EE 210

HW#: 04

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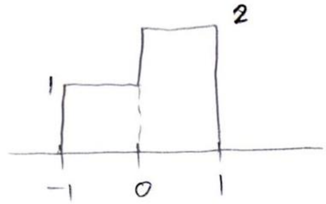
ID: **011510317**

Date: **9/25/2020**

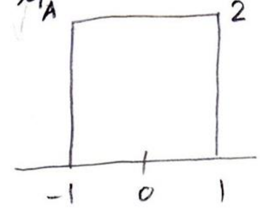
Assigned question #s: **7**

HW04

① $x_1(t)$

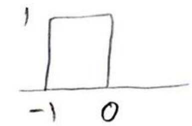


$=$ x_{1A}



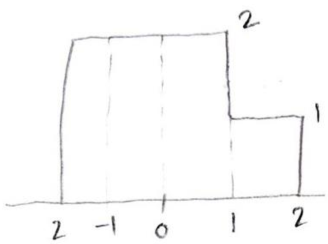
$2\,\mathrm{rect}\left(\frac{t}{2}\right)$

$\ominus$ x_{1B}

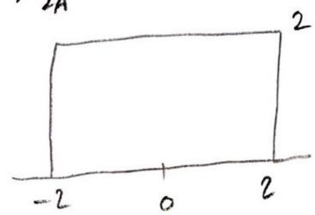


$1\,\mathrm{rect}\left(\frac{t}{1}\right) * \delta\left(t+\frac{1}{2}\right)$

$x_2(t)$

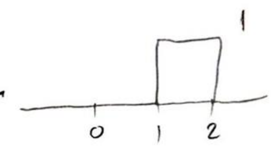


$=$ x_{2A}



$2\,\mathrm{rect}\left(\frac{t}{4}\right)$

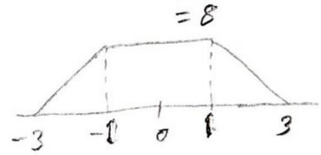
$\ominus$ x_{2B}



$1\,\mathrm{rect}\left(\frac{t}{1}\right) * \delta\left(t-3/2\right)$

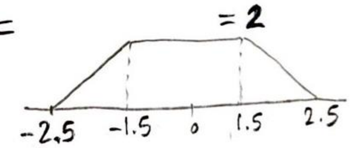
$x_{1A} * x_{2A}$ $=$

$2.2.2 = 8$



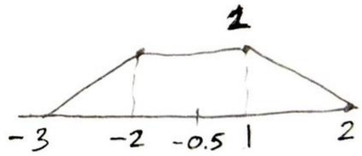
$\ominus$ $x_{1B} * x_{2A}$ $=$

$1.2.1 = 2$



$* \delta\left(t+\frac{1}{2}\right)$

$=$



$x_{1A} * x_{2B}$ $=$

$\ominus$

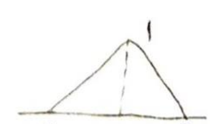
$* \delta(t-1.5)$

$=$

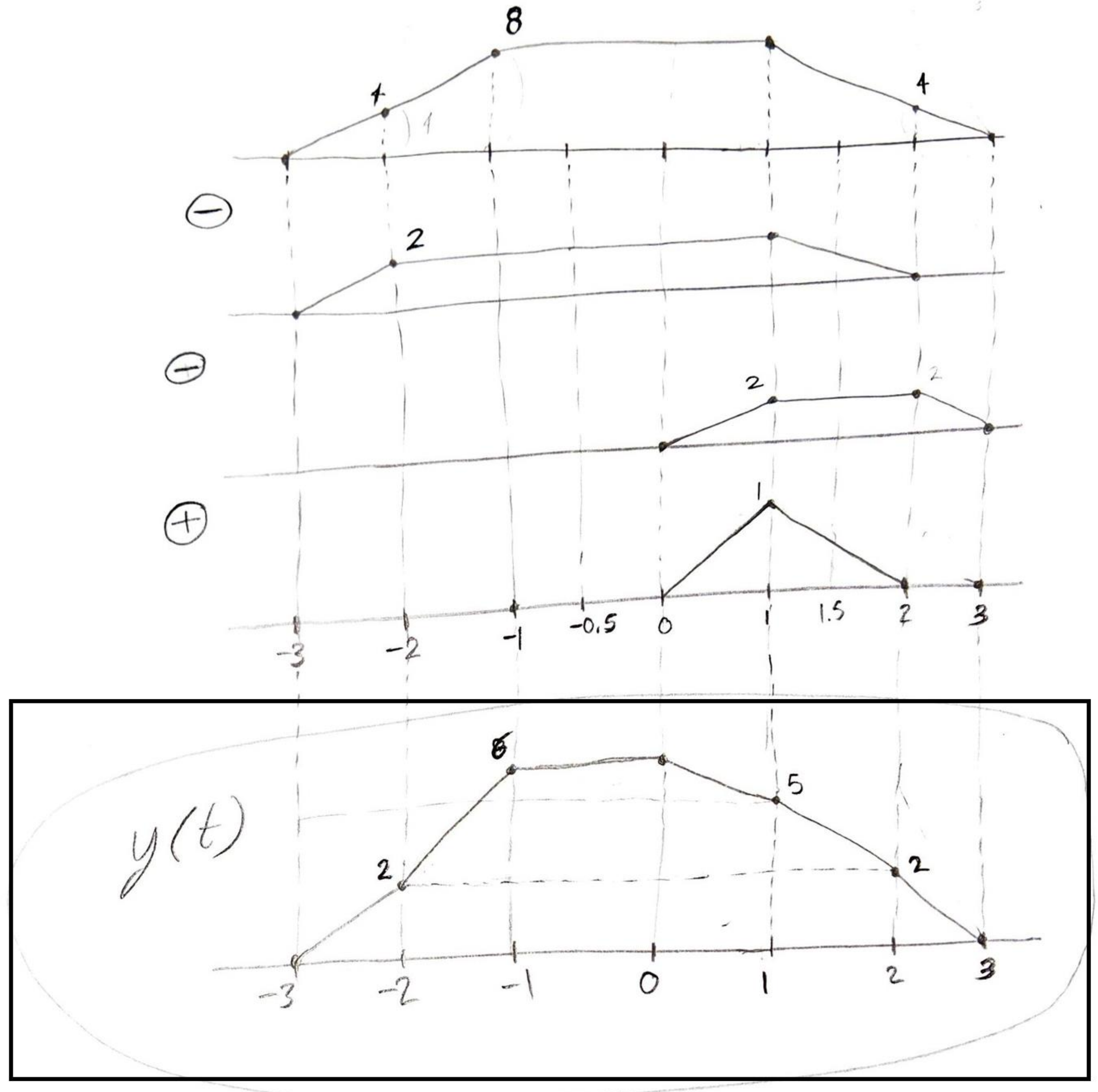
$x_{1B} * x_{2B}$ $=$

$* \delta(t-1)$

$=$



$$y(t) = (x_{1A} * x_{2A}) - (x_{1B} * x_{2A}) - (x_{1A} * x_{2B}) + (x_{1B} * x_{2B})$$



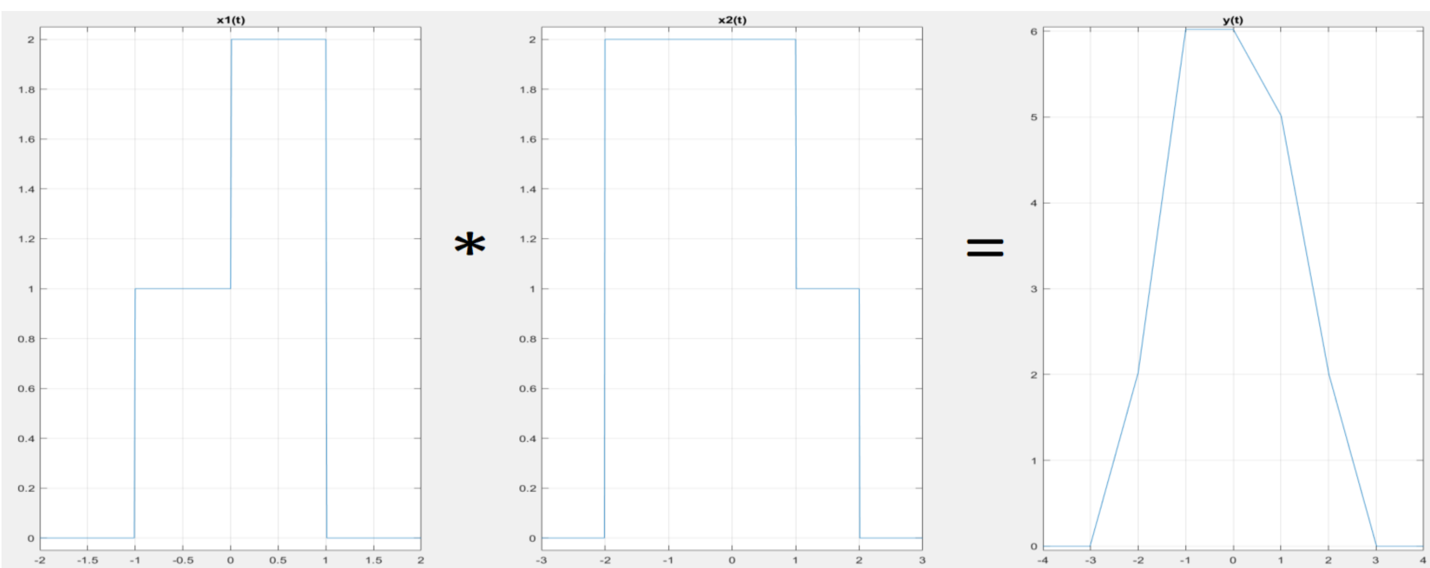
$$\mathcal{F}\left\{ (x_1(t) * x_2(t)) \right\} = \mathcal{F}\left\{ \left[2\,\text{rect}\left(\tfrac{t}{2}\right) * 2\,\text{rect}\left(\tfrac{t}{4}\right)\right] - \left[\text{rect}(t) * 2\,\text{rect}\left(\tfrac{t}{4}\right) * \delta\left(t+\tfrac{1}{2}\right)\right] \right.$$
$$\left. + \left[2\,\text{rect}\left(\tfrac{t}{2}\right) * \text{rect}(t) * \delta\left(t-\tfrac{3}{2}\right)\right] + \left[\text{rect}(t) * \text{rect}(t) * \delta(t-1)\right] \right.$$

$$\boxed{\begin{aligned} Y(f) &= 32\,\text{Sinc}(2f)\,\text{Sinc}(4f) - 8\,\text{Sinc}(f)\,\text{Sinc}(4f)\,e^{j(\pi f)} \\ &\quad - 4\,\text{Sinc}(2f)\,\text{Sinc}(f)\,e^{-j3\pi f} + \text{Sinc}^2(f)\,e^{-j2\pi f} \end{aligned}}$$

Matlab Code for Q1

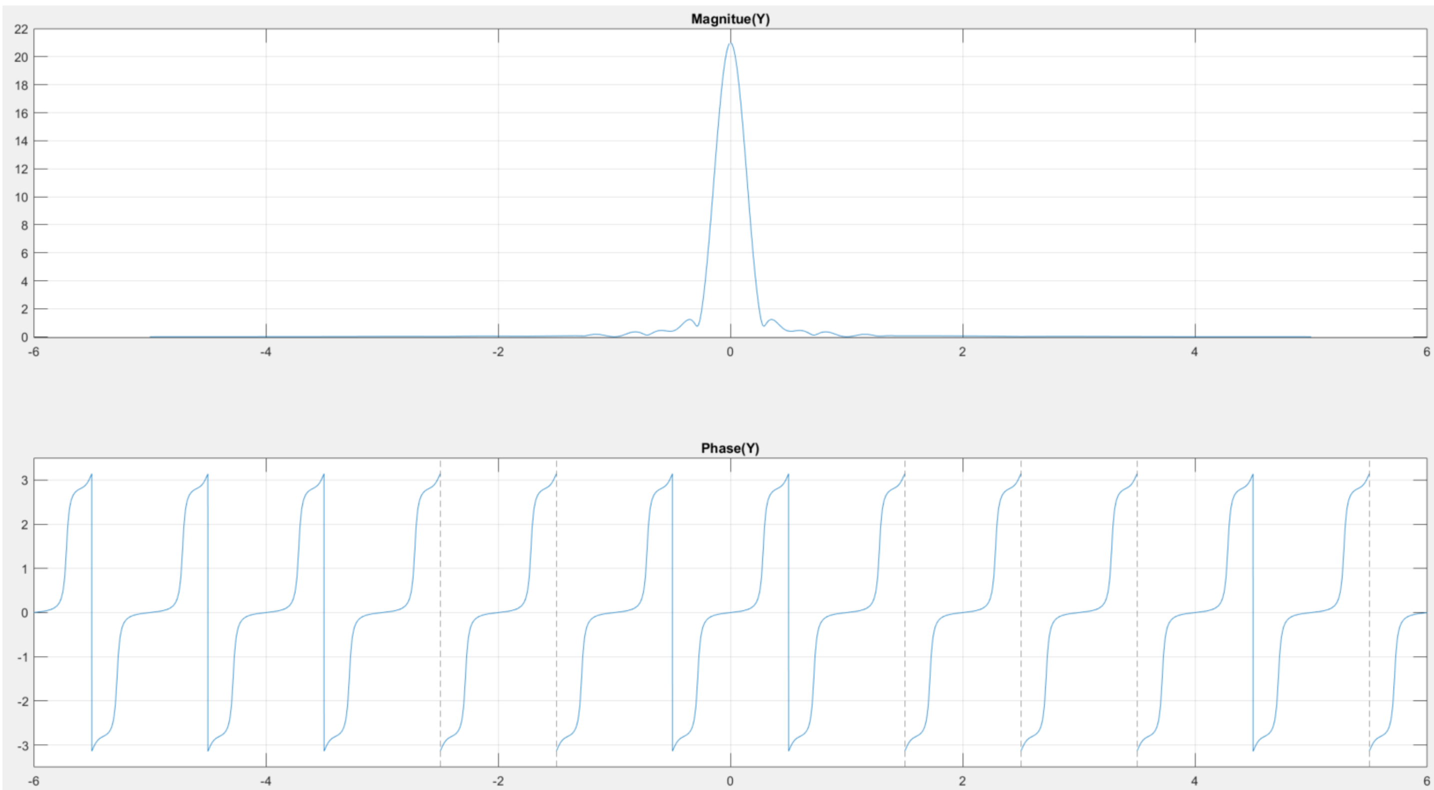
a) Time Domain:

```
tstep=0.01; tstart=-3; tend=3;
t=tstart:tstep:tend;

%x1(t) Function
for m=1:1:length(t)
    if (t(m) >= -1)&&(t(m) <= 0)
        x1(m) = 1;
    elseif (t(m) > 0)&&(t(m) <= 1)
        x1(m) = 2;
    else
        x1(m) = 0;
    end
end
subplot(1,3,1); plot(t,x1); axis([-2 2 -0.05 2.05]); title("x1(t)"); grid on;

%x2(t) Function
for m=1:1:length(t)
    if (t(m) >= -2)&&(t(m) <= 1)
        x2(m) = 2;
    elseif (t(m) > 1)&&(t(m) <= 2)
        x2(m) = 1;
    else
        x2(m) = 0;
    end
end
subplot(1,3,2); plot(t,x2); axis([-3 3 -0.05 2.05]); title("x2(t)"); grid on;

y = conv(x1,x2)
y = y.*tstep
t_2x=(tstart*2):tstep:(tend*2);
subplot(1,3,3); plot(t_2x,y); axis([-4 4 -0.05 6.05]); title("y(t)"); grid on;
```

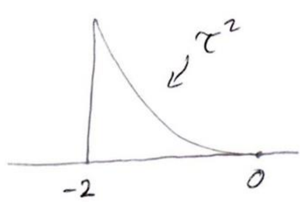


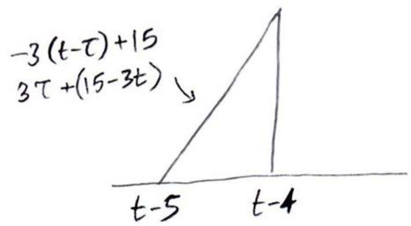
b) <u>Frequency Domain:</u>

```
syms f t
ex = exp(-j*2*pi*f*t);
x1a = 1; x1b = 2;
X1 = int(x1a*ex,t,-1,0)+int(x1b*ex,t,0,1);

x2a = 2; x2b = 1;
X2 = int(x2a*ex,t,-2,1)+int(x2b*ex,t,1,2);

Y = X1*X2;
Y = simplify(Y)
subplot(2,1,1); fplot(f,abs(Y)); axis([-6 6 -0.08 22]); title('Magnitue(Y)'); grid on;
subplot(2,1,2); fplot(angle(Y)); axis([-6 6 -3.5 3.5]); title('Phase(Y)'); grid on;
```



② $x_3(\tau)$ $\qquad\qquad\qquad$ $x_4(t-\tau)$





at $t-4<-2 \rightarrow t<2 \rightarrow y(t)=0$

at $-2\leq t-4\leq 0 \rightarrow 2\leq t\leq 3 \rightarrow y(t)=\int_{-2}^{t-4}\tau^2\,(3\tau+(15-3t))\,d\tau$

$t-5\leq -2$

$$=\int_{-2}^{t-4}\left[3\tau^3+(15-3t)\,\tau^2\right]d\tau$$

$$=\left[\frac{3}{4}\tau^4+\frac{(15-3t)}{3}\tau^3\right]_{-2}^{t-4}$$

$$\boxed{y(t)=\frac{3}{4}(t-4)^4+\frac{(15-3t)}{3}(t-4)^3-12+\frac{8}{3}(15-3t)}$$

$y(2)=0$, $y(3)=2.75$

at $t-4<0$, $t-5>-2$ $\rightarrow 3\leq t\leq 4 \rightarrow y(t)=\int_{t-5}^{t-4}\tau^2\,(3\tau+(15-3t))\,d\tau$

$$=\left[\frac{3}{4}\tau^4+\frac{(15-3t)}{3}\tau^3\right]_{t-5}^{t-4}$$

$$\boxed{\begin{aligned}y(t)=\frac{3}{4}(t-4)^4&+\left(\frac{15-3t}{3}\right)(t-4)^3\\&-\frac{3}{4}(t-5)^4-\left(\frac{15-3t}{3}\right)(t-5)^3\end{aligned}}$$

$y(3)=2.75$, $y(4)=0.25$

at $-2\leq t-5\leq 0 \rightarrow 4\leq t\leq 5 \rightarrow y(t)=\int_{t-5}^{0}\tau^2\,(3\tau+(15-3\tau))\,d\tau$

$t-4>0$

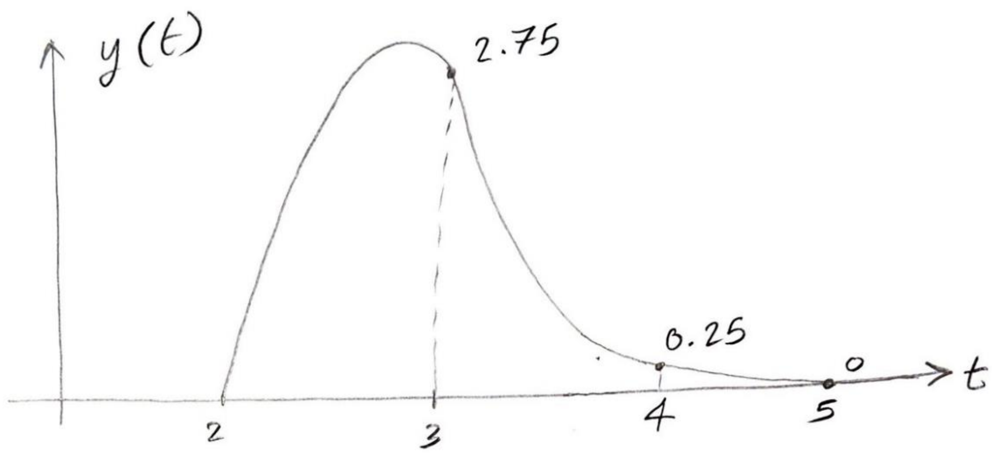
$$=\left[\frac{3}{4}\tau^4+\frac{(15-3t)}{3}\tau^3\right]_{t-5}^{0}$$

$$\boxed{y(t)=-\frac{3}{4}(t-5)^4-\left(\frac{15-3t}{3}\right)(t-5)^3}$$

$y(4)=0.25$, $y(5)=0$

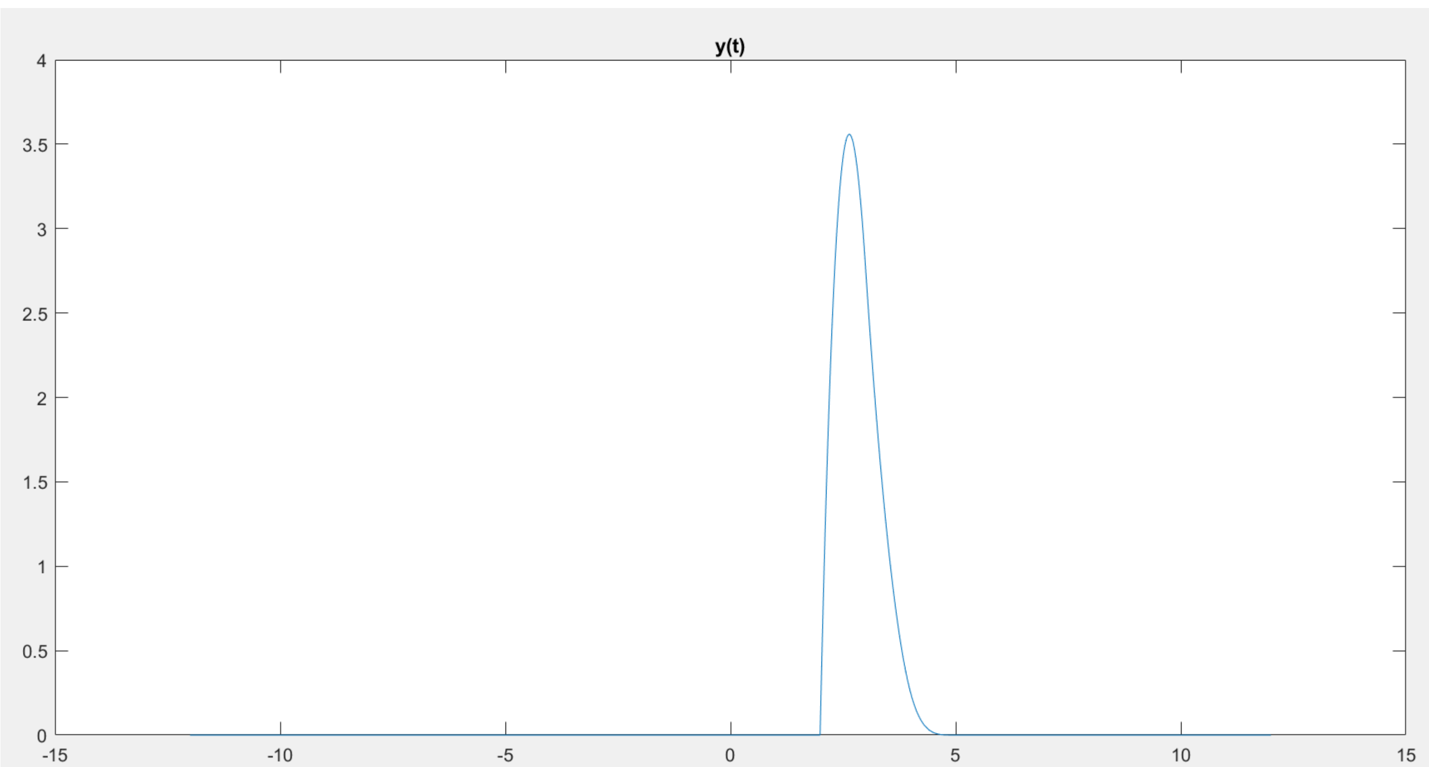
at $t>5 \rightarrow y(t)=0$

$$y(t) = \begin{cases} 0 & t < 2 \\ \frac{3}{4}(t-4)^4 + (5-t)(t-4)^3 + \frac{8}{3}(15-3t) - 12 & 2 \leq t \leq 3 \\ \frac{3}{4}(t-4)^4 + (5-t)(t-4)^3 - \frac{3}{4}(t-5)^4 - (5-t)(t-5)^3 & 3 \leq t \leq 4 \\ -\frac{3}{4}(t-5)^4 - (5-t)(t-5)^3 & 4 \leq t \leq 5 \\ 0 & t > 5 \end{cases}$$

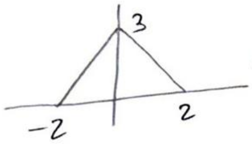


Matlab Code for Q2

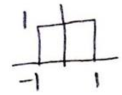
```
clc; clear all; close all;

tstart=-6; tstep=0.01; tend=6;
t=tstart:tstep:tend;

%x1(t) Function
for m=1:1:length(t)
    if (t(m) >= -2)&&(t(m) <= 0)
        x1(m) = t(m)^2;
    else
        x1(m) = 0;
    end
end
figure; plot(t,x1);

%x2(t) Function
for m=1:1:length(t)
    if (t(m) >= 4)&&(t(m) <= 5)
        x2(m) = (-3*t(m))+15;
    else
        x2(m) = 0;
    end
end
figure; plot(t,x2);

y = conv(x1,x2)
y = y.*tstep
t_2x=(tstart*2):tstep:(tend*2);
figure; plot(t_2x,y)
```



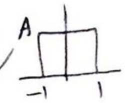
③ a) $\boxed{y(t) = 3\,\Delta\left(\frac{t}{4}\right) * \delta(t-5)} = x(t) * h(t)$?



$* \, \delta(t-5) =$



$* \, \delta(t-1) \; *$



$* \, \delta(t-t_0)$

$\int_{-1}^{1} (1)(A)\, d\tau = 3$

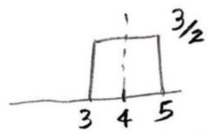
$2A = 3$

$\therefore A = 3/2$

$5 = 1 + t_0$

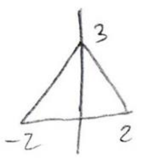
$\therefore t_0 = 4$

$$\boxed{h(t) = \frac{3}{2}\,\mathrm{rect}\left(\frac{t-4}{2}\right)}$$

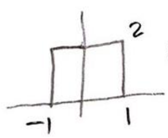


$x(t) = 2\,\mathrm{rect}\left(\frac{t}{2}\right) * \delta(t-7) \quad * \quad h(t)$?

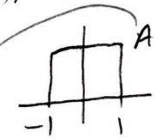
b)



$* \, \delta(t-5) =$



$* \, \delta(t-7)$



$* \, \delta(t-t_0)$

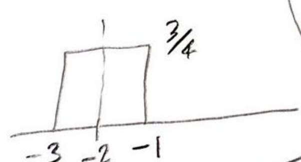
$\int_{-1}^{1} (2)(A)\, d\tau = 3$

$\therefore A = \frac{3}{4}$

$5 = 7 + t_0$

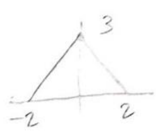
$\therefore t_0 = -2$

$$\boxed{\therefore h(t) = \frac{3}{4}\,\mathrm{rect}\left(\frac{t+2}{2}\right)}$$

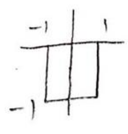


$x(t) = -1\,\mathrm{rect}\left(\frac{t}{2}\right) * \delta(t+5) \quad * \quad h(t)$?

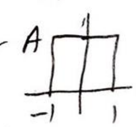
c)



$* \, \delta(t-5) =$



$* \, \delta(t+5)$



$* \, \delta(t-t_0)$

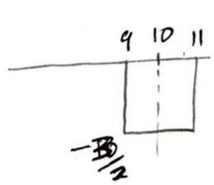
$\int_{-1}^{1} (-1)(A)\, d\tau = 3$

$\therefore A = -3/2$

$5 = -5 + t_0$

$\therefore t_0 = 10$

$$\boxed{\therefore h(t) = \frac{-3}{2}\,\mathrm{rect}\left(\frac{t-10}{2}\right)}$$



(4) $y(t) = 1\,\text{rect}\left(\frac{t}{2}\right) * \delta(t-1) * \left[2\,\text{rect}\left(\frac{t}{2}\right) * \delta(t-2)\right] \longrightarrow y_A(t)$

$+ 1\,\text{rect}\left(\frac{t}{2}\right) * \delta(t-1) * \left[2\,\text{rect}(t) * \delta(t-5)\right] \longrightarrow y_B(t)$

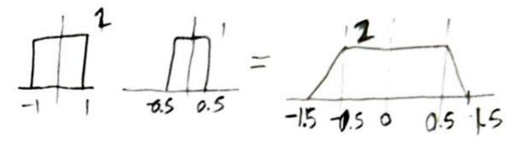
$+ 1\,\text{rect}\left(\frac{t}{2}\right) * \delta(t-1) * \left[2\,\delta(t-7)\right] \longrightarrow y_C(t)$

$y_A(t) = 2\,\text{rect}\left(\frac{t}{2}\right) * \text{rect}\left(\frac{t}{2}\right) * \delta(t-3)$

$= 4\,\Delta\left(\frac{t}{4}\right) * \delta(t-3)$

$= 4\,\Delta\left(\frac{t-3}{4}\right)$

$y_B(t) = 2\,\text{rect}\left(\frac{t}{2}\right) * \text{rect}\left(\frac{t}{1}\right) * \delta(t-6)$



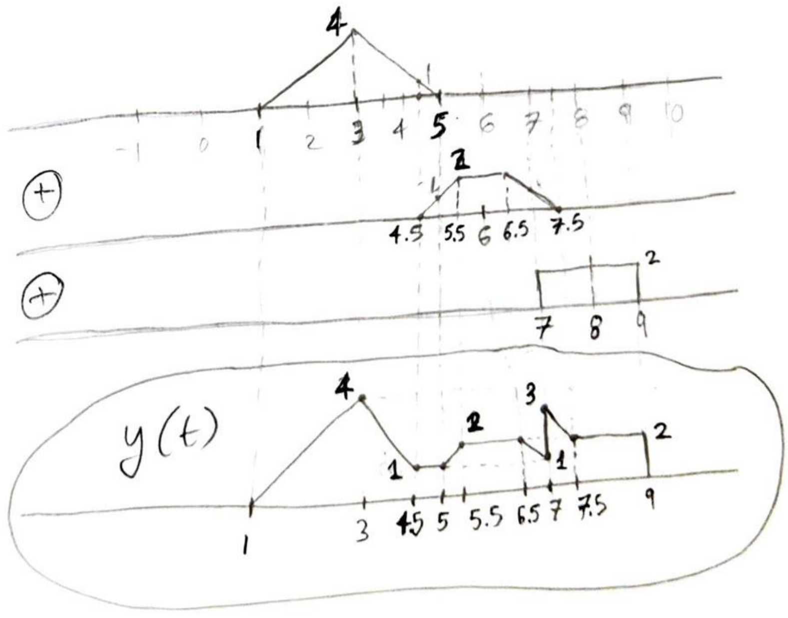
$y_C(t) = 2\,\text{rect}\left(\frac{t}{2}\right) * \delta(t-8)$

$= 2\,\text{rect}\left(\frac{t-8}{2}\right)$

a)

$$\boxed{\therefore\, y(t) = 4\,\Delta\left(\frac{t-3}{4}\right) + 2\,\underbrace{\text{rect}\left(\frac{t}{2}\right) * \text{rect}(t)}_{\text{Trapezoid}} * \delta(t-6) + 2\,\text{rect}\left(\frac{t-8}{2}\right)}$$

b)



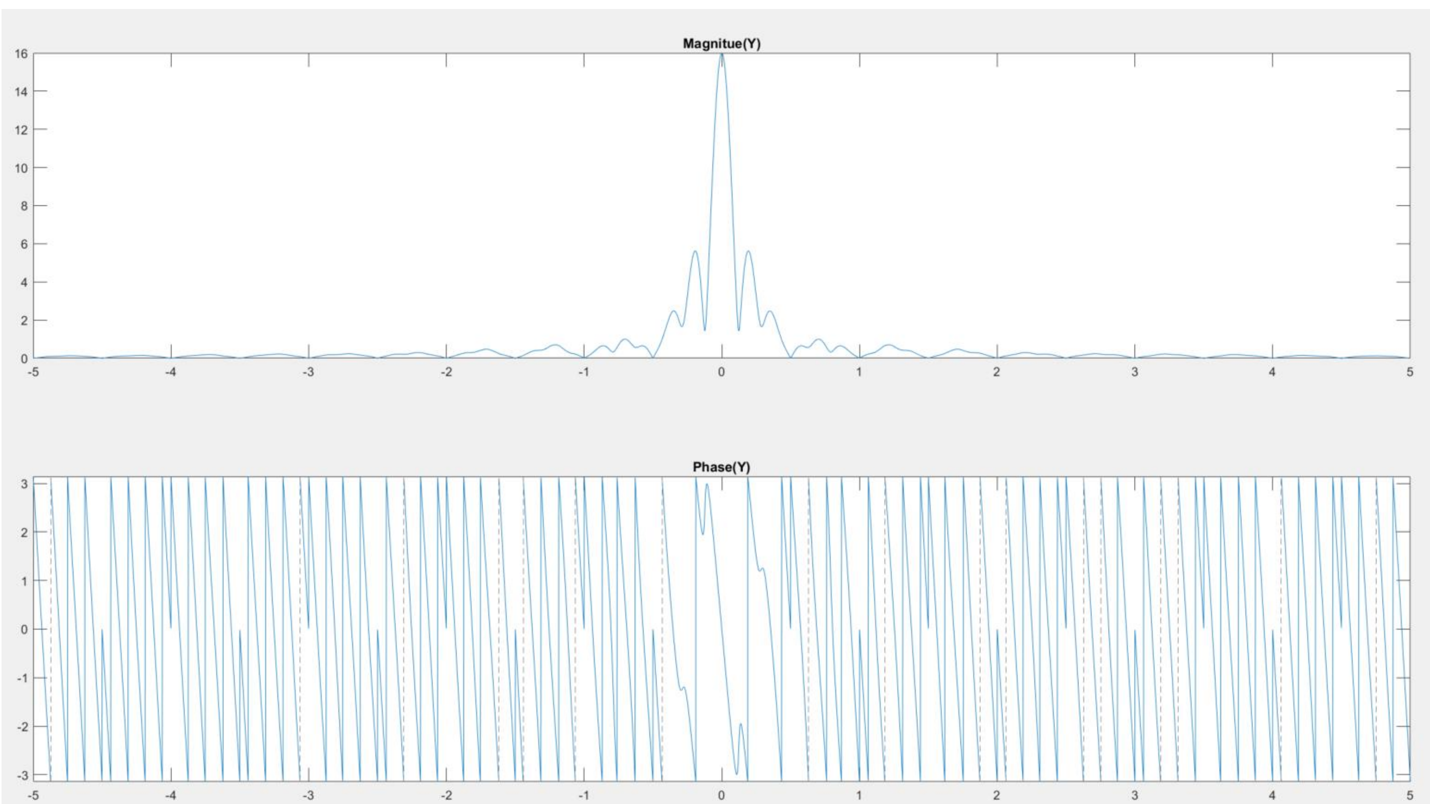
c) $$\mathcal{F}\{y(t)\} = 2\times 2\ \text{Sinc}(2f) \times 2 \times \text{Sinc}(2f) \times e^{-j2\pi f(3)} + 2\times 2\ \text{Sinc}(2f) \times \text{Sinc}(f) \times e^{-j2\pi f(6)} + 2\times 2\ \text{Sinc}(2f) \times e^{-j2\pi f(8)}$$

$$\boxed{Y(f) = 8\ \text{Sinc}^2(2f)\ e^{-j6\pi f} + 4\ \text{Sinc}(2f)\ \text{Sinc}(f)\ e^{-j12\pi f} + 4\ \text{Sinc}(2f)\ e^{-j16\pi f}}$$

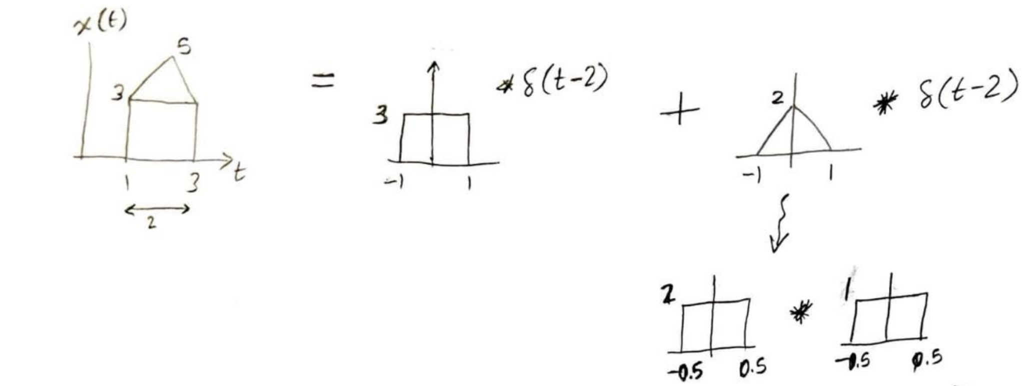
d)

Matlab Code for Q4

```
syms f t

ex = exp(-j*2*pi*f*t);
x = 1; X = int(x*ex,t,0,2);
h1 = 2; H1 = int(h1*ex,t,1,3);
h2 = 2; H2 = int(h2*ex,t,4.5,5.5);
ya = 2; Ya = int(ya*ex,t,7,9);

Y = (X*H1)+(X*H2)+Ya;
Y = simplify(Y)
subplot(2,1,1); fplot(abs(Y)); title('Magnitue(Y)');
subplot(2,1,2); fplot(angle(Y)); title('Phase(Y)');
```



④* Find Fourier Transform of $x(t)$?



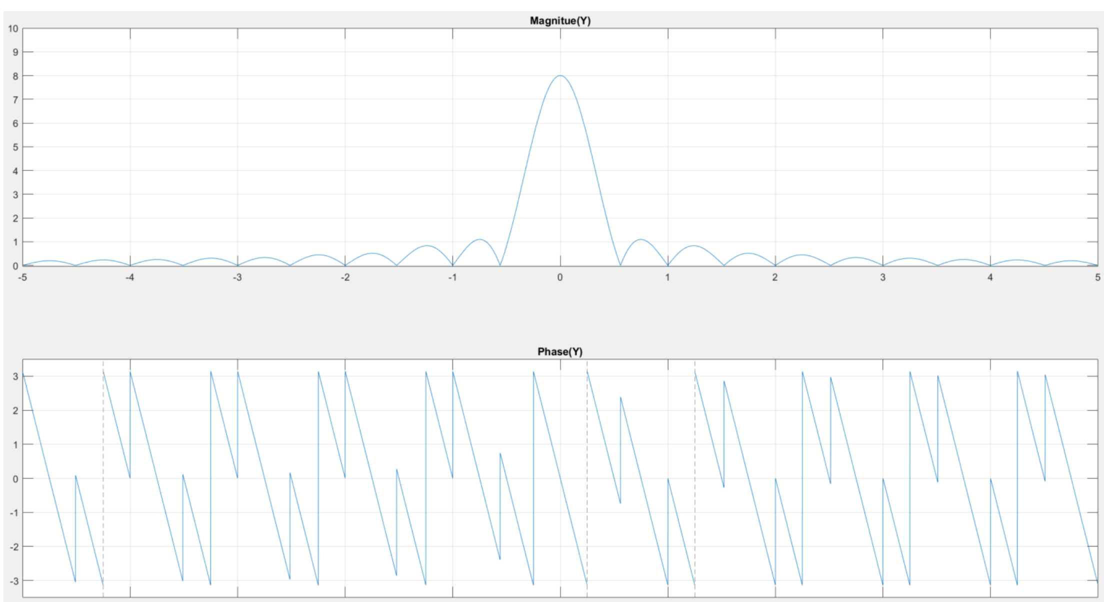
$$x(t) = \left[3\,\mathrm{rect}\left(\tfrac{t}{2}\right) * \delta(t-2)\right] + \left[2\,\mathrm{rect}\left(\tfrac{t}{1}\right) * 1\,\mathrm{rect}\left(\tfrac{t}{1}\right) * \delta(t-2)\right]$$

$$X(f) = \left[3\times 2\ \mathrm{Sinc}(2f)\ e^{-j2\pi f(2)}\right] + \left[2\ \mathrm{Sinc}(1f)\cdot 1\ \mathrm{Sinc}(1f)\cdot e^{-j2\pi f(2)}\right]$$

$$\boxed{X(f) = \left(6\ \mathrm{Sinc}(2f) + 2\ \mathrm{Sinc}^2(f)\right) e^{-j4\pi f}}$$

<u>Matlab Code for Q4 (extra problem)</u>

```
syms f t
ex = exp(-j*2*pi*f*t);
x1 = 3;
X1 = int(x1*ex,t,1,3);
x2 = 2*triangularPulse(1,3,t);
X2 = int(x2*ex,t,1,3);

Y = X1 + X2;
Y = simplify(Y)
subplot(2,1,1); fplot(abs(Y)); title('Magnitue(Y)'); axis([-5 5 -0.05 15])
subplot(2,1,2); fplot(angle(Y)); title('Phase(Y)'); axis([-5 5 -3.5 3.5])
```



5. $$Y(f) = X(f) * H(f) = \int_{\tau=-\infty}^{\infty} H(\tau)\, X(f-\tau)\, d\tau$$

$$\mathcal{F}^{-1}\{Y(f)\} = \int_{f=-\infty}^{\infty} X(f) * H(f)\, e^{j2\pi f t}\, df$$

$$= \int_{f=-\infty}^{\infty} \left(\int_{\tau=-\infty}^{\infty} H(\tau)\, X(f-\tau)\, d\tau \right) e^{j2\pi f t}\, df$$

$$e^{j2\pi f t} = e^{j2\pi t((f-\tau)+\tau)}$$

$$= \int_{\tau=-\infty}^{\infty} H(\tau)\, e^{j2\pi t \tau} \underbrace{\left(\int_{f=-\infty}^{\infty} X(f-\tau)\, e^{j2\pi t (f-\tau)}\, df \right)}_{\int_{a=-\infty}^{\infty} X(a)\, e^{j2\pi t a}\, da \,=\, x(t)} d\tau$$

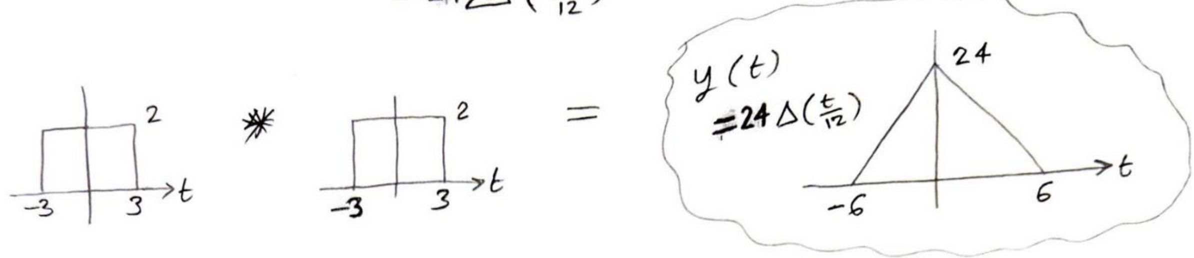
Let $f-\tau = a$
$df = da$

$$= x(t) \cdot \int_{\tau=-\infty}^{\infty} H(\tau)\, e^{j2\pi t \tau}\, d\tau$$

$$\therefore \boxed{y(t) = x(t) \cdot h(t)}$$

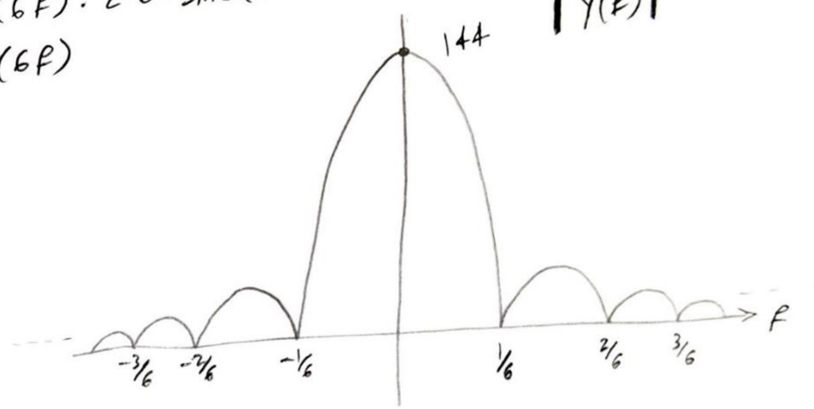
⑥ a) $y(t) = x(t) * x(t) = 2\,\mathrm{rect}\left(\frac{t}{6}\right) * 2\,\mathrm{rect}\left(\frac{t}{6}\right)$

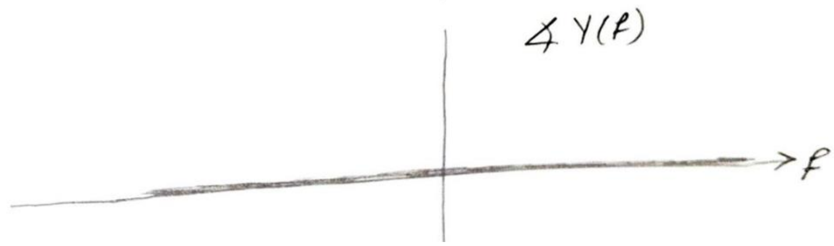
$$= 24\,\Delta\left(\frac{t}{12}\right)$$

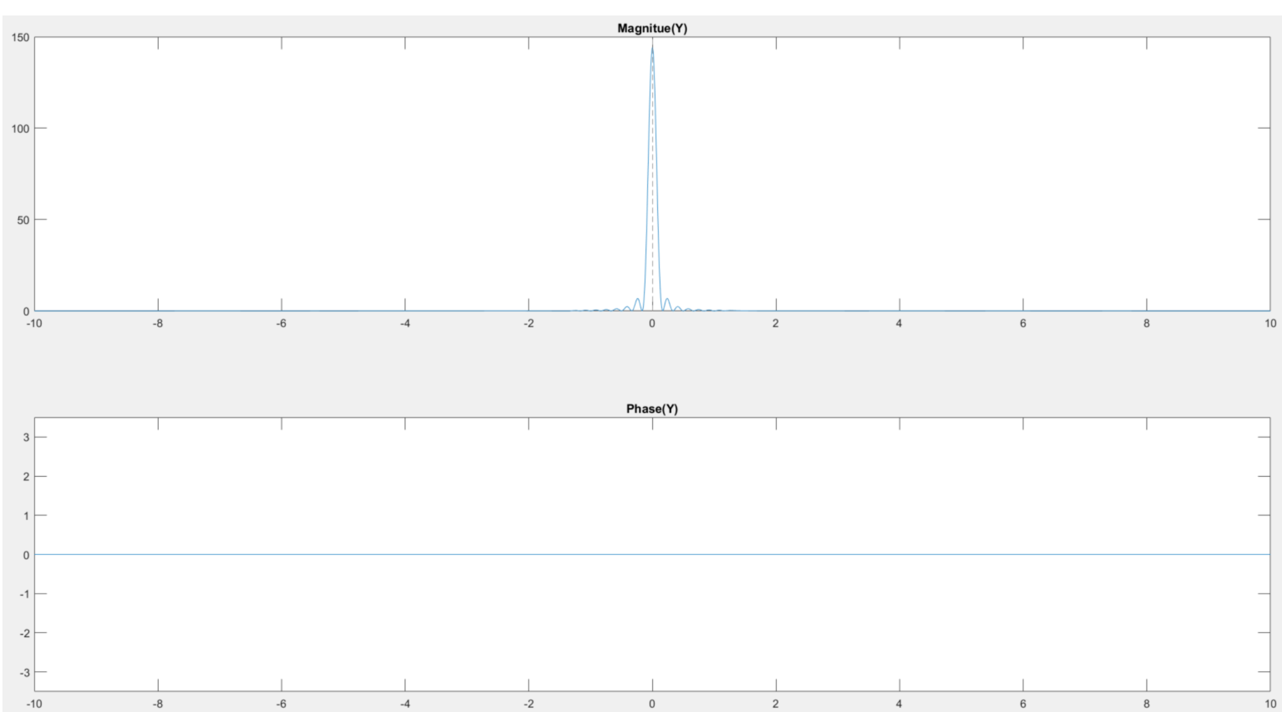


$Y(f) = 2 \times 6 \times \mathrm{Sinc}(6f) \cdot 2 \times 6 \times \mathrm{Sinc}(6f)$

$= 144\ \mathrm{Sinc}^2(6f)$



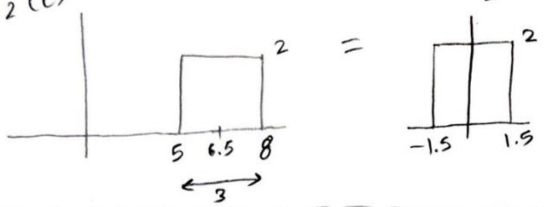




b) $x_1(t)$ $\qquad 3\,\mathrm{rect}\left(\frac{t}{3}\right)$

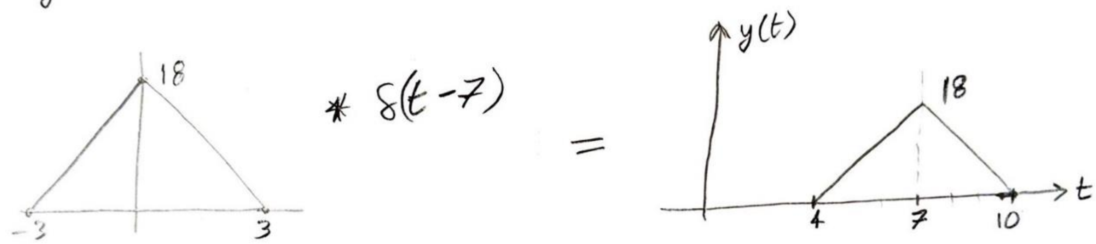
$* \,\delta(t-0.5)$

$x_2(t)$ $\qquad 2\,\mathrm{rect}\left(\frac{t}{3}\right)$

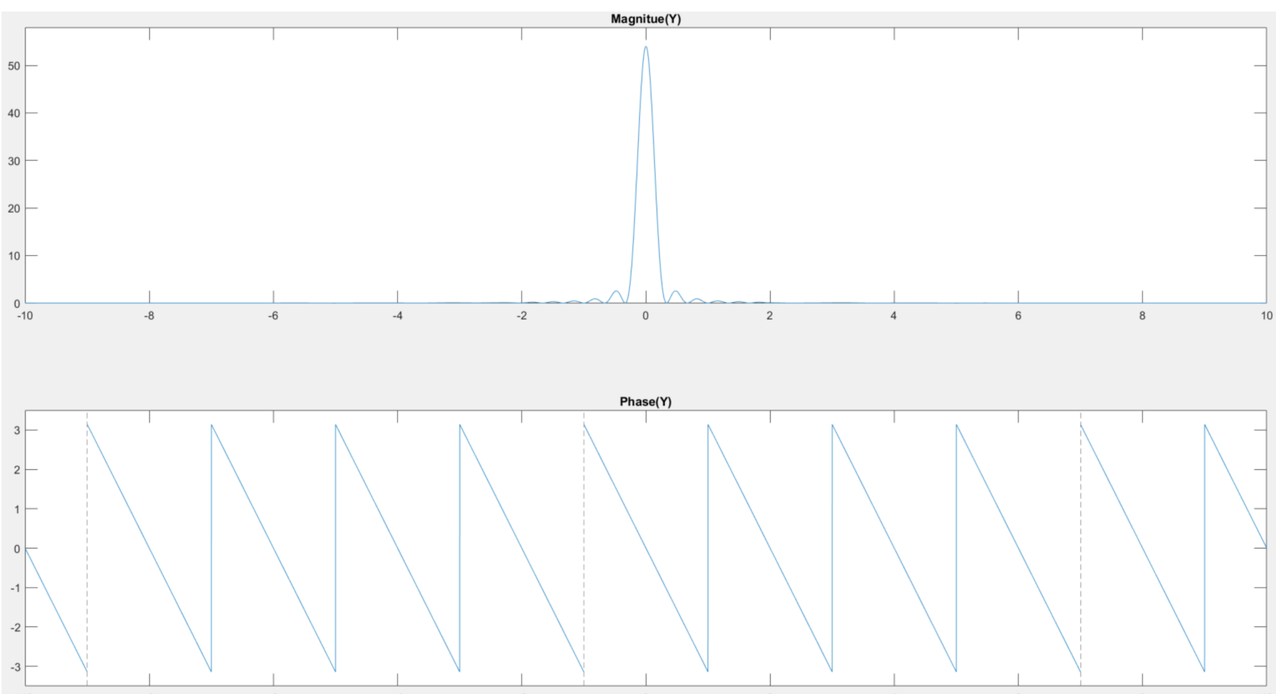


$* \,\delta(t-6.5)$

$$y(t) = x_1(t) * x_2(t) = 18\,\Delta\left(\frac{t-7}{6}\right)$$



$$Y(f) = 3\times 3\ \mathrm{Sinc}(3f)\cdot 2\times 3\ \mathrm{Sinc}(3f)\cdot e^{-j2\pi f(7)}$$
$$= 54\ \mathrm{Sinc}^2(3f)\, e^{-j14\pi f}$$



Matlab Code for Q6

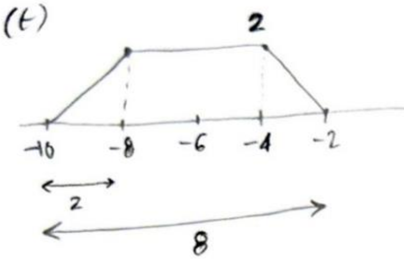
- Part a):

```
syms f t
ex = exp(-j*2*pi*f*t);
x1 = 2;
X1 = int(x1*ex,t,-3,3);

x2 = 2;
X2 = int(x2*ex,t,-3,3);

Y = X1*X2;
Y = simplify(Y)

subplot(2,1,1); fplot(abs(Y)); title('Magnitue(Y)'); axis([-10 10 -0.05 150])
subplot(2,1,2); fplot(angle(Y)); title('Phase(Y)'); axis([-10 10 -3.5 3.5])
```

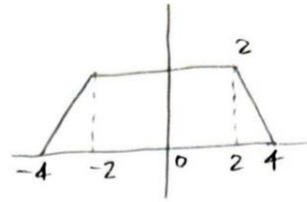
- Part b):

```
syms f t
ex = exp(-j*2*pi*f*t);
x1 = 3;
X1 = int(x1*ex,t,-1,2);

x2 = 2;
X2 = int(x2*ex,t,-1.5,1.5);

Y = X1*X2;
Y = simplify(Y)

subplot(2,1,1); fplot(abs(Y)); title('Magnitue(Y)'); axis([-10 10 -0.05 58])
subplot(2,1,2); fplot(angle(Y)); title('Phase(Y)'); axis([-10 10 -3.5 3.5])
```

7 A . $y(t)$



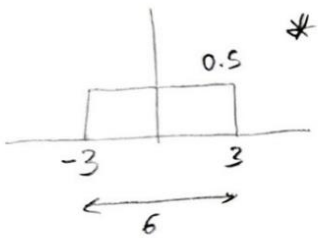
$=$



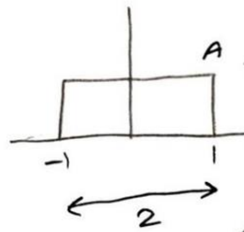
$* \ \delta(t+6)$

a) $=$

$x(t) = \frac{1}{2}\,\text{rect}\left(\frac{t}{6}\right) * \delta(t-5) \quad * \quad h(t)$



$* \ \delta(t-5)$



$* \ \delta(t-t_0)$

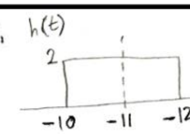
$5 + t_0 = -6$

$\underline{\underline{t_0 = -11}}$

$\int_{-1}^{1} A(0.5)\, d\tau = 2$

$A(0.5)(\cancel{2}) = \cancel{2}$

$\therefore \underline{\underline{A = 2}}$

$\therefore$ 

$h(t) = 2\,\text{rect}\left(\frac{t}{2}\right) * \delta(t+11)$

$= 2\,\text{rect}\left(\frac{t+11}{2}\right)$

---

b) $y(t) = x(t) * h(t)$

$= \frac{1}{\cancel{2}}\,\text{rect}\left(\frac{t}{6}\right) * \delta(t-5) * \cancel{2}\,\text{rect}\left(\frac{t}{2}\right) * \delta(t+11)$

$= \text{rect}\left(\frac{t}{6}\right) * \text{rect}\left(\frac{t}{2}\right) * \delta(t+6)$

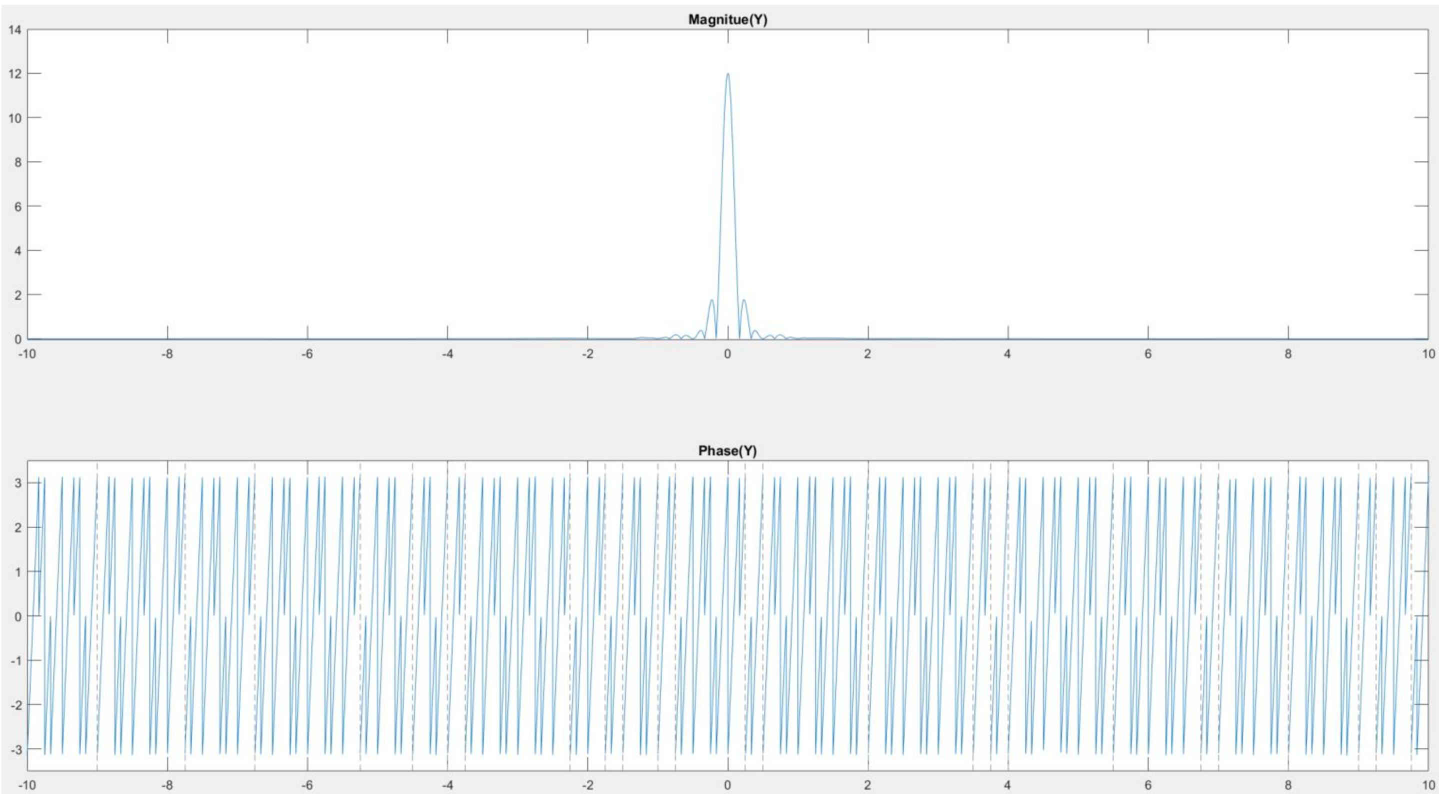
$$\therefore \boxed{\begin{aligned} Y(f) &= 6\,\text{Sinc}(6f) \cdot 2\,\text{Sinc}(2f) \cdot e^{j2\pi f(6)} \\ &= 12\,\text{Sinc}(6f)\,\text{Sinc}(2f)\,e^{j12\pi f} \end{aligned}}$$

---

c)

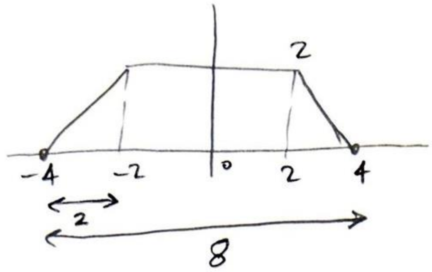
Matlab Code for Q7 (c):

```
syms f t
ex = exp(-j*2*pi*f*t);
X = int(0.5*ex,t,2,8);
H = int(2*ex,t,-10,-12);

Y = X*H;
Y = simplify(Y)

subplot(2,1,1); fplot(abs(Y)); title('Magnitude(Y)'); axis([-10 10 -0.05 14])
subplot(2,1,2); fplot(angle(Y)); title('Phase(Y)'); axis([-10 10 -3.5 3.5])
```



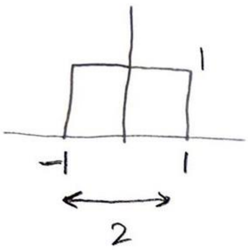
7) $\boxed{B}$

$y(t) =$

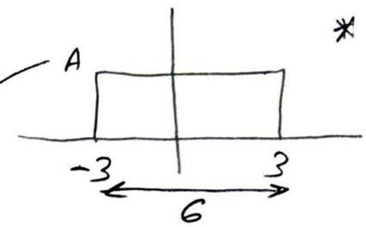


$* \delta(t+6)$

d) $h(t) = \qquad * \qquad x(t)$



$* \delta(t-1)$



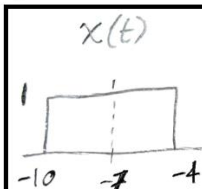
$* \delta(t-t_0)$

$\int_{-1}^{1} A(1)\,d\tau = 2$

$\underline{A=1}$

$1+t_0=-6$

$\underline{t_0=-7}$



$x(t) = 1\,\text{rect}\left(\frac{t}{6}\right) * \delta(t+7)$

$= 1\,\text{rect}\left(\frac{t+7}{6}\right)$

---

e) $y(t) = 1\,\text{rect}\left(\frac{t}{2}\right) * \delta(t-1) * 1\,\text{rect}\left(\frac{t}{6}\right) * \delta(t+7)$

$= \text{rect}\left(\frac{t}{2}\right) * \text{rect}\left(\frac{t}{6}\right) * \delta(t+6)$

$\therefore \boxed{Y(f) = 12\,\text{sinc}(2f)\cdot\text{sinc}(6f)\,e^{j12\pi f}}$

---

f)

<u>Matlab Code for Q7 (f):</u>

```
syms f t
ex = exp(-j*2*pi*f*t);
X = int(0.5*ex,t,2,8);
H = int(2*ex,t,-10,-12);

Y = X*H;
Y = simplify(Y)

subplot(2,1,1); fplot(abs(Y)); title('Magnitue(Y)'); axis([-10 10 -0.05 14])
subplot(2,1,2); fplot(angle(Y)); title('Phase(Y)'); axis([-10 10 -3.5 3.5])
```