Lec07

Definitions of filter

$$\frac{1}{2}\text{Power}$$

Filter shape depends on the length of the filter coefficients

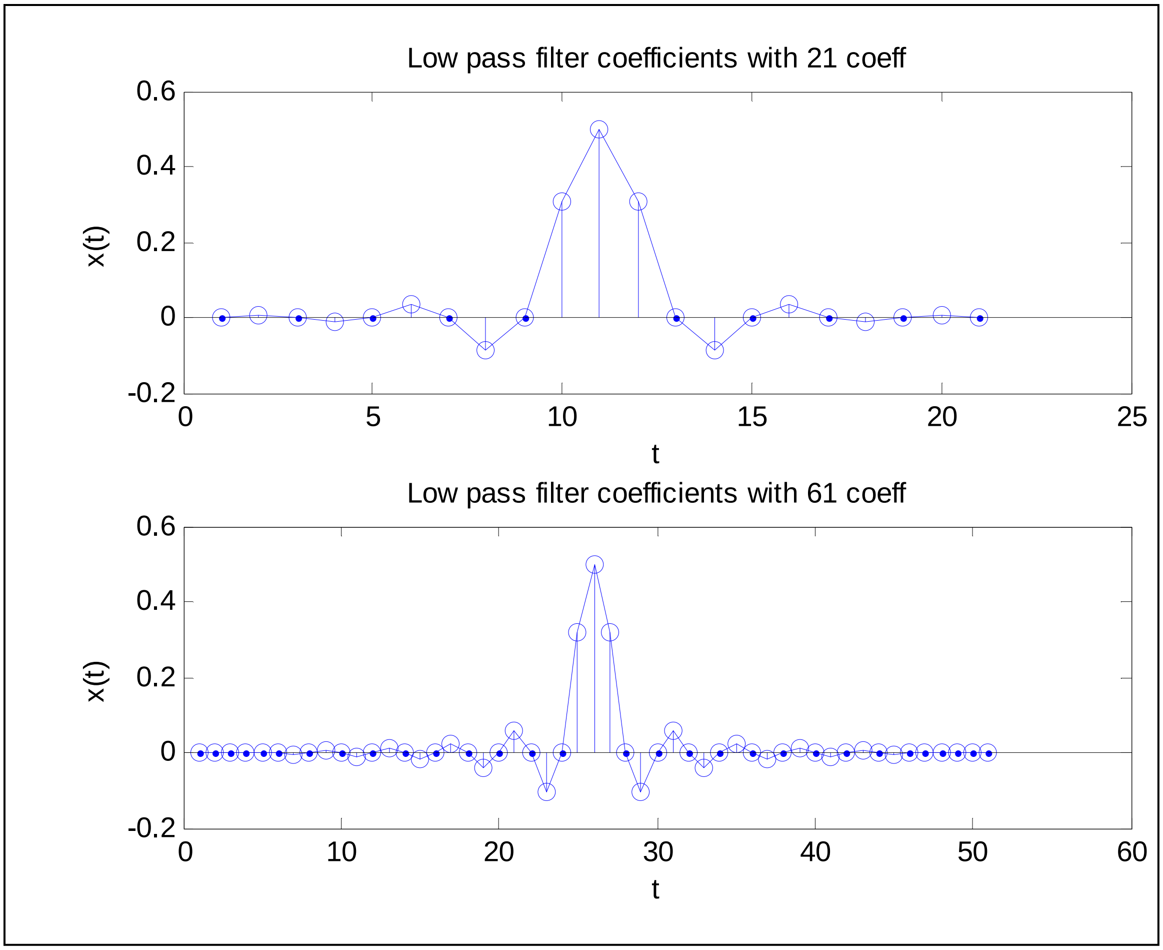
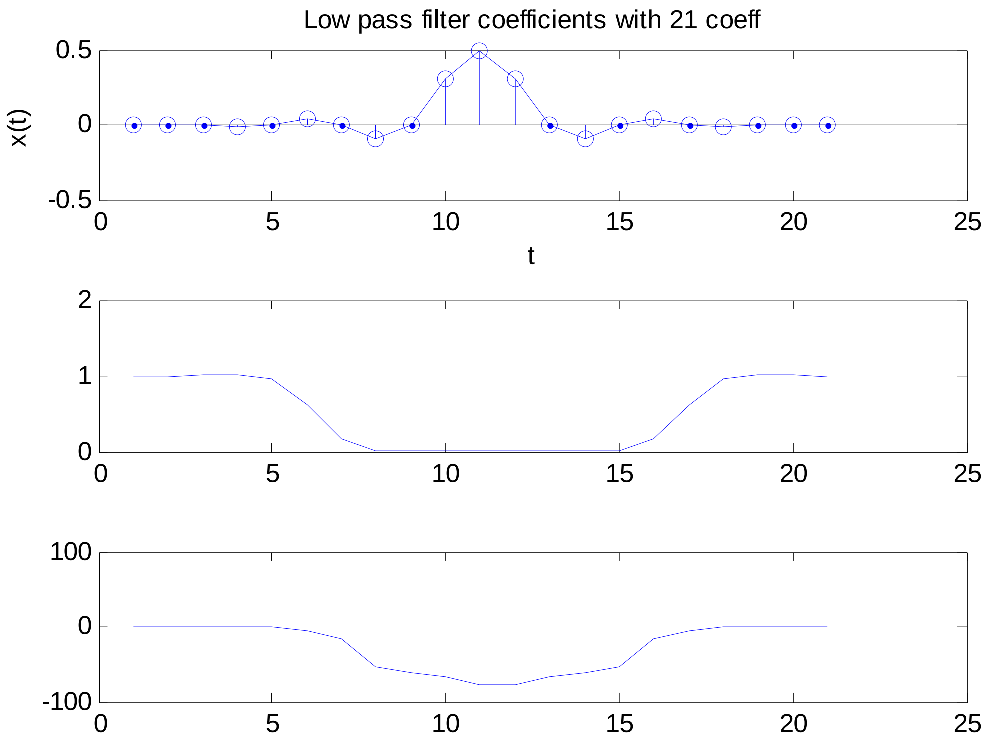


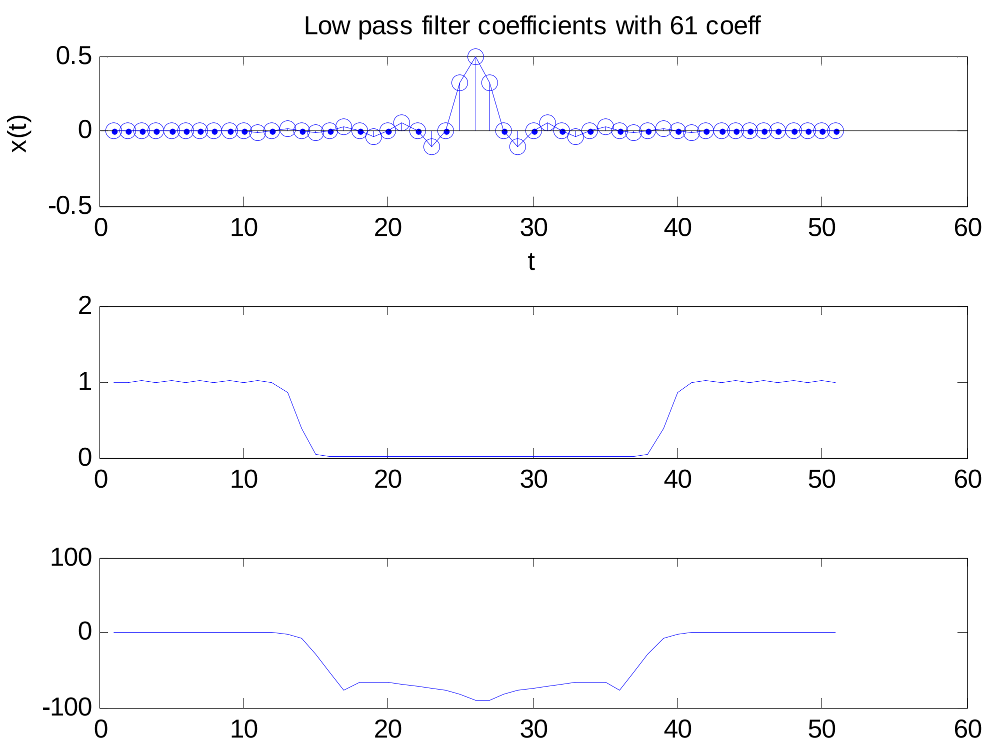
Figure 1: Filter coefficients 21 vs. 61.

-0.0001
0.0035
-0.0002
-0.0125
-0.0004
0.0338
-0.0007
-0.0867
-0.0009
0.3099
0.4986
0.3099
-0.0009
-0.0867
-0.0007
0.0338
-0.0004
-0.0125
-0.0002
0.0035
-0.0001

Filter coefficients: 21 vs. 61 taps.

0.0009
-0.0001
-0.0014
-0.0001
0.0019
-0.0002
-0.0036
-0.0002
0.0052
-0.0003
-0.0088
-0.0005
0.0120
-0.0006
-0.0187
-0.0007
0.0253
-0.0008
-0.0387
-0.0009
0.0572
-0.0009
-0.1037
-0.0010
0.3162
0.4991
0.3162
-0.0010
-0.1037
-0.0009
0.0572
-0.0009
-0.0387
-0.0008
0.0253
-0.0007
-0.0187
-0.0006
0.0120
-0.0005
-0.0088
-0.0003
0.0052
-0.0002
-0.0036
-0.0002
0.0019
-0.0001
-0.0014
-0.0001
0.0009


Low pass filter coefficients with 21 coeff
x(t)
t
0.5
0
-0.5
0
5
10
15
20
25
2
1
100
-100


Low pass filter coefficients with 61 coeff
x(t)
t
0.5
0
-0.5
10
20
30
40
50
60
2
1
100
-100

List of items to look

- Gain of the filter in certain frequency
- Pass band and gain factor
- Stop band
- Cut off frequency
- Bandwidth of the filter

Low pass filter: Smooth signals by averaging out

- Stock market chart with MA

High pass filter: Tend to emphasize sharp transition

Linear Time Invariant (LTI) and Causal system

Linear systems obey **superposition** property

- When input x_1 produces output y_1 and input x_2 produces output y_2, then an input that is sum of x_1 and x_2 will produce an output that is the sum of y_1 and y_2.

Time invariant system gives the same output for an input no matter when that input is applied. If input is delayed, then the output is delayed by the same amount.

Causal system: Output depends on present and previous data. Never the future data

A discrete time system can be thought of as a transformation or operator that maps an input sequence $x[n]$ to an output sequence $y[n]$. By placing various conditions on system, we can define different classes of systems, such as **linear, non-linear, time invariant, time variant**, etc.

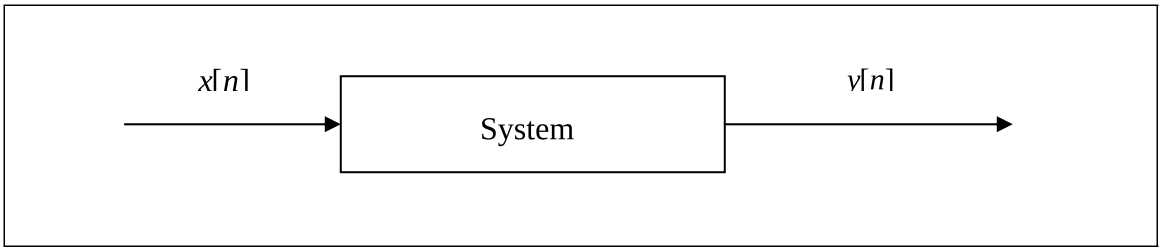


Figure 3: LTI system

Linearity

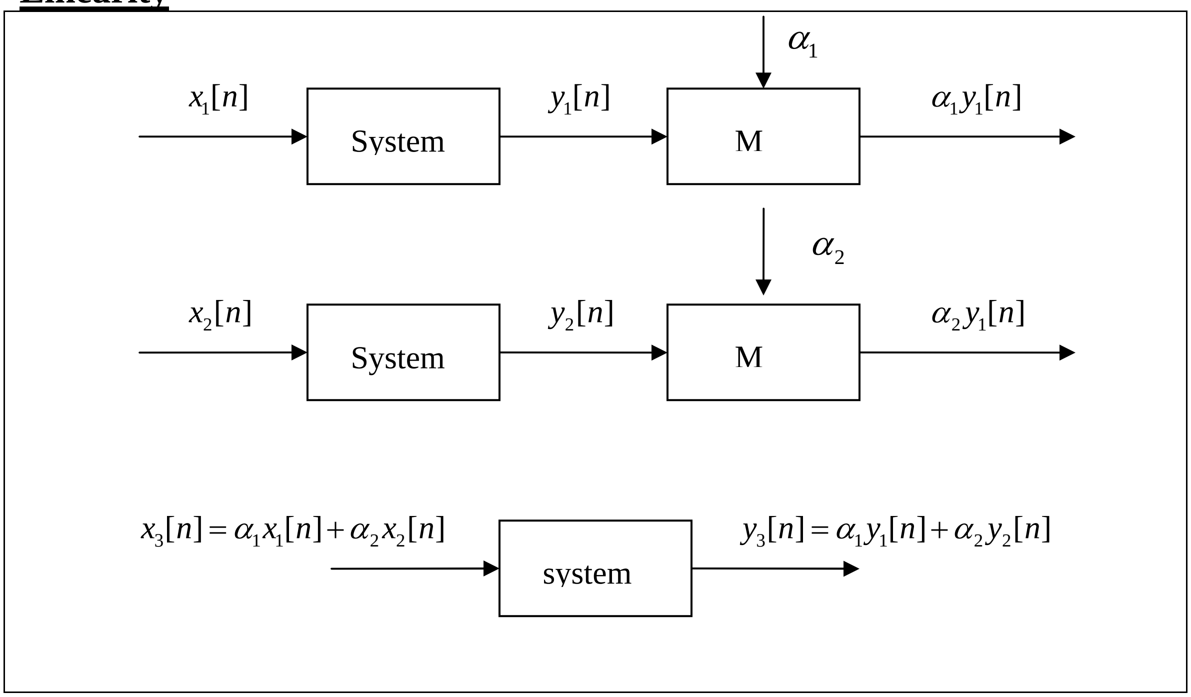


Figure 4: Linearity checking diagram

Ex] Suppose that the system in the above figure is described by

$$y[n] = [x[n]]^2 \tag{1}$$

Is this system **linear or non-linear**?

The input $x_1[n]$ produces the output $y_1[n]$ and the value of $y_1[n] = [x_1[n]]^2$.

Multiplying this output by α_1 gives $\alpha_1 y_1[n] = \alpha_1 \left[x_1[n]\right]^2$. Similarly,

$$\alpha_2 y_2[n] = \alpha_2 \left[x_2[n]\right]^2 \tag{2}$$

and

$$\begin{aligned} x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n] \xrightarrow{System} y_3[n] &= \left[\alpha_1 x_1[n] + \alpha_2 x_2[n]\right]^2 \\ &= \underline{\underline{\left[\alpha_1 x_1[n]\right]^2 + \left[\alpha_2 x_2[n]\right]^2}} + \boxed{2\alpha_1\alpha_2 x_1[n] x_2[n]} \end{aligned} \tag{3}$$

This output $y_3[n]$ is different from $\alpha_1 y_1[n] + \alpha_2 y_2[n]$.
So the system is **not** linear.

Time-Invariance

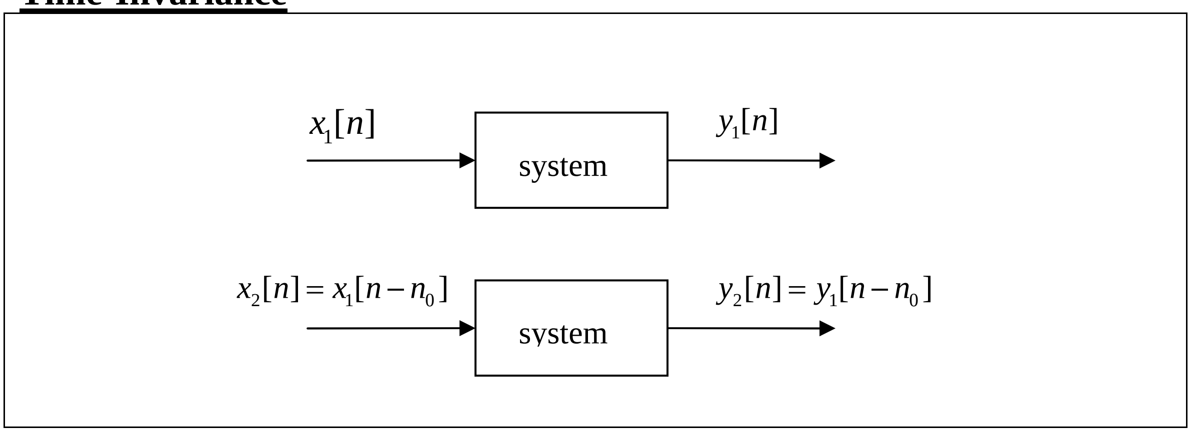


Figure 5: Time invariance checking

An input $x_1[n]$ produces the output $y_1[n]$. Consider a second input $x_2[n]$ which is a shifted version of $x_1[n]$, that is

$$x_2[n] = x_1[n-n_0]. \tag{4}$$

If the output $y_2[n]$ caused by $x_2[n]$ is a delayed replica of $y_1[n]$, then

$$y_2[n] = y_1[n-n_0] \tag{5}$$

for all n and for arbitrary $x_1[n]$ and n_0, then the system is said to be time-invariant or shift invariant.

Linear time-Invariant (LTI) System

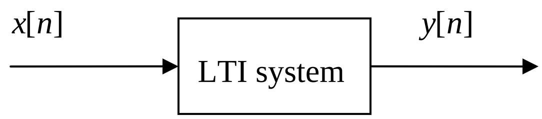


Figure 6: Linear time-Invariant (LTI) System

Ex] A system is described by the relationship

$$y[n] = n^2 \left| x[n] \right|, \quad 0 \le n \le \infty \tag{6}$$

Is this **LTI system**?

We need to **test for linearity** and time-invariance

$$\begin{aligned} y_1[n] &= n^2 \left| x_1[n] \right| \\ \alpha_1 y_1[n] &= \alpha_1 n^2 \left| x_1[n] \right| \\ \\ y_2[n] &= n^2 \left| x_2[n] \right| \\ \alpha_2 y_2[n] &= \alpha_2 n^2 \left| x_2[n] \right| \end{aligned} \tag{7}$$

Now we assume that the input is

$$x_3[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]. \tag{8}$$

The output for this input is

$$n^2 \left| x_3[n] \right| = n^2 \left| \alpha_1 x_1[n] + \alpha_2 x_2[n] \right|. \tag{9}$$

We need to test for

$$n^2 \left| \alpha_1 x_1[n] + \alpha_2 x_2[n] \right| \stackrel{?}{=} \underbrace{\alpha_1 n^2 \left| x_1[n] \right| + \alpha_2 n^2 \left| x_2[n] \right|}_{y_3} \tag{10}$$

The answer is **no**.

Now test for the time invariance

The shifted input

$$x_2[n] = x_1[n - n_0] \tag{11}$$

produces the output

$$\begin{aligned} y_2[n] &= n^2 \left| x_2[n] \right| \\ &= n^2 \left| x_1[n - n_0] \right| . \end{aligned} \tag{12}$$

But

$$y_2[n - n_0] = [n - n_0]^2 \left| x_1[n - n_0] \right| . \tag{13}$$

This is not equal to

$$n^2 \left| x_1[n - n_0] \right| \tag{14}$$

So this system is <u>**time varying**</u>**.**

<u>Stability</u>

A sequence $x[n]$ is bounded if there exists a finite M such that $\left| x[n] \right| < M$ for all n.

A discrete-time system is bounded input-bounded output (**BIBO**) stable if every bounded input sequence $x[n]$ produces a bounded output sequences.

Ex] Consider a system shown above that

$$y[n] = n^2 x[n], \qquad 0 \le n \le \infty \tag{15}$$

where the input

$$x[n] = A \cdot u[n] . \tag{16}$$

Is this stable system?

Causality

A discrete time system is causal if the output at $n = n_0$ depends only on the input for $n \le n_0$.

Difference equation structures

The most general expression of the difference equation is

$$
\begin{aligned}
& a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] + \ldots + a_N y[n-N] \\
& = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \ldots + b_M x[n-M]
\end{aligned}
\tag{17}
$$

$$\sum_{k=0}^{N} a_k y\left[n-k\right] = \sum_{k=0}^{M} b_k x\left[n-k\right] \tag{18}$$

N : # of past output

M : # of past input

Once a_0 is equal to one, the above equation can be re-organized to obtain a new general expression for $y[n]$

$$
\begin{aligned}
y[n] &= -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b[k] x[n-k] \\
&= -a_1 y[n-1] - a_2 y[n-2] - \ldots - a_N y[n-N] \\
&\quad + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \ldots + b_M x[n-M]
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
y[n] &= \sum_{k=0}^{M} b[k] x[n-k] \\
&= b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \ldots + b_M x[n-M]
\end{aligned} \tag{20}
$$

What is difference between two shown above?

Ex 4.3]

$$y[n] = 0.5x[n] - 0.3x[n-1] \tag{21}$$

a. Identify all coefficients a_k and b_k

b. Is this recursive or non-recursive difference equation?

c. For input $x[n] = \sin\left(\frac{n2\pi}{9}\right)u[n]$, find the first 20 samples of the output.

Ans

a. $a_0 = 1,\ b_0 = 0.5,\ b_1 = \text{-}0.3$

b. Non-recursive

c.

```
y =

  Columns 1 through 9

         0    0.3214    0.2996    0.1376   -0.0888   -0.2736   -0.3304   -0.2326   -0.0260

  Columns 10 through 18

    0.1928    0.3214    0.2996    0.1376   -0.0888   -0.2736   -0.3304   -0.2326   -0.0260

  Columns 19 through 20

    0.1928    0.3214
```

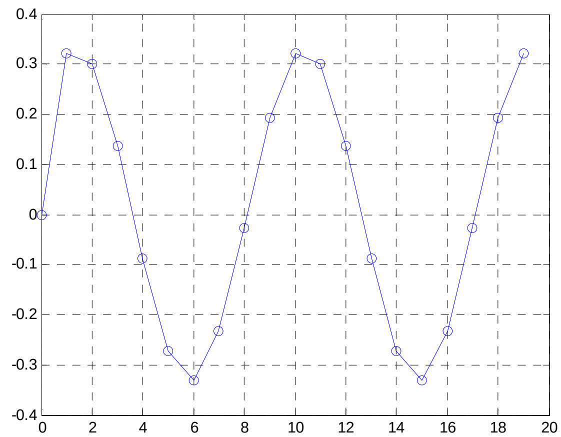


Figure 7: $x[n] = \sin\left(\frac{n2\pi}{9}\right)u[n]$

Ex 4.2] A filter has the difference equation

$$y[n] = 0.5y\left[n-1\right] + x\left[n\right] \tag{22}$$

a. Identify all coefficients a_k and b_k

b. Is this recursive or non-recursive difference equation?

c. If the input $x[n]$ is as given in the figure, find the first 12 samples of the output starting with $n = 0$.

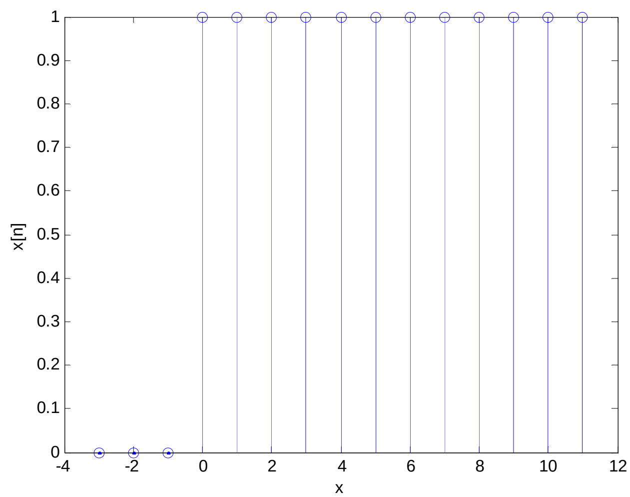


Figure 8: Input to the system

<u>**Ans**</u>

a. $a_0 = 1,\ a_1 = -0.5,\ b_0 = 1$

b. Recursive.

c.

```
ans =

  Columns 1 through 6

    1.0000    1.5000    1.7500    1.8750    1.9375    1.9688

  Columns 7 through 12

    1.9844    1.9922    1.9961    1.9980    1.9990    1.9995
```

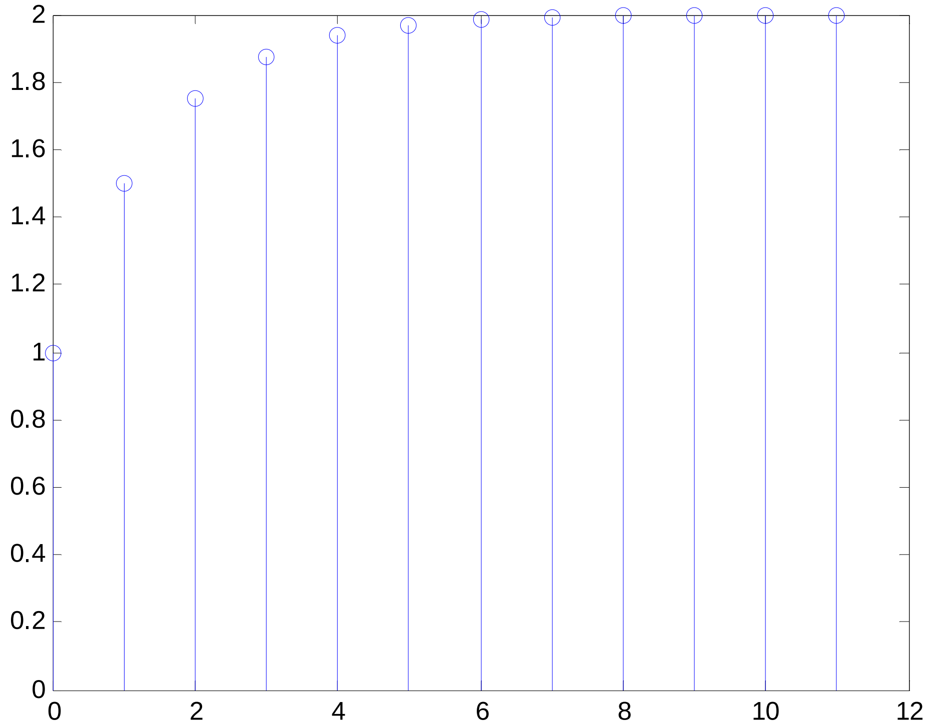


Figure 9: Output of the system