# Path-loss and Shadowing (Large-scale Fading)

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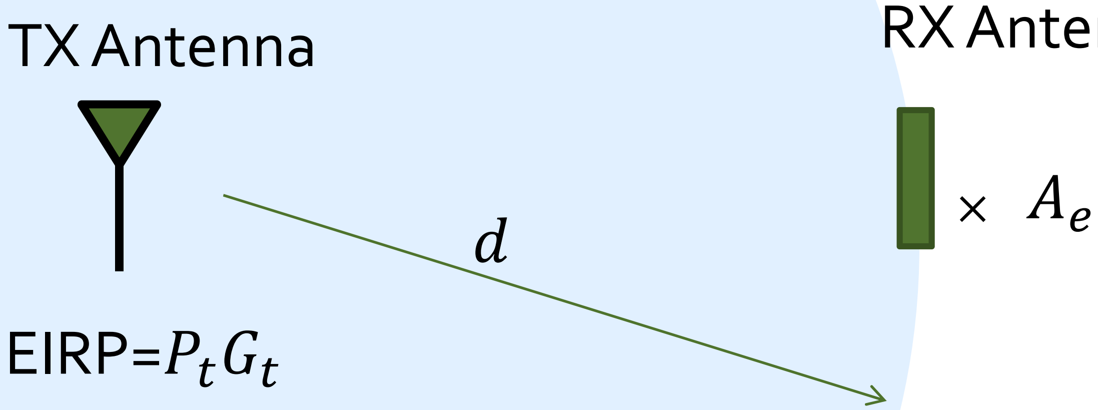
# Friis Formula




$$\Rightarrow P_r = \frac{P_t G_t A_e}{4\pi d^2}$$

Power spatial density$\left(\frac{W}{m^2}\right)$

$$\times\ \frac{1}{4\pi d^2}$$

# Antenna Aperture

$$\boxed{P_r = \frac{P_t G_t A_e}{4\pi d^2}}$$

- **Antenna Aperture=Effective Area**
- **Isotropic Antenna's effective area** $A_{e,iso} \doteq \frac{\lambda^2}{4\pi}$
- **Isotropic Antenna's Gain=1**
- $G = \frac{4\pi}{\lambda^2} A_e$
- **Friis Formula becomes:** $P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2} = \frac{P_t A_t A_r}{\lambda^2 d^2}$

# Friis Formula

$$\boxed{P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi d)^2}}$$

- $\frac{\lambda^2}{(4\pi d)^2}$ is often referred as "Free-Space Path Loss" (FSPL)
- Only valid when d is in the "far-field" of the transmitting antenna
- Far-field: when $d > d_f$, Fraunhofer distance
- $d_f = \frac{2D^2}{\lambda}$, and it must satisfies $d_f \gg D$ and $d_f \gg \lambda$
  - D: Largest physical linear dimension of the antenna
  - $\lambda$: Wavelength
- We often choose a $d_0$ in the far-field region, and smaller than any practical distance used in the system
- Then we have $P_r(d) = P_r(d_0)\left(\frac{d}{d_0}\right)^2$

# Received Signal after Free-Space Path Loss

$$r(t) = Re\left\{ \frac{\lambda\sqrt{G_t G_r}\, \overbrace{exp\left(-\frac{j2\pi d}{\lambda}\right)}^{\text{phase difference due to propagation distance}}}{4\pi d} \underbrace{\tilde{g}(t)}_{\text{Complex envelope}} \underbrace{exp(j2\pi f_c t)}_{\text{Carrier (sinusoid)}} \right\}$$

Free-Space Path Loss

# Example: Far-field Distance

- **Find the far-field distance of an antenna with maximum dimension of 1m and operating frequency of 900 MHz (GSM 900)**
- **Ans:**
- **Largest dimension of antenna: D=1m**
- **Operating Frequency: f=900 MHz**
- **Wavelength:** $\boldsymbol{\lambda = \frac{c}{f} = \frac{3\times 10^{8}}{900\times 10^{6}} = 0.33}$
- $\boldsymbol{d_f = \frac{2D^2}{\lambda} = \frac{2}{0.33} = 6.06}$ **(m)**

# Example: FSPL

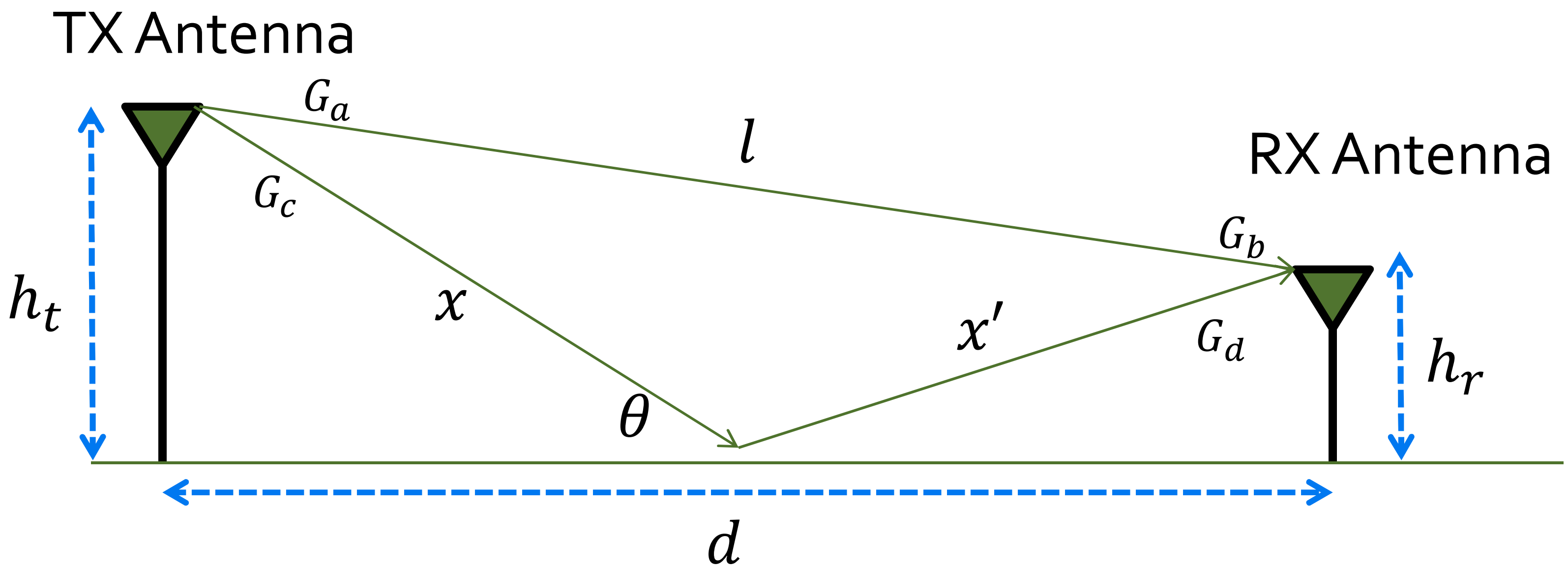
- If a transmitter produces 50 watts of power, express the transmit power in units of (a) dBm and (b) dBW.
- If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency, find the received power in dBm at a free space distance of 100 m from the antenna. What is the received power at 10 km? Assume unity gain for the receiver antenna.
- Ans:
- $10\ log_{10}(50) = 17\ dBW = 47\ \mathrm{dBm}$
- Received Power at 100m

$$P_r(100m) = \frac{50 \times 1 \times 1 \times \left(\frac{3 \times 10^8}{900 \times 10^6}\right)^2}{(4\pi \times 100)^2} = 3.5 \times 10^{-6}\ (W)$$
$$= -54.5\ (dBW)$$

- Received Power at 10km

$$P_r(10km) = P_r(100m)\left(\frac{100}{10000}\right)^2 = 3.5 \times 10^{-10}(W)$$
$$= -94.5\ (dBW)$$

# Two-ray Model



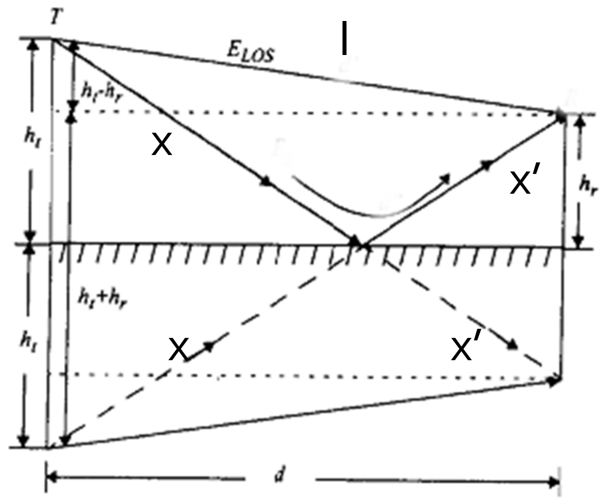

Delayed since x+x′ is longer. $\tau = (x + x' - l)/c$

$$r_{2-ray}(t) =$$

$$Re\left\{\frac{\lambda}{4\pi}\left[\frac{\sqrt{G_a G_b}\tilde{g}(t)\exp\left(-\frac{j2\pi l}{\lambda}\right)}{l} + \frac{R\sqrt{G_c G_d}\overbrace{\tilde{g}(t-\tau)}\exp\left(-\frac{j2\pi(x+x')}{\lambda}\right)}{x+x'}\right]\exp(j2\pi f_c t)\right\}$$

R: ground reflection coefficient (phase and amplitude change)

# Two-ray Model: Received Power

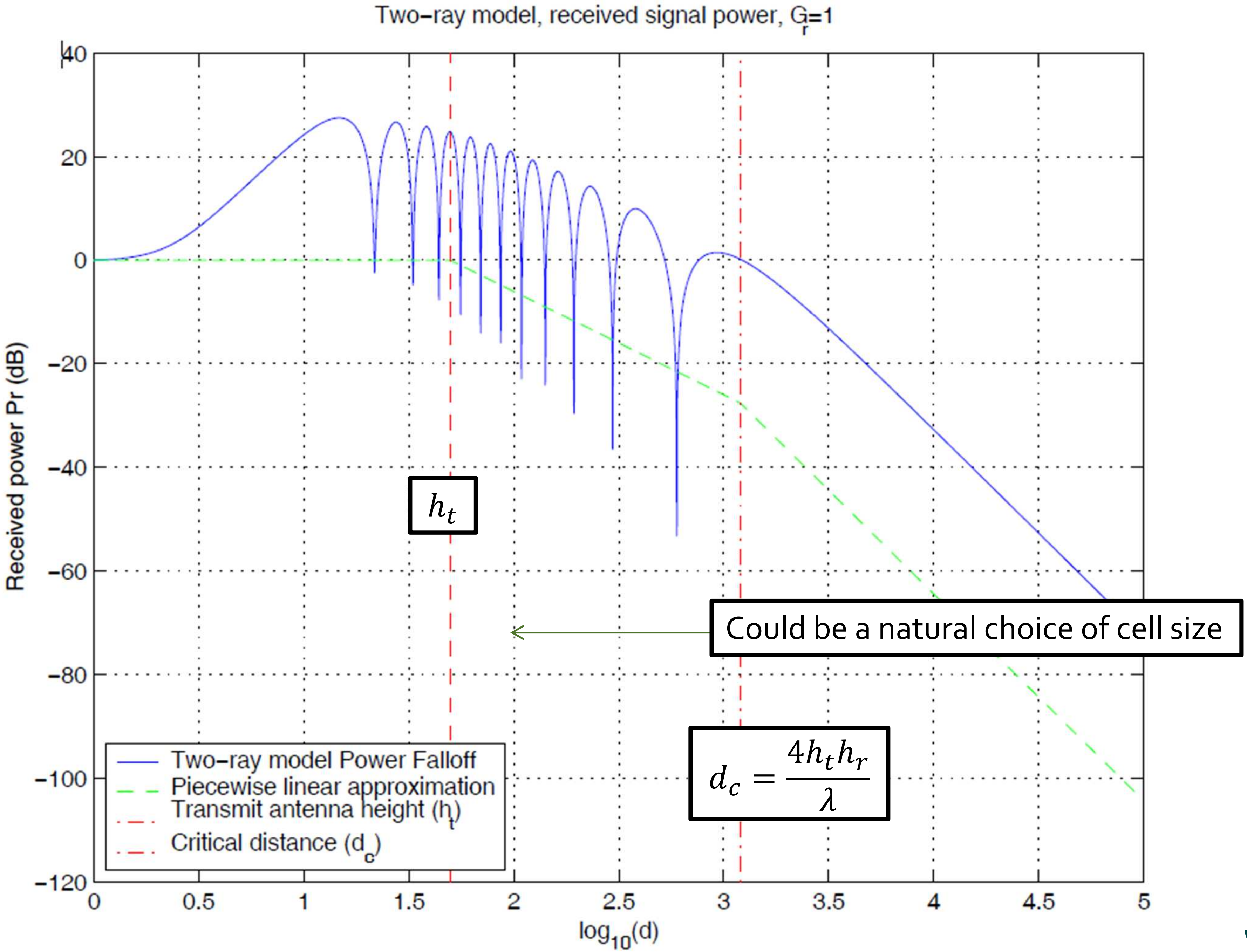



- $P_r = P_t \left[\frac{\lambda}{4\pi}\right]^2 \left|\frac{\sqrt{G_a G_b}}{l} + \frac{\sqrt{G_c G_d} exp(-j\Delta\phi)}{x+x'}\right|^2$
- **The above is verified by empirical results.**
- $\Delta\phi = 2\pi(x + x' - l)/\lambda$
- $x + x' - l = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$

# Two-ray Model: Received Power

- When $d \gg h_t + h_r$, $\Delta\phi = \frac{2\pi(x+x'-l)}{\lambda} \approx \frac{4\pi h_t h_r}{\lambda d}$
- For asymptotically large d, $x + x' \approx \mathrm{l} \approx d$, $\theta \approx 0$, $\mathrm{G_a G_b} \approx G_c G_d$, $\mathrm{R} \approx -1$ (phase is inverted after reflection)
- $\left|\frac{\sqrt{G_a G_b}}{l} + \frac{\sqrt{G_c G_d} exp(-j\Delta\phi)}{x+x\prime}\right|^2 \approx \left|\frac{\sqrt{G_a G_b}}{d}\right|^2 |1 + exp(-j\Delta\phi)|^2$
- $|1 + exp(-j\Delta\phi)|^2 = \left(1 - cos(\Delta\phi)\right)^2 + sin^2\Delta\phi = 2 - 2\cos(\Delta\phi) = 4sin^2(\frac{\Delta\phi}{2}) \approx \Delta\phi^2$
- $P_r \approx \left[\frac{\lambda\sqrt{G_a G_b}}{4\pi d}\right]^2 \left[\frac{4\pi h_t h_r}{\lambda d}\right]^2 \mathrm{P_t} = \left[\frac{\sqrt{G_a G_b} h_t h_r}{d^2}\right]^2 \mathrm{P_t}$

Independent of $\lambda$ now



Two-ray model, received signal power, $G_r$=1
Received power Pr (dB)
$\log_{10}(d)$
Two-ray model Power Falloff
Piecewise linear approximation
Transmit antenna height ($h_t$)
Critical distance ($d_c$)
$h_t$
Could be a natural choice of cell size
$d_c = \frac{4h_t h_r}{\lambda}$

# Indoor Attenuation

- **Factors which affect the indoor path-loss:**
  - Wall/floor materials
  - Room/hallway/window/open area layouts
  - Obstructing objects' location and materials
  - Room size/floor numbers
- **Partition Loss:**

| Partition type | Partition Loss (dB) for 900-1300 MHz |
|---|---|
| Floor | 10-20 for the first one,<br>6-10 per floor for the next 3,<br>A few dB per floor afterwards. |
| Cloth partition | 1.4 |
| Double plasterboard wall | 3.4 |
| Foil insulation | 3.9 |
| Concrete wall | 13 |
| Aluminum siding | 20.4 |
| All metal | 26 |

# Simplified Path-Loss Model

- **Back to the simplest:**

$$P_r = P_t K \left[\frac{d_0}{d}\right]^{\gamma}$$

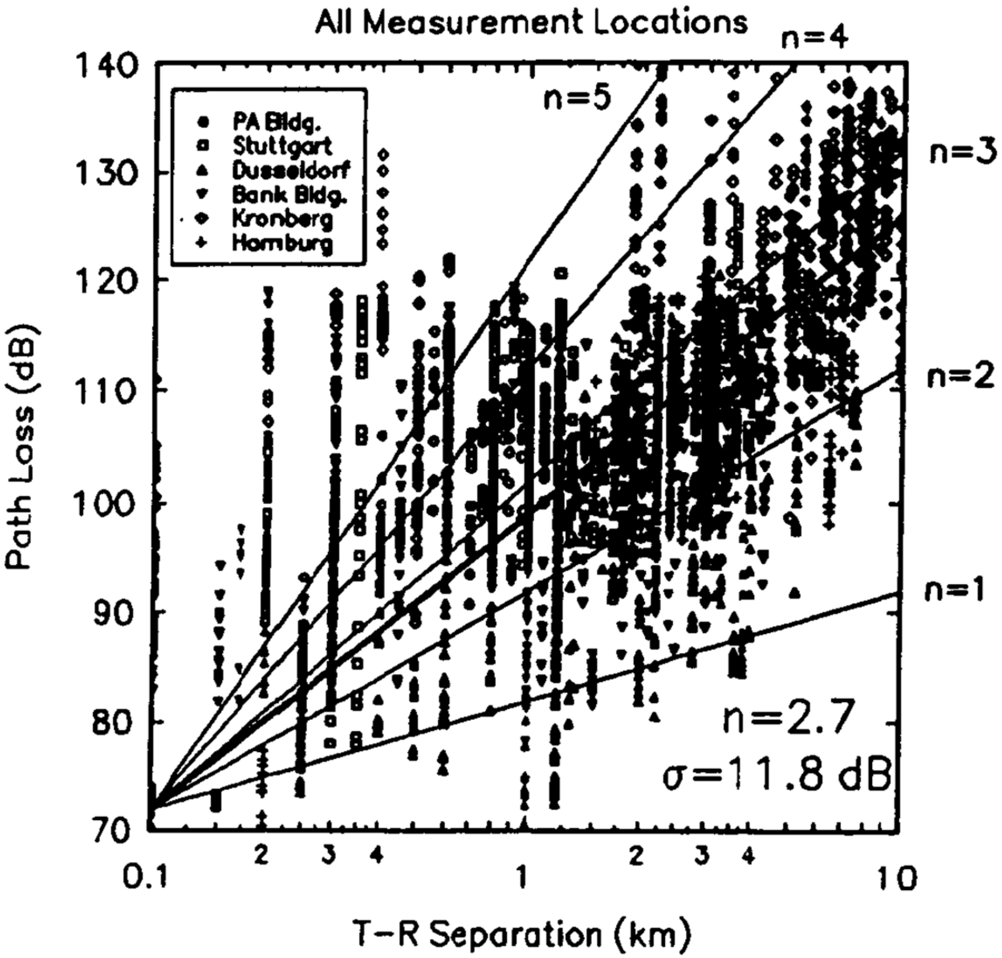
  - $d_0$: reference distance for the antenna far field (usually 1-10m indoors and 10-100m outdoors)
  - $K$: constant path-loss factor (antenna, average channel attenuation), and sometimes we use

$$K = \left(\frac{\lambda}{4\pi d_0}\right)^2$$

  - $\gamma$: path-loss exponent

# Some empirical results

Measurements in Germany Cities




| Environment | Path-loss Exponent |
|---|---|
| Free-space | 2 |
| Urban area cellular radio | 2.7-3.5 |
| Shadowed urban cellular radio | 3-5 |
| In building LOS | 1.6 to 1.8 |
| Obstructed in building | 4 to 6 |
| Obstructed in factories | 2 to 3 |

# Empirical Path-Loss Model

- **Based on empirical measurements**
  - over a given distance
  - in a given frequency range
  - for a particular geographical area or building
- **Could be applicable to other environments as well**
  - Less accurate in a more general environment
- **Analytical model: $P_r/P_t$ is characterized as a function of distance.**
- **Empirical Model: $P_r/P_t$ is a function of distance including the effects of path loss, shadowing, and multipath.**
  - Need to average the received power measurements to remove multipath effects → Local Mean Attenuation (LMA) at distance d.

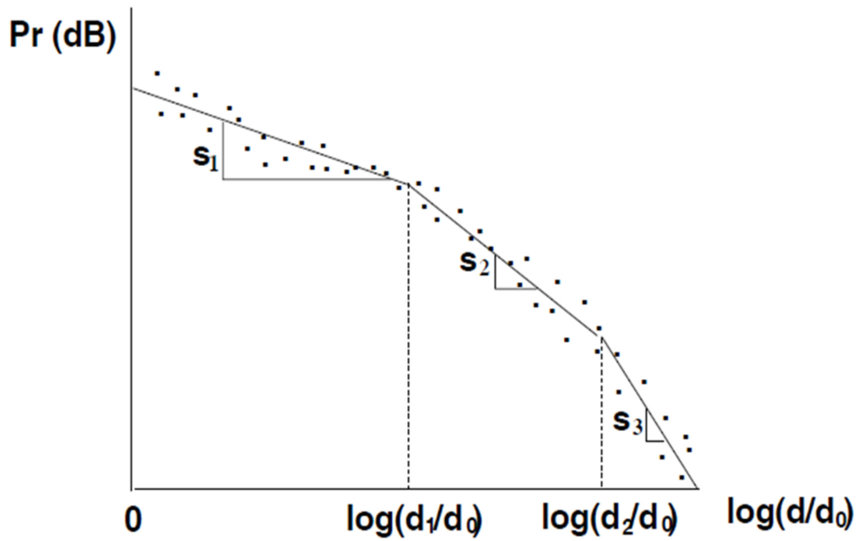
# Example: Okumura Model

- **Okumura Model:**
  $$\boldsymbol{P_L(d)dB = L(f_c, d) + A_\mu(f_c, d) - G(h_t) - G(h_r) - G_{AREA}}$$
  - $\boldsymbol{L(f_c, d)}$**: FSPL,** $\boldsymbol{A_\mu(f_c, d)}$**: median attenuation in addition to FSPL**
  - $G(h_t) = 20\log_{10}(\frac{h_t}{200}), G(h_r) = \begin{cases} 10\log_{10}\left(\frac{h_r}{3}\right), h_r \le 3m, \\ 20\log_{10}\left(\frac{h_r}{3}\right), 3m < h_r < 10m. \end{cases}$

    :antenna height gain factor.
  - $\boldsymbol{G_{AREA}}$: gain due to the type of environment

# Example: Piecewise Linear Model

- **N segments with N-1 "breakpoints"**
- **Applicable to both outdoor and indoor channels**
- **Example – dual-slope model**




$$P_r(d) = \begin{cases} P_t K \left(\frac{d_0}{d}\right)^{\gamma_1} & d_0 \le d \le d_c, \\ P_t K \left(\frac{d_c}{d}\right)^{\gamma_1} \left(\frac{d}{d_c}\right)^{\gamma_2} & d > d_c. \end{cases}$$

- $K$: constant path-loss factor
- $\gamma_1$: path-loss exponent for $d_0 \sim d_c$
- $\gamma_2$: path-loss exponent after $d_c$

# Shadow Fading

- **Same T-R distance usually have different path loss**
  - Surrounding environment is different
- **Reality: simplified Path-Loss Model represents an "average"**
- **How to represent the difference between the average and the actual path loss?**
- **Empirical measurements have shown that**
  - it is random (and so is a random variable)
  - Log-normal distributed

# Log-normal distribution

- **A log-normal distribution is a probability distribution of a random variable whose logarithm is normally distributed:**

logarithm of the random variable

$$f_X(x;\mu,\sigma^2) = \frac{1}{\boxed{x}\sigma\sqrt{2\pi}}\exp\left(-\frac{(\boxed{\log x} - \mu)^2}{2\sigma^2}\right)$$

normalized so that the integration of the pdf=1

- $x$: the random variable (linear scale)
- $\mu, \sigma^2$: mean and variance of the distribution (in dB)

# Log-normal Shadowing

- **Expressing the path loss in dB, we have**

$$P_L(d)[dB] = \overline{P_L}\,(d) + X_\sigma = \overline{P_L}\,(d_0) + 10\gamma \log\left(\frac{d}{d_0}\right) + X_\sigma$$

- $\boldsymbol{X_\sigma}$**: Describes the random shadowing effects**
- $\boldsymbol{X_\sigma \sim N(0, \sigma^2)}$
  **(normal distribution with zero mean and** $\boldsymbol{\sigma^2}$ **variance)**
- **Same T-R distance, but different levels of clutter.**
- **Empirical Studies show that** $\boldsymbol{\sigma}$ **ranges from 4 dB to 13dB in an outdoor channel**

# Why is it log-normal distributed?

- **Attenuation of a signal when passing through an object of depth *d is approximately:***

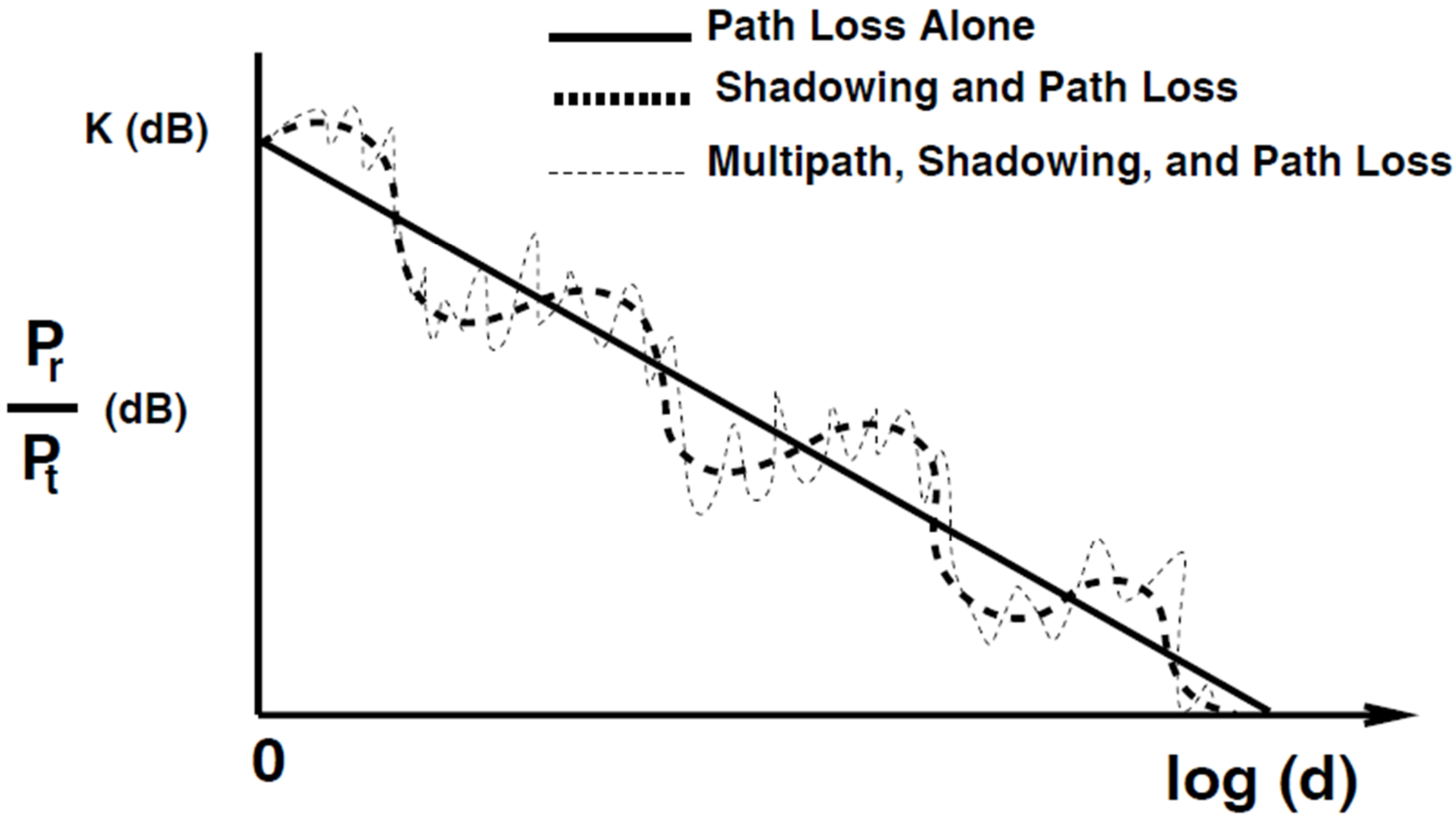
$$s(d) = \exp(-\alpha d)$$

  - $\alpha$: Attenuation factor which depends on the material
- **If $\boldsymbol{\alpha}$ is approximately the same for all blocking objects:**

$$s(d_t) = \exp\left(-\alpha \sum_i d_i\right) = \exp(-\alpha d_t)$$

  - $d_t = \sum_i d_i$: sum of all object depths
- **By central limit theorem, $\boldsymbol{d_t \sim N(\mu, \sigma^2)}$ when the number of object is large (which is true).**

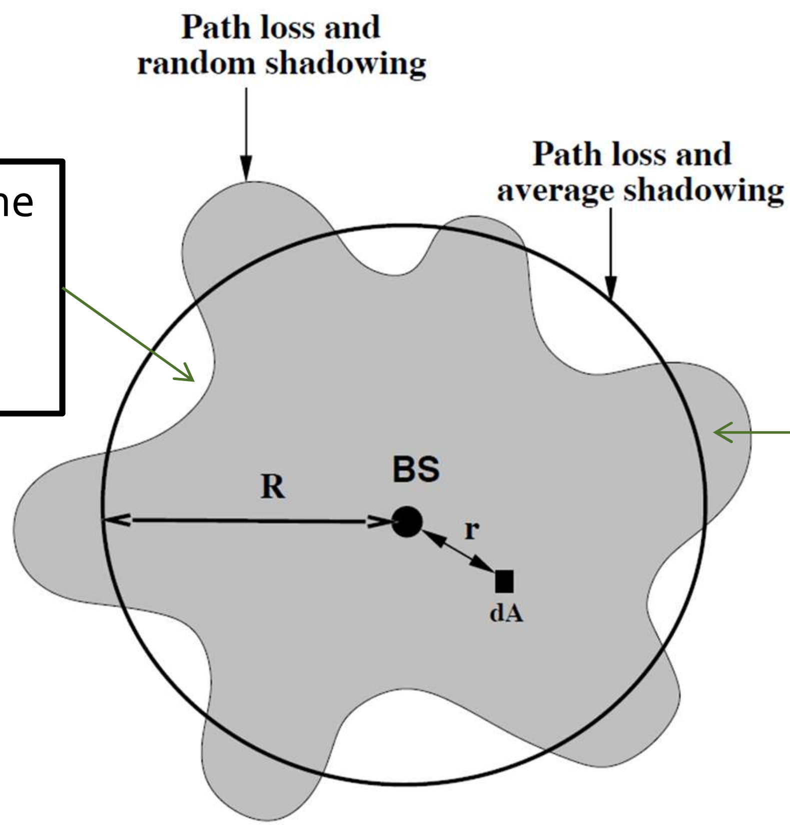
# Path Loss, Shadowing, and Multi-Path



# Cell Coverage Area

- **Cell coverage area: expected percentage of locations within a cell where the received power at these locations is <u>above a given minimum</u>.**

Some area within the cell has received power lower than $P_{min}$




Some area outside of the cell has received power higher than $P_{min}$

# Cell Coverage Area

- **We can boost the transmission power at the BS**
  - Extra interference to the neighbor cells
- **In fact, <u>any mobile</u> in the cell has a <u>nonzero probability</u> of having its received power below $P_{min}$.**
  - Since Normal distribution has infinite tails
  - Make sense in the real-world:
    in a tunnel, blocked by large buildings, doesn't matter if it is very close to the BS

# Cell Coverage Area

- **Cell coverage area is given by**

$$C = E\left[\frac{1}{\pi R^2}\int_{cell\ area} \overbrace{1[P_r(r) > P_{min}\ in\ dA]}^{\text{1 if the statement is true, 0 otherwise. (indicator function)}} dA\right]$$

$$= \frac{1}{\pi R^2}\int_{cell\ area} E\big[1[P_r(r) > P_{min}\ in\ dA]\big] dA$$

- $P_A \doteq p(P_r(r) > P_{min})$

$$C = \frac{1}{\pi R^2}\int_{cell\ area} P_A dA = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R P_A\, r dr\ d\theta$$

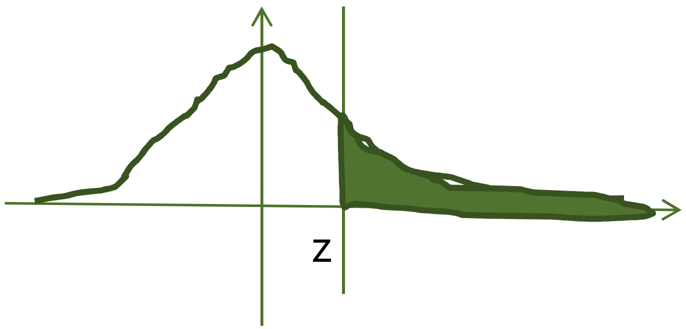
# Cell Coverage Area

$$P_A = p(\overline{P_r}(r) \geq P_{min}) = Q\left(\frac{P_{min} - (P_t - \overline{P_L}(r))}{\sigma}\right)$$

Log-normal distribution's standard deviation

- **Q-function:**

$$Q(z) \doteq p(X > z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy$$



# Cell Coverage Area

- **Solving the equations yield:**

$$C = Q(a) + \exp\left[\frac{2-2ab}{b^2}\right] Q\left(\frac{2-ab}{b}\right)$$

- $\boldsymbol{a = \frac{P_{min} - \overline{P_r}(R)}{\sigma}, b = \frac{10\gamma log_{10}(e)}{\sigma}}$

average received power at cell boundary (distance=R)

- **If $\boldsymbol{P_{min} = \overline{P_r}(R)}$**

$$C = \frac{1}{2} + \exp\left[\frac{2}{b^2}\right] Q\left(\frac{2}{b}\right)$$

$$C = Q(a) + \exp\left[\frac{2-2ab}{b^2}\right] Q\left(\frac{2-ab}{b}\right)$$

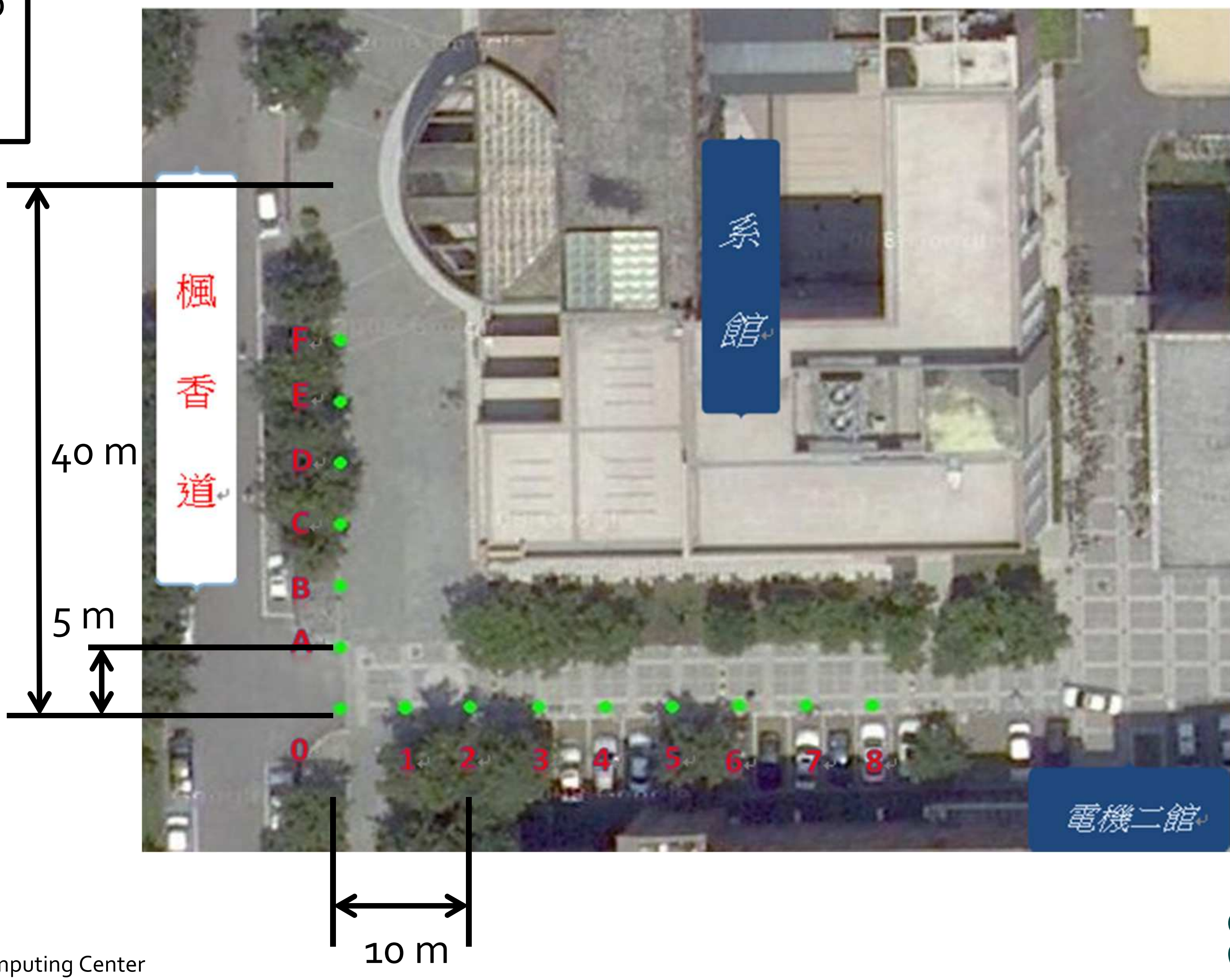
# Example

$$\boldsymbol{a = \frac{P_{min} - \overline{P_r}(R)}{\sigma}, b = \frac{10\gamma log_{10}(e)}{\sigma}}$$

- **Find the coverage area for a cell with**
  - a cell radius of 600m
  - a base station transmission power of 20 dBm
  - a minimum received power requirement of -110 dBm.
  - path loss model: $P_r(d) = P_t K \left(\frac{d_0}{d}\right)^{\gamma}, \gamma = 3.71, K = -31.54\ dB, d_0 = 1$, shadowing standard deviation $\sigma = 3.65$ dB
- **Ans:**
- $\boldsymbol{\overline{P_r}(R) = 20 - 31.54 - 10 \times 3.71 \times log_{10}(600) = -114.6\ dBm}$
- $\boldsymbol{a = \frac{-110+114.6}{3.65} = 1.26, b = \frac{10\times3.71\times0.434}{3.65} = 4.41}$
- $\boldsymbol{C = Q(1.26) + exp(-0.46)Q(-0.807) = 0.6}$ **(not good)**
- **If we calculate C for a minimum received power requirement of -120 dBm**
  - C=0.988!

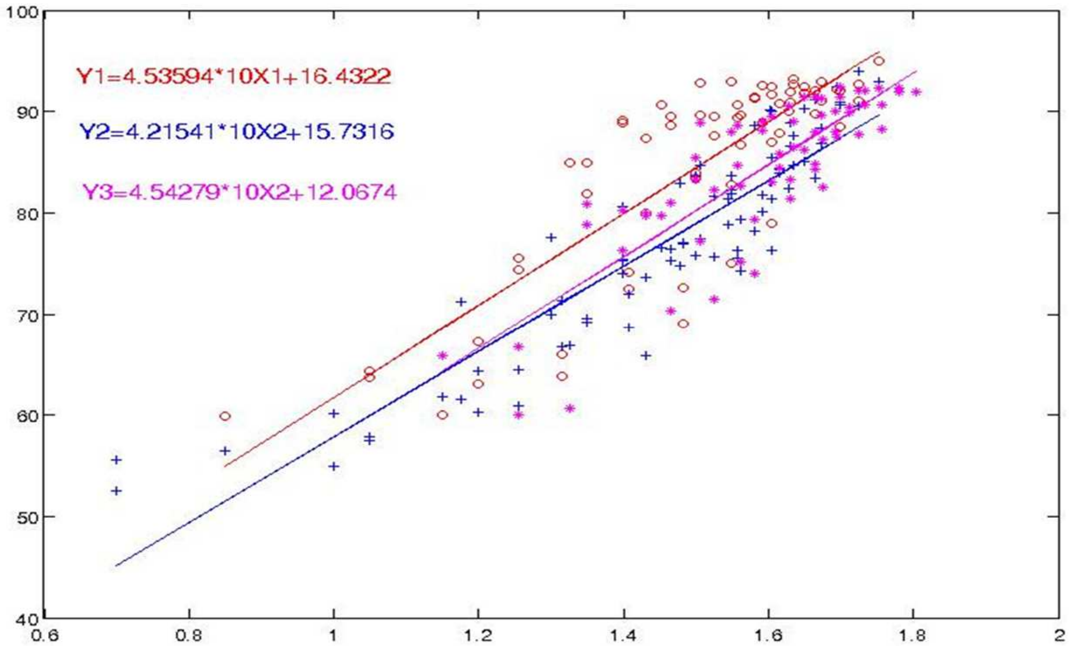
# Example: road corners path loss

Radio: Chipcon CC2420
IEEE 802.15.4, 2.4 GHz
TX pwr: 0 dBm




8 dBi peak gain omni-directional antenna

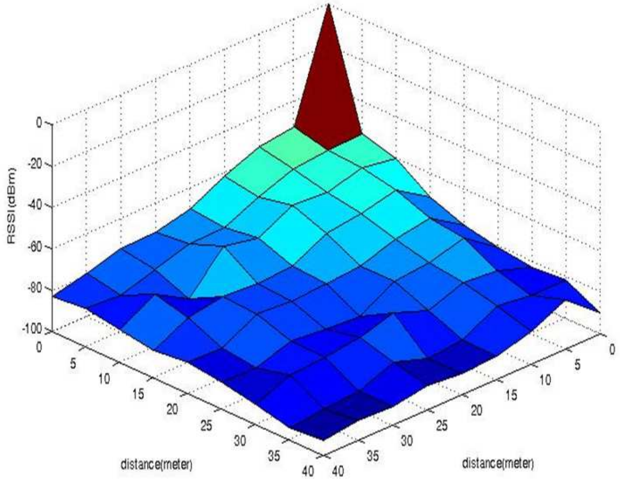
# Link Measurements –
# Path loss around the corner building




- **Compare the path-loss exponent of three different locations:**

1. **Corner of NTU_CSIE building**
2. **XinHai-Keelong intersection**
3. **FuXing-HePing intersection**

1

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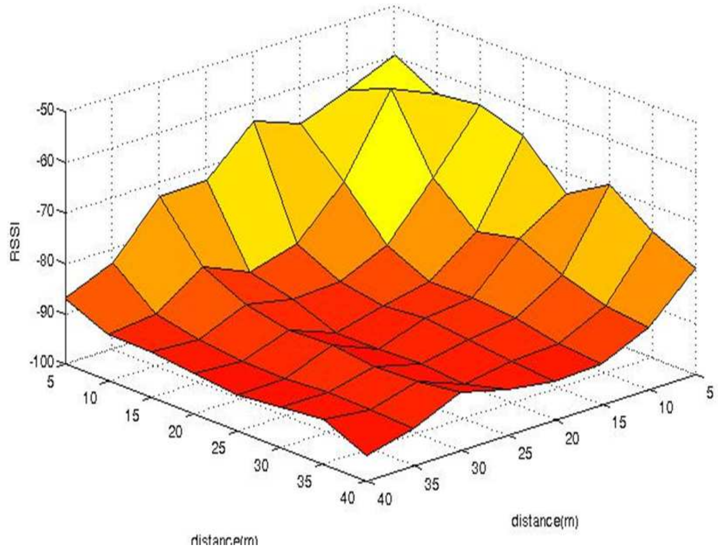


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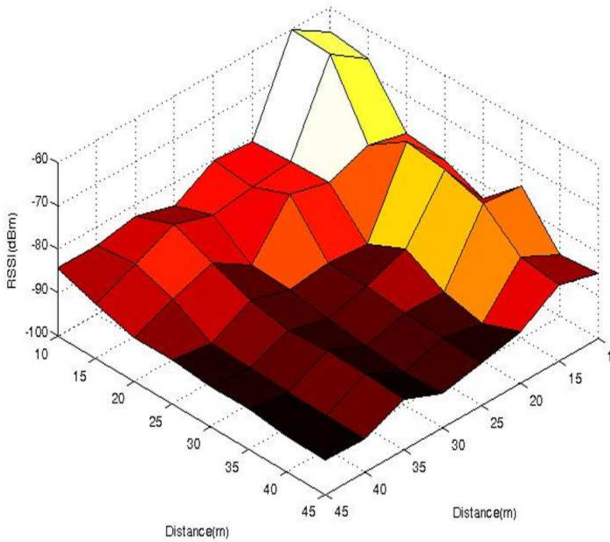
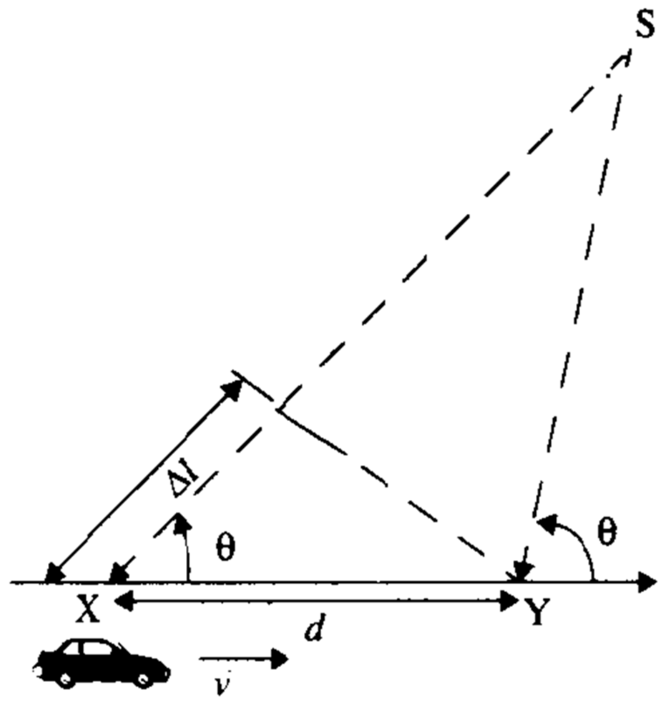



| | 1 | 2 | 3 |
|---|---|---|---|
| Passing-by vehicles | Occasionally | Frequently | Frequently |
| Buildings Around | No | Few buildings | Some high buildings |
| Intersection | Narrow | Wide | Wide |

# Doppler Effect




- Difference in path lengths $\Delta \mathrm{l} = d\ cos\theta = v\Delta \mathrm{t}\ \mathrm{cos}\theta$
- Phase change $\Delta\phi = \frac{2\pi\Delta \mathrm{l}}{\lambda} = \frac{2\pi v \Delta \mathrm{t}}{\lambda}\mathrm{cos}\theta$
- Frequency change, or Doppler shift,

$$f_d = \frac{1}{2\pi}\ \frac{\Delta\phi}{\Delta \mathrm{t}} = \frac{v}{\lambda} cos\theta$$

# Example

$$f_d = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta$$

- **Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving**
  1. directly toward the transmitter.
  2. directly away from the transmitter
  3. in a direction which is perpendicular to the direction of arrival of the transmitted signal.
- **Ans:**
  - Wavelength=$\lambda = \frac{c}{f_c} = \frac{3\times10^8}{1850\times10^6} = 0.162\ (m)$
  - Vehicle speed $v = 60mph = 26.82\ \left(\frac{m}{s}\right)$

  1. $f_d = \frac{26.82}{0.162}\cos(0) = 160\ (Hz)$
  2. $f_d = \frac{26.82}{0.162}\cos(\pi) = -160\ (Hz)$
  3. Since $\cos\left(\frac{\pi}{2}\right) = 0$, there is no Doppler shift!

# Doppler Effect

- If the car (mobile) is moving toward the direction of the arriving wave, the Doppler shift is positive
- Different Doppler shifts if different $\theta$ (incoming angle)
- Multi-path: all different angles
- Many Doppler shifts → Doppler spread