EE 210

HW#: 06

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Assigned question #s: **5**

HW06

① $f_s = 11.025 \, kHz \longrightarrow T_s = \dfrac{1}{f_s} = 90.7 \, \mu s$

$T_{sampler \atop acquisition} = 10 \, \mu s$

∴ Time available for quantization & digitization $= T_s - T_{sampler \atop acquisition} = 80.7 \, \mu s$

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② a) $x(t) = \cos(20t + 12°)$

$= \cos(2\pi f_m t + 12°) \longrightarrow 2\pi f_m = 20$

∴ $f_m = \dfrac{20}{2\pi} = 3.18 \, Hz$

∴ Nyquist rate $= f_N = 2 \times f_m = 6.366 \, Hz$

b) $x(t) = 2\sin(5000\pi t/3)$

$= 2\sin\left(2\pi\left(\dfrac{2500}{3}\right)t\right) \longrightarrow$ ∴ $f_m = \dfrac{2500}{3} = 833.33 \, Hz$

∴ Nyquist rate $= f_N = 2 \times 833.33 = 1666.67 \, Hz$

c) $x(t) = \sin\left(\dfrac{3000}{7}\pi t + \dfrac{\pi}{10}\right)$

$= \sin\left(2\pi\left(\dfrac{1500}{7}\right)t + \dfrac{\pi}{10}\right) \longrightarrow$ ∴ $f_m = \dfrac{1500}{7} = 214.2857 \, Hz$

∴ Nyquist rate $= f_N = 2 \times 214.2857 = 428.57 \, Hz$

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③
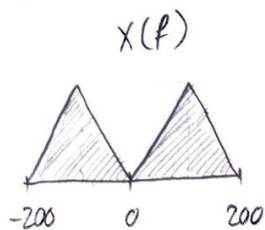
④ $f_s = 1000 \, Hz$

a)

$X(f)$

after sampling

-3400	-3100	-2000		-900	-600	$\Rightarrow -2f_s$
-2400	-2100	-1000		100	400	$\Rightarrow -1f_s$
-1400	-1100	0		1100	1400	$\Rightarrow 0f_s$
-400	-100	1000		2100	2400	$\Rightarrow 1f_s$
600	900	2000		3100	3400	$\Rightarrow 2f_s$
1600	1900	3000		4100	4400	$\Rightarrow 3f_s$
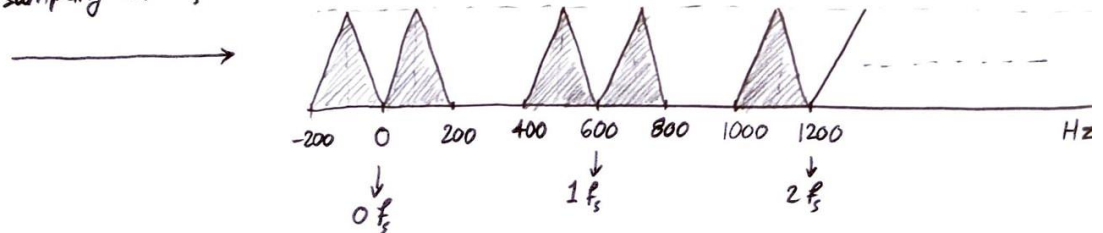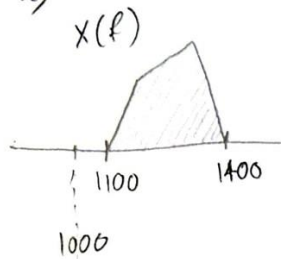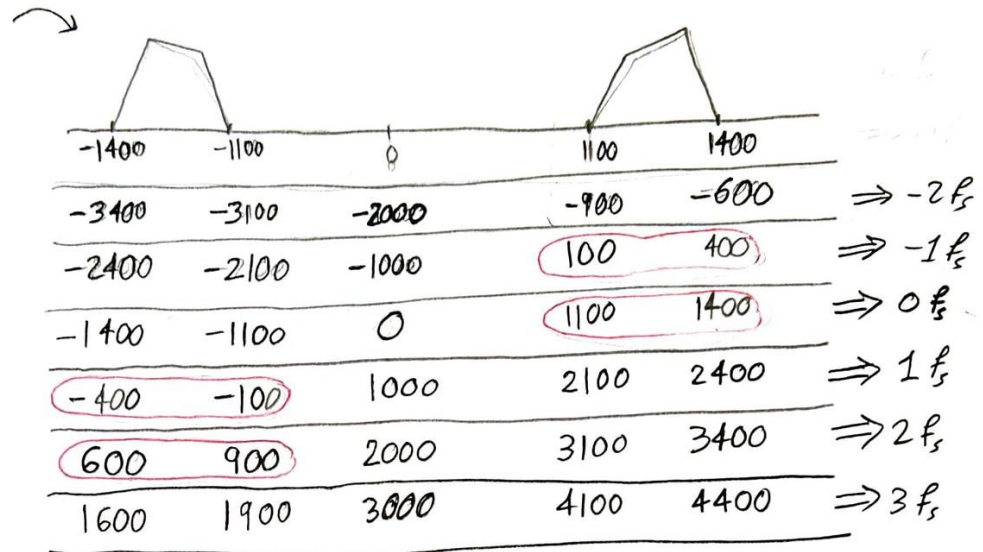
✳ Spectrum after Sampling (between -500 Hz to 1500 Hz)
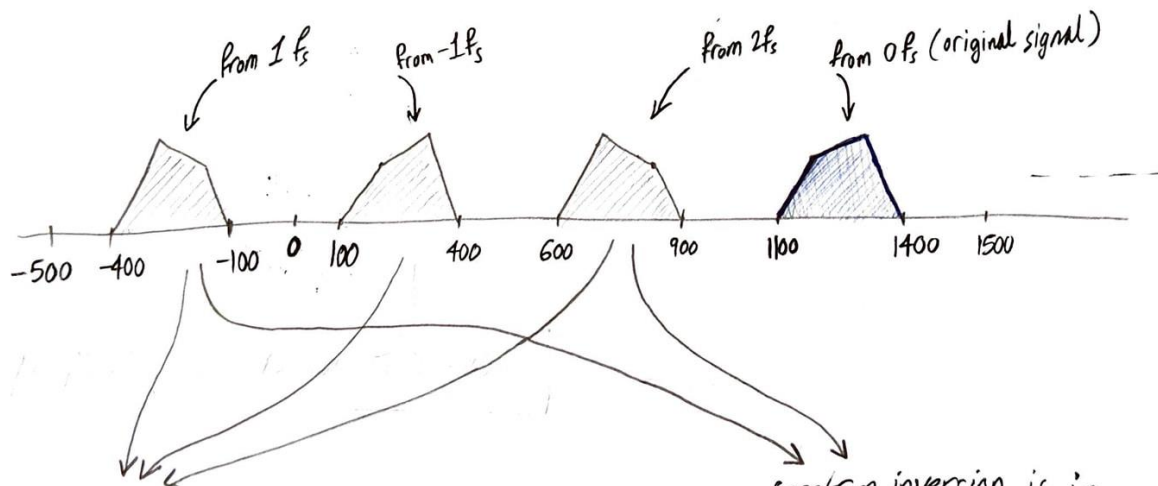
Aliased Spectrum is in

-400 → -100
& 100 → 400
& 600 → 900

(within the Baseband)

spectrum inversion is in

-400 → -100   (due to 1 f_s sample)
& 600 → 900   (due to 2f_s sample)

(within the Baseband)

b) $f_s = 1000\,Hz$

$X(f)$

					$\Rightarrow -2f_s$
-950	-800	0	800	950	
-2950	-2800	-2000	-1200	-1050	$\Rightarrow -2f_s$
-1950	-1800	-1000	-200	-50	$\Rightarrow -1f_s$
-950	-800	0	800	950	$\Rightarrow 0f_s$
50	200	1000	1800	1950	$\Rightarrow 1f_s$
1050	1200	2000	2800	2950	$\Rightarrow 2f_s$
2050	2200	3000	3800	3950	$\Rightarrow 3f_s$

✳ Spectrum after Sampling (between $-500\,Hz$ to $+500\,Hz$)

from $-1f_s$ from $1f_s$ from $0f_s$ (original signal)   from $2f_s$

Aliased spectrum in
$-200 \rightarrow -50$
& $50 \rightarrow 200$
(within the Baseband)

Spectrum inversion in
$50 \rightarrow 200$ (from $1f_s$)
(within the Baseband)

⑤ $x(t) = \sin(2\pi f_m t)$

(A)

$f_s - f_m = 150$ or $f_s - f_m = -150$

$600 - f_m = 150$ $600 - f_m = -150$
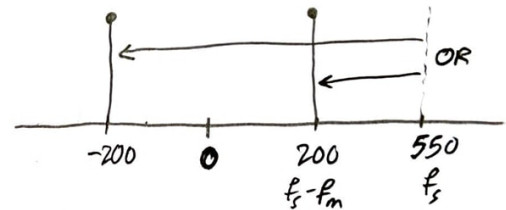
$\boxed{f_m = 450}$ OR $f_m = 750$

(A)
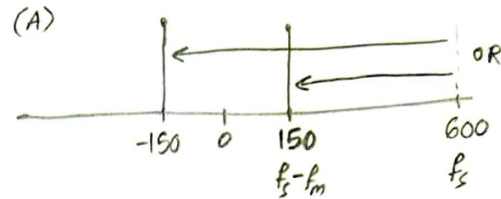
⤷ if we consider $2 f_s$, f_m will be greater than $1k$.

(B)

$f_s - f_m = 200$ or $f_s - f_m = -200$

$550 - f_m = 200$ $550 - f_m = -200$

$\boxed{f_m = 350}$ OR $f_m = 750$

from (A) & (B), The frequency of the signal $x(t)$ is $\underline{750\ Hz}$.

$$\boxed{\begin{array}{l} f_m = 750\ Hz \\ x(t) = \sin(2\pi(750)t) \end{array}}$$