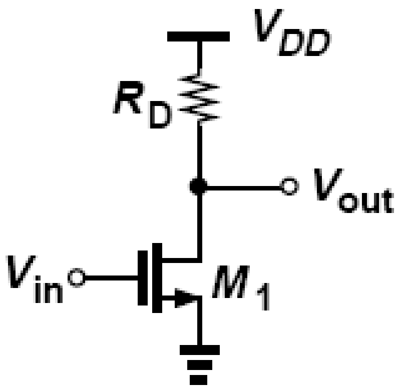
# Example of Polynomial Approximation




$$
\begin{aligned}
V_{out} &= V_{DD} - I_D R_D \\
&= V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2 R_D
\end{aligned}
$$

# Example of Polynomial Approximation

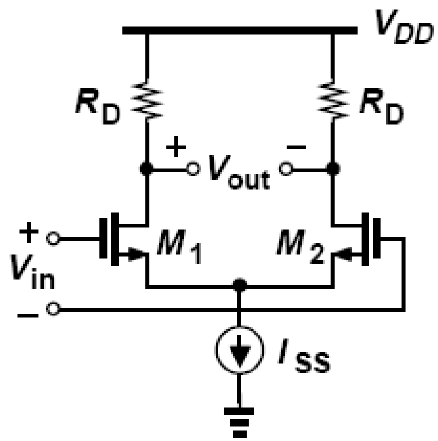
**For square-law MOS transistors operating in saturation, the characteristic of the circuit can be expressed as**

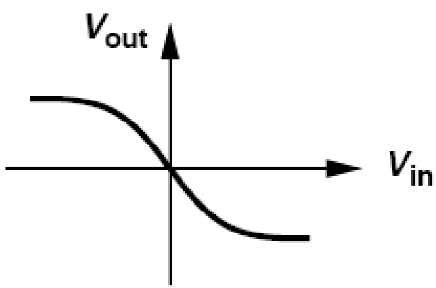
$$V_{out} = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{in}^2}\,R_D$$

**If the differential input is small, approximate the characteristic by a polynomial.**

**Factoring $4I_{ss} / (\mu_n C_{ox} W/L)$ out of the square root and assuming**

$$V_{in}^2 \ll \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$




**Approximation gives us:**

$$V_{out} \approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}V_{in}}\left(1 - \frac{\mu_n C_{ox}\frac{W}{L}}{8I_{SS}}V_{in}^2\right)R_D$$

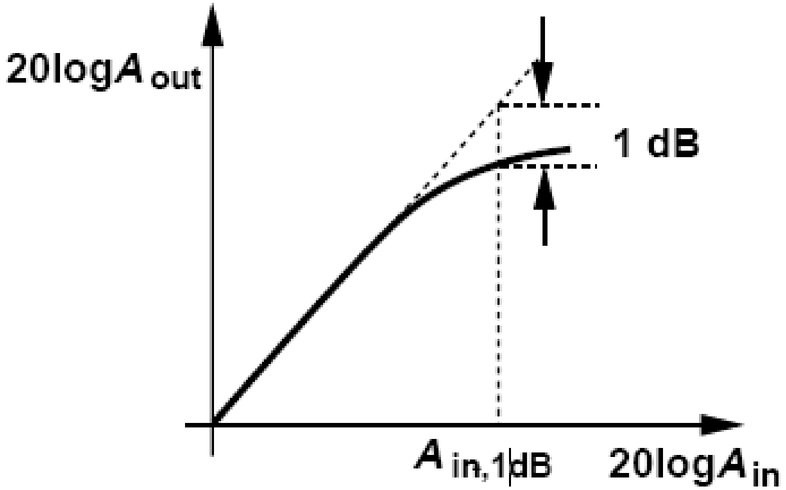
$$\approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}R_D V_{in}} + \frac{\left(\mu_n C_{ox}\frac{W}{L}\right)^{3/2}}{8\sqrt{I_{SS}}}R_D V_{in}^3$$

# Effects of Nonlinearity: Harmonic Distortion

$$
\begin{aligned}
y(t) &= \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t \\
&= \alpha_1 A \cos \omega t + \frac{\alpha_2 A^2}{2}(1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3 \cos \omega t + \cos 3\omega t) \\
&= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right) \cos \omega t + \frac{\alpha_2 A^2}{2} \cos 2\omega t + \frac{\alpha_3 A^3}{4} \cos 3\omega t.
\end{aligned}
$$

**DC** | **Fundamental** | **Second Harmonic** | **Third Harmonic**

# Gain Compression: 1-dB Compression Point




$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{in,1dB}^2\right| = 20\log|\alpha_1| - 1\text{ dB}.$$

$$\boxed{A_{in,1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}}.$$

- **Output falls below its ideal value by 1 dB at the 1-dB compression point**

# Gain Compression: 1-dB Compression Point

$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{in(1dB)}^2\right| = 20\lg|\alpha_1| - 1\,dB$$

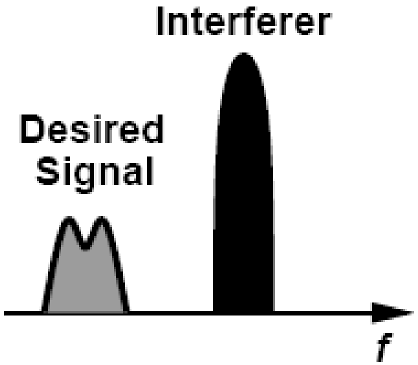
$$\lg\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{in(1dB)}^2\right| = \lg|\alpha_1| - 0.05$$

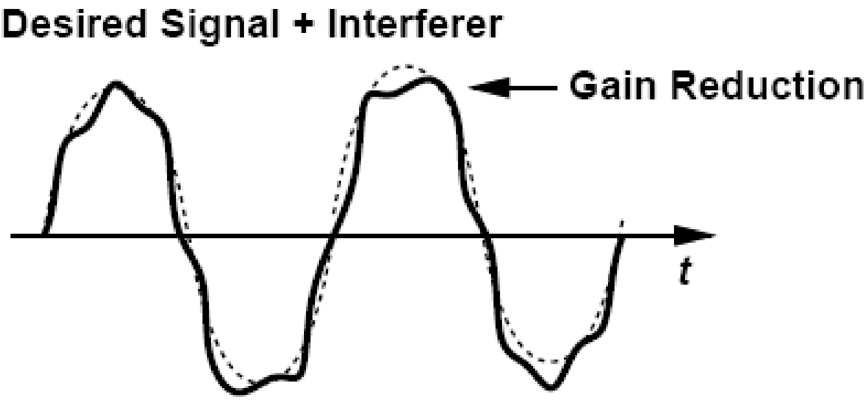
$$\alpha_1 + \frac{3}{4}\alpha_3 A_{in}^2(1dB) = \alpha_1 / 10^{0.05} = \frac{\alpha_1}{1.122}$$

$$\frac{3}{4}\alpha_3 A_{in(1dB)}^2 = -0.1087\alpha_1$$

$$A_{in(1dB)}^2 = -0.145\frac{\alpha_1}{\alpha_3} = 0.145\left|\frac{\alpha_1}{\alpha_3}\right| \quad \text{since } \alpha_1\alpha_3 < 0$$

# Gain Compression: Desensitization




$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

$$y(t) = \left(\alpha_1 + \frac{3}{4}\alpha_3 A_1^2 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos \omega_1 t + \cdots .$$

For $A_1 << A_2$

$$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos \omega_1 t + \cdots$$

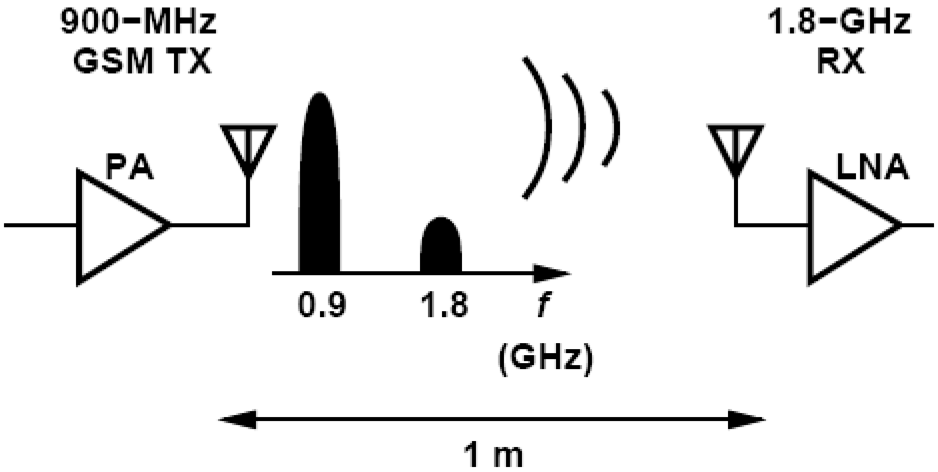
- **Desensitization: the receiver gain is reduced by the large excursions produced by the interferer even though the desired signal itself is small.**
- **When A2 is big enough, the overall gain may drop to zero, or the signal is "blocked"**

# Example I of Gain Compression

**A 900-MHz GSM transmitter delivers a power of 1 W to the antenna. By how much must the second harmonic of the signal be suppressed (filtered) so that it will be below $P_{1dB}$ of a 1.8-GHz receiver having $P_{1dB}$ = -25 dBm? Assume the receiver is 1 meter away and both 900-MHz and 1.8-GHz signals are attenuated by 10 dB as it propagates across this distance.**

## *Solution:*

**The output power at 900 MHz is equal to +30 dBm. With an attenuation of 10 dB, the second harmonic must not exceed -15 dBm at the transmitter antenna so that it is below $P_{1dB}$ of the receiver. Thus, the second harmonic must remain at least 45 dB below the fundamental at the TX output. In practice, this interference must be another several dB lower to ensure the RX does not compress.**



# Example II of Gain Compression

**Bluetooth specification requires that the maximum acceptable RX input power should be at least -10dBm. Assuming that this maximum acceptable RX input power is 3dB below the input power where "zero" gain happens, determine the P1dB of the receiver RX.**

***Solution:***

**Step 1: recall the P1dB equation in terms of $\alpha 1$ and $\alpha 3$**

**Step 2: derive power level at which the gain is zero, in terms of $\alpha 1$ and $\alpha 3$**

**Step 3: compare P1dB and zero-gain power level**

**Step 4: relate P1dB to -10dBm**

# Example II of Gain Compression

**Bluetooth specification requires that the maximum acceptable RX input power should be at least -10dBm. Assuming that this maximum acceptable RX input power is 3dB below the input power where "zero" gain happens, determine the P1dB of the receiver RX.**

***Solution:***

Step 1: $y(x) = (\alpha_1 A + \frac{3}{4}\alpha_3 A^3)\cos\omega t$

$$A_{1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

Step 2: $\alpha_1 A + \frac{3}{4}\alpha_3 A^3 = 0 \Rightarrow A_{Gain=0} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$

Step 3: $\frac{P_{Gain=0}}{P_{1dB}} = 20\lg\left(\frac{A_{Gain=0}}{A_{1dB}}\right) = 9.6dB$

Step 4: $P_{1dB} + 9.6dB = P_{Gain=0} = P_{in,max} + 3dB$

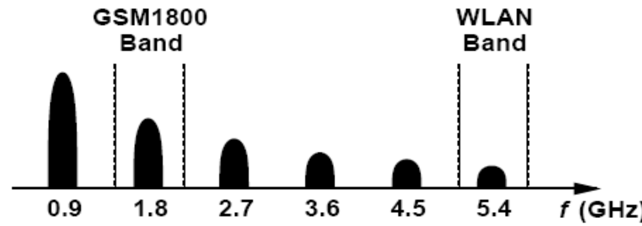
$$\Rightarrow P_{1dB} + 6.6dB = P_{in,max} = -10dBm$$

$$\Rightarrow P_{1dB} = -16.6dBm \ \#$$

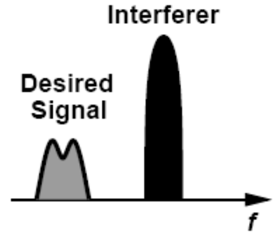
# Effects of Nonlinearity: Intermodulation— Recall Previous Discussion

**So far we have considered the case of:**

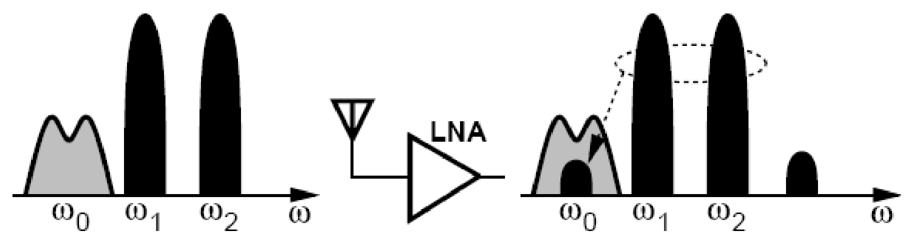
- **Single Signal** → **Harmonic distortion**




- **Signal + one large interferer** → **Desensitization**




- **Signal + two large interferers** → **Intermodulation**



# Effects of Nonlinearity: Intermodulation

**assume** $$x(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t$$

**Thus**

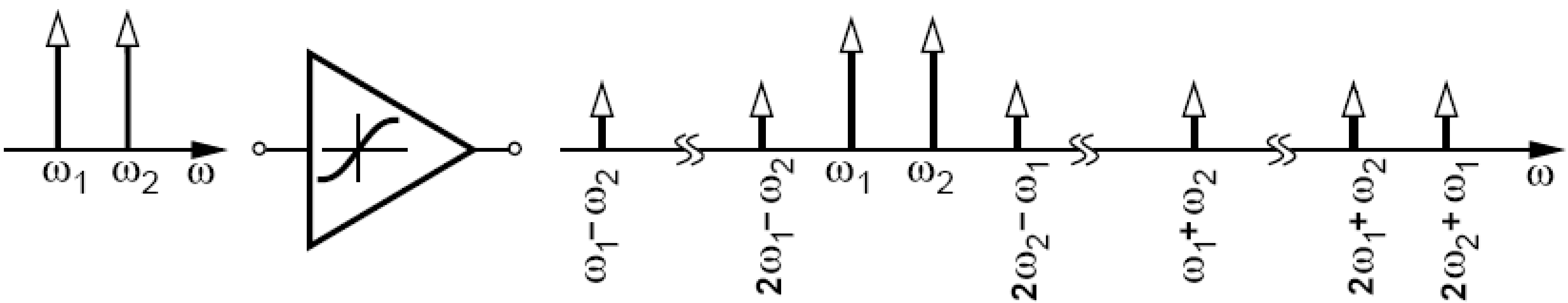
$$y(t) = \alpha_1(A_1\cos\omega_1 t + A_2\cos\omega_2 t) + \alpha_2(A_1\cos\omega_1 t + A_2\cos\omega_2 t)^2 + \alpha_3(A_1\cos\omega_1 t + A_2\cos\omega_2 t)^3$$

**Intermodulation products:**

$$\begin{aligned}
\omega &= 2\omega_1 \pm \omega_2 : \; \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1+\omega_2)t + \boxed{\frac{3\alpha_3 A_1^2 A_2}{4}}\cos(2\omega_1-\omega_2)t \\
\omega &= 2\omega_2 \pm \omega_1 : \; \frac{3\alpha_3 A_1 A_2^2}{4}\cos(2\omega_2+\omega_1)t + \boxed{\frac{3\alpha_3 A_1 A_2^2}{4}}\cos(2\omega_2-\omega_1)t
\end{aligned}$$

**Fundamental components:**

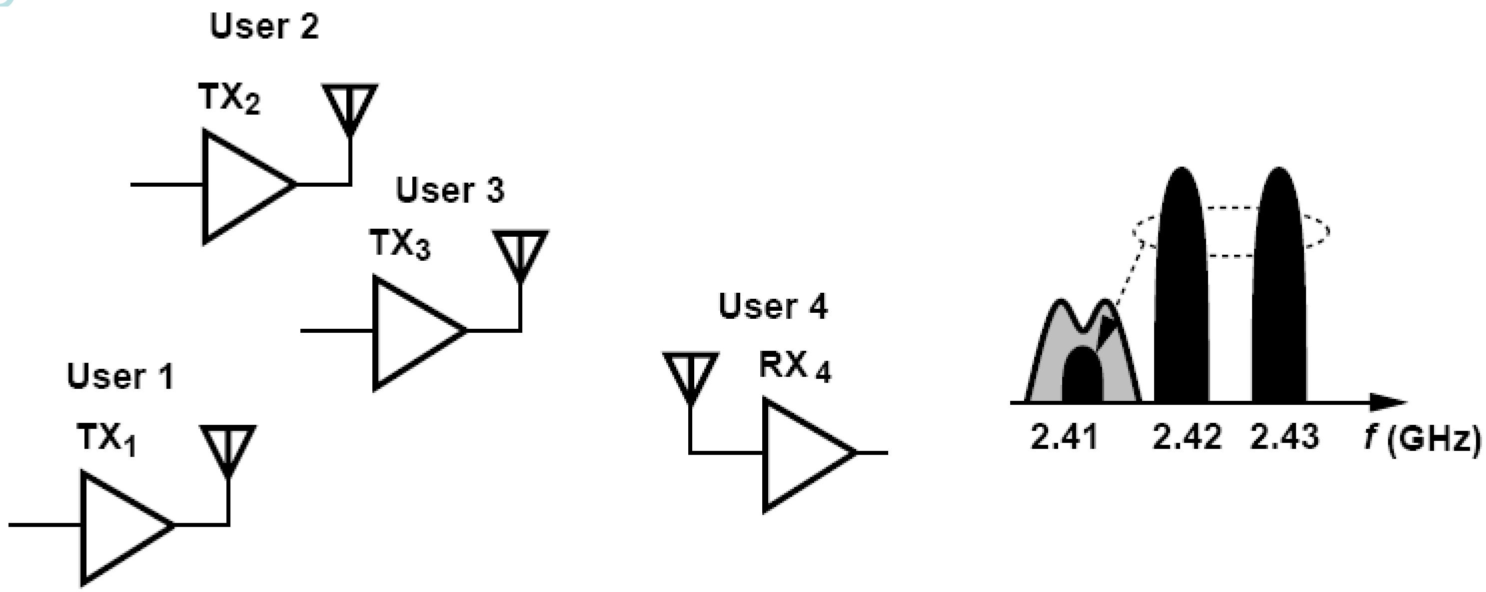
$$\omega = \omega_1,\ \omega_2 : \; \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos\omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right)\cos\omega_2 t$$



# Example of Intermodulation

**Suppose four Bluetooth users operate in a room as shown in figure below. User 4 is in the receive mode and attempts to sense a weak signal transmitted by User 1 at 2.410 GHz. At the same time, Users 2 and 3 transmit at 2.420 GHz and 2.430 GHz, respectively. Explain what happens.**

***Solution:***




**Since the frequencies transmitted by Users 1, 2, and 3 happen to be equally spaced, the intermodulation in the LNA of $R_{X4}$ corrupts the desired signal at 2.410 GHz.**

# Example of Gain Compression and Intermodulation

**A Bluetooth receiver employs a low-noise amplifier having a gain of 10 (or 20dB gain) and an input impedance of 50 Ω. The input of LNA senses a desired signal level of -80 dBm at 2.410 GHz and two interferers of equal levels at 2.420 GHz and 2.430 GHz . For simplicity, assume the LNA drives a 50-Ω load.**

**(a) Determine the value of $\alpha_3$ that yields a $P_{1dB}$ of -30 dBm at LNA input.**

**(b) If each interferer is 10 dB below the input $P_{1dB}$, determine the corruption experienced by the desired signal at the LNA output.**

***Solution:***

**(a) From previous equation,** $A_{in,1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$. **$\alpha_3$ = 14.500 $V^{-2}$**

**(b) Each interferer has a level of -40 dBm (= 6.32 m $V_{pp}$), we determine the amplitude of the IM product at 2.410 GHz (at LNA output) as:**

$$\frac{3\alpha_3 A_1^2 A_2}{4} = 0.343\,\mathrm{mV_p} = -59.3\ \mathrm{dBm}.$$

# Example of Gain Compression and Intermodulation

A Bluetooth receiver employs a low-noise amplifier having a gain of 10 (or 20dB gain) and an input impedance of 50 Ω. The input of LNA senses a desired signal level of -80 dBm at 2.410 GHz and two interferers of equal levels at 2.420 GHz and 2.430 GHz . For simplicity, assume the LNA drives a 50-Ω load.

(a) Determine the value of $\alpha_3$ that yields a $P_{1dB}$ of -30 dBm at LNA input.

(b) If each interferer is 10 dB below the input $P_{1dB}$, determine the corruption experienced by the desired signal at the LNA output.

(c) Can the receiver receive -80dBm signal when two -40dBm interferer signals exist at LNA inputs?

(d) How low the interferer signals should be in order to have the desired signal be 10 dB stronger than the unwanted intermodulation signal, at LNA output?

***Solution:***

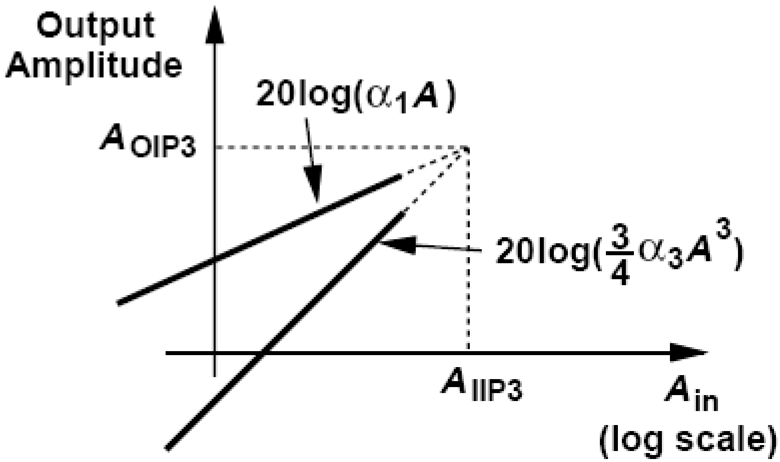
(c) No. The desired signal of -80dBm will become -60dBm at LNA output, after 20dB gain. This -60dBm signal is at the similar level of the unwanted IM3 signal which is -59.3dBm.

(d) Use the following equation in (b), but try to reduce A1 and A2 so that the unwanted IM product at LNA output is -70dBm (which is 10dB lower than -60dBm of the desired signal at LNA output).

$$\frac{3\alpha_3 A_1^2 A_2}{4} = 0.343\ \mathrm{mV_p} = -59.3\ \mathrm{dBm}.$$

(Answer to d) Since we want to reduce the IM3 at LNA output from -59.3dBm to -70dBm, the A1 and A2 needs to be reduced by (-70dBm – (-59.3dBm))/3 = 3.6dB. So the interferer signals at LNA input should be at -43.6dBm.

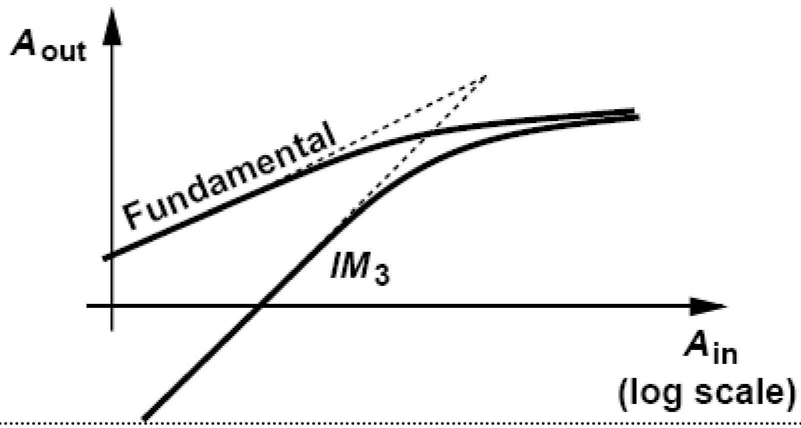
# Intermodulation: Third Intercept Point

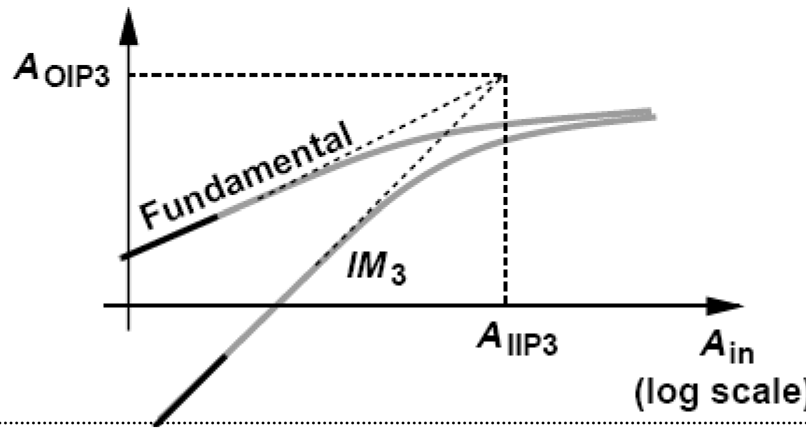



$$|\alpha_1 A_{IIP3}| = \left|\frac{3}{4}\alpha_3 A_{IIP3}^3\right|$$

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

$$\begin{aligned}\frac{A_{IIP3}}{A_{1dB}} &= \sqrt{\frac{4}{0.435}} \\ &\approx 9.6 \text{ dB.}\end{aligned}$$




- IP3 is not a directly measureable quantity, but a point obtained by extrapolation
- IP 3 needs to be calculated in linear region

# Example of Third Intercept Point

**A low-noise amplifier senses a -80-dBm signal at 2.410 GHz and two -20-dBm interferers at 2.420 GHz and 2.430 GHz. What $IIP_3$ is required if the IM products must remain 20 dB below the signal? For simplicity, assume 50-Ω interfaces at the input and output.**

***Solution:***

**Let Asig be desired signal amplitude, and Aint be interferer amplitude**

**At the LNA output:**

$$20\log|\alpha_1 A_{sig}| - 20\ \mathrm{dB} = 20\log\left|\frac{3}{4}\alpha_3 A_{int}^3\right|$$

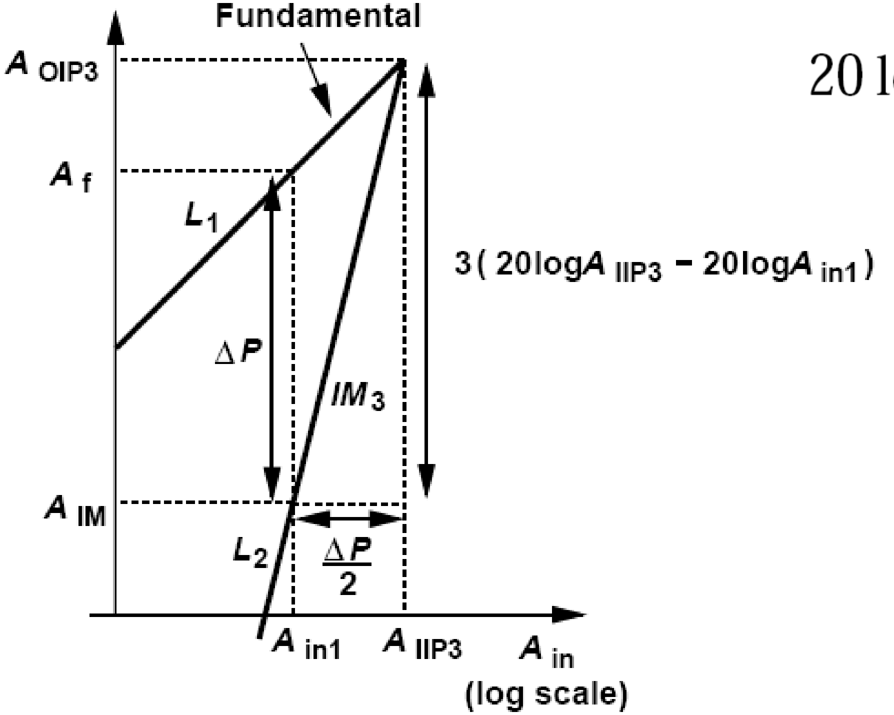
$$|\alpha_1 A_{sig}| = \left|\frac{30}{4}\alpha_3 A_{int}^3\right|$$

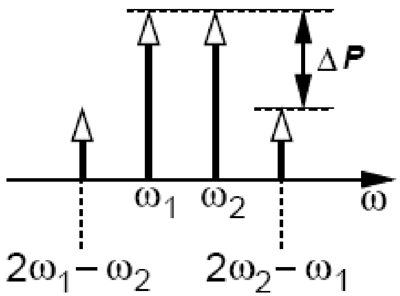
**From $\alpha 1/\alpha 3$, one can obtain** $\mathrm{IIP}_3 = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$ **= 3.16Vp = +20dBm**

# Third Intercept Point: A reasonable estimate

$$\Delta P = 20\log A_f - 20\log A_{IM} = 2(20\log A_{IIP3} - 20\log A_{in1}),$$

$$20\log A_{IIP3} = \frac{\Delta P}{2} + 20\log A_{in1}$$



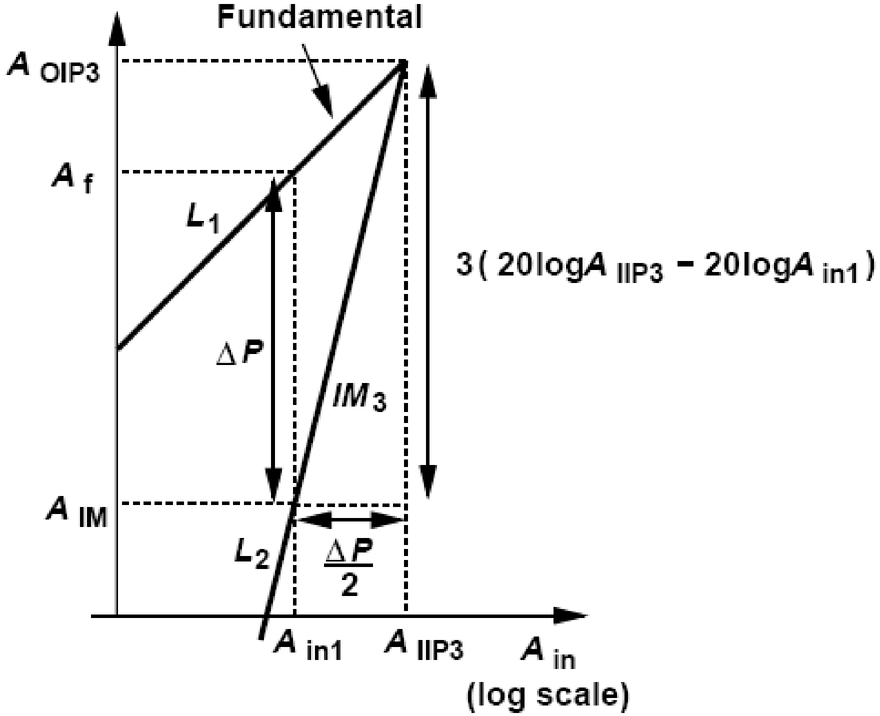

$$IIP_3\big|_{dBm} = \frac{\Delta P\big|_{dB}}{2} + P_{in}\big|_{dBm}$$

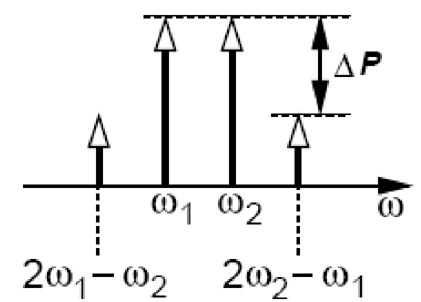
- **For a given input level (well below $P_{1dB}$), the $IIP_3$ can be calculated by halving the difference between the output fundamental and IM levels and adding the result to the input level, where all values are expressed as logarithmic quantities.**

# Example of Third Intercept Point

**A low-noise amplifier senses a -80-dBm signal at 2.410 GHz and two -20-dBm interferers at 2.420 GHz and 2.430 GHz. What $IIP_3$ is required if the IM products must remain 20 dB below the signal? For simplicity, assume 50-Ω interfaces at the input and output.**

***Solution:***




$$IIP_3\big|_{dBm} = \frac{\Delta P\big|_{dB}}{2} + P_{in}\big|_{dBm}$$

# Recall the Example of Polynomial Approximation

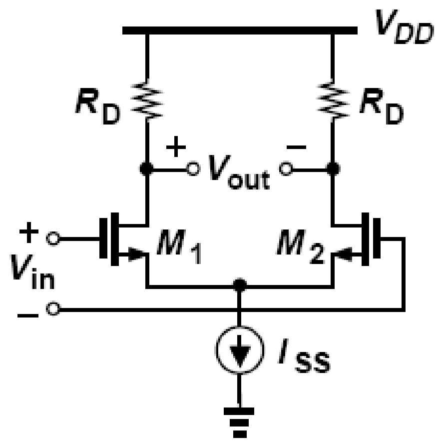
**For square-law MOS transistors operating in saturation, the characteristic above can be expressed as**

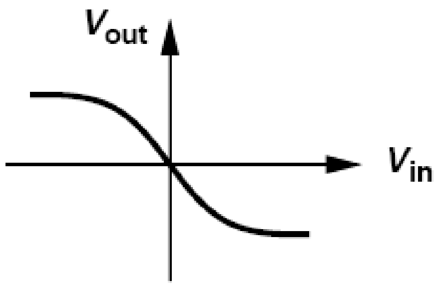
$$V_{out} = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{in}^2}\,R_D$$

**If the differential input is small, approximate the characteristic by a polynomial.**

**Factoring $4I_{ss} / (\mu_n C_{ox} W/L)$ out of the square root and assuming**

$$V_{in}^2 \ll \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$



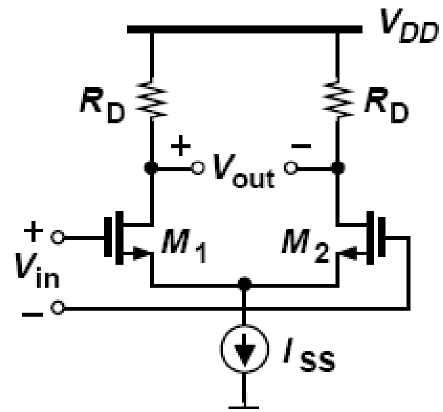



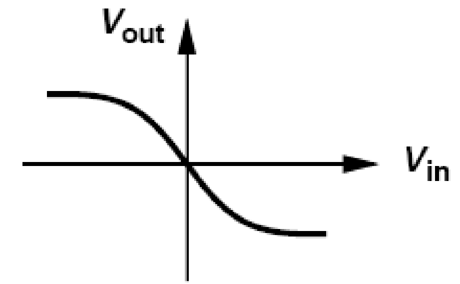

**Approximation gives us:**

$$V_{out} \approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}V_{in}\left(1 - \frac{\mu_n C_{ox}\frac{W}{L}}{8I_{SS}}V_{in}^2\right)R_D$$

$$\approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}R_D V_{in} + \frac{\left(\mu_n C_{ox}\frac{W}{L}\right)^{3/2}}{8\sqrt{I_{SS}}}R_D V_{in}^3$$

# Recall the Example of Polynomial Approximation



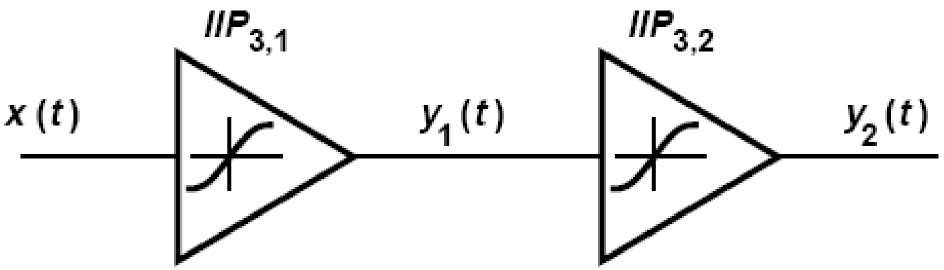

$$V_{out} \approx -\sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}} V_{in}\left(1 - \frac{\mu_n C_{ox}\frac{W}{L}}{8 I_{SS}} V_{in}^2\right) R_D$$

$$\approx -\sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}} R_D V_{in} + \frac{\left(\mu_n C_{ox}\frac{W}{L}\right)^{3/2}}{8\sqrt{I_{SS}}} R_D V_{in}^3$$

**What is the $A_{IIP3}$ at Vin?**

**Show that *IIP3* is proportional to $I_{SS}$.**

# Cascaded Nonlinear Stages: Generalized Result




**For two stages:**

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$

**For more stages:**

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IP3,3}^2} + \cdots$$

# Example of Cascaded Nonlinear Stages

**A low-noise amplifier having an input $IP_3$ of -10 dBm and a gain of 20 dB is followed by a mixer with an input $IP_3$ of +4 dBm. Which stage limits the $IP_3$ of the cascade more? What is the total input IP3 ?**

***Solution:***

**With $\alpha_1$ = 20 dB, we note that**

$$A_{IP3,1} = -10\text{ dBm}$$
$$\frac{A_{IP3,2}}{\alpha_1} = -16\text{ dBm}$$

**Since the scaled $IP_3$ of the second stage is lower than the $IP_3$ of the first stage, we say the second stage limits the overall $IP_3$ more.**

**What is the total input IP3?**

# Example of Cascaded Nonlinear Stages

**A low-noise amplifier having an input $IP_3$ of -10 dBm and a gain of 20 dB is followed by a mixer with an input $IP_3$ of +4 dBm. Which stage limits the $IP_3$ of the cascade more? What is the total input IP3 ?**

***Solution:***

$$A_{IP3,1} = -10 \text{ dBm}$$

$$\frac{A_{IP3,2}}{\alpha_1} = -16 \text{ dBm}$$

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}$$

$$1/IP_3 = 1/IP_{3,1} + 1/(IP_{3,2} / \alpha_1^2)$$

$$1/(-16.9\text{dBm}) = 1/(-10\text{dBm}) + 1/(-16\text{dBm})$$