

Lecture #9: MOS Device Operation

Long-channel

In linear region:

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

for small V_{DS} ,

$$I_D = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T) \cdot V_{DS} \Rightarrow \text{voltage controlled resistor}$$

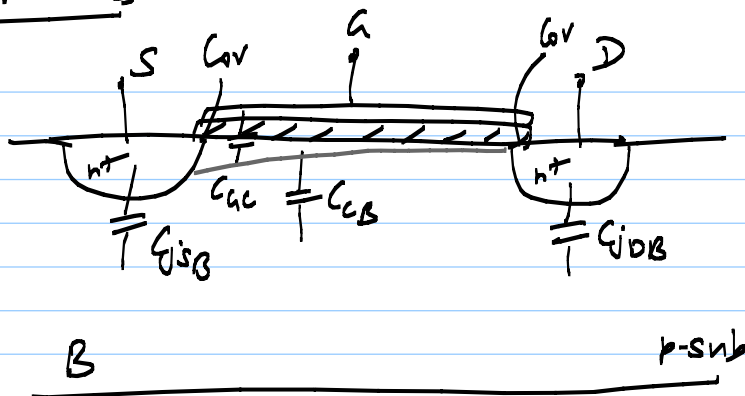
In saturation:

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2 \quad \text{square law dependence}$$

At "low" frequencies: ignore device caps (e.g. bias)

At "high" frequencies: consider all appropriate device caps.

Capacitances



L_D = overlap of gate over S & D

$$L_{eff} = L - 2L_D$$

$$C_{gc} = C_{ox} \cdot W \cdot L_{eff}$$

$$C_{cb} = \frac{\epsilon_{si}}{\epsilon_d} \cdot W \cdot L_{eff}$$

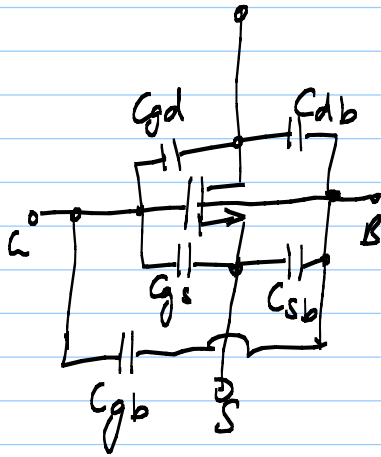
cap	OFF	TRIODE	SAT.
C_{gs}	C_{ov}	$C_{ov} + \frac{C_{gc}}{2}$	$\frac{2}{3} C_{gc} + C_{ov}$
C_{gd}	C_{ov}	$C_{ov} + \frac{W L_{eff} C_{ox}}{2}$	C_{ov}
C_{gb}	$C_{gb} < C_{gc}$	0	0
C_{sb}	C_{gsb}	$C_{gsb} + C_{cb}/2$	$C_{gsb} + 2C_{cb}/3$
C_{db}	C_{gdb}	$C_{gdb} + \frac{C_{cb}}{2}$	C_{gdb}

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

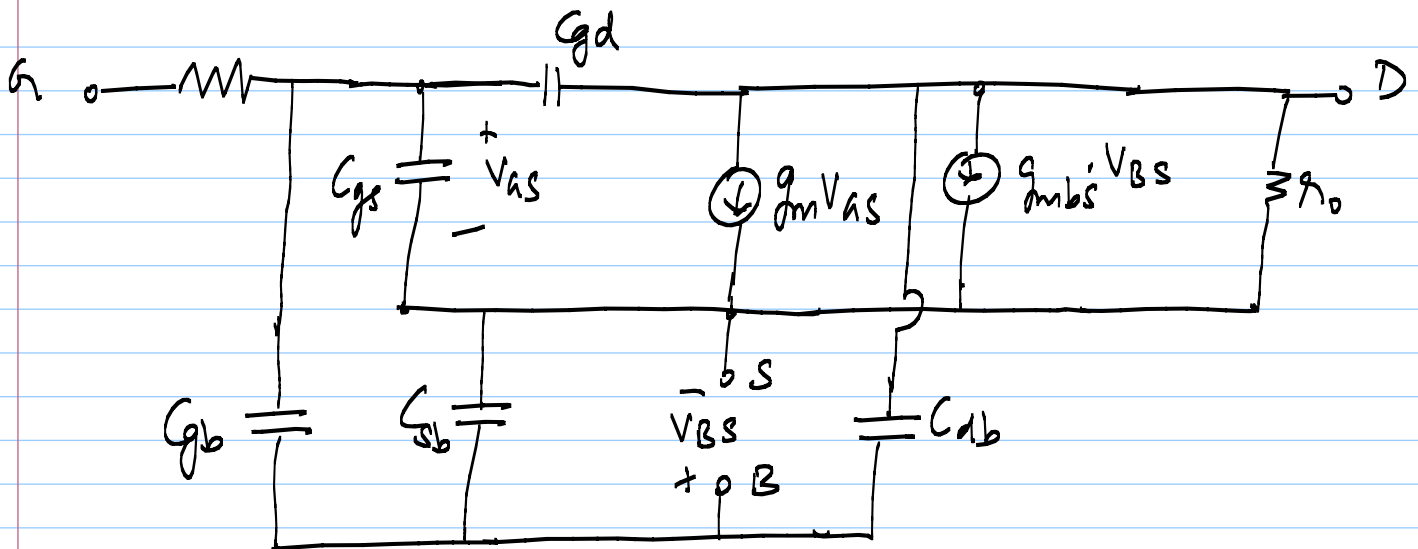
$$C_{ov} = W L_D C_{ox}$$

$$x_d = \sqrt{\frac{2\epsilon_{si}}{qN_{sub}} |\phi_s - \phi_f|}$$

{depletion layer depth}



MOSFET MODEL



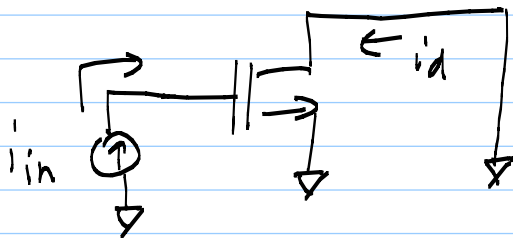
$$\begin{aligned}
 1) \quad g_m &= \left. \frac{\partial I_D}{\partial V_{gs}} \right|_{V_{DS} = \text{const.}} = \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_T) \\
 &= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \\
 &= \frac{2I_D}{V_{gs} - V_T} = \frac{2I_D}{V_{DSAT.}}
 \end{aligned}$$

$$2) g_o = \frac{\partial I_{DS}}{\partial V_{DS}} = \frac{1}{\lambda I_D}$$

$$3) g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \eta g_m = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} \cdot g_m$$

4) f_T : "Transition Frequency"
 \equiv Frequency at which current gain = 1

$\omega_T = 2\pi f_T$ is often used



* neglects C_{db} , r_g
 * Effect of C_{gd} only on Z_{in} is considered.

$$\left| \frac{i_d}{i_{in}} \right| \approx \frac{g_m}{\omega (C_{gs} + C_{gd})}$$

$$\left| \frac{i_d}{i_{in}} \right| = 1 \quad \text{at} \quad \omega_T = \frac{g_m}{C_{gs} + C_{gd}}$$

$$\left| \frac{i_d}{i_{in}} \right| \approx \frac{\omega_T}{\omega}$$

Long-channel devices:

$$\omega_T \approx \frac{g_m}{C_{gs}} = \frac{\mu_n C_{ox} \frac{W}{L} (V_{as} - V_T)}{\frac{2}{3} W L C_{ox}}$$

$$= \frac{3}{2} \frac{\mu_n (V_{as} - V_T)}{L^2} \Rightarrow \omega_T \uparrow \text{ when } \begin{matrix} * V_{DSAT} \uparrow \\ * L \downarrow \end{matrix}$$

5) $f_{\max} (\omega_{\max}) \equiv$ freq. at which power gain becomes 1

It can be shown that

$$g_{out} = g_m \cdot \frac{C_d}{C_d + C_s} = \omega_T \cdot C_d$$

conjugate match @ output implies

$$g_L = g_{out}$$

i_d is divided equally between load & device g_{out} itself

assume gate resistance of device is r_g

$$P_{in} = \frac{i_{in}^2 \cdot r_g}{2}$$

$$P_L = \frac{1}{2} (i_d/2)^2 \cdot g_L$$

$$= \frac{1}{2} \left(\frac{\omega_T}{\omega} i_{in} \cdot \frac{1}{2} \right)^2 \cdot \frac{1}{\omega_T C_d}$$

$$\Rightarrow \frac{P_L}{P_{in}} = \frac{\omega_T}{\omega^2 4 r_g C_d}$$

$$\boxed{\omega_{\max} = \frac{1}{2} \sqrt{\frac{\omega_T}{r_g C_d}}}$$

6) NQS effects: consider transit time effects

$$g_g = \frac{\omega^2 C_s^2}{5 g_{d0}} \approx \frac{g_m}{5} \left(\frac{\omega}{\omega_T} \right)^2 \leftarrow \text{this impedance is seen @ gate}$$

Brief note on CMOS "Constant Field Scaling"

1) Reduce all lateral & vertical dimensions
by $\alpha (>1) \Rightarrow W, L, t_{ox}, \text{ depth \& perimeter}$
of S.D junctions

2) Reduce V_{DDs} and V_{Ts} by α

3) Increase all doping levels by α

(1) & (2) \Rightarrow Electric fields stay constant inside
the semiconductor

Impact on device parameters :

1) Device current : $I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \Rightarrow C_{ox,\alpha} = \frac{\epsilon_{ox}}{t_{ox}/\alpha} = \alpha \cdot C_{ox}$$

$$I_{D,\alpha} = \frac{1}{2} \mu_n (\alpha C_{ox}) \cdot \frac{W/\alpha}{L/\alpha} \left(\frac{V_{GS}}{\alpha} - \frac{V_T}{\alpha} \right)^2$$
$$= I_D \cdot \frac{1}{\alpha}$$

2) Capacitances :

channel
cap $C_{ch} = WL C_{ox}$

$$C_{ch,\alpha} = \frac{W}{\alpha} \cdot \frac{L}{\alpha} (\alpha C_{ox}) = C_{ch} \cdot \frac{1}{\alpha}$$

Depl. region
width $w_d = \sqrt{\frac{2 \epsilon_{si}}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) (\Phi_B + V_R)}$

$$\Phi_B = V_T \ln \left(\frac{N_A N_D}{n_i^2} \right), \quad V_R = \text{reverse bias voltage}$$

assuming $V_R \gg \phi_B$,

$$w_{d,\alpha} \approx \sqrt{\frac{2\epsilon_{Si}}{q} \left(\frac{1}{\alpha N_A} + \frac{1}{\alpha N_D} \right) \cdot \frac{V_R}{\alpha}}$$
$$= w_d \cdot \frac{1}{\alpha}$$

$$\Rightarrow C_{depl,\alpha} = \alpha \cdot C_{depl}$$

* in general, all caps \downarrow by a factor of $\frac{1}{\alpha}$

3) Gate delay (CMOS inverter)

$$T_d = \frac{C}{I} \cdot V_{DD}$$

$$T_{d,\alpha} = \frac{C/\alpha}{I/\alpha} \cdot \frac{V_{DD}}{\alpha} = T_d \cdot \frac{1}{\alpha}$$

4) Power consumption (digital)

$$P = f C V_{DD}^2$$

$$P_\alpha = f \cdot \frac{C}{\alpha} \cdot \left(\frac{V_{DD}}{\alpha} \right)^2 = P \cdot \frac{1}{\alpha^3}$$

5) Layout Density:

$$\text{area}_\alpha = \frac{1}{\alpha^2} (\text{area})$$

6) Transconductance:

$$g_m = \mu C_{ox} \frac{W}{L} (V_{as} - V_T)$$

$$g_{m,\alpha} = \mu (\alpha C_{ox}) \cdot \frac{W/\alpha}{L/\alpha} \cdot \frac{V_{as} - V_T}{\alpha}$$
$$= g_m$$

Short-channel MOS operation

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{as} - V_T)^2$$

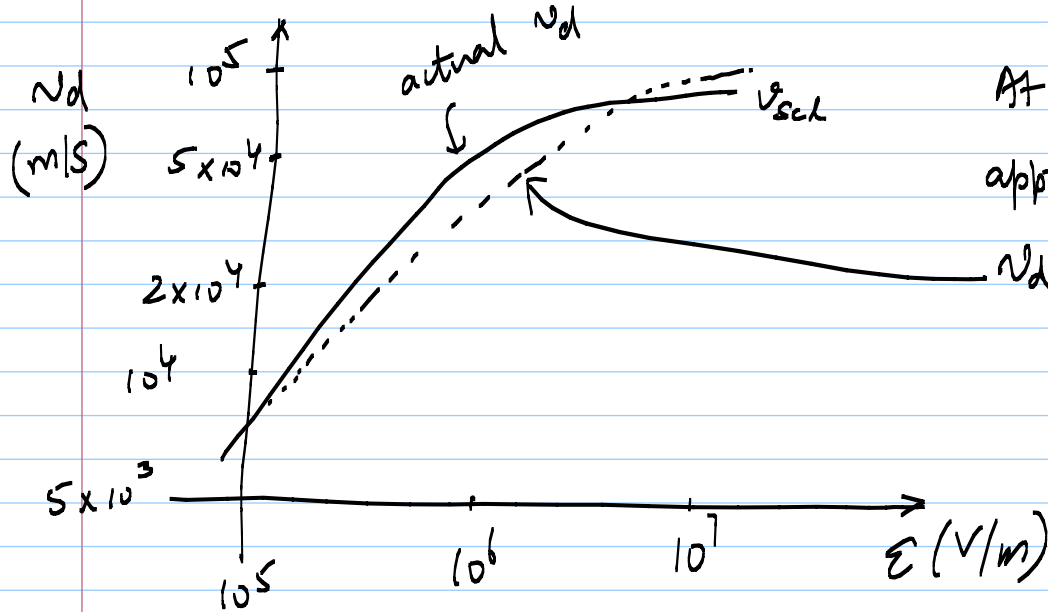
⇒ Derived using

$$v_d(y) = \mu_n E(y)$$

drift velocity

horizontal electric field

length along channel



At high fields, an approximate relation is:

$$v_d = \frac{\mu_n E}{1 + E/E_c}$$

scattering limited velocity:

$$v_{sd} = \mu_n E_c$$

$$E_c \approx 1.5 \times 10^6 \text{ V/m}$$

$$\mu_n \approx 0.07 \text{ m}^2/\text{Vs}$$

long-channel device in sat.:

$$I_D = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} (V_{as} - V_T) \cdot V_{DSAT}$$

$$V_{DSAT} (\text{long ch.}) = (V_{as} - V_T)$$

In general,

$$V_{DSAT} = (V_{as} - V_T) \parallel (L \cdot E_c)$$

$$= \frac{(V_{as} - V_T) (L E_c)}{(V_{as} - V_T) + (L E_c)}$$

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{as} - V_T) \left[(V_{as} - V_T) \parallel (L \cdot E_c) \right]$$

prominence of short channel effects

\Rightarrow compare $\frac{V_{as} - V_T}{L}$ and E_c (ratio)

If small \Rightarrow long channel approx. is valid

(actual length of gate is irrelevant)

* as L decreases, smaller $(V_{as} - V_T)$ is needed to exhibit short-channel effects!

Rewrite:

$$I_D = W C_{ox} (V_{as} - V_T) \cdot v_{sd} \left[1 + \frac{L E_c}{V_{as} - V_T} \right]^{-1}$$

In deep velocity saturation,

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{as} - V_T) [L \cdot E_c]$$

$$= \frac{\mu_n C_{ox}}{2} \cdot W (V_{as} - V_T) \cdot E_c$$

\Rightarrow Drain current independent of L !

\Rightarrow $I_D - V_{as}$ relationship is linear!

$$* g_m = \frac{\partial I_D}{\partial V_{as}} = \frac{\mu_n C_{ox}}{2} \cdot W \cdot E_c$$

$$* C_{gs} = \frac{2}{3} W L C_{ox}$$

$$* \omega_T = \frac{g_m}{C_{gs}} = \frac{3}{4} \frac{\mu_n E_c}{L}$$

$$\Rightarrow \mu_T \propto \frac{1}{L} \quad \left[\text{was } \frac{1}{L^2} \text{ for long-channel} \right]$$

$\Rightarrow \mu_T$ does not depend on bias conditions (as long as device is in sat.), or oxide thickness (t_{ox} , C_{ox})

* PMOS devices show saturation effects at higher fields (holes vs e^- s)