Midterm Exam II

11/24/2020

If you provide only solutions then you don’t get credits.

You are not allowed to use of computer tools, such as Matlab, Octave, Maple, Mathematica, etc..… If there are any traces then you get zero points.

Calculator is ok (Cell phone is not going to be used as a calculator)

1. [7 7 6 20 pts]

Frequency response of $x[n]$

$\{X[k=0]=12,\ \ X[k=6]=3+j36.3731,\ \ X[k=12]=3-j36.3731\}$

and other values of $X[k]$ are zeros.

a) Find 6 point DFT
b) Find 4 point FFT
c) Find $DFT\{x[-4]\ \ x[-3]\ \ x[-2]\}$

$X_6[k]=\begin{bmatrix}4\\0\\1+j12.1244\\0\\1-j12.1244\\0\end{bmatrix}$	$FFT\{x[n]\}=\begin{bmatrix}2\\-3+j3\\8\\-3-j3\end{bmatrix}$	$X[k]=\begin{bmatrix}2\\0.5+j6.0622\\0.5-j6.0622\end{bmatrix}$

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a)	Since only non-zero values of $[X[k]\ \ k=0,\ 6,\ 12]$, it is a periodic function. $X[k]=F\{\mathrm{x\ x\ x\ x\ x\ x}\}$ $x[n]=F^{-1}\left\{\frac{1}{6}\cdot[12\ \ 3+j36.3731\ \ 3-j36.3731]\right\}$ $=[1\ \ -3\ \ 4]$ $X_6[k]=F\{\mathrm{x\ x}\}$ $=\begin{bmatrix}4\\0\\1+j12.1244\\0\\1-j12.1244\\0\end{bmatrix}$
b)	$FFT\{x[n]\}=F\{[1\ \ -3\ \ 4\ \ 0]\}$ $=\begin{bmatrix}2\\-3+j3\\8\\-3-j3\end{bmatrix}$
c)	$X[k]=\sum_{n=-4,-3,-2}[4,1,-3]\cdot e^{\frac{-j2\pi kn}{N}}=\sum_{n=-1,0,1}[4,1,-3]\cdot e^{\frac{-j2\pi kn}{N}}$ $=\begin{bmatrix}2\\0.5+j6.0622\\0.5-j6.0622\end{bmatrix}$

```
clc; clear;
% x = [1 -3 4];
x = [4 1 -3];

N = length(x);

n = [-1 0 1];
n = [-4 -3 -2]
k = [0 1 2];


X = exp(-j*2*pi*k'*n/N)*x'
X
[real(k') X]
```

2. [7 7 6 20 pts]

Signal $x[n]$ & $h[n]$ are defined as

$$x[n]=-2\delta[-n-1]+3\delta[-n]-4\delta[-n+1] \qquad h[n]=-3\delta[n+5]-4\delta[n+4]-3\delta[n+3]$$

a) Do the circular convolution, $y[n]=x[n]\otimes_3 h[n]$ where $y[n=0,1,2]$

b) Do the discrete convolution (DTFT domain), $y[n]=x[n]*h[n]$

c) Find DFT (discrete FT) response of $x[n]$ & $h[n]$ and write $\left[X[k]\ \&\ Y[k]\ \ where\ k=0,1,2\right]$

<table>
<tr><td>a)</td><td colspan="2">$$\begin{aligned} x[n]&=-2\delta[-n-1]+3\delta[-n]-4\delta[-n+1]\\ &=-2\delta[n+1]+3\delta[n]-4\delta[n-1]\end{aligned}$$
$$h[n]=-3\delta[n+5]-4\delta[n+4]-3\delta[n+3]$$
$$\begin{aligned} y[n]&=x[n]\otimes_3 h[n]\\ &=[3\ \ -4\ \ -2]\otimes_3[-3\ \ -3\ \ -4]\\ &=[13\ \ 11\ \ 6]\end{aligned}$$</td></tr>
<tr><td>b)</td><td colspan="2">$$x[n]=-2\delta[n+1]+3\delta[n]-4\delta[n-1]$$
$$h[n]=-3\delta[n+5]-4\delta[n+4]-3\delta[n+3]$$
$$\begin{aligned} y[n]&=x[n]*h[n]\\ &=6\delta[n+6]+8\delta[n+5]+6\delta[n+4]\\ &\qquad -9\delta[n+5]-12\delta[n+4]-9\delta[n+3]\\ &\qquad\qquad +12\delta[n+4]+16\delta[n+3]+12\delta[n+2]\\ &=6\delta[n+6]-\delta[n+5]+6\delta[n+4]+7\delta[n+3]+12\delta[n+2]\end{aligned}$$</td></tr>
<tr><td>c)</td><td>$$x[n]=-2\delta[n+1]+3\delta[n]-4\delta[n-1]$$
$$X[k=0,1,2]=\sum_{n=-1,0,1}[-2\ 3\ -4]\cdot e^{\frac{-j2\pi nk}{3}}$$
$$=\begin{bmatrix}-3\\ 6\ +\ j1.7321\\ 6\ -\ j1.7321\end{bmatrix}$$</td><td>$$h[n]=-3\delta[n+5]-4\delta[n+4]-3\delta[n+3]$$
$$H[k=0,1,2]=\sum_{n=-5,-4,-3}[-3\ -4\ -3]\cdot e^{\frac{-j2\pi nk}{3}}$$
$$=\begin{bmatrix}-10\\ 0.5\ -\ j0.866\\ 0.5\ +\ j0.866\end{bmatrix}$$</td></tr>
</table>

```
clc; clear;

x = [-2 3 -4];
n = [-1 0 1];
k = [0 1 2];
X = exp(-j*2*pi*k'*n/3)*x'

y = [-3 -4 -3];
n = [-5 -4 -3];
k = [0 1 2];
Y = exp(-j*2*pi*k'*n/3)*y'
```

3. [10 5 pts]

A couple of sinusoidal signals are buried in the AWGN (Additive White Gaussian Noise) signals. You are going to recover the two sinusoidal signals and clean up the noise. The two sinusoidal signals shown in the frequency domain are at $36Hz$ and $105Hz$.

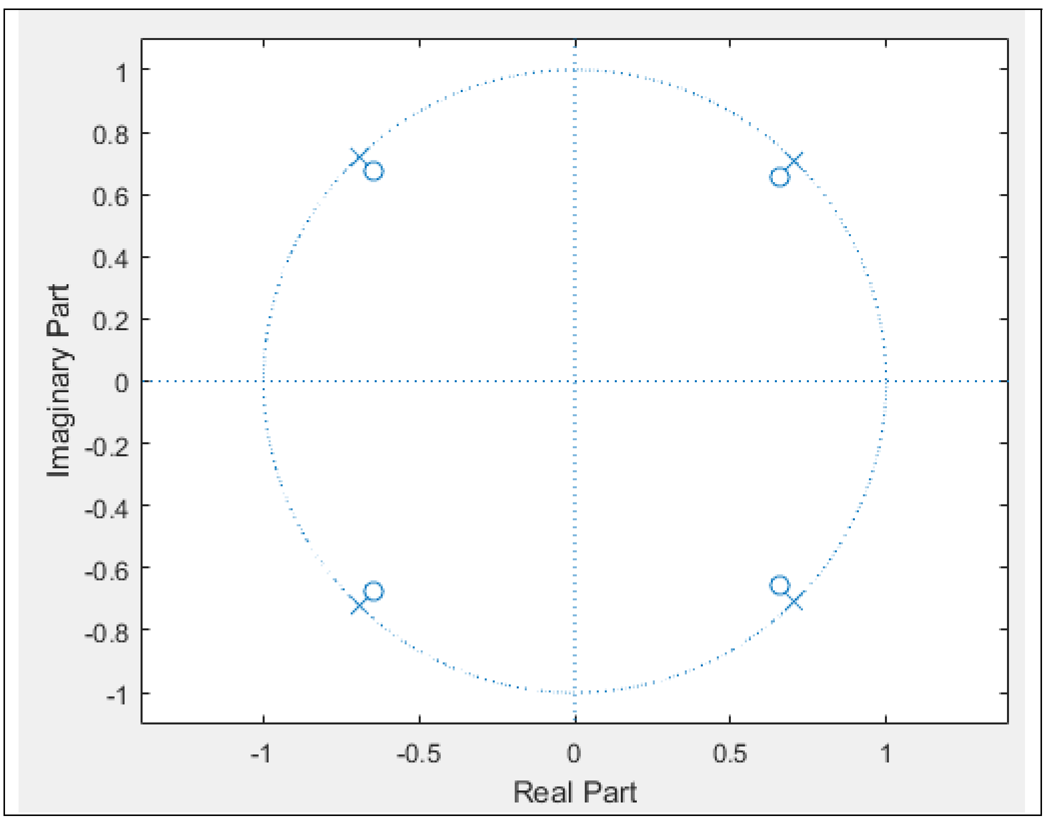
<u>Design</u> <u>a stable fourth order IIR filter</u> that can do the recovery of the signals. The signals were sampled with sampling rate of $fs = 280$.

a) You don't have to expand the system function. Simply write the system function in the given space.

$$36Hz \Rightarrow \frac{36}{140}\cdot\pi = 0.257\pi \qquad 105Hz \Rightarrow \frac{105}{140}\cdot\pi = 0.75\pi$$

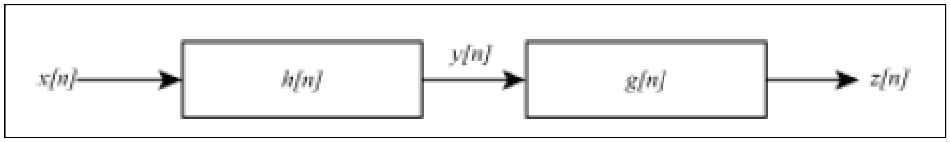
$$H(z) = \frac{\left(z-0.99e^{j0.257\pi}\right)\left(z-0.99e^{-j0.257\pi}\right)\left(z-0.99e^{j0.75\pi}\right)\left(z-0.99e^{-j0.75\pi}\right)}{\left(z-0.999e^{j0.257\pi}\right)\left(z-0.999e^{-j0.257\pi}\right)\left(z-0.999e^{j0.75\pi}\right)\left(z-0.999e^{-j0.75\pi}\right)}$$

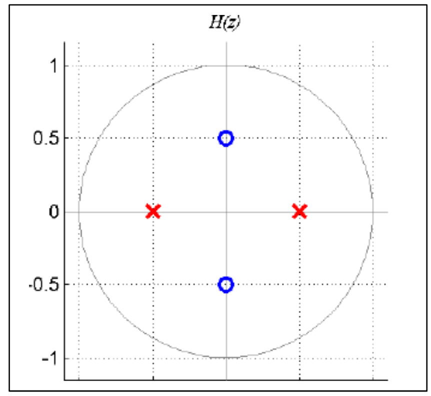
b) Locate the zeros and poles in the pole zero plots



4. [10 5 15 pts]

A real-time system comprises a cascade of two discrete-time filters. The first filter, with impulse response $h[n]$, is defined by the pole-zero plot shown below.





a) Find $h[n]$ and values of $h[n]$ $where\ n = 0,1,2,3$ assume that $h[n<0] = 0$

b) Find the difference equation of a second system, characterized by $g[n]$, such that $z[n] = x[n]$

a)	$H(z) = \frac{Y(z)}{X(z)} = \frac{(z-j0.5)(z+j0.5)}{(z-0.5)(z+0.5)} = \frac{z^2+0.25}{z^2-0.25}$ $Y(z)(z^2-0.25) = X(z)(z^2+0.25)$ $y[n]-0.25y[n-2] = x[n]+0.25x[n-2]$ $h[n] = 0.25h[n-2]+\delta[n]+0.25\delta[n-2]$ $h[0]=1,\ \ h[1]=0,\ \ h[2]=0.25+0.25=0.5,\ \ h[3]=0$	$H(z) = \frac{z^2+0.25}{z^2-0.25} = 1+\frac{0.5}{z^2-0.25}$ $=1+\frac{0.5}{(z-0.5)(z+0.5)}$ $=1+\frac{0.5}{(z-0.5)}-\frac{0.5}{(z+0.5)}$ $h[n] = \delta[n]+\left(\left[0.5\cdot(0.5)^{(n-1)}\right]-\left[0.5\cdot(-0.5)^{(n-1)}\right]\right)u[n-1]$ $h[0]=1,\ \ h[1]=0+0.5-0.5=0,$ $h[2]=0.25+0.25=0.5,\ \ h[3]=0.25-0.25=0$
b)	It has to be $H(z)\cdot G(z)=1\ such\ that\ Z^{-1}\{H(z)\cdot G(z)\}=\delta[n]$ $H(z)=\frac{(z^2+0.25)}{(z^2-0.25)}=\frac{1}{G(z)}$ $G(z)=\frac{(z^2-0.25)}{(z^2+0.25)}=\frac{Z(z)}{Y(z)}$ $Z(z)(z^2+0.25)=Y(z)(z^2-0.25)$ $z[n]=-0.25z[n-2]+y[n]-0.25y[n-2]$	

5. [7 7 6 20 pts]

A transfer function of causal system is defined as

$$H(z)=\frac{(z-2)^2\cdot(z-1)}{(z^2-4)^2}$$

a) Find impulse response using partial fraction expansion

b) Find $h[0]$, $h[1]$, and $h[2]$

c) Find input and output difference equation

a)	$h[n]=\left(n(-2)^{n-1}u[n]\right)-\left((n-1)(-2)^{n-2}u[n-1]\right)$
b)	$h[0]=0,\ h[1]=1,\ h[2]=2(-2)-1=-5$
c)	$y[n]=-4y[n-1]-4y[n-2]+x[n-1]-x[n-2]$

<table>
<tr><td>a)</td><td>$H(z)=\frac{(z-2)^2\cdot(z-1)}{(z^2-4)^2}$
$=\frac{(z-2)^2\cdot(z-1)}{(z^2-4)(z^2-4)}$
$=\frac{(z-2)(z-2)\cdot(z-1)}{(z-2)(z+2)(z-2)(z+2)}$
$=\frac{(z-1)}{(z+2)(z+2)}=\frac{(z-1)}{(z+2)^2}$</td><td>$Z^{-1}\left[\frac{(z-1)}{(z+2)^2}\right]=Z^{-1}\left[\frac{z}{(z+2)^2}-\frac{z}{(z+2)^2}\cdot z^{-1}\right]$

$h[n]=\left(n(-2)^{n-1}u[n]\right)-\left((n-1)(-2)^{n-2}u[n-1]\right)$</td></tr>
<tr><td>b)</td><td colspan="2">$h[n]=\left(n(-2)^{n-1}u[n]\right)-\left((n-1)(-2)^{n-2}u[n-1]\right)$

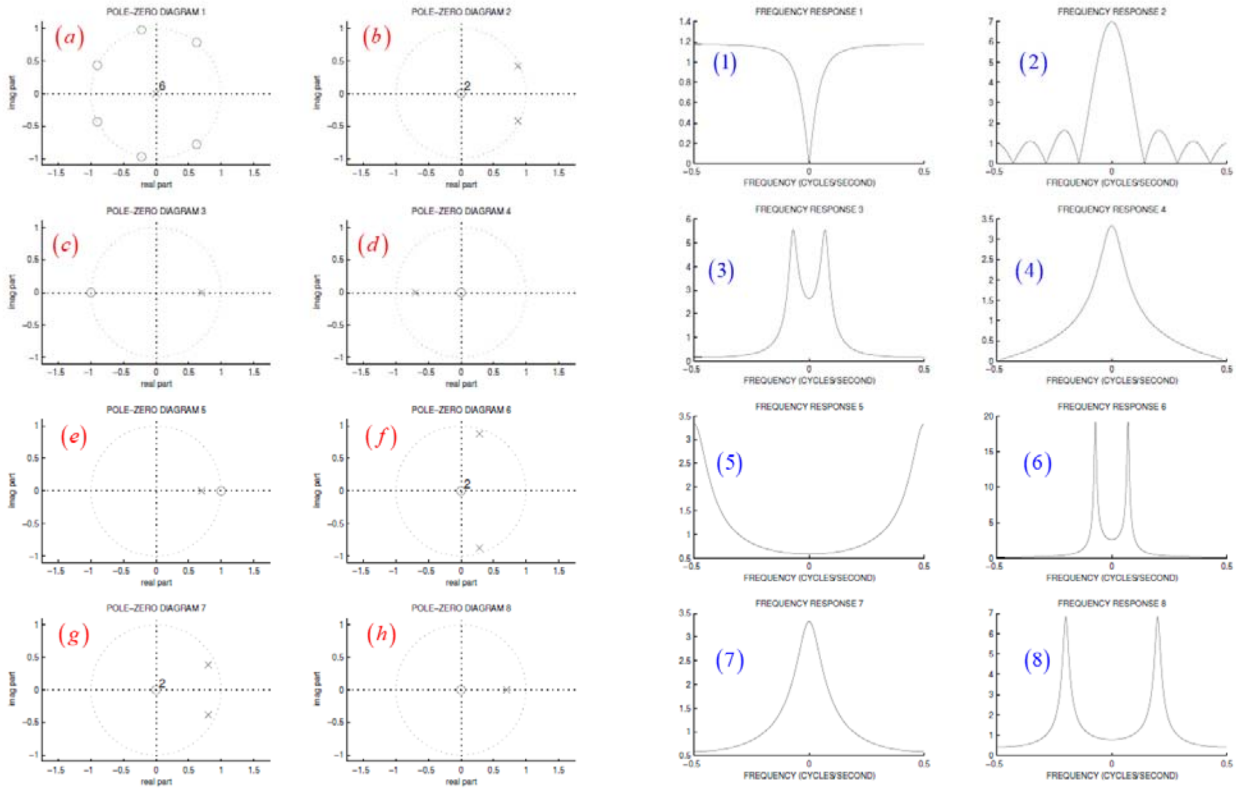
$h[0]=0,\ h[1]=1,\ h[2]=2(-2)-1=-5$</td></tr>
<tr><td>c)</td><td colspan="2">$H(z)=\frac{(z-1)}{(z+2)^2}=\frac{Y(z)}{X(z)}$

$Y(z)(z+2)^2=X(z)(z-1)$
$Y(z)(z^2+4z+4)=X(z)(z-1)$
$y[n+2]+4y[n+1]+4y[n]=x[n+1]-x[n]$

$y[n]=-4y[n-1]-4y[n-2]+x[n-1]-x[n-2]$</td></tr>
</table>

6. [16 pts]

The diagrams show the pole-zero plots, and frequency responses magnitudes of 8 discrete-time causal LTI systems. But the diagrams are out of order. Match each diagram by filling out the following table.



(a)	2
(b)	6
(c)	7
(d)	5
(e)	1
(f)	8
(g)	3
(h)	4