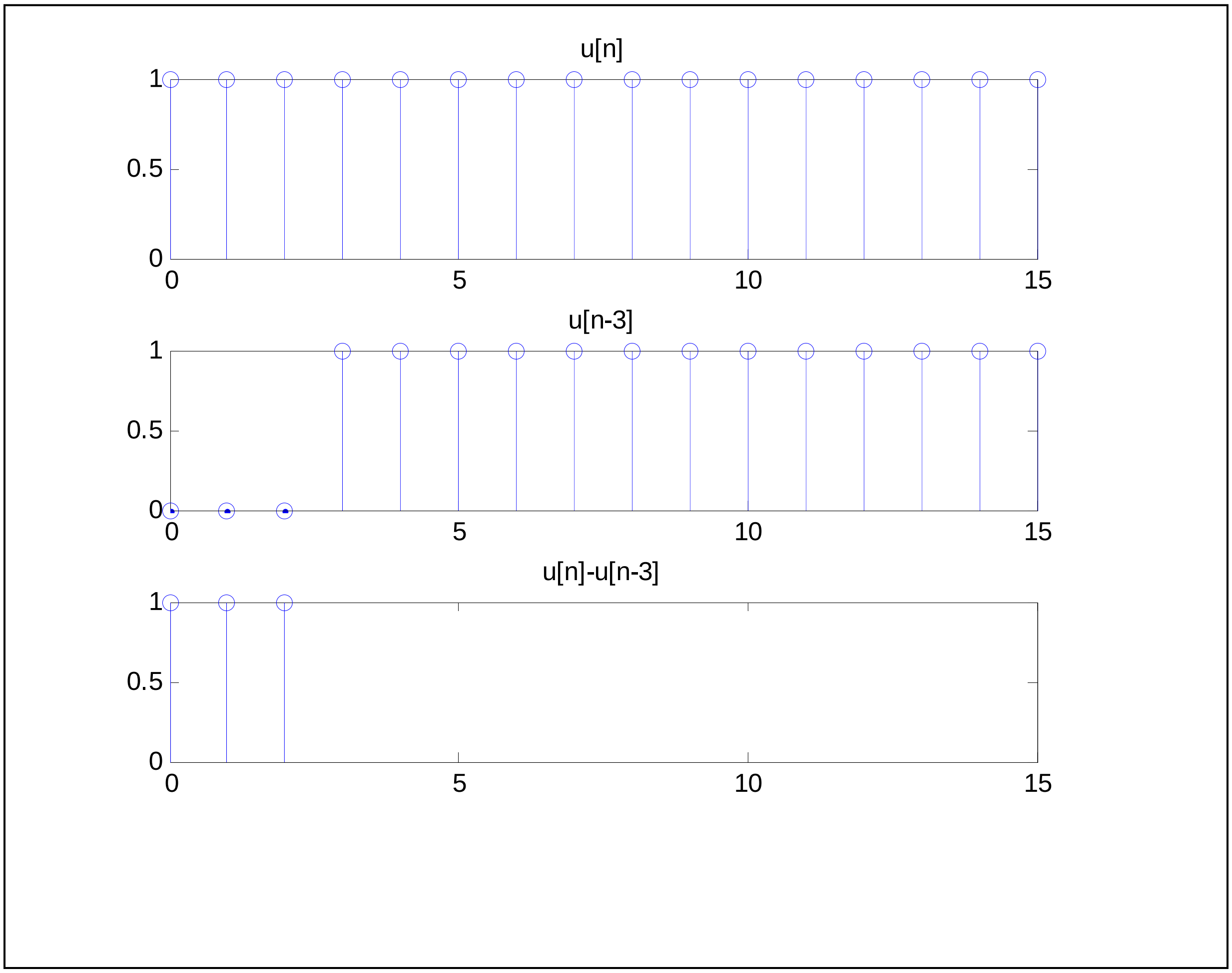


Figure 5: $u[n]$, $u[n\text{-}3]$ and $\left[u[n]-u[n-3]\right]$

Power and exponential function

$$x[n] = A\alpha^{\beta n} \tag{1}$$

The special case of exponential function is

$$x[n] = Ae^{\beta n} \tag{2}$$

where $e = 2.71828...$

Ex 4] Draw

$$x[n] = e^{-0.5n}u(n)$$

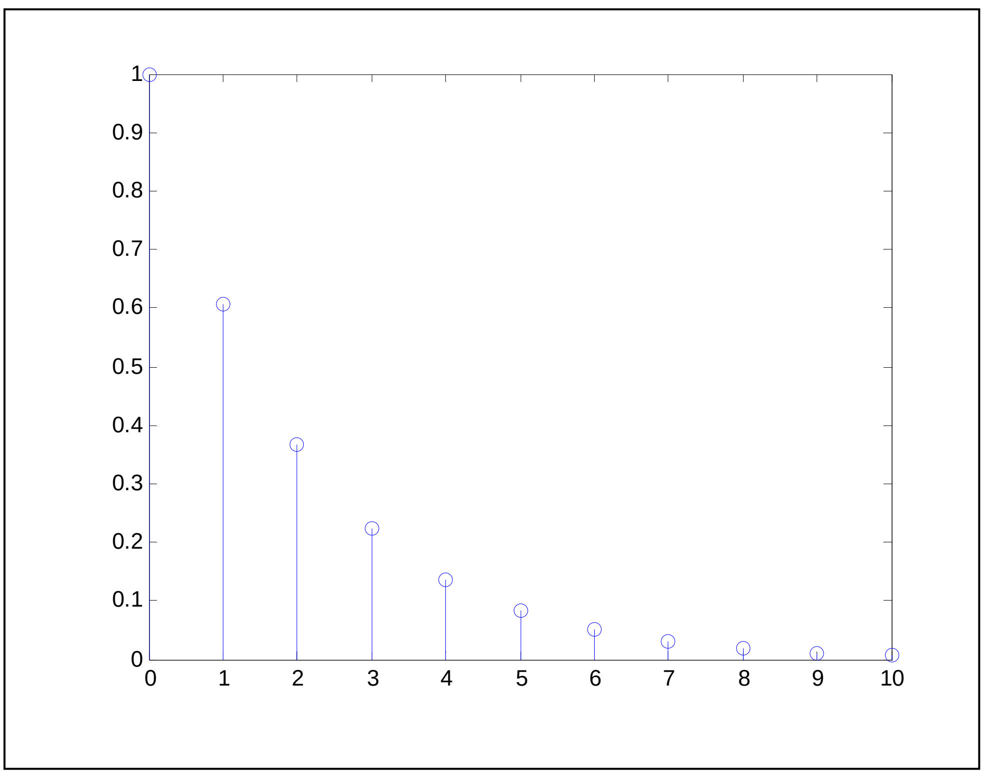


Figure 6: $x[n] = e^{-0.5n}u[n]$

Ex 5] Draw

$$x[n] = (-0.6)^n u[n]$$

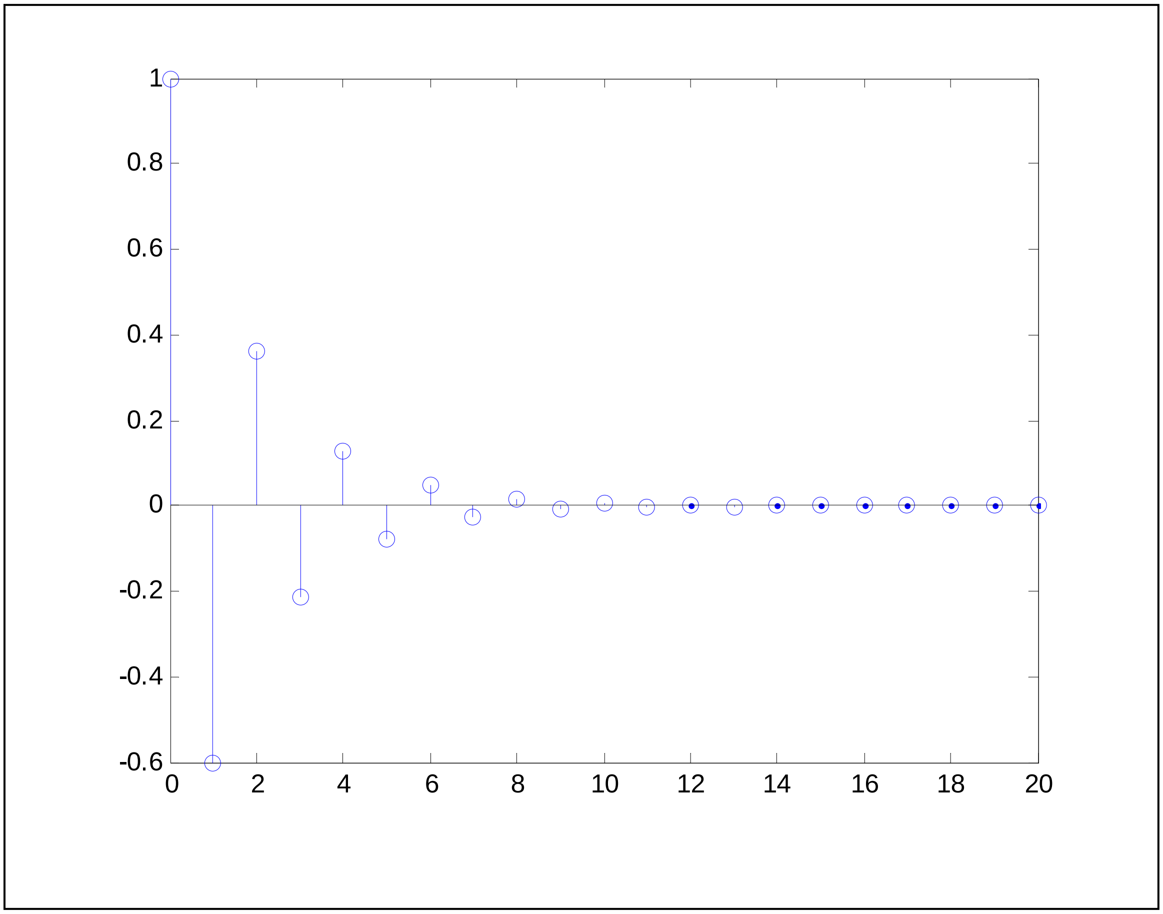


Figure 7: $x[n] = (-0.6)^n u[n]$

The complex exponential funciton

$$x[n] = e^{j\beta n} \tag{3}$$

- DFS
- DTFT
- DFT

$$\begin{aligned} x[n] &= e^{j\beta n} \\ &= \cos(\beta n) + j\sin(\beta n) \end{aligned} \tag{4}$$

Sine and cosine function

$$x[n] = A\sin(n\Omega) \tag{5}$$

or

$$x[n] = A\cos(n\Omega) \tag{6}$$

where

- A: amplitude
- Ω: Frequency repetition of the digital sequence
- Digital sine and cosine functions are **not necessarily periodic**.
- Ω is **not equal to the frequency of the analog signal being sampled.**

Let **analog** sine wave can be described by the function

$$x(t)=\sin(\omega t) \tag{7}$$

where $(\omega = 2\pi f)$ is the frequency of the sine wave in rad/sec.

When this sine wave is sampled, samples are collected every T_s second.

Sample time

$$t = nT_s \rightarrow x(nT_s) = x[n] \tag{8}$$

Let $x(nT_s) = A\sin(\omega\, nT_s)$

- $\omega = 2\pi f$
- $T_s = \dfrac{1}{fs}$ $\qquad fs$: sampling frequency.

$$\begin{aligned} x(nTs) &= A\sin(\omega \cdot nTs) \\ &= A\sin(2\pi f \cdot nTs) \\ &= A\sin\left(2\pi f \cdot n\frac{1}{fs}\right) \\ &= A\sin\left(n \cdot 2\pi \frac{f}{fs}\right) \\ &= A\sin(n \cdot \Omega) \end{aligned} \tag{9}$$

where $\left[\Omega = 2\pi \dfrac{f}{fs}\right]$: **digital frequency** in radian.

For a sequence to repeat, N sampling intervals T_s must fit exactly into M period T of the analog waveform being sampled.

For some pair of integers N and M,

$$NT_s = MT \tag{10}$$

or

$$\frac{N}{M} = \frac{T}{Ts} = \frac{\frac{1}{f}}{\frac{1}{fs}} = \frac{fs}{f} = \frac{2\pi}{2\pi \frac{f}{fs}} = \frac{2\pi}{\Omega} \tag{11}$$

- $f = \frac{1}{T}$: frequency of analog signal
- $fs = \frac{1}{Ts}$: sampling frequency

$$\Omega = 2\pi \frac{f}{fs}$$

$$\Rightarrow \frac{fs}{f} = \frac{2\pi}{\Omega}$$

To find N and M, fraction $\frac{2\pi}{\Omega}$ must be reduced to its lowest term.

Ex 6] Is this function periodic?

$$x[n] = \cos(2n)$$

$$\frac{2\pi}{\Omega} = \frac{2\pi}{2} = \pi$$

This is **irrational #**. And it is not expressed as ratio of integer so it is not periodic.

Ex 7] Is this function periodic?

$$x[n] = \cos\left[n\frac{4\pi}{5}\right]$$

$$\Rightarrow \frac{2\pi}{\Omega} = \frac{2\pi}{\frac{4\pi}{5}} = \frac{5}{2}$$

The sequence repeats every 5 samples and these 5 samples are collected over 2 complete cycles of analog signal.

Most general from of digital sinusoid

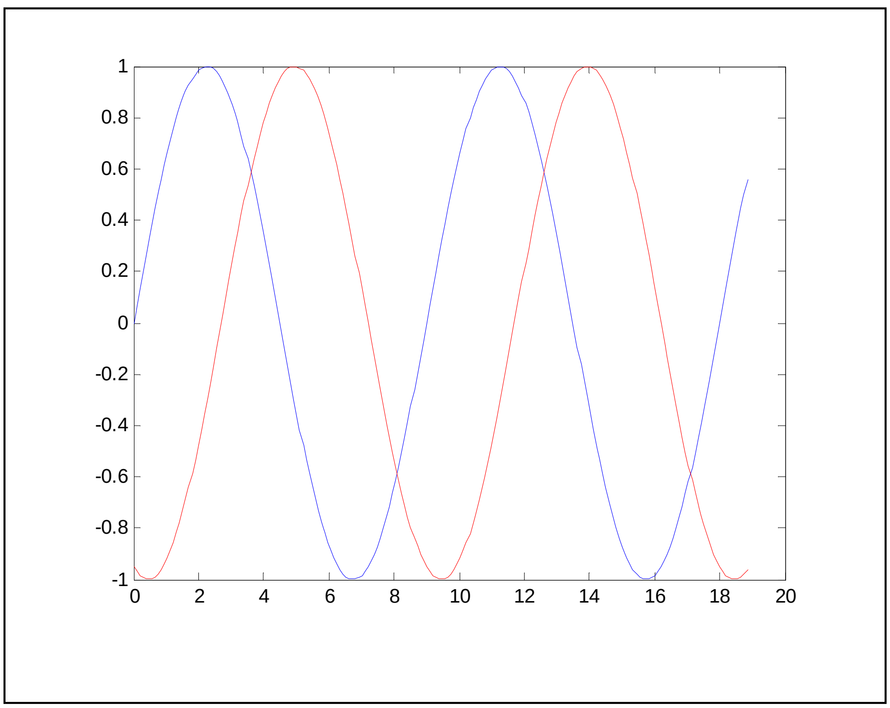
$$\begin{aligned} &x[n] = A\sin(n\Omega - \Theta) \\ &or \\ &x[n] = A\cos(n\Omega - \Theta) \end{aligned} \tag{12}$$

Ex 8]

$$x_1[n] = \sin\left(n\frac{2\pi}{9}\right)$$

$$x_2[n] = \sin\left(n\frac{2\pi}{9} - \frac{3\pi}{5}\right) \quad \text{: red color}$$

$\dfrac{2\pi}{\Omega} = \dfrac{2\pi}{\frac{2\pi}{9}} = 9$ Both sequences repeat every 9 samples.



<u>Composite function</u>

$$x[n] = u[n]u[3-n]$$

$$x[n] = e^{-2n}u[n] \tag{13}$$

$$x[n-2] = e^{-2[n-2]}u[n-2]$$

CH 4 Difference equations and filtering

Most common categories of filter

- Low pass
- Band pass
- Band stop
- High pass

The **cutoff frequency** of the filter

$$Gain = 20\log_{10}\left(\frac{1}{\sqrt{2}}\right)$$

$$=10\log_{10}\left(\frac{1}{2}\right) \qquad (14)$$

$$=-3\text{dB}$$

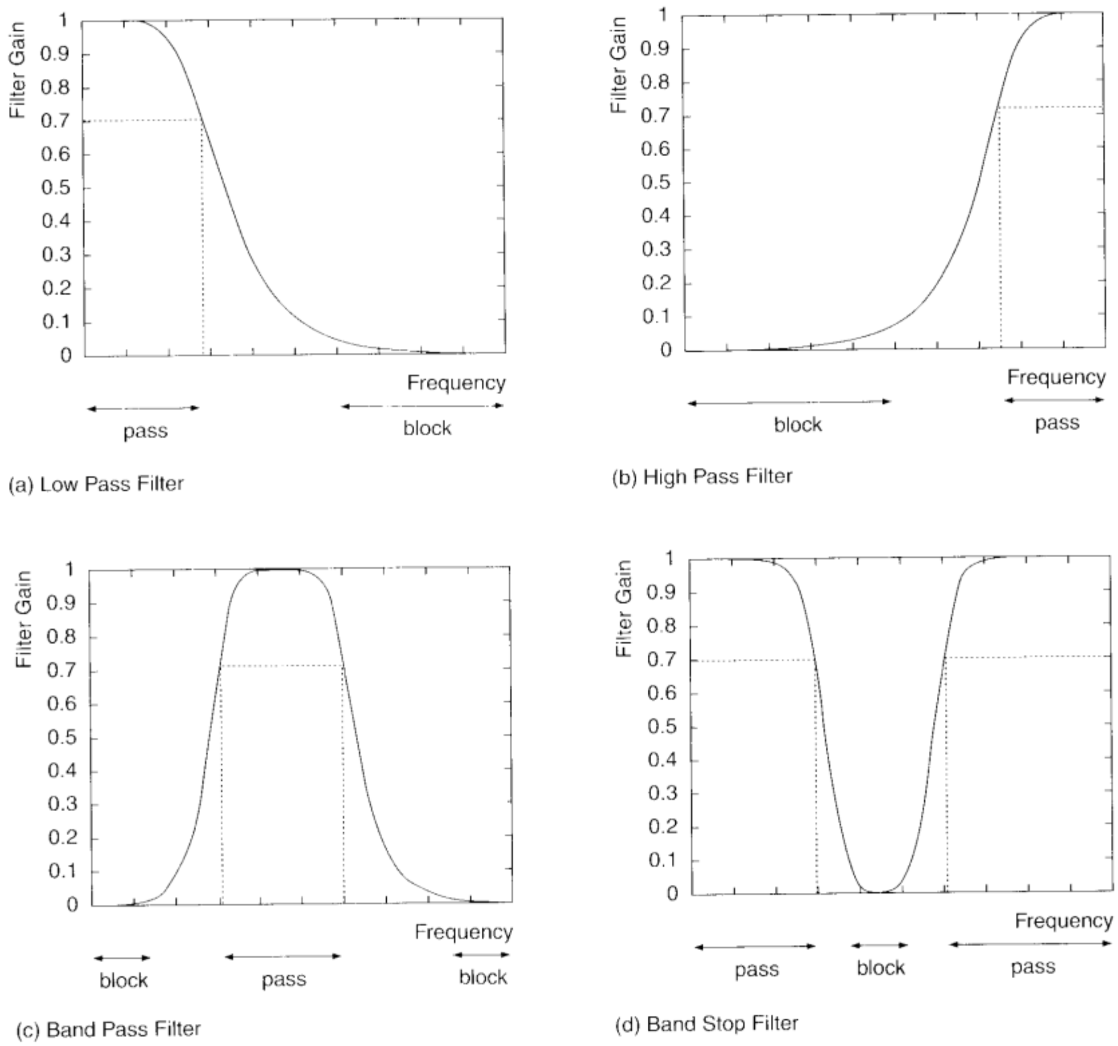


Figure 10: Low pass, High pass, Band pass, and Band stop filters

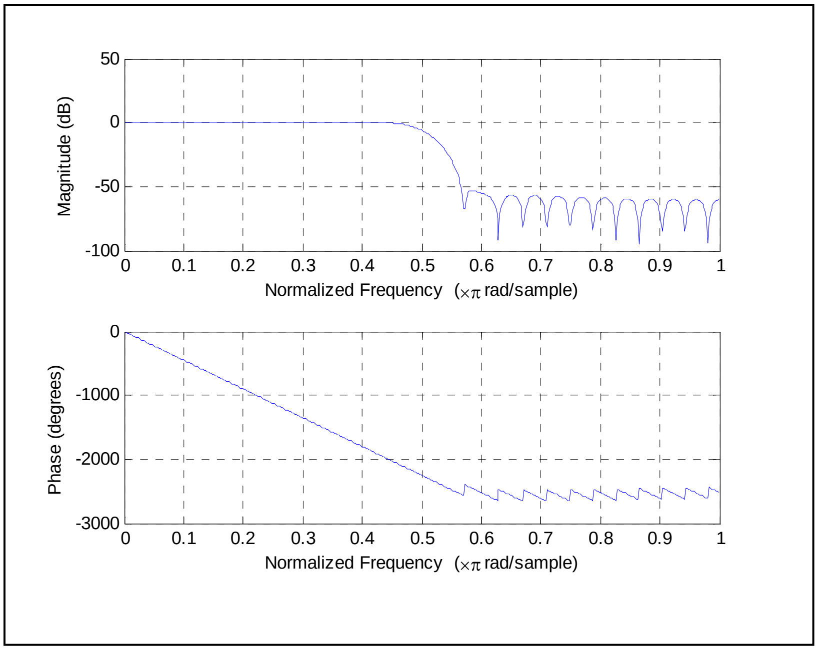
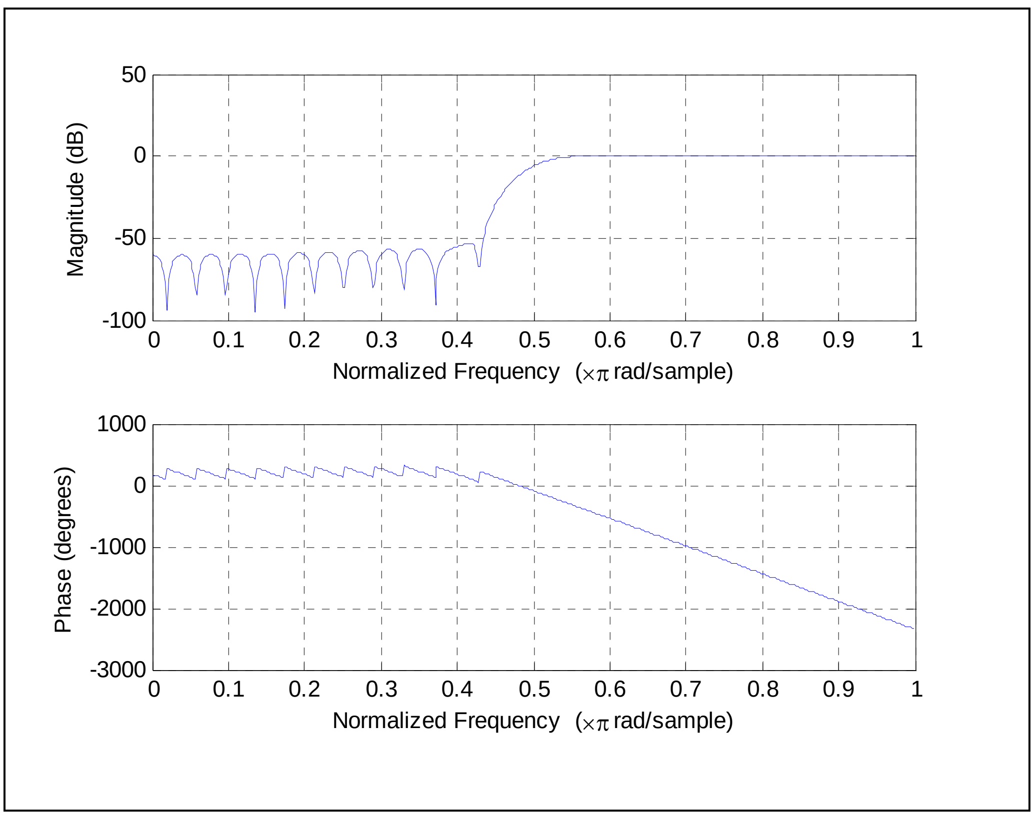
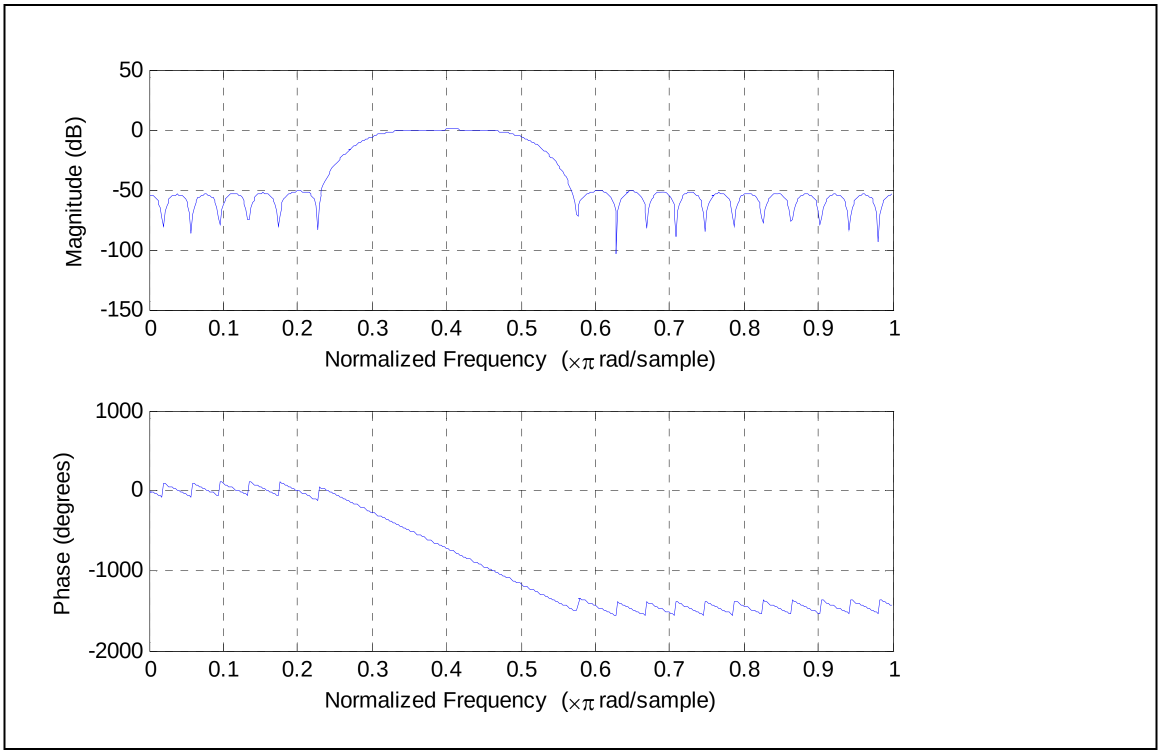
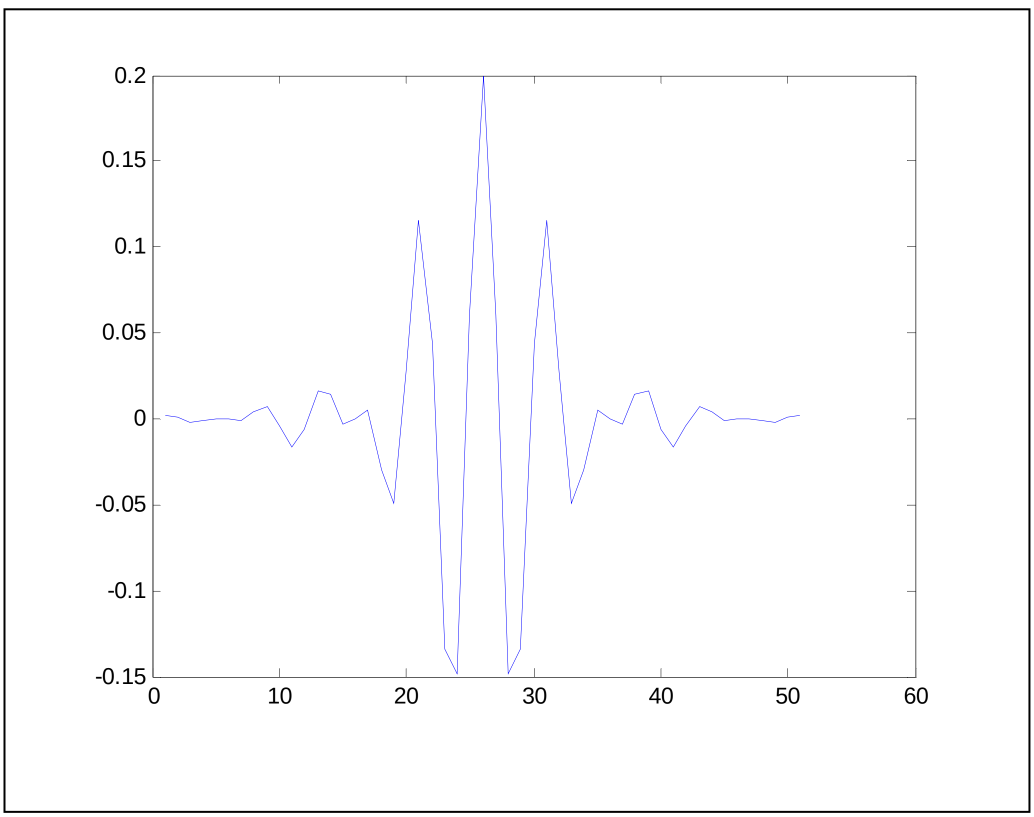


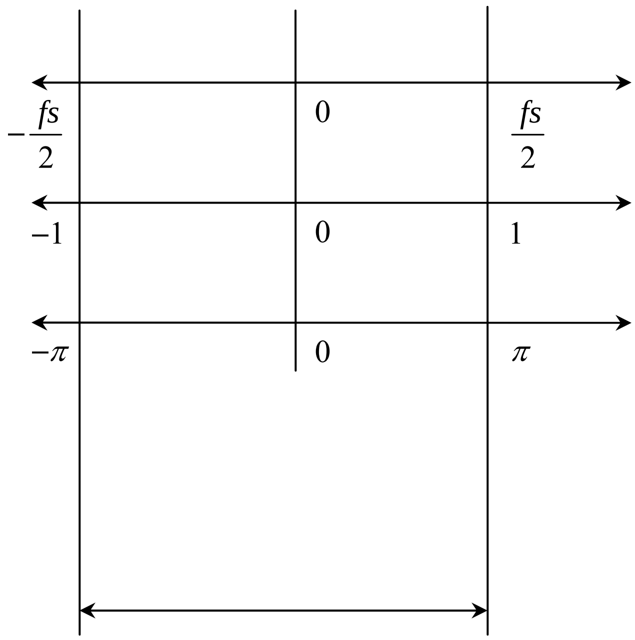
Figure 11: Low pass filter representation with “freqz” in Matlab.




50
0
-50
-100
-150
Magnitude (dB)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Normalized Frequency (×π rad/sample)
1000
0
-1000
-2000
Phase (degrees)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Normalized Frequency (×π rad/sample)


0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0 10 20 30 40 50 60

Frequency units in DTFT



```
clc; clear all;

f1 = 1;
f2 = 2;

fs = 5;
n = 0:fs*10;

x1 = cos(2*pi*f1*n/fs);
x2 = 2*cos(2*pi*f2*n/fs);
y = x1 + x2;

h = fir1(50,0.5,'low');

y_f = conv(h,y)
stem(n,y)

figure;
plot(y_f)

OM = 0:0.001: pi;
len_y = length(h);
n = 0:len_y-1;

Y = exp(-j*OM'*n)*h';
plot(OM,abs(Y))
```