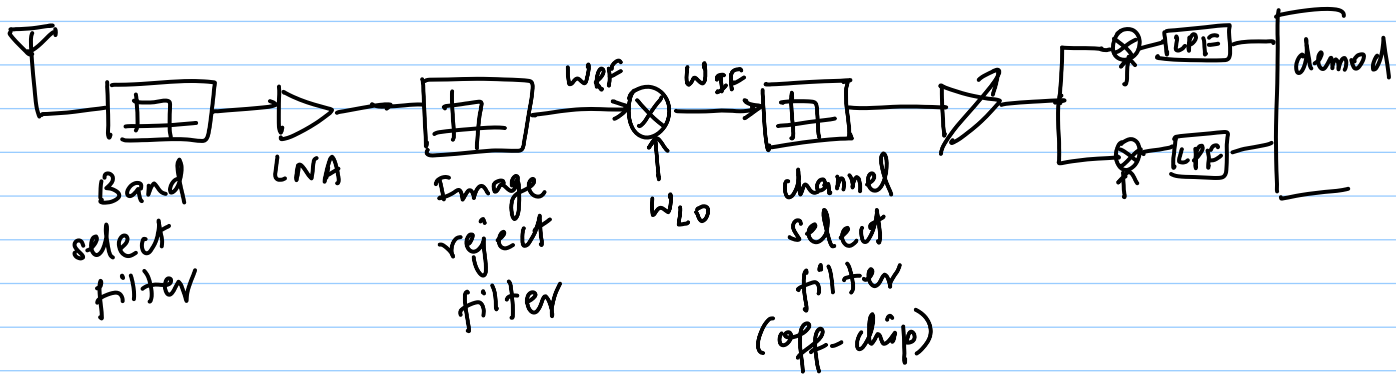
Lecture 26: Heterodyne Rx ; Image-Reject Rx

II Heterodyne Receivers

{studied this in Lecture #10}

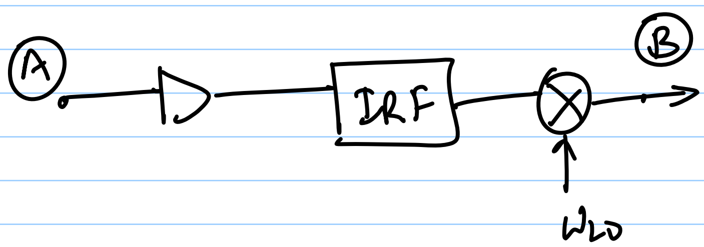


* Image problem ⇒ IR Filter
* NO DC problems

* high IF = easy IR filtering
  low IF = easy IF design (high gain, high-Q filtering)

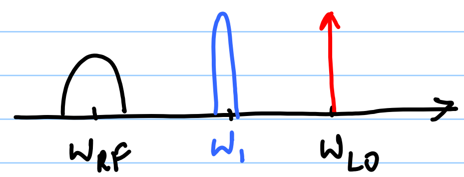
  i.e. Tradeoff between
  image rejection (sensitivity) and
  channel-selection (selectivity)

Half-IF problem



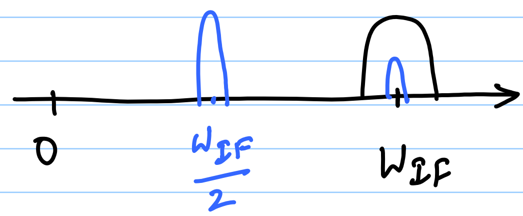
* Mixer has significant second order distortion
* LO has 2^{nd} harmonic

(A)



$\omega_1 = \dfrac{\omega_{RF} + \omega_{LO}}{2}$

(B)



* IF component @ $|2\omega_1 - 2\omega_{LO}| = \omega_{IF}$

second-order distortion in mixer

LO has second harmonic

* If BB has second-order distortion,
  IF component @ $|\omega_1 - \omega_{LO}| = \omega_{IF}/2$
  BB component @ ω_{IF} (2^{nd} order distortion)

Solutions:

→ minimise 2^{nd}-order distortion in RF/IF path

→ 50% LO duty cycle (no $2\omega_{LO}$)

<u>Dual-IF topology</u> ≡ two stages of down-conversion

→ partial channel selection @ each IF

→ each filter Q is relaxed

→ Second IF has image problem too!

III Image-reject Receivers

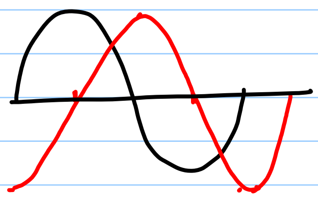
* IR filters → off-chip, 50 Ω etc.

→ impact on noise, power consumption

⇒ motivates other RF system architectures for image rejection

Basic idea: process signal & image differently: cancel image with negative replica (IM & RF on opposite sides of LO)

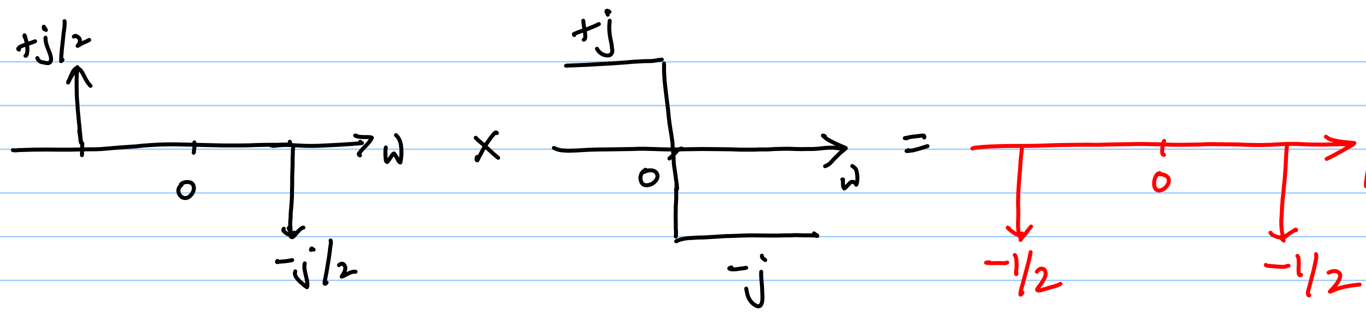
Shift-by-90° operation



— = $\sin \omega t$

— = $-\cos \omega t$

} Shifting in time domain



shifting by 90° in freq. domain

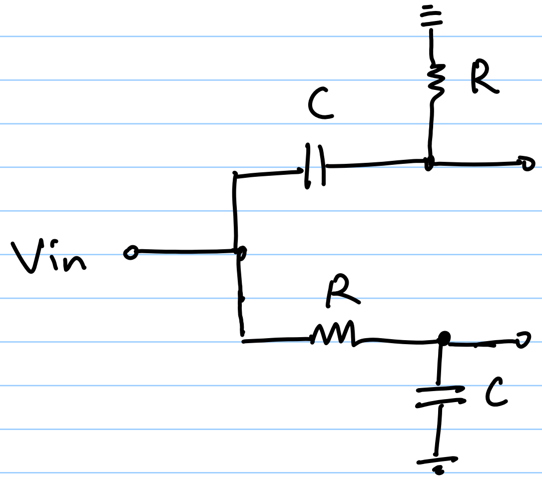
multiply by $G(\omega) = -j\,\mathrm{sgn}(\omega)$ } equivalent to taking Hilbert Transform

$t \rightarrow t - T/4$

$\sin \omega t \rightarrow -\cos \omega t$

$\cos \omega t \rightarrow \sin \omega t$

One Ckt implementation



V_{out_1} → phase shift $= \pi/2 - \tan^{-1}(\omega RC)$

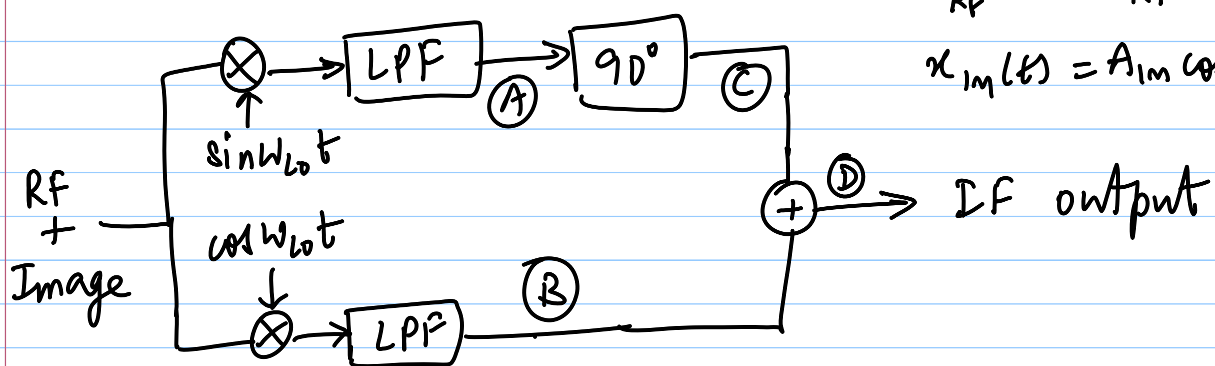
V_{out_2} → phase shift $= -\tan^{-1}(\omega RC)$

* V_{out_1} & V_{out_2} have a 90° phase difference @ all ω
* Amplitudes are equal only @ $\omega = \frac{1}{RC}$ $\left\{ \begin{array}{l} 45^\circ \text{ on} \\ \text{each} \\ \text{side} \end{array} \right\}$

1) Hartley Architecture

$x_{RF}(t) = A_{RF} \cos \omega_{RF} t$

$x_{IM}(t) = A_{IM} \cos \omega_{IM} t$



assume low-side injection: $\omega_{RF} - \omega_{LO} = \omega_{LO} - \omega_{IM}$

$$x_A(t) = \frac{A_{RF}}{2} \sin(\omega_{LO} - \omega_{RF})t + \frac{A_{IM}}{2} \sin(\omega_{LO} - \omega_{IM})t$$

$$= -\frac{A_{RF}}{2} \sin(\omega_{RF} - \omega_{LO})t + \frac{A_{IM}}{2} \sin(\omega_{LO} - \omega_{IM})t$$

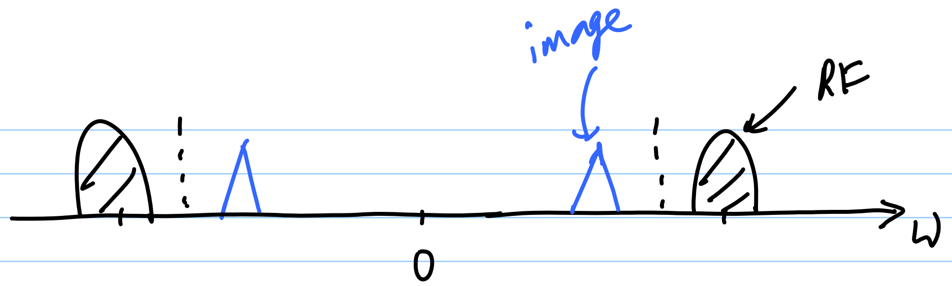
$$x_B(t) = \frac{A_{RF}}{2} \cos(\omega_{LO} - \omega_{RF})t + \frac{A_{IM}}{2} \cos(\omega_{LO} - \omega_{IM})t$$

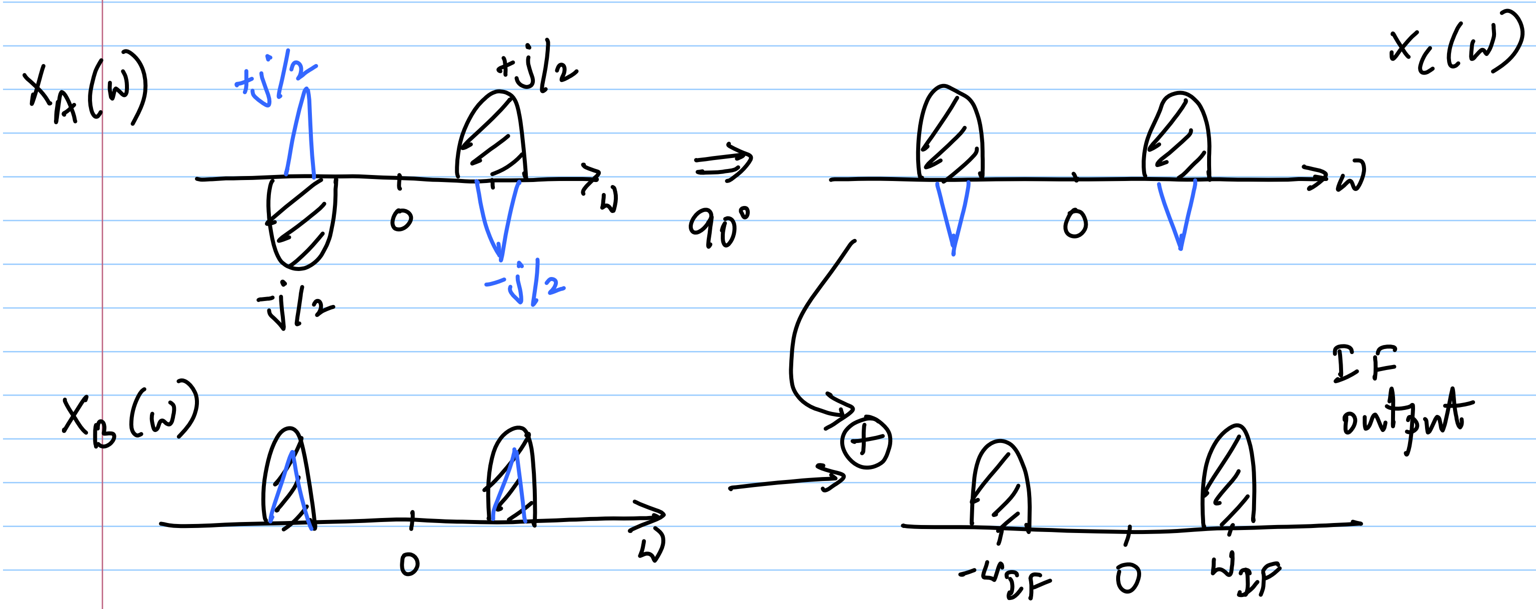
$$\Rightarrow x_C(t) = +\frac{A_{RF}}{2}\cos(\omega_{RF}-\omega_{LO})t - \frac{A_{IM}}{2}\cos(\omega_{LO}-\omega_{IM})t$$

$$x_D(t) = x_B(t) + x_C(t)$$

$$= A_{RF}\cos(\omega_{LO}-\omega_{RF})t$$

* RF signal – same polarity; IM signal – opposite polarity
* 90° shift operation distinguishes between $\omega_{LO}-\omega_{RF}<0$ & $\omega_{LO}-\omega_{IM}>0$
* In practice, 90° is achieved by applying 45° & −45° phase shifts on $x_A(t)$ & $x_B(t)$ respectively





* Key drawback: Sensitivity to mismatches
  → LOs not in quadrature
  → gains & phase shifts in two paths are not identical
  ⇒ Image cancellation is incomplete & signal is corrupted

Assume: LO inputs are $A_{LO}\sin\omega_{LO}t$ and $(A_{LO}+\epsilon)(\cos\omega_{LO}t+\theta)$

$\epsilon \rightarrow$ amplitude mismatch

$\theta \rightarrow$ phase imbalance

$$x_A(t) = \frac{A_{LO}A_{RF}}{2}\sin(\omega_{LO}-\omega_{RF})t + \frac{A_{LO}A_{IM}}{2}\sin(\omega_{LO}-\omega_{IM})t$$

$$x_B(t) = (A_{LO}+\epsilon)\frac{A_{RF}}{2}\cos\left[(\omega_{LO}-\omega_{RF})t+\theta\right] + (A_{LO}+\epsilon)\frac{A_{IM}}{2}\cos\left[(\omega_{LO}-\omega_{IM})t+\theta\right]$$

$$x_C(t) = A_{LO}\left[\frac{A_{RF}}{2}\cos(\omega_{LO}-\omega_{RF})t - \frac{A_{IM}}{2}\cos(\omega_{LO}-\omega_{IM})t\right]$$

$$x_D(t) = x_B(t) + x_C(t)$$

$$= y_{IF}(t) + y_{IM}(t)$$

desired signal

fraction of down-converted image

$$y_{IF}(t) = (A_{LO}+\epsilon)\frac{A_{RF}}{2}\cos[(\omega_{LO}-\omega_{RF})t+\theta]$$

$$+ \frac{A_{LO}A_{RF}}{2}\cos(\omega_{LO}-\omega_{RF})t$$

$$y_{IM}(t) = (A_{LO}+\epsilon)\frac{A_{IM}}{2}\cos[(\omega_{LO}-\omega_{IM})t+\theta]$$

$$- \frac{A_{LO}A_{IM}}{2}\cos(\omega_{LO}-\omega_{IM})t$$

$$\left.\begin{matrix}\text{Image-to-}\\ \text{signal ratio}\end{matrix}\right\} = \frac{\text{image power @ IF port}}{\text{signal power @ IF port}}$$

$$= \frac{A_{IM}^2}{A_{RF}^2}\cdot\frac{A^2-2AB\cos\theta+B^2}{A^2+2AB\cos\theta+B^2}$$

where $A = A_{LO}$; $B = A_{LO}+\epsilon$

Note that : image-to-signal ratio @ input $= \dfrac{A_{IM}^2}{A_{RF}^2}$

Define Image Rejection Ratio (IRR) as

$$IRR = \frac{\text{Image-to-signal ratio @ output}}{\text{'' \quad '' \quad @ input}}$$

$$= \frac{A^2-2AB\cos\theta+B^2}{A^2+2AB\cos\theta+B^2}$$

If $\epsilon \ll A_{LO}$ & $\theta \ll 1$ rad,

$$\boxed{IRR = \frac{(\Delta A/A)^2+\theta^2}{4}}$$ where $\dfrac{\Delta A}{A} = \dfrac{\epsilon}{A_{LO}}$

typical IC matching:

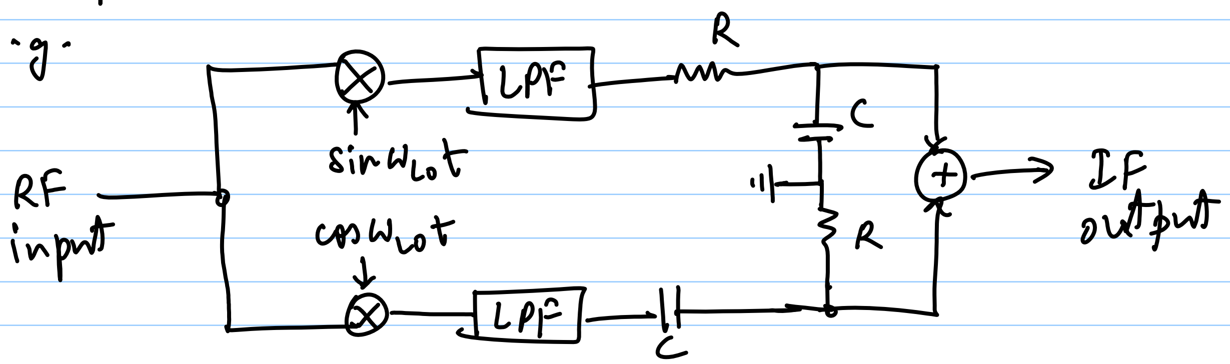
IRR ~ 30–40dB

$\frac{\Delta A}{A}$ ~ 0.2–0.6dB

θ ~ 1 to 5°

* Gain mismatch from 90° phase shift operation is <u>critical</u>

e.g.



say, resistors are R, $R+\Delta R$

caps are C, $C+\Delta C$

Remember: equal gains only @ $\omega_{IF} = \frac{1}{RC}$

$\Rightarrow$ Gain imbalance is a function of process, temperature

$$\frac{\Delta A}{A} = \frac{(R+\Delta R)(C+\Delta C)\omega - 1}{\sqrt{1+(R+\Delta R)^2(C+\Delta C)^2\omega^2}} \div \frac{1}{\sqrt{1+R^2C^2\omega^2}}$$

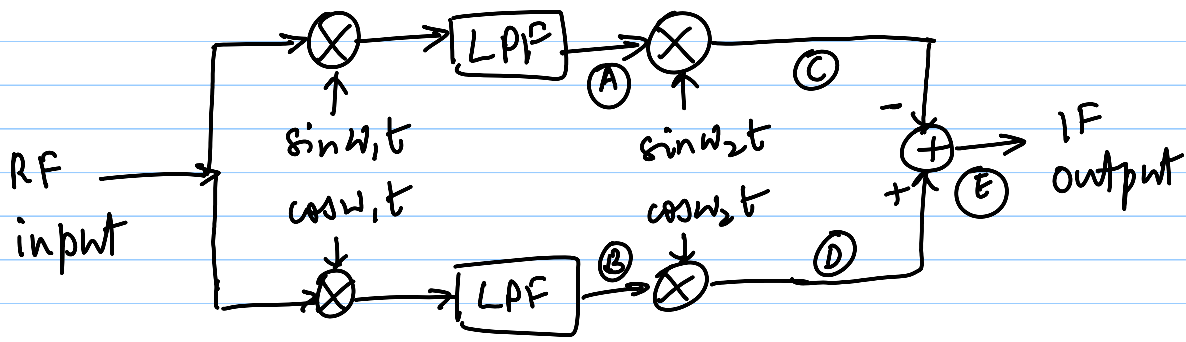
using $RC\omega \approx 1$ around ω_{IF},

$$\frac{\Delta A}{A} \approx \frac{\Delta R/R + \Delta C/C}{\sqrt{2+\frac{\Delta R}{R}+\frac{\Delta C}{C}}} \div \frac{1}{\sqrt{2}} \approx \frac{\Delta R}{R} + \frac{\Delta C}{C}$$

$\frac{\Delta R}{R} = 20\% \Rightarrow$ IRR ~ 20dB !

* Frequency deviation can cause gain imbalance
  → image cancellation only @ $\omega_{IF} = \frac{1}{RC}$
  → if $\omega_{channel} \cancel{<}< \omega_{IF}$, IRR can degrade near edges of channel
* We want overall image suppression ~ 60-70dB
  → large IF = better image attenuation + IR arch. can give 60-70dB
* Linearity of adder is critical (adjacent channel interfer)
* Noise + loss of 90° stage are significant

2) Weaver Architecture

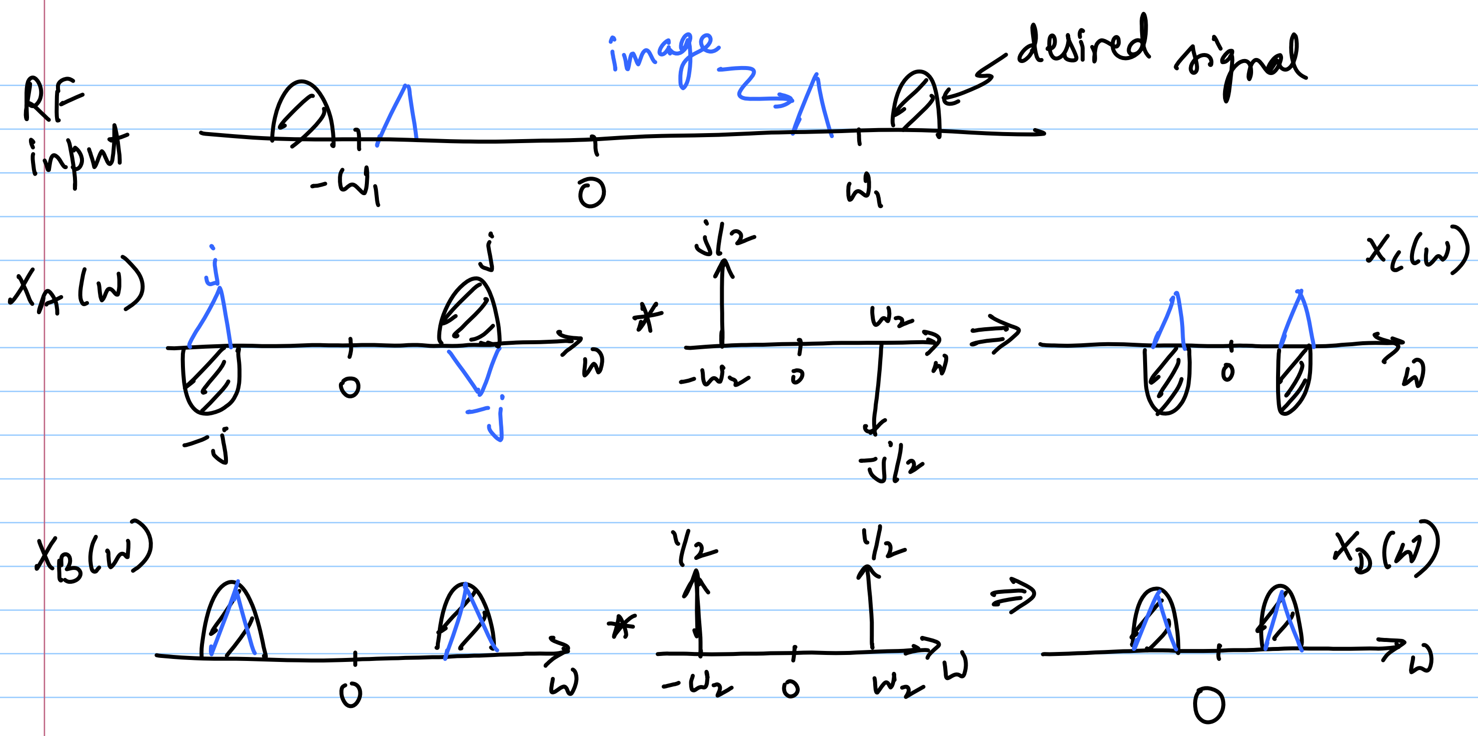


Second quadrature mixer → performs 90° operation

assume $\omega_2 << \omega_1$

$X_A(\omega)$ is convolved with $\frac{j}{2}[\delta(\omega+\omega_2) - \delta(\omega-\omega_2)] \rightarrow X_C(\omega)$

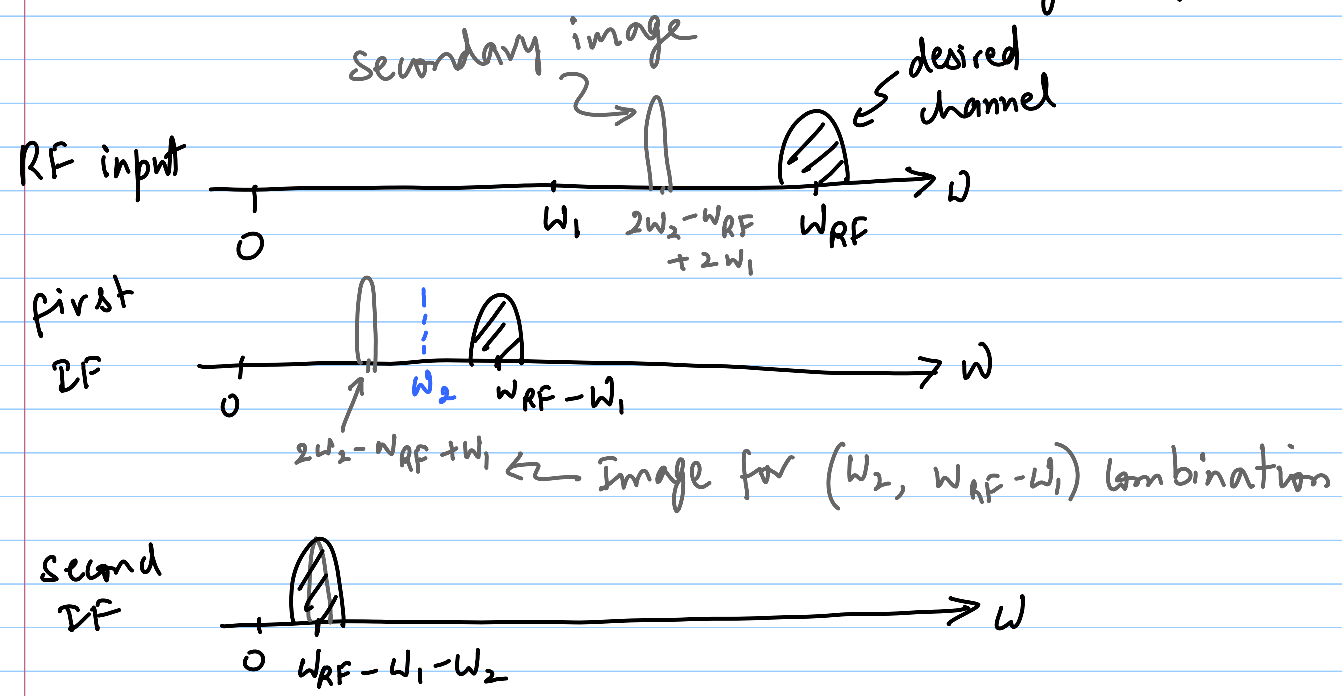
$X_B(\omega)$ is convolved with $\frac{1}{2}[\delta(\omega+\omega_2) + \delta(\omega-\omega_2)] \rightarrow X_D(\omega)$



$X_D(\omega) - X_C(\omega)$ gives the desired output
(signal additive, image cancels)

* image is still present @ $\omega_2 + \omega_{IF}$ & $-(\omega_2 + \omega_{IF})$
→ LPF @ output

* If final $\omega_{IF} \neq 0$ → "secondary image" problem



* interferer is on same side of LO as desired RF
* LPF should be replaced by BPFs to suppress "secondary image"
* $\omega_1 \pm \omega_2 = \omega_{RF}$ $\Rightarrow$ final output is @ BB
  $\rightarrow$ no secondary image