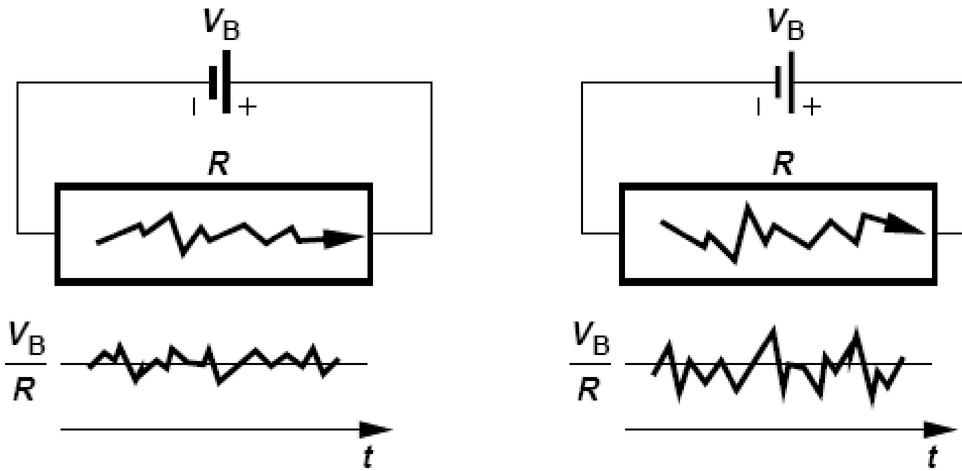
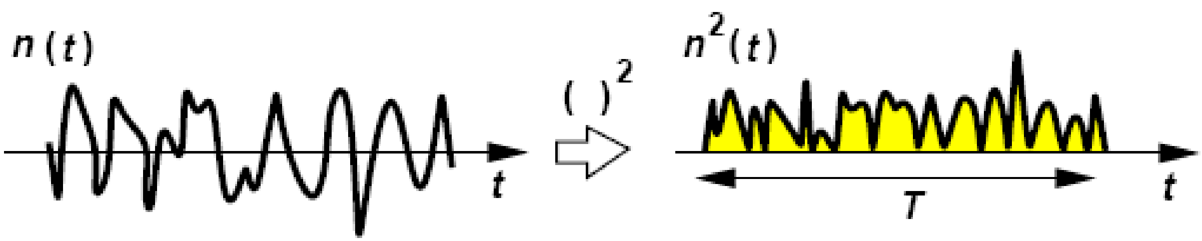
Noise: Noise as a Random Process



Higher temperature

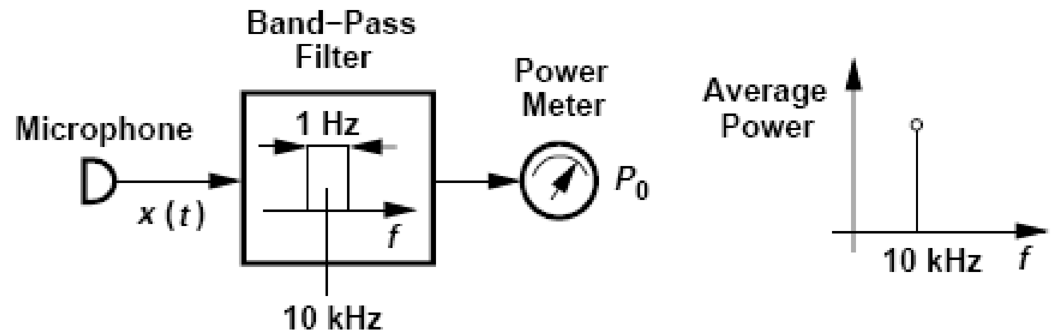
- The average current remains equal to V_B/R but the instantaneous current displays random values

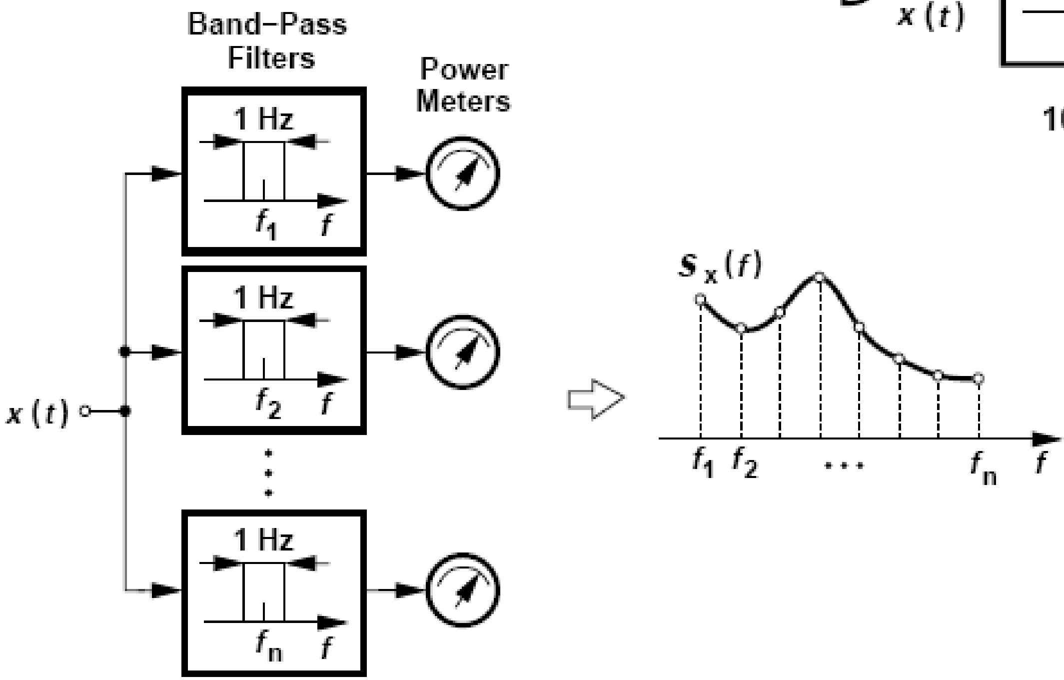


$$P_n = \lim_{T\to\infty} \frac{1}{T}\int_0^T n^2(t)dt$$

- *T* must be long enough to accommodate several cycles of the lowest frequency.

Measurement of Noise Spectrum

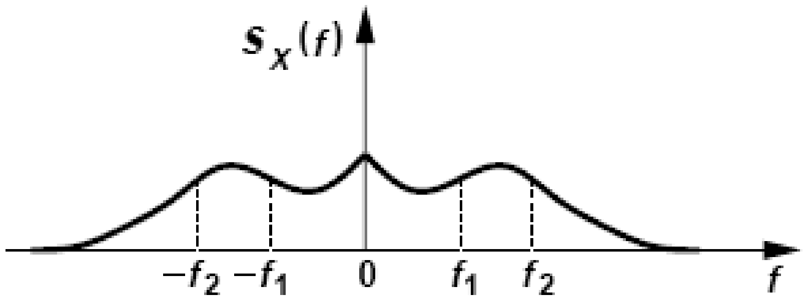




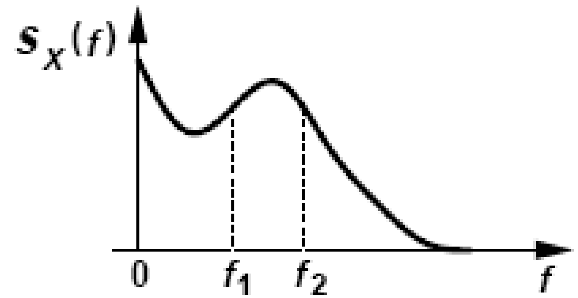
- To measure signal's frequency content at 10 kHz, we need to filter out the remainder of the spectrum and measure the average power of the 10-kHz component.

Noise Spectrum: Power Spectral Density (PSD)

Two-Sided



One-Sided



$$\int_0^\infty S_x(f)df = \lim_{T\to\infty} \frac{1}{T}\int_0^T x^2(t)dt$$

- **Total area under $S_x(f)$ represents the average power carried by $x(t)$**

Example of Noise Spectrum

A resistor of value R_1 generates a noise voltage whose one-sided PSD is given by

$$S_v(f) = 4kTR_1$$

where $k = 1.38 \times 10^{-23}$ J/K denotes the Boltzmann constant and T the absolute temperature. Such a flat PSD is called "white" because, like white light, it contains all frequencies with equal power levels.
(a) What is the total average power carried by the noise voltage?
(b) What is the dimension of $S_v(f)$?
(c) Calculate the noise voltage for a 50-Ω resistor in 1 Hz at room temperature.

(a) The area under $S_v(f)$ appears to be infinite, an implausible result because the resistor noise arises from the finite ambient heat. In reality, $S_v(f)$ begins to fall at f > 1 THz, exhibiting a finite total energy, i.e., thermal noise is not quite white.

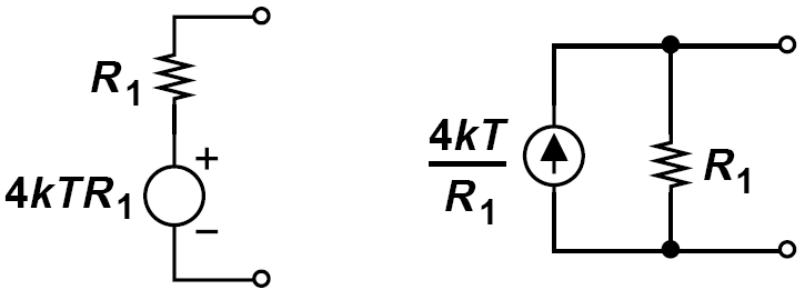
(b) The dimension of $S_v(f)$ is voltage squared per unit bandwidth (V^2/Hz)

(c) For a 50-Ω resistor at T = 300 K

$$\boxed{\overline{V_n^2} = 8.28 \times 10^{-19}\ \mathrm{V}^2/\mathrm{Hz} = 4kTR_1}$$

$$\sqrt{\overline{V_n^2}} = 0.91\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$$

Noise Floor (-174dBm/Hz)



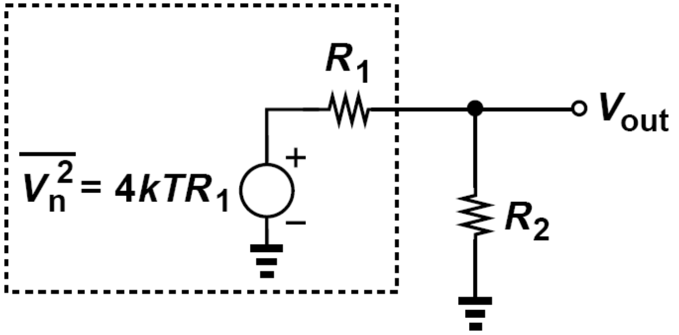
$$\overline{V_n^2} = 4kTR_1$$

- Noise can be modeled by a series voltage source or a parallel current source
- Polarity of the sources is unimportant but must be kept same throughout the calculations

- When a loading of R1 is added, the power delivered to the load R1 is kT.
- $kT = 1.38 \times 10^{-23} * 300 = 4.14 \times 10^{-21}$ Joule $= 4.14 \times 10^{-18}$ mW/Hz = -174dBm/Hz

Maximum Available Noise Power

Suppose R_2 is held at T = 0 K



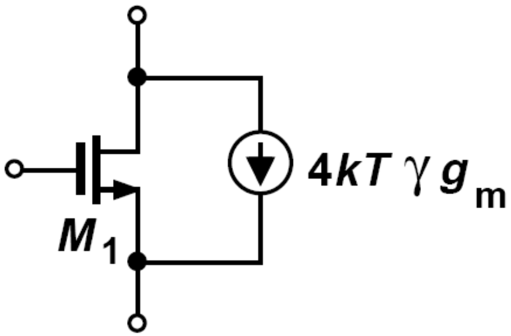
$$
\begin{aligned}
P_{R2} &= \frac{\overline{V_{out}^2}}{R_2} \\
&= \overline{V_n^2}\left(\frac{R_2}{R_1+R_2}\right)^2\frac{1}{R_2} \\
&= 4kT\frac{R_1R_2}{(R_1+R_2)^2}.
\end{aligned}
$$

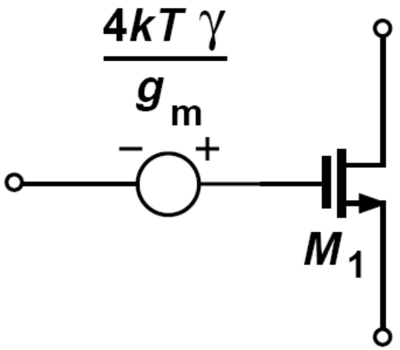
This quantity reaches a maximum if $R_2 = R_1$:

$$P_{R2,max} = kT$$

Available noise power

Noise in MOSFETS





$$\overline{I_n^2} = 4kT\gamma g_m$$

$$\overline{V_n^2} = 4kT\gamma / g_m$$

- **Thermal noise of MOS transistors operating in the saturation region is approximated by a current source tied between the source and drain terminals, or can be modeled by a voltage source in series with gate.**

Noise Figure

$$\mathrm{NF} = \frac{SNR_{in}}{SNR_{out}}$$

$$\mathrm{NF}|_{\mathrm{dB}} = 10 \log \frac{SNR_{in}}{SNR_{out}}.$$

- Depends on not only the noise of the circuit under consideration but the SNR provided by the preceding stage
- If the input signal contains no noise, NF=∞

Calculation of Noise Figure - I

$$SNR_{in} = \frac{|\alpha|^2 V_{in}^2}{|\alpha|^2 \overline{V_{RS}^2}}$$

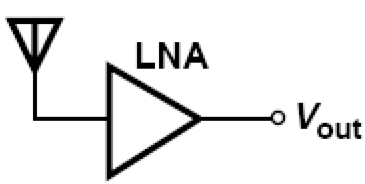
$$SNR_{out} = \frac{V_{in}^2 |\alpha|^2 A_v^2}{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}$$

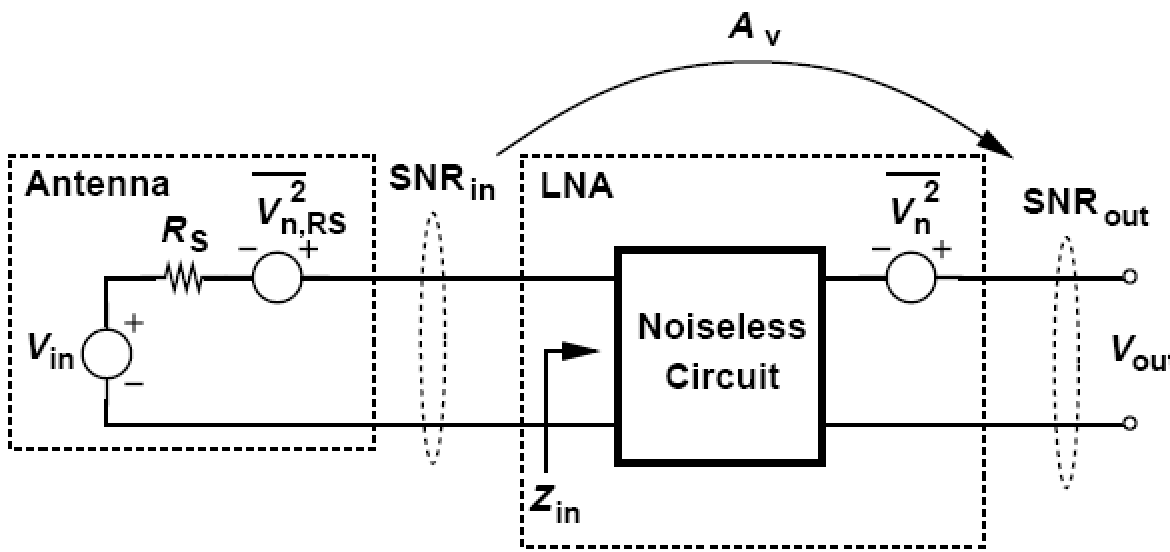
$$\alpha = Z_{in}/(Z_{in} + R_S)$$

$$\mathrm{NF} = \frac{1}{4kTR_S} \cdot \frac{\overline{V_{n,out}^2}}{A_0^2}$$

$$\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2} = \overline{V_{n,out}^2}$$

$$|\alpha|^2 A_v^2 = A_0^2$$





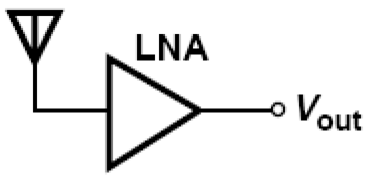
Calculation of Noise Figure - II

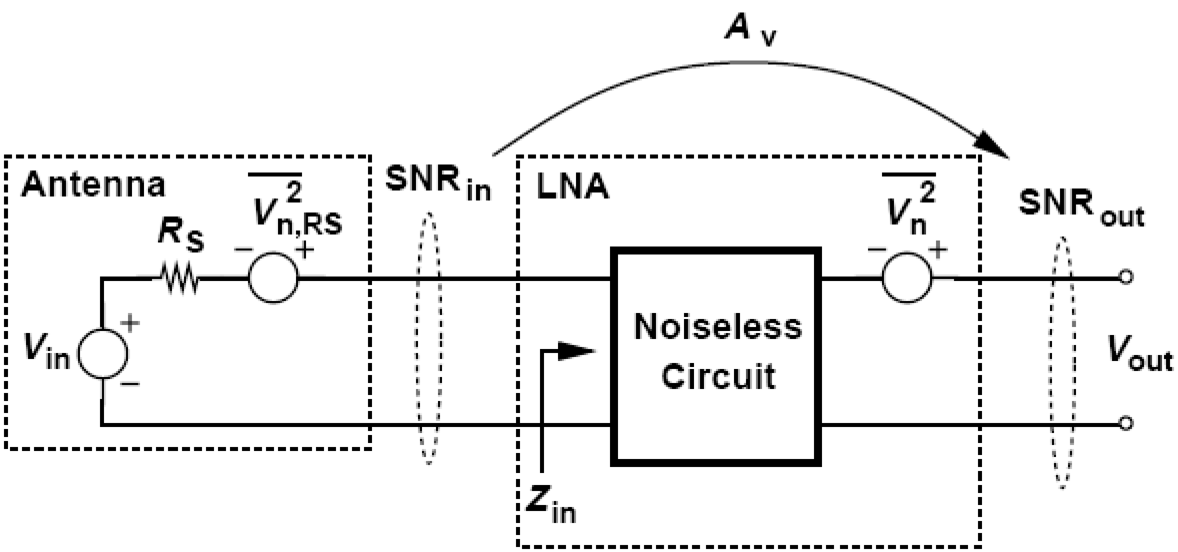
$$SNR_{in} = \frac{|\alpha|^2 V_{in}^2}{|\alpha|^2 \overline{V_{RS}^2}}$$

$$SNR_{out} = \frac{V_{in}^2 |\alpha|^2 A_v^2}{\overline{V_{RS}^2} |\alpha|^2 A_v^2 + \overline{V_n^2}}$$

$$\alpha = Z_{in}/(Z_{in} + R_S)$$

$$\begin{aligned} \mathrm{NF} &= \frac{V_{in}^2}{4kTR_S} \cdot \frac{\overline{V_{RS}^2} |\alpha|^2 A_v^2 + \overline{V_n^2}}{V_{in}^2 |\alpha|^2 A_v^2} \\ &= \frac{1}{\overline{V_{RS}^2}} \cdot \frac{\overline{V_{RS}^2} |\alpha|^2 A_v^2 + \overline{V_n^2}}{|\alpha|^2 A_v^2} \\ &= 1 + \frac{\overline{V_n^2}}{|\alpha|^2 A_v^2} \cdot \frac{1}{\overline{V_{RS}^2}}. \end{aligned}$$





Calculation of NF: Summary

Calculation of NF

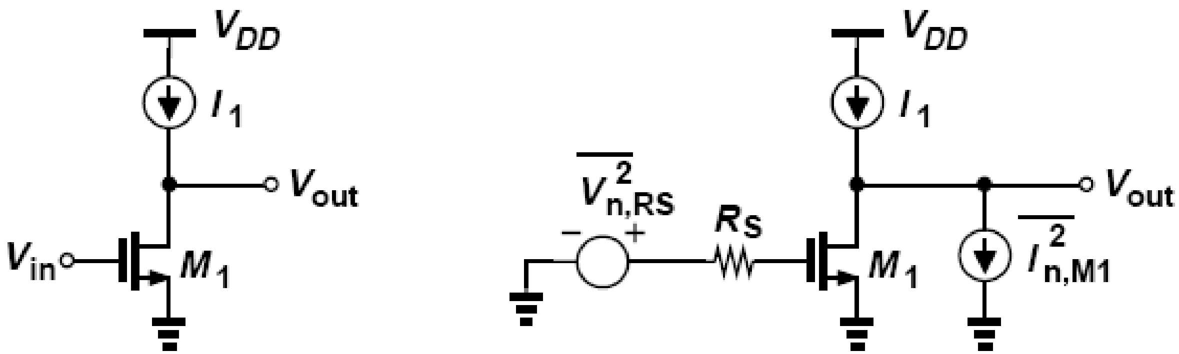
- **Divide total output noise by the gain from V_{in} to V_{out} and normalize the result to the noise of R_s**

- **Calculate the output noise due to the amplifier, divide it by the gain, normalize it to $4kTR_s$ and add 1 to the result**

Another Example of Noise Figure Calculation

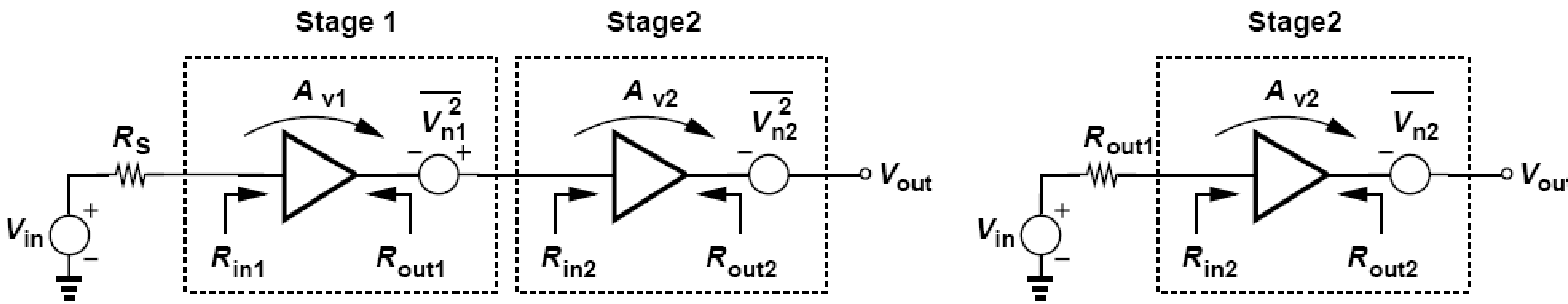
Determine the noise figure of the common-source stage shown in below (left) with respect to a source impedance R_S. Neglect the capacitances and flicker noise of M_1 and assume I_1 is ideal.

Solution:



$$\begin{aligned} \mathrm{NF} &= \frac{4kT\gamma g_m r_O^2 + 4kTR_S(g_m r_O)^2}{(g_m r_O)^2} \cdot \frac{1}{4kTR_S} \\ &= \frac{\gamma}{g_m R_S} + 1. \end{aligned}$$

Noise Figure of Cascaded Stages (Ⅰ)

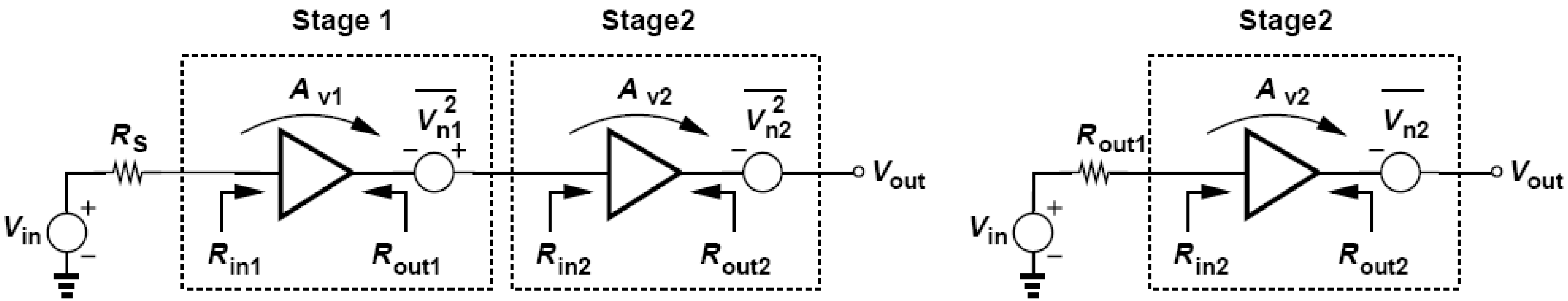


$$A_0 = \frac{V_{out}}{V_{in}} = \frac{R_{in1}}{R_{in1}+R_S} A_{v1} \frac{R_{in2}}{R_{in2}+R_{out1}} A_{v2}$$

$$\overline{V_{n,out}^2} = \overline{V_{n2}^2} + \overline{V_{n1}^2} \frac{R_{in2}^2}{(R_{in2}+R_{out1})^2} A_{v2}^2$$

$$\begin{aligned}
\mathrm{NF_{tot}} &= 1 + \frac{\overline{V_{n,out}^2}}{A_0^2} \cdot \frac{1}{4kTR_S} \\
&= 1 + \frac{\overline{V_{n1}^2}}{\left(\frac{R_{in1}}{R_{in1}+R_S}\right)^2 A_{v1}^2} \cdot \frac{1}{4kTR_S} \\
&+ \frac{\overline{V_{n2}^2}}{\left(\frac{R_{in1}}{R_{in1}+R_S}\right)^2 A_{v1}^2 \left(\frac{R_{in2}}{R_{in2}+R_{out1}}\right)^2 A_{v2}^2} \cdot \frac{1}{4kTR_S}
\end{aligned}$$

Noise Figure of Cascaded Stages (Ⅱ)



$$NF_2 = 1 + \frac{\overline{V_{n2}^2}}{\frac{R_{in2}^2}{(R_{in2}+R_{out1})^2}A_{v2}^2}\frac{1}{4kTR_{out1}} \qquad NF_{tot} = NF_1 + \frac{NF_2 - 1}{\frac{R_{in1}^2}{(R_{in1}+R_S)^2}A_{v1}^2\frac{R_S}{R_{out1}}}$$

$$\boxed{NF_{tot} = NF_1 + \frac{NF_2 - 1}{A_{P1}}}$$

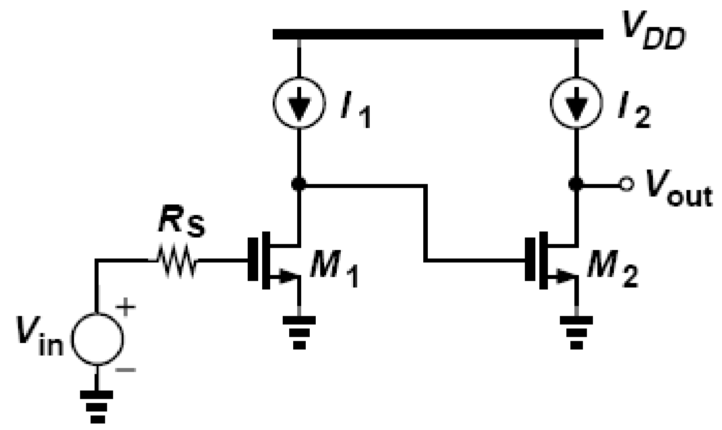
$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{P1}} + \cdots + \frac{NF_m - 1}{A_{P1}\cdots A_{P(m-1)}}.$$

This result suggests that the noise contributed by each stage decreases as the total gain preceding that stage increases, implying that the first few stages in a cascade are the most critical.

Example of Noise Figure of Cascaded Stages

Determine the *NF* of the cascade of common-source stages shown in figure below. Neglect the transistor capacitances and flicker noise.

Solution:



$$R_{in1} = R_{in2} = \infty$$

$$NF = 1 + \frac{\overline{V_{n1}^2}}{A_{v1}^2}\frac{1}{4kTR_S} + \frac{\overline{V_{n2}^2}}{A_{v1}^2 A_{v2}^2}\frac{1}{4kTR_S}$$

where

$$\overline{V_{n1}^2} = 4kT\gamma g_{m1} r_{O1}^2,\ \overline{V_{n2}^2} = 4kT\gamma g_{m2} r_{O2}^2,\ A_{v1} = g_{m1} r_{O1},\ \text{and}\ A_{v2} = g_{m2} r_{O2}.$$

$$NF = 1 + \frac{\gamma}{g_{m1}R_S} + \frac{\gamma}{g_{m1}^2 r_{O1}^2 g_{m2} R_S}$$

Sensitivity and Dynamic Range: Sensitivity

- **The sensitivity is defined as the minimum signal level that a receiver can detect with "acceptable quality."**

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{sig}/P_{RS}}{SNR_{out}}$$

$$P_{sig} = P_{RS} \cdot NF \cdot SNR_{out}$$

$$P_{sig,tot} = P_{RS} \cdot NF \cdot SNR_{out} \cdot B$$

$$P_{sen}|_{dBm} = P_{RS}|_{dBm/Hz} + NF|_{dB} + SNR_{min}|_{dB} + 10\log B$$

$$P_{sen} = \underbrace{-174\ \mathrm{dBm/Hz} + NF + 10\log B}_{\text{Noise Floor}} + SNR_{min}$$

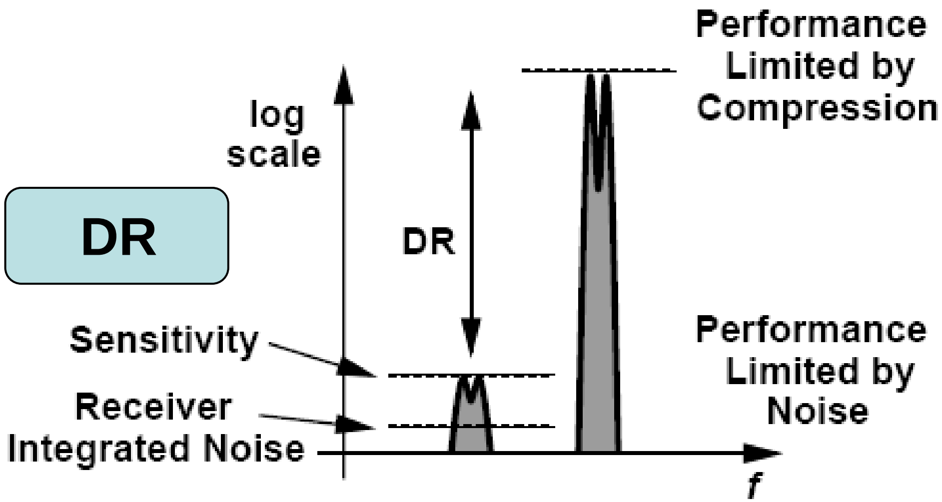
Example of Sensitivity

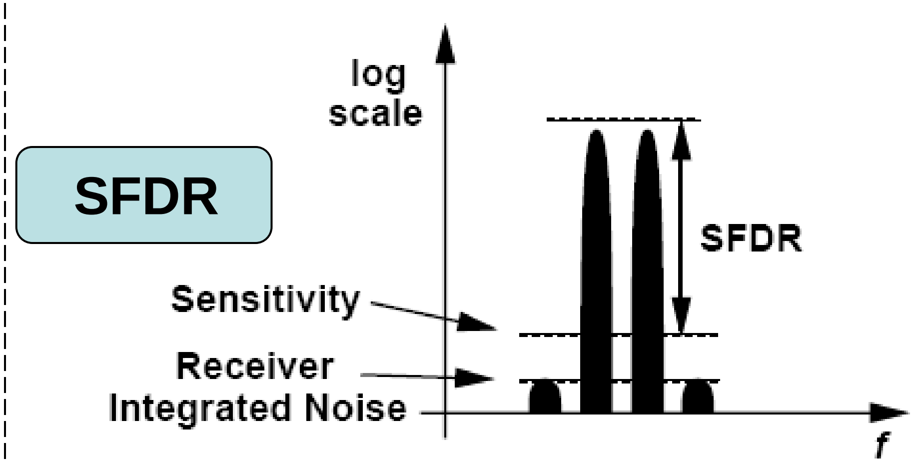
A GSM receiver requires a minimum *SNR* of 12 dB and has a channel bandwidth of 200 kHz. A wireless LAN receiver, on the other hand, specifies a minimum *SNR* of 23 dB and has a channel bandwidth of 20 MHz. Compare the sensitivities of these two systems if both have an *NF* of 7 dB.

Solution:

For the GSM receiver, P_{sen} = -102 dBm, whereas for the wireless LAN system, P_{sen} = -71 dBm. Does this mean that the latter is inferior? No, the latter employs a much wider bandwidth and a more efficient modulation to accommodate a data rate of 54 Mb/s. The GSM system handles a data rate of only 270 kb/s. In other words, specifying the sensitivity of a receiver without the data rate is not meaningful.

Dynamic Range Compared with SFDR





- **Dynamic Range:**

  Maximum tolerable desired signal power divided by the minimum tolerable desired signal power

- **SFDR:**

  Lower end equal to sensitivity.

  Higher end defined as maximum input level in a *two-tone* test for which the third-order IM products do not exceed the integrated noise of the receiver

SFDR Calculation

Refer output IM magnitudes to input:

$$P_{IIP3} = P_{in} + \frac{P_{out} - P_{IM,out}}{2}$$

$$P_{IM,in} = P_{IM,out} - G. \qquad P_{in} = P_{out} - G.$$

$$\begin{aligned} P_{IIP3} &= P_{in} + \frac{P_{in} - P_{IM,in}}{2} \\ &= \frac{3P_{in} - P_{IM,in}}{2}, \end{aligned}$$

$$P_{in} = \frac{2P_{IIP3} + P_{IM,in}}{3}.$$

$$P_{in,max} = \frac{2P_{IIP3} + (-174\ \text{dBm} + NF + 10\log B)}{3}.$$

$$\begin{aligned} SFDR &= P_{in,max} - (-174\ \text{dBm} + NF + 10\log B + SNR_{min}) \\ &= \frac{2(P_{IIP3} + 174\ \text{dBm} - NF - 10\log B)}{3} - SNR_{min}. \end{aligned}$$

Example Comparing SFDR and DR

The upper end of the dynamic range is limited by intermodulation in the presence of two interferers or desensitization in the presence of one interferer. Compare these two cases and determine which one is more restrictive.

Solution:

$$P_{1-dB} \overset{?}{\gtrless} P_{in,max}$$

Since $P_{1-dB} = P_{IIP3} - 9.6\ \text{dB}$

$$P_{IIP3} - 9.6\ \text{dB} \overset{?}{\gtrless} \frac{2P_{IIP3} + (-174\ \text{dBm} + NF + 10\log B)}{3}$$

$$P_{IIP3} - 28.8\ \text{dB} \overset{?}{\gtrless} -174\ \text{dBm} + NF + 10\log B$$

Noise floor

$$P_{1-dB} > P_{in,max}$$

- **SFDR is a more stringent characteristic of system than DR**