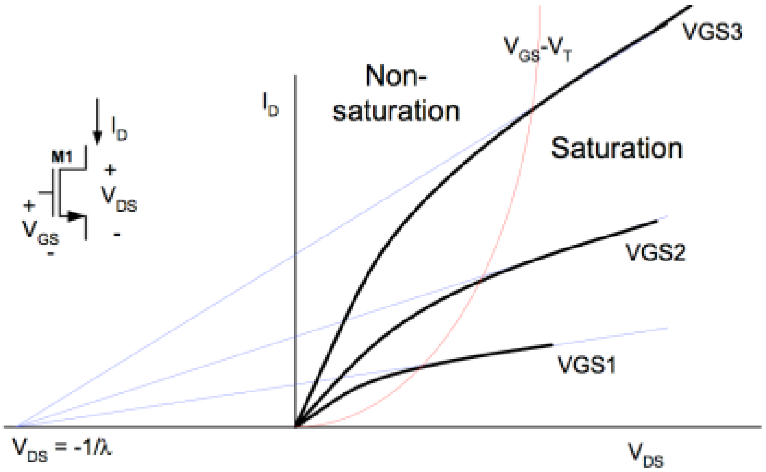
Nonlinearity: MOS Equation

1. MOSFET Equations

a) N-channel MOSFET

Cut Off	$V_{GS} \le V_T$	$I_{DS} = 0$
Linear	$V_{GS} > V_T$, $V_{DS} \le V_{GS} - V_T$	$I_{DS} = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right](1 + \lambda V_{DS})$
Saturation	$V_{GS} > V_T$, $V_{DS} > V_{GS} - V_T$	$I_{DS} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_T)^2(1 + \lambda V_{DS})$



The simplest model in SPICE (Level 1 or default model) uses the above equations.

Parameter	SPICE Parameter	Units	Typical Values
$\mu_n C_{ox}$	KP	A/V^2	200µ
V_{T0}	VTO	V	0.5 – 1.0
λ	LAMBDA	V^{-1}	0.05 – 0.005

Nonlinearity: Memoryless and Static System

$$y(t) = \alpha x(t),$$

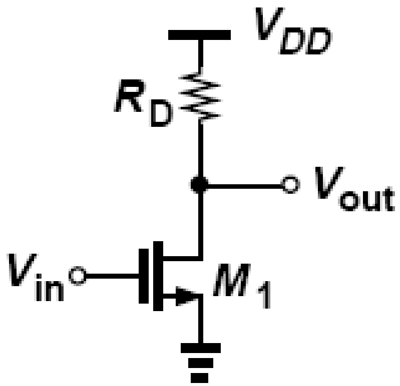
linear

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$

nonlinear

- **The input/output characteristic of a memoryless nonlinear system can be approximated with a polynomial**

$$\begin{aligned} V_{out} &= V_{DD} - I_D R_D \\ &= V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2 R_D \end{aligned}$$



- **In this idealized case, the circuit displays only second-order nonlinearity**

Example of Polynomial Approximation

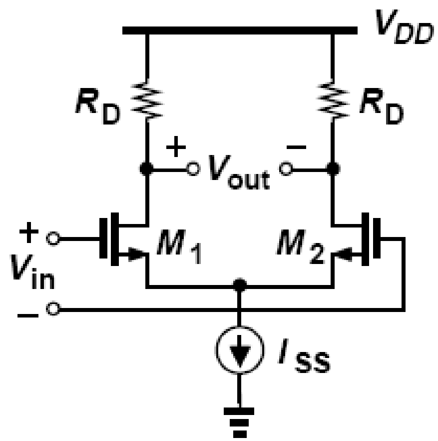
For square-law MOS transistors operating in saturation, the characteristic above can be expressed as

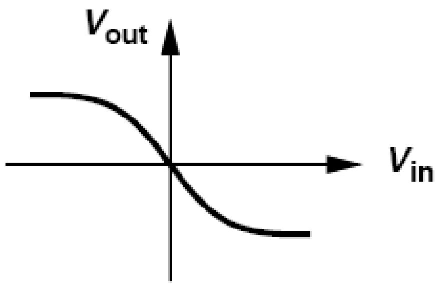
$$V_{out} = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{in}^2}\,R_D$$

If the differential input is small, approximate the characteristic by a polynomial.

Factoring $4I_{ss} / (\mu_n C_{ox} W/L)$ out of the square root and assuming

$$V_{in}^2 \ll \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$





Approximation gives us:

$$V_{out} \approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}V_{in}}\left(1 - \frac{\mu_n C_{ox}\frac{W}{L}}{8I_{SS}}V_{in}^2\right)R_D$$

$$\approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}R_D V_{in} + \frac{\left(\mu_n C_{ox}\frac{W}{L}\right)^{3/2}}{8\sqrt{I_{SS}}}R_D V_{in}^3$$

Example of Polynomial Approximation

$$V_{out} = V_{out1} - V_{out2}$$

$$= \left(V_{DD} - \frac{1}{2}\mu C\frac{W}{L}(V_{in1} - V_{th})^2 R_D\right) - \left(V_{DD} - \frac{1}{2}\mu C\frac{W}{L}(V_{in2} - V_{th})^2 R_D\right)$$

$$= \frac{-1}{2}\mu C\frac{W}{L}(V_{in1} + V_{in2} - 2V_{th})(V_{in1} - V_{in2})R_D$$

$$(V_{in1} + V_{in2} - 2V_{th}) = \sqrt{\frac{4I_{ss}}{\mu C\frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

$$\therefore V_{out} = -\frac{1}{2}\mu C\frac{W}{L} V_{in}\sqrt{\frac{4I_{ss}}{\mu C\frac{W}{L}} - V_{in}^2}\; R_D$$

$$I_{ss} = I_{D1} + I_{D2} = \frac{1}{2}\mu C\frac{W}{L}\left[(V_1 - V_{th})^2 + (V_2 - V_{th})^2\right]$$

$$\frac{4I_{ss}}{\mu C\frac{W}{L}} = 2\left[(V_1 - V_t)^2 + (V_2 - V_t)^2\right]$$

$$\therefore \sqrt{\frac{4I_{ss}}{\mu C\frac{W}{L}} - (V_{in})^2} = \sqrt{2\left[(V_1 - V_t)^2 + (V_2 - V_t)^2\right] - (V_1 - V_2)^2}$$

$$= \sqrt{2V_1^2 - 4V_1V_t + 2V_t^2 + 2V_2^2 - 4V_2V_t + 2V_t^2 - (V_1^2 + V_2^2 - 2V_1V_2)}$$

$$= (V_1 + V_2 - 2V_t)\ \#$$

Example of Polynomial Approximation

$$I_D = \frac{1}{2}\mu C \frac{W}{L}(V_{in} - V_{th})^2$$

$$g_m = \frac{\Delta I_D}{\Delta V_{gs}} \simeq \frac{\Delta I_D}{\Delta V_{in}} = \mu C \frac{W}{L}(V_{in} - V_{th})$$

also $(V_{in} - V_{th}) = \sqrt{\frac{2 I_D}{\mu C}\left(\frac{L}{W}\right)}$

$$\therefore g_m = \sqrt{\mu C \frac{W}{L}(2 I_D)} = \sqrt{\mu C \frac{W}{L} I_{SS}} \quad \#$$

Design Example

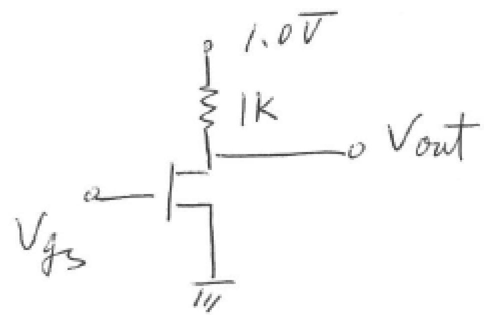
$$I = 100\mu \left(\frac{W}{L}\right)(V_{GS} - V_t)^2$$

$$g_m = 200\mu \left(\frac{W}{L}\right)(V_{GS} - V_t)$$

Let $V_{GS} - V_t = 0.3\,\mathrm{V}$

$$I = 100\mu \left(\frac{W}{L}\right)(0.09), \quad g_m = 200\mu \left(\frac{W}{L}\right)(0.3)$$

Analyze $\frac{W}{L} = 10,\ 100$ for $\left(\frac{V_{out}}{V_{gs}}\right) = \text{Gain}$



Effects of Nonlinearity: Harmonic Distortion

$$
\begin{aligned}
y(t) &= \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t \\
&= \alpha_1 A \cos \omega t + \frac{\alpha_2 A^2}{2}(1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3 \cos \omega t + \cos 3\omega t) \\
&= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right) \cos \omega t + \frac{\alpha_2 A^2}{2} \cos 2\omega t + \frac{\alpha_3 A^3}{4} \cos 3\omega t.
\end{aligned}
$$

Effects of Nonlinearity: Harmonic Distortion

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y(t) &= \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t \\
&= \alpha_1 A \cos \omega t + \frac{\alpha_2 A^2}{2}(1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3 \cos \omega t + \cos 3\omega t) \\
&= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right) \cos \omega t + \frac{\alpha_2 A^2}{2} \cos 2\omega t + \frac{\alpha_3 A^3}{4} \cos 3\omega t.
\end{aligned}$$

DC **Fundamental** **Second Harmonic** **Third Harmonic**

- **Even-order harmonics result from α_j with even j**
- **nth harmonic grows in proportion to A^n**

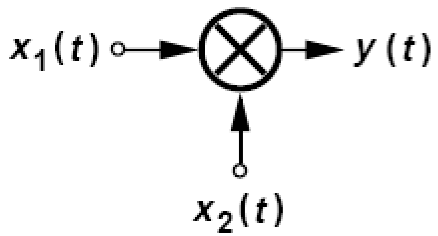
Example of Harmonic Distortion in Mixer

An analog multiplier "mixes" its two inputs below, ideally producing *$y(t)$* = *$kx_1(t)x_2(t)$*, where *k* is a constant. Assume *$x_1(t) = A_1$* cos *$\omega_1 t$* and *$x_2(t) = A_2$* cos *$\omega_2 t$*.

(a) If the mixer is ideal, determine the output frequency components.

(b) If the input port sensing *$x_2(t)$* suffers from third-order nonlinearity, determine the output frequency components.

Solution:



(a)

$$\begin{aligned} y(t) &= k(A_1 \cos \omega_1 t)(A_2 \cos \omega_2 t) \\ &= \frac{kA_1A_2}{2}\cos(\omega_1+\omega_2)t + \frac{kA_1A_2}{2}\cos(\omega_1-\omega_2)t. \end{aligned}$$

(b)

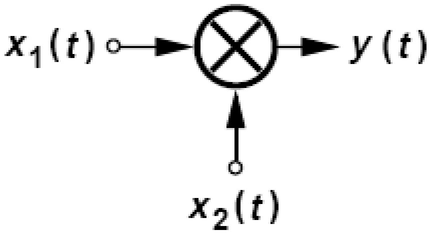
$$\begin{aligned} y(t) &= k(A_1\cos\omega_1 t)\left(A_2\cos\omega_2 t + \frac{\alpha_3 A_2^3}{4}\cos 3\omega_2 t\right) \\ &= \frac{kA_1A_2}{2}\cos(\omega_1+\omega_2)t + \frac{kA_1A_2}{2}\cos(\omega_1-\omega_2)t + \frac{k\alpha_3A_1A_2^3}{8}\cos(\omega_1+3\omega_2)t \\ &+ \frac{k\alpha_3A_1A_2^3}{8}\cos(\omega_1-3\omega_2)t. \end{aligned}$$

Example of Harmonic Distortion in Mixer

Find out the frequency components in the previous example (a) and (b)

Assuming a down conversion mixer where ω1 = 2.45GHz and ω2 = 2.451GHz.

Solution:



(a)
$$y(t) = k(A_1 \cos\omega_1 t)(A_2 \cos\omega_2 t) = \frac{kA_1A_2}{2}\cos(\omega_1+\omega_2)t + \frac{kA_1A_2}{2}\cos(\omega_1-\omega_2)t.$$

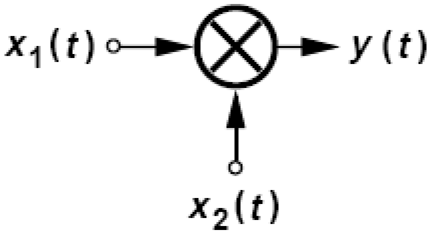
(b)
$$y(t) = k(A_1\cos\omega_1 t)\left(A_2\cos\omega_2 t + \frac{\alpha_3 A_2^3}{4}\cos 3\omega_2 t\right)$$
$$= \frac{kA_1A_2}{2}\cos(\omega_1+\omega_2)t + \frac{kA_1A_2}{2}\cos(\omega_1-\omega_2)t + \frac{k\alpha_3A_1A_2^3}{8}\cos(\omega_1+3\omega_2)t$$
$$+ \frac{k\alpha_3A_1A_2^3}{8}\cos(\omega_1-3\omega_2)t.$$

Example of Harmonic Distortion in Mixer

Find out the frequency components in the previous example (a) and (b)

Assuming a up conversion mixer where ω1 = 1MHz and ω2 = 2.45GHz.

Solution:



(a)
$$\begin{aligned} y(t) &= k(A_1\cos\omega_1 t)(A_2\cos\omega_2 t) \\ &= \frac{kA_1A_2}{2}\cos(\omega_1+\omega_2)t + \frac{kA_1A_2}{2}\cos(\omega_1-\omega_2)t. \end{aligned}$$

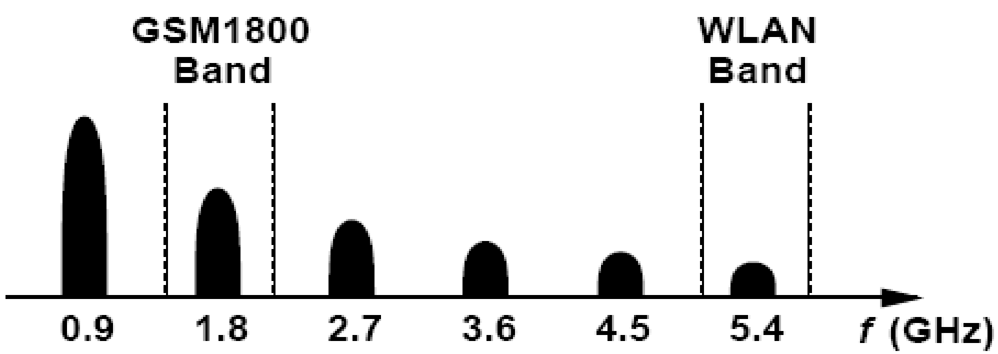
(b)
$$\begin{aligned} y(t) &= k(A_1\cos\omega_1 t)\left(A_2\cos\omega_2 t + \frac{\alpha_3A_2^3}{4}\cos 3\omega_2 t\right) \\ &= \frac{kA_1A_2}{2}\cos(\omega_1+\omega_2)t + \frac{kA_1A_2}{2}\cos(\omega_1-\omega_2)t + \frac{k\alpha_3A_1A_2^3}{8}\cos(\omega_1+3\omega_2)t \\ &+ \frac{k\alpha_3A_1A_2^3}{8}\cos(\omega_1-3\omega_2)t. \end{aligned}$$

Example of Harmonics on GSM Signal

The transmitter in a 900-MHz GSM cellphone delivers 1 W of power to the antenna. Explain the effect of the harmonics of this signal.

Solution:

The second harmonic falls within another GSM cellphone band around 1800 MHz and must be sufficiently small to negligibly impact the other users in that band. The third, fourth, and fifth harmonics do not coincide with any popular bands but must still remain below a certain level imposed by regulatory organizations in each country. The sixth harmonic falls in the 5-GHz band used in wireless local area networks (WLANs), e.g., in laptops. Figure below summarizes these results.

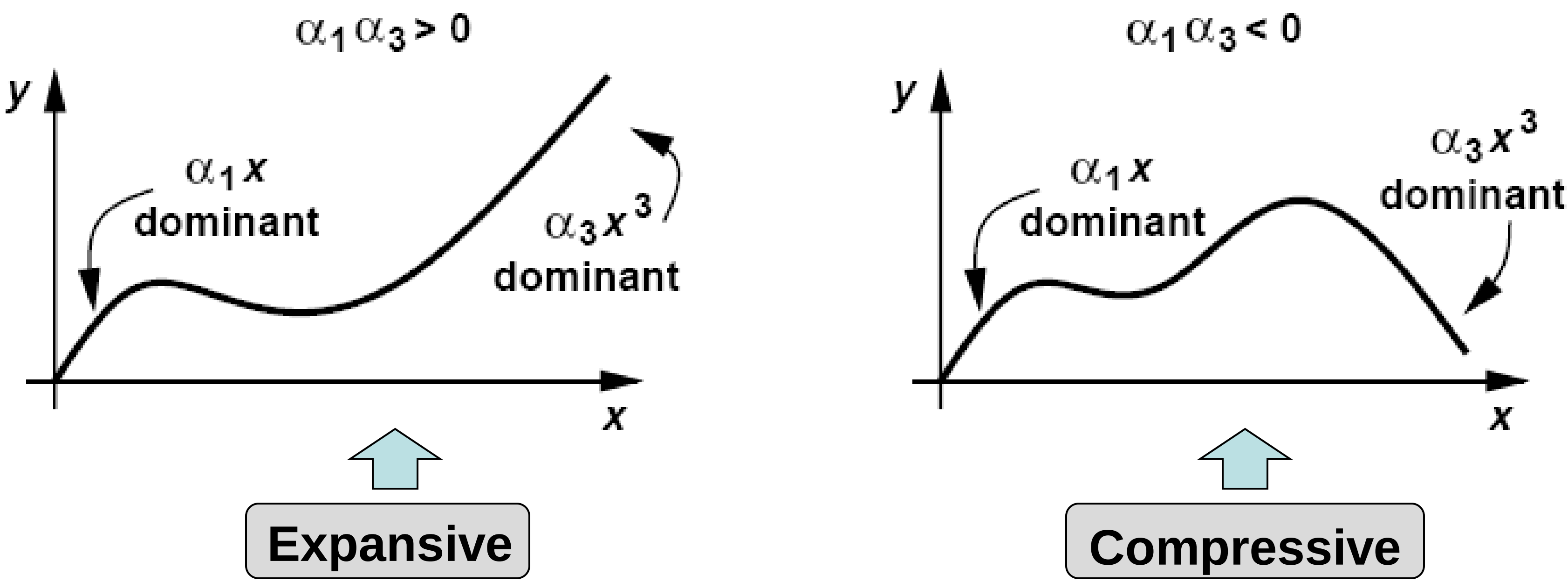


Example of Harmonics on GSM Signal

GSM-850	850	824.2 – 849.2	869.2 – 893.8	128 – 251	5
P-GSM-900	900	890.0 – 915.0	935.0 – 960.0	1 – 124	
E-GSM-900	900	880.0 – 915.0	925.0 – 960.0	975 – 1023, 0 - 124	8
R-GSM-900	900	876.0 – 915.0	921.0 – 960.0	955 – 1023, 0 - 124	
T-GSM-900	900	870.4 – 876.0	915.4 – 921.0	dynamic^	
DCS-1800	1800	1710.2 – 1784.8	1805.2 – 1879.8	512 – 885	3
PCS-1900	1900	1850.2 – 1909.8	1930.2 – 1989.8	512 – 810	2

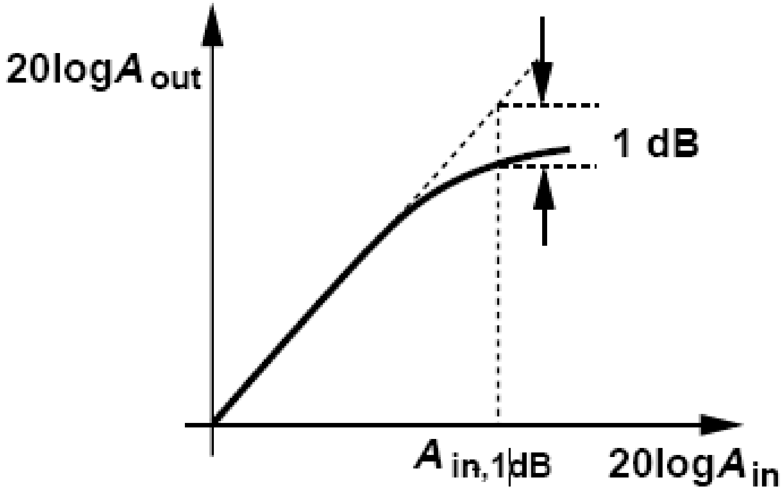
- bands 2 and 5 (shaded in blue) have been deployed in NAR and CALA (North American Region [Canada and the US], Caribbean and Latin America)
- bands 3 and 8 (shaded in yellow) have been deployed in EMEA and APAC (Europe, the Middle East and Africa, Asia-Pacific)

Gain Compression– Sign of α_1, α_3



- **Most RF circuit of interest are compressive, we focus on this type.**

Gain Compression: 1-dB Compression Point



$$20 \log \left| \alpha_1 + \frac{3}{4} \alpha_3 A_{in,1dB}^2 \right| = 20 \log |\alpha_1| - 1 \text{ dB}.$$

$$\boxed{A_{in,1dB} = \sqrt{0.145 \left| \frac{\alpha_1}{\alpha_3} \right|}.}$$

- **Output falls below its ideal value by 1 dB at the 1-dB compression point**

Gain Compression: 1-dB Compression Point

$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{in(1dB)}^2\right| = 20\log|\alpha_1| - 1\,dB$$

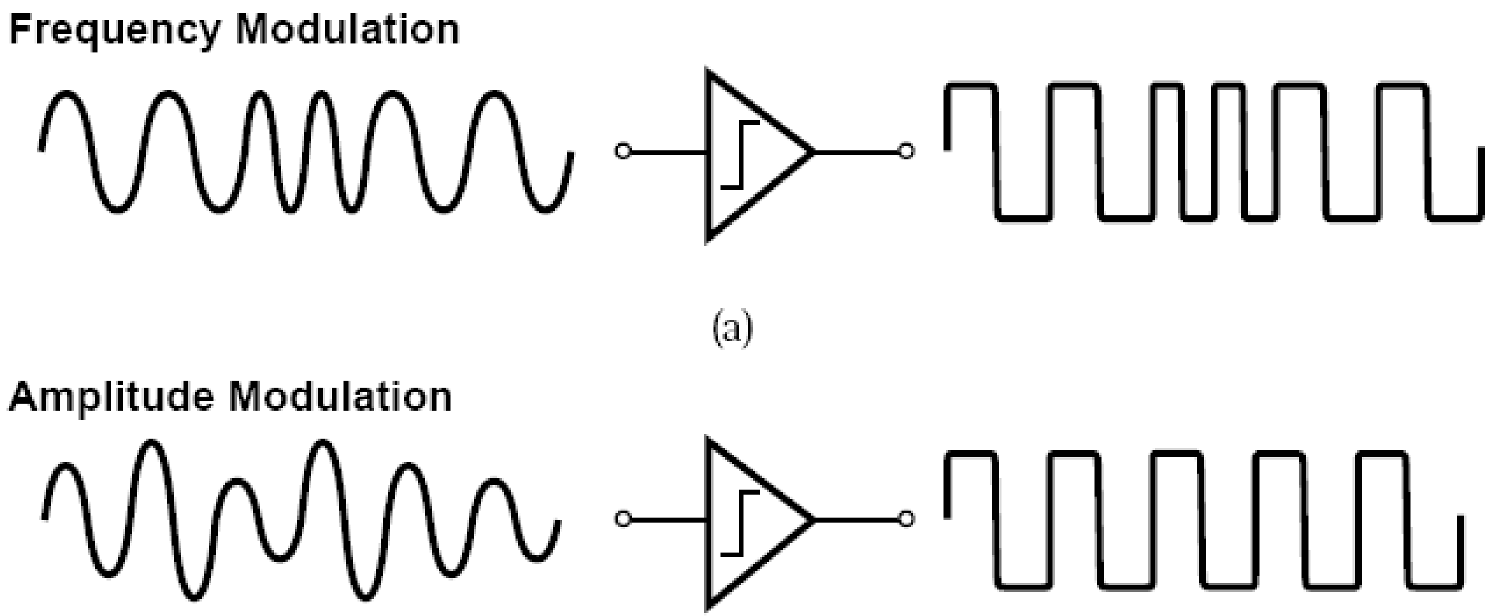
$$\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{in(1dB)}^2\right| = \log|\alpha_1| - 0.05$$

$$\alpha_1 + \frac{3}{4}\alpha_3 A_{in}^2(1dB) = \alpha_1 / 10^{0.05} = \frac{\alpha_1}{1.122}$$

$$\frac{3}{4}\alpha_3 A_{in(1dB)}^2 = -0.1087\alpha_1$$

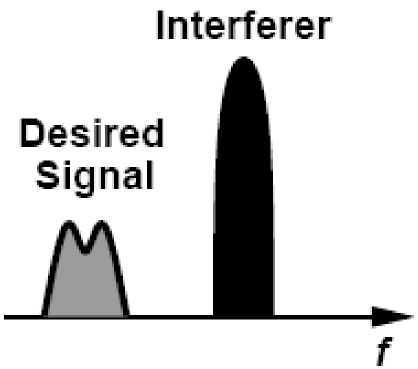
$$A_{in(1dB)}^2 = -0.145\frac{\alpha_1}{\alpha_3} = 0.145\left|\frac{\alpha_1}{\alpha_3}\right| \quad \text{since } \alpha_1\alpha_3 < 0$$

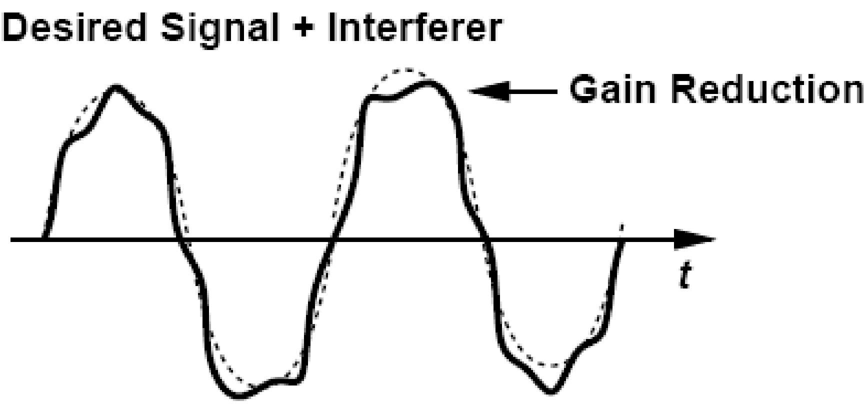
Gain Compression: Effect on FM and AM Waveforms



- FM signal carries no information in its amplitude and hence tolerates compression.
- AM contains information in its amplitude, hence distorted by compression

Gain Compression: Desensitization





Prove this $$y(t) = \left(\alpha_1 + \frac{3}{4}\alpha_3 A_1^2 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots.$$

For $A_1 << A_2$ $$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots$$

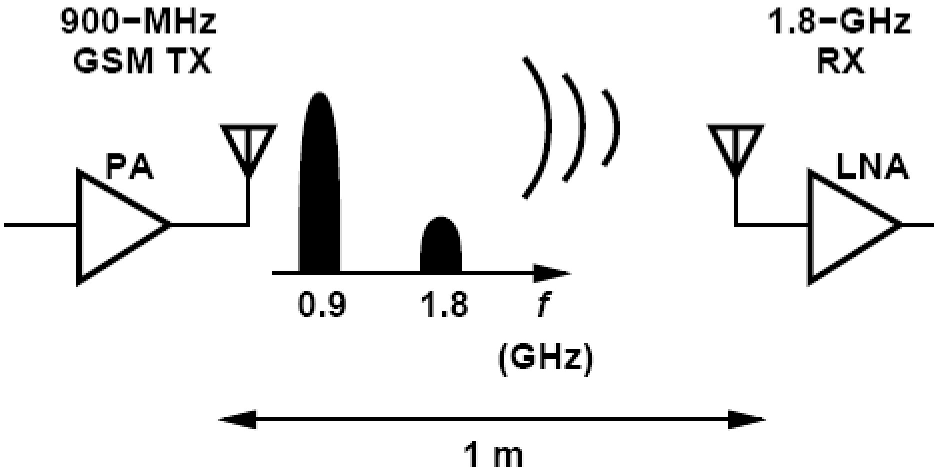
- **Desensitization: the receiver gain is reduced by the large excursions produced by the interferer even though the desired signal itself is small.**
- **When A2 is big enough, the overall gain may drop to zero, or the signal is "blocked"**

Example I of Gain Compression

A 900-MHz GSM transmitter delivers a power of 1 W to the antenna. By how much must the second harmonic of the signal be suppressed (filtered) so that it does not desensitize a 1.8-GHz receiver having P_{1dB} = -25 dBm? Assume the receiver is 1 m away and the 1.8-GHz signal is attenuated by 10 dB as it propagates across this distance.

Solution:

The output power at 900 MHz is equal to +30 dBm. With an attenuation of 10 dB, the second harmonic must not exceed -15 dBm at the transmitter antenna so that it is below P_{1dB} of the receiver. Thus, the second harmonic must remain at least 45 dB below the fundamental at the TX output. In practice, this interference must be another several dB lower to ensure the RX does not compress.



Example II of Gain Compression

Bluetooth specification requires that the maximum acceptable RX input power should be at least -10dBm. Assuming that this maximum acceptable RX input power is 3dB below the input power where "zero" gain happens, determine the P1dB of the receiver RX.

Solution:

Step 1: recall the P1dB equation in terms of $\alpha 1$ and $\alpha 3$

Step 2: derive power level at which the gain is zero, in terms of $\alpha 1$ and $\alpha 3$

Step 3: compare P1dB and zero-gain power level

Step 4: relate P1dB to -10dBm

Example II of Gain Compression

Bluetooth specification requires that the maximum acceptable RX input power should be at least -10dBm. Assuming that this maximum acceptable RX input power is 3dB below the input power where "zero" gain happens, determine the P1dB of the receiver RX.

Solution:

Step 1: $y(x) = (\alpha_1 A + \frac{3}{4}\alpha_3 A^3)\cos\omega t$

$A_{1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$

Step 2: $\alpha_1 A + \frac{3}{4}\alpha_3 A^3 = 0 \Rightarrow A_{Gain=0} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$

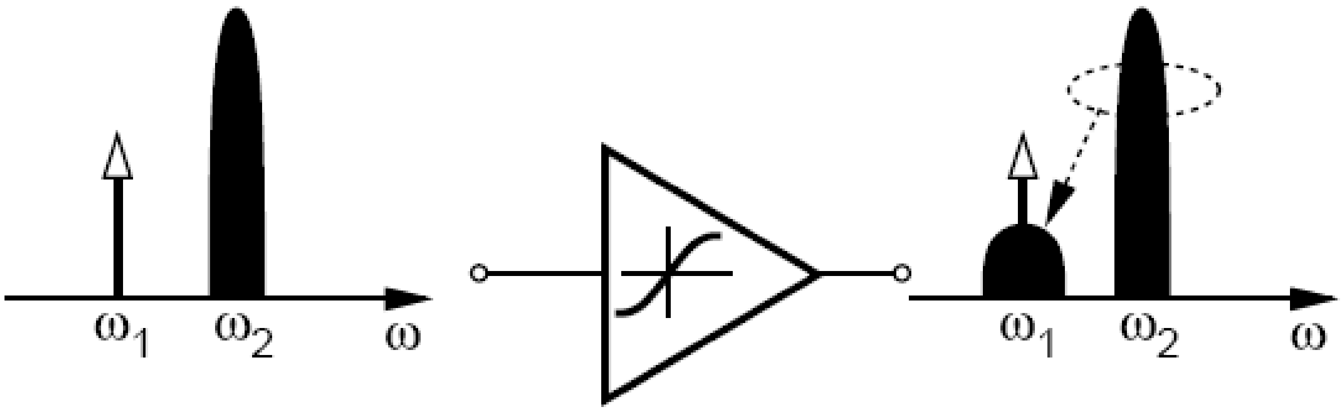
Step 3: $\frac{P_{Gain=0}}{P_{1dB}} = 20\lg\left(\frac{A_{Gain=0}}{A_{1dB}}\right) = 9.6dB$

Step 4: $P_{1dB} + 9.6dB = P_{Gain=0} = P_{in,max} + 3dB$

$\Rightarrow P_{1dB} + 6.6dB = P_{in,max} = -10dBm$

$\Rightarrow P_{1dB} = -16.6dBm$ #

Effects of Nonlinearity: Cross Modulation



Suppose that the interferer is an amplitude-modulated signal

$$A_2(1 + m \cos \omega_m t) \cos \omega_2 t$$

Thus

$$y(t) = \left[\alpha_1 + \frac{3}{2}\alpha_3 A_2^2 \left(1 + \frac{m^2}{2} + \frac{m^2}{2}\cos 2\omega_m t + 2m \cos \omega_m t\right)\right] A_1 \cos \omega_1 t + \cdots .$$

- **Desired signal at output suffers from amplitude modulation**

Example of Cross Modulation

Suppose an interferer contains phase modulation but not amplitude modulation. Does cross modulation occur in this case?

Solution:

Expressing the input as: $$x(t) = A_1 \cos \omega_1 t + A_2 \cos(\omega_2 t + \phi)$$

where the second term represents the interferer (*A_2* is constant but *Φ* varies with time)

$$\begin{aligned} y(t) &= \alpha_1[A_1 \cos \omega_1 t + A_2 \cos(\omega_2 t + \phi)] + \alpha_2[A_1 \cos \omega_1 t + A_2 \cos(\omega_2 t + \phi)]^2 \\ &+ \alpha_3[A_1 \cos \omega_1 t + A_2 \cos(\omega_2 t + \phi)]^3. \end{aligned}$$

We now note that (1) the second-order term yields components at $\omega_1 \pm \omega_2$ but not at ω_1; (2) the third-order term expansion gives $3\alpha_3 A_1 \cos \omega_1 t\, A_2^2 \cos^2(\omega_2 t+\Phi)$, which results in a component at ω_1. Thus,

$$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos \omega_1 t + \cdots.$$

The desired signal at ω_1 does not experience cross modulation

Effects of Nonlinearity

1. **Harmonics distortion**

2. **Gain compression**

3. **Cross modulation**

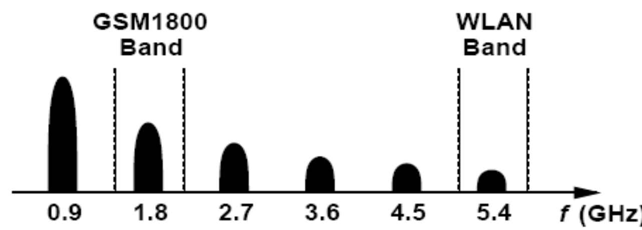
Effects of Nonlinearity

1. Harmonics distortion
2. Gain compression
3. Cross modulation
4. Inter-modulation
5. Cascaded effects
6. AM/PM conversion

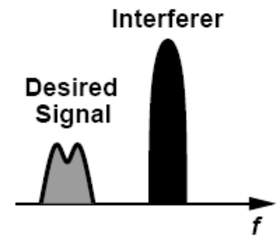
Effects of Nonlinearity: Intermodulation— Recall Previous Discussion

So far we have considered the case of:

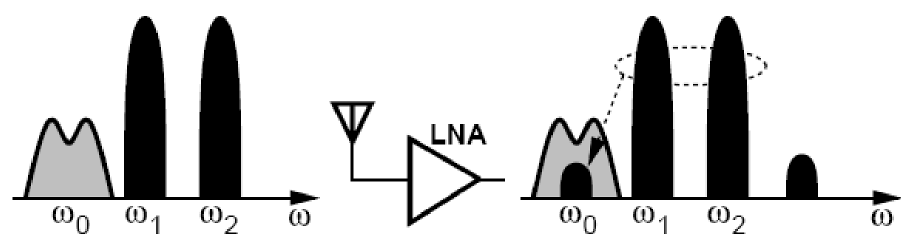
- **Single Signal** ⟶ **Harmonic distortion**



- **Signal + one large interferer** ⟶ **Desensitization**



- **Signal + two large interferers** ⟶ **Intermodulation**



Effects of Nonlinearity: Intermodulation

assume $x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$

Thus

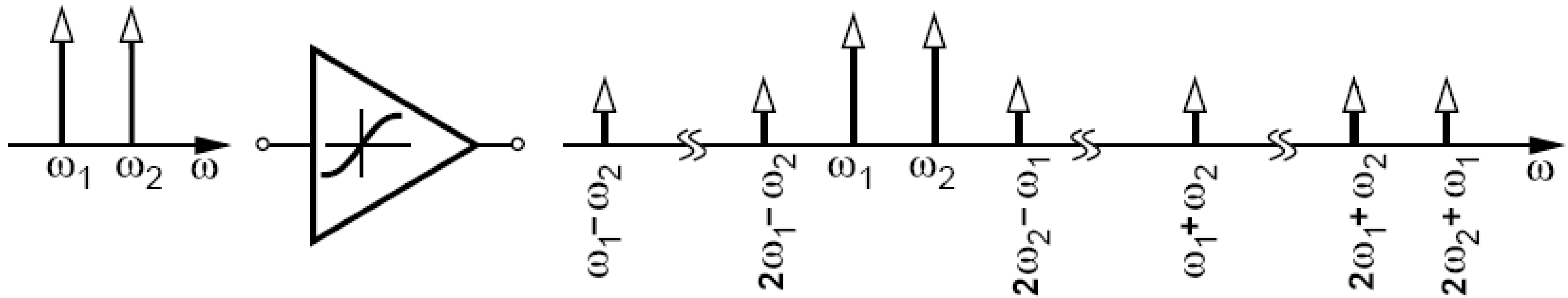
$$y(t) = \alpha_1(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + \alpha_2(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 + \alpha_3(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3$$

Intermodulation products:

$$\begin{aligned} \omega &= 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 - \omega_2)t \\ \omega &= 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2 - \omega_1)t \end{aligned}$$

Fundamental components:

$$\omega = \omega_1,\ \omega_2 : \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right) \cos \omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right) \cos \omega_2 t$$



Effects of Nonlinearity: Intermodulation

assume $x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$

Thus

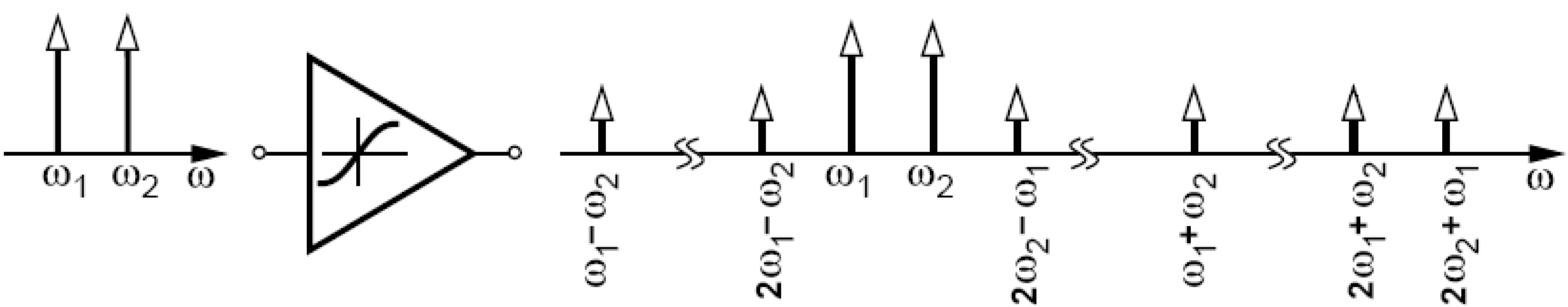
$$y(t) = \alpha_1 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + \alpha_2 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 + \alpha_3 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3$$

Intermodulation products:

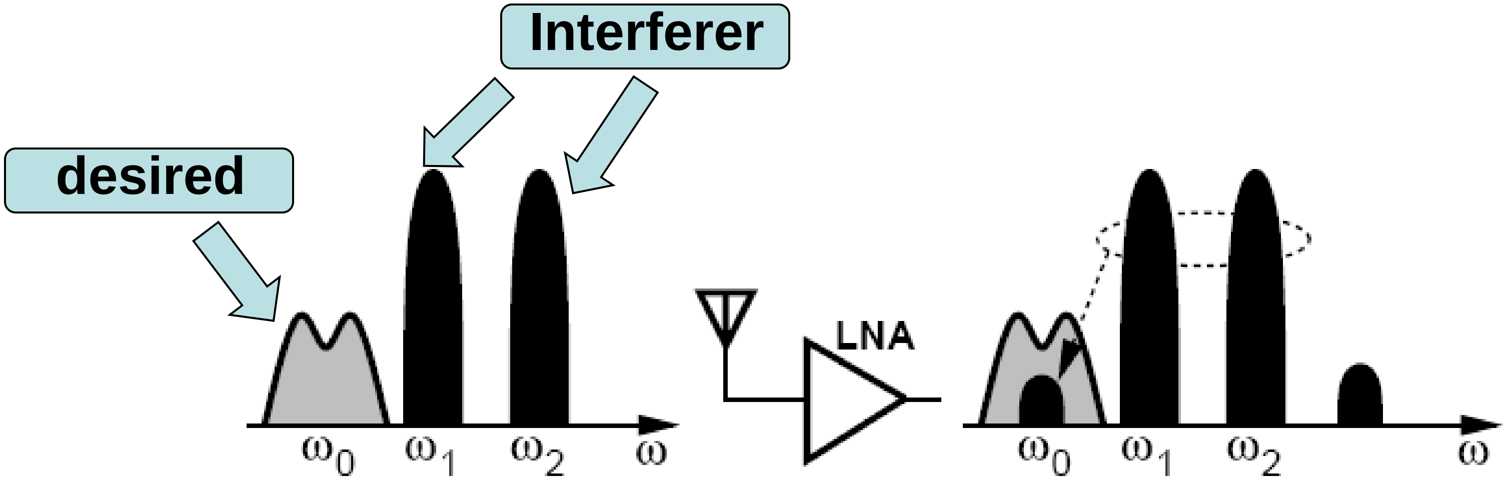
$$\begin{aligned} \omega &= 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 + \omega_2)t + \boxed{\frac{3\alpha_3 A_1^2 A_2}{4}} \cos(2\omega_1 - \omega_2)t \\ \omega &= 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_1 A_2^2}{4} \cos(2\omega_2 + \omega_1)t + \boxed{\frac{3\alpha_3 A_1 A_2^2}{4}} \cos(2\omega_2 - \omega_1)t \end{aligned}$$

Fundamental components:

$$\omega = \omega_1, \omega_2 : \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right) \cos \omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right) \cos \omega_2 t$$



Intermodulation Product Falling on Desired Channel



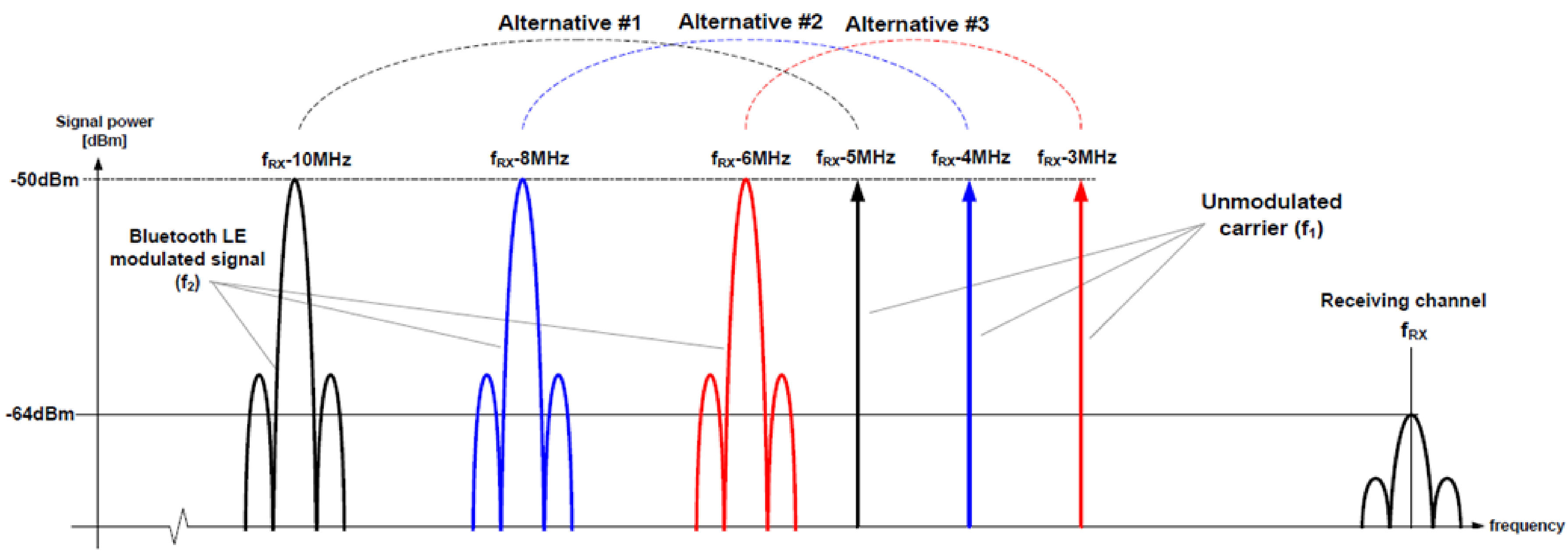
$$2\omega_1 - \omega_2 = \omega_0$$

- **A received small desired signal along with two large interferers**
- **Intermodulation product falls onto the desired channel, corrupts signal.**

Intermodulation Product Falling on Desired Channel

3. Intermodulation performance (related to LNA/Mixer linearity)
This test verifies that the receiver inter-modulation performance is satisfactory.

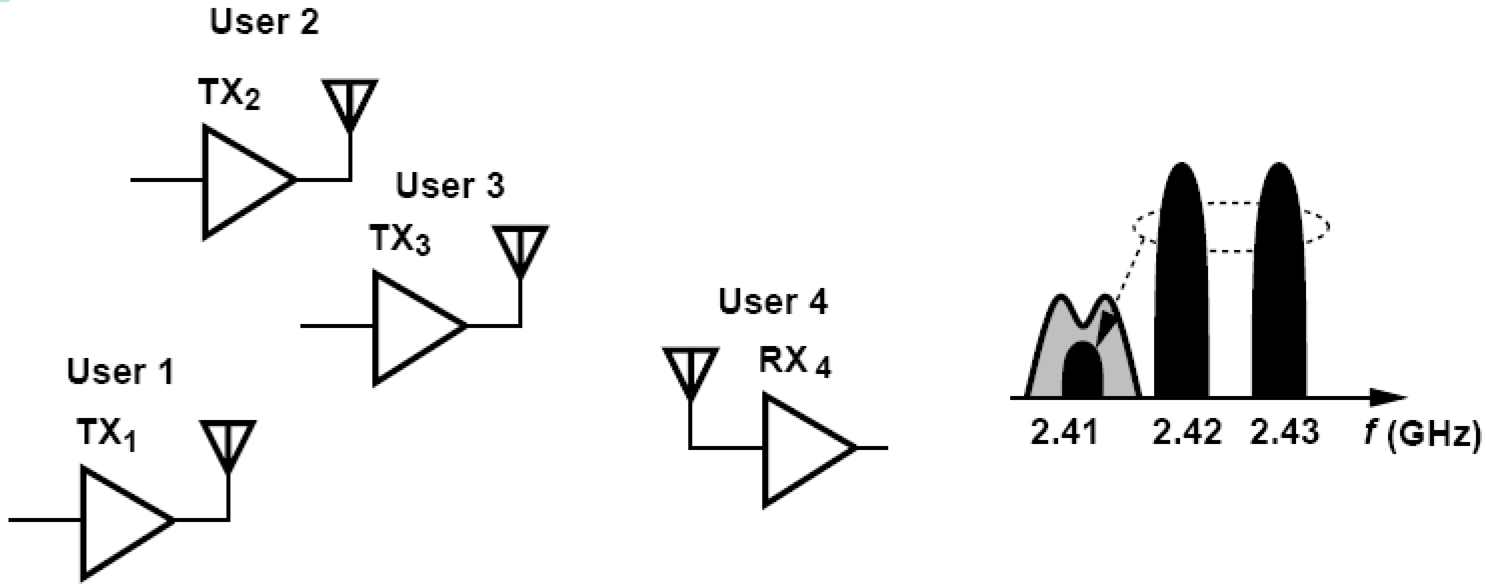
Expected outcome
PER (packet error rate) better than 30.8% for a minimum of 1500 packets transmitted by the tester.



Example of Intermodulation

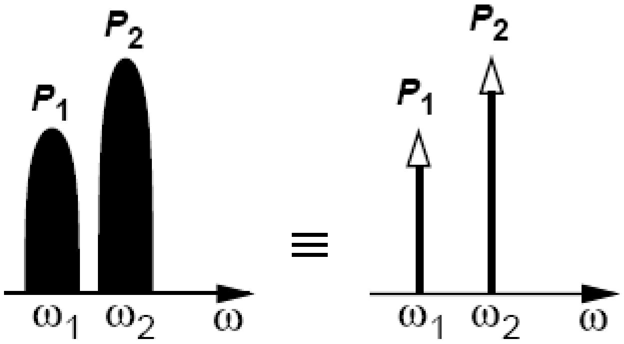
Suppose four Bluetooth users operate in a room as shown in figure below. User 4 is in the receive mode and attempts to sense a weak signal transmitted by User 1 at 2.410 GHz. At the same time, Users 2 and 3 transmit at 2.420 GHz and 2.430 GHz, respectively. Explain what happens.

Solution:

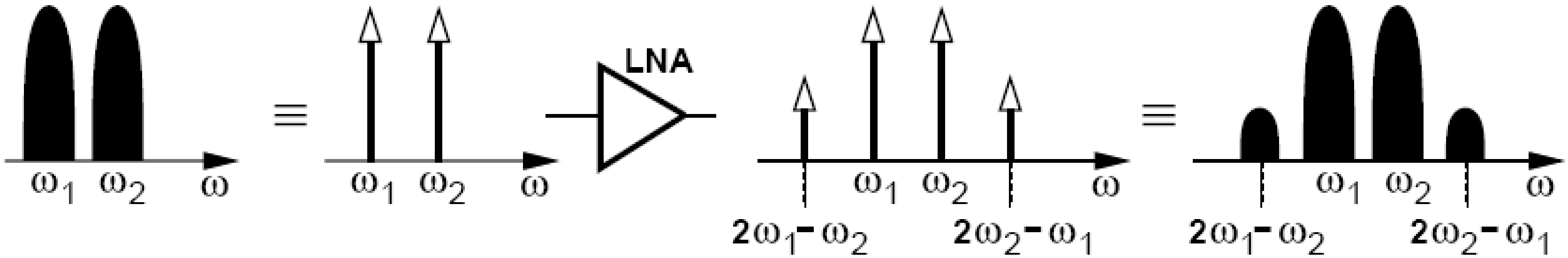


Since the frequencies transmitted by Users 1, 2, and 3 happen to be equally spaced, the intermodulation in the LNA of R_{X4} corrupts the desired signal at 2.410 GHz.

Intermodulation: Tones and Modulated Interferers

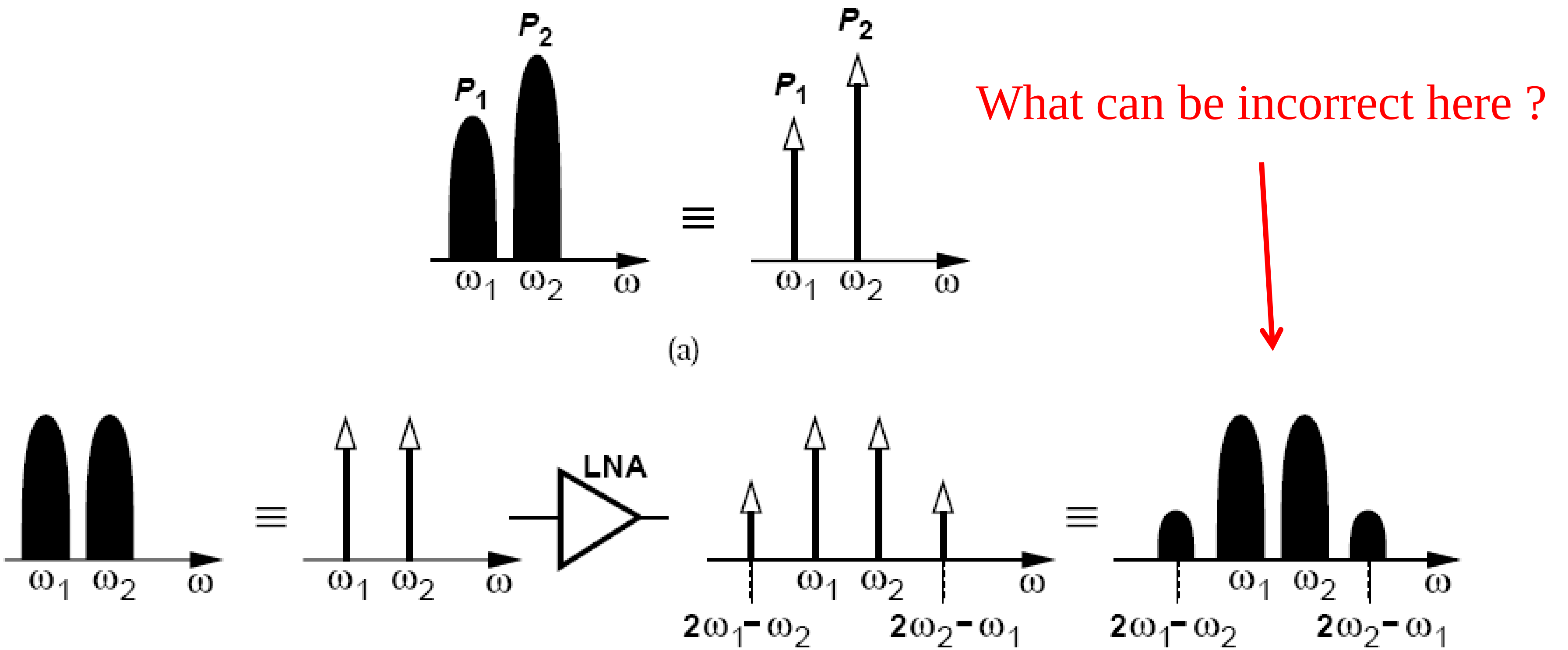


(a)



- **In intermodulation Analyses:**
  (a) approximate the interferers with tones
  (b) calculate the level of intermodulation products at the output
  (c) mentally convert the intermodulation tones back to modulated components

Intermodulation: Tones and Modulated Interferers



- **In intermodulation Analyses:**

  (a) approximate the interferers with tones

  (b) calculate the level of intermodulation products at the output

  (c) mentally convert the intermodulation tones back to modulated components

Example of Gain Compression and Intermodulation

A Bluetooth receiver employs a low-noise amplifier having a gain of 10 and an input impedance of 50 Ω. The LNA senses a desired signal level of -80 dBm at 2.410 GHz and two interferers of equal levels at 2.420 GHz and 2.430 GHz. For simplicity, assume the LNA drives a 50-Ω load.

(a) Determine the value of α_3 that yields a P_{1dB} of -30 dBm.

(b) If each interferer is 10 dB below P_{1dB}, determine the corruption experienced by the desired signal at the LNA output.

Example of Gain Compression and Intermodulation

A Bluetooth receiver employs a low-noise amplifier having a gain of 10 and an input impedance of 50 Ω. The LNA senses a desired signal level of -80 dBm at 2.410 GHz and two interferers of equal levels at 2.420 GHz and 2.430 GHz. For simplicity, assume the LNA drives a 50-Ω load.

(a) Determine the value of α_3 that yields a P_{1dB} of -30 dBm.

(b) If each interferer is 10 dB below P_{1dB}, determine the corruption experienced by the desired signal at the LNA output.

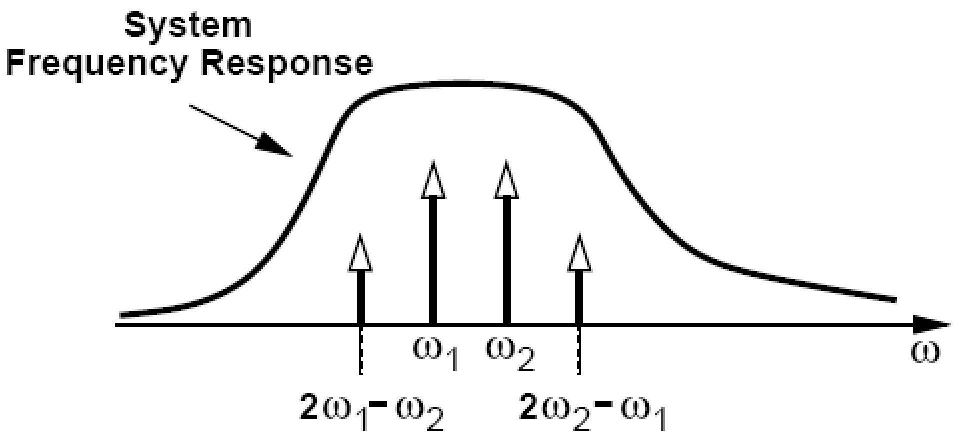
Solution:

(a) From previous equation, $A_{in,1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$. **$\alpha_3$ = 14.500 V^{-2}**

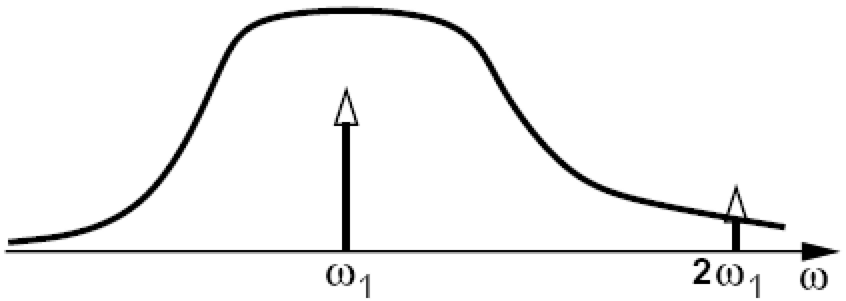
(b) Each interferer has a level of -40 dBm (= 6.32 m V_{pp}), we determine the amplitude of the IM product at 2.410 GHz as:

$$\frac{3\alpha_3 A_1^2 A_2}{4} = 0.343\,\mathrm{mV_p} = -59.3\ \mathrm{dBm}.$$

Intermodulation: Two-Tone Test and Relative IM



Two-Tone Test (useful for non-linearity)



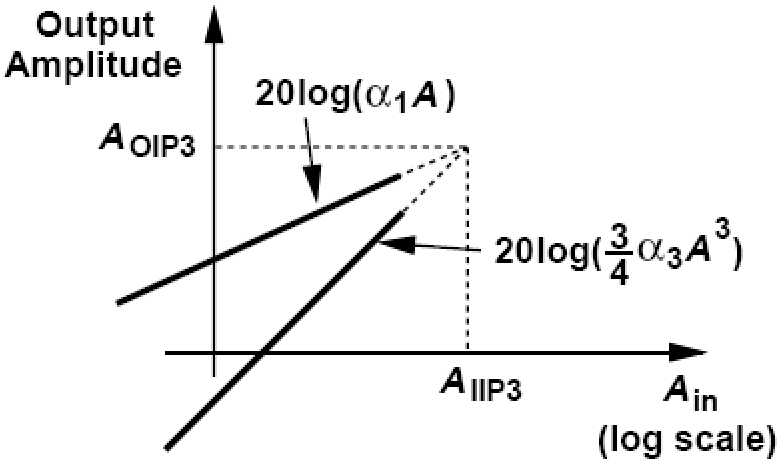
Harmonic Test (not useful)

- **Two-Tone Test can be applied to systems with arbitrarily narrow bandwidths to examine the non-linearity, where the second harmonics may not be as meaningful regarding the non-linearity.**

$$\text{Relative IM} = 20\log\left(\frac{3}{4}\frac{\alpha_3}{\alpha_1}A^2\right)\ \text{dBc}$$

Meaningful only when A is given

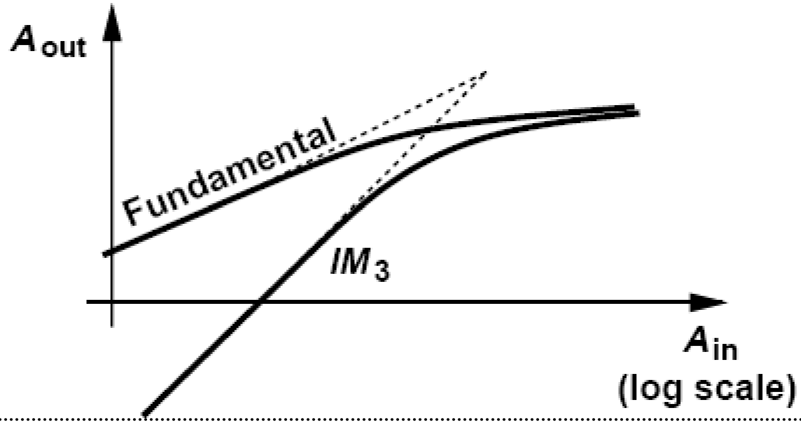
Intermodulation: Third Intercept Point

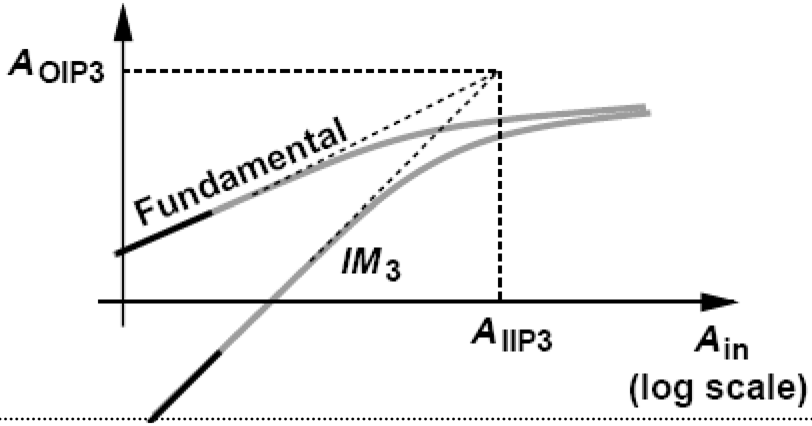


$$|\alpha_1 A_{IIP3}| = \left|\frac{3}{4}\alpha_3 A_{IIP3}^3\right|$$

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

$$\frac{A_{IIP3}}{A_{1dB}} = \sqrt{\frac{4}{0.435}} \approx 9.6 \text{ dB}.$$





- IP3 is not a directly measureable quantity, but a point obtained by extrapolation
- IP 3 needs to be calculated in linear region

Example of Third Intercept Point

A low-noise amplifier senses a -80-dBm signal at 2.410 GHz and two -20-dBm interferers at 2.420 GHz and 2.430 GHz. What IIP_3 is required if the IM products must remain 20 dB below the signal? For simplicity, assume 50-Ω interfaces at the input and output.

Solution:

Let Asig be desired signal amplitude, and Aint be interferer amplitude

At the LNA output:

$$20\log|\alpha_1 A_{sig}| - 20\text{ dB} = 20\log\left|\frac{3}{4}\alpha_3 A_{int}^3\right|$$

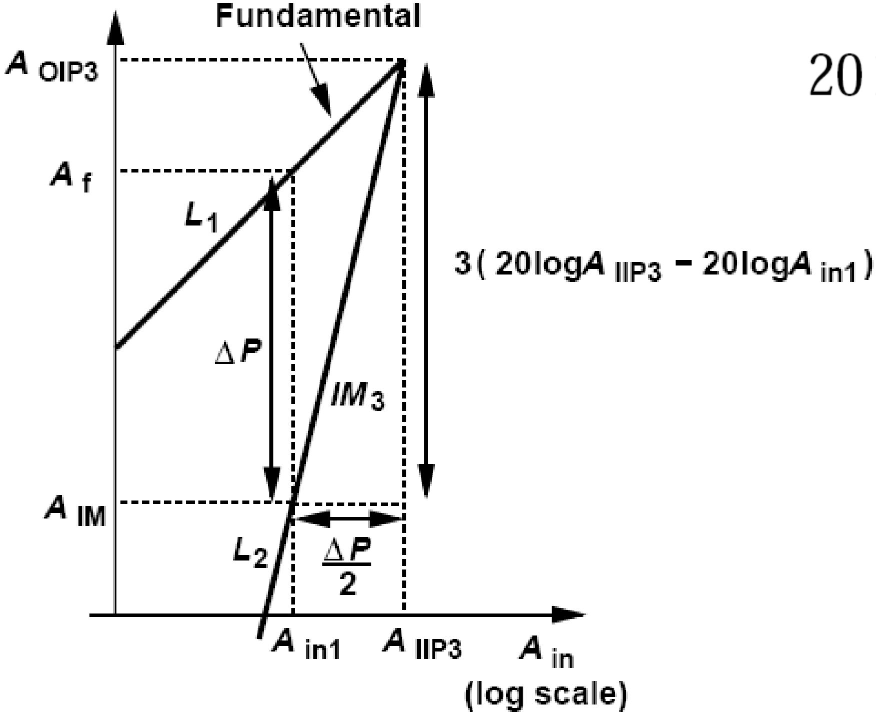
$$|\alpha_1 A_{sig}| = \left|\frac{30}{4}\alpha_3 A_{int}^3\right|$$

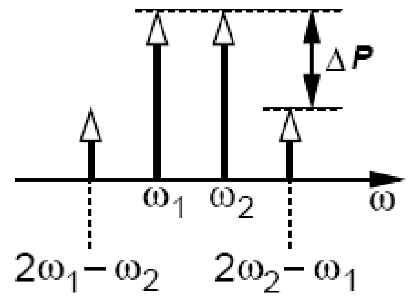
From α1/α3, one can obtain $\mathrm{IIP}_3 = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$ **= 3.16Vp = +20dBm**

Third Intercept Point: A reasonable estimate

$$\Delta P = 20\log A_f - 20\log A_{IM} = 2(20\log A_{IIP3} - 20\log A_{in1}),$$

$$20\log A_{IIP3} = \frac{\Delta P}{2} + 20\log A_{in1}$$





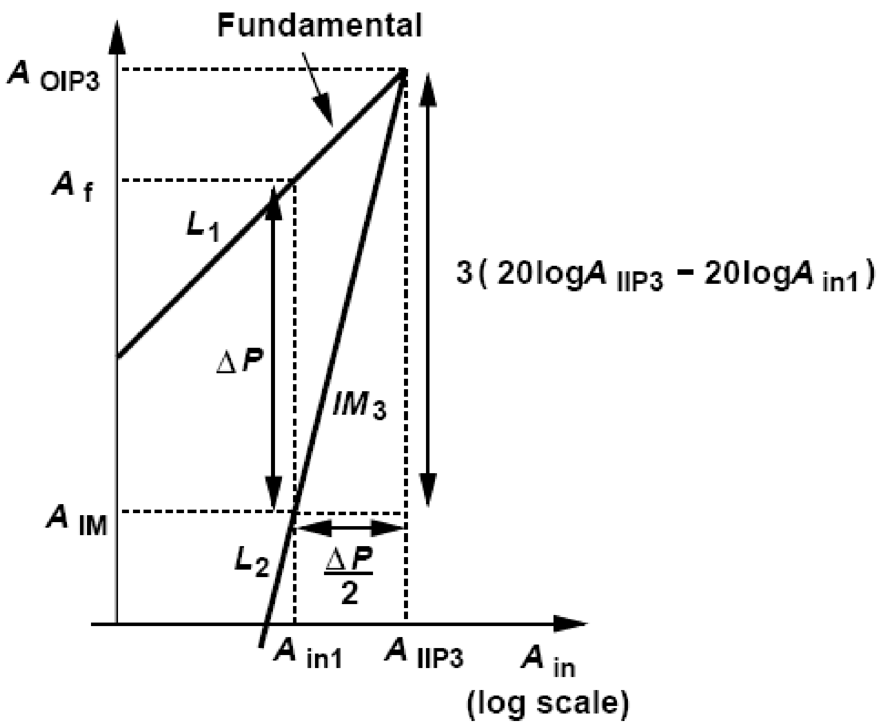
$$IIP_3\big|_{dBm} = \frac{\Delta P\big|_{dB}}{2} + P_{in}\big|_{dBm}$$

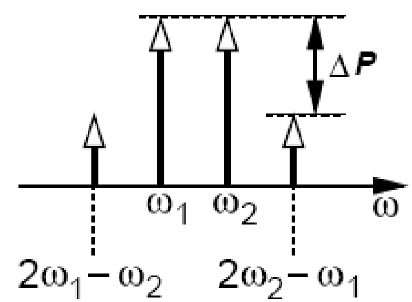
- For a given input level (well below P_{1dB}), the IIP$_3$ can be calculated by halving the difference between the output fundamental and IM levels and adding the result to the input level, where all values are expressed as logarithmic quantities.

Example of Third Intercept Point

A low-noise amplifier senses a -80-dBm signal at 2.410 GHz and two -20-dBm interferers at 2.420 GHz and 2.430 GHz. What IIP$_3$ is required if the IM products must remain 20 dB below the signal? For simplicity, assume 50-Ω interfaces at the input and output.

Solution:





$$IIP_3\big|_{dBm} = \frac{\Delta P\big|_{dB}}{2} + P_{in}\big|_{dBm}$$

Recall the Example of Polynomial Approximation

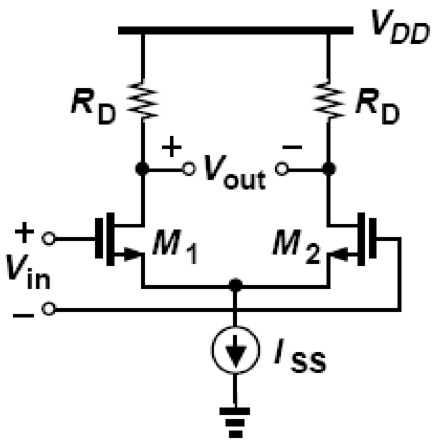
For square-law MOS transistors operating in saturation, the characteristic above can be expressed as

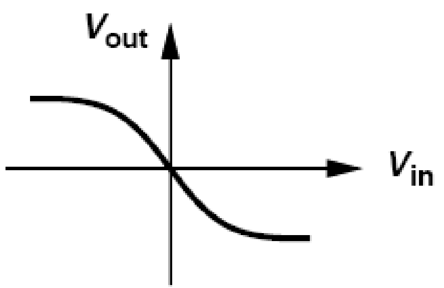
$$V_{out} = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{in}^2}\,R_D$$

If the differential input is small, approximate the characteristic by a polynomial.

Factoring $4I_{ss} / (\mu_n C_{ox} W/L)$ out of the square root and assuming

$$V_{in}^2 \ll \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$



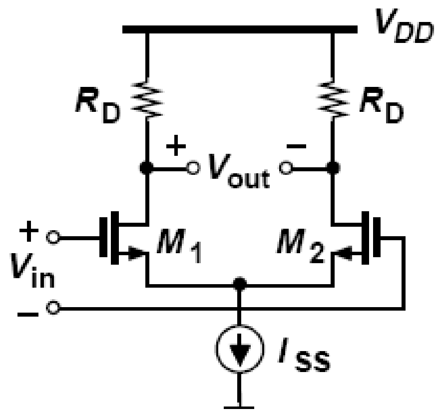


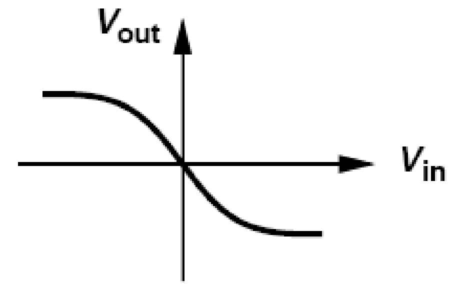
Approximation gives us:

$$V_{out} \approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}V_{in}}\left(1 - \frac{\mu_n C_{ox}\frac{W}{L}}{8I_{SS}}V_{in}^2\right)R_D$$

$$\approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}R_D V_{in} + \frac{\left(\mu_n C_{ox}\frac{W}{L}\right)^{3/2}}{8\sqrt{I_{SS}}}R_D V_{in}^3$$

Recall the Example of Polynomial Approximation



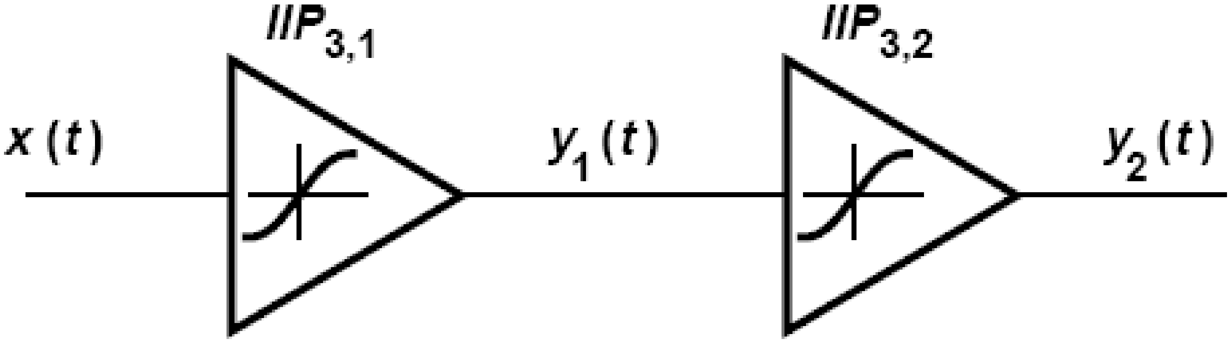


$$
\begin{aligned}
V_{out} &\approx -\sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS} V_{in}}\left(1 - \frac{\mu_n C_{ox}\frac{W}{L}}{8 I_{SS}} V_{in}^2\right) R_D \\
&\approx -\sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}} R_D V_{in} + \frac{\left(\mu_n C_{ox}\frac{W}{L}\right)^{3/2}}{8\sqrt{I_{SS}}} R_D V_{in}^3
\end{aligned}
$$

What is the A_{IIP3} at Vin?

Show that *IIP3* is proportional to I_{SS}.

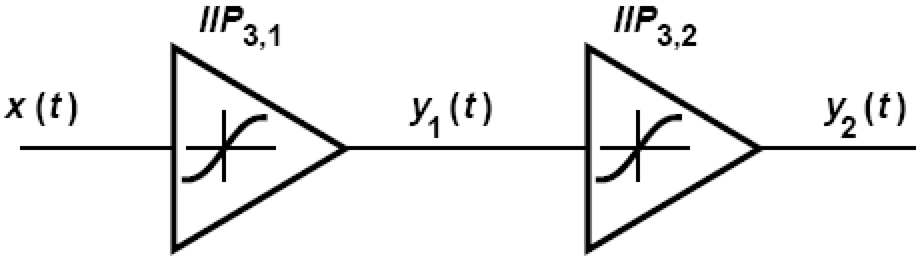
Effects of Nonlinearity: Cascaded Nonlinear Stages



Will the *IIP*3_2 affect the overall *IIP*3?

How will *IIP*3_2 affect the overall *IIP*3?

Effects of Nonlinearity: Cascaded Nonlinear Stages



$$
\begin{aligned}
y_1(t) &= \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) \\
y_2(t) &= \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)
\end{aligned}
$$

$$
\begin{aligned}
y_2(t) &= \beta_1[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 \\
&+ \beta_3[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3.
\end{aligned}
$$

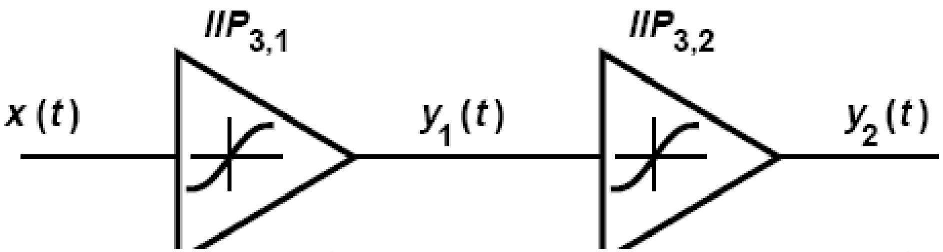
Considering only the first- and third-order terms, we have:

$$y_2(t) = \alpha_1 \beta_1 x(t) + (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) x^3(t) + \cdots$$

Thus,

$$A_{IP3} = \sqrt{\frac{4}{3} \left| \frac{\alpha_1 \beta_1}{\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3} \right|}.$$

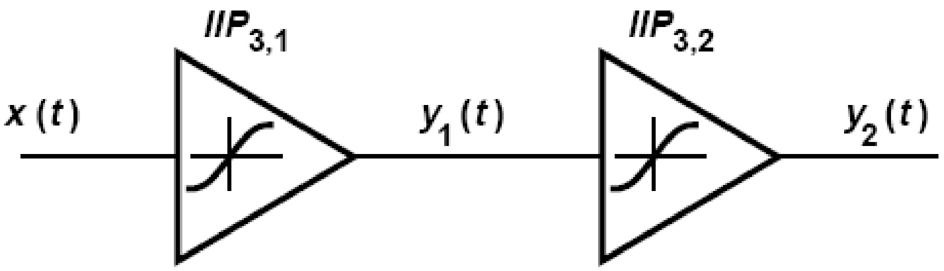
Cascaded Nonlinear Stages: Intuitive results



$$\begin{aligned}\frac{1}{A_{IP3}^2} &= \frac{3}{4}\left|\frac{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}{\alpha_1\beta_1}\right| \\ &= \frac{3}{4}\left|\frac{\alpha_3}{\alpha_1} + \frac{2\alpha_2\beta_2}{\beta_1} + \frac{\alpha_1^2\beta_3}{\beta_1}\right| \\ &= \left|\frac{1}{A_{IP3,1}^2} + \frac{3\alpha_2\beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{IP3,2}^2}\right|\end{aligned}$$

- **To "refer" the IP_3 of the second stage to the input of the cascade, we must divide it by α_1. Thus, the higher the gain of the first stage, the more nonlinearity is contributed by the second stage.**
- **In balanced circuits (differential circuits), $\alpha 2$ and $\beta 2$ are small and can be ignored. *IP*3 becomes a function of *IP*3,1 and *IP*3,2.**

Cascaded Nonlinear Stages: Generalized Result



For two stages:

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$

For more stages:

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IP3,3}^2} + \cdots$$

- **Thus, if each stage in a cascade has a gain greater than unity, the nonlinearity of the latter stages becomes increasingly more critical because the IP3 of each stage is equivalently scaled down by the total gain preceding that stage.**

Example of Cascaded Nonlinear Stages

A low-noise amplifier having an input IP_3 of -10 dBm and a gain of 20 dB is followed by a mixer with an input IP_3 of +4 dBm. Which stage limits the IP_3 of the cascade more? What is the total input IP3 ?

Solution:

With α_1 = 20 dB, we note that

$$
\begin{aligned}
A_{IP3,1} &= -10\ \text{dBm} \\
\frac{A_{IP3,2}}{\alpha_1} &= -16\ \text{dBm}
\end{aligned}
$$

Since the scaled IP_3 of the second stage is lower than the IP_3 of the first stage, we say the second stage limits the overall IP_3 more.

What is the total input IP3?

Example of Cascaded Nonlinear Stages

A low-noise amplifier having an input IP_3 of -10 dBm and a gain of 20 dB is followed by a mixer with an input IP_3 of +4 dBm. Which stage limits the IP_3 of the cascade more? What is the total input IP3 ?

Solution:

$$\begin{aligned} A_{IP3,1} &= -10 \text{ dBm} \\ \frac{A_{IP3,2}}{\alpha_1} &= -16 \text{ dBm} \end{aligned}$$

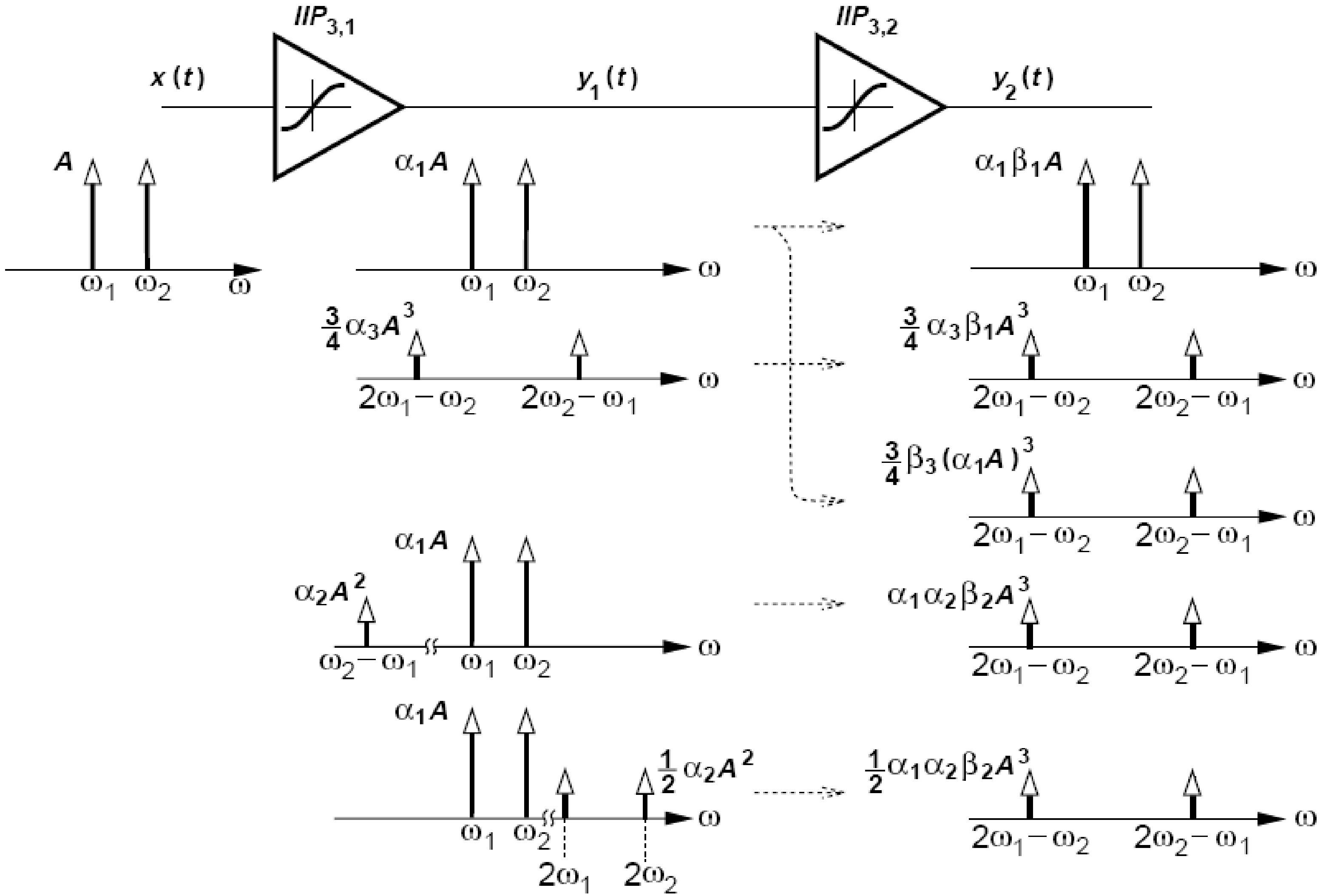
$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$

$$1/IP_3 = 1/IP_{3,1} + 1/IP_{3,2}$$

$$1/(-16.9\text{dBm}) = 1/(-10\text{dBm}) + 1/(-16\text{dBm})$$

IM Spectra in a Cascade (I)

Let us assume X(t) =$A\cos\omega_1 t + A\cos\omega_2 t$ and identify the IM products in a cascade:



IM Spectra in a Cascade (Ⅱ)

Adding the amplitudes of the IM products, we have the following:

$$
\begin{aligned}
y_2(t) &= \alpha_1\beta_1 A(\cos\omega_1 t + \cos\omega_2 t) \\
&+ \left(\frac{3\alpha_3\beta_1}{4} + \frac{3\alpha_1^3\beta_3}{4} + \frac{3\alpha_1\alpha_2\beta_2}{2}\right) A^3[\cos(\omega_1 - 2\omega_2)t + \cos(2\omega_2 - \omega_1)t] + \cdots
\end{aligned}
$$

The above equation can be also derived from X(t) =Acos ω1t + Acos ω2t :

$$
y_2(t) = \alpha_1\beta_1 x(t) + (\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3)x^3(t) + \cdots
$$