EE 538B CMOS RF IC DESIGN

Midterm Examination No. 2: May 24, 2004

Time Allowed: 110 Minutes

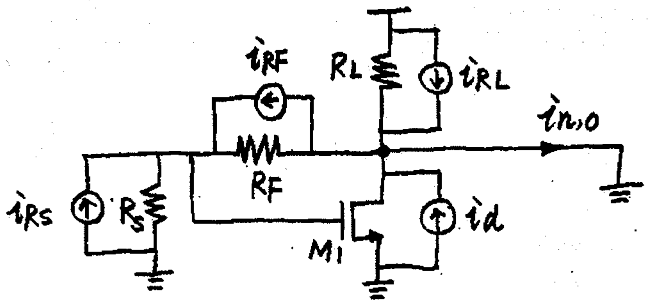
Student Name: Solution

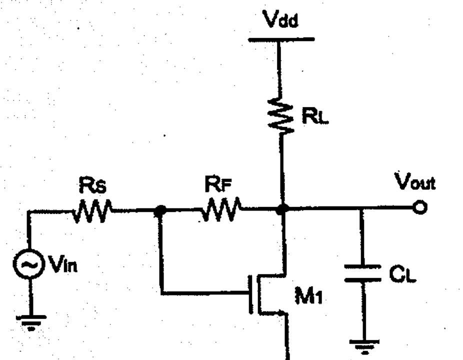
UW Student ID #: 1234567

You are allowed two sheets of notes. Write legibly. Show all work. State assumptions.

Problem #	Points Possible	Points
1	**15**	
2	**15**	
3	**30**	
4	**25**	
5	**15**	

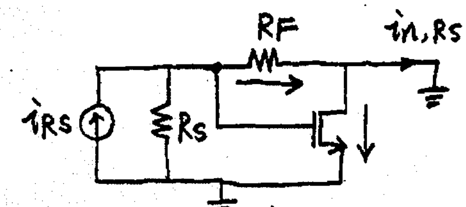
1. (15 points) Determine the noise factor of the circuit shown with respect to the source resistance R_S. Neglect channel length modulation effects, all parasitic capacitors and induced gate noise of the MOSFETs. Assume that all the MOSFETs operate in the saturation region.





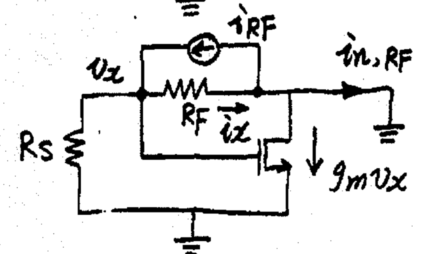
There are four noise sources:

$$\overline{i_{RS}^2} = \frac{4kT}{R_S}\Delta f,\quad \overline{i_{RF}^2} = \frac{4kT}{R_F}\Delta f,\quad \overline{i_{RL}^2} = \frac{4kT}{R_L}\Delta f,\quad \overline{i_d^2} = 4kT\gamma g_{do}\Delta f$$



• $$\overline{i_{n,RS}^2} = \overline{i_{RS}^2}\cdot\left(\frac{R_F R_S}{R_F+R_S}\right)^2\cdot\left(\frac{1}{R_F} - g_m\right)^2$$

• $\overline{i_{n,RF}^2}$:
$$v_x = i_{RF}\cdot\left(\frac{R_F R_S}{R_F+R_S}\right)$$
$$i_x = i_{RF}\cdot\left(\frac{R_S}{R_S+R_F}\right)$$



$$\begin{aligned} i_{n,RF} &= i_x - i_{RF} - g_m v_x \\ &= i_{RF}\cdot\frac{R_S}{R_S+R_F} - i_{RF} - g_m\cdot i_{RF}\cdot\frac{R_F R_S}{R_F+R_S} \\ &= -i_{RF}\cdot\frac{R_F}{R_F+R_S}\cdot(1+g_m R_S) \end{aligned}$$

$$\overline{i_{n,RF}^2} = \overline{i_{RF}^2}\cdot\left(\frac{R_F}{R_F+R_S}\right)^2(1+g_m R_S)^2$$

• $$\overline{i_{n,RL}^2} = \frac{4kT}{R_L}\Delta f$$

• $$\overline{i_{n,d}^2} = 4kT\gamma g_{do}\Delta f$$

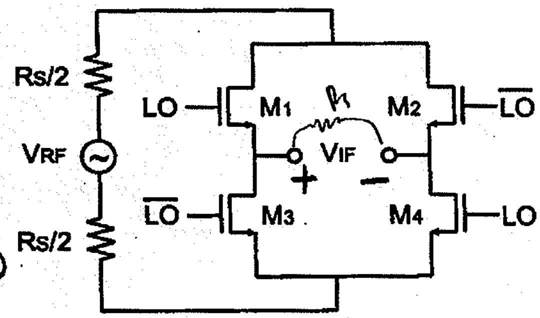
$$F = 1 + \frac{\overline{i_{n,RF}^2} + \overline{i_{n,RL}^2} + \overline{i_{n,d}^2}}{\overline{i_{n,RS}^2}}$$

$$= 1 + \frac{\frac{4kT}{R_F}\Delta f\left(\frac{R_F}{R_F+R_S}\right)^2(1+g_m R_S)^2 + \frac{4kT}{R_L}\Delta f + 4kT\gamma g_{do}\Delta f}{\frac{4kT}{R_S}\Delta f\cdot\left(\frac{R_F R_S}{R_F+R_S}\right)^2\cdot\left(g_m - \frac{1}{R_F}\right)^2}$$

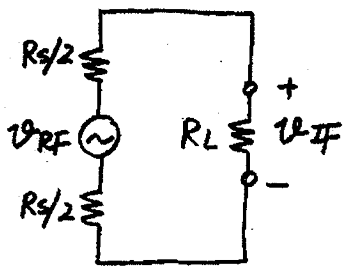
$$\Rightarrow \boxed{F = 1 + \frac{R_F}{R_S}\cdot\frac{(1+g_m R_S)^2}{(1-g_m R_F)^2} + \frac{1}{R_S}\left(\frac{1}{R_L} + \gamma g_{do}\right)\cdot\frac{(R_F+R_S)^2}{(1-g_m R_F)^2}}$$

2. (15 points) A double balanced passive mixer is shown below. The *IF* port is terminated by a resistance $R_L = R_S$. What is the conversion gain? Assume the *LO* is driven by square wave the switching is infinitely fast. The *RF* signal is sinusoidal with peak value *A* and the LO square wave has peak value *B*.

LO port is driven by ideal square wave.

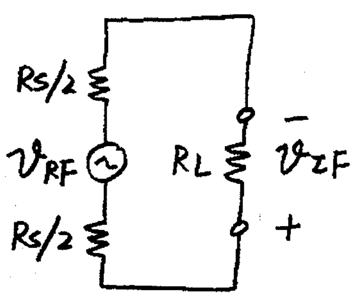


i) If $LO(t) > \overline{LO}(t)$:



$$\Rightarrow v_{IF}(t) = \frac{1}{2} v_{RF}(t)$$

ii) If $LO(t) < \overline{LO}(t)$:



$$\Rightarrow v_{IF}(t) = -\frac{1}{2} v_{RF}(t)$$

Therefore,
$$v_{IF}(t) = \frac{1}{2} v_{RF}(t) \cdot \frac{4B}{\pi}\left[\cos\omega_{LO}t + \frac{1}{3}\cos 3\omega_{LO}t + \frac{1}{5}\cos 5\omega_{LO}t + \cdots\right]$$
$$= \frac{1}{2} A\cos\omega_{RF}t \cdot \frac{4B}{\pi}\left[\cos\omega_{LO}t + \frac{1}{3}\cos 3\omega_{LO}t + \frac{1}{5}\cos 5\omega_{LO}t + \cdots\right]$$

Desired IF output term is $\frac{1}{2} \cdot A \cdot \frac{4B}{\pi} \cdot \frac{1}{2} \cos(\omega_{RF} - \omega_{LO})t$

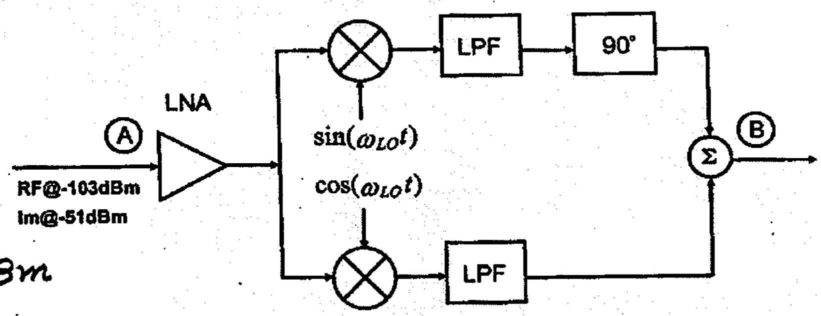
$$\Rightarrow \text{Conversion Gain } G_c = \frac{\frac{1}{2} A \cdot \frac{4B}{\pi} \cdot \frac{1}{2}}{A}$$

$$\therefore \boxed{G_c = \frac{B}{\pi}}$$

3. (30 points) (a) A Hartley receiver is shown below. The RF signal power (@ node *A*) is -103dBm. The power of image interferer (also @ node *A*) is as high as -51dBm. The gain from node *A* to node *B* is 50dB. After down-conversion, the acceptable interference from image frequency can be 12dB higher than the desired signal. Determine the required image reject ratio (*IRR*).

$P_{RF}|_A = -103\,dBm$

$P_{IM}|_A = -51\,dBm$



Without image rejection:

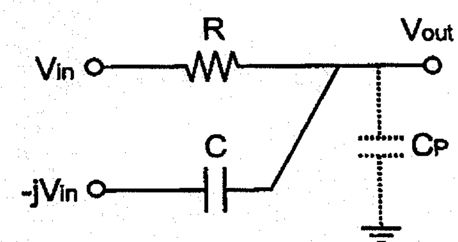
$P_{RF}|_B = -103+50 = -53\,dBm$

$P_{IM}|_B = -51+50 = -1\,dBm$

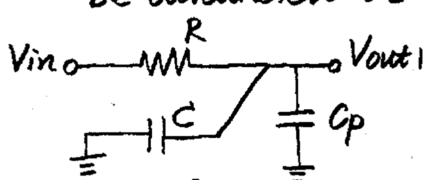
$P_{IM}|_{B,\ acceptable} = -53+12 = -41\,dBm$

∴ The required IRR $= -1-(-41) = 40\,dB$

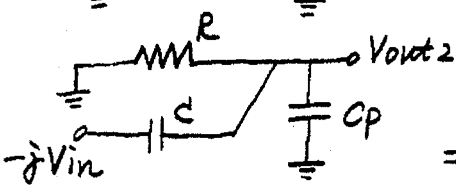
(b) Suppose the phase shift of 90° is realized by the one stage polyphase filter, as shown below. That is, -45° is realized in one branch and +45° is realized in the other branch. The parasitic capacitance at the output node is C_P=0.1pF. R=1KΩ, C=1pF. What is the *IRR* of the Hartley receiver? Assume that *IRR* degradation is only caused by the existence of C_P.



Considering C_p, the transfer function can be calculated as



$$\frac{V_{out1}}{V_{in}} = \frac{1}{1+j\omega R(C+C_p)}$$



$$\frac{V_{out2}}{(-jV_{in})} = \frac{Rj\omega C}{1+j\omega R(C+C_p)}$$

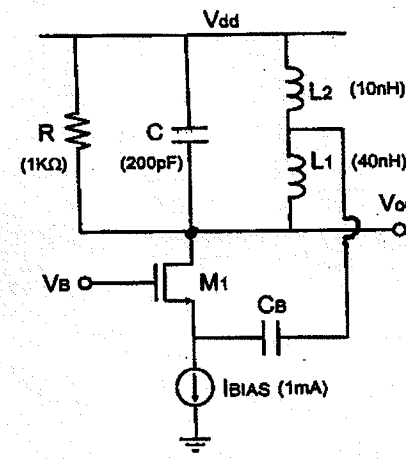
$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{1+\omega RC}{1+j\omega R(C+C_p)}$$

Therefore, C_p affects the phase of transfer function, not the amplitude.

At $\omega_0 = -\frac{1}{RC}$, $|H(j\omega_0)| = 0$. So image signal is suppressed.

$\Rightarrow$ IRR $= \infty\ dB$. phase errors cause IRR degradation too!

4. (25 points) For the Hartley oscillator shown below, calculate (a) frequency of oscillation, (b) peak output amplitude, (c) loaded and unloaded Qs of the tank, and (d) minimum transconductance required for startup (assume a safety margin of 3X to ensure start-up).



(a) $L_{eq} = L_1 + L_2 = 50nH$

$$\omega_0 = \frac{1}{\sqrt{L_{eq} \cdot C}} = \frac{1}{\sqrt{50\times10^{-9}\times200\times10^{-12}}}$$

$$= 3.16\times10^{8}\ rad/s$$

$$\boxed{f_0 = \frac{\omega_0}{2\pi} = 50.3\,MHz}$$

(b) $V_{tank} \approx 2 I_{BIAS} R (1-n)$

$$n = \frac{L_2}{L_1+L_2} = \frac{10}{10+40} = 0.2$$

$$V_{tank} \approx 2\times10^{-3}\times10^{3}\times(1-0.2)$$

$$\therefore \quad \boxed{V_{tank} \approx 1.6V}$$

(c) $G_m = \dfrac{2 I_{BIAS}}{V_1}$, $\quad V_1 = n \cdot V_{tank} = 0.2\times1.6 = 0.32V$

$$G_m = \frac{2\times10^{-3}}{0.32} = 6.25\,mS$$

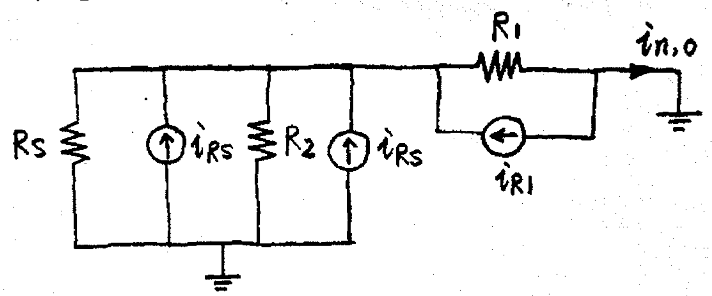
$$R_{eq} = R \,\|\, \frac{1}{n^2 G_m} = 800\,\Omega$$

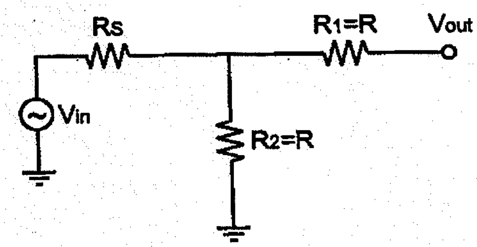
$$\boxed{\begin{aligned} Q_{unloaded} &= \omega_0 R C = 63.25 \\ Q_{loaded} &= \omega_0 R_{eq} C = 50.60 \end{aligned}}$$

(d) $$g_m \geq \frac{1}{R(n-n^2)} = \frac{1}{10^3\times(0.2-0.2^2)} = 6.25\,mS$$

$$\boxed{g_{m,min} = 3\times6.25 = 18.75\,mS}$$

5. (15 points) A resistive network is shown below:





(a) What is the noise factor of the above circuit with respect to source resistance R_S?

There are three noise sources:

$$\overline{i_{Rs}^2} = \frac{4kT}{R_s}\Delta f,\quad \overline{i_{R1}^2} = \frac{4kT}{R_1}\Delta f,\quad \overline{i_{R2}^2} = \frac{4kT}{R_2}\Delta f$$

$$\overline{i_{n,Rs}^2} = \overline{i_{Rs}^2}\cdot\left(\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_s}}\right)^2 = \frac{4kT}{R_s}\Delta f\cdot\frac{R_s^2}{(2R_s+R)^2}$$

$$\overline{i_{n,R_2}^2} = \overline{i_{R2}^2}\cdot\left(\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_s}}\right)^2 = \frac{4kT}{R}\Delta f\cdot\frac{R_s^2}{(2R_s+R)^2}$$

$$\overline{i_{n,R1}^2} = \overline{i_{R1}^2}\cdot\left(1-\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_s}}\right)^2 = \frac{4kT}{R}\Delta f\cdot\left(\frac{R_s+R}{2R_s+R}\right)^2$$

$$\therefore\ F = 1+\frac{\overline{i_{n,R1}^2}+\overline{i_{n,R2}^2}}{\overline{i_{n,Rs}^2}} = 1+\frac{R_s}{R}+\frac{R_s}{R}\cdot\left(\frac{R_s+R}{R_s}\right)^2$$

$$\Longrightarrow\ \boxed{F = 3+2\cdot\frac{R_s}{R}+\frac{R}{R_s}}$$

(b) What value of R minimizes the noise factor F?

$$\frac{\partial F}{\partial R} = -2\frac{R_s}{R^2}+\frac{1}{R_s} = 0$$

$$\Longrightarrow\ \boxed{R_{opt} = \sqrt{2}\,R_s}$$