EE223 Analog Integrated Circuits
Fall 2018

Lecture 8: CS with Source Degeneration
Source Follower

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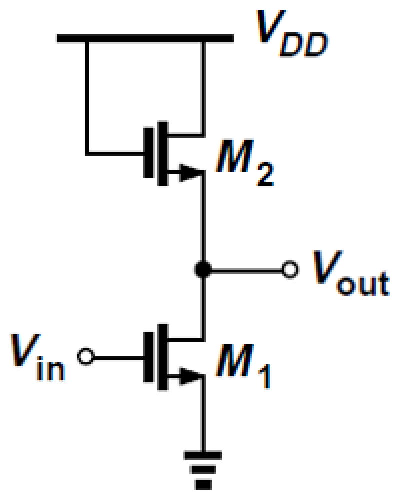
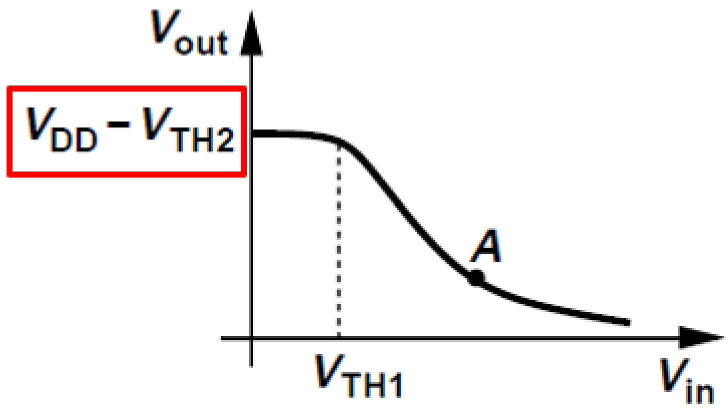
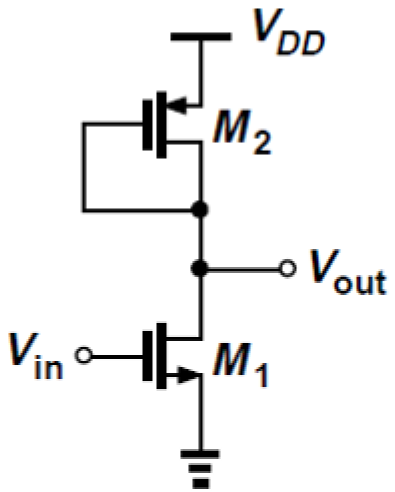
CS Amp with Diode-Connected Load





- For $V_{in} < V_{TH1}$, $V_{out} = V_{DD} - V_{TH2}$
- When $V_{in} > V_{TH1}$, previous large-signal analysis predicts that V_{out} approximately follows a single line
- As V_{in} exceeds $V_{out} + V_{TH1}$ (to the right of point A), M_1 enters the triode region and the characteristic becomes nonlinear.

CS Amp with Diode-Connected PMOS



- Diode-connected load can be implemented as a PMOS device, free of body-effect
- Small-signal voltage gain neglecting channel-length modulation

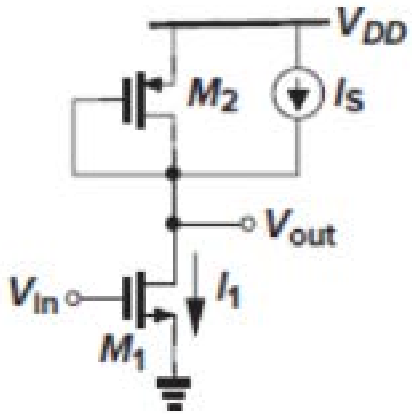
$$A_v = -\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}}$$

- Gain is a relatively weak function of device dimensions
- Since $\mu_n \approx 2\mu_p$, high gain requires "strong" input device wide and "weak" load device narrow
- This limits voltage swings since for $\lambda = 0$, we get

$$\frac{|V_{GS2} - V_{TH2}|}{V_{GS1} - V_{TH1}} = A_v$$

- For diode-connected loads, swing is constrained by both required overdrive voltage and threshold voltage, i.e., for small overdrive, output cannot exceed $V_{DD} - |V_{TH}|$.

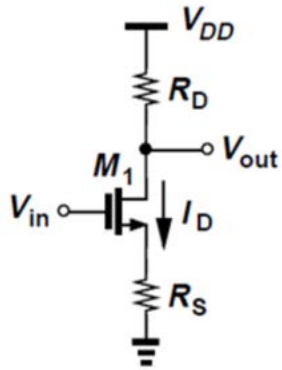
CS Amp with Diode-Connected PMOS and Current Source

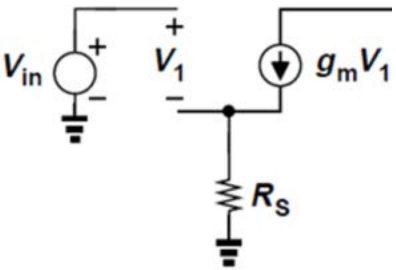


What is the gain of the Amp if $(W/L)_1 = 2(W/L)_2$ and $Is = 0.75\ I_1$? Assume $\mu_n = 2\ \mu_p$

$$A_v = \frac{v_{out}}{v_{in}} = -\frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_1}}{\sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_2 I_2}} = -\frac{\sqrt{2\cdot 2\mu_p\cdot 2\left(\frac{W}{L}\right)_2 I_1}}{\sqrt{2\mu_p\left(\frac{W}{L}\right)_2 0.25 I_1}} = -4$$

CS Stage with Source Degeneration





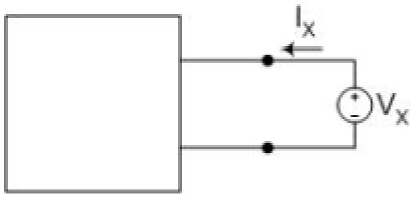
$$A_v = -G_m R_D = \frac{-g_m R_D}{1 + g_m R_S}$$

Complete the small-signal model and derive the gain expression.

Circuit Impedance - 1

- It is often useful to determine the impedance of a circuit seen from a specific pair of terminals

- The following is the recipe to do so:
  1. Connect a voltage source, V_x, to the port
  2. Suppress all independent sources
  3. Measure or calculate I_x
  4. $R = V_x / I_x$ from Ohm's law

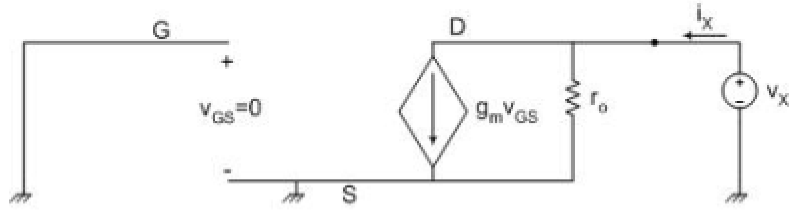


In general, $g_m > g_{mb} > g_o$

Circuit Impedance - 2

- Find the small-signal impedance of the following circuits

- We draw the small-signal model, which is the same for both circuits, and connect a voltage source as shown below:

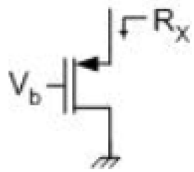


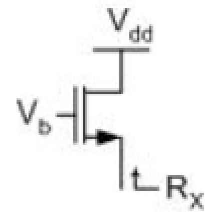
$$i_X = \frac{v_X}{r_o} + g_m \cdot v_{GS} = \frac{v_X}{r_o}$$

$$R_X = \frac{v_X}{i_X} = r_o$$

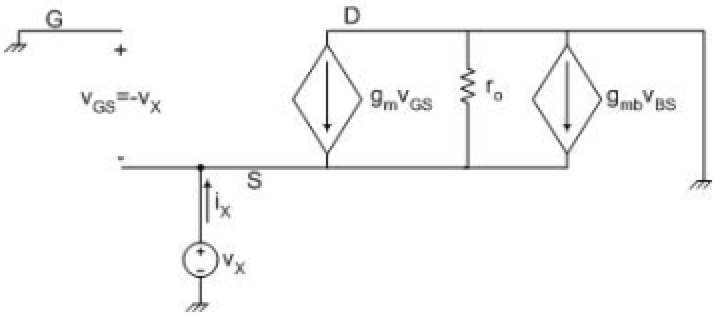
Circuit Impedance - 3

- Find the small-signal impedance of the following circuits





- We draw the small-signal model, which is the same for both circuits, and connect a voltage source as shown below:

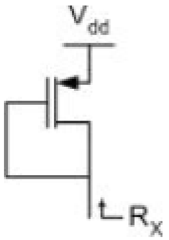


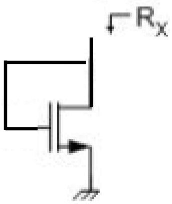
$$i_X = \frac{v_X}{r_o} - g_m \cdot v_{GS} - g_{mb} \cdot v_{BS} = \frac{v_X}{r_o} + g_m \cdot v_X + g_{mb} \cdot v_X$$

$$R_X = \frac{v_X}{i_X} = \frac{1}{\frac{1}{r_o} + g_m + g_{mb}} = r_o \left\| \frac{1}{g_m} \right\| \frac{1}{g_{mb}} \approx 1/g_m$$ $\Leftarrow$ **In general, $g_m > g_{mb} > g_o$**

Circuit Impedance - 4

- Find the small-signal impedance of the following circuits

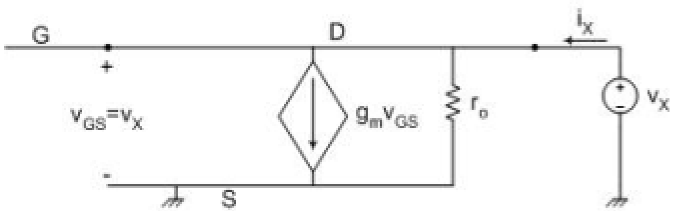




- We draw the small-signal model and connect a voltage source as shown below:

$$i_X = \frac{v_X}{r_o} + g_m \cdot v_{GS} = \frac{v_X}{r_o} + g_m \cdot v_X = v_X \cdot \left(\frac{1}{r_o} + g_m \right)$$

$$R_X = \frac{v_X}{i_X} = \frac{1}{\frac{1}{r_o} + g_m} = r_o \Big\| \frac{1}{g_m}$$



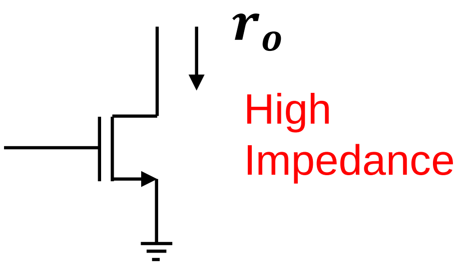
If channel length modulation is ignored ($r_o = \infty$) we get:

$$R_X = r_o \Big\| \frac{1}{g_m} = \infty \Big\| \frac{1}{g_m} = \frac{1}{g_m}$$

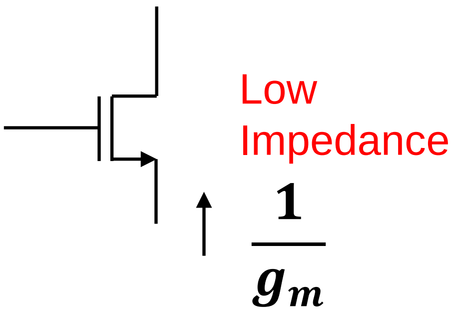
$\Leftarrow$ **In general, $g_m > g_{mb} > g_o$**

Circuit Impedance Summary

- Looking into the Drain
  - $g = g_o = g_{ds}$ $\quad r = r_o = r_{ds}$
  - $C = C_{gd} + C_{db}$
  - “High” Impedance node

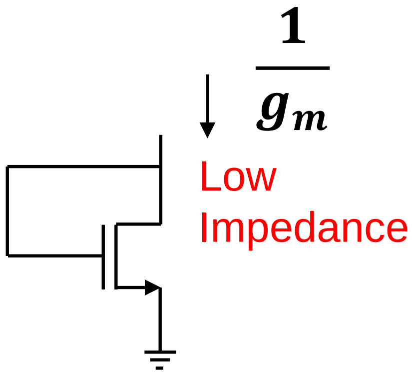


- Looking into the Source
  - $g = g_m + g_{mb} + g_{ds} \sim g_m$ $\quad r = 1/g_m$
  - $C = C_{gs} + C_{sb}$
  - “Low” Impedance node

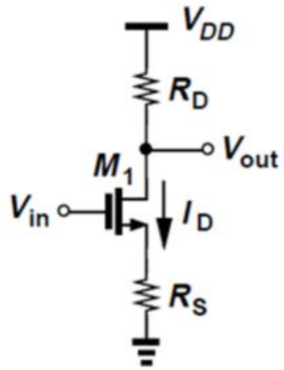


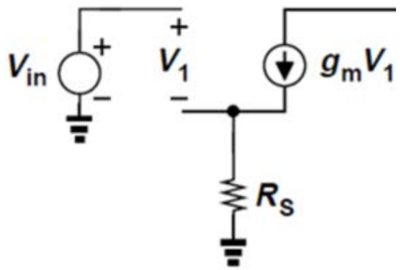
- Looking into the Source with Well diode
  - $g = g_m + g_{ds} \sim g_m$ $\quad r = 1/g_m$
  - $C = C_{gs} + C_{well}$

- Looking into the Diode (Drain & Gate are connected)
  - $g = g_m + g_o \sim g_m$ $\quad r = 1/g_m$
  - $C = C_{gs} + C_{db}$
  - “Low” Impedance node



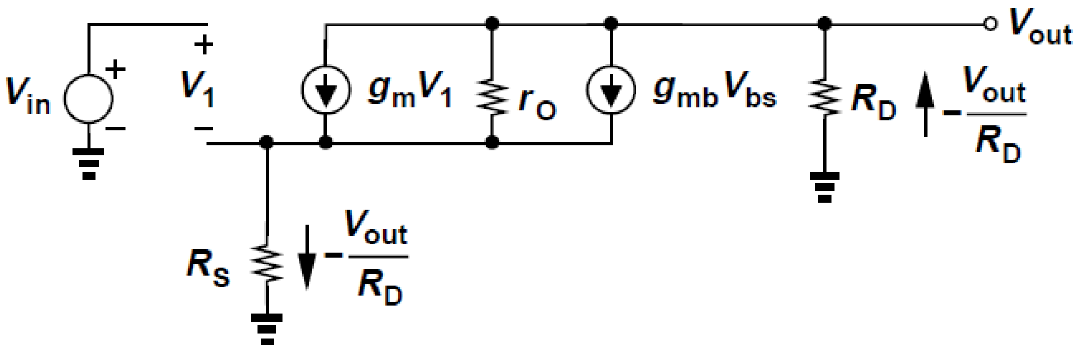
CS Stage with Source Degeneration





Complete the small-signal model and derive the gain expression.

CS Stage with Source Degeneration



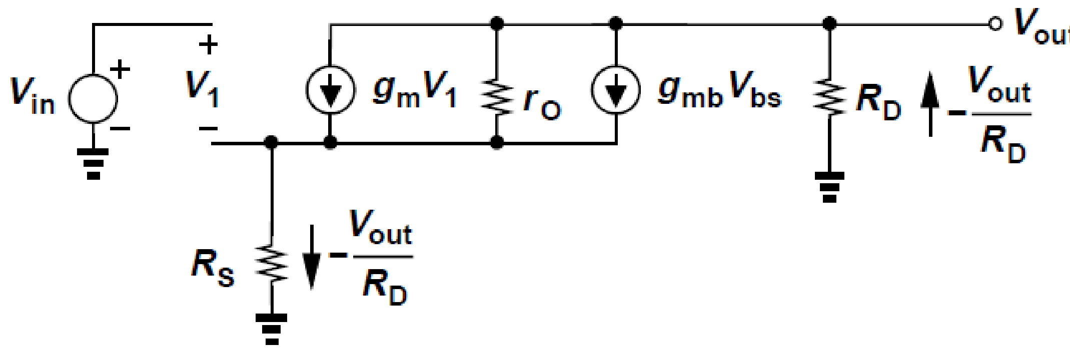
- **To compute gain in the general case including body effect and channel-length modulation, consider above small-signal model**
- **From KVL at input,**

$$V_1 = V_{in} + V_{out} R_S / R_D$$

- **KCL at output gives**

$$\begin{aligned} I_{ro} &= -\frac{V_{out}}{R_D} - (g_m V_1 + g_{mb} V_{bs}) \\ &= -\frac{V_{out}}{R_D} - \left[g_m \left(V_{in} + V_{out} \frac{R_S}{R_D} \right) + g_{mb} V_{out} \frac{R_S}{R_D} \right] \end{aligned}$$

CS Stage with Source Degeneration



- **Since voltage drops across r_O and R_S must add up to V_{out},**

$$\begin{aligned} V_{out} &= I_{ro} r_O - \frac{V_{out}}{R_D} R_S \\ &= -\frac{V_{out}}{R_D} r_O - \left[g_m \left(V_{in} + V_{out} \frac{R_S}{R_D} \right) + g_{mb} V_{out} \frac{R_S}{R_D} \right] r_O - V_{out} \frac{R_S}{R_D} \end{aligned}$$

- **Voltage gain is therefore**

$$\frac{V_{out}}{V_{in}} = \frac{-g_m r_O R_D}{R_D + R_S + r_O + (g_m + g_{mb}) R_S r_O} \approx \frac{-g_m R_D}{1 + (g_m + g_{mb}) R_s} \approx \frac{-g_m R_D}{1 + g_m R_s}$$

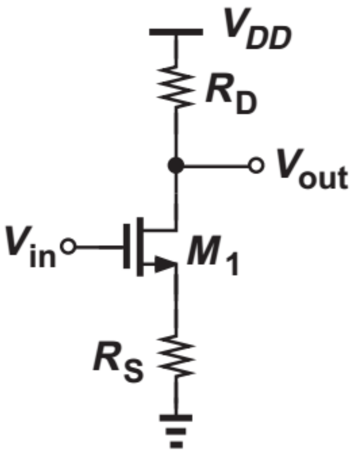
CS Stage with Source Degeneration

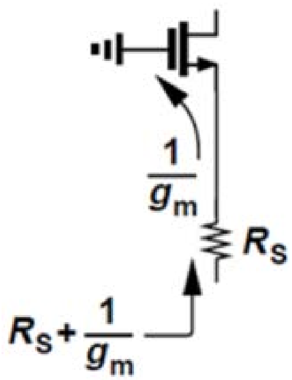
- Small-signal gain can be written as

$$A_v = -\frac{g_m R_D}{1 + g_m R_S} = \boxed{-\frac{R_D}{\frac{1}{g_m} + R_S}}$$

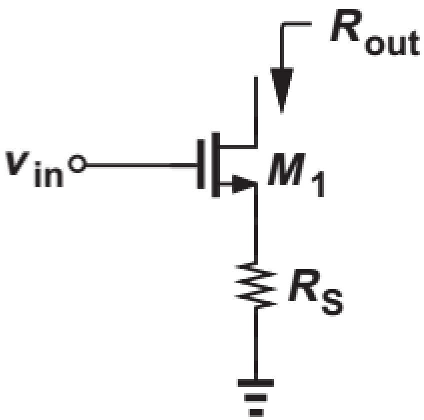
- Magnitude of gain

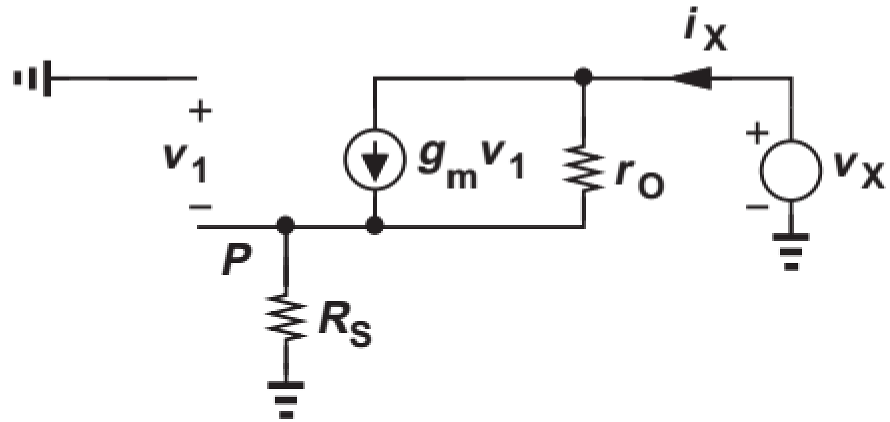
$$\frac{Resistance\ seen\ at\ the\ drain}{Total\ resistance\ seen\ in\ the\ source\ path}$$





Output Impedance of CS Amp with R_s





$$v_1 = -i_X R_S$$

$$v_X = (i_X - g_m v_1) r_o - v_1$$

$$= (i_X + g_m i_X R_S) r_o + i_X R_S$$

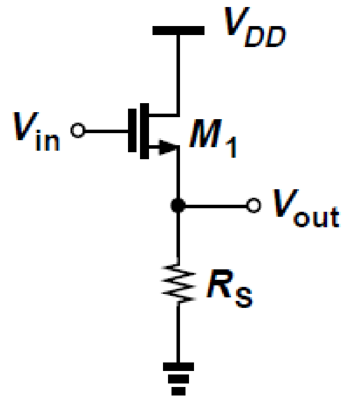
$$= i_X [(1 + g_m R_S) r_o + R_S]$$

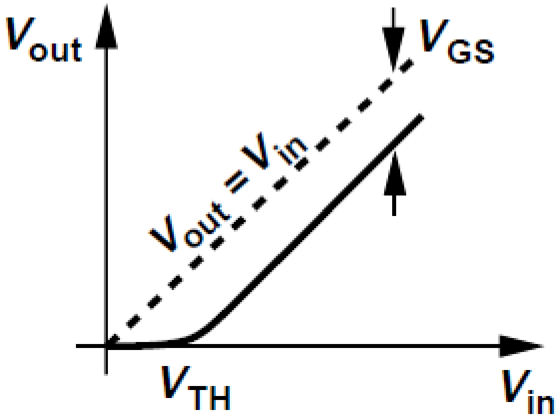
$$R_{out} = \frac{v_X}{i_X} = (1 + g_m R_S) r_o + R_S$$

$$= r_o + R_S + (g_m r_o) \cdot R_S$$

$$\approx (g_m r_o) \cdot R_S$$

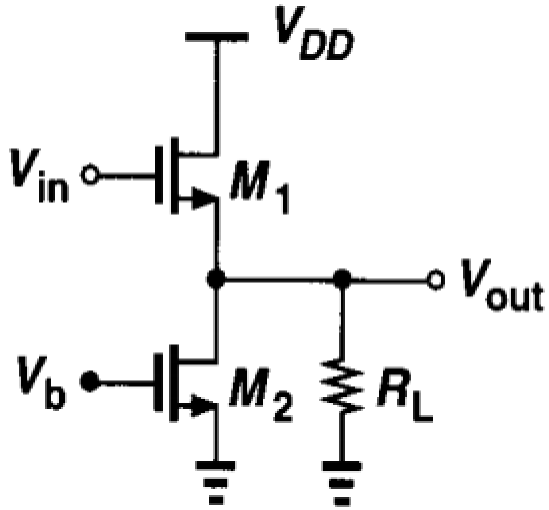
Source Follower

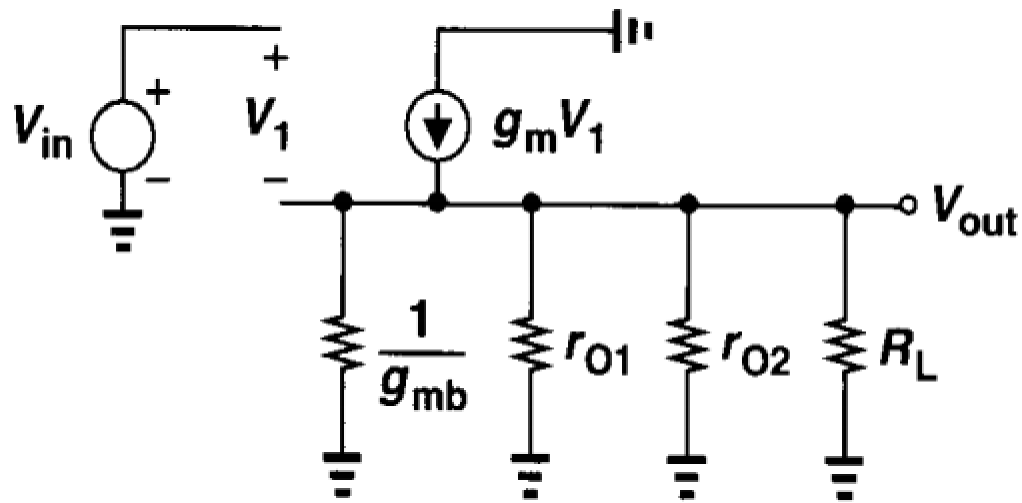




- For V_{in} < V_{TH}, M_1 is off and V_{out} = 0
- As V_{in} increases, V_{out} follows the input with a difference (level shift) equal to V_{GS}

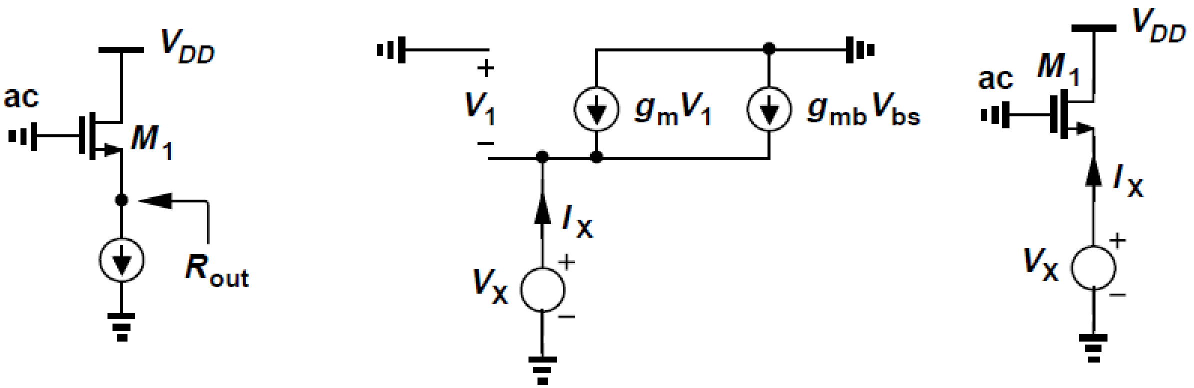
Source Follower Gain





$$A_v = g_m R_{out} = g_m(R\uparrow // R\downarrow) = \frac{g_m}{g\uparrow + g\downarrow}$$

Source Follower Output Impedance

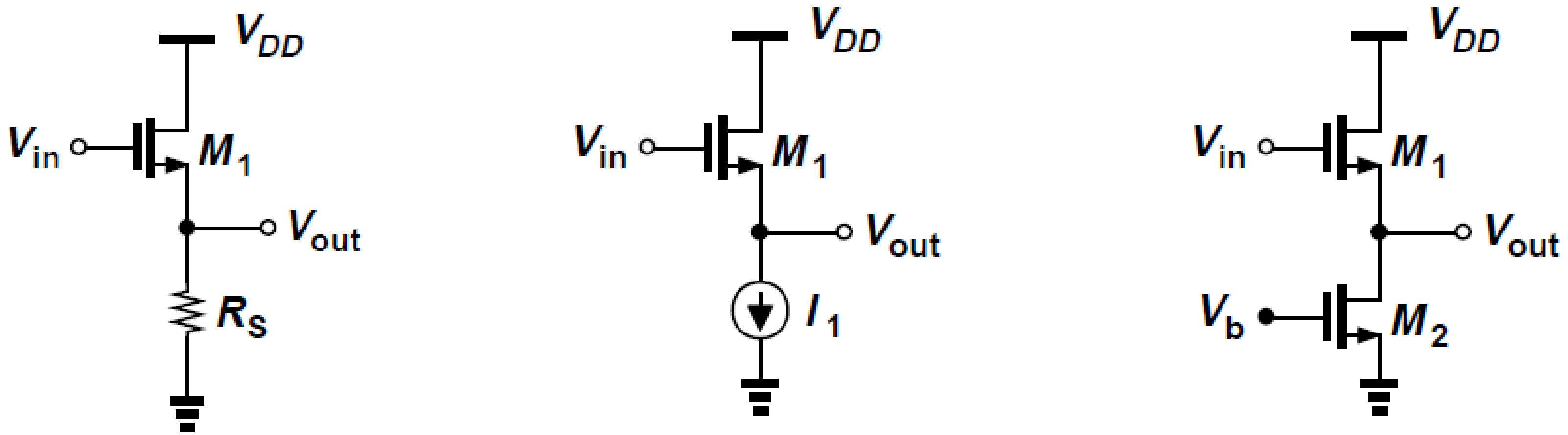


$$V_X = -V_{bs}$$

$$I_X - g_m V_X - g_{mb} V_X = 0$$

$$R_{out} = \frac{1}{g_m + g_{mb}}$$

Issue with Source Follower



- **Voltage headroom limitation**
- **Nonlinear dependence of V_{TH} on the source potential**
- **r_O changes substantially with V_{DS}**

Issue with Source Follower

- **Nonlinearity can be eliminated if the bulk is tied to the source**

- **PMOS source follower employing two separate n-wells can eliminate the body effect of M_1**