

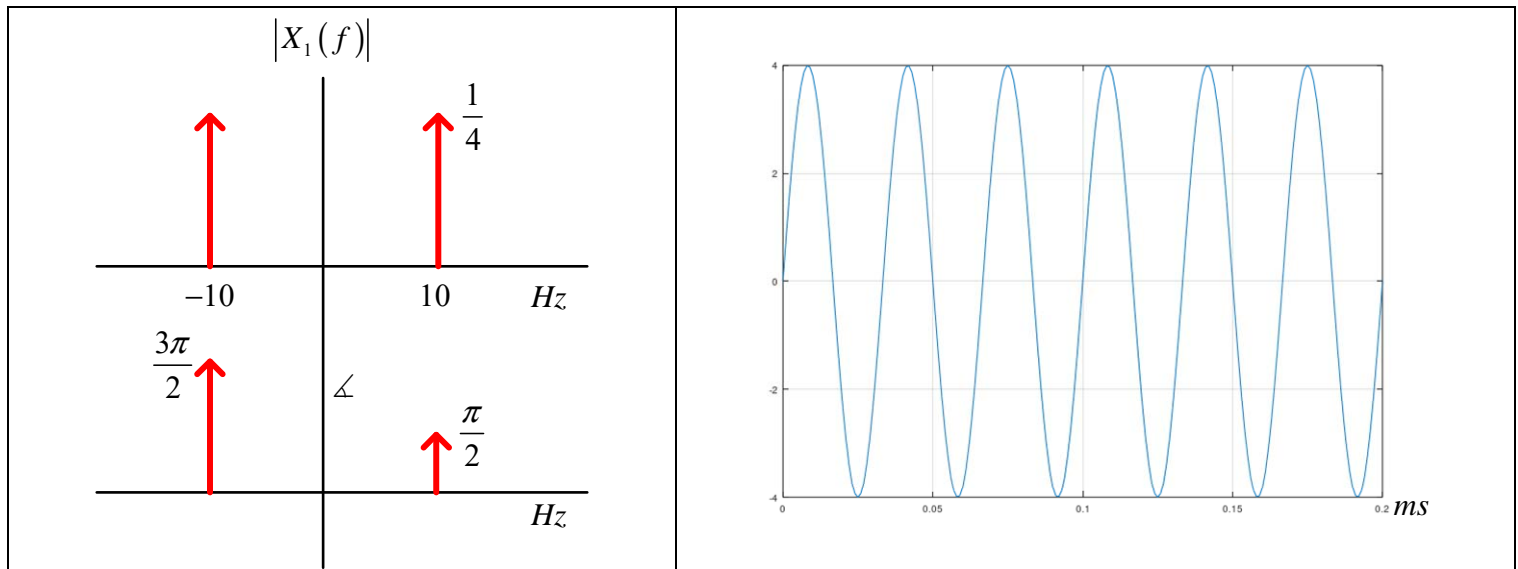
Last Name: First Name:

Q01

Name of the file is your last name before you submit.

Two functions are shown,  $X_1(f)$  in frequency (magnitude & phase) and  $x_2(t)$  in time domain.

Note:  $x_2(t)$  is written in from 0 to 0.2 ms (millisecond) in time and amplitude is 4. In this quiz, the amplitude of sinusoid is in peak values (not in RMS)



a) Write two signals in time domain.

b) Find  $[X(f) = F\{x_1(t) \cdot x_2(t)\}]$

(frequency response of multiplication of two functions)

c) Plot part b) in frequency domain

a)

$$x_1(t) = -\frac{1}{2} \sin(2\pi \cdot 10 \cdot t)$$

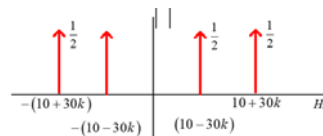
$$x_2(t) = 4 \sin(2\pi \cdot 30 \cdot t)$$

b)

$$X(f) = \frac{1}{2} [\delta(f - (10 + 30k)) + \delta(f + (10 + 30k))] - \frac{1}{2} [\delta(f - (10 - 30k)) + \delta(f + (10 - 30k))]$$

c)

Magnitude plot  $|X(f)|$



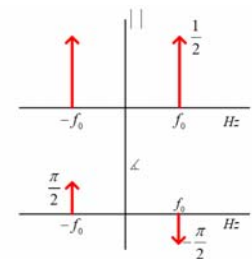
Phase plot  $\angle X(f)$



a)

From the Euler's equation,

$$\begin{cases} F\{A \sin(2\pi f_0 t)\} = \frac{1}{2j} [\delta(f - f_0) - \delta(f + f_0)] \\ @ f = f_0 : \left| \frac{1}{2j} \right| e^{j(\frac{\pi}{2})} & \& \quad f = -f_0 : \left| -\frac{1}{2j} \right| e^{j(\frac{\pi}{2})} \end{cases}$$



So

	$x_1(t) = -\frac{1}{2}\sin(2\pi \cdot 10 \cdot t) \text{ and } x_2(t) = 4\sin(2\pi \cdot 30k \cdot t)$
b)	$  \begin{aligned}  X(f) &= F\{x_1(t) \cdot x_2(t)\} \\  &= \left(-\frac{1}{2}\sin(2\pi \cdot 10 \cdot t)\right) \cdot (4\sin(2\pi \cdot 30k \cdot t)) \quad \text{where } [x = 2\pi \cdot 10 \cdot t \text{ \& } y = 2\pi \cdot 30k \cdot t] \\  &= \left(-\frac{1}{2}\right) \cdot (4) \left[ \left( \frac{e^{jx} - e^{-jx}}{j2} \right) \left( \frac{e^{jy} - e^{-jy}}{j2} \right) \right] \\  &= \left(-\frac{1}{2}\right) \cdot (4) \cdot \left(\frac{1}{2j}\right) \cdot \left(\frac{1}{2j}\right) [(e^{jx} - e^{-jx})(e^{jy} - e^{-jy})] \\  &= \left(-\frac{1}{2}\right) \cdot (4) \cdot \left(\frac{1}{2j}\right) \cdot \left(\frac{1}{2j}\right) [(e^{j(x+y)} - e^{j(x-y)} - e^{-j(x-y)} + e^{-j(x+y)})] \\  &= \left(\frac{1}{2}\right) \cdot (4) \cdot \left(\frac{1}{2}\right) \left[ \frac{e^{j(x+y)} + e^{-j(x+y)}}{2} - \left( \frac{e^{j(x-y)} + e^{-j(x-y)}}{2} \right) \right] \\  &= \cos(x+y) - \cos(x-y) \\  &= \cos(2\pi \cdot (10+30k) \cdot t) - \cos(2\pi \cdot (10-30k) \cdot t)  \end{aligned}  $
c)	$  \begin{aligned}  &F\{\cos(2\pi \cdot (10+30k) \cdot t) - \cos(2\pi \cdot (10-30k) \cdot t)\} \\  &= \frac{1}{2} [\delta(f - (10+30k)) + \delta(f + (10+30k))] - \frac{1}{2} [\delta(f - (10-30k)) + \delta(f + (10-30k))]  \end{aligned}  $