EE 210

HW#: 05

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Date: **10/3/2020**

Assigned question #s: **2**

Q1

HW05

FS

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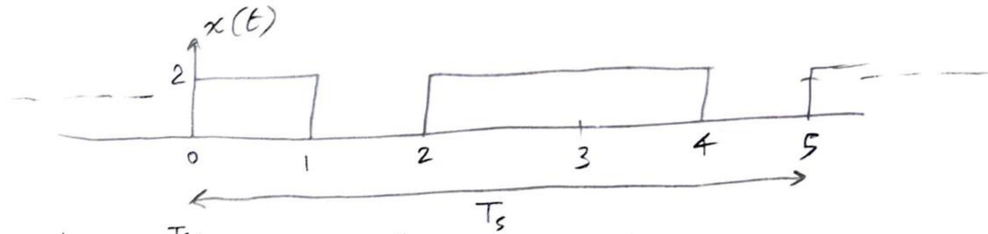


$T_s = 5ms$

$$X_k = \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} x(t)\, e^{-j2\pi\frac{k}{T_s}t}\,dt$$

$$= \frac{1}{5}\left[\int_0^1 (2)\, e^{-j2\pi\frac{k}{5}t}\,dt + \int_2^4 (2)\, e^{-j2\pi\frac{k}{5}t}\,dt\right]$$

$$= \frac{\cancel{2}}{\cancel{5}}\,\frac{1}{-j\cancel{2}\pi\frac{k}{\cancel{5}}}\left[\left(e^{-j2\pi\frac{k}{5}t}\right)_0^1 + \left(e^{-j2\pi\frac{k}{5}t}\right)_2^4\right]$$

$$= \frac{1}{-j\pi k}\left[e^{-j2\pi\frac{k}{5}(1)} - e^{-j2\pi\frac{k}{5}(0)} + e^{-j2\pi\frac{k}{5}(4)} - e^{-j2\pi\frac{k}{5}(2)}\right]$$

$$= \left(\frac{2}{2}\right)\frac{1}{-j\pi k}\left[e^{-j2\pi\frac{k}{5}(\frac{1}{2})}\left(e^{-j2\pi\frac{k}{5}(+\frac{1}{2})} - e^{-j2\pi\frac{k}{5}(-\frac{1}{2})}\right) + e^{-j2\pi\frac{k}{5}(3)}\left(e^{-j2\pi\frac{k}{5}(1)} - e^{-j2\pi\frac{k}{5}(-1)}\right)\right]$$

$$= \frac{2}{\pi k}\left[e^{-j2\pi\frac{k}{5}(\frac{1}{2})}\left(\frac{1}{2j}\left(e^{+j2\pi\frac{k}{5}(\frac{1}{2})} - e^{-j2\pi\frac{k}{5}(\frac{1}{2})}\right)\right) + e^{-j2\pi\frac{k}{5}(3)}\left(\frac{1}{2j}\left(e^{+j2\pi\frac{k}{5}(1)} - e^{-j2\pi\frac{k}{5}(1)}\right)\right)\right]$$

$$= \frac{2}{\pi k}\,\mathrm{Sin}\left(2\pi\frac{k}{5}(\tfrac{1}{2})\right) e^{-j2\pi\frac{k}{5}(\frac{1}{2})} + \frac{2}{\pi k}\,\mathrm{Sin}\left(2\pi\frac{k}{5}(1)\right) e^{-j2\pi\frac{k}{5}(3)}$$

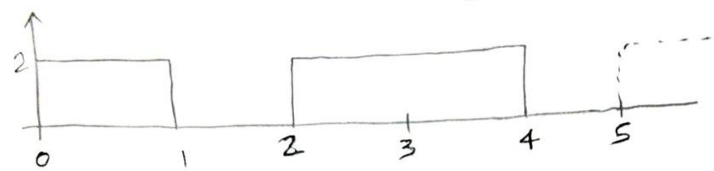
$$= \frac{2}{5}\,\frac{1}{\pi\frac{k}{5}}\,\mathrm{Sin}\left(\pi\frac{k}{5}\right) e^{-j2\pi\frac{k}{5}(\frac{1}{2})} + \frac{2\times 2}{5}\,\frac{1}{2\pi\frac{k}{5}}\,\mathrm{Sin}\left(2\pi\frac{k}{5}\right) e^{-j2\pi\frac{k}{5}(3)}$$

$$\boxed{X_k = \frac{2}{5}\,\mathrm{Sinc}\left(\frac{k}{5}\right) e^{-j\pi\frac{k}{5}} + \frac{4}{5}\,\mathrm{Sinc}\left(\frac{2}{5}k\right) e^{-j\pi\frac{6}{5}k}}$$

for k = integer

Another way

$$\boxed{X_k = \frac{1}{T_s} X(f)\Big|_{f=\frac{k}{T_s}}}$$ where $X(f)$ is the Fourier Transform of 1 period



For 1 period:

$$x(t) = \left[2\,\text{rect}\left(\frac{t}{1}\right) * \delta\left(t-\tfrac{1}{2}\right)\right] + \left[2\,\text{rect}\left(\frac{t}{2}\right) * \delta(t-3)\right]$$

$$\therefore X(f) = \left[2\times1.\,\text{Sinc}(f).\,e^{-j2\pi f(\frac{1}{2})}\right] + \left[2.2.\,\text{Sinc}(2f).\,e^{-j2\pi f(3)}\right]$$

$$\boxed{\begin{aligned}\therefore X_k &= \frac{1}{T_s} X(f)\Big|_{f=\frac{k}{T_s}} \\ &= \left[\frac{2}{5}\,\text{Sinc}\left(\frac{k}{5}\right) e^{-j\pi\frac{k}{5}}\right] + \left[\frac{4}{5}\,\text{Sinc}\left(\frac{2}{5}k\right) e^{-j\pi\frac{6}{5}k}\right]\end{aligned}}$$

$-\infty < k < \infty$
integer

$\rightarrow$ @ $k=-2$: $X_k = \left[\frac{2}{5}\,\frac{\sin\left(\frac{\pi}{5}(-2)\right)}{\frac{\pi}{5}(-2)}\,e^{-j\frac{\pi}{5}(-2)}\right] + \left[\frac{4}{5}\,\frac{\sin\left(\frac{2\pi}{5}(-2)\right)}{\frac{2\pi}{5}(-2)}\,e^{-j\pi\frac{6}{5}(-2)}\right]$

$= 0.3027\;e^{j(1.2566)} + 0.187\;e^{j(7.5398)}$

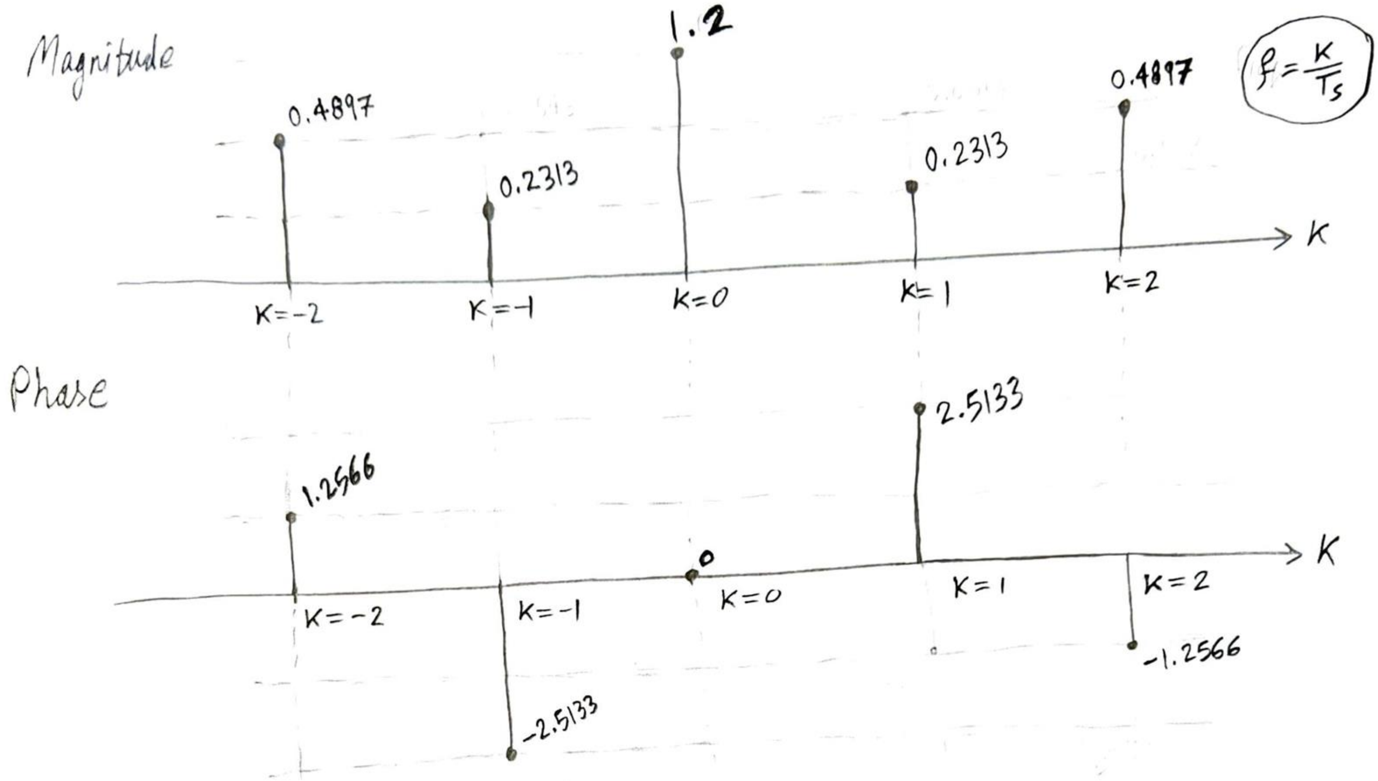
$= \boxed{0.4897\,e^{j(1.2566)} = 0.4897\,e^{j\frac{2\pi}{5}}}$

$\rightarrow$ @ $k=-1$: $X_k = \boxed{0.2313\,e^{j(-2.5133)}}$

$\rightarrow$ @ $k=0$: $X_k = \boxed{1.2\;e^{j\emptyset}}$

$\rightarrow$ @ $k=1$: $X_k = \boxed{0.2313\,e^{j(2.5133)}}$

$\rightarrow$ @ $k=2$: $X_k = \boxed{0.4897\,e^{j(-1.2566)}}$


Magnitude
1.2
0.4897
0.2313
0.2313
0.4897
$f = \frac{K}{T_s}$
K
K=-2
K=-1
K=0
K=1
K=2
Phase
2.5133
1.2566
0
K
K=-2
K=-1
K=0
K=1
K=2
-1.2566
-2.5133

Q2: Assuming that x(t) is periodic & continuous in time

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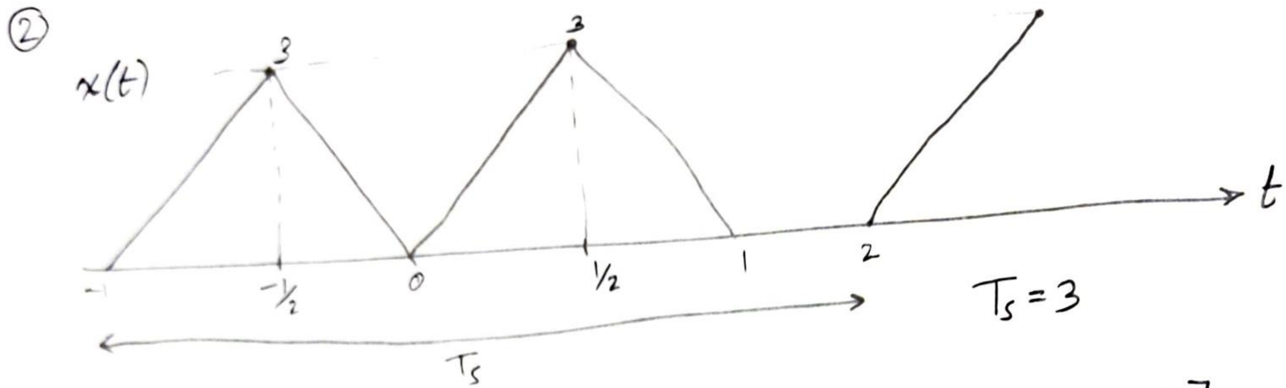
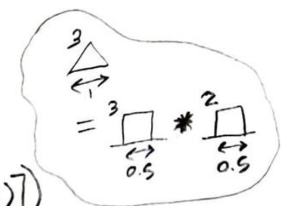


* Taking 1 period from -1 to 2:

$$x(t) = \left[3\Delta\left(\tfrac{t}{1}\right) * \delta(t+0.5)\right] + \left[3\Delta\left(\tfrac{t}{1}\right) * \delta(t-0.5)\right]$$

$$= \left(3\,\text{rect}\left(\tfrac{t}{0.5}\right) * 2\,\text{rect}\left(\tfrac{t}{0.5}\right) * \delta(t+0.5)\right) + \left(3\,\text{rect}\left(\tfrac{t}{0.5}\right) * 2\,\text{rect}\left(\tfrac{t}{0.5}\right) * \delta(t-0.5)\right)$$



$$= 6\left(\text{rect}\left(\tfrac{t}{0.5}\right) * \text{rect}\left(\tfrac{t}{0.5}\right) * \left[\delta(t+0.5) + \delta(t-0.5)\right]\right)$$

$$\therefore X(f) = 6\left(0.5\,\text{Sinc}\left(\tfrac{f}{2}\right) . \,0.5\,\text{Sinc}\left(\tfrac{f}{2}\right) . \left[e^{j2\pi f(0.5)} + e^{-j2\pi f(0.5)}\right]\right)$$

$$= 3\,\text{Sinc}^2\left(\tfrac{f}{2}\right) . \,\text{Cos}(\pi f)$$

$$\therefore \boxed{X_K = \frac{1}{T_s} X(f)\Big|_{f=\frac{K}{T_s}} = \text{Sinc}^2\left(\tfrac{K}{6}\right)\,\text{Cos}\left(\tfrac{\pi}{3}K\right)}$$

$-\infty \leq K \leq \infty$ integer

$f = \frac{K}{T_s} = \frac{K}{3}$

@ $K = -2 \rightarrow X_K = -1.0259$

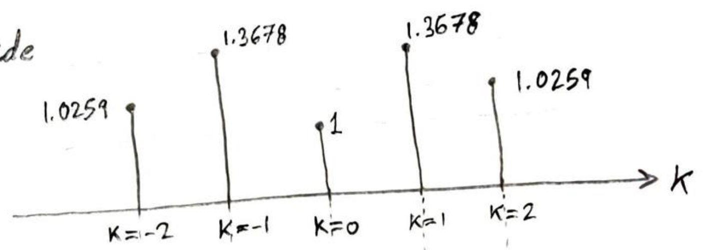
@ $K = -1 \rightarrow X_K = 1.3678$

@ $K = 0 \rightarrow X_K = 1$

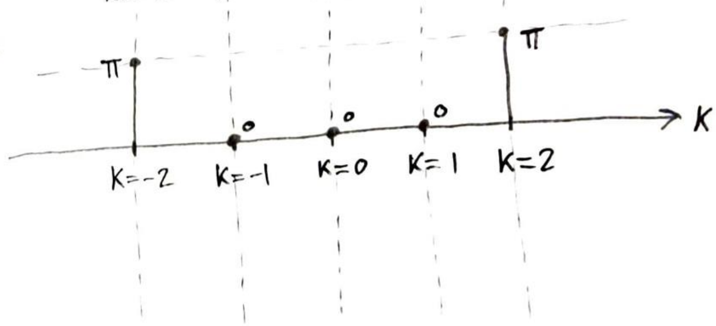
@ $K = 1 \rightarrow X_K = 1.3678$

@ $K = 2 \rightarrow X_K = -1.0259$

Magnitude

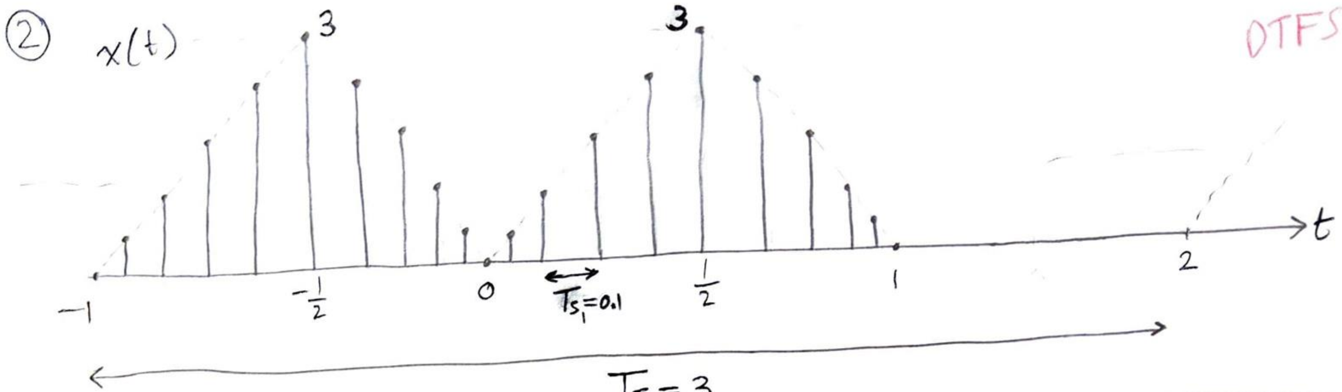


Phase



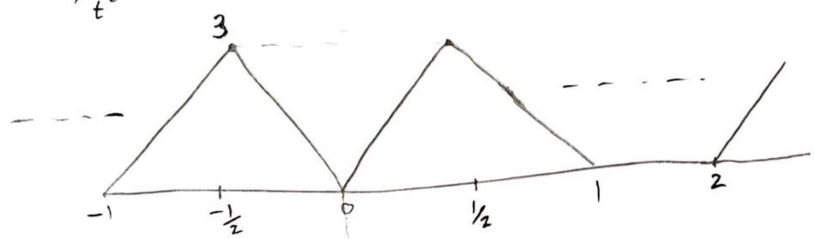
Fall 2020

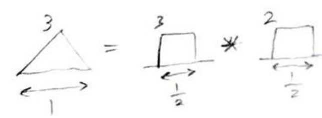
Q2: Assuming that x(t) is periodic & discrete in time



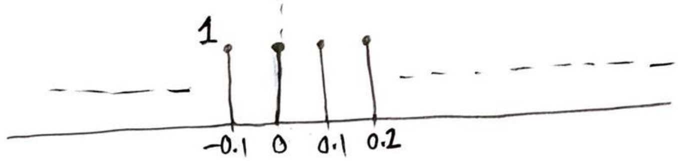
$$x(t) = x_{tri}(t) \cdot x_{pulses}(t)$$

$=$ $x_t(t)$





• $x_p(t)$



* from $-1 \le t \le 2$:

For the triangular signal

$$x_t(t) = \left[3\,tri\left(\tfrac{t}{1}\right) * \delta(t+0.5)\right] + \left[3\,tri\left(\tfrac{t}{1}\right) * \delta(t-0.5)\right]$$

$$= \left[3\,rect\left(\tfrac{t}{0.5}\right) * 2\,rect\left(\tfrac{t}{0.5}\right) * \delta(t+0.5)\right] + \left[3\,rect\left(\tfrac{t}{0.5}\right) * 2\,rect\left(\tfrac{t}{0.5}\right) * \delta(t-0.5)\right]$$

$$\therefore \boxed{x_t(t) = 6\left(rect\left(\tfrac{t}{0.5}\right) * rect\left(\tfrac{t}{0.5}\right) * [\delta(t+0.5) + \delta(t-0.5)]\right)}$$

$$\therefore X_t(f) = 6\left(0.5\,Sinc\left(\tfrac{1}{2}f\right) . 0.5\,Sinc\left(\tfrac{1}{2}f\right) . \left[e^{j2\pi f(0.5)} + e^{-j2\pi f(0.5)}\right]\right)$$

$$= 3\,Sinc^2(f/2) . Cos(\pi f)$$

$$\therefore \boxed{X_{k_{(tri)}} = \frac{1}{T_{s_2}} X_t(f)\Big|_{f=\frac{k}{T_{s_2}}} = Sinc^2\left(\frac{k}{6}\right) . Cos\left(\frac{\pi}{3}k\right)}$$

$-\infty \le k \le \infty$ integer

$$f = \frac{k}{T_{s_2}} = \frac{k}{3}$$

* from $-0.05 \leq t \leq 0.05$: For the pulse train

$$X_{\underset{(pulses)}{k}} = \frac{1}{T_{s_1}} \int_{-\frac{T_{s_1}}{2}}^{\frac{T_{s_1}}{2}} \delta(t)\, e^{-j2\pi \frac{k}{T_{s_1}} t}\, dt \qquad \underline{\underline{T_{s_1} = 0.1}}$$

$$= 10 \int_{-0.05}^{0.05} e^{-j2\pi \frac{k}{0.1}(0)}\, dt$$

$$\therefore \boxed{X_{\underset{(pulses)}{k}} = \frac{1}{T_{s_1}} = 10} \text{ at every } k \quad \text{where } f = \frac{k}{T_{s_1}} = 10k$$

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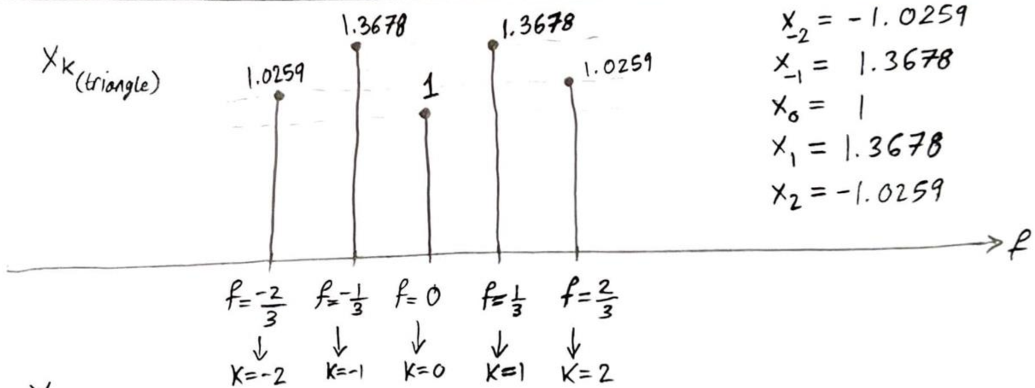
* In freq domain:

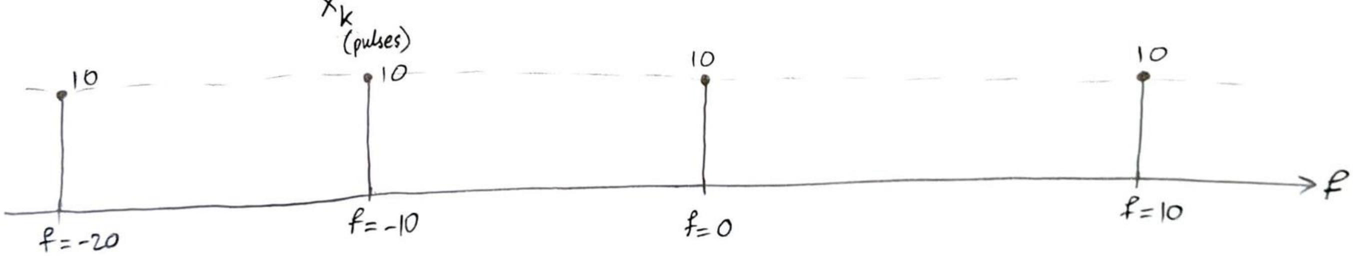
$$\boxed{X_k = X_{\underset{(tri)}{k}} * X_{\underset{(pulses)}{k}}}$$



$X_{-2} = -1.0259$
$X_{-1} = 1.3678$
$X_0 = 1$
$X_1 = 1.3678$
$X_2 = -1.0259$

Convolution

$*$



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