Shifting, reflecting and time-scaling operations

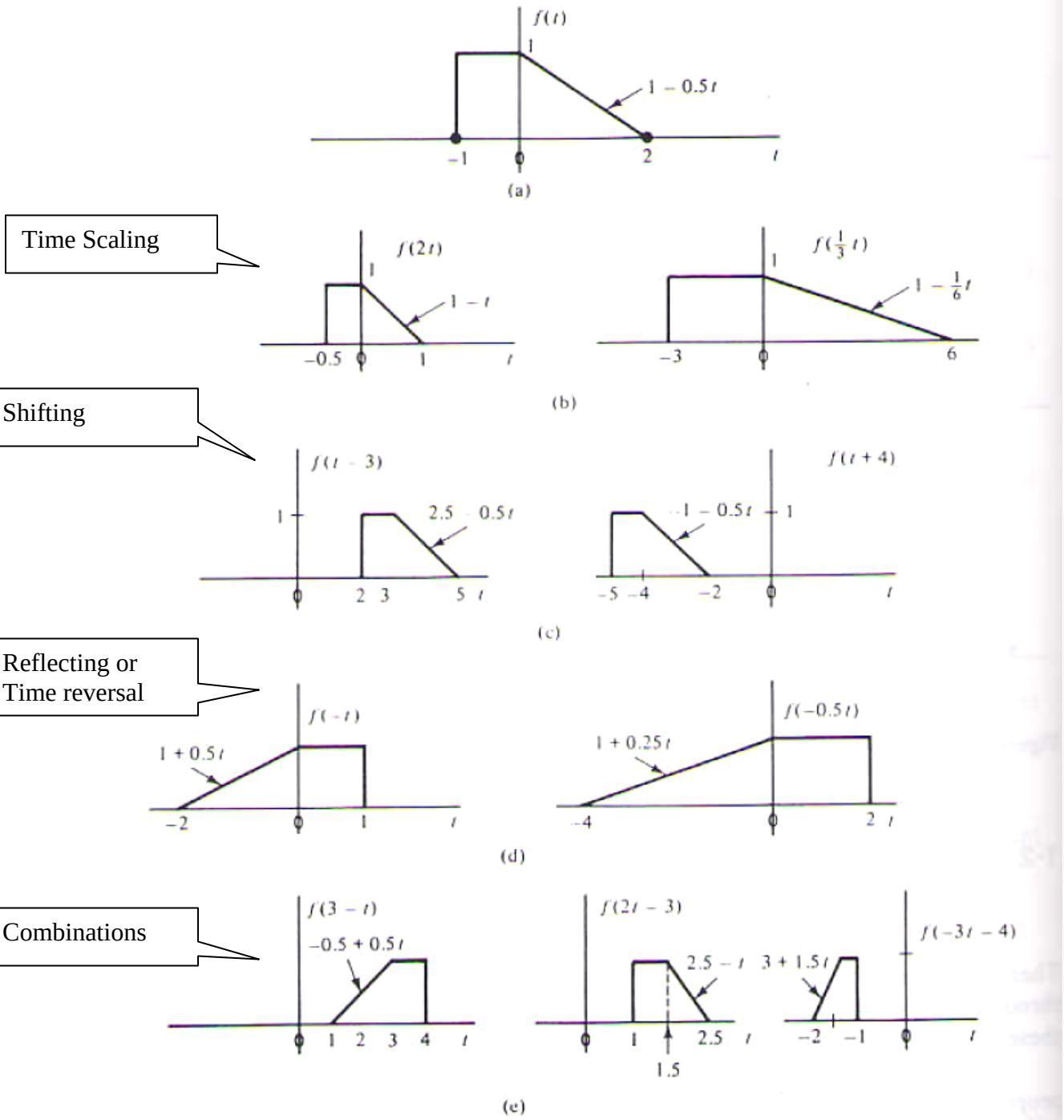


Figure 1-4 Reflecting, time-scaling, and shifting the continuous test function of Example 1-2.

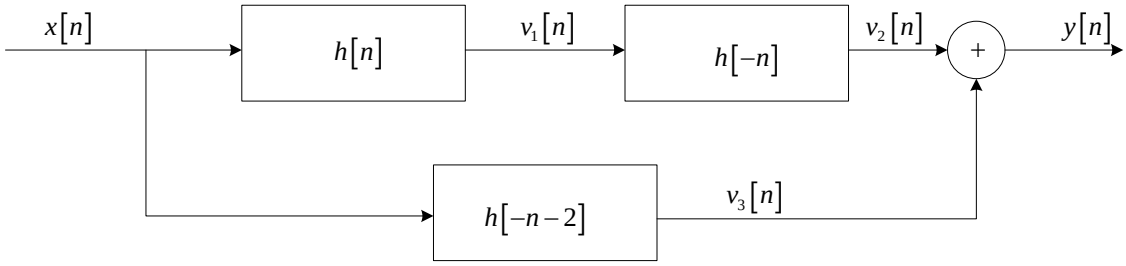
Ex. $f(at \pm b) = f\left[\pm a\left(t \pm \frac{b}{a}\right)\right]$

Scaling and reflecting

Shifting to right or left

An input and impulse response are shown below, find the output of the system.

$$h[n] = \delta(n-1) + 2\delta(n-2)$$
$$x[n] = \delta(n-2) + 2\delta(n-3)$$

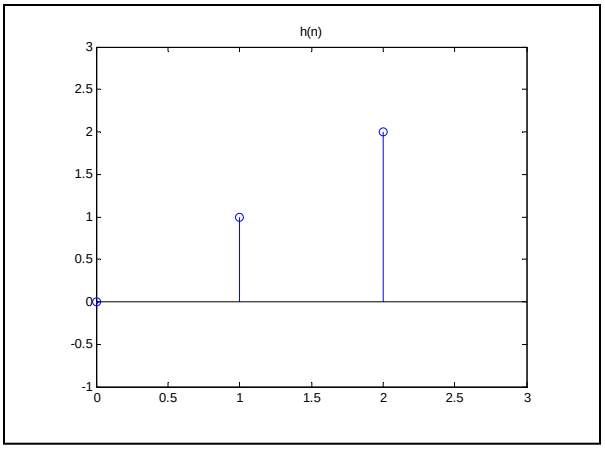


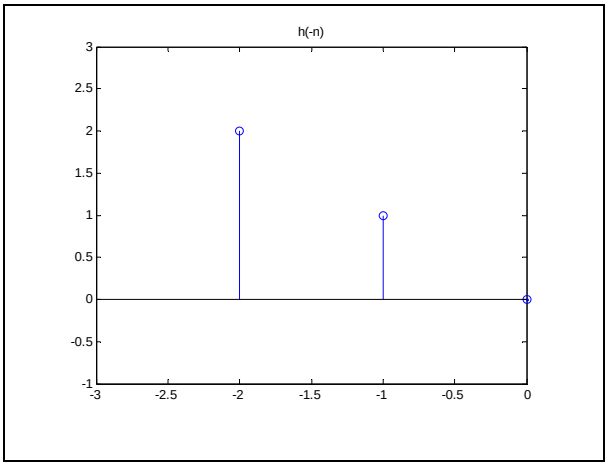
The overall system is shown below

$$\begin{aligned} y[n] &= v_2[n] + v_3[n] \\ &= \underbrace{\left[h[-n] * h[n] * x[n]\right]}_{v_2(n)} + \underbrace{\left[h[-n-2] * x[n]\right]}_{v_3(n)} \\ &= \left[h[-n] * h[n] + h[-n-2]\right] * x[n] \end{aligned}$$

First of all, let's do the top branch convolution first. That is

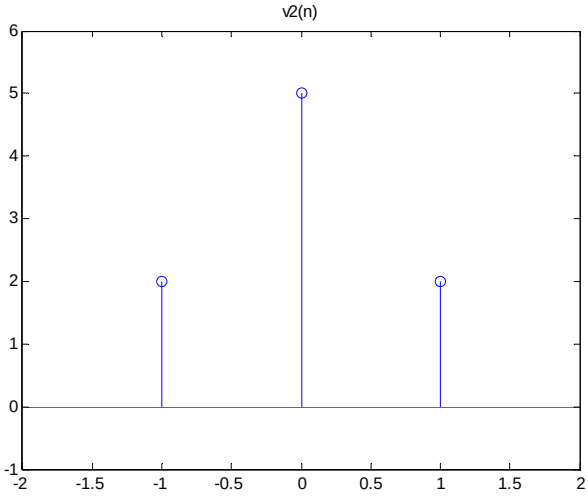
$$\begin{aligned} v_2[n] &= h[-n] * h[n] \\ &= h[n] * h[-n] \end{aligned}$$





The convolution of $v_2[n] = h[-n] * h[n]$ is

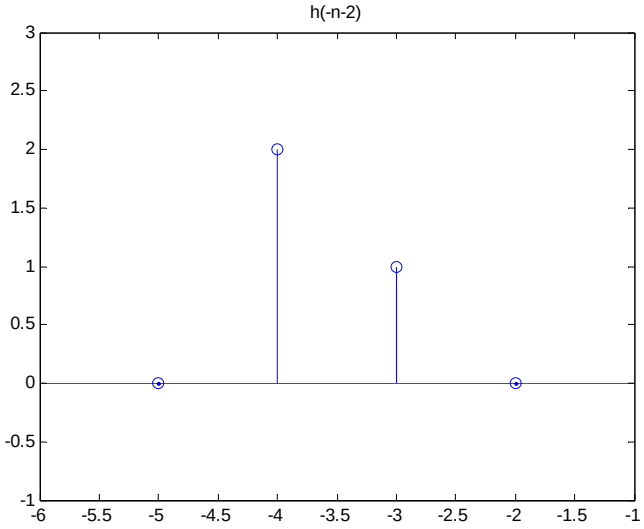
$$\begin{aligned} v_2[n] &= h[-n] * h[n] \\ &= 2\delta(n+1) + 5\delta(n) + 2\delta(n-1) \end{aligned}$$



Plot of

$$\begin{aligned} h[-n-2] &= h[-(n+2)] \\ &= 2\delta(n+4) + \delta(n+3) \end{aligned}$$

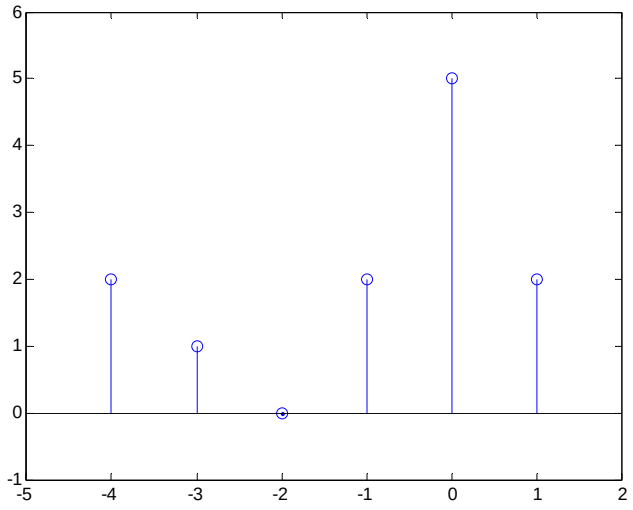
is



Finding the value of $v(n)$ in the below equation is

$$\begin{aligned} y(n) &= v_2(n) + v_3(n) \\ &= \left[h(-n) * h(n) * x(n)\right] + \left[h(-n-2) * x(n)\right] \\ &= \underbrace{\left[h(-n) * h(n) + h(-n-2)\right]}_{v(n)} * x(n) \end{aligned}$$

$$v(n) = \left[2\delta(n+1) + 5\delta(n) + 2\delta(n-1)\right] + \left[2\delta(n+4) + \delta(n+3)\right]$$



The output is

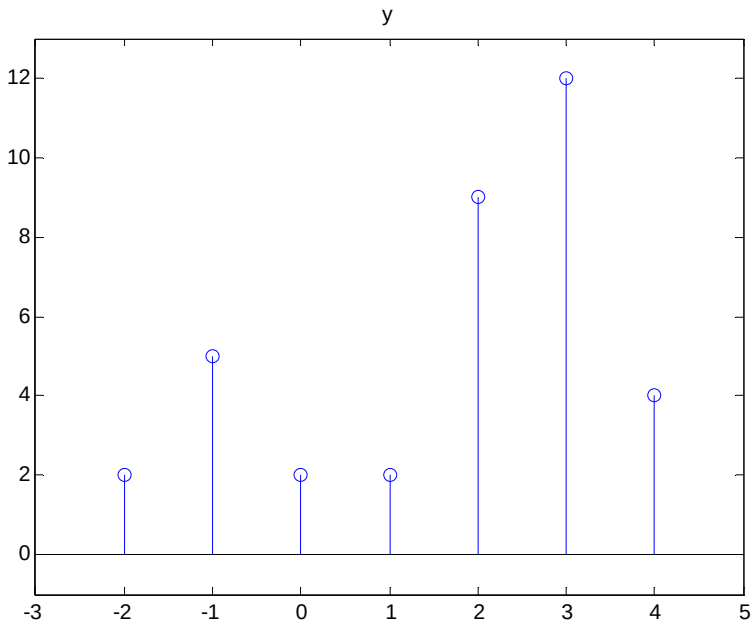
$$y(n) = \underbrace{\left[h(-n) * h(n) + h(-n-2)\right]}_{v(n)} * x(n)$$

$$= \left[2\delta(n+4) + \delta(n+3) + 2\delta(n+1) + 5\delta(n) + 2\delta(n-1)\right] * \left[\delta(n-2) + 2\delta(n-3)\right]$$

$$= \left[2\delta(n+4) + \delta(n+3) + 2\delta(n+1) + 5\delta(n) + 2\delta(n-1)\right] * \left[\delta(n-2)\right]$$
$$+ \left[2\delta(n+4) + \delta(n+3) + 2\delta(n+1) + 5\delta(n) + 2\delta(n-1)\right] * \left[2\delta(n-3)\right]$$

$$= \left[2\delta(n-2+4) + \delta(n-2+3) + 2\delta(n-2+1) + 5\delta(n-2) + 2\delta(n-2-1)\right]$$
$$+ 2\left[2\delta(n-3+4) + \delta(n-3+3) + 2\delta(n-3+1) + 5\delta(n-3) + 2\delta(n-3-1)\right]$$

$$= \left[2\delta(n+2) + \delta(n+1) + 2\delta(n-1) + 5\delta(n-2) + 2\delta(n-3) \right]$$
$$+2\left[2\delta(n+1) + \delta(n) + 2\delta(n-2) + 5\delta(n-3) + 2\delta(n-4) \right]$$
$$= 2\delta(n+2) + 5\delta(n+1) + 2\delta(n) + 2\delta(n-1) + 9\delta(n-2) + 12\delta(n-3) + 4\delta(n-4)$$



The output of the system