Axioms ① $P(A) \geq 0$, ② $P(S) = 1$

③ if A & B mutually exclusive $A \cap B = \emptyset$, $\boxed{P(A\cup B) = P(A) + P(B)}$

③ if A_i & A_j mutually exc., for $i \neq j$ ∴ $P[\bigcup_{i=1}^{\infty} A_i] = \sum_{i=1}^{\infty} P(A_i)$

Corollaries ① $P(A^c) = 1 - P(A)$ ② $P(A) \leq 1$ ③ $P(\emptyset) = 0$

④ if $A_1 \dots A_n$ mut. exc., ∴ $P[\bigcup_{i=1}^{n} A_i] = \sum_{i=1}^{n} P(A_i)$

⑤ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

⑥ $P[\bigcup_{i=1}^{n} A_i] = \sum_{j=1}^{n} P(A_j) - \sum_{j<k} P(A_j \cap A_k) + \dots (-1)^{n+1} P(A_1 \cap \dots \cap A_n)$

$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3)$
$- [P[A_1 \cap A_2] + P[A_2 \cap A_3] + P[A_3 \cap A_1]]$
$+ (-1)^4 P(A_1 \cap A_2 \cap A_3)$

⑦ $A \subset B \rightsquigarrow$ ∴ $P(A) \leq P(B)$

★ **Cond. Prob** $\boxed{P(A|B) = \frac{P(A\cap B)}{P(B)}}$ $\left.\begin{matrix} A = B \\ B \subset A \end{matrix}\right\} P(A|B) = 1$

★ **Theorem on Total Prob.** $P(A) = P(A|B_1)P(B_1) + \dots P(A|B_n)P(B_n)$

a) B_js partition S , b) B_js are mutually exclusive

★ **Baye's Rule** $P(B_j|A) = \frac{P(B_j \cap A)}{P(A)} = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{n} P(A|B_i)P(B_i)}$

* **if A & B independent:** $P(A\cap B) = P(A)P(B)$

$\underset{B|A}{P(A|B)} = \frac{P(A\cap B)}{P(B)} = \underset{B}{P(A)}$

* For n independent event: $2^n - n - 1$ conditions

① **Binomial Prob law** Fail $1-p$, Pass p $k = 0, 1, \dots n$

[1] $p(0 \text{ success}) = (1-p)^n$

[2] $p(0 \text{ fail}) = p^n = p(\text{all success})$

[3] $p(k \text{ success}) = \binom{n}{k} p^k (1-p)^{n-k}$ $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

② **Multinomial Prob law** p_1 p_2 ...... p_M M outcomes

$P[(k_1, k_2, \dots k_M)]$
$= \frac{n!}{k_1! k_2! \dots k_M!} p_1^{k_1} p_2^{k_2} \dots p_M^{k_M}$

done n times

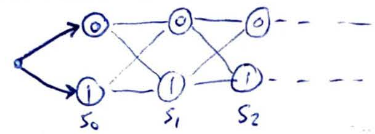
③ **Geo Prob law** Fail $1-p$, pass p $n = \infty$

[1] $p[1^{st} \text{ success in } m^{th} \text{ exp}] = p[m] = (1-p)^{m-1} p$

[2] $P[1^{st} \text{ pass after } k] = p[m = k+1] + p[m = k+2] + \dots$
$= p(1-p)^k (1 + (1-p) + (1-p)^2 + \dots)$
$= p(1-p)^k \sum_{r=0}^{\infty} (1-p)^r$ $\sum_{r=0}^{\infty} q^r = \frac{1}{1-q}$

$P[1^{st} k \text{ all fails}] = (1-p)^k$

★ **Seq. of dependent experiments**



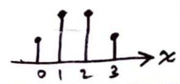
$s_2|s_1$

$P(s_0 s_1 s_2) = P(s_0 \cap s_1 \cap s_2) = P(s_2|s_0 \cap s_1) P(s_0 \cap s_1)$
$= P(s_2|s_1) P(s_1|s_0) P(s_0)$

$P(s_0 \cap s_1 \cap s_2 \dots s_n) = P(s_n|s_{n-1}) P(s_{n-1}|s_{n-2}) \dots P(s_0)$

★ **Discrete RV** $S_X = \{x_1, x_2, x_3, \dots\}$

pmf $p_X(x) = P[X = x]$



① $p_X(x) \geq 0$, ② $\sum_{x \in S_X} p_X(x) = 1$

③ $P[X \text{ in } B] = \sum_{x \in B} p_X(x)$ $B \subset S$

* **Expected Value:** $m_X = E[X] = \sum_{x \in S_X} x \, p(x)$

$E[|x|] = \sum_{x \in S_X} |x| p(x) < \infty$

$E[g(x)] = \sum g(x) p(x)$

$Z = a g(x) + b h(x) + c$

$E[Z] = a E[g(x)] + b E[h(x)] + c$

* $E[X+Y] = E[X] + E[Y]$
* $E[aX] = a E[X]$
* $E[X+c] = E[X] + c$

* **Variance:** $\sigma^2 = VAR[X] = E[(X - m_X)^2]$
$= \sum (x - m_x)^2 p(x)$
$= E[x^2] - m_x^2$

n^{th} moment $= E[x^n]$

* $Var[cX] = c^2 Var[X]$
* $std(x) = \sigma = \sqrt{Var[X]}$
* $Var[X+c] = Var[X]$

[1] **Bernoulli RV:** $E[X] = p$

$Var[X] = p(1-p)$

[2] **Binomial RV:** k successes in n experiments
pass → p
fail → 1−p

$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$ $k = 0, 1, 2, \dots n$

$E[X=k] = \sum_{k=0}^{n} k \, p_X(k)$
$= \sum_{k=1}^{n} k \binom{n}{k} p^k (1-p)^{n-k}$
$= n \sum_{k=1}^{k=n} \binom{n-1}{k-1} p^k (1-p)^{n-k}$
$= np \underbrace{\sum_{j=0}^{n-1} \binom{n-1}{j} p^j (1-p)^{(n-1)-j}}_{1}$

$j = k-1$
$k = j+1$

$\boxed{E(k) = np}$

$Var[X] = E[X^2] - (E[X])^2$

$E[x^2] = \sum_{k=0}^{n} k^2 p(k)$
$= n \sum_{j=0}^{n-1} (j+1) \binom{n-1}{j} p^j p (1-p)^{n-1-j}$

$\boxed{Var[X] = np(1-p)}$

[3] **Geometric RV:** 1^{st} success in k^{th} experiment
pass → p
fail → 1−p

$p_X(k) = (1-p)^{k-1} p$ $k = 1, 2, 3, \dots$

$E[X] = \sum_{k=1}^{\infty} k \, p_X(k) = \frac{1}{p}$

$Var[X] = \frac{2-p}{p^2} - \frac{1}{p^2} = \frac{1-p}{p^2}$

$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

d

$\sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}$

d

$\sum_{n=1}^{\infty} n(n-1) x^{n-2} = \frac{2}{(1-x)^3}$

* **Cond PMF:**

$p_X(x|C) = P[X = x|C]$
$= \frac{P[X = x \cap C]}{P[C]}$

$p_X(x) = \sum_{i=1}^{n} p_X(x|B_i) P(B_i)$ Theorem of total Prob.

$$\boxed{E[X|C] = \sum_{x} x\, P_X(x|C)}$$

$$E[X] = \sum_x x\, p(x)$$
$$= \sum_x x \sum_i p(x|B_i)\, p(B_i)$$
$$= \sum_i p(B_i) \sum_x x\, p(x|B_i) = \sum_i E(x|B_i)\, p(B_i)$$

$$E[g(x)] = \sum_i E(g(x)|B_i)\, p(B_i)$$

In Geometric $(1-p)^{k-1}$ $j,k \to +ve$

$$\Rightarrow P[M \geq k+j \mid M > j] = P[M \geq k]$$ memoryless RV

4 Poisson RV:

$$\boxed{P[X=k] = e^{-\alpha} \frac{\alpha^k}{k!}}$$ $k = 0, 1, 2, ----$ (N)

$$\boxed{E[X] = \alpha = Var[X]}$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$ Taylor series

5 Uniform RV:

$$E[X] = \sum_{k=0}^{L-1} k \cdot \frac{1}{L} = \frac{1}{L} \sum_{k=0}^{L-1} k$$

$$\boxed{E[X] = \frac{1}{L} \frac{L(L-1)}{2} = \frac{L-1}{2}}$$

$$\boxed{Var[X] = \frac{L^2-1}{12}}$$

6 Zipf RV:

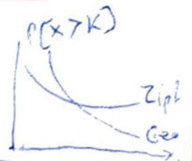
$$S_X = \{1, 2, ---- L\}$$

$$\boxed{P_X(k) = \frac{1}{C_L} \frac{1}{k}}$$ $k = 1, 2, ---L$

$$\sum_{k=1}^{L} P_X(k) = 1 \longrightarrow C_L = \sum_{k=1}^{L} \frac{1}{k}$$

$$\boxed{E[X] = \frac{L}{C_L}} \qquad \boxed{Var[X] = \frac{L(L+1)}{2C_L} - \frac{L^2}{C_L^2}}$$

$$* \; P[X > m] = 1 - P[X \leq m]$$
$$= 1 - \sum_{k=1}^{m} P_X(k) = 1 - \frac{C_m}{C_L}$$



★ Binomial $K = 0 \longrightarrow n$ $\longrightarrow$ Poisson $K = 0 \longrightarrow \infty$

$$\left.\begin{matrix} n \to \infty \\ p \to 0 \end{matrix}\right\} \alpha = np$$

Discontinuous

$$\lim_{x \to \alpha^-} F(x) \neq \lim_{x \to \alpha^+} F(x)$$

PMF: $P_X(x) = P[X = x]$

CDF: $F_X(x) = P[X \leq x]$

(a) $\lim_{x \to -\infty} F_X(x) = 0$ (b) $\lim_{x \to +\infty} F_X(x) = 1$

(c) $\alpha < \beta$, $F(\alpha) \leq F(\beta)$ → motonically increasing

(d) $P[\alpha < X \leq \beta] = F(\beta) - F(\alpha^+)$

(e) $P[X = \alpha] = F(\alpha^+) - F(\alpha^-)$

$$P[\alpha \leq X \leq \beta] = P[X = \alpha] + P[\alpha < X \leq \beta]$$

PDF: $f_X(x) = \frac{dF_X(x)}{dx} \simeq \frac{P[x < X < x+h]}{h}$

(a) $f_X(x) \geq 0$

(b) $P[a < x < b] = F(b) - F(a)$
$= \int_a^b f_X(x)\, dx$

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\, dx$$

(c) $\int_{-\infty}^{\infty} f_X(x)\, dx = F(+\infty) - F(-\infty)$
$= 1$

$$F_X(x|A) = P[X \leq x | A]$$
$$= \frac{P[X \leq x \cap A]}{P[A]} \xrightarrow{d} f_X(x|A)$$

$$F_X(x) = \sum_{i=1}^{n} F_X(x|B_i)\, p(B_i) \xrightarrow{d} f_X(x)$$

$$E_X(X) = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

Trig:

$\cos^2\theta + \sin^2\theta = 1$
$\sin 2\theta = 2 \sin\theta \cos\theta$
$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= 2\cos^2\theta - 1$ $= 1 - 2\sin^2\theta$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\boxed{\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}}$$

$$\sec\theta = \frac{1}{\cos\theta}, \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$
$$\cos x \cos y = \frac{1}{2}(\cos(x-y) + \cos(x+y))$$
$$\cos x \sin y = \frac{1}{2}(\sin(x+y) - \sin(x-y))$$

Derivatives:

$\frac{d}{dx} F(x)^n = n\, F(x)^{n-1} F'(x)$
$\frac{d}{dx} fg = f'g + fg'$
$\frac{d}{dx} e^{f(x)} = f'(x)\, e^{f(x)}$
$\frac{d}{dx} \ln g(x) = \frac{g'}{g}$

$d \sin x = \cos x$
$d \cos x = -\sin x$
$d \tan x = \sec^2 x$
$d \sec x = \sec x \tan x$
$d \operatorname{cosec} x = -\operatorname{cosec} x \cot x$
$d \cot x = -\operatorname{cosec}^2 x$

$\int$

Integrals:

$\int dx = x$	$\int x^n dx = \frac{1}{n+1} x^{n+1}$
$\int \frac{1}{x} dx = \ln(x)$	$\int x^{-n} dx = \frac{1}{-n+1} x^{-n+1}$
$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln\lvert ax+b \rvert$	$\int e^{nx} dx = \frac{1}{n} e^{nx}$

$$\boxed{\int u v' dx = \int u\, dv = uv - \int v\, du}$$

$$\int \frac{1}{x^2 + \alpha^2} dx = \int \frac{\alpha}{\alpha^2} d\theta$$
$x \to \alpha \tan^2\theta$

$$\boxed{\sum_{j=0}^{n} q^j = \frac{1 - q^{n-1}}{1 - q}} \qquad \sum_{j=0}^{n} \binom{n}{j} a^n = (1+a)^n$$

$$\boxed{P[X > Y] = \sum P[X > Y | Y = k]\; P[Y = k]}$$

$$\frac{20 \times 19 \times 18 \times 17 \times 16}{20 \times 20 \times 20 \times 20 \times 20}$$

Circle
$x^2 + y^2 = R^2$
Area $= \pi R^2$