EE223 Analog Integrated Circuits
Fall 2018

Lecture 17: Frequency Response

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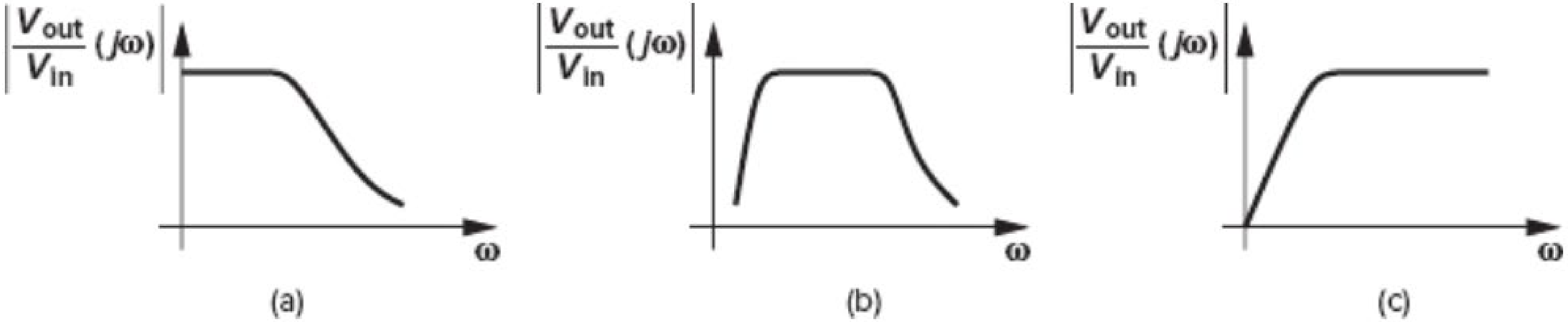
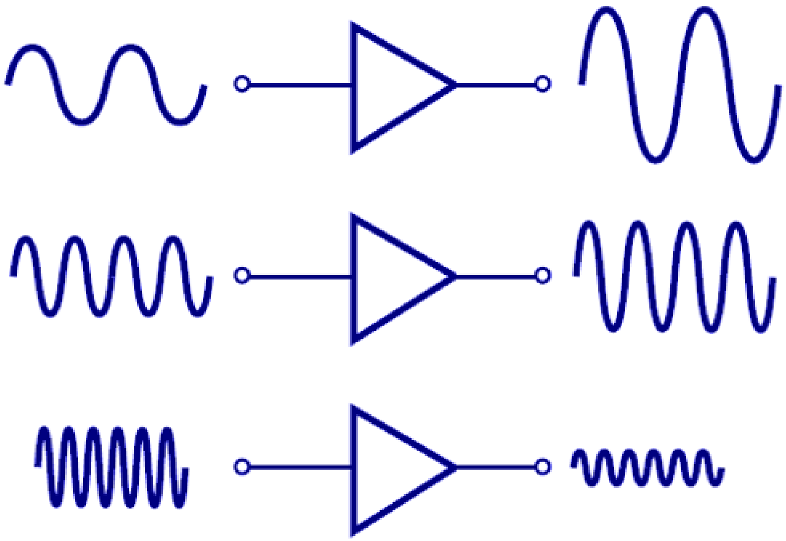
ENG-259

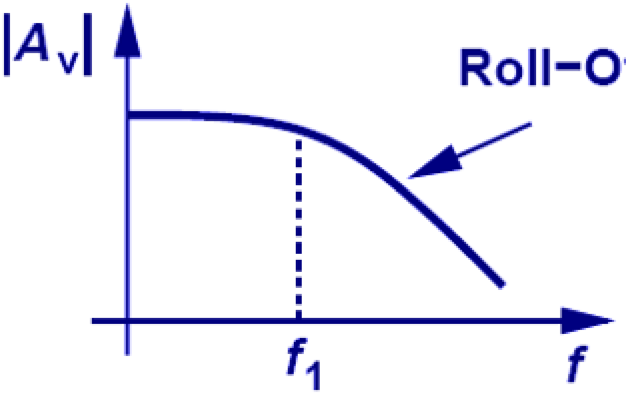
Frequency Response



- For the time being, we are primarily interested in the magnitude of the transfer function.
- The magnitude of a complex number a + jb is given by $\sqrt{a^2 + b^2}$.
- Zeros and poles are respectively defined as the roots of the numerator and denominator of the transfer function.

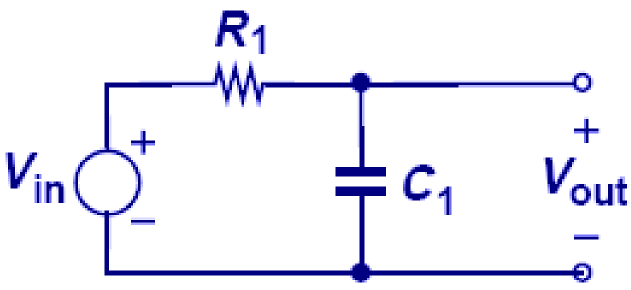
High Frequency Roll-Off of Amplifier Gain

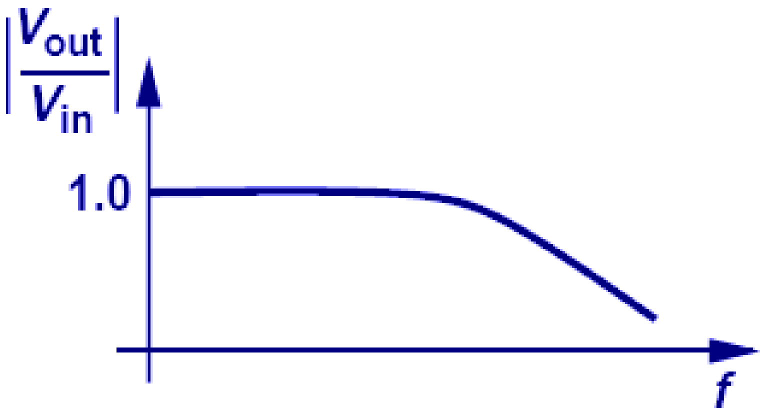




- As frequency increases, the gain of amplifier decreases.
- Gain drop in high frequency is caused by capacitive effect.

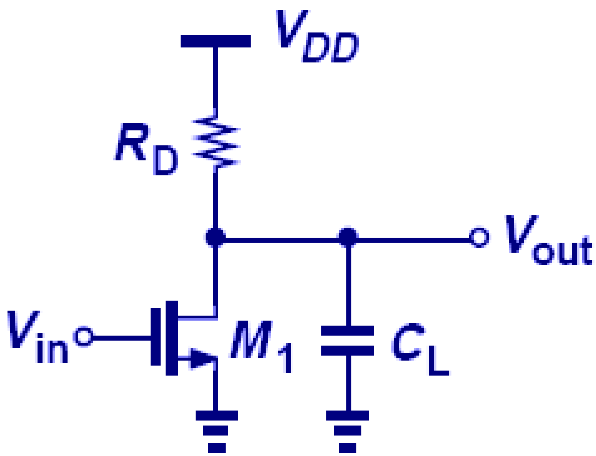
Simple Low Pass Filter Example



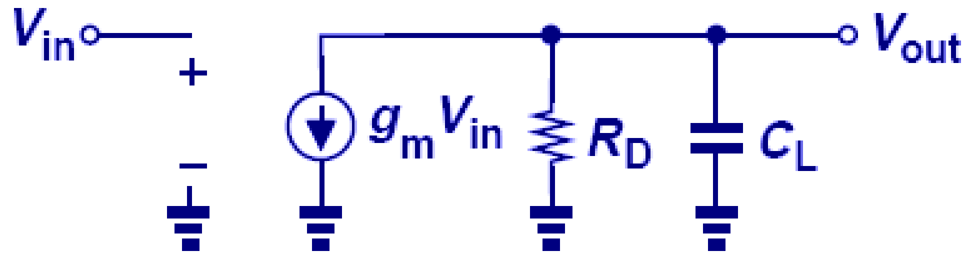


- In this simple example, as frequency increases the impedance of C_1 decreases and the voltage divider consists of C_1 and R_1 attenuates V_{in} to a greater extent at the output.

Simple Common Source Amplifier



(a)

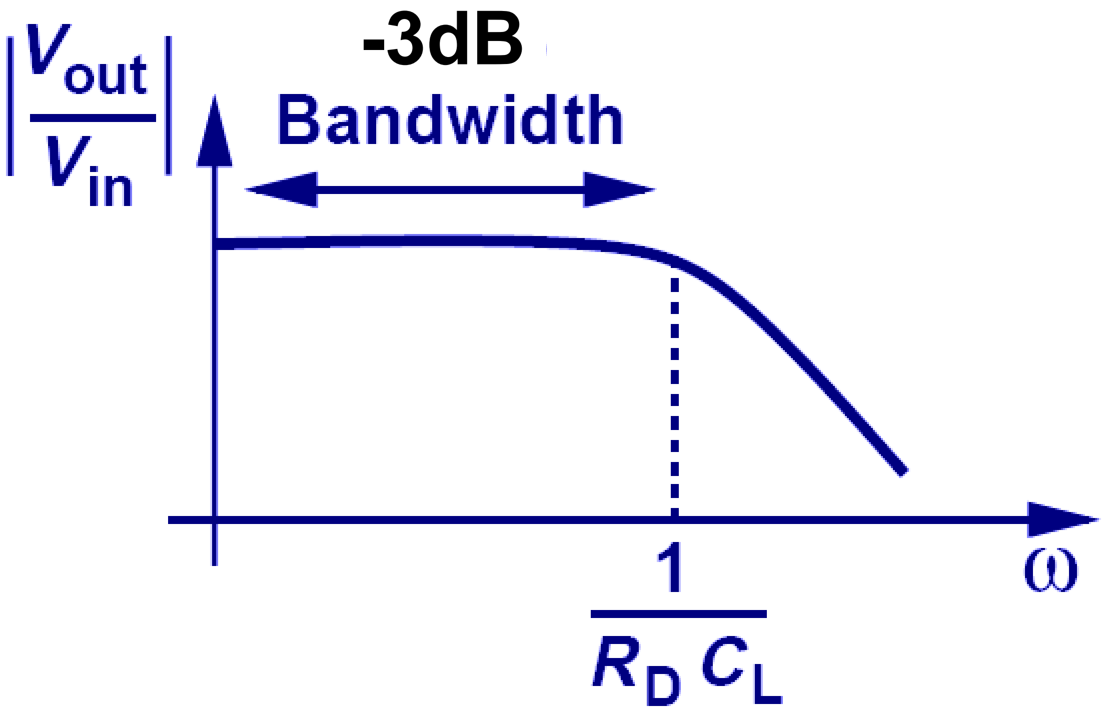


(b)

$$V_{out} = -g_m V_{in}\left(R_D \parallel \frac{1}{C_L s}\right)$$

- The capacitive load, C_L, is the culprit for gain roll-off since at high frequency, it will “steal” away some signal current and shunt it to ground.

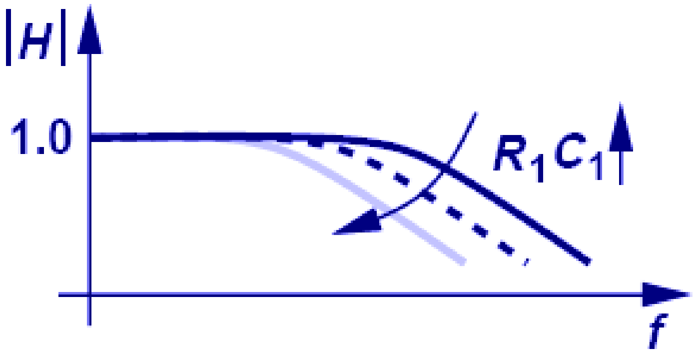
Frequency Response of the CS Amplifier



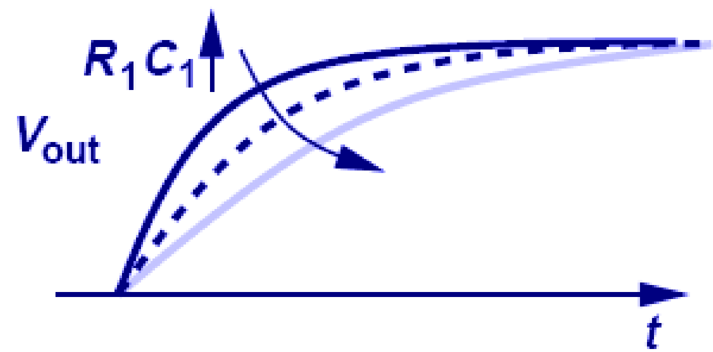
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}$$

- At low frequency, the capacitor is effectively open and the gain is flat. As frequency increases, the capacitor tends to a short and the gain starts to decrease. A special frequency is $\omega=1/(R_DC_L)$, where the gain drops by 3dB.

Relationship Between Frequency Response and Step Response



(a)



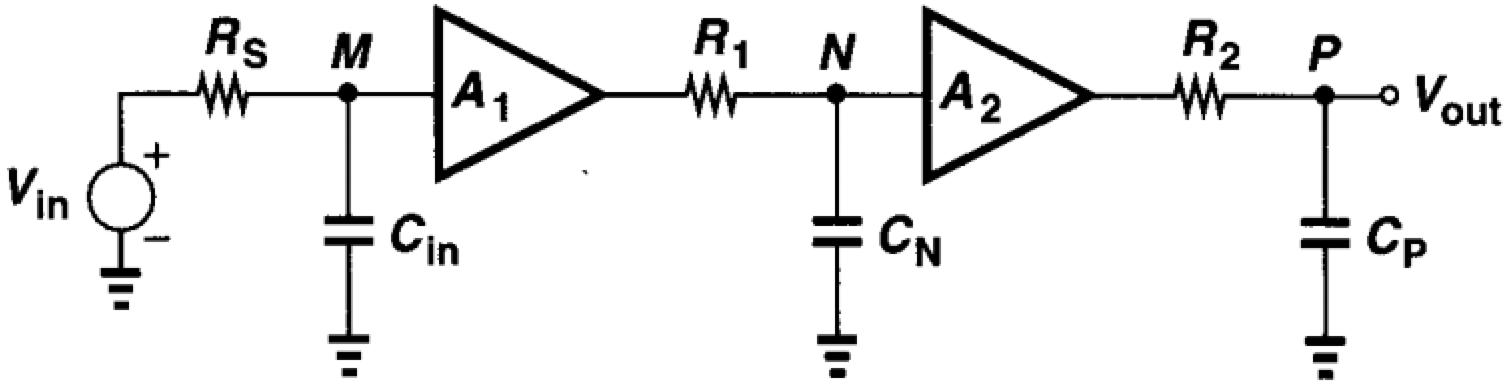
(b)

$$\left|H(s=j\omega)\right| = \frac{1}{\sqrt{R_1^2 C_1^2 \omega^2 + 1}}$$

$$V_{out}(t) = V_0\left(1-\exp\frac{-t}{R_1C_1}\right)u(t)$$

- **The relationship is such that as R_1C_1 increases, the bandwidth *drops* and the step response becomes *slower.***

Association of Poles with Nodes



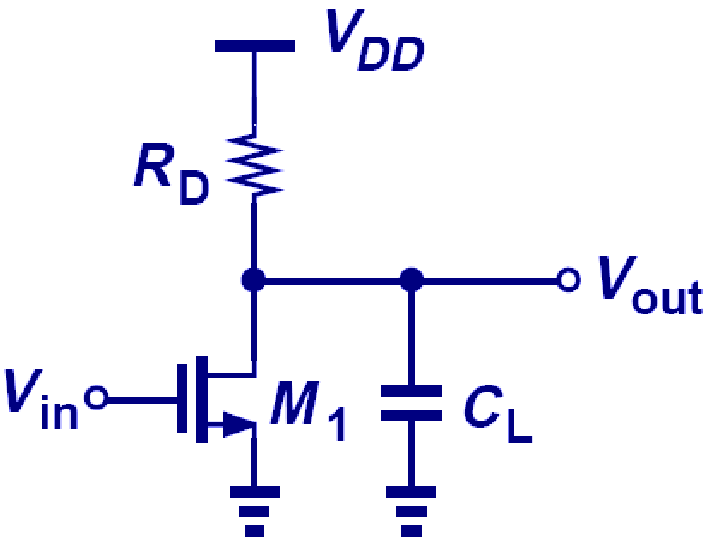
$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + R_S C_{in} s} \cdot \frac{A_2}{1 + R_1 C_N s} \cdot \frac{1}{1 + R_2 C_P s}$$

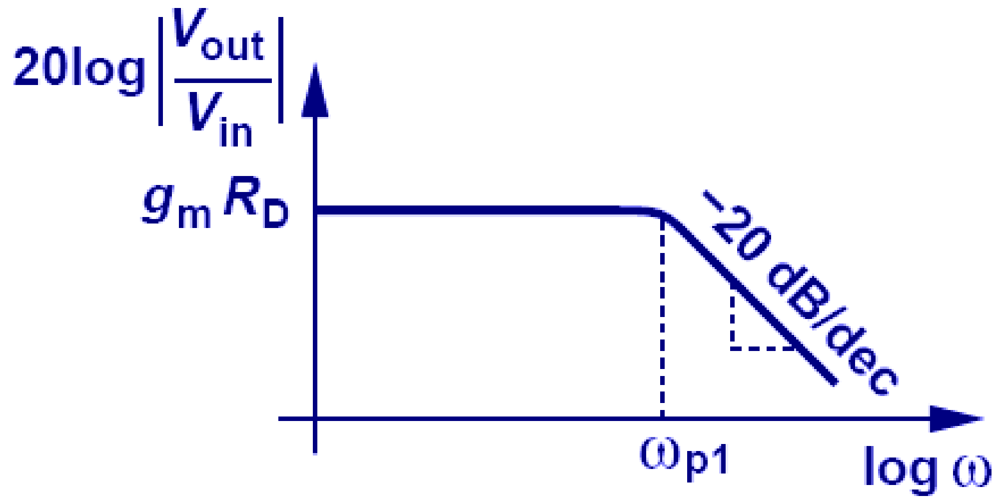
Bode Plot

$$H(s) = A_0 \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)\cdots}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)\cdots}$$

- **When we hit a zero, ω_{zj}, the Bode magnitude rises with a slope of +20dB/dec.**
- **When we hit a pole, ω_{pj}, the Bode magnitude falls with a slope of -20dB/dec**

Example Bode Plot

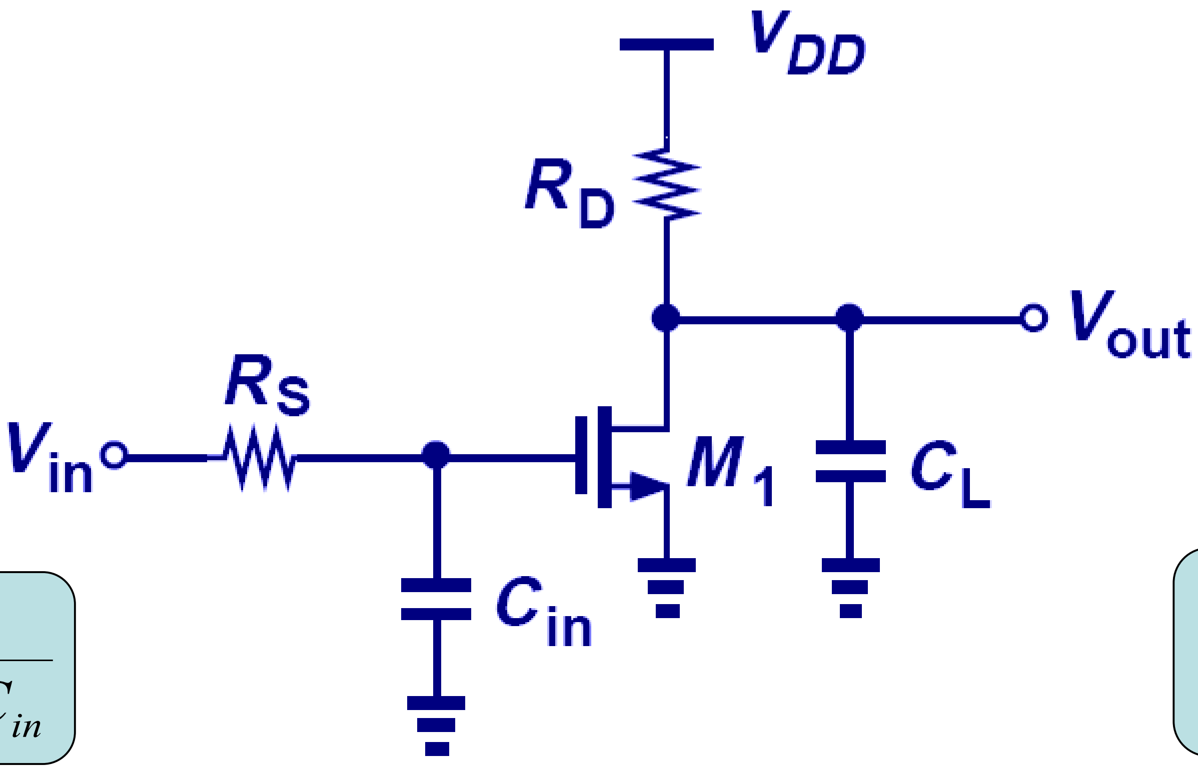




$$\left|\omega_{p1}\right| = \frac{1}{R_D C_L}$$

- The circuit has one pole (no zero) at $1/(R_D C_L)$, so the slope drops from 0 to -20dB/dec as we pass ω_{p1}.

Pole Identification Example

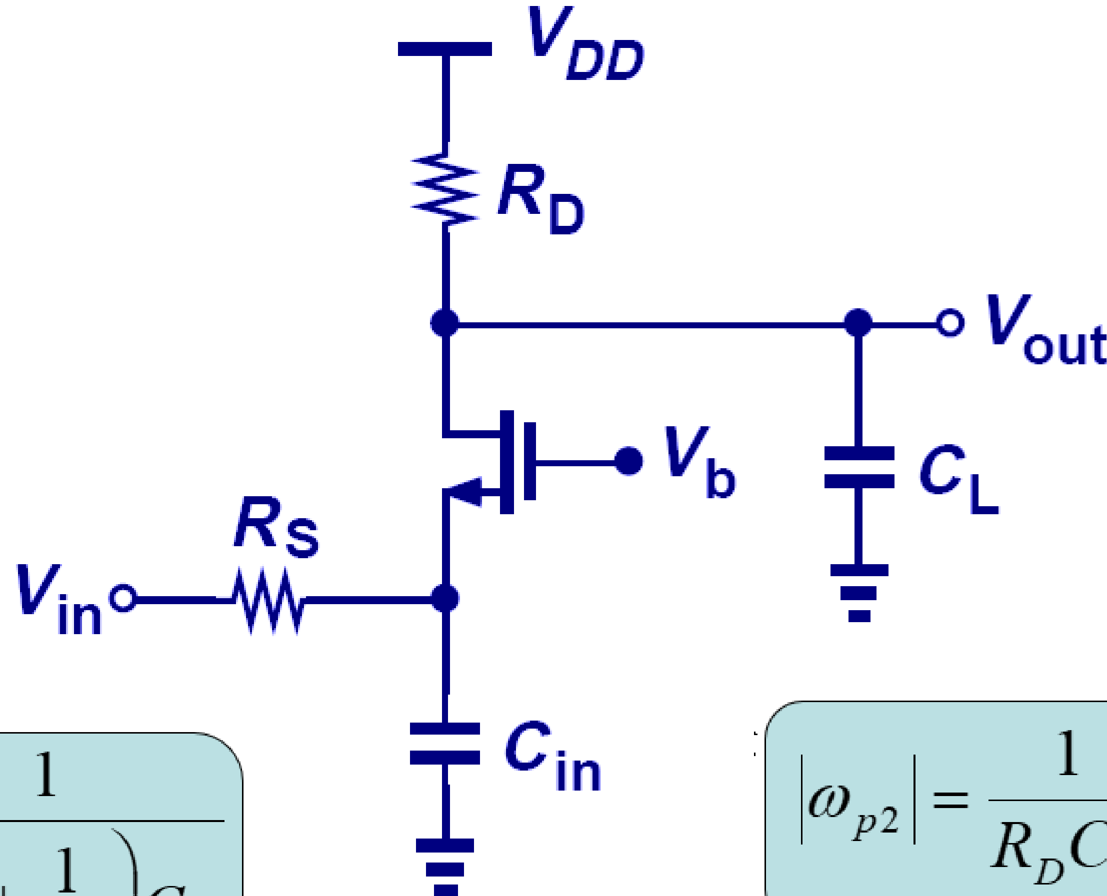


$$|\omega_{p1}| = \frac{1}{R_S C_{in}}$$

$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{\left(1+\omega^2/\omega_{p1}^2\right)\left(1+\omega^2/\omega_{p2}^2\right)}}$$

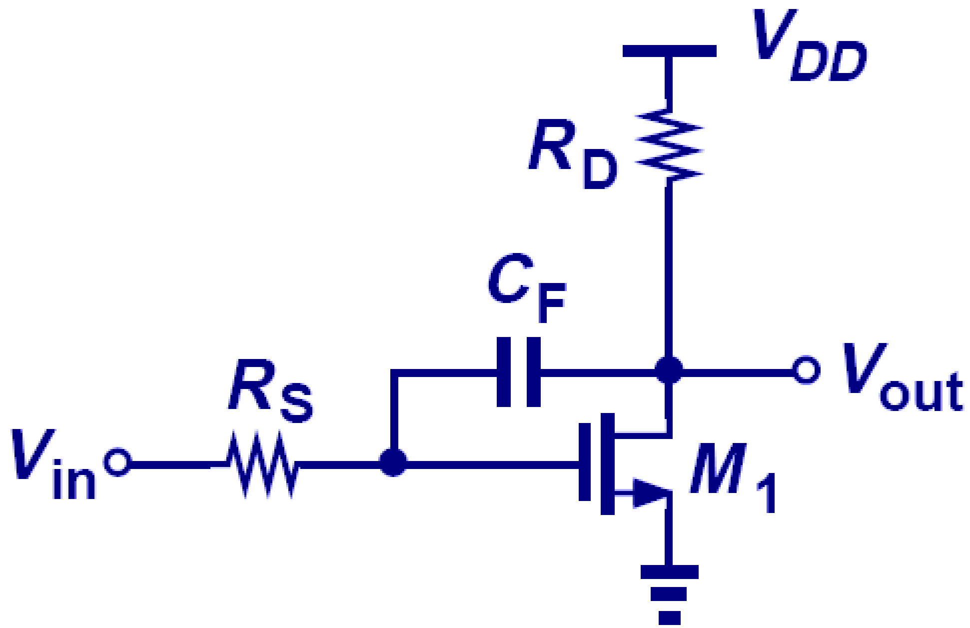
Pole Identification Example



$$|\omega_{p1}| = \frac{1}{\left(R_S \parallel \frac{1}{g_m}\right)C_{in}}$$

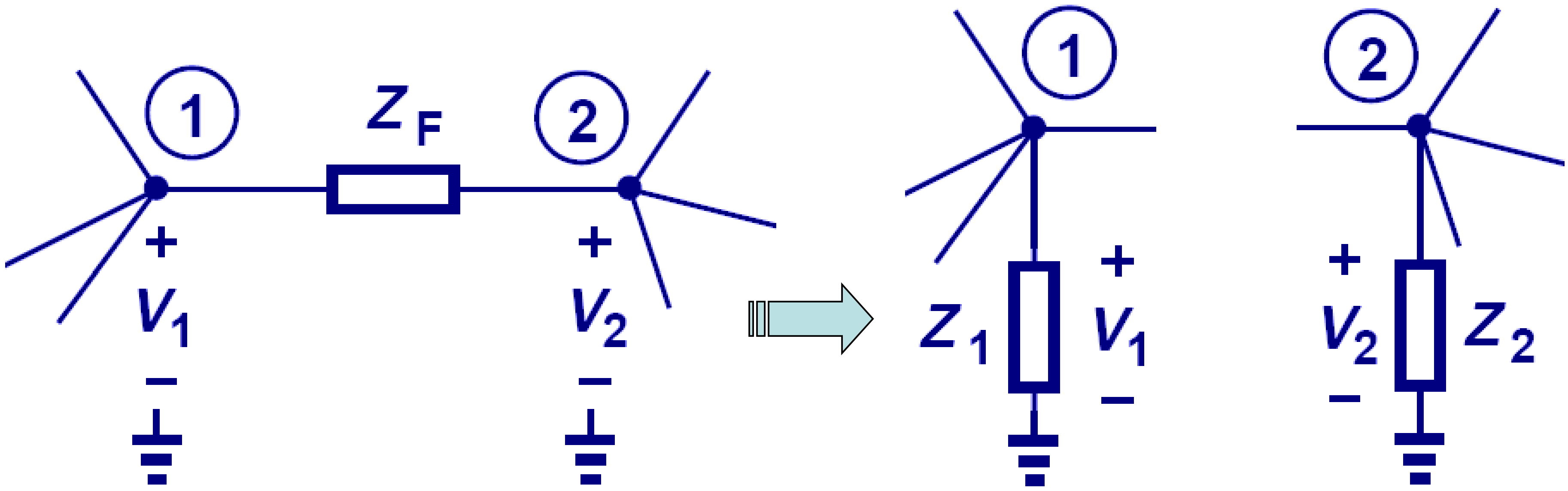
$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

Circuit with Floating Capacitor



- The pole of a circuit is computed by finding the effective resistance and capacitance from a node to GROUND.
- The circuit above creates a problem since neither terminal of C_F is grounded.

Miller's Theorem

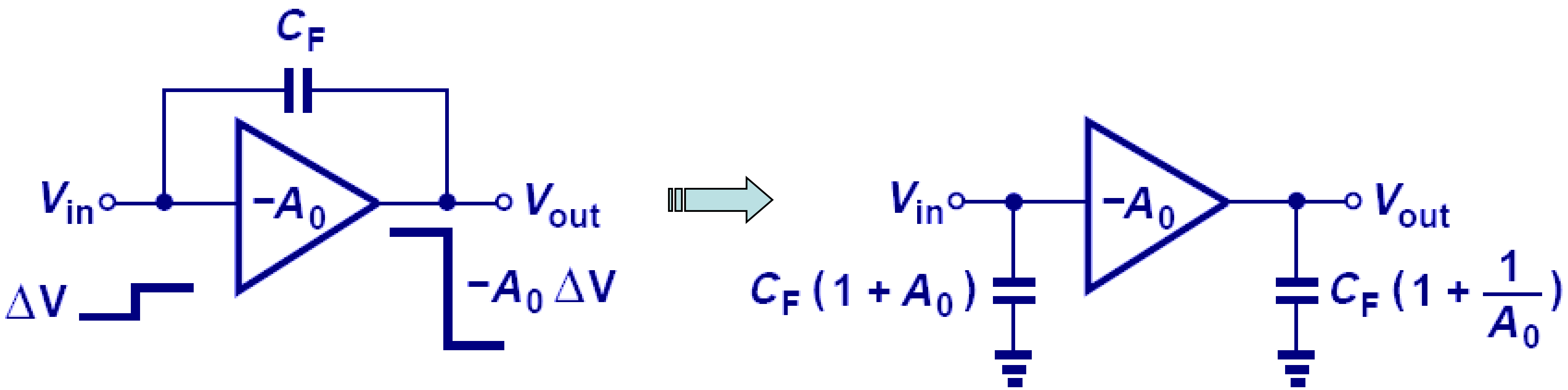


$$Z_1 = \frac{Z_F}{1 - A_v}$$

$$Z_2 = \frac{Z_F}{1 - 1/A_v}$$

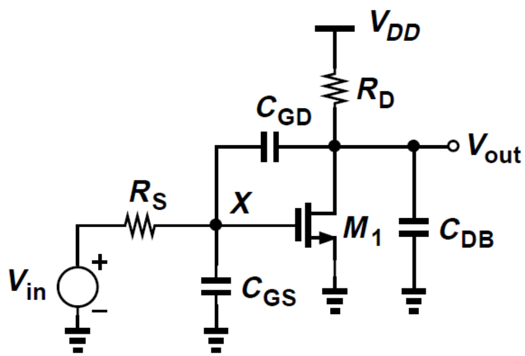
- If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2.

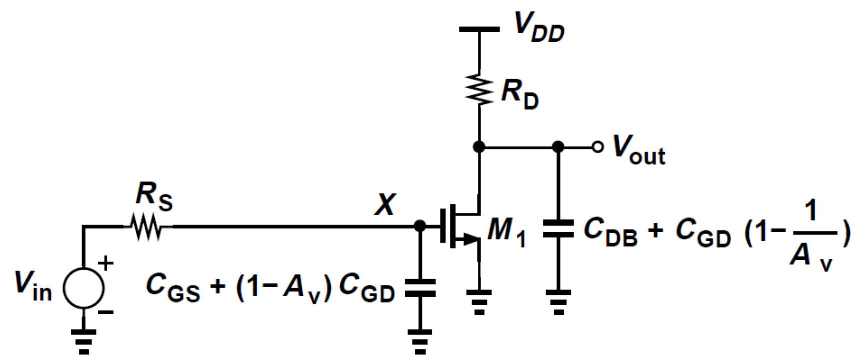
Miller Multiplication



- **With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.**

CS Frequency Response using Miller's Theorem





- **The magnitude of the "input" pole**

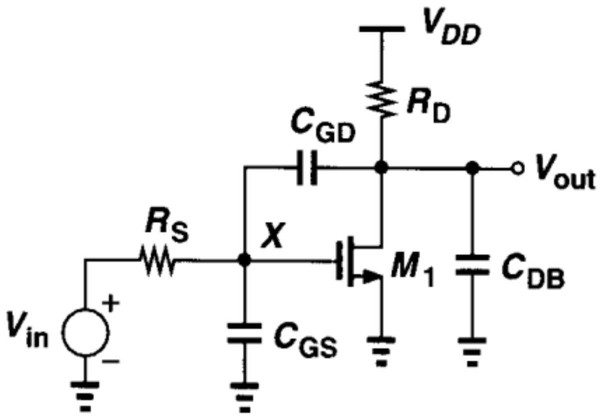
$$\omega_{in} = \frac{1}{R_S[C_{GS} + (1 + g_m R_D)C_{GD}]}$$

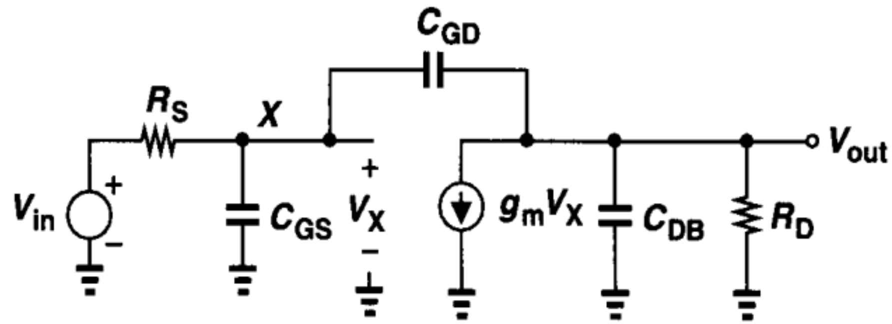
- **At the output node**

$$\omega_{out} = \frac{1}{R_D(C_{DB} + C_{GD})}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

Exact Analysis of Common Source Frequency Response





$$\frac{V_X - V_{in}}{R_S} + V_X C_{GS} s + (V_X - V_{out}) C_{GD} s = 0$$

$$(V_{out} - V_X) C_{GD} s + g_m V_X + V_{out} \left(\frac{1}{R_D} + C_{DB} s \right) = 0.$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD} s - g_m) R_D}{R_S R_D \xi s^2 + [R_S (1 + g_m R_D) C_{GD} + R_S C_{GS} + R_D (C_{GD} + C_{DB})] s + 1}$$

$$\xi = C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB}$$

Dominant Pole Approximation

$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})]s + 1}$$

$$D = \left(\frac{s}{\omega_{p1}} + 1\right)\left(\frac{s}{\omega_{p2}} + 1\right)$$
$$= \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1 \quad = as^2 + bs + 1$$

If we assume $|\omega_{p1}| \ll |\omega_{p2}|$ $\rightarrow$ $b = \frac{1}{\omega_{p1}}$

$$\omega_{p1} = \frac{1}{R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})} \approx \omega_{in} = \frac{1}{R_S[C_{GS} + (1+g_m R_D)C_{GD}]}$$

Estimation of Second Pole

$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{GD}s - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}$$

$$D \approx \frac{s^2}{\omega_{p1}\omega_{p2}} + \frac{1}{\omega_{p1}} s + 1 = as^2 + bs + 1$$

$$\omega_{p2} = \frac{b}{a} = \frac{R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})}{R_S R_D(C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})}$$

If $C_{GS} \gg (1 + g_m R_D)C_{GD} + R_D(C_{GD} + C_{DB})/R_S$,

$$\omega_{p2} \approx \frac{R_S C_{GS}}{R_S R_D(C_{GS}C_{GD} + C_{GS}C_{DB})}$$

$$= \frac{1}{R_D(C_{GD} + C_{DB})},$$