Chapter 3.
Modulation and Detection

2009-2 RF IC 공학

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Contents

Overview

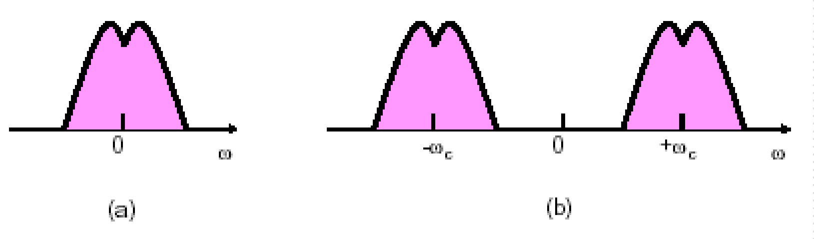
- **Introduction**
  - Why do we need modulation
  - Modulation process
  - Modems characterization
- **Analog modulation**
  - AM, PM, FM
- **Digital modulation & demodulation**
  - BASK, BPSK, BFSK
  - Digital I/Q formats
  - Bit rates and Symbol rates
  - Filtering
- **Different ways to see a modulated signal**
  - time, frequency, power, trajectory, constellation and eye diagrams

General Consideration

- **The transmitted waveform in RF communications**
  - a high frequency carrier modulated by the original baseband signal
- **Modulation is needed because:**
  - **Shielding:** In wired systems, coaxial cables exhibit superior shielding at higher frequency
  - **Antenna size:** In wireless systems, the antenna size should be a significant fraction of the wavelength to achieve a reasonable gain
  - **Regulation:** In most cases, the communication must occur in a certain part of the spectrum because of existent regulations (FCC, etc.)
  - **Simple detection:** In some applications, modulation allows simpler detection at the receive end in the presence of nonideal communication channel (noise, interference, fading, etc.)

Baseband and Passband

- RF communications two types of signals are defined:
  - a "baseband" signal (modulating wave)
  - a "passband" signal (modulated wave)



- Modulation converts a baseband signal to a passband.
- counterpart by variating some characteristics of a carrier in accordance with the baseband signal
- Demodulation (detection)
  - The reverse operation of restoring the original
- Baseband signal with minimum noise, distortion, etc.

Passband Signal

- **Modulation Process**
- **A modulated signal can always be expressed as**

$$x(t) = A_c(t)\cos[\omega_c t + \phi(t)]$$

- $\omega_c t + \phi(t)$ - the total phase (angle)
- $\phi(t)$ - the excess phase
- $\omega_c t$ - the phase of the unmodulated carrier

Instantaneous frequency is defined as the time derivative of the phase

- $\omega_c + d\phi(t)/dt$ - the total frequency (angular frequency)
- $d\phi(t)/dt$ - the excess frequency (or the frequency deviation)
- ω_c - the frequency of the unmodulated carrier

We assume that $\phi(t)|_{t=0} = 0$

Important Aspects of Modem

- **Signal Quality at Detector**
  - Q: What is the quality of the detector output signal in presence of attenuation and interference in the channel as well as noise at the input of the detector?
  - Analog modulation - Signal to Noise Ratio (SNR)
  - Digital modulation - Bit Error Rate (BER)
  - Goal: minimize BER ($<10^{-3}$ for voice, $<10^{-6}$ data)

- **Spectral Efficiency**
  - Q: What bandwidth requires the modulated RF signal?
  - Very important role in choosing a modem in bandlimited communications
  - Goal: minimize the needed bandwidth

Important Aspects of Modem –cont.

- **Power Efficiency (of the power amplifier that can be used in the transmitter)**
  - Q: It is needed a linear or a nonlinear power amplifier for the modulation?
  - Linear power amplifier
    - maximal (theoretical) power efficiency limited to 50% (Class A, A/B)
    - good for all modulation types (also variable amplitude signal)
  - Nonlinear power amplifier
    - maximal (theoretical) power efficiency is 100% (Class C, D, E, F, …)
    - good only for constant envelope modulated signal (distorts the variable amplitude signal)
    - Goal: nonlinear amplifier (higher efficiency)!

Additive White Gaussian Noise Channel (AWGN)

- **Additive : $r(t) = s(t) + n(t)$**
- **White : $S_n(f) = No / 2$**
- **Gaussian : Gaussian distribution**

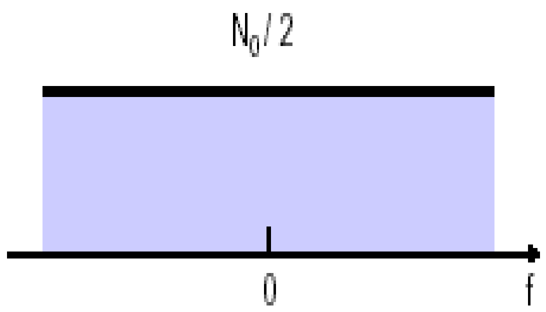


Figure 3.3 White spectrum assumed for additive Gaussian noise.

Amplitude Modulation

- The modulation signal $mx_{BB}(t)$ contains the information and varies the amplitude of the carrier.
- Local oscillator: $\cos\omega_c t$

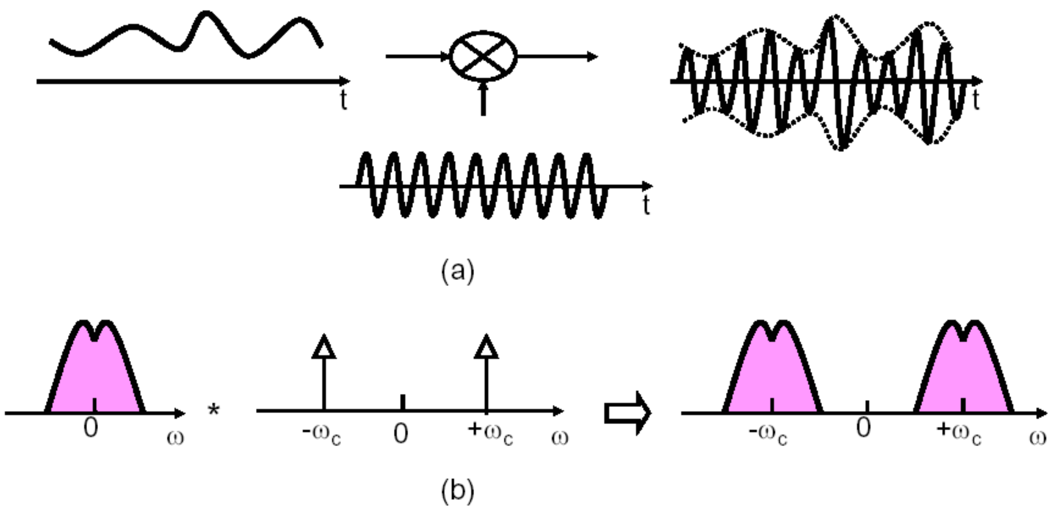


Figure 3.4 Amplitude modulation in (a) time domain, (b) frequency domain.

$$x_{AM}(t) = A_c[1 + mx_{BB}(t)]\cos(\omega_c t)$$

Demodulation of Amplitude Modulation

- **Local oscillator frequency**
  - The same frequency as the carrier frequency of $X_{am}(t)$
- **If $1+mx_{BB}(t)$ remains positive for all t**
  - The envelop does not cross zero.
  - Envelope detection is possible.
- **SNR at the output of the detector**

$$SNR_{out} = \frac{A_c^2 m^2 \overline{x_{BB}^2(t)}}{2N_0 B}, \qquad (3.2)$$

  - Normally SNR> 25 dB
- **Limited usage**
  - Signal susceptible to noise
  - Highly linear power amplifier required

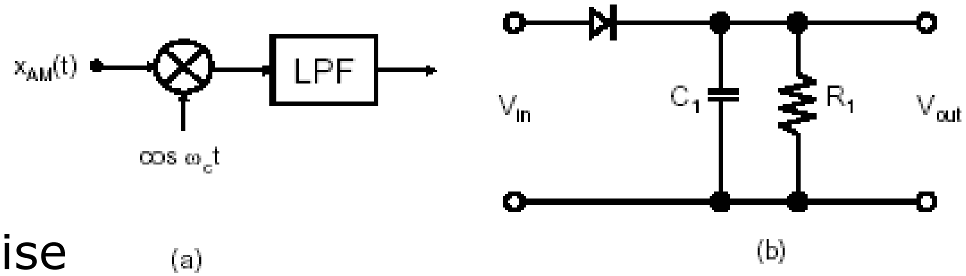
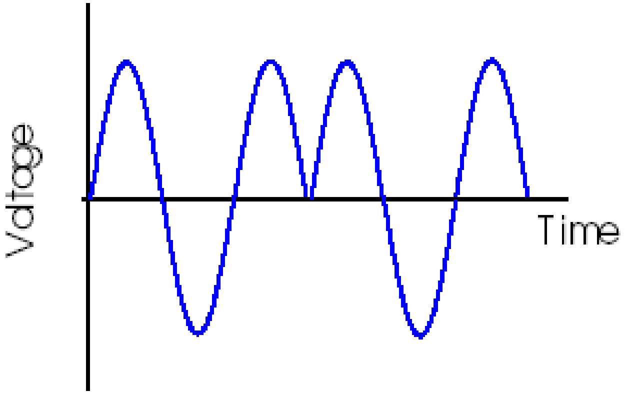


Figure 3.5 AM detectors.

Phase and Frequency Modulation



$$x_{PM}(t) = A_c \cos[\omega_c t + m x_{BB}(t)]$$

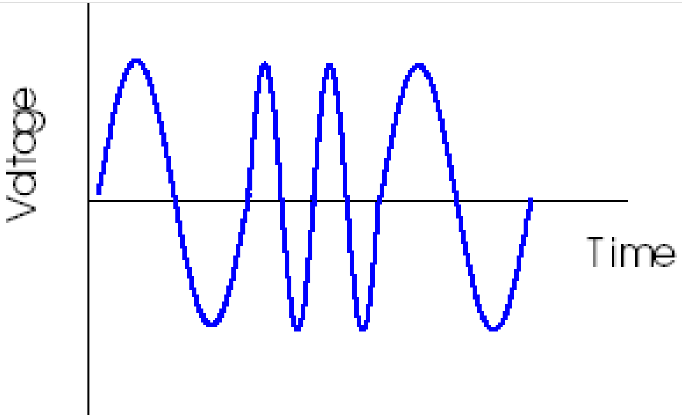
m - the phase sensitivity of the modulator (rad/V)

- When the excess phase is linearly proportional to the baseband signal **$\phi(t) = m x_{BB}(t)$** we say **the carrier is phase modulated**
- If $x_{BB}(t) = kt$

$$x_{PM}(t) = A_c \cos(\omega_c t + mkt) = A_c \cos(\omega_c + mk)t \qquad \text{FM}$$

- A ramp baseband waveform shifts the carrier frequency by a constant value in PM
- The frequency deviation is equal to $m dx_{BB}(t)/dt$

Frequency Modulation



$$x_{FM}(t) = A_c \cos\left[\omega_c t + m\int_0^t x_{BB}(\tau)d\tau\right]$$

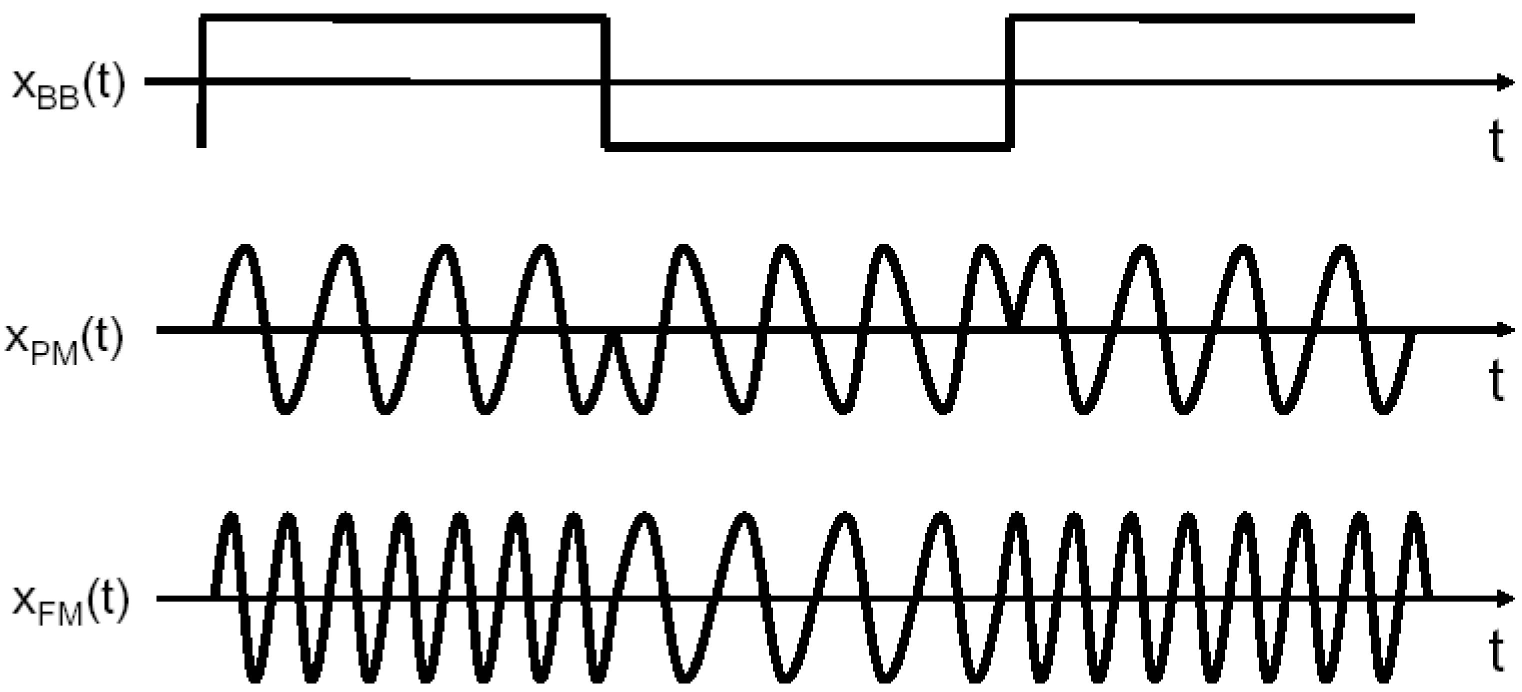
m - the frequency sensitivity of the modulator (Hz/V)

- When the excess frequency is linearly proportional to the baseband signal **$d\phi(t)/dt = mx_{BB}(t)$** we say **the carrier is frequency modulated**
- If $x_{BB}(t) = k$

$$x_{FM}(t) = A_c \cos(\omega_c t + mkt) = A_c \cos(\omega_c + mk)t$$

- A dc baseband waveform shifts the carrier frequency by a constant value in FM
- The frequency deviation is equal to $mx_{BB}(t)$

FM and PM for Digital Signal



FM Generation and Detection

- The resonance frequency of an LC oscillator is varied in proportion to the baseband signal.

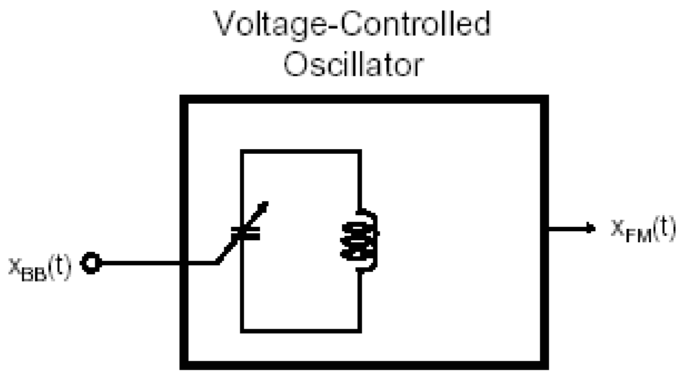


Figure 3.7 Simple frequency modulator.

- Converting FM to AM and then using envelope detector

$$V_{in}(t) = A_c \cos\left[\omega_c t + mA_m \int_0^t x_{BB}(\tau)d\tau\right]$$

$$V_{out}(t) = A_c R_1 C_1 [\omega_c + m x_{BB}(t)] \sin\left[\omega_c t + mA_m \int_0^t x_{BB}(\tau)d\tau\right]$$

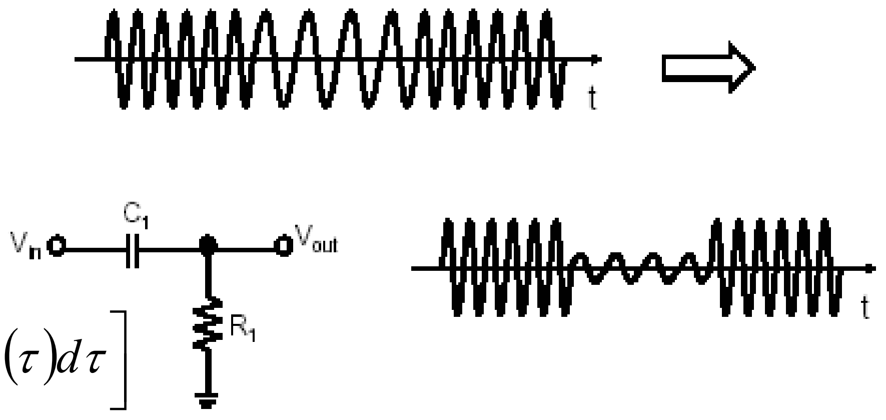


Figure 3.8 Simple frequency demodulator.

Narrowband FM

- **Bandwidth estimation**

$$m\int_0^t x_{BB}(\tau)d\tau \quad \textbf{<<1 then} \quad x_{FM,NB}(t) \approx A_c \cos\omega_c t - A_m A_c \frac{m}{\omega_m}\sin\omega_c t \sin\omega_m t$$

- **A special case $x_{BB}(t)=A_m\cos\omega_m t$**

$$x_{FM}(t) = A_c \cos\left[\omega_c t + mA_m \int_0^t \cos\omega_m \tau d\tau_s\right] = A_c \cos\left[\omega_c t + \frac{mA_m}{\omega_m}\sin\omega_m t\right]$$

  - Frequency deviation=dϕ/dt=m$A_m$cosω_mt
  - ΔF_{peak}=mA_m
  - Instantaneous frequency=ω_c+ ΔF_{peak}cosω_mt
  - ΔF_{peak} is the maximum departure of frequency from ω_c

Narrowband FM – cont.

FM Signal frequency

$$x_{FM,NB}(t) \approx A_c \cos\omega_c t - A_m A_c \frac{m}{\omega_m} \sin\omega_c t \sin\omega_m t$$

$$= A_c \cos\omega_c t - A_m A_c \frac{m}{2\omega_m} \cos(\omega_c - \omega_m)t$$

$$+ A_m A_c \frac{m}{2\omega_m} \cos(\omega_c + \omega_m)t$$

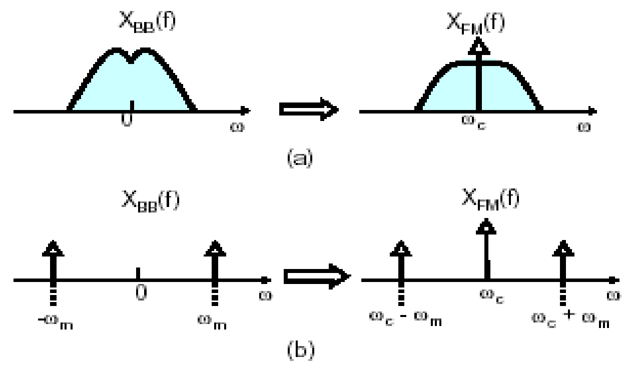


Figure 3.9 Narrowband FM with (a) random and (b) sinusoidal modulation.

- Spectrum at $\pm\omega_c$ and $\pm(\omega_c \pm \omega_m)$
- The equation is valid when $mA_m/\omega_m << 1$
- If ω_m increases, the sideband decreases
- Maximum frequency deviation mA_m
  - no effect on the sideband which is $\pm\omega_m$ apart

Wideband FM

- **Consider an FM signal**

$$x_{BB}(t) = A_m \cos \omega_m t$$

$$x_{FM}(t) = A_c \cos\left[\omega_c t - mA_m \int_0^t \cos \omega_m \tau d\tau\right] = A_c \cos\left[\omega_c t - \frac{mA_m}{\omega_m} \sin \omega_m t\right]$$

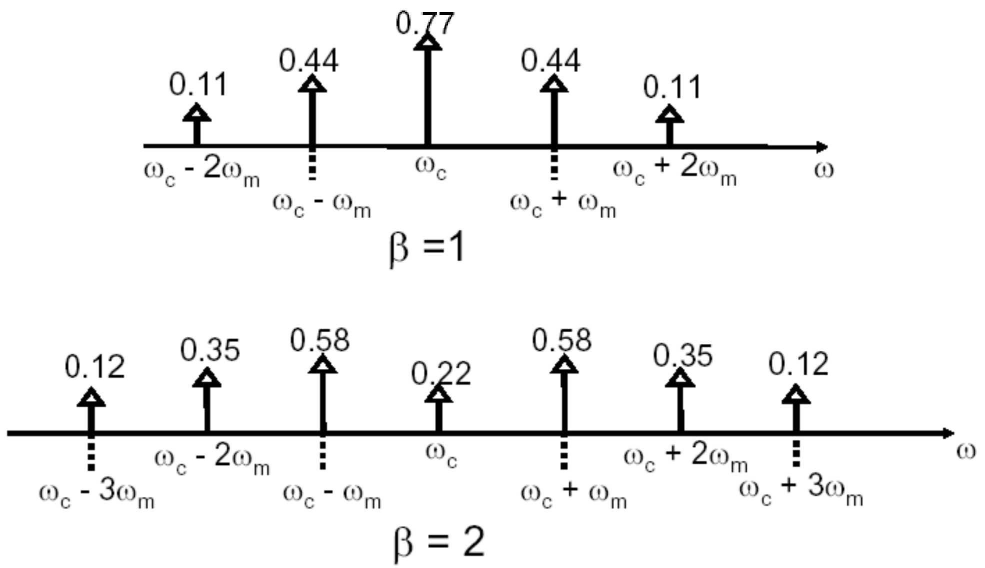
$$x_{FM}(t) = A_c \sum_{n=-\infty}^{+\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

  - $J_n(t)$:nth order Bessel function of the first kind
  - $\beta = mA_m/\omega_m$
- **If β<<1 rad (narrow FM case)**
  - $J_o(t) \approx 1$
  - $J_{\pm 1}(t) \approx \pm\beta/2$
  - $J_{\pm n}(t) \approx 0 \quad |n|>1$

$$x_{FM,NB}(t) \approx A_c \cos \omega_c t - \frac{A_m \beta}{2} \cos(\omega_c - \omega_m)t + \frac{A_m \beta}{2} \cos(\omega_c + \omega_m)t$$

Wideband FM – Cont.

- **As β increases (β>1), $\omega_c+n\omega_m$ appears**
  - $J_n(t)\neq 0$
- **f_c:Carrier and f_c+nf_m**
- **Carson's rule**
  - BBF is the bandwidth containing 98% of the signal power
  - $B_{Bf}\approx 2(\beta+1)B_{BB}$



Preemphasis and deemphasis in FM

- **As ω_m increases, signal decreases**

$$x_{FM,NB}(t) = A_c \cos\omega_c t - A_m A_c \frac{m}{2\omega_m}\cos(\omega_c - \omega_m)t + A_m A_c \frac{m}{2\omega_m}\cos(\omega_c + \omega_m)t$$

- **Increase gain at high frequency**
  - Amplifies the high frequency noise at the same time
    - High f noise > midband noise
  - Decrease overall SNR
  - Not a good solution
- **Shape baseband signal**
  - Preemphasis and deemphasis
  - Without pre- and de-emphasis

$$\frac{SNR_{out}}{SNR_{in}} = 6\beta^2(\beta+1)\frac{\overline{x_{BB}^2(t)}}{V_P^2} \quad (3.8)$$

  - With pre- and de-emphasis

$$\frac{SNR_{out}}{SNR_{in}} = 2\beta^2(\beta+1)\left(\frac{B}{f1}\right)^2\frac{\overline{x_{BB}^2(t)}}{V_P^2} \quad (3.9)$$

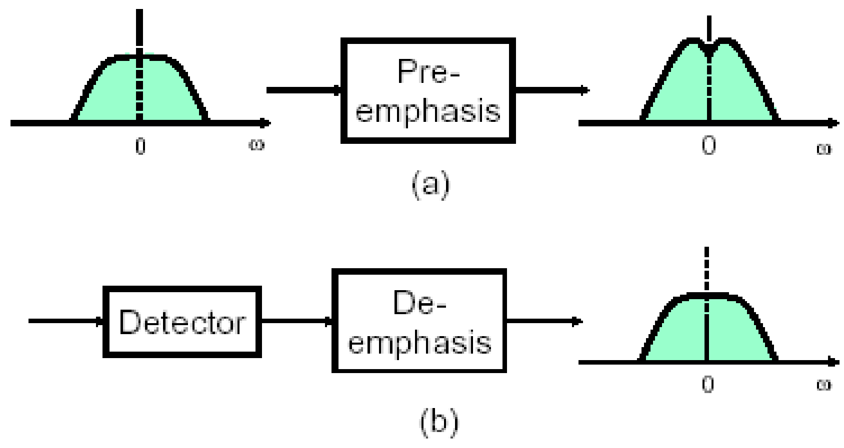


Figure 3.11 Preemphasis and deemphasis in FM.

10-15 dB increase in SNR

✓10~15dB 향상(f1=-3dB)

Contents

Digital Modulation

- **Communication system**
  - Limited bandwidth
  - Limited permissible power in the desired frequency range
  - Inherent noise level
- **Digital modulation scheme**
  - More information capacity
  - Compatibility with digital data services
  - Higher data security
  - Better signal quality
  - Quicker system availability
- **Analog vs Digital**
  - AM → ASK
  - FM → FSK
  - PM → PSK

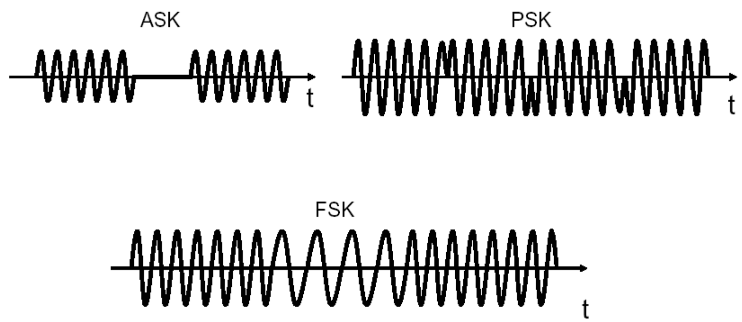
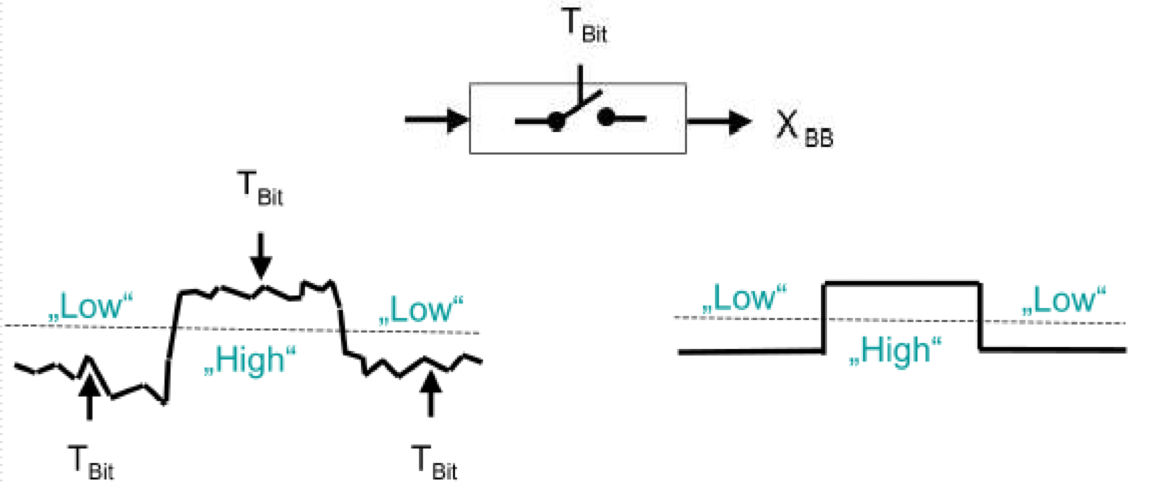


Figure 3.12 (a) Amplitude, (b) phase, and (c) frequency shift keying.

Basic Concepts

- **Binary**
  - b_n is the bit value in the time interval [nTb, (n+1)Tb]
  - One of two value (0,1) or (-1,1): binary

$$x_{BB}(t) = \sum_n b_n p(t - nT_b)$$



- **Multilevel Digital (M-ary)**
  - Symbol rate reduction

M-ary Signaling

- 2 levels → 4 levels

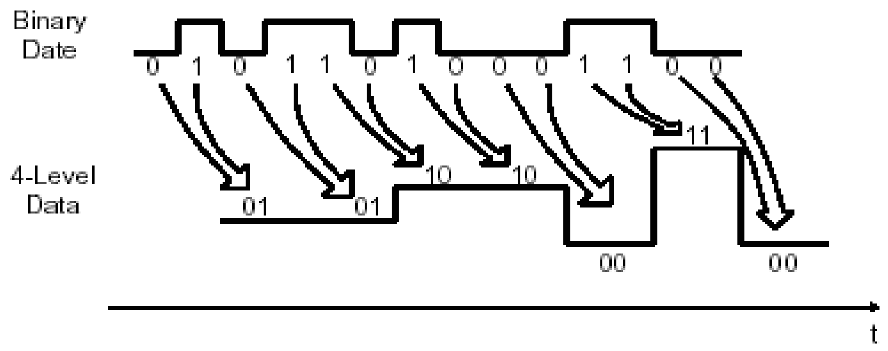


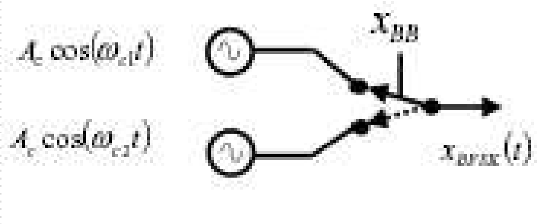
Figure 3.13 Four-level representation of a binary data stream.

  - The symbol rate is half the bit rate.
  - If more bits can be sent with each symbol, then the same amount of data can be sent in a narrower spectrum.
  - Modulation formats that are more complex and use a higher number of states can send the same information over a narrower band of the RF spectrum.
  - The needed signal bandwidth for the communications channel depends on the symbol rate, not on the bit rate.

Basis Functions

- Binary FSK signal

$$x_{FSK}(t) = A_c \cos\omega_1 t \quad \text{if } b_n = 0$$
$$= A_c \cos\omega_2 t \quad \text{if } b_n = 1$$



- Inner product: linear combination

$$x_{FSK}(t) = \alpha_1\phi_1(t) + \alpha_2\phi_2(t) = [\alpha_1 \ \ \alpha_2]\cdot[\phi_1 \ \ \phi_2]$$

  - $[\alpha_1 \ \ \alpha_2]=[0 \ A_c]$ or $[A_c \ \ 0]$

$$\phi_1(t) = \cos\omega_1 t, \ \ \phi_2(t) = \cos\omega_2 t$$

Signal Constellation

- **From $x(t) = [\alpha_1 \ \ \alpha_2 \ldots][\phi_1 \ \phi_2 \ldots]$**
  - plot the vector $[\alpha_1 \ \ \alpha_2 \ldots]$

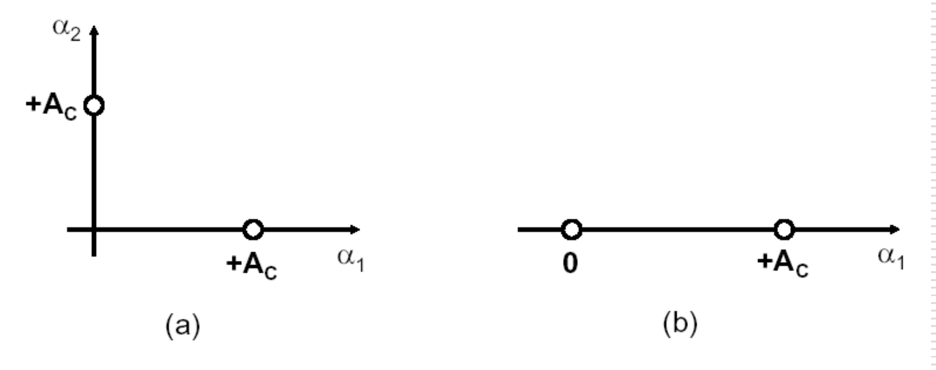
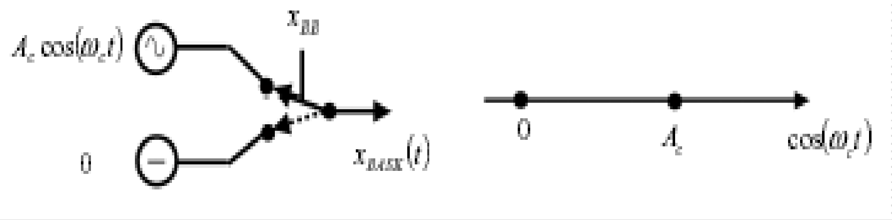


Figure 3.14 Signal constellations of (a) FSK, (b) ASK.

- **Binary ASK**

$$x_{ASK}(t) = A_c \cos\omega_C t \quad \text{if } b_n = 1$$
$$= 0 \quad \text{if } b_n = 0$$
$$x_{ASK}(t) = \alpha_1 \phi_1(t)$$



Noise on Constellation

- Effect of noise on ASK

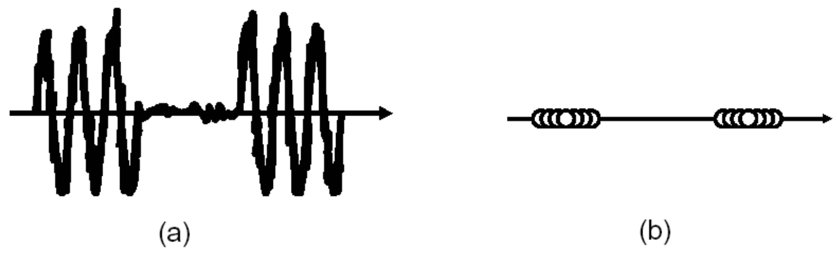


Figure 3.15 Effect of noise on (a) an ASK signal and (b) its constellation.

- Effect of noise on FSK

- Increase in noise power
  - Signal crosses decision boundary
  - Increase in BER

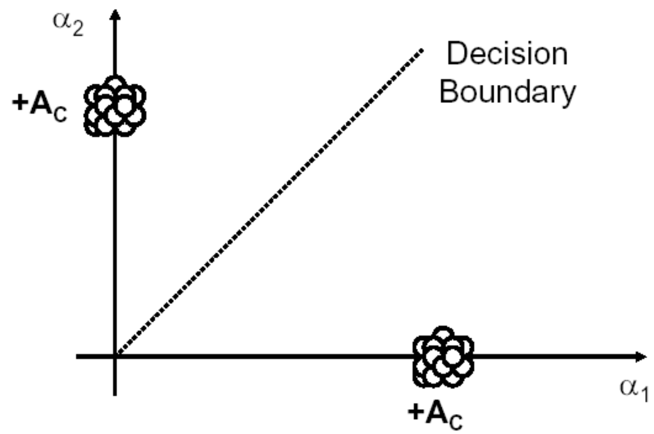


Figure 3.16 Effect of noise on FSK constellation.

Optimum Detection

Digital signal sampling

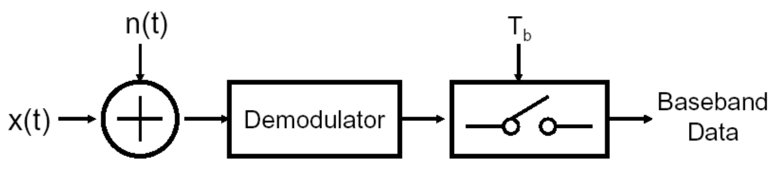


Figure 3.17 Signal detection by sampling.

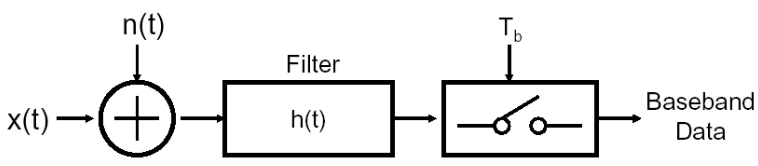
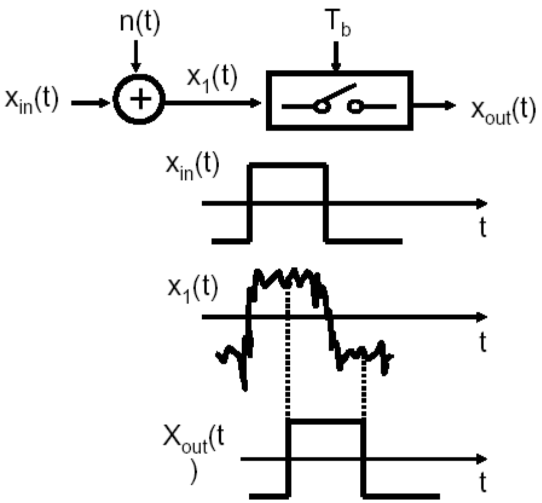
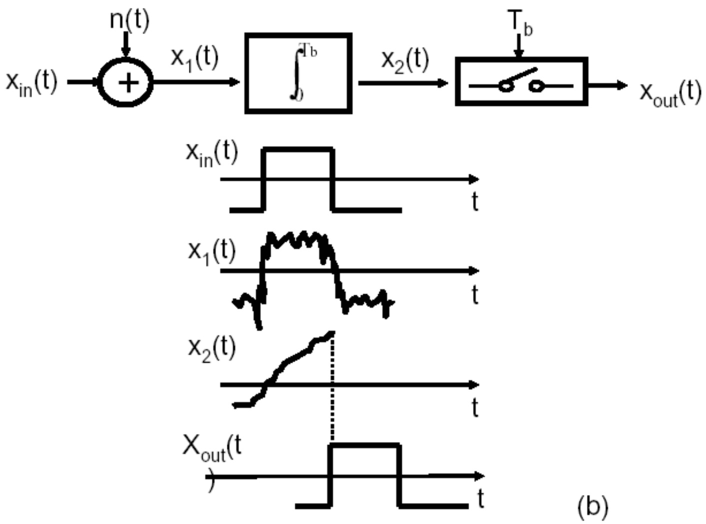


Figure 3.18 Use of filter in detector.

Sampling the peak value and integration over on bit



(a)



(b)

Matched Filter

- A matched filter maximizes SNR of a pulse p(t)

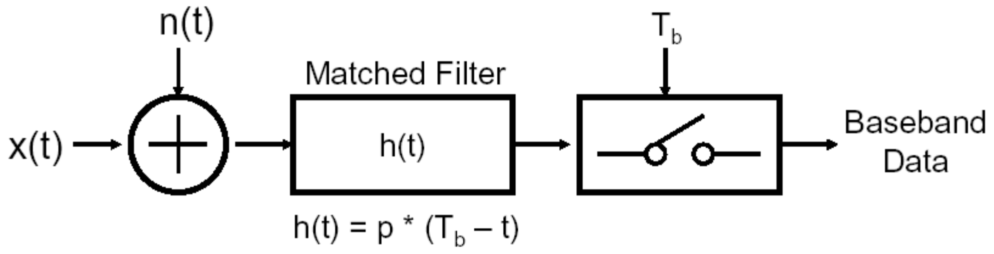


Figure 3.20 Detection using matched filtering.

- **h(t):** $h(t) = p^*(T_b - t)$
- **y(t) is maximum at t=T$_b$**

$$y(t) = p(t) * h(t) = \int_{-\infty}^{+\infty} p(t-\tau)h(\tau)d\tau$$

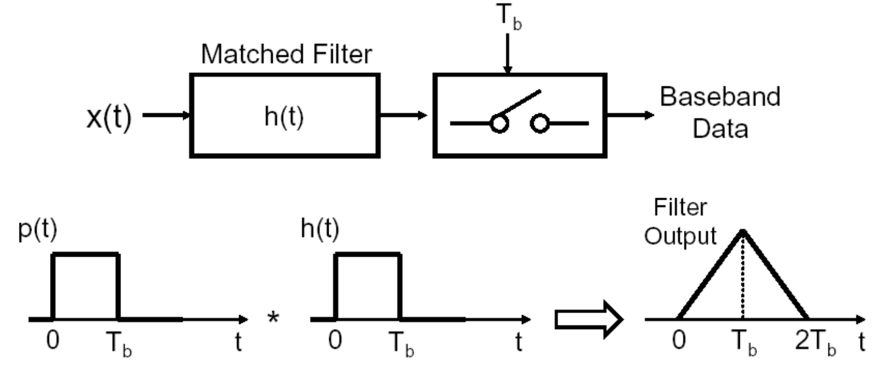


Figure 3.21 Optimum detection of a rectangular pulse.

- **SNR$_{max}$**

$$SNR_{\max} = \frac{2E_p}{N_o}$$

  - Noise spectral density $N_o/2$, $E_p = \int_{-\infty}^{+\infty} |p(t)|^2 dt$

Matched Filter with Modulated Signal

- **Modulated signal** $x(t) = p(t) + n(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$y(T_b) = \int_{-\infty}^{\infty} x(\tau) h(T_b-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) p(\tau) d\tau$$

- p(t) is non-zero in [0 T_b],
- p(t) can be any shape

$$y(T_b) = \int_{0}^{T_b} x(\tau) p(\tau) d\tau$$

- Correlation function

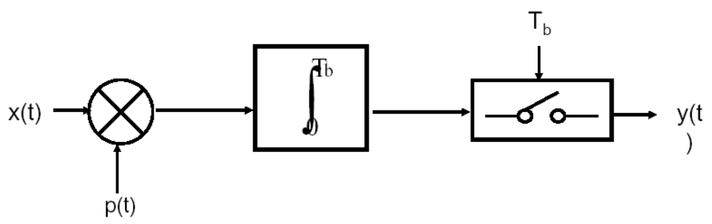


Figure 3.22 Optimum detection using a correlator.

- For p(t) is a rectangular pulse, Fig 3.22 → Fig 3.21
- Bit (or symbol) synchronization is important.

Two Dimensional Signal Space

- **FSK**

$$x_{FSK}(t) = \alpha_1\phi_1(t) + \alpha_2\phi_2(t) = [\alpha_1 \;\; \alpha_2]\cdot[\phi_1 \;\; \phi_2]$$

$$\phi_1(t) = \cos\omega_1 t, \;\; \phi_2(t) = \cos\omega_2 t$$

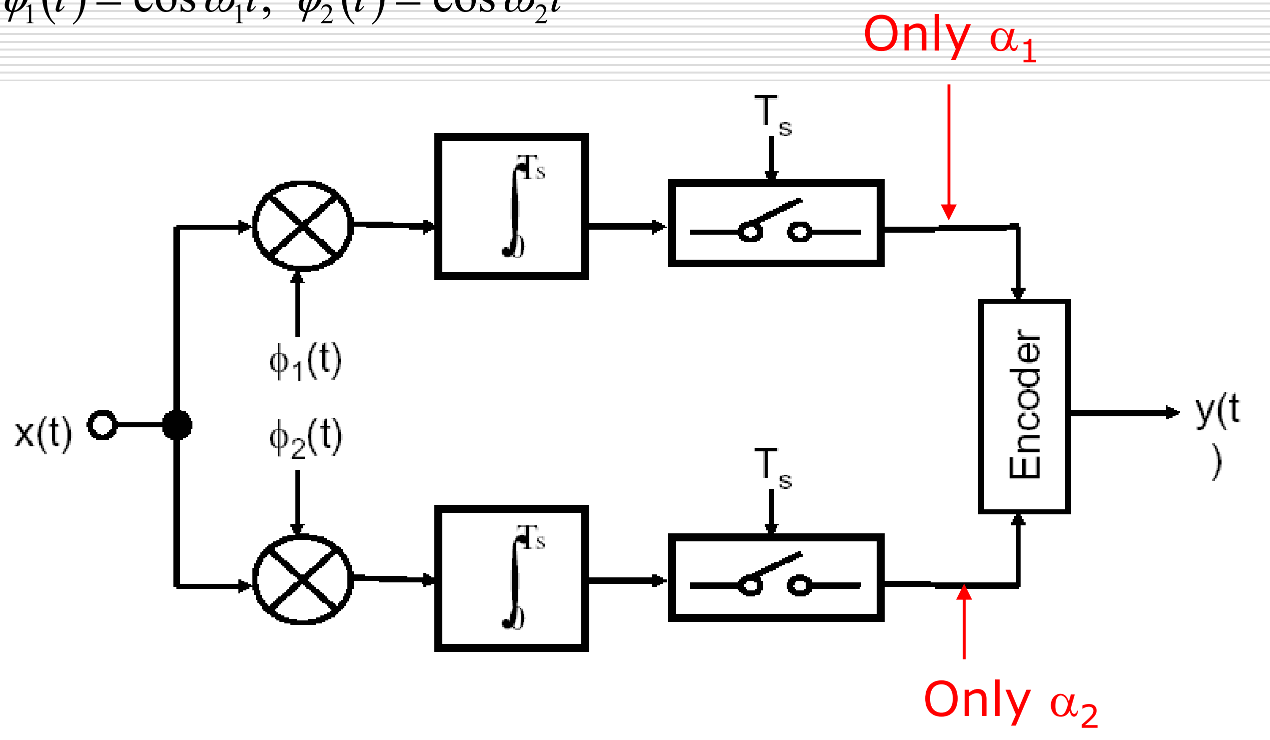


Figure 3.23 Correlation receiver for two-dimensional signal space.

Coherent Detection

Coherent Detection

- Phase synchronization between the carrier and the oscillator
- Lower bit error rate than do their noncoherent counterparts

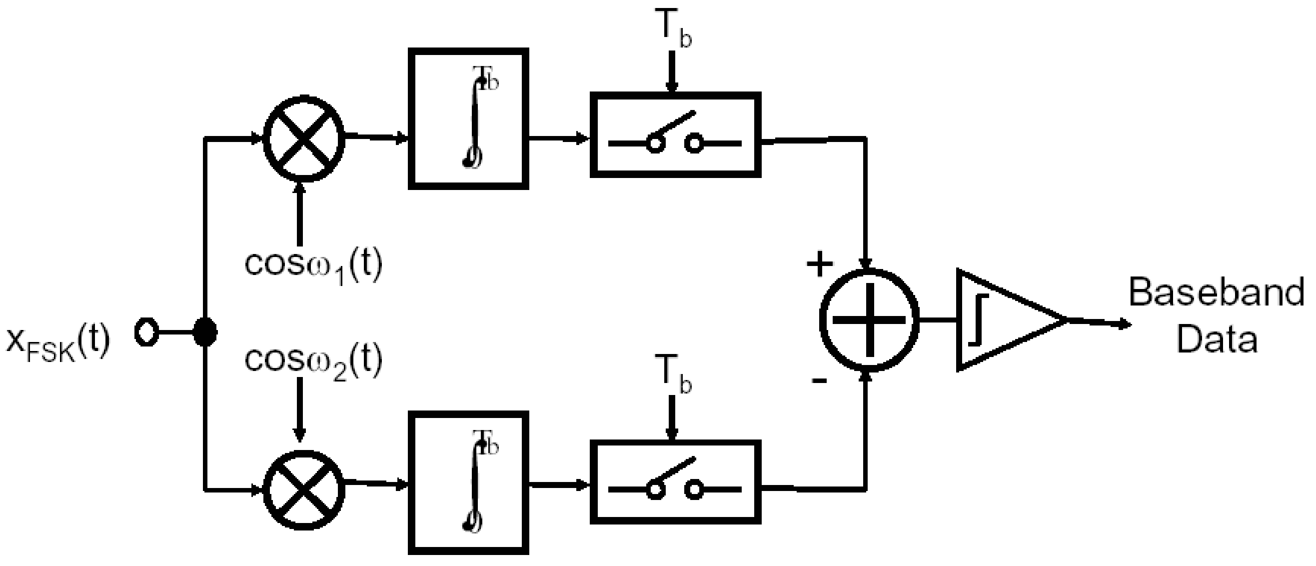


Figure 3.24 Coherent FSK detector.

Noncoherent Detection

- Noncoherent detections are more widely used in RF design
- Less complexity

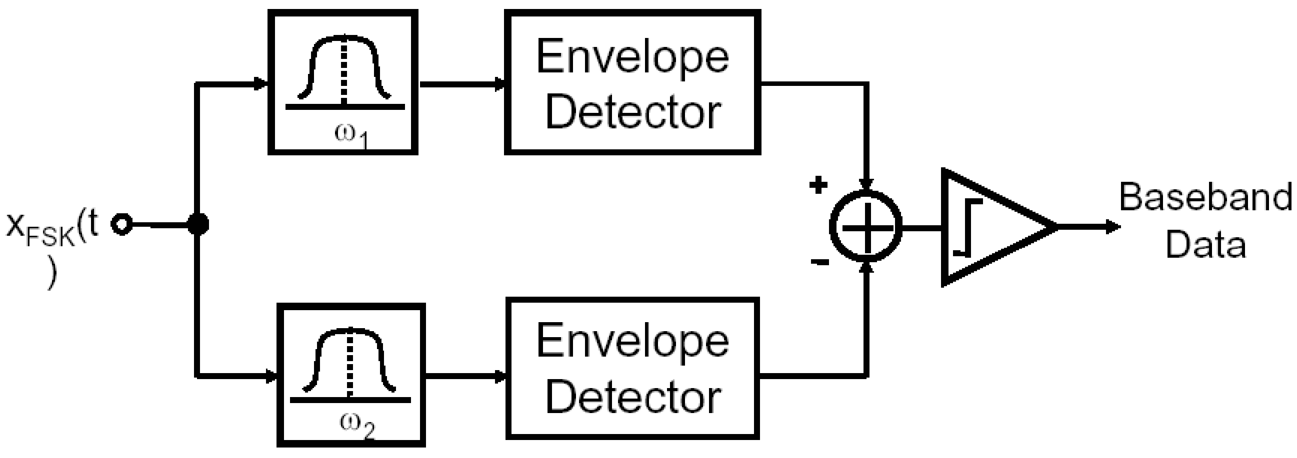
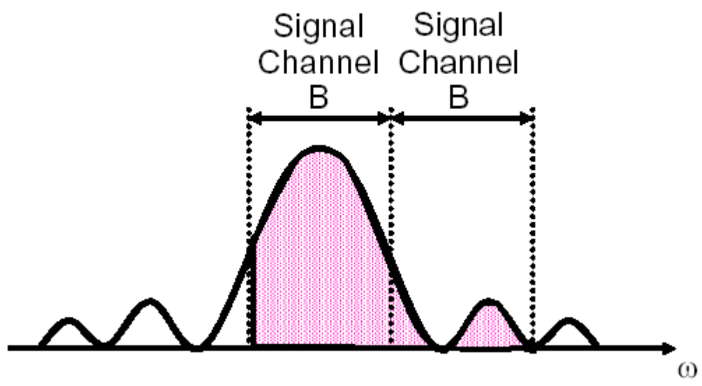


Figure 3.25 Noncoherent FSK detector.

Definition of Bandwidth

- **In analog FM**
  - 98% of the signal power lies in a bandwith 2(ß+1)B
- **In digital modulation**
  - "99% bandwidth"
- **Signal bandwidth by the power at the adjacent channel**
  - For a bandwidth 30kHz, the signal exhibits an "adjacent channel Power"(ACP) of –50dBc
  - The power in the adjacent 30kHz channel is 50 dB lower



Contents

Binary Modulation

- **Bianry ASK(BASK), binary PSK (BPSK), binary FSK (BFSK)**
  - BASK is seldom used.
- **$p(t) \rightarrow p_1(t), p_2(t)$**
  - Logical zero is not always "low"
  - $p_1(t)$, $p_2(t)$ are not necessarily orthogonal.
  - In BFSK, $p_1(t)=A_c\cos\omega_1 t$, $p_2(t)= A_c\cos\omega_2 t$

Coherent Binary Receiver

Coherent Binary Receiver

- Sampled value without noise: A_1 and A_2 are +1 or -1
- Sampled value with noise
  - $A_1+n(T_b)$, $A_2+n(T_b)$
- Threshold level $(A_1+A_2)/2$

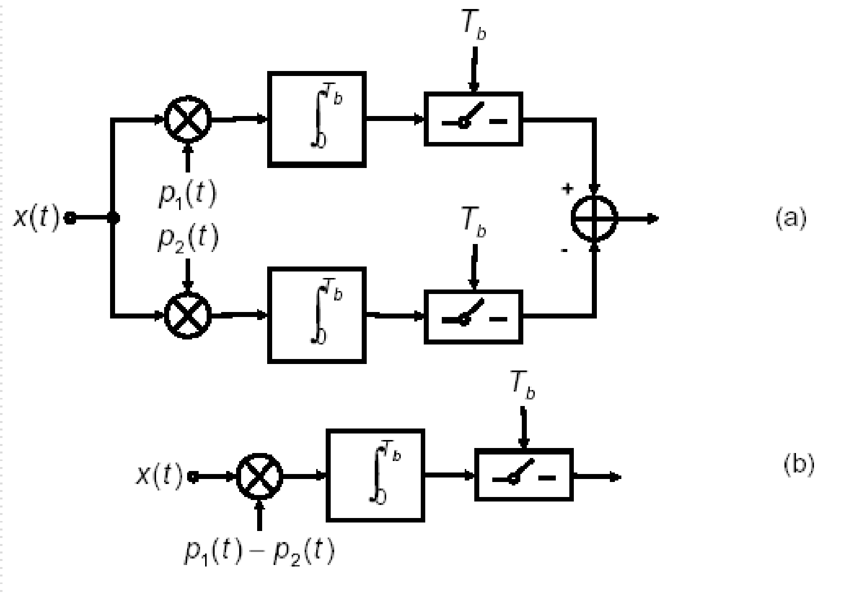


Fig. 3.27 (a) Coherent binary receiver, (b) simplified version of (a)

$$SNR_{max} = \frac{2E_d}{N_0} \qquad (3.30)$$

$$E_d = \int_{-\infty}^{+\infty} [p_1(t) - p_2(t)]^2 dt \qquad (3.31)$$

E_d is maximum if $p_1(t)=-p_2(t)$

Probability Density Function

Probability of error event & Q function

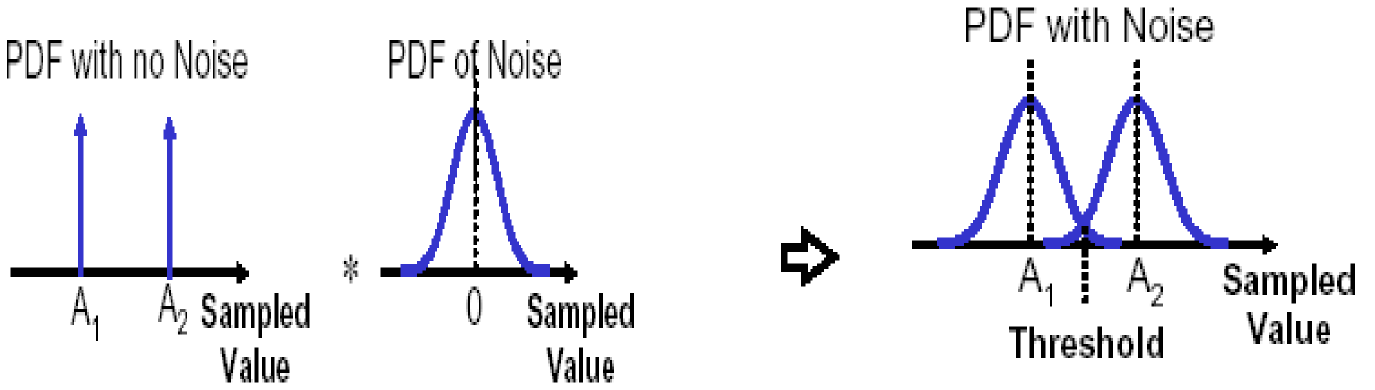


Fig. 3.28 Calculation of PDF for binary data with additive noise

$$P_{e1} = \frac{1}{2}\int_{(A_1+A_2)/2}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\frac{-(u-A_1)^2}{2\sigma_n^2} du \qquad (3.32)$$

$$P_e = \int_{(A_2-A_1)/2\sigma_n}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\frac{-v^2}{2} d \quad = Q\left(\frac{A_2 - A_1}{2\sigma_n}\right) \qquad (3.34)$$

$$P_e = Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right)$$

Where Q() is $$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\frac{-u^2}{2} du \qquad (3.35)$$

Q Function Calculation

- **Q function can be approximated as**

$$Q(x) \approx \frac{1}{x\sqrt{2\pi}} \int_x^\infty \exp\frac{-x^2}{2} \qquad (3.36)$$

(For x > 3)

  - By linear superposition the quantity $A_2 - A_1$ is the response of the receiver to an input $x(t) = p_1(t) - p_2(t)$
  - Since the filter is matched to $p_1(t) - p_2(t)$,if $x(t) = p_1(t) - p_2(t)$ is applied, the SNR at the sampling instant is equal to $(A_2 - A_1)^2 / \sigma_n{}^2$, which reaches a maximum given by (3.30)

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) \qquad (3.37)$$

    - Only for AWGN channel and Matched-filter(coherent) detection
    - Error rate depends only on signal energy and noise spectral density

Binary PSK

- Binary Phase Shift Keying

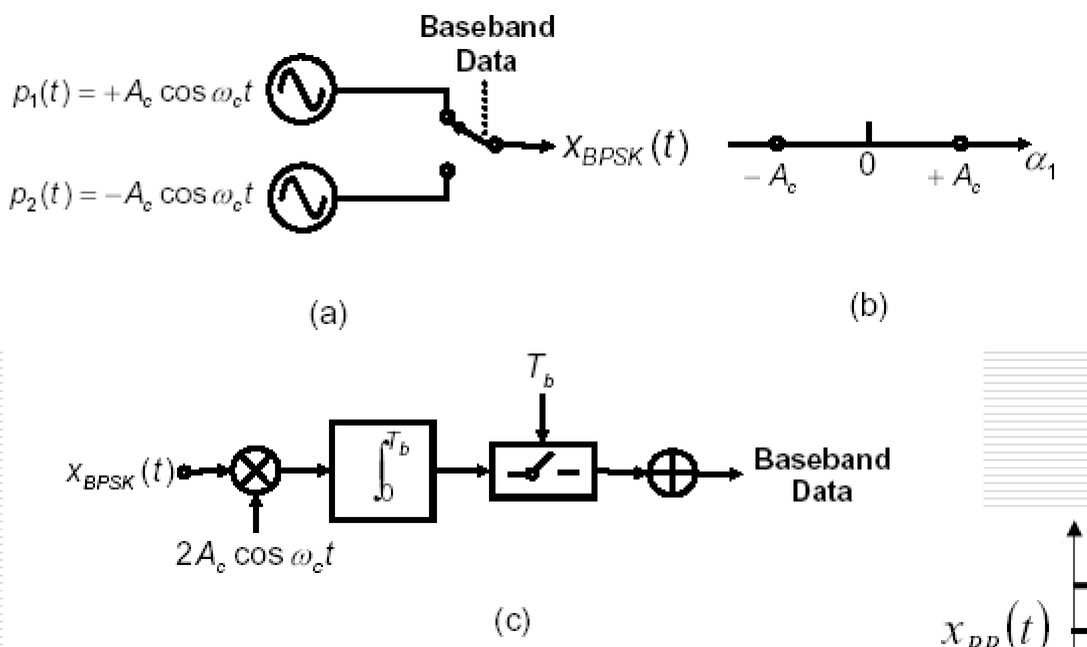
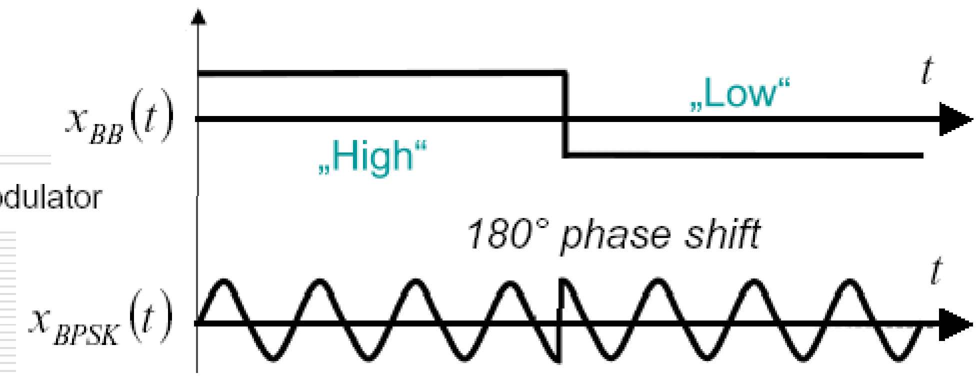


Fig. 3.29 BPSK (a) modulator, (b) constellation,and (c) demodulator



Binary Phase Shift Keying P_e

- **BPSK Equation**

$$x_{BPSK}(t) = A_c \cos[\omega_c t + \phi]$$ where ϕ = 0 or 180° (π rad)

- **$P_1(t)$ and $P_2(t)$**

$$p_1(t) = -p_2(t) \qquad p_1(t) - p_2(t) = 2p_1(t) = 2A_c \cos\omega_c t$$

- **Output of the integrator at $t=T_b$**

$$V_{int} = \int_0^{T_b} \pm 2A_c^2 \cos^2 \omega_c t dt \qquad (3.38) \xrightarrow{\text{for large } \omega_c} V_{int} \approx \pm A_c{}^2 T_b$$

- **E_d**

$$E_d = \int_{-\infty}^{+\infty} [p_1(t) - p_2(t)]^2 dt \qquad (3.39)$$

$$= \int_0^{T_b} (2A_c \cos\omega_c t)^2 dt \qquad (3.40)$$

$$= 2A_c^2 T_b \qquad (3.41)$$

$$P_e = Q\left(\sqrt{\frac{A_c^2 T_b}{N_0}}\right) \qquad (3.42)$$

Binary Phase Shift Keying

- **Average Energy/bit E_b**
  - To make fair comparisons with other modulation schemes

$$E_b = A_c^{\,2} T_b / 2$$

- **P_e with E_b**

$$P_{e,BPSK} = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

Binary Phase Shift Keying

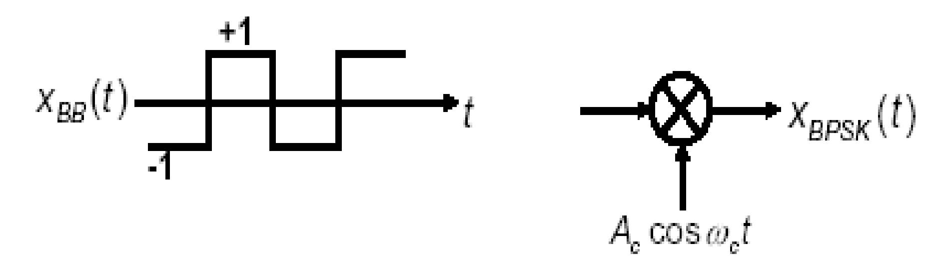
- Spectrum of a BPSK waveform

$$x_{BPSK} = x_{BB}(t)A_c \cos\omega_c t$$

- Spectrum of x_{BPSK} is simply translation of $x_{BB}(t)$ at $\pm\omega$
- For $P(\omega)$ Fourier transform of p(t)

$$S_x(\omega) = \frac{1}{T_b}\left|P(\omega)\right|^2$$

  - For a rectangular wave with duration T_b and unit amplitude



$$P(\omega) = \frac{2\sin\frac{\omega T_b}{2}}{\omega}$$

$$S_{BPSK} = \frac{A_c^2}{T_b}\frac{\sin^2\left[(\omega+\omega_c)T_b/2\right]}{(\omega+\omega_c)^2} + \frac{A_c^2}{T_b}\frac{\sin^2\left[(\omega-\omega_c)T_b/2\right]}{(\omega-\omega_c)^2}$$

Binary Frequency Shift Keying

- Baseband data selects one of two frequency ω_1 and ω_2 with equal amplitude

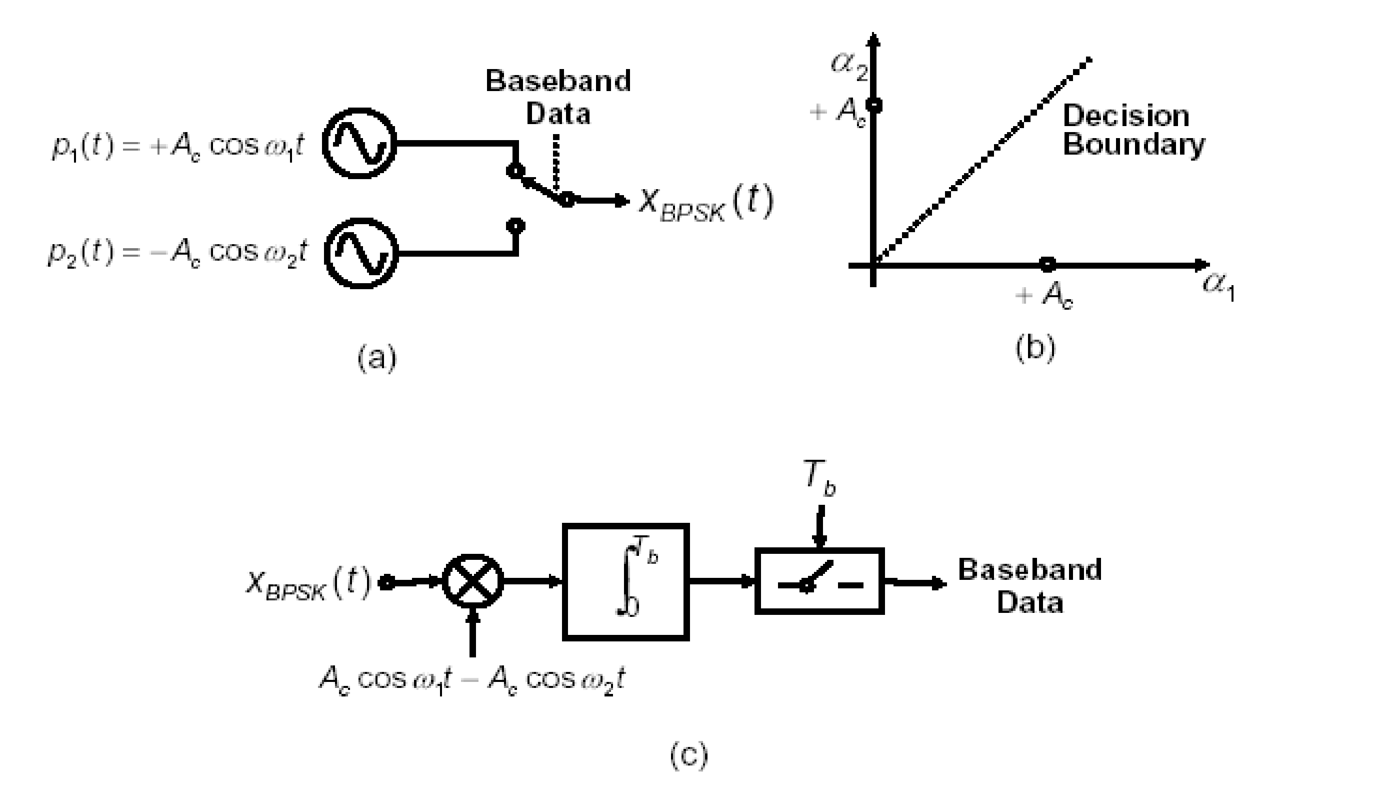


Fig. 3.31 (a) modulator, (b) constellation, (c) detection

BFSK Bases Function

- **Bases function**
  - cos ω_1t and cos ω_2t: orthogonal to each other

$$\int_0^{T_b} \cos\omega_1 t \cos\omega_2 t dt = 0$$

  - Since

$$(\omega_1 - \omega_2)T_b = n\pi$$

$$\int_0^{T_b} \cos\omega_1 t \cos\omega_2 t dt \approx \frac{\sin[(\omega_1 - \omega_2)T_b]}{(\omega_1 - \omega_2)} = 0$$

  - Minimum Spacing

$$(\omega_1 - \omega_2) = \pi / T_b \quad \text{or} \quad (f_1 - f_2) = 1/2T_b$$

- **x_{BFSK}**

$$x_{BFSK}(t) = \alpha_1 \cos\omega_1 t + \alpha_2 \cos\omega_2 t$$

$$[\alpha_1 \ \ \alpha_2] = [0 \ \ A_c] \ \text{ or } \ [A_c \ \ 0]$$

BFSK Error Rate

- **Error rate**
  - Average energy per bit $E_b = A_c^2 T_b / 2$

$$E_d = \int_0^{T_b} \left[p_1^2(t) + p_2^2(t) \right] dt = A_c^2 T_b$$

  - P_e

$$P_e = Q\left(\sqrt{\frac{A_c^2 T_b}{2N_o}} \right)$$

  - Rearrange with E_b

$$P_e = Q\left(\sqrt{\frac{E_b}{N_o}} \right)$$

- **Higher P_e than BPSK**
  - Minimum distance in constellation is greater in BPSK
- **Advantages of BFSK**
  - Simplicity in detection (noncoherent detection)
  - Power efficiency (discussed later)

Contents

Quadrature Modulation

- Pairs of two bits into I and Q stream
- cos ω_ct and sin ω_ct are orthogonal

$$x(t) = b_m A_c \cos\omega_c t - b_{m+1} A_c \sin\omega_c t$$

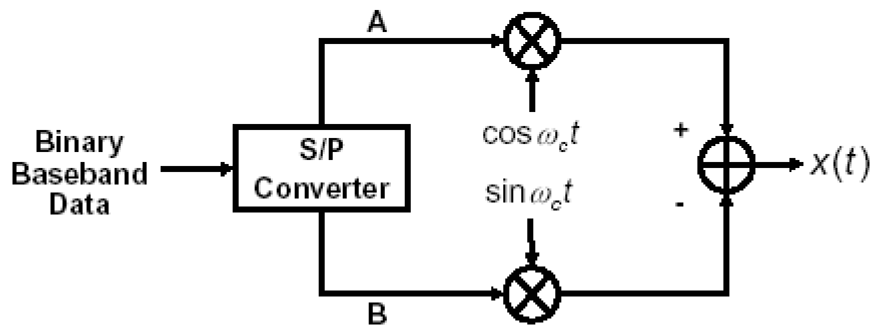
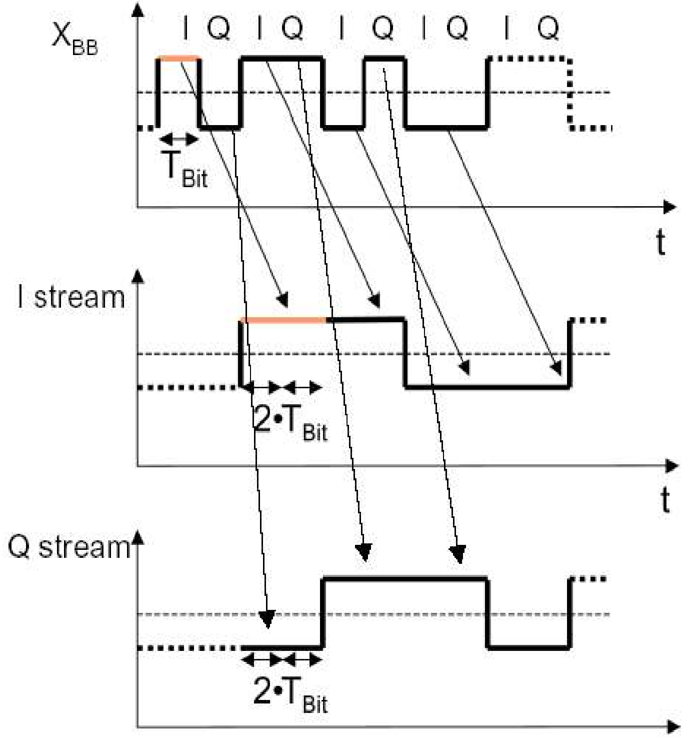


Fig. 3.32 Quadrature modulator.



The period of the I/Q signals is doubled $\Rightarrow$ the frequency is halved $\Rightarrow$ reduce the occupied bandwidth!

Quadrature Modulation Constellation

- b_m and b_{m+1} are rectangular pulse with a height +1,-1
- Modulated signal $x(t) = \alpha_1 \cos \omega_c t + \alpha_2 \sin \omega_c t$
- α_1 and α_2 can each take on value of $+A_c$ or $-A_c$.

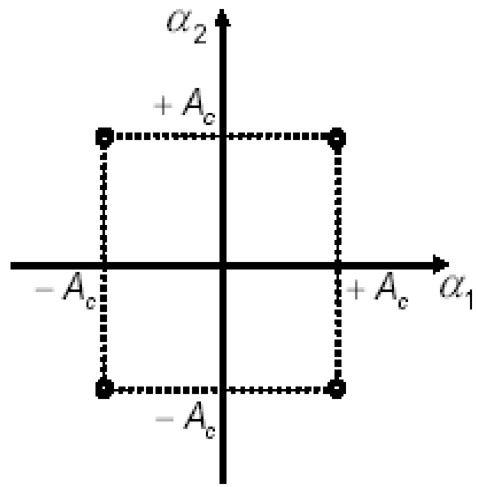


Fig. 3.33 Signal constellation for quadrature modulation

Quadrature Modulation Category

- **Quadrature Phase Shift Keying (QPSK)**
  - Offset QPSK
  - π/4-QPSK
- **Minimum shift keying (MSK)**
  - Gaussian MSK

Quadrature Phase Shift Keying

$$x(t) = b_m A_c \cos \omega_c t - b_{m+1} A_c \sin \omega_c t$$

$$\longrightarrow \quad x_{QPSK}(t) = \sqrt{2} A_c \cos(\omega_c t + k\pi / 4) \qquad k = 1,3,5,7$$

- **b_m, b_{m+1}=±1**

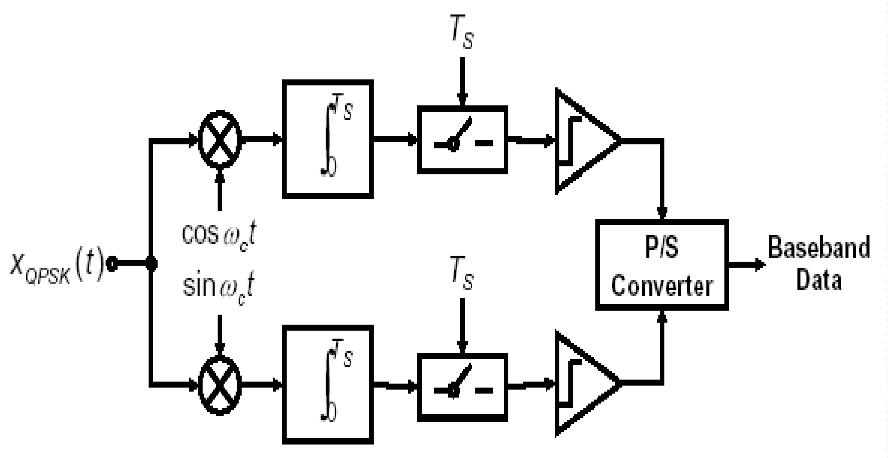


Fig. 3.34 Coherent QPSK detection

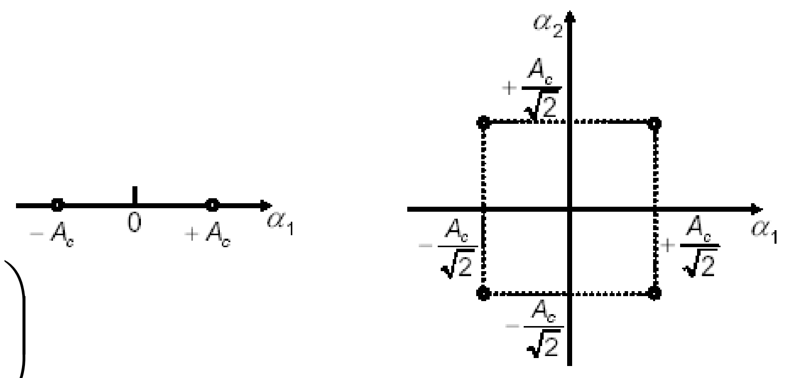
QPSK Probability of Errors

- **Comparison between BPSK and QPSK**
  - For a fair comparison, the same output power between BPSK and QPSK
  - Average power $A_c^2/2$
    - BPSK

      $$x_{BPSK}(t) = \pm A_c \cos \omega_c t$$

    - QPSK

      $$x_{QPSK}(t) = A_c \cos\left(\omega_c t + \frac{k\pi}{4}\right)$$



$$= \pm(A_c/\sqrt{2})\cos(\omega_c t) \pm (A_c/\sqrt{2})\sin(\omega_c t)$$

    - QPSK constellation is closer
  - Energy in a symbol
    - QPSK has longer integration time $2T_b$
    - BPSK $A_c^2T_b/2$, QPSK $A_c^2/2\ (2T_b)=A_c^2T_b$
  - BER of BPSK and QPSK is similar.

Drawback of QPSK

- An Important drawback of QPSK
  - Large phase change in a transition between two diagonal points [-1 -1]→ [1 1]

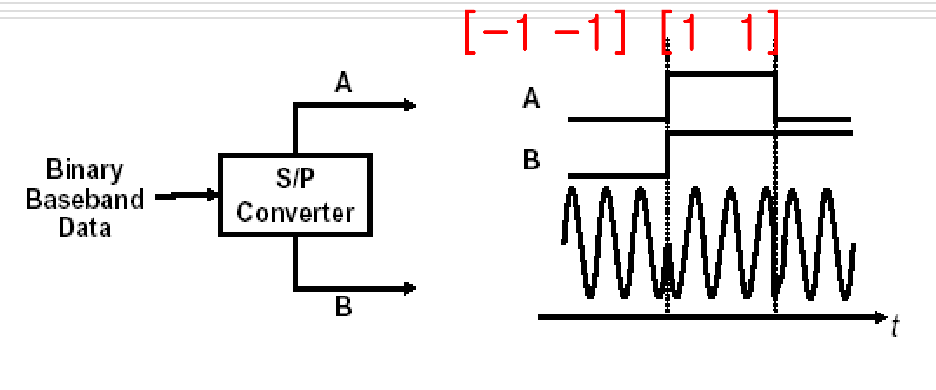
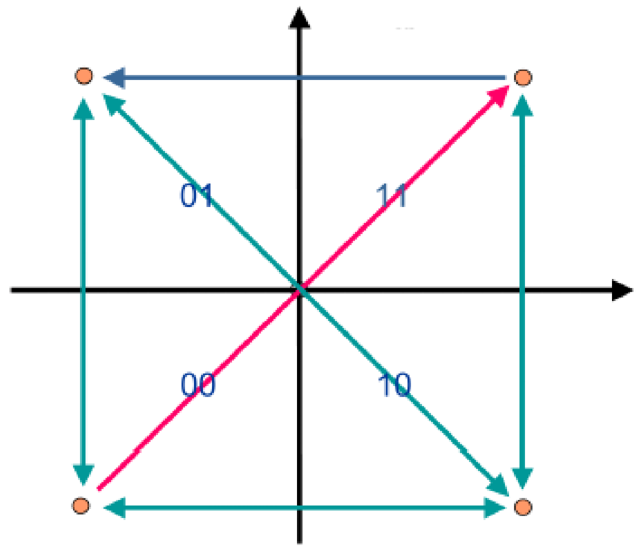


Fig. 3.36 Phase transitions in a QPSK waveform



Offset QPSK

- **To avoid a large phase change**
  - The data streams are offset in time by half the symbol period after S/P conversion
  - No simultaneous transitions in waveforms at node A, B.
  - Thus phase step is only ±90 degrees.

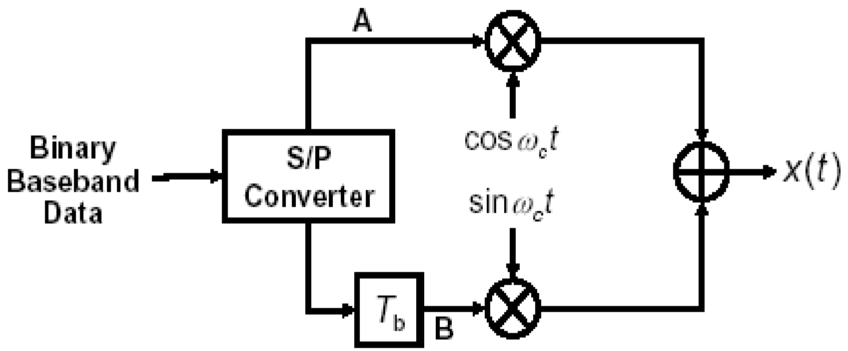


Fig. 3.37 Offset QPSK modulator

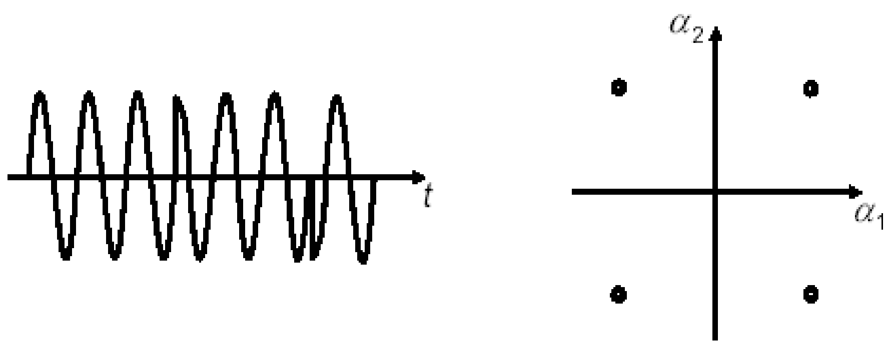


Fig. 3.38 Phase transitions in OQPSK

- **Drawback of OQPSK**
  - Differential encoding (important for noncoherent detection) is not possible.

π/4-QPSK

- Two QPSKs, one rotated π/4 with respect to the other

$$x_1(t) = A_c \cos\left(\omega_c t + \frac{k\pi}{4}\right) \quad k \text{ odd}$$

$$x_2(t) = A_c \cos\left(\omega_c t + \frac{k\pi}{4}\right) \quad k \text{ even}$$

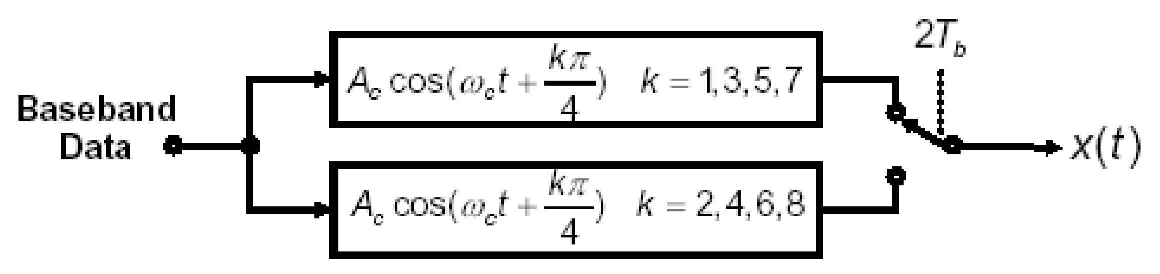


Fig. 3.39 Conceptual generation of π/4-QPSK signals

Implementation of π/4-QPSK

$$x_1(t) = A_c \cos\left(\omega_c t + \frac{k\pi}{4}\right) \quad k \text{ odd}$$

$$x_2(t) = A_c \cos\left(\omega_c t + \frac{k\pi}{4}\right) \quad k \text{ even}$$

$$x_1(t) = \alpha_1 \cos\omega_c t + \alpha_2 \sin\omega_c t$$

$$x_2(t) = \beta_1 \cos\omega_c t + \beta_2 \sin\omega_c t$$

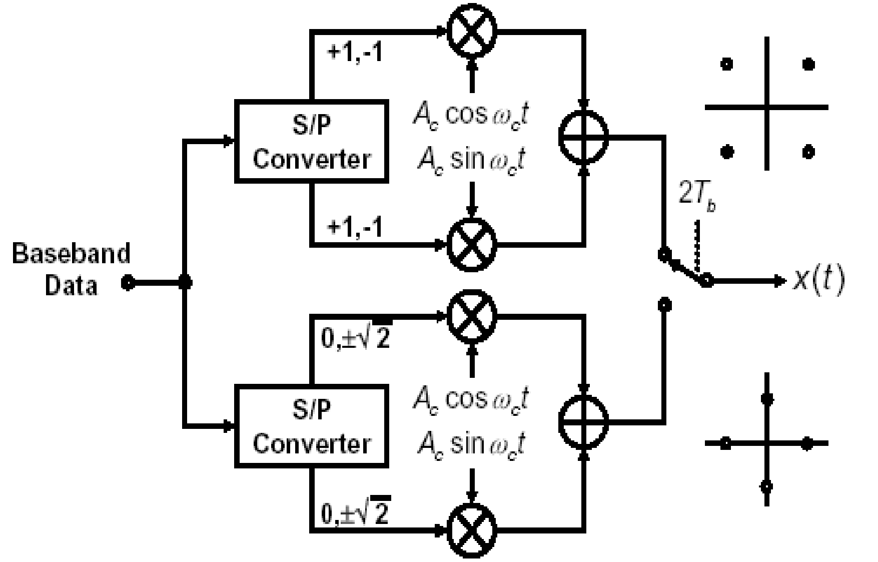


Fig. 3.40 Generation of π/4-QPSK signals

$$[\alpha_1, \alpha_2] = [\pm A_c, \pm A_c]$$

$$[\beta_1, \beta_2] = [0 \ \ \pm\sqrt{2}A_c] \text{ or } [\pm\sqrt{2}A_c \ \ 0]$$

- Alternating between two schemes

Implementation of π/4-QPSK

- π/4- QPSK
- **No two consecutive points from the constellation**
  - Always changing positions
  - Maximum phase step 135° (45° less than QPSK)
- **BER of π/4-QPSK is identical to that of QPSK**
- **Raised-cosine signal to minimize ISI**
  - Raised-cosine signal rather than rectangular pulses
  - Effective in band-limiting

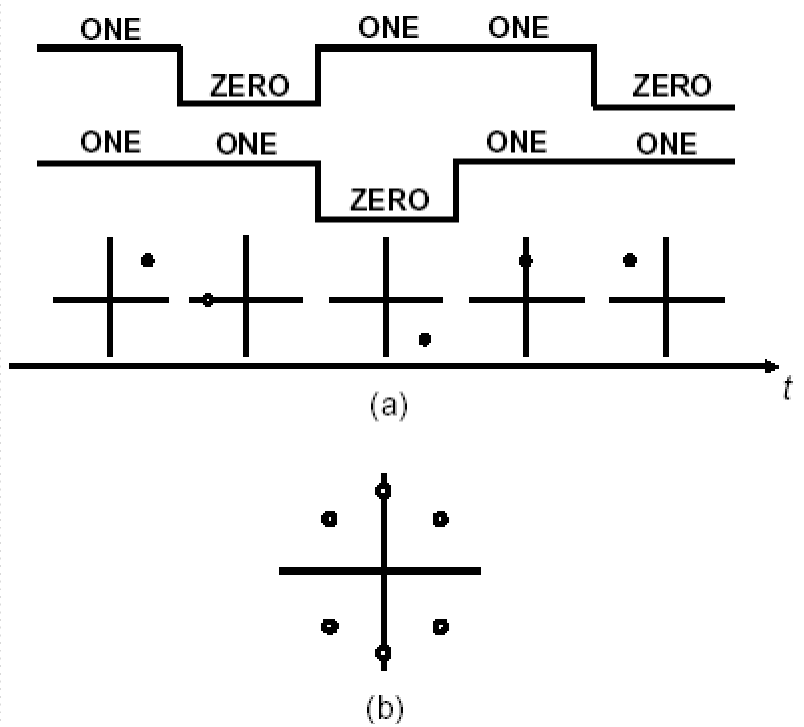
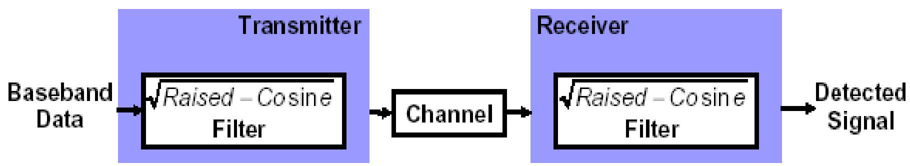


Fig. 3.41 (a) Evolution of π/4-QPSK in time domain, (b) possible phase transitions in the constellation



Minimum Shift Keying

- One of "continuous phase modulation"

$$x(t) = a_m \cos(\omega_1 t)\cos(\omega_c t)$$
$$- a_{m+1} \sin(\omega_1 t)\sin(\omega_c t)$$

$$\omega_1 = \pi/(2T_b)$$

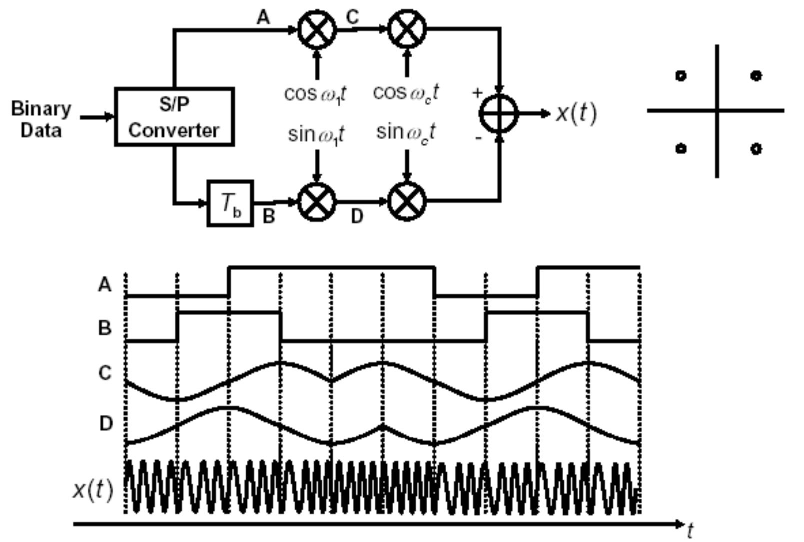


Fig. 3.43 MSK modulation and signal constellation

- When a_m goes from +1 to -1, while a_{m+1}=1

$$x(t) = 2\cos(\omega_1 + \omega_c)t \rightarrow 2\cos\left[(\omega_c - \omega_1)t + \pi\right]$$

$$\Delta\phi = (\omega_c - \omega_1)t + \pi - (\omega_1 + \omega_c)t = -2k\pi$$

  - **Phase change is continuous**

MSK Spectral Density

- BER of MSK is the same as QPSK.
- Shaper decay in spectrum than rectangular-pulse QPSK

$$S_{MSK}(f) = \frac{16A_c^2 T_b}{\pi^2}\left\{\frac{\cos^2[2\pi T_b(f-f_c)]}{[1-16T_b^2(f-f_c)^2]^2} + \frac{\cos^2[2\pi T_b(f+f_c)]}{[1-16T_b^2(f+f_c)^2]^2}\right\} \qquad (3.55)$$

- Decay is proportional to f^4.
  - Faster decay than BPSK

Gaussian Minimum Shift Keying

- **Spectral efficiency**
  - From smoother phase change
- **MSK**
  - To arrive at GMSK, we state without proof that the MSK signal of Fig.3.43 can also be written as

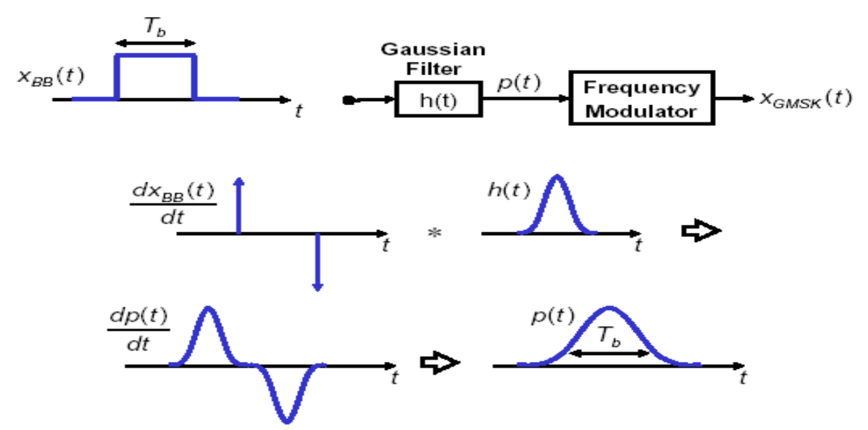
$$x_{MSK}(t) = \sqrt{2}A_c \cos\left[\omega_c t + \int_{-\infty}^{t} \sum_m b_m p(t - mT_b)dt\right] \qquad (3.56)$$

  - Summation represents the baseband signal: $b_m = \pm 1$, and p(t) is a rectangular pulse of width T_b

Gaussian Minimum Shift Keying

- GMSK
  - Filter with Gaussian impulse response h(t) $h(t) = \exp(-\alpha t^2)$
  - p(t) is obtained by passing the filter with Gaussian impulse response h(t)
    - $p(t) = x_{BB}(t) * h(t)$

Fig. 3.44 GMSK modulation



- Choice of α
  - For large α, output is closer to rectangular pulse.
  - For α=1, 99% bandwidth < 1.2/Tb
  - The lower the value of α, the narrower the spectrum!
  - For a small α, ISI is significant.
  - Typical α is in the vicinity of 0.3.

Contents

Power Efficiency of Modulation Schemes

- **Constant-and Variable-Envelope Signal**

$$x(t) = A(t)\cos[\omega_c t + \phi(t)]$$

  - Variable-envelope signal if A(t) varies with time
  - constant-envelope signal if A(t)=Ac constant
- **Bandwidth Consideration**
  - Constant-envelope signal
    - 3rd order

$$\begin{aligned} y(t) &= \alpha_3 x^3(t) + \cdots \\ &= \alpha_3 A_c^3 \cos^3[\omega_c t + \phi(t)] + \cdots \\ &= \frac{\alpha_3 A_c^3}{4}\cos[3\omega_c t + 3\phi(t)] + \frac{3\alpha_3 A_c^3}{4}\cos[\omega_c t + \phi(t)] \end{aligned}$$

    - No change in BW at ω_c
    - Additional BW at $3\omega_c$, but small
  - Variable-envelope signal
    - 3rd order

$$x(t) = x_I(t)\cos\omega_c t - x_Q(t)\sin\omega_c t \qquad (3.60)$$

$$\begin{aligned} y(t) &= \alpha_3[x_I(t)\cos\omega_c t - x_Q(t)\sin\omega_c t]^3 + \cdots \\ &= \alpha_3 x_I^3(t)\frac{\cos 3\omega_c t + 3\cos\omega_c t}{4} \\ &\quad - \alpha_3 x_Q^3(t)\frac{-\sin 3\omega_c t + 3\sin\omega_c t}{4} + \cdots \end{aligned}$$

    - x_I^3 and x_Q^3 around ω_c
    - Spectrum grows at ω_c
    - Additional band at $3\omega_c$

Spectral Regrowth

- **Effect of filtering**
  - LPF limits BW
  - Smooth out in time domain
    - Abrupt phase change smooth out
  - Envelope variation
  - Spectral regrowth from nonlinearity of power amp
- **Linear PA required to minimize spectral regrowth**
  - Less efficiency
    - ~40 % at best for linear PA
    - ~60 % for nonlinear PA
  - More power consumption

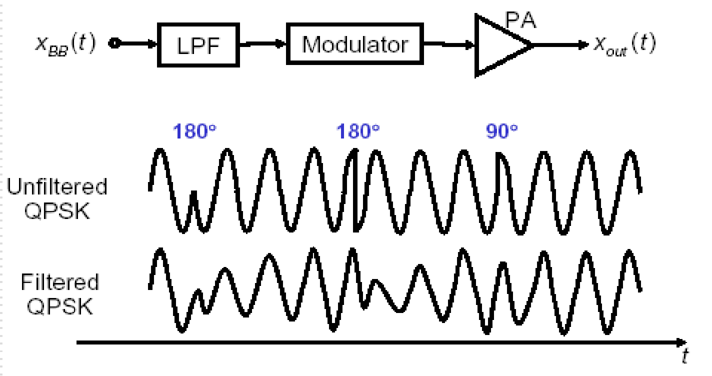


Fig. 3.45 Elope of bandlimiting on the envelope of a QPSK waveform

Spectral Efficiency and Power Efficiency

- **Trade off between spectral and power efficiency**
  - Narrow band (for high spectral efficiency)

  → Envelope variation

  → Spectral regrowth

  → Highly linear PA required

  → Low power efficiency
- **Power efficient modulation**
  - FM, FSK
    - Continuous phase change
    - No abrupt phase change
    - No envelope variation
    - Nonlinear PA applicable
  - OQPSK, π/4-QPSK
    - Less severe envelope variation than QPSK
  - MSK
    - No abrupt phase change, better power eff. than QPSK
    - Wider band required than QPSK

Noncoherent Detection

- **Coherent detection**
  - Highest SNR
  - Complex ← carrier recovery is difficult.
- **Noncoherent detection**
  - Inferior performance, but simple
- **Noncoherent FSK Detection**
  - Two bandpass filters : to detect binary modulation
  - Two envelope detectors : to determine which freq. is received

$$P_e = \frac{1}{2}\exp\left[\frac{-E_b}{2T_bB_pN_o}\right] \underset{\text{if } T_bB_p=1}{=} \frac{1}{2}\exp\left[\frac{-E_b}{2N_o}\right]$$

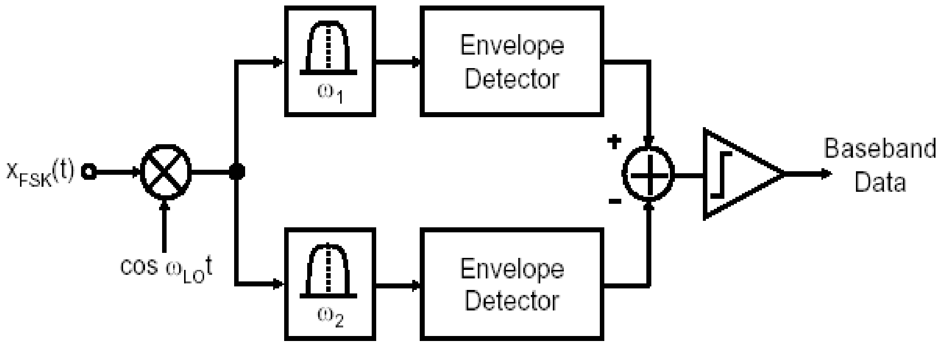


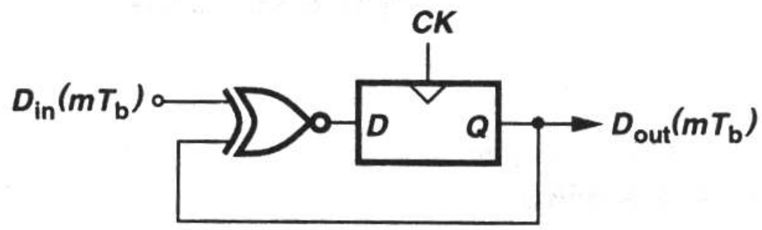
Figure 3.46 Noncoherent FSK detection

Noncoherent Detection-Differential PSK

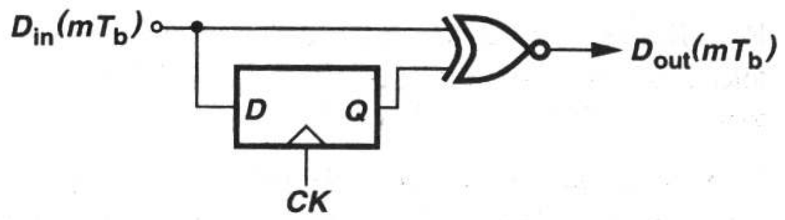
- Differential Phase Shift Keying (DPSK)
- Simple PSK cannot be detected noncoherently.
- Noncoherent detection of PSK is accomplished through "differential"

$$D_{out}[(m+1)T_b] = \overline{D_{in}(mT_b) \oplus D_{out}(mT_b)} \qquad (3.65)$$

- **Pe** $$P_e = \frac{1}{2}\exp\left[\frac{-E_b}{N_o}\right]$$
  - 3 dB gain over noncoherent FSK
  - 3 dB loss over coherent PSK



(a)



(b)

Input Data		0	1	1	1	0	0	1	1	0	1
Encoded Data	1	0	0	0	0	1	0	0	0	1	1
Decoded Data		0	1	1	1	0	0	1	1	0	1

(c)