

# Homework 8

3.6) a) 1 pole:  $x[n] = (-\frac{1}{2})^n u[n]$

$$\begin{array}{r} 1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} \dots \\ 1 + \frac{1}{2}z^{-1} \overline{) 1} \\ \underline{-1 + \frac{1}{2}z^{-1}} \\ -\frac{1}{2}z^{-1} \\ \underline{-(-\frac{1}{2}z^{-1} - \frac{1}{4}z^{-2})} \\ \frac{1}{4}z^{-2} \end{array}$$

b) 1 pole:  $x[n] = -(-\frac{1}{2})^n u[-n-1]$

$$\begin{array}{r} 2z - 4z^2 + 8z^3 \dots \\ \frac{1}{2}z^{-1} + 1 \overline{) 1} \\ \underline{-(1+2z)} \\ 2z \\ \underline{-(2z+4z^2)} \end{array}$$

c)  $X(z) = \frac{-3}{1+\frac{3}{4}z^{-1}} + \frac{4}{1+\frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2}$

$x[n] = [-3(-\frac{3}{4})^n + 4(-\frac{1}{2})^n] u[n]$

$$\begin{array}{r} 1 + (-\frac{3}{4} - \frac{1}{2})z^{-1} \\ 1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2} \overline{) 1 - \frac{1}{2}z^{-1}} \\ \underline{-(1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2})} \\ (-\frac{3}{4} - \frac{1}{2})z^{-1} - \frac{1}{8}z^{-2} \\ \underline{-[(-\frac{3}{4} - \frac{1}{2})z^{-1} + \frac{3}{4}(-\frac{3}{4} - \frac{1}{2})z^{-2} + \frac{1}{8}(-\frac{3}{4} + \frac{1}{2})z^{-3}]} \\ [-\frac{1}{8} + \frac{3}{4}(-\frac{3}{4} - \frac{1}{2})]z^{-2} + \frac{1}{8}(\frac{3}{4} + \frac{1}{2})z^{-3} \end{array}$$

$x[n] = [-3(-\frac{3}{4})^n + (-\frac{1}{2})^{n-2}] u[n]$

3.7) a)  $x[n] = u[-n-1] + (\frac{1}{2})^n u[n]$

$$X(z) = \frac{-1}{1-z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}} \quad \frac{1}{2} < |z| < 1$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{-\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1+z^{-1})} \quad \frac{(1-z^{-1})(1-\frac{1}{2}z^{-1})}{-\frac{1}{2}z^{-1}}$$

$$\boxed{H(z) = \frac{1-z^{-1}}{1+z^{-1}}} \quad \text{Causal ROC } |z| > 1$$

b)  $Y(z)$  converges  $|z| > 1$

c)  $Y(z) = \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{3}}{1+z^{-1}} \quad |z| > 1$

$$y[n] = -\frac{1}{3}(\frac{1}{2})^n u[n] + \frac{1}{3}(-1)^n u[n]$$

3.8) a)  $H(z) = \frac{1-z^{-1}}{1+\frac{3}{4}z^{-1}}$

$$h[n] = (-\frac{3}{4})^n u[n] - (-\frac{3}{4})^{n-1} u[n-1]$$

b)  $Y(z) = X(z)H(z) = \frac{-\frac{2}{3}z^{-1}}{(1-\frac{1}{3}z^{-1})(1+\frac{3}{4}z^{-1})} \quad |z| > \frac{3}{4}$

$$= \frac{-\frac{8}{13}}{1-\frac{1}{3}z^{-1}} + \frac{\frac{8}{13}}{1+\frac{3}{4}z^{-1}}$$

$$y[n] = -\frac{8}{13}(\frac{1}{3})^n u[n] + \frac{8}{13}(-\frac{3}{4})^n u[n]$$

c) Stable, causal if ROC  $|z| > \frac{3}{4} \neq$  absolutely summable

3.9) a)  $h[n]$  is causal ; ROC  $|z| > \frac{1}{2}$

b) ROC includes 1 ; stable

c)  $y[n] = -\frac{1}{3} \left(-\frac{1}{4}\right)^n u[n] - \frac{4}{3} (2)^n u[-n-1]$

$$Y(z) = \frac{-\frac{1}{3}}{1 + \frac{1}{4}z^{-1}} + \frac{\frac{4}{3}}{1 - 2z^{-1}} \quad \frac{1}{4} < |z| < 2$$

$$X(z) = \frac{Y(z)}{H(z)} = \frac{(1 - \frac{1}{2}z^{-1})}{(1 - 2z^{-1})} \quad |z| < 2$$

$$x[n] = -(2)^n u[n-1] + \frac{1}{2} (2)^{n-1} u[-n]$$

d)  $h[n] = 2\left(\frac{1}{2}\right)^n u[n] - \left(-\frac{1}{4}\right)^n u[n]$

3.10) d)  $x[n] = \left[\left(\frac{1}{4}\right)^{n+4} - \left(e^{j\frac{\pi}{3}}\right)^n\right] u[n-1]$

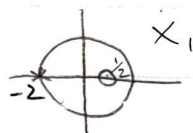
Right sided; non zero @  $n = -1$ ;

$$\text{ROC} = 1 < |z| < \infty$$

3.11) b)  $\lim_{x \rightarrow \infty} \frac{(z-1)^2}{(z-\frac{1}{2})} = \infty$  NOT CAUSAL

3.12) a)  $X_1(z) = \frac{1 - \frac{1}{2}z^{-1}}{1 + 2z^{-1}}$

Pole @ -2 zero @  $\frac{1}{2}$



3.16) a)  $X(z) = \frac{1}{1 - \frac{1}{3}z^{-1}} - \frac{1}{1 - 2z^{-1}}$   
 $= \frac{-\frac{5}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - 2z^{-1})} \quad \frac{1}{3} < |z| < 2$

$$Y(z) = \frac{5}{(1 - \frac{1}{3}z^{-1})} - \frac{5}{(1 - \frac{2}{3}z^{-1})}$$

$$= \frac{-\frac{5}{3}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - \frac{2}{3}z^{-1})} \quad |z| > \frac{2}{3}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 2z^{-1}}{1 - \frac{2}{3}z^{-1}} \quad |z| > \frac{2}{3}$$



$$3.16) b) h[n] = \left(\frac{2}{3}\right)^n u[n] - 2\left(\frac{2}{3}\right)^{n-1} u[n-1] \\ = \left(\frac{2}{3}\right)^n (u[n] - 3u[n-1])$$

$$c) H(z) = \frac{Y(z)}{X(z)} = \frac{1-2z^{-1}}{1-\frac{2}{3}z^{-1}} \\ Y(z)\left(1-\frac{2}{3}z^{-1}\right) = X(z)(1-2z^{-1}) \\ y[n] - \frac{2}{3}y[n-1] = x[n] - 2x[n-1]$$

d) Stable, Roc in unit circle

Impulse response  $h[n] = 0 \quad n < 0$

$$3.17) Y(z)\left(1-\frac{5}{2}z^{-1}+z^{-2}\right) = X(z)(1-z^{-1})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-z^{-1}}{1-\frac{5}{2}z^{-1}+z^{-2}} \\ = \frac{1-z^{-1}}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})} \\ = \frac{\frac{2}{3}}{1-2z^{-1}} + \frac{\frac{1}{3}}{1-\frac{1}{2}z^{-1}}$$

$$Roc = |z| < \frac{1}{2} \quad h[n] = -\frac{2}{3}2^n u[n-1] - \frac{1}{3}\left(\frac{1}{2}\right)^n u[-n-1] \\ h[0] = 0$$

$$Roc = \frac{1}{2} < |z| < 2 \quad h[n] = -\frac{2}{3}2^n u[n-1] + \frac{1}{3}\left(\frac{1}{2}\right)^n u[n] \\ h[0] = \frac{1}{3}$$

$$Roc = |z| > 2 \quad h[n] = \frac{2}{3}2^n u[n] + \frac{1}{3}\left(\frac{1}{2}\right)^n u[n] \\ h[0] = 1$$

$$Roc \quad |z| > 2 \quad |z| < \frac{1}{2} \quad h[n] = \frac{2}{3}2^n u[n] - \frac{1}{3}\left(\frac{1}{2}\right)^n u[n-1] \\ h[0] = \frac{2}{3}$$

3.19) c) 
$$Y(z) = \frac{1}{(1 - \frac{1}{3}z^{-1})(1 + \frac{1}{3}z^{-1})}$$
$$Roc = |z| > \frac{1}{3}$$