EE 210

HW#: 02

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Assigned question #s: **5**

1. One of the key points of this exercise is to find frequencies of two or more sinusoids are multiplied or summed.
   a) Find the following sinusoidal multiplications using Euler's identities.
      i. $\sin(3x)\cdot\cos(5x)$
      ii. $\sin^2(3x)\cdot\cos(2x)$
      iii. $\cos(3x)\cdot\cos^2(3x)$

   b) Now plug in if the value of $[x = 13\pi t]$, find the frequencies after the multiplications
   c) Plot frequency responses of b) in magnitude and phase.

① a) i.

$$\sin(3x)\cos(5x) = \frac{1}{2j}\left(e^{j3x} - e^{-j3x}\right)\cdot\frac{1}{2}\left(e^{j5x} + e^{-j5x}\right)$$

$$= \frac{1}{2}\cdot\frac{1}{2j}\left(e^{j8x} + e^{-j2x} - e^{j2x} - e^{-j8x}\right)$$

$$= \frac{1}{2}\cdot\frac{1}{2j}\left(e^{j8x} - e^{-j8x} - \left(e^{j2x} - e^{-j2x}\right)\right)$$

$$= \boxed{\frac{1}{2}\left(\sin(8x) - \sin(2x)\right)}$$

ii.

$$\sin^2(3x)\cdot\cos(2x) = \frac{1}{2j}\frac{1}{2j}\left(e^{j3x} - e^{-j3x}\right)\left(e^{j3x} - e^{-j3x}\right)\cdot\frac{1}{2}\left(e^{j2x} + e^{-j2x}\right)$$

$$= \frac{1}{2}\frac{1}{2j}\frac{1}{2j}\left(e^{j6x} + e^{-j6x} - 2\right)\left(e^{j2x} + e^{-j2x}\right)$$

$$= \frac{1}{2}\frac{1}{2j}\frac{1}{2j}\left(e^{j8x} + e^{j4x} + e^{-j4x} + e^{-j8x} - 2\left(e^{j2x} + e^{-j2x}\right)\right)$$

$$= \boxed{\frac{-1}{4}\left(\cos(8x) + \cos(4x) - 2\cos(2x)\right)}$$

iii.

$$\cos(3x)\cdot\cos^2(3x) = \frac{1}{2}\frac{1}{2}\frac{1}{2}\left(e^{j3x} + e^{-j3x}\right)\left(e^{j3x} + e^{-j3x}\right)\left(e^{j3x} + e^{-j3x}\right)$$

$$= \frac{1}{4}\cdot\frac{1}{2}\left(e^{j6x} + e^{-j6x} + 2\right)\cdot\left(e^{j3x} + e^{-j3x}\right)$$

$$= \frac{1}{4}\cdot\frac{1}{2}\left(e^{j9x} + e^{j3x} + e^{-j3x} + e^{-j9x} + 2\left(e^{j3x} + e^{-j3x}\right)\right)$$

$$= \frac{1}{4}\left(\cos(9x) + \cos(3x) + 2\cos(3x)\right)$$

$$= \boxed{\frac{1}{4}\left(\cos(9x) + 3\cos(3x)\right)}$$

b) i. $\sin(3x)\cos(5x) = \frac{1}{2}\sin(8x) - \frac{1}{2}\sin(2x)$ $\qquad x = 13\pi t$

$$= \frac{1}{2}\sin(2\pi(52)t) - \frac{1}{2}\sin(2\pi(13)t)$$

$\therefore$ $\boxed{\text{frequencies are at } \underline{52\text{ Hz}} \ \& \ \underline{13\text{ Hz}}}$

ii. $\sin^2(3x)\cos(2x) = \frac{-1}{4}\left(\cos(8x) + \cos(4x) - 2\cos(2x)\right)$

$$= \frac{-1}{4}\cos(2\pi(52)t) - \frac{1}{4}\cos(2\pi(26)t) + \frac{1}{2}\cos(2\pi(13)t)$$

$\therefore$ $\boxed{\text{frequencies are at } \underline{52, 26, \text{ and } 13 \text{ Hz}}}$

iii. $\cos(3x)\cos^2(3x) = \frac{1}{4}\cos(9x) + \frac{3}{4}\cos(3x)$

$$= \frac{1}{4}\cos(2\pi(58.5)t) + \frac{3}{4}\cos(2\pi(19.5)t)$$

$\therefore$ $\boxed{\text{frequencies are at } \underline{58.5 \text{ and } 19.5 \text{ Hz}}}$

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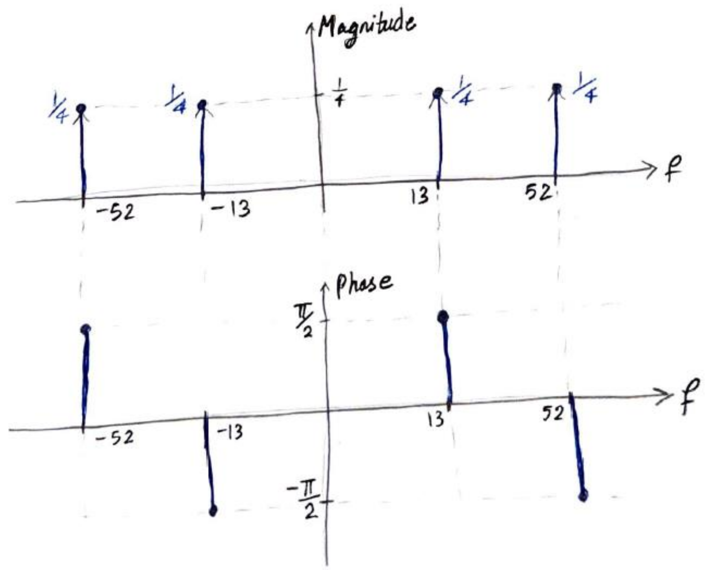
c) i. $\mathcal{F}\left\{\sin(3x)\cos(5x)\right\}$ $\leftarrow$ Fourier Transform

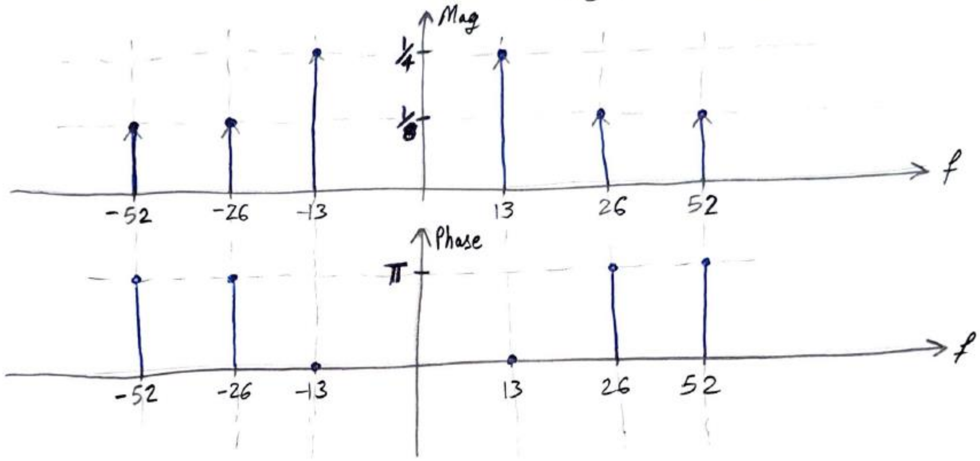
$$= -\frac{1}{4}j\left[\delta(f-52) - \delta(f+52)\right] + \frac{1}{4}j\left[\delta(f-13) - \delta(f+13)\right]$$

$$= -\frac{1}{4}j\,\delta(f-52) + \frac{1}{4}j\,\delta(f+52) + \frac{1}{4}j\,\delta(f-13) - \frac{1}{4}j\,\delta(f+13)$$



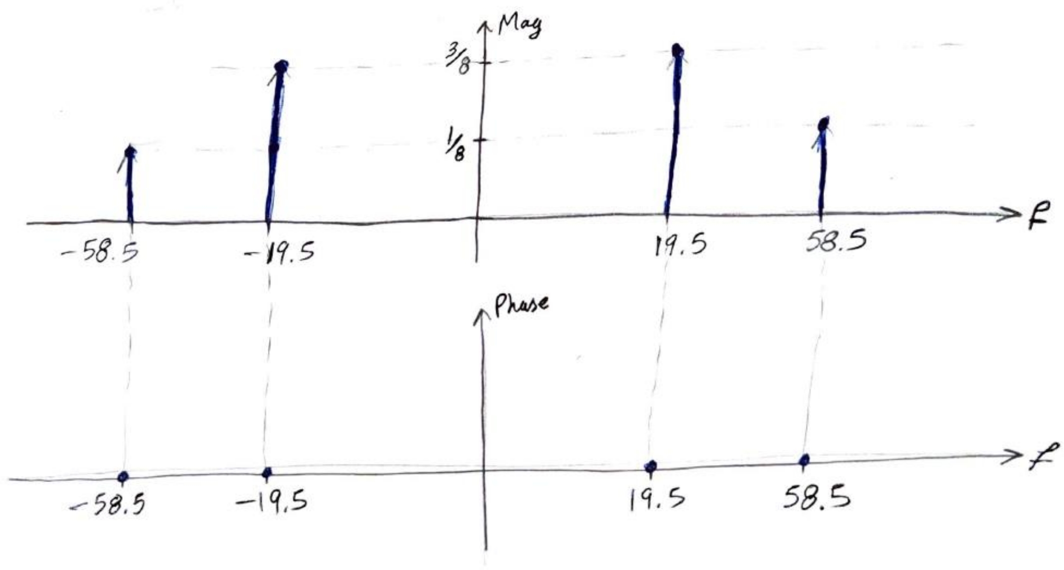
ii. $\mathcal{F}\left\{ \sin^2(3x)\cos(2x) \right\}$

$$= -\frac{1}{4}\left[\frac{1}{2}\left(\delta(f-52)+\delta(f+52)\right)\right] - \frac{1}{4}\left[\frac{1}{2}\left(\delta(f-26)+\delta(f+26)\right)\right] + \frac{1}{2}\left[\frac{1}{2}\left(\delta(f-13)+\delta(f+13)\right)\right]$$



iii. $\mathcal{F}\left\{ \cos(3x)\cos^2(3x) \right\}$

$$= \frac{1}{4}\left[\frac{1}{2}\left(\delta(f-58.5)+\delta(f+58.5)\right)\right] + \frac{3}{4}\left[\frac{1}{2}\left(\delta(f-19.5)+\delta(f+19.5)\right)\right]$$



2. I recommend using tools (Matlab, octave, python, etc…) to plot the frequency responses and the ranges of the frequencies are $\left[-5Hz \le f \le 5Hz\right]$

   a) Plot $F\left\{2\cdot\Pi\left(\frac{t}{4}\right)\right\}$ in magnitude and phase. (it means frequency response of rectangular function with amplitude 2, center at 0, and width of 4)

   b) Plot $F\left\{2\cdot\Pi\left(\frac{t-3}{4}\right)\right\}$ in magnitude and phase.

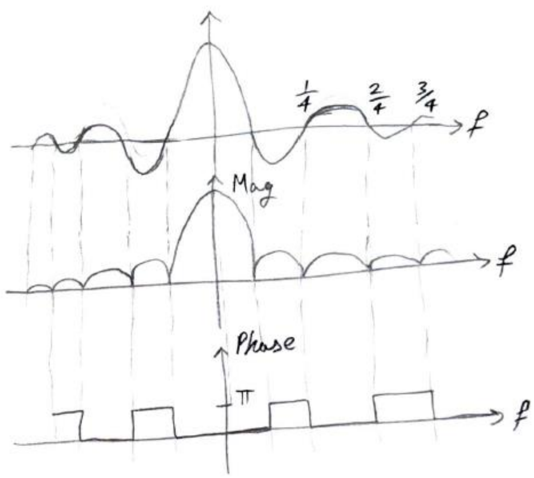
   c) Plot $F\left\{2\cdot\Pi\left(\frac{t+3}{4}\right)\right\}$ in magnitude and phase.

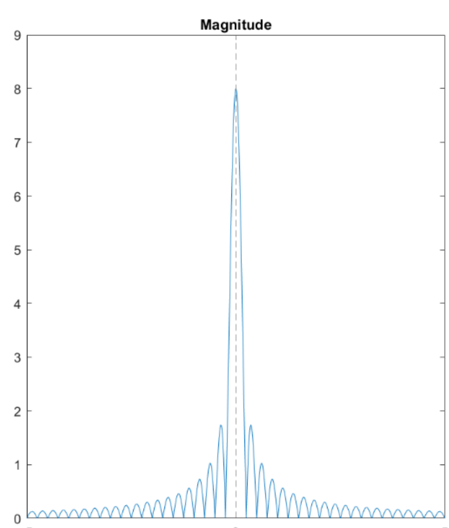
   d) What is the difference between a) & b) in the frequency domain? Compare magnitude vs. magnitude & phase vs. phase of two signals in the in the frequency domain.

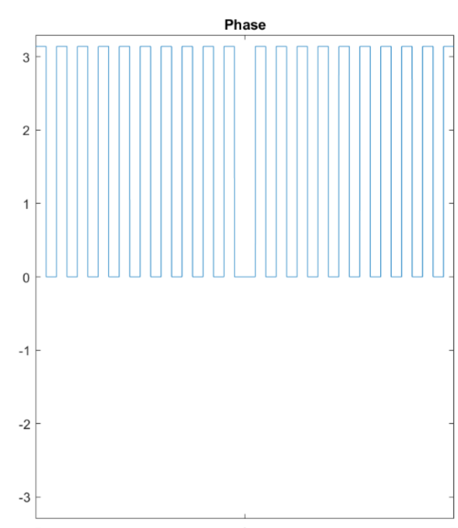
2 a) $\mathcal{F}\left\{2.\Pi\left(\frac{t}{4}\right)\right\} = 2.2.2\ \text{Sinc}\,(2.2.f)$

$A = 2$
$t_0 = 2$

$= 8\ \text{Sinc}(4f)$ $\rightsquigarrow$ zero crossings at $\underline{4f = n}$



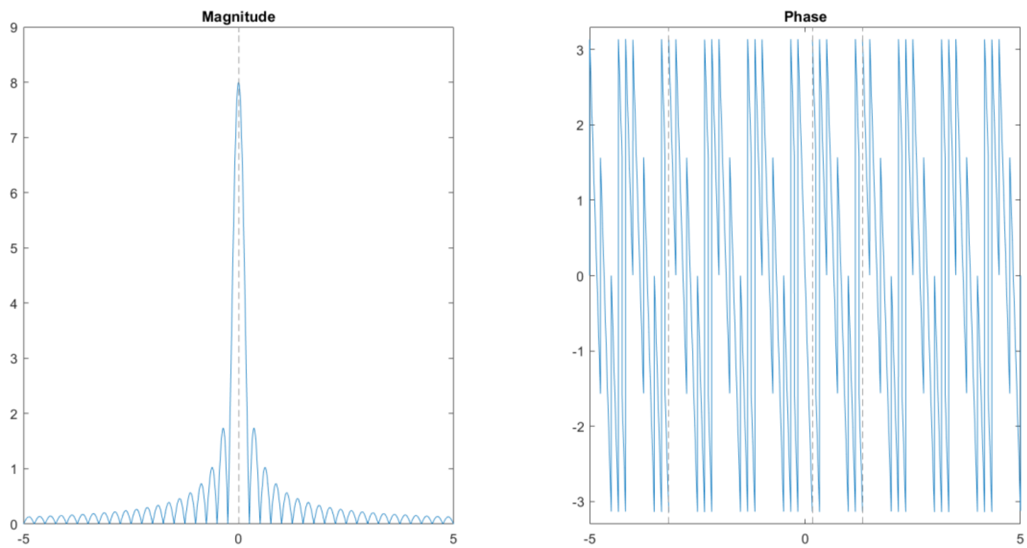




b) $\mathcal{F}\left\{2 \cdot \Pi\left(\frac{t-3}{4}\right)\right\} = 2.2.2 \text{ Sinc}\,(2.2f)\; e^{-j2\pi(3)f}$

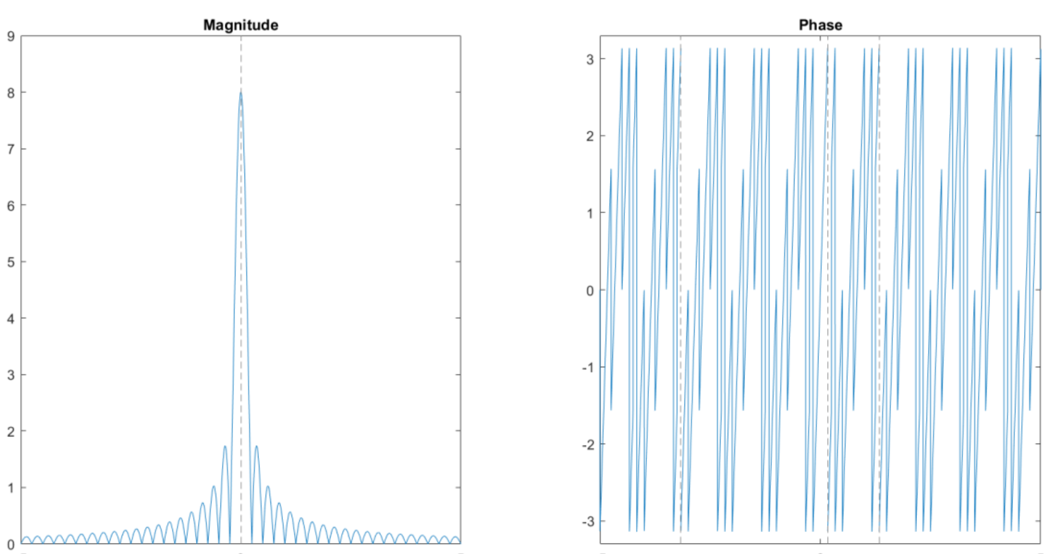
$= 8 \text{ Sinc}\,(4f)\; \underbrace{e^{-j6\pi f}} \longrightarrow \theta = -6\pi f$

(negative slope)

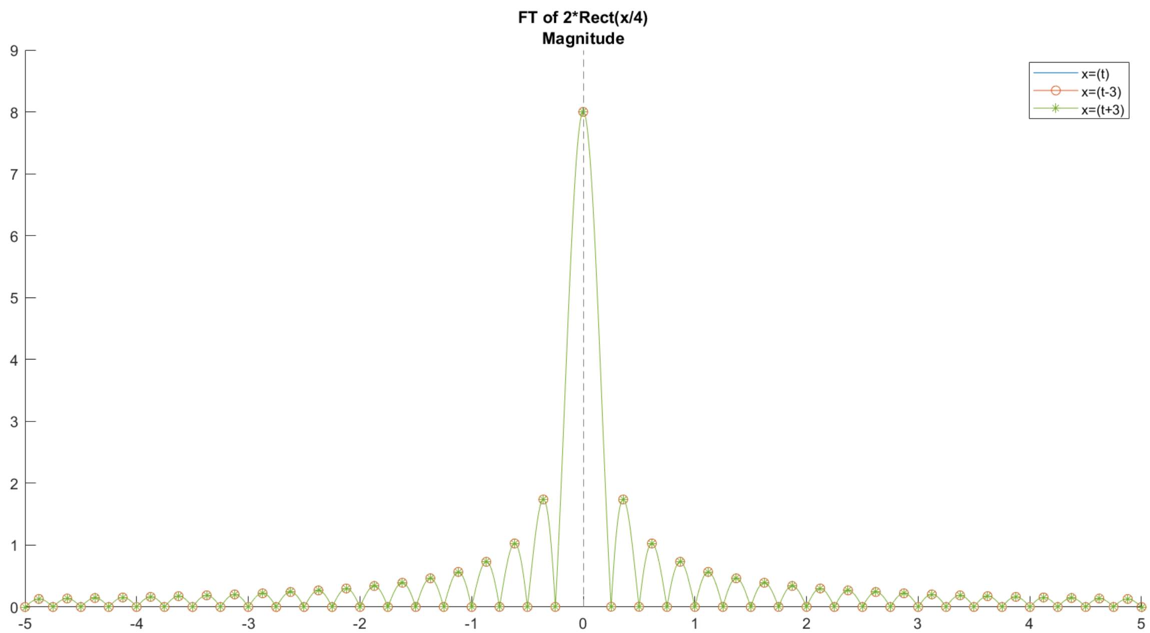


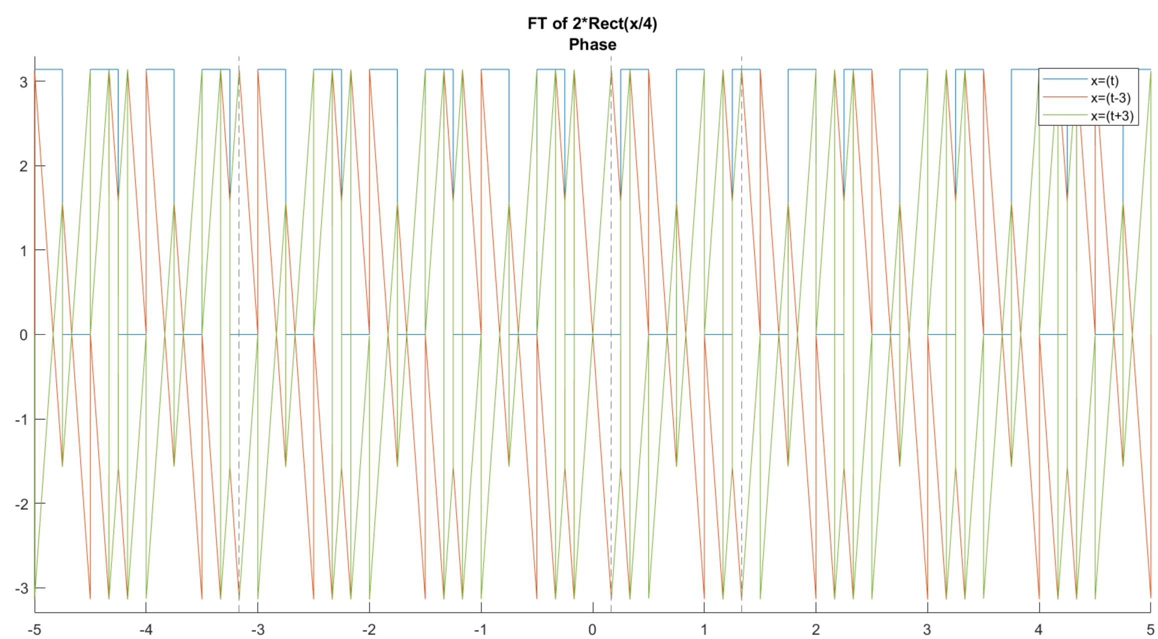
c) $\mathcal{F}\left\{2\,\Pi\left(\frac{t+3}{4}\right)\right\} = 8 \text{ Sinc}\,(4f)\; \underbrace{e^{j6\pi f}} \longrightarrow \theta = 6\pi f$

(positive slope)



d) The magnitude stays the same for all 3 cases ⓐ, ⓑ, & ⓒ.
The phase takes only 2 values in case ⓐ, which is 0 and π.
There is a phase shift due to the delay in both case ⓑ (with negative slope)
& case ⓒ (with positive slope).





Matlab Code for Q2:

1) 1st Method:

```
syms t f
A = 2
%Fourier Transform of rectangular function
%Y=integrating(A*exp(-j*2*pi*f*t)) from t = -2 to +2
X=int((A*exp(-j*2*pi*f*t)),t,-2,2);

figure(1)
subplot(2,2,1); fplot(X); title('FT of 2*Rect(x/4)'); xlim([-5 5])
legend('show','Location','best')

subplot(2,2,3); fplot(abs(X)); title('Magnitude'); axis([-5 5 0 9])
subplot(2,2,4); fplot(angle(X)); title('Phase'); axis([-5 5 -3.3 3.3])
%-----------------------------------------------------

% hold on
Y=int((A*exp(-j*2*pi*f*t)),t,1,5);
Z=int((A*exp(-j*2*pi*f*t)),t,-5,-1);

figure(2)
subplot(1,2,1); fplot(abs(X)); title('Magnitude'); axis([-5 5 0 9])
subplot(1,2,2); fplot(angle(X)); title('Phase'); axis([-5 5 -3.3 3.3])
figure(3)
subplot(1,2,1); fplot(abs(Y)); title('Magnitude'); axis([-5 5 0 9])
subplot(1,2,2); fplot(angle(Y)); title('Phase'); axis([-5 5 -3.3 3.3])
figure(4)
subplot(1,2,1); fplot(abs(Z)); title('Magnitude'); axis([-5 5 0 9])
subplot(1,2,2); fplot(angle(Z)); title('Phase'); axis([-5 5 -3.3 3.3])

figure(5) %Magnitudes
hold on
fplot(abs(X)); fplot(abs(Y),'-o'); fplot(abs(Z),'-*','Color','#77AC30');
title({'FT of 2*Rect(x/4)','Magnitude'}); axis([-5 5 0 9]); legend('x=(t)','x=(t-
3)','x=(t+3)')

figure(6) %Phases
hold on
fplot(angle(X)); fplot(angle(Y),'-'); fplot(angle(Z),'-','Color','#77AC30');
title({'FT of 2*Rect(x/4)','Phase'}); axis([-5 5 -3.3 3.3]); legend('x=(t)','x=(t-
3)','x=(t+3)')
```

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2) 2nd Method:

```
%Creating Rectangular Function
A=2;
t=-6:0.01:6;

for m=1:1:length(t)
 if (t(m) >= -2)&&(t(m) <= 2)
     x(m) = A;
 else
     x(m) = 0;
 end
end
figure; subplot(2,2,1); plot(t,x); title('2*Rect(t/4)')
```

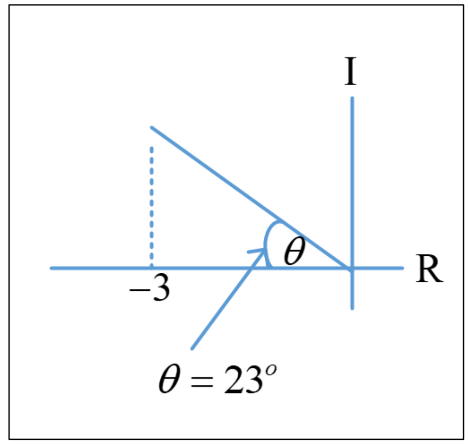
```
f=linspace(-5,5,length(t));

for k=1:1:length(f)
 X(k)=trapz(t,x.*exp(-j*2*pi*f(k)*t));
end

subplot(2,2,2); plot(f,X); title('FT of 2*Rect(t/4)')
subplot(2,2,3); plot(f,abs(X)); title('Magnitude')
subplot(2,2,4); plot(f,angle(X)); title('Phase')
%----------------------------------
%----------------------------------

for m=1:1:length(t)
 if (t(m) >= 1)&&(t(m) <= 5)
     y(m) = A;
 else
     y(m) = 0;
 end
end
figure; subplot(2,2,1); plot(t,y); title('2*Rect((t-3)/4)')

f=linspace(-5,5,length(t));

for k=1:1:length(f)
 Y(k)=trapz(t,y.*exp(-j*2*pi*f(k)*t));
end

subplot(2,2,2); plot(f,Y); title('FT of 2*Rect((t-3)/4)')
subplot(2,2,3); plot(f,abs(Y)); title('Magnitude')
subplot(2,2,4); plot(f,angle(Y)); title('Phase')

%----------------------------------
%----------------------------------

for m=1:1:length(t)
 if (t(m) >= -5)&&(t(m) <= -1)
     z(m) = A;
 else
     z(m) = 0;
 end
end
figure; subplot(2,2,1); plot(t,z); title('2*Rect((t+3)/4)')

f=linspace(-5,5,length(t));

for k=1:1:length(f)
 Z(k)=trapz(t,z.*exp(-j*2*pi*f(k)*t));
end

subplot(2,2,2); plot(f,Z); title('FT of 2*Rect((t+3)/4)')
subplot(2,2,3); plot(f,abs(Z)); title('Magnitude')
subplot(2,2,4); plot(f,angle(Z)); title('Phase')
```

3.

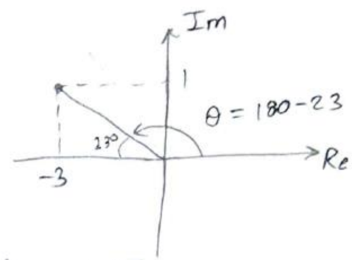
The angle $\theta = 23^o$ is in degrees.

a) Write given plot in rectangular form. $(a + jb)$

b) Write given plot in polar form (in radian) $(|\ |e^{j\theta})$



③ a) Rectangular form: $\longrightarrow \boxed{-3 + 1j}$



b) Polar form:

Magnitude:

$\sqrt{a^2 + b^2}$

$= 3.162$

$|\ |e^{j\theta} = \boxed{3.162\, e^{(0.87)\pi j}}$

Phase:

$\theta\ (rad) \longrightarrow 157$

$\pi \longrightarrow 180$

$\theta = \frac{157}{180}\pi = 0.87\pi$

4.

Convert the following function in polar form.

a) $1+j,\ 1-j,\ -1+j,\ -1-j,\ 2,\ -2,\ j,\ -j$

b) $\frac{1+j}{1-j},\ \frac{-1+j}{-1-j},\ \frac{2}{-j},\ \frac{j}{-2}$

4 a)

$1+j \longrightarrow |\,|e^{j\theta} = \sqrt{(1)^2+(1)^2}\, e^{j\tan^{-1}(1/1)} = \boxed{\sqrt{2}\, e^{j(\pi/4)}}$

$\theta = \tan^{-1}(1) = 45$ "1st quad"

$1-j \longrightarrow |\,|e^{j\theta} = \sqrt{(1)^2+(1)^2}\, e^{j\tan^{-1}(-1/1)} = \boxed{\sqrt{2}\, e^{j(\frac{7}{4}\pi)}}$

$\theta = \tan^{-1}(\frac{-1}{1}) = 360-45 = 315$

$\theta = \frac{7}{4}\pi$

$-1+j \longrightarrow |\,|e^{j\theta} = \boxed{\sqrt{2}\, e^{j(\frac{3}{4}\pi)}}$

$\theta = \tan^{-1}(\frac{1}{-1}) = 180-45 = 135$

$\theta = \frac{3}{4}\pi$

$-1-j \longrightarrow |\,|e^{j\theta} = \boxed{\sqrt{2}\, e^{j(\frac{5}{4}\pi)}}$

$\theta = \tan^{-1}(\frac{-1}{-1}) = 180+45 = 225$

$\theta = \frac{5}{4}\pi$

$2 \longrightarrow |\,|e^{j\theta} = \boxed{2\, e^{j0}}$

$-2 \longrightarrow |\,|e^{j\theta} = \boxed{2\, e^{j\pi}}$

$j \longrightarrow |\,|e^{j\theta} = \boxed{1\, e^{j(\pi/2)}}$

$-j \longrightarrow |\,|e^{j\theta} = \boxed{1\, e^{-j\pi/2}}$

b) $$\frac{1+j}{1-j} = \frac{(1+j)(1+j)}{(1-j)(1+j)}$$

$$= \frac{1+j^2+2j}{1-j^2} = \frac{2j}{2} = j \longrightarrow \boxed{\text{polar} = 1\, e^{j\pi/2}}$$

$$\frac{-1+j}{-1-j} = \frac{(-1+j)(-1+j)}{(-1-j)(-1+j)}$$

$$= \frac{1+j^2-2j}{1-j^2} = \frac{-2j}{2} = -j \longrightarrow \boxed{\text{polar} = 1\, e^{-j\pi/2}}$$

$$\frac{2}{-j} = \frac{2\times j}{-j\times j}$$

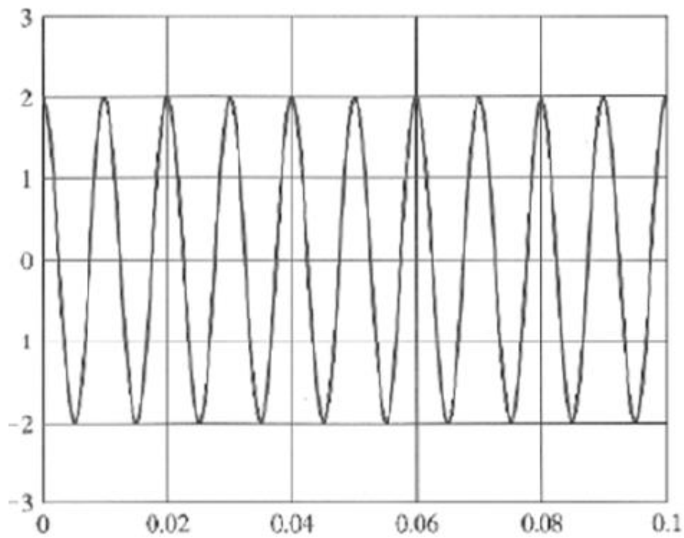
$$= 2j \longrightarrow \boxed{\text{polar} = 2\, e^{j\pi/2}}$$

$$\frac{j}{-2} = -\frac{1}{2}j \longrightarrow \boxed{\text{polar} = \frac{1}{2}\, e^{-j\pi/2}}$$

5. For the given sinusoid figure below, determine amplitude (A), period $\left(T = \frac{1}{f}\right)$, frequency (f), and phase (θ).

   a) Write this figure in terms of **sine & cosine** function using RMS (do you remember how to represent RMS (root mean square) values.-or you can verity with simple calculation)

   b) And also represent this function in terms of exponential function too (You know that $\left[\mathrm{Re}\{e^{jx}\} = \cos(x)\ \&\ \mathrm{Im}\{e^{jx}\} = \sin(x)\right]$)



5 a) $A = 2$, $T = \frac{0.1}{10} = 0.01s$, $f = 100\ Hz$, $\theta = 0$

$$\boxed{V_{rms}(t) = \frac{2}{\sqrt{2}} \cos(2\pi(100)t)}$$

for Sin or Cos:

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T [A \sin(2\pi f t + \theta)]^2 dt}$$

$$= A\sqrt{\frac{1}{T}\int_0^T \left[\frac{1}{2}(1 - \cos(2\pi f t + \theta))\right] dt}$$

$$= A\sqrt{\frac{1}{2T}\int_0^T dt}$$

$$= \frac{A}{\sqrt{2}}$$

b) $V_{rms}(t) = \frac{2}{\sqrt{2}} \cos(2\pi(100)t)$

$$= \frac{2}{\sqrt{2}} \frac{1}{2}\left(e^{j2\pi(100)t} + e^{-j2\pi(100)t}\right)$$

$$= \boxed{\frac{1}{\sqrt{2}}\left(e^{j200\pi t} + e^{-j200\pi t}\right)}$$