# EE210

# Midterm I

Name and ID:
- Last:
- First:
- Student ID #:
- Email:

4/14/2020

- Write your signature on the bottom right corner of front page
- Draw a box around your final answers otherwise you will NOT get any credits and move your solutions to the given boxes.
- Your phone must be turned off and kept in your bag.
- One (8.5x11) cheat sheet is allowed.
- Calculator is not ok.
- No cell phone for calculator
- Only pencil and eraser
- If you use pen, 20 pts will be deducted.

1. [7 7 6 20 pts]

Two signals are defined as

$$x_1(t) = \cos(20 \cdot \pi \cdot t) \qquad x_2(t) = \sin(30 \cdot \pi \cdot t)$$
$$y(t) = -x_1(t) \cdot x_2(t)$$

a) Plot frequency response of $y(t)$ in magnitude and phase in the frequency domain

b) If $y(t)$ is sampled with $fs = 12$, plot magnitude and phase in $[-fs \le f \le fs]$

c) If $y(t)$ is sampled with $fs = 10$, plot magnitude and phase in $[-fs \le f \le fs]$

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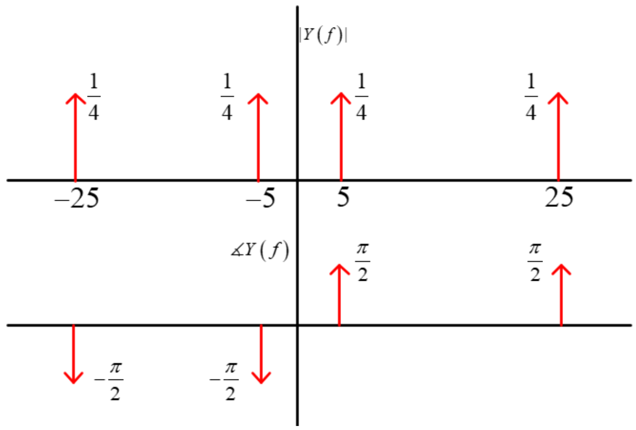
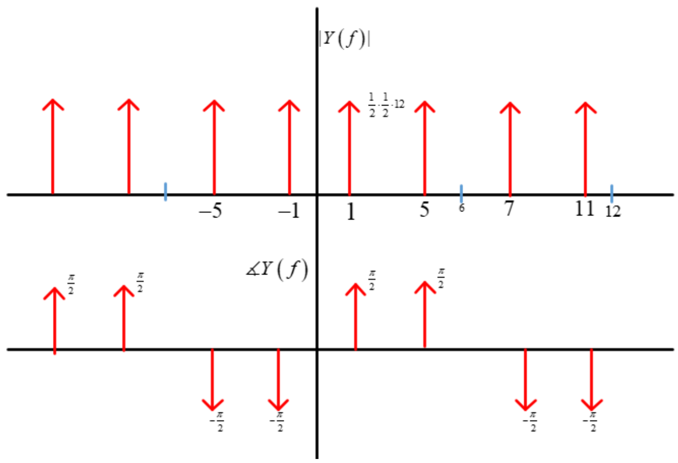
| | |
|---|---|
| a) | Approach from the frequency domain<br>$X_1(f) = F\{\cos(20\pi t)\} = \frac{1}{2}[\delta(f-10) + \delta(f+10)]$<br>$X_2(f) = F\{\sin(30\pi t)\} = \frac{1}{2j}[\delta(f-15) - \delta(f+15)]$<br>$F\{y(t) = -x_1(t) \cdot x_2(t)\}$<br>$= -\left(\frac{1}{2}[\delta(f-10) + \delta(f+10)]\right) \cdot \left(\frac{1}{2j}[\delta(f-15) - \delta(f+15)]\right)$<br>$= -\frac{1}{2} \cdot \frac{1}{2j}([\delta(f-25) - \delta(f+25) + \delta(f-5) - \delta(f+5)])$<br>$= \frac{j}{4} \cdot ([\delta(f-25) - \delta(f+25) + \delta(f-5) - \delta(f+5)])$<br>$\Rightarrow -\frac{1}{2} \cdot [\sin(2\pi 25t) + \sin(2\pi 5t)]$ |

<table>
<tr><td></td><td>


</td></tr>
<tr><td>b)</td><td>
Two new frequencies are $f_1 = 5$ & $f_2 = 25$ before sampling process<br>
If $fs = 12$, then $f_1 = 5$ is not aliased but $f_2 = 25$ is aliased with new frequency of $f_2 = 1$



</td></tr>
<tr><td></td><td>
Two new frequencies are $f_1 = 5$ & $f_2 = 25$ before sampling process<br>
If $fs = 10$, then $f_1 = 5$ becomes 5Hz but $f_2 = 25$ also becomes 5Hz<br>
But because of phases, it cancels each other and there is nothing so all are zeros.
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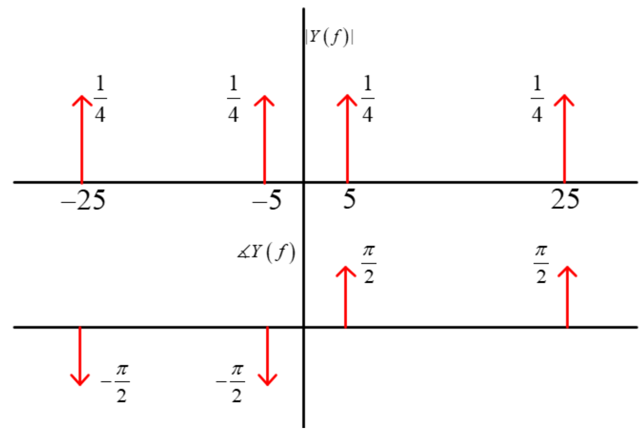
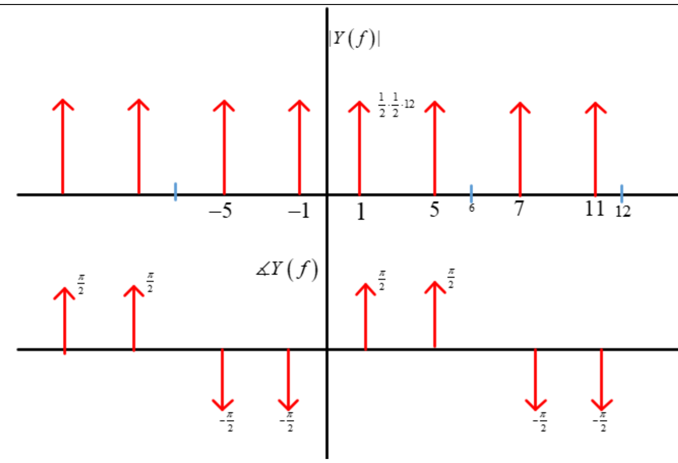
<table>
<tr><td>a)</td><td><br></td></tr>
<tr><td>b)</td><td><br></td></tr>
<tr><td>c)</td><td>all are zeros.</td></tr>
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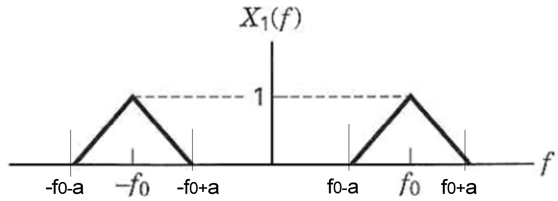
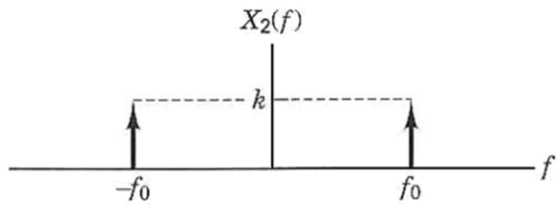
2. [5 5 10 20 pts]

Two signals are shown in the frequency domain.



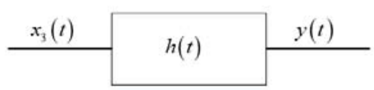




Let's define $\left[X_3(f) = X_1(f) * X_2(f)\right]$ (Convolutions of two functions)

a) Write equations for $X_1(f)$ & $X_2(f)$ Note: Use $\Delta$ symbol for $X_1(f)$

b) Plot the convolution of two signals, $\left[X_3(f) = X_1(f) * X_2(f)\right]$ assume $f_0 >> a$

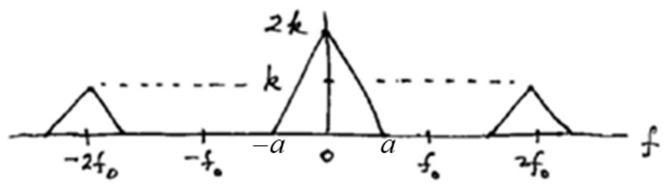
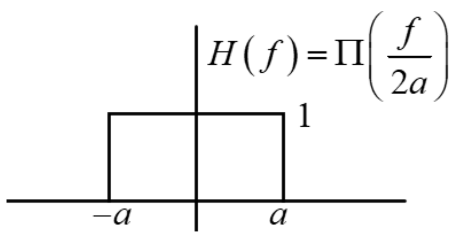
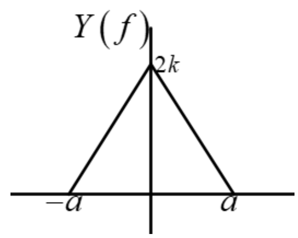
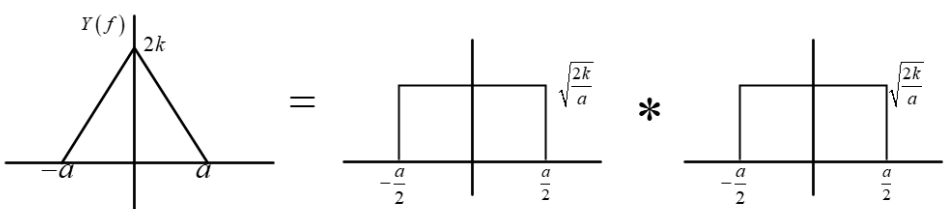
c) Now $X_3(f)$ is passed the following filter and the filter, $h(t)$, in the time domain is defined as $\left[h(t) = 2 \cdot a \cdot \text{sinc}(2 \cdot a \cdot t)\right]$




i. Write $Y(f)$ (Note: This is part of (c))

ii. Write $y(t)$ (Note: This is part of (c))

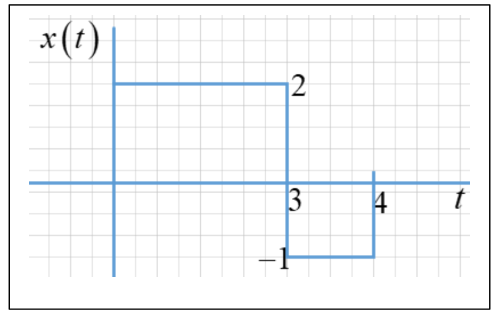
| | | |
|---|---|---|
| a) | | |
| b) | | |
| c-i) | | |
| c-ii) | | |

| | |
|---|---|
| a) | $X_1(f) = \Delta\left(\frac{f-f_0}{2a}\right) + \Delta\left(\frac{f+f_0}{2a}\right)$<br>$X_2(f) = k\cdot\left[\delta(f-f_0)+\delta(f+f_0)\right]$ |
| b) | $X_2(f) = k\left[\delta(f-f_0)+\delta(f+f_0)\right]$<br>$X_1(f) * X_2(f) = X_1(f) * k\left[\delta(f-f_0)+\delta(f+f_0)\right]$<br> |
| c) | <br>$Y(f) = 2k\cdot\Delta\left(\frac{f}{2a}\right)$ |
| | Convolution of two rec function is triangle function<br><br>$y(t) = \left[2\cdot\sqrt{\frac{2k}{a}}\cdot\frac{a}{2}\cdot\mathrm{sinc}\left(2\cdot\frac{a}{2}\cdot t\right)\right]^2$<br>$= \left[a\cdot\sqrt{\frac{2k}{a}}\cdot\mathrm{sinc}(a\cdot t)\right]^2$ |

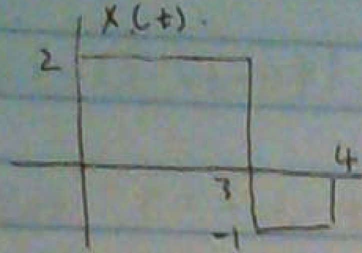
3. [10 pts]

a) Plot $-x(-2t-4)$

b) Write $F\{-x(-2t-4)\}$




| | |
|---|---|
| a) | |
| b) | |



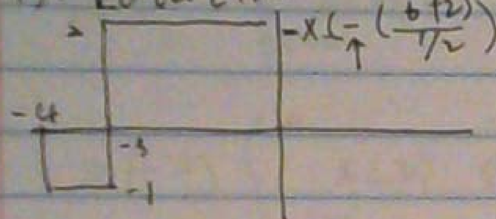

(5.5 pts).

a). $-x(-2t-4)$.

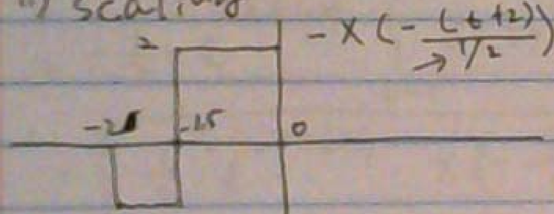
b). $\mathcal{F}\{-x(-2t-4)\}$.

a). $-x(-2t-4) = -x(-2(t+2)) = -x\left(\frac{t+2}{-\frac{1}{2}}\right)$

i). Rotation

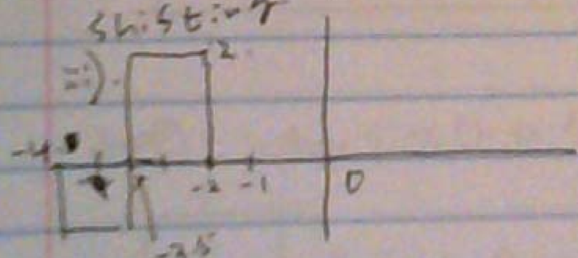



ii) scaling

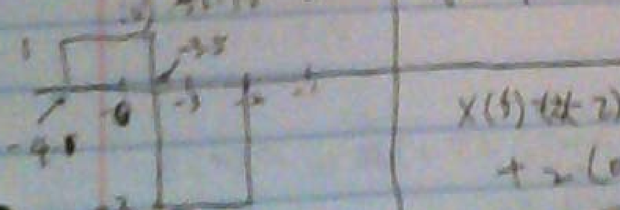



shifting

iii).




iv) flipping ⇐ final Answer.




$X(f) = (2)\left(\frac{1.5}{2}\right)\text{sinc}(2(0.75)f)\,e^{-j2\pi f(-2.75)} + 2(0.25)(1)\,\text{sinc}(2(0.25)f)\,e^{-j2\pi f(-3.25)}$

$$\boxed{X(f) = \frac{3}{2}\,\text{sinc}(1.5f)\cdot e^{-j2\pi f(-2.75)} + 0.5\,\text{sinc}(0.5f)\cdot e^{-j2\pi f(-3.25)}}$$

$F\{-x\{-2t-4\}\} = F\{Y(t)\}$

Let $\tau = -2t-4 \rightarrow \tau+4 = -2\cdot t \rightarrow t = \frac{\tau+4}{-2}$

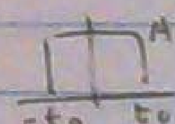
$d\tau = -2\,dt \qquad dt = -\frac{1}{2}\cdot d\tau$

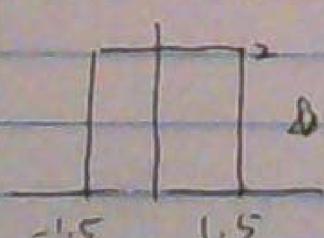
$$Y(f) = -\int_{t=-\infty}^{\infty} x(t)\cdot e^{-j2\pi f t}\,dt = -\int_{\tau=-2\cdot(-\infty)=+\infty}^{-\infty} x(\tau)\cdot e^{-j2\pi f(-\frac{1}{2}\cdot(\tau+4))}\cdot\left(-\frac{1}{2}\right)d\tau$$

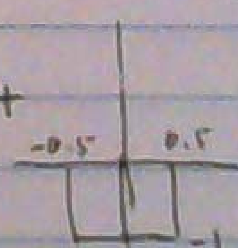
$$= \left(-\frac{1}{2}\right)\int_{t=-\infty}^{\infty} x(\tau)\cdot e^{-j2\pi f(-\frac{1}{2})\tau}\cdot \underbrace{e^{-j2\pi f(-2)}}_{\text{constant}}\,d\tau$$

$$= -\frac{1}{2}\cdot e^{-j2\pi f(-2)}\cdot\int_{\tau=\infty}^{\infty} x(\tau)\cdot e^{-j2\pi(-\frac{f}{2})\cdot\tau}\,d\tau$$

$$= \boxed{-\frac{1}{2}\cdot e^{-j2\pi f(-2)}\cdot X\left(-\frac{f}{2}\right)}$$

$\cancel{x(t)} = 2\cdot A t_0\, \mathrm{sinc}(2 t_0 f)$ for 

$\circledast\ \delta(t-5)$ + 

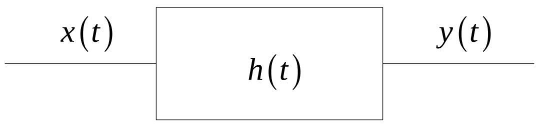
$\circledast\ \delta(t-3.5)$

$$X(f) = 2\cdot\cancel{A}^{2}(1.5)\cdot \mathrm{sinc}(2\cdot(1.5)\cdot f)\cdot e^{-j2\pi f\cdot 5}$$
$$+\ 2\cdot(-1)\cdot(0.5)\cdot \mathrm{sinc}(2\cdot(0.5)\cdot f)\cdot e^{-j2\pi f(3.5)}$$

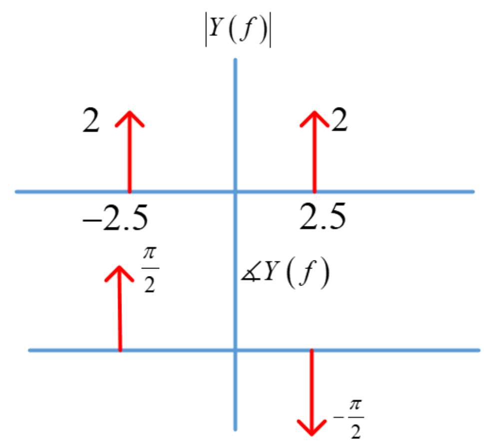
4. [5 10(5 5) 15 pts]

It is given that

$$x(t) = 4\sin\left(\frac{10}{2}\pi t\right)$$
$$h(t) = \delta(t-2)$$




a) Find $F\{y(t)\}$
b) Write the magnitude response of the $Y(f)$ and plot it.
c) Find the phase response of $Y(f)$ and plot it.

| | |
|---|---|
| a) | $-2j\cdot\left[\delta(f-2.5)-\delta(f+2.5)\right]\cdot e^{-j2\pi f(2)}$ |
| b) |  |

| | |
|---|---|
| a) | $y(t) = x(t) * h(t)$<br>$= 4\sin\left(\frac{10}{2}\pi t\right) * \delta(t-2) = 4\sin\left(5\pi(t-2)\right) = 4\sin(5\pi t - 10\pi)$<br>$Y(f) = 4\cdot\left[\frac{\delta(f-2.5)-\delta(f+2.5)}{j2}\right]\cdot e^{-j2\pi f(2)}$<br>$= -2j\cdot\left[\delta(f-2.5)-\delta(f+2.5)\right]\cdot e^{-j2\pi f(2)}$ |

b)

$$Y(f)=4\cdot\left[\frac{\delta(f-2.5)-\delta(f+2.5)}{j2}\right]\cdot e^{-j2\pi f(2)}$$

$$=-2j\cdot\left[\delta(f-2.5)-\delta(f+2.5)\right]\cdot e^{-j2\pi f(2)}$$

$$|Y(f)|=\left|-2j\cdot\left[\delta(f-2.5)-\delta(f+2.5)\right]\cdot e^{-j2\pi f(2)}\right|$$

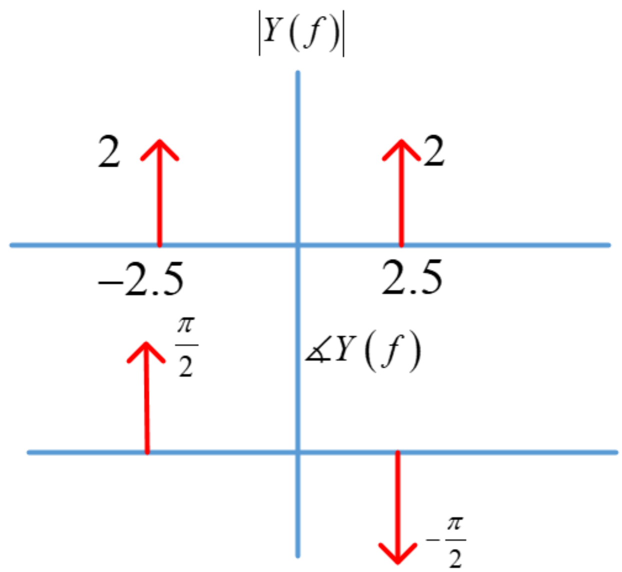
$$=2 @ f=2.5 \quad \& \quad f=-2.5$$

$$\measuredangle(Y(f))=\left|-2j\cdot\left[\delta(f-2.5)-\delta(f+2.5)\right]\cdot e^{-j2\pi f(2)}\right|$$

$$=-j\cdot e^{-j2\pi(2.5)\cdot(2)} @ f=2.5 \quad \& \quad j\cdot e^{-j2\pi(-2.5)\cdot(2)} @ f=-2.5$$

$$=-j\cdot e^{-j10\pi} @ f=2.5 \quad \& \quad j\cdot e^{j10\pi} @ f=-2.5$$

$$=-j @ f=2.5 \quad \& \quad j @ f=-2.5$$



5. [15 pts]

An equation is given as

$$y(t) = -2\cdot\cos(10\pi t)\cdot\left[\cos(10\pi t) - j\cdot\sin(10\pi t)\right]$$

a) Find magnitude response of $Y(f)$ and plot it.

b) Find phase response of $Y(f)$ and plot it.

| | |
|---|---|
| a) | |
| b) | |

Using Euler's identity

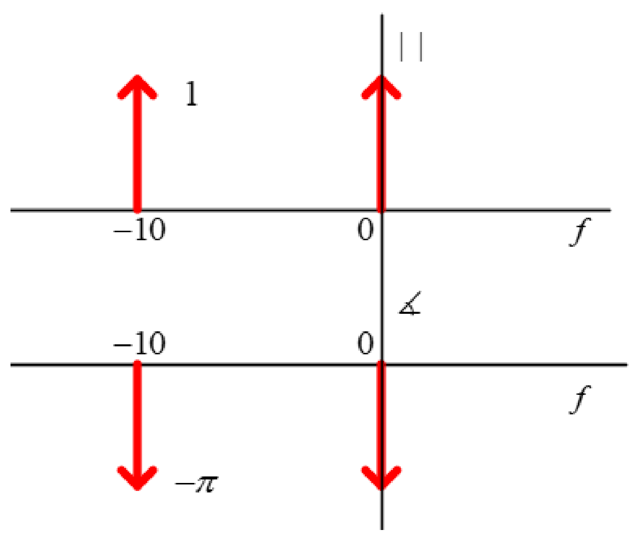
$$e^{jx} = \cos(x) + j\sin(x)$$

$$e^{-j10\pi t} = \cos(-10\pi t) + j\sin(-10\pi t) = \cos(10\pi t) - j\sin(10\pi t)$$

$$-2\cdot\cos(10\pi t)\cdot\left[\cos(10\pi t) - j\cdot\sin(10\pi t)\right] = -2\cdot\cos(10\pi t)\cdot e^{-j10\pi t}$$

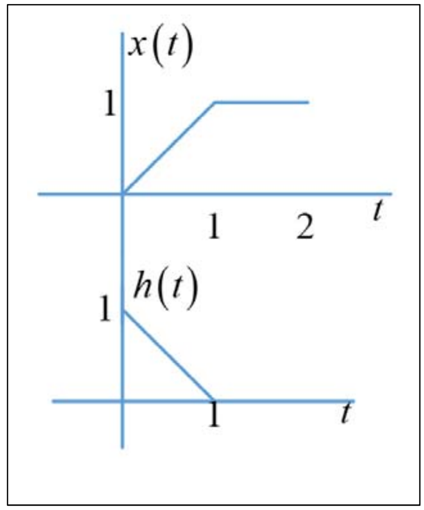
$$F\left\{-2\cdot\cos(10\pi t)\cdot e^{-j10\pi t}\right\} = -2\cdot\frac{1}{2}\cdot\left[\delta(f-5)+\delta(f+5)\right]*\delta(f+5)$$

$$= -\left[\delta(f)+\delta(f+10)\right]$$



6. [20 pts]

Do the convolution of two functions and write complete output of the function for each range of t. (You need to carry out the calculation of the integrations.)




| | |
|---|---|
| $0 \le t < 1$ | $y(t)=\int_{\tau=0}^{t} \tau\cdot\left(-(t-\tau)+1\right)d\tau$ <br> $=\frac{-t^2(t-3)}{6}$ <br> $=-\frac{t^3}{6}+\frac{t^2}{2}$ |
| $1 \le t < 2$ | $y(t)=\int_{\tau=t-1}^{1} \tau\cdot\left(-(t-\tau)+1\right)d\tau+\int_{\tau=1}^{t} 1\cdot\left(-(t-\tau)+1\right)d\tau$ <br> $=\frac{(t+1)(t-2)^2}{6}-\frac{(t-1)(t-3)}{2}$ <br> $=\frac{t^3}{6}-t^2+2t-\frac{5}{6}$ |
| $2 \le t < 3$ | $y(t)=\int_{\tau=t-1}^{2} 1\cdot\left(-(t-\tau)+1\right)d\tau$ <br> $=\frac{(t-3)^2}{2}$ <br> $=\frac{t^2}{2}-3t+\frac{9}{2}$ |