## CH9 Finite impulse response filters (Nonrecursive filters)

- Rely only on past input information
- Never on past output information

A nonrecursive filter, the difference equation takes the form

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \dots + b_M x[n-M]$$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$Y(z) = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

$$h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{M} b_k z^{-k} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_{M-1} z^{-(M-1)} + b_M z^{-M}$$

$$H(\Omega) = b_0 + b_1 e^{-j\Omega} + b_2 e^{-j2\Omega} + \dots + b_{M-1} e^{-j(M-1)\Omega} + b_M e^{-jM\Omega}$$

$$H(z) = \frac{b_0 z^M + b_1 z^{M-1} + b_2 z^{M-2} + \dots + b_{M-1} z + b_M}{z^M}$$

**Moving average filter**

- nonrecursive filter
- compute a running average of a sequence of digital samples

The general form is

$$y[n]=\frac{1}{M}\Big(x[n]+x[n-1]+x[n-2]+x[n-3]+...+x[n-(M-1)]\Big)=\frac{1}{M}\sum_{k=0}^{M-1}x[n-k]$$

$$h[n]=\frac{1}{M}\Big(\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3]+...+\delta[n-(M-1)]\Big)=\frac{1}{M}\sum_{k=0}^{M-1}\delta[n-k]$$

**Ex)** Moving average filter with five terms

$$y[n]=\frac{1}{5}\big(x[n]+x[n-1]+x[n-2]+x[n-3]+x[n-4]\big)$$

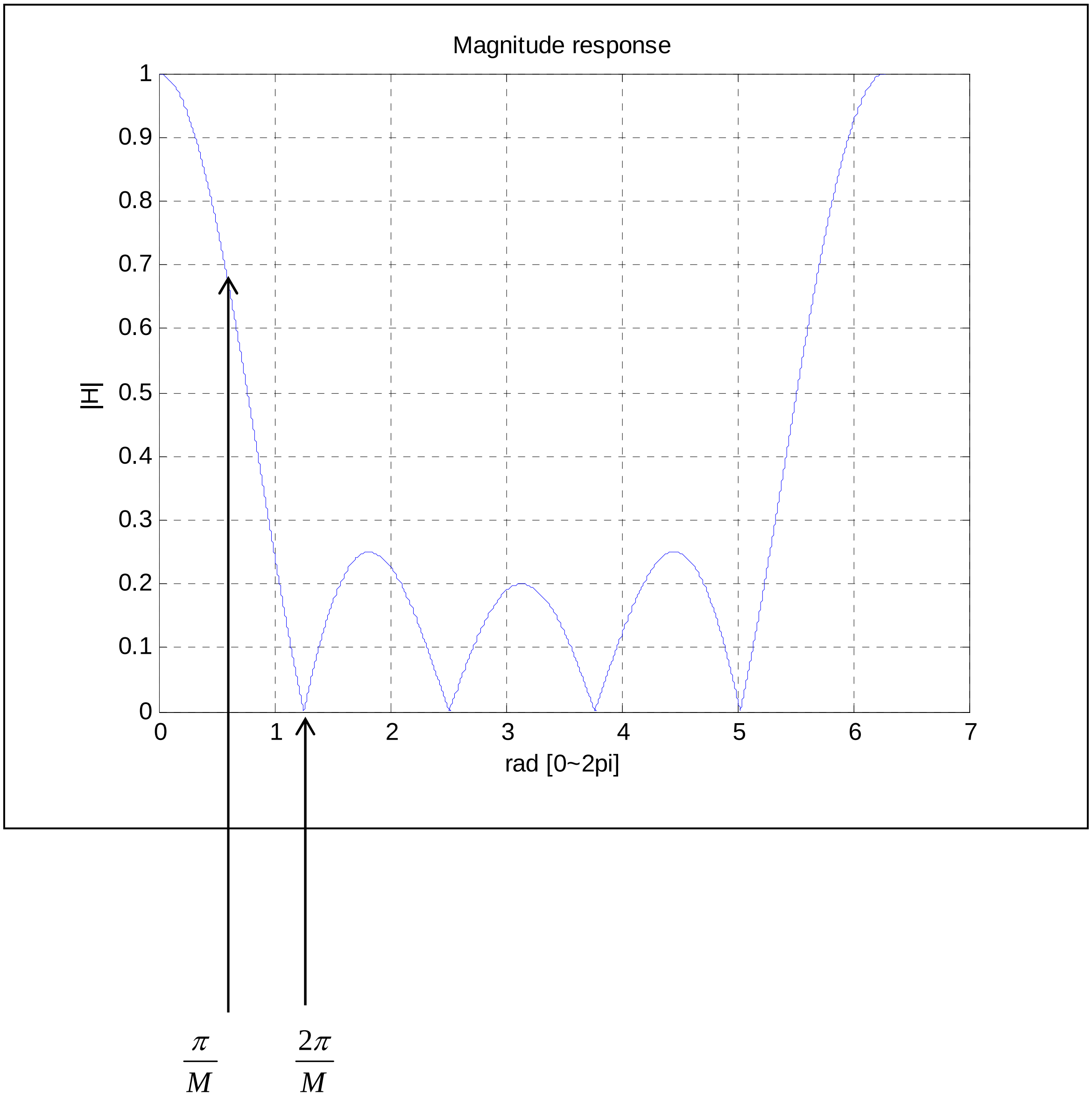
The equation for the impulse response is

$$h[n]=\frac{1}{5}\big(\delta[n]+\delta[n-1]+\delta[n-2]+\delta[n-3]+\delta[n-4]\big)$$

$$H(z)=0.2\left(1+z^{-1}+z^{-2}+z^{-3}+z^{-4}\right)$$

The frequency response is

$$H(z)=0.2\left(1+e^{-j\Omega}+e^{-j2\Omega}+e^{-j3\Omega}+e^{-j4\Omega}\right)$$




```
clc; clear all;

OM = 0:0.001:2*pi;
H = 0.2*(1+exp(-j*OM)+exp(-j*2*OM)+exp(-j*3*OM)+exp(-j*4*OM));

plot(OM, abs(H))
xlabel('rad [0~2pi]');
ylabel('|H|');
title('Magnitude response');
grid;
```

**Ex 9.2)**

Design a moving average filter with a -3dB frequency of 480 Hz if the sampling frequency is 10 kHz

$$\Omega = 2\pi \frac{f}{fs} = 2\pi \frac{480}{10000} = 0.302 \text{ rad}$$

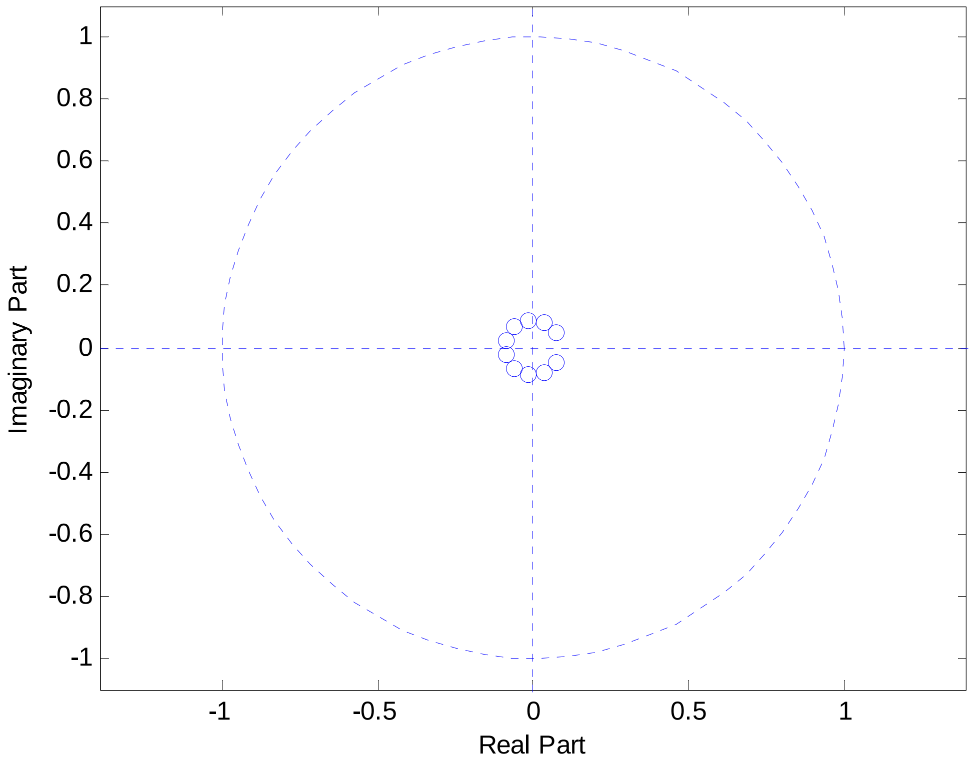
$$\frac{\pi}{M} = 0.302 \Rightarrow \frac{\pi}{0.302} = 10.4 \Rightarrow M = 11$$

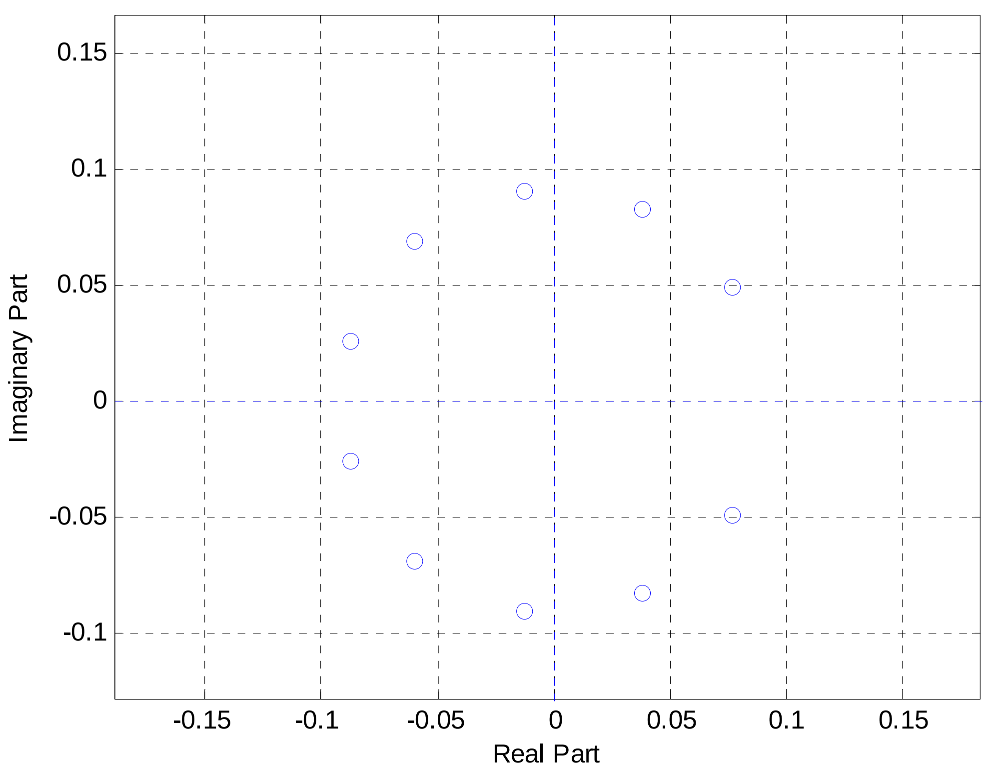
$$\Rightarrow \frac{\pi}{11} = 0.286$$

So the impulse response for this filter is

$$h\left[n\right] = \frac{1}{11}\sum_{k=0}^{10} \rho\left[n-k\right]$$

$$y[n] = \frac{1}{11}\sum_{k=0}^{10} x\left[n-k\right]$$


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Imaginary Part
-1
-0.5
0
0.5
1
Real Part


0.15
0.1
0.05
0
-0.05
-0.1
Imaginary Part
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
Real Part

z =

0.076478 + 0.049149i
0.076478 - 0.049149i
0.037765 + 0.082694i
0.037765 - 0.082694i
-0.087227 + 0.025612i
-0.087227 - 0.025612i
-0.059533 + 0.068705i
-0.059533 - 0.068705i
-0.012938 + 0.089984i
-0.012938 - 0.089984i

Phase distortion

When a sinusoidal signal passes through a linear digital filter, both its amplitude and phase are modified $|H(\Omega)|$ and phases difference of the filter $\theta(\Omega)$.

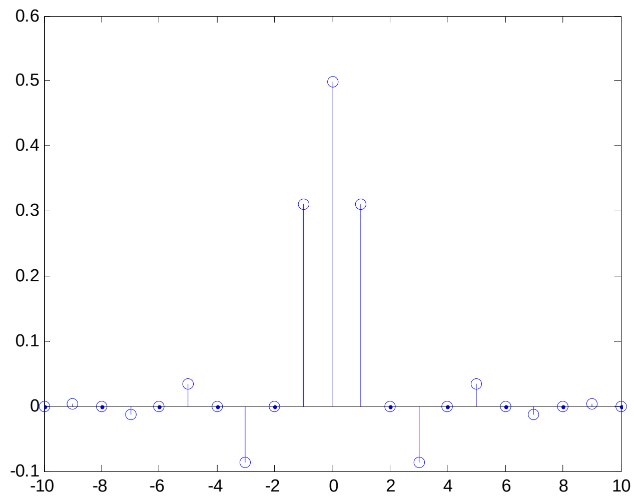
The only way to ensure that phase distortion will not occur is to make sure that signals of different frequencies have the same delays as they pass through the filter.

- Linear phase delay

So the goal is to construct the filter with linear phase characteristics so there is no phase distortion.

**Assume, an impulse response that is symmetrical about zero, equal # of terms on either side of zero so odd number of terms, is**

$$h_1[n] = h_1[-M]\delta[n+M] + \ldots + h_1[-1]\delta[n+1] + h_1[0]\delta[n] + h_1[1]\delta[n-1] + \ldots + h_1[M]\delta[n-M]$$




$h_1[-M] = h_1[M]$ and so on so that

$$h_1[n] = h_1[M]\delta[n+M] + \ldots + h_1[1]\delta[n+1] + h_1[0]\delta[n] + h_1[1]\delta[n-1] + \ldots + h_1[M]\delta[n-M]$$

$$H_1(z) = h_1[M]z^M + \ldots + h_1[1]z + h_1[0] + h_1[1]z^{-1} + \ldots + h_1[M]z^{-M}$$

$$H_1(\Omega) = h_1[M]e^{jM\Omega} + \ldots + h_1[1]e^{jM\Omega} + h_1[0] + h_1[1]e^{-jM\Omega} + \ldots + h_1[M]e^{-jM\Omega}$$

$$= h_1[0] + h_1[1]\left(e^{j\Omega} + e^{-j\Omega}\right) + \ldots + h_1[M]\left(e^{jM\Omega} + e^{-jM\Omega}\right)$$

$$= h_1[0] + 2h_1[1]\cos(\Omega) + \ldots + 2h_1[M]\cos(M\Omega)$$

When the **impulse response is symmetrical around zero, the Fourier transform is purely real**.

Purely real numbers can only have two possible phases: zero (for positive real numbers) and 180 (for negative real numbers)

**Time shifting has no effect on the filter coefficients, but does affect how the filter is implemented (Making non-causal to causal)**.

$$H(\Omega) = F\{h[n]\} = F\{h_1[n-M]\} = e^{-jM\Omega}H_1(\Omega)$$

$$|H(\Omega)|e^{j\theta(\Omega)} = e^{-jM\Omega}|H_1(\Omega)|e^{j\theta_1(\Omega)}$$

$$= |H_1(\Omega)|e^{j(\theta_1(\Omega) - M\Omega)}$$

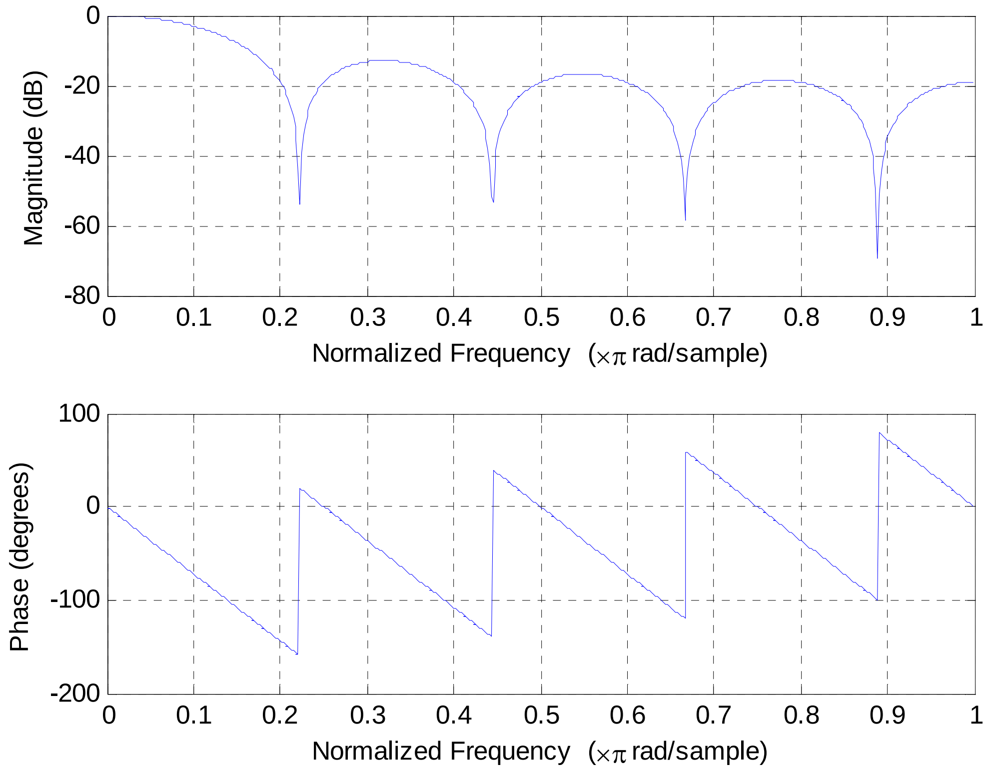
No effect on magnitude response of the filter

-$M\Omega$ to the phase response – the phase in the pass band is **linear in frequency** $\Omega$

**Ex 9.3)** Consider

$$h[n] = \frac{1}{9}\sum_{k=0}^{8}\delta(n-k)$$

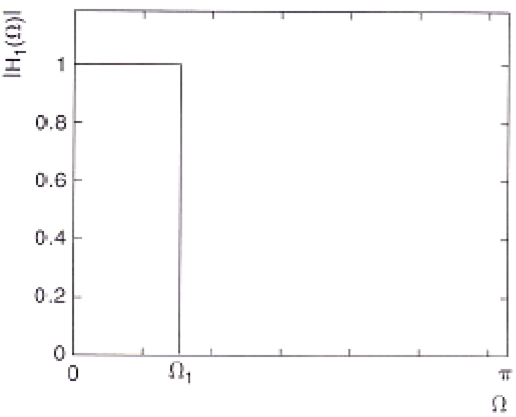
Symmetrical around a midpoint so the phase response should be linear in frequency within the pass band.

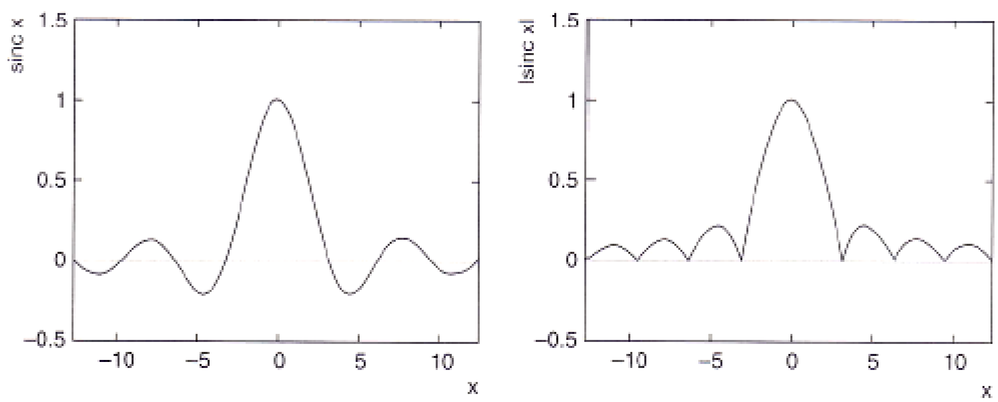


## Approximating an ideal low pass filter

SECTION 9.4 Approximating An Ideal Low Pass Filter | 327

FIGURE 9.10
Ideal low pass filter magnitude response.




(a) Sinc Function

(b) Absolute Value of Sinc Function

FIGURE 9.11
Sinc function.

$$h_1[n] = \frac{1}{n\pi}\sin(n\Omega_1) = \frac{1}{n\pi}(n\Omega_1)\frac{\sin(n\Omega_1)}{n\Omega_1} = \frac{\Omega_1}{\pi}\,\text{sinc}(n\Omega_1)$$

Proof:

$$\begin{aligned} h[n] &= \frac{1}{2\pi}\int_{-\Omega_1}^{\Omega_1} 1\cdot e^{j\Omega n}\,d\Omega \\ &= \frac{1}{2\pi}\frac{1}{jn}e^{j\Omega n}\Big|_{-\Omega_1}^{\Omega_1} \\ &= \frac{1}{n\pi}\left(\frac{e^{j\Omega_1 n} - e^{-j\Omega_1 n}}{2j}\right) \\ &= \frac{1}{n\pi}\sin(\Omega_1 n) \end{aligned}$$

**FIGURE 9.12**
Impulse response of ideal low pass filter.

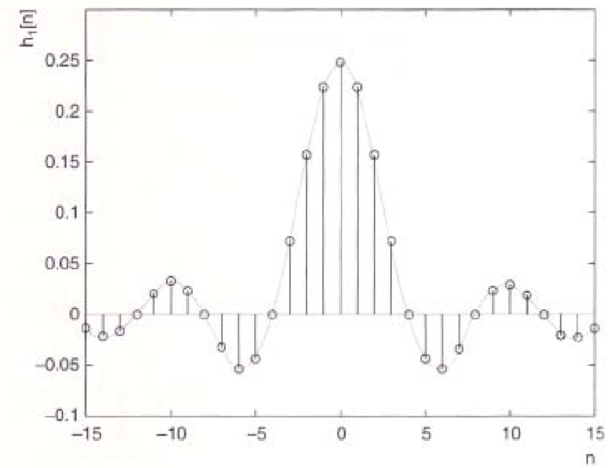



**FIGURE 9.13**
Truncated impulse response.

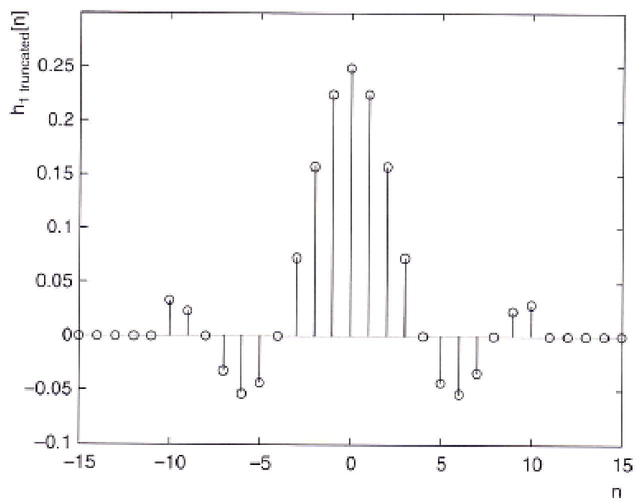
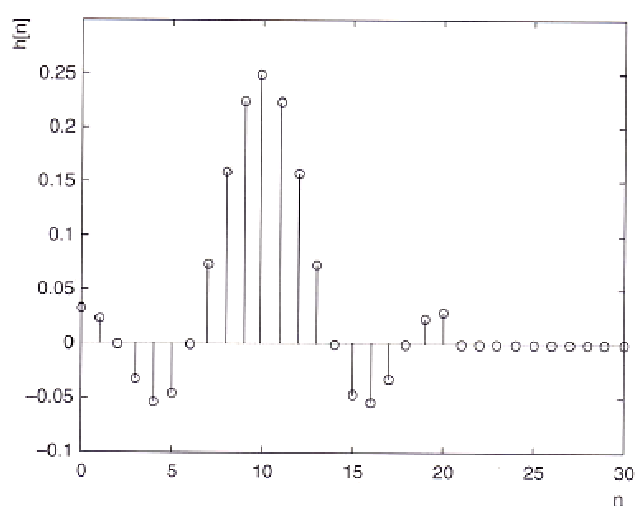
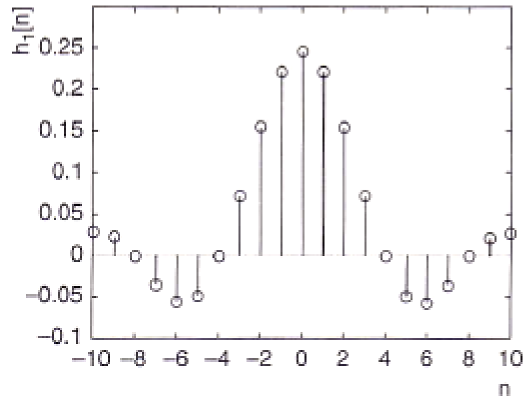
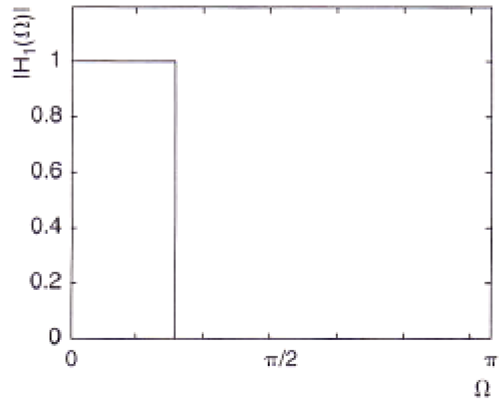



**FIGURE 9.14**
Truncated and shifted impulse response.

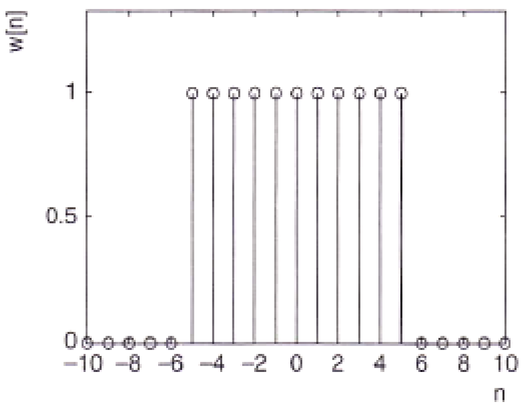




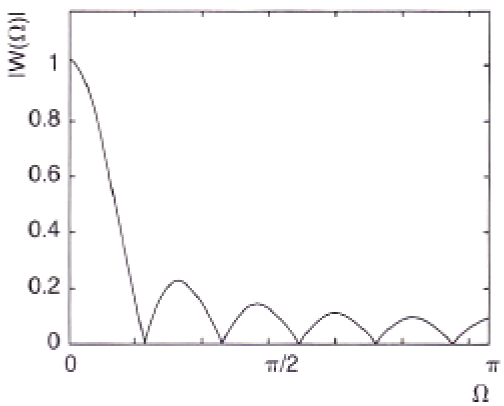

(a)(i) Impulse Response for Ideal Low Pass Filter



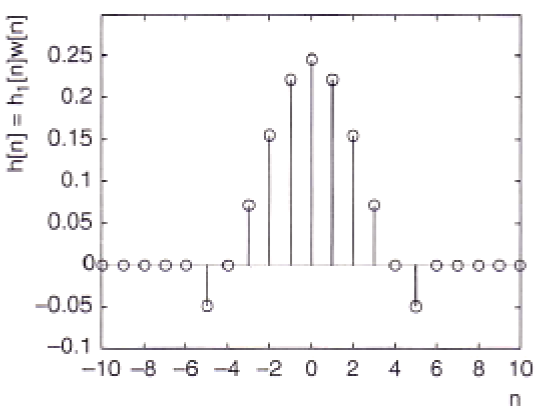

(ii) Filter Shape for Ideal Low Pass Filter



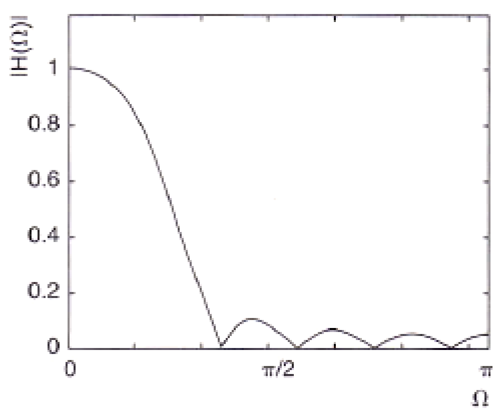

(b)(i) Rectangular Window




(ii) Magnitude Response for Rectangular Window




(c)(i) Impulse Response for Non-Ideal Low Pass Filter




(ii) Filter Shape for Non-Ideal Low Pass Filter

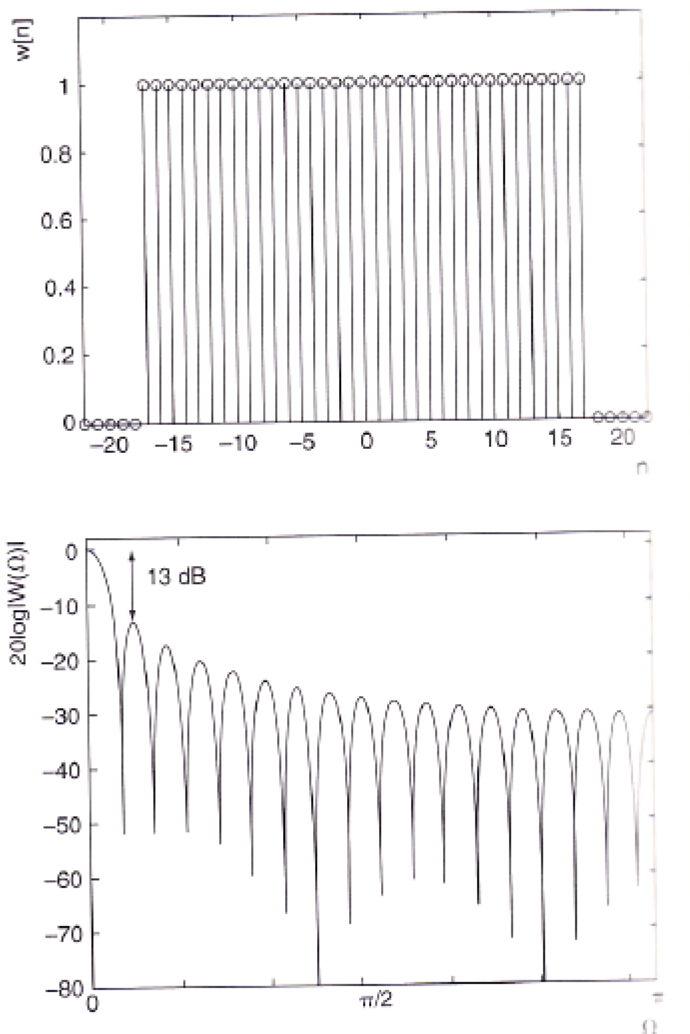

w[n]
1
0.8
0.6
0.4
0.2
0
−20
−15
−10
−5
0
5
10
15
20
n
20log|W(Ω)|
0
−10
−20
−30
−40
−50
−60
−70
−80
13 dB
0
π/2
π
Ω

**FIGURE 9.15**
Nonideal low pass filter magnitude response.

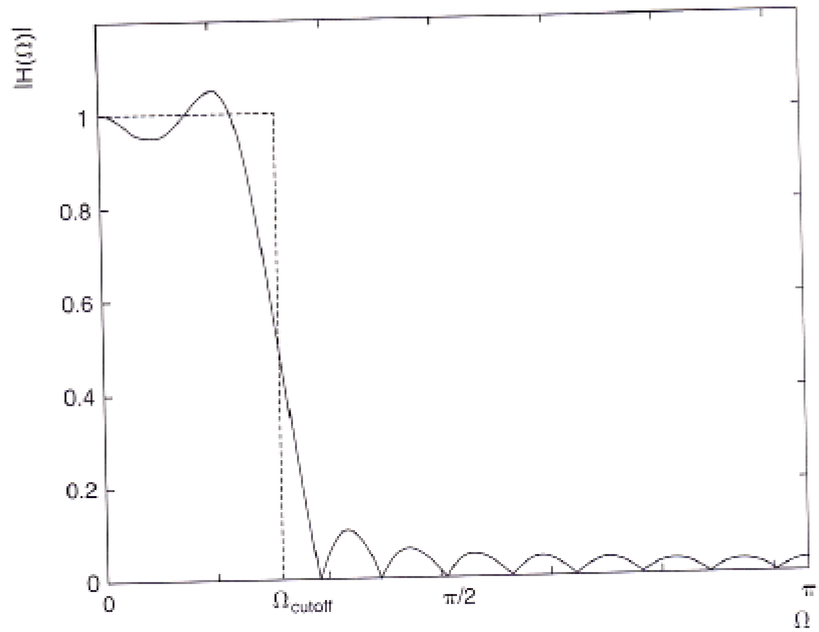



**FIGURE 9.16**
Filter features.

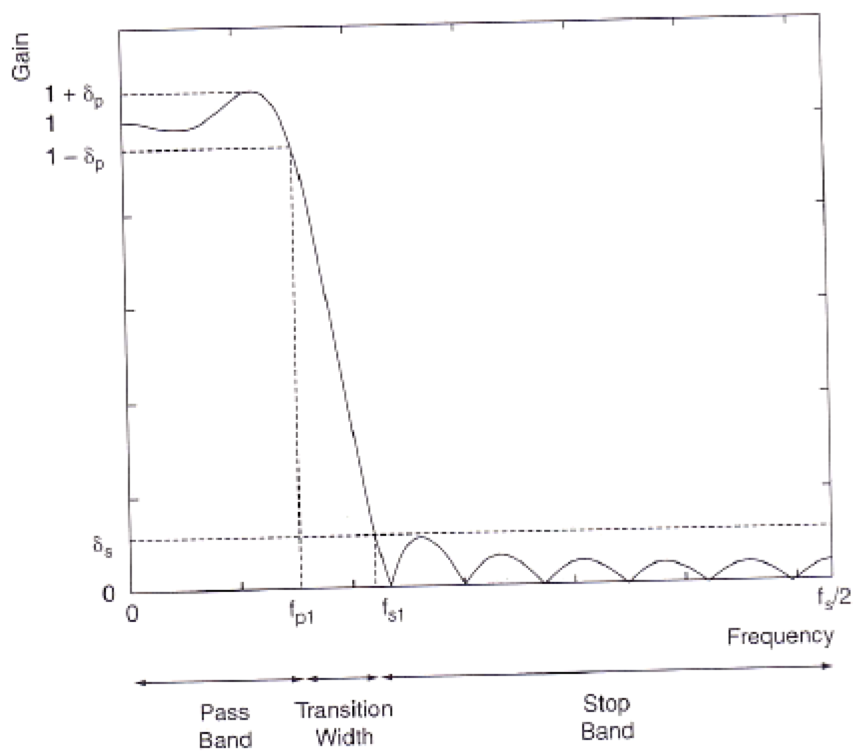


**FIGURE 9.19**
Impulse response for rectangular window.

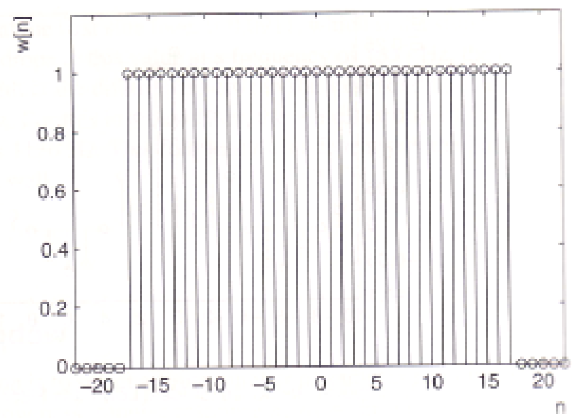



**FIGURE 9.20**
Magnitude response of rectangular window.

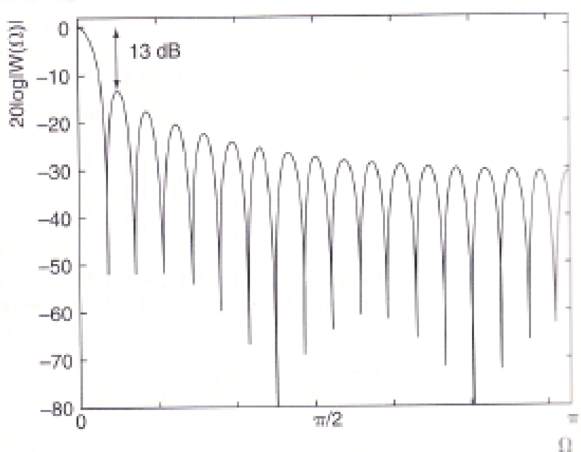
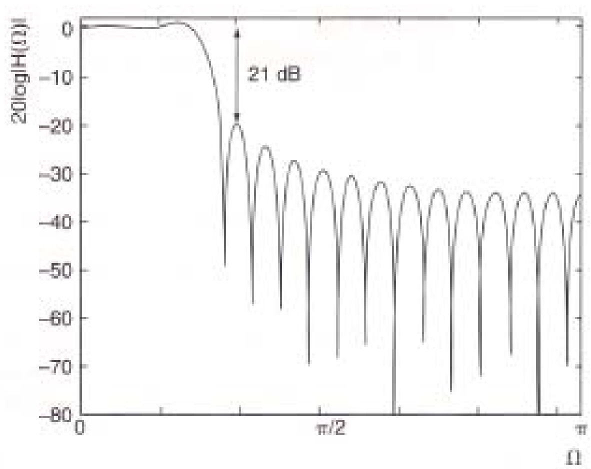



**FIGURE 9.21**
Filter shape for filter made with rectangular window.



## Hanning Window

The $N$ -term Hanning windows is defined by the equation

$$w[n] = 0.5 + 0.5\left(\cos\frac{2\pi n}{N-1}\right)$$

**FIGURE 9.22**
Impulse response for Hanning window.

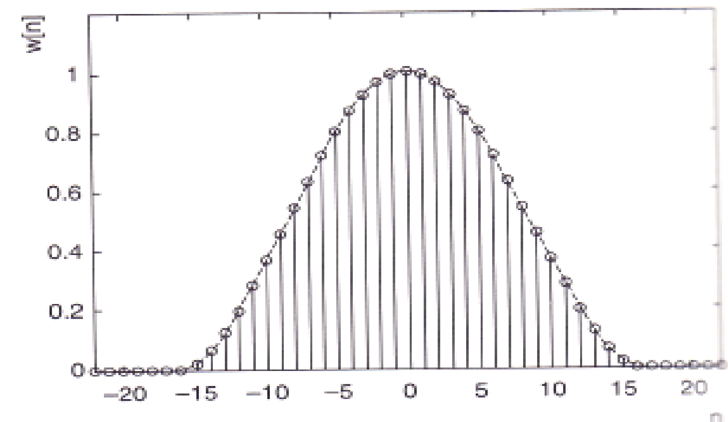



**FIGURE 9.23**
Magnitude response for Hanning window.

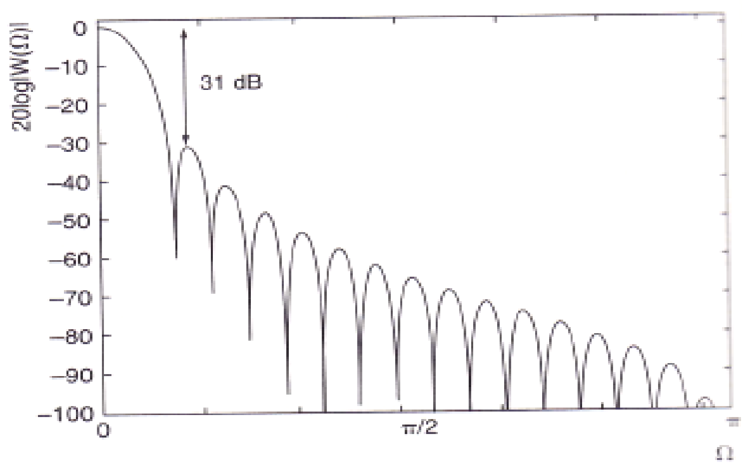
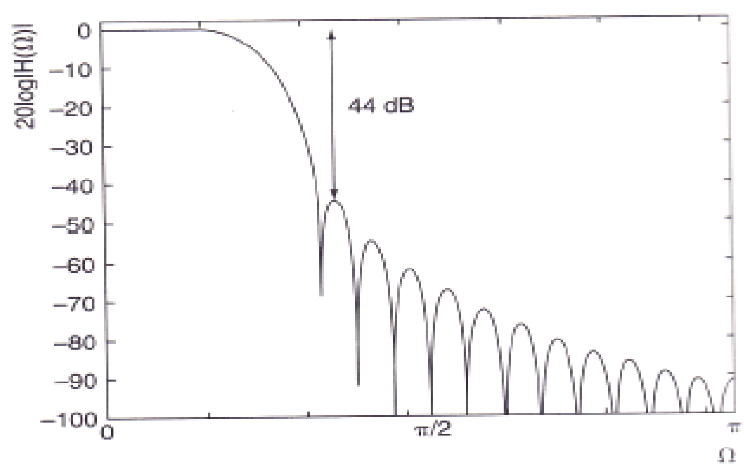



**FIGURE 9.24**
Filter shape for filter made with Hanning window.



## Hamming Window

The $N$ -term Hamming windows is defined by the equation

$$w[n] = 0.54 + 0.46\left(\cos\frac{2\pi n}{N-1}\right)$$

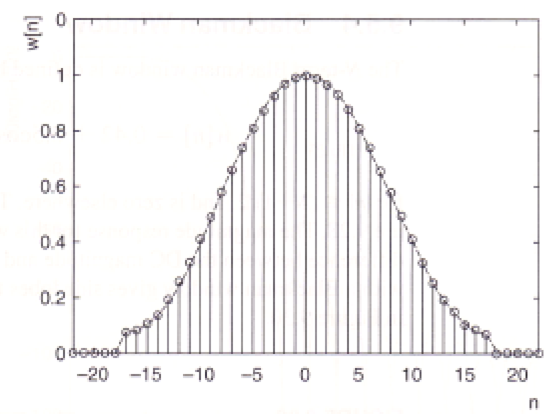



**FIGURE 9.25**
Impulse response for Hamming window.

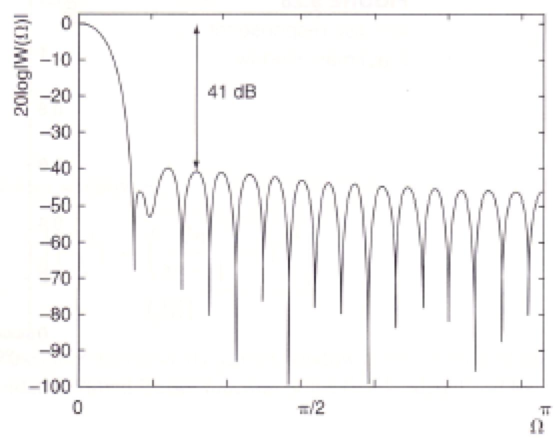



**FIGURE 9.26**
Magnitude response for Hamming window.

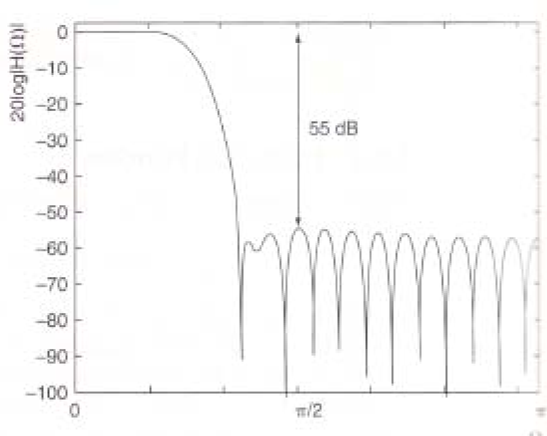



**FIGURE 9.27**
Filter shape for filter made with Hamming window.

## Blackman Window

The $N$-term Blackman windows is defined by the equation

$$w[n] = 0.42 + 0.5\left(\cos\frac{2\pi n}{N-1}\right) + 0.08\left(\cos\frac{4\pi n}{N-1}\right)$$

FIGURE 9.28
Impulse response for Blackman window

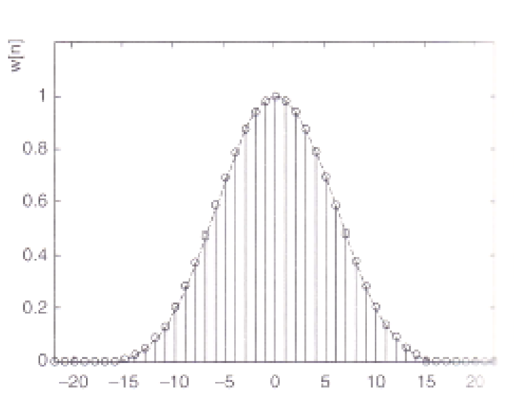



FIGURE 9.29
Magnitude response for Blackman window.

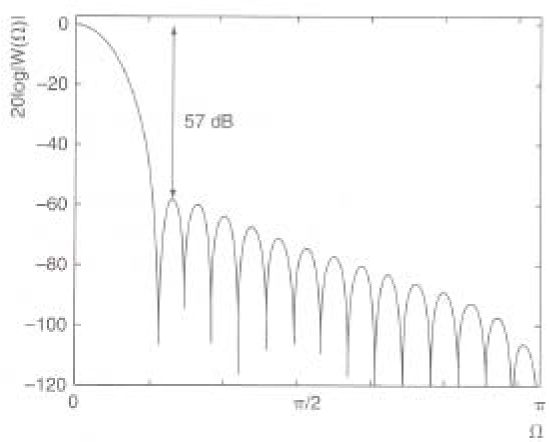
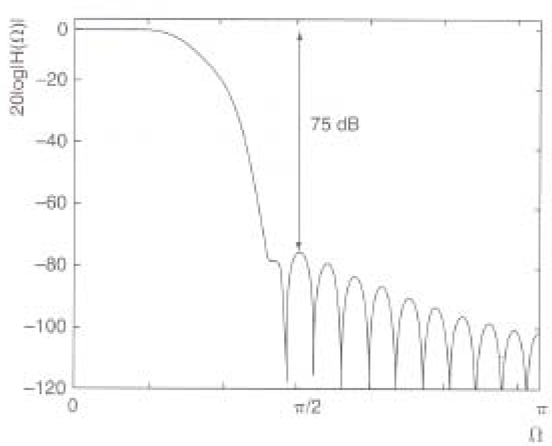



FIGURE 9.30
Filter shape for filter made with Blackman window.



## Kaiser Window

The $N$-term Kaiser windows is defined by the equation

$$w[n] = \frac{I_0\left[\beta\sqrt{1-\left(\frac{2n}{N-1}-1\right)^2}\right]}{I_0\left[\beta\right]}$$

$$I_0\left(x\right) = 1+\sum_{j=1}^{\infty}\left[\frac{\left(\frac{x}{2}\right)^j}{j!}\right]^2$$

**FIGURE 9.31**
Impulse response for Kaiser window, $\beta = 8$.

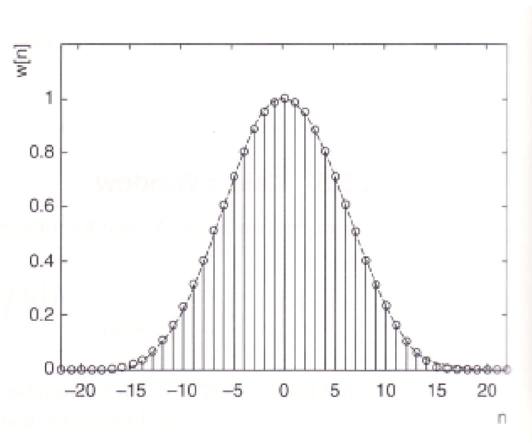



**FIGURE 9.32**
Magnitude response for Kaiser window, $\beta = 8$.

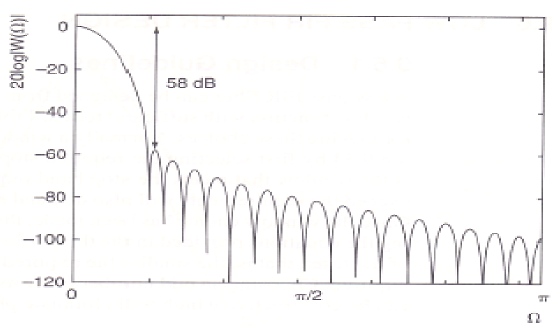
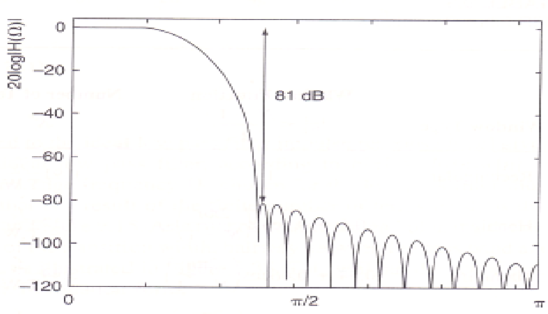



**FIGURE 9.33**
Filter shape for filter made with Kaiser window, $\beta = 8$.




## Low pass FIR filter design

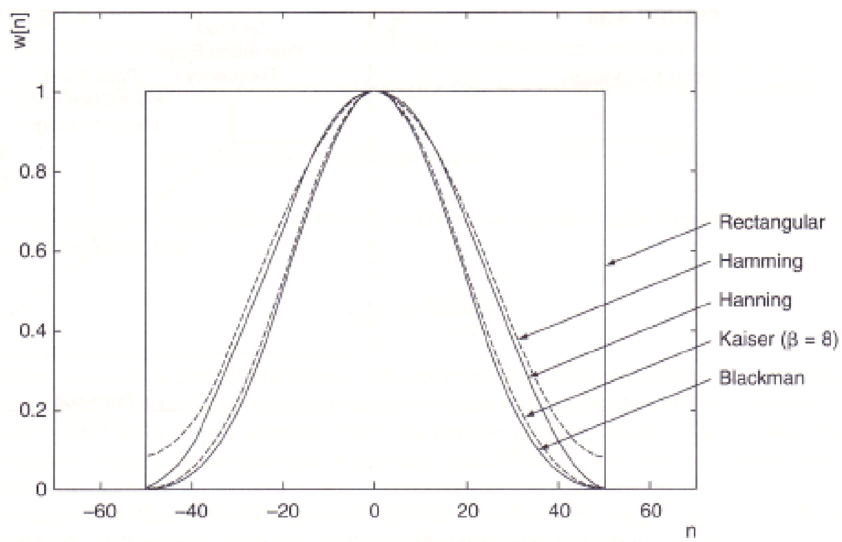



**FIGURE 9.34**
Window shape envelopes ($N$ = 101).

**TABLE 9.3**
FIR Filter Guidelines

| Window Type | Window Function $\lvert n\rvert \le \frac{N-1}{2}$ | Number of Terms, $N^*$ | Filter Stop Band Attenuation (dB) | Gain at Edge of Pass Band $20\log(1-\delta_p)$ (dB) |
|---|---|---|---|---|
| Rectangular | 1 | $0.91\frac{f_S}{\text{T.W.}}$ | 21 | −0.9 |
| Hanning | $0.5 + 0.5\cos\left(\frac{2\pi n}{N-1}\right)$ | $3.32\frac{f_S}{\text{T.W.}}$ | 44 | −0.06 |
| Hamming | $0.54 + 0.46\cos\left(\frac{2\pi n}{N-1}\right)$ | $3.44\frac{f_S}{\text{T.W.}}$ | 55 | −0.02 |
| Blackman | $0.42 + 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$ | $5.98\frac{f_S}{\text{T.W.}}$ | 75 | −0.0014 |
| Kaiser | $\frac{I_0\left(\beta\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right)}{I_0(\beta)}$ | $4.33\frac{f_S}{\text{T.W.}}$ ($\beta$ = 6) | 64 | −0.0057 |
| | | $5.25\frac{f_S}{\text{T.W.}}$ ($\beta$ = 8) | 81 | −0.00087 |
| | | $6.36\frac{f_S}{\text{T.W.}}$ ($\beta$ = 10) | 100 | −0.000013 |

$^*N$ = number of terms in window, $f_S$ = sampling frequency, T.W. = transition width.

**TABLE 9.4**

**Design Steps for Windowed Low Pass FIR Filter Design**

1. Choose a pass band edge frequency in Hz for the filter in the middle of the transition width:

$$f_1 = \text{Desired pass band edge frequency} + \frac{\text{Transition width}}{2}$$

2. Calculate $\Omega_1 = \dfrac{2\pi f_1}{fs}$ and substitute this value into $h_1[n]$, the infinite impulse response for an ideal low pass filter:

$$h_1[n] = \frac{\sin\left(n\Omega_1\right)}{n\pi}$$

3. Choose a window from Table 9.3 that will satisfy the stop band attenuation and other filter requirements. Calculate the number of nonzero window terms required using the formula for *N* in the table. **<u>Choose an odd number so that the impulse response can be perfectly symmetrical, thereby avoiding phase distortion in the final filter</u>**. Evaluate the window function $w[n]$ for $|n| \le \dfrac{(N-1)}{2}$.

4. Calculate the finite impulse response $h[n]$ for the filter from $h[n] = h_1[n]w[n]$ for $|n| \le (N-1)/2$, with $h[n] = 0$ for other values of *n*. This impulse response is non-causal.

5. Shift the impulse response values to the right by $\dfrac{(N-l)}{2}$ steps to ensure that the first nonzero value occurs at $n = 0$, thereby making the low pass filter causal.

**EXAMPLE 9.6**

A filter must have a stop band attenuation of 75 dB and a transition width of 1 kHz for 16 kHz sampling. Which window should be selected, and how many terms should be used?

---

The two windows in Table 9.3 that best satisfy the stop band requirement are the **Blackman** window and the $\beta = 8$ **Kaiser** window. The Blackman window require 5.98(16000/1000) = 95.68 = 95 terms. The ( $\beta = 8$ Kaiser window require 5.25(16000/1000) = 84 = 85 terms. In general, the number of terms required is rounded to the nearest **odd integer**. Rounding up produces a filter that exceeds specifications slightly, while rounding down gives a filter that falls a little short of the requirements.

### EXAMPLE 9.7

A low pass filter must be designed according to the following specifications:

| | |
|---|---|
| Pass band edge | 2 kHz |
| Stop band edge | 3 kHz |
| Stop band attenuation | 40 dB |
| Sampling frequency | 10 kHz |

Transition width = Stop band edge frequency - Pass band edge frequency

= 3 - 2 = 1 *kHz*

For step 1, the design method then yields

$$f_1 = 2000 + \frac{1000}{2} = 2500 Hz$$

For step 2,

$$\Omega_1 = 2\pi \frac{f_1}{fs} = 2\pi \frac{2500}{10000} = 0.5\pi$$

$$h_1[n] = \frac{\sin(n\Omega_1)}{n\pi} = \frac{\sin(0.5\pi n)}{n\pi}$$

For step 3, Hanning Window

$$N = 3.32 \frac{fs}{T.W} = 3.32 \frac{10}{1} = 33.2$$

Choosing $N = 33$, the window function becomes

$$w[n] = 0.5 + 0.5\cos\left(\frac{2\pi n}{N-1}\right)$$

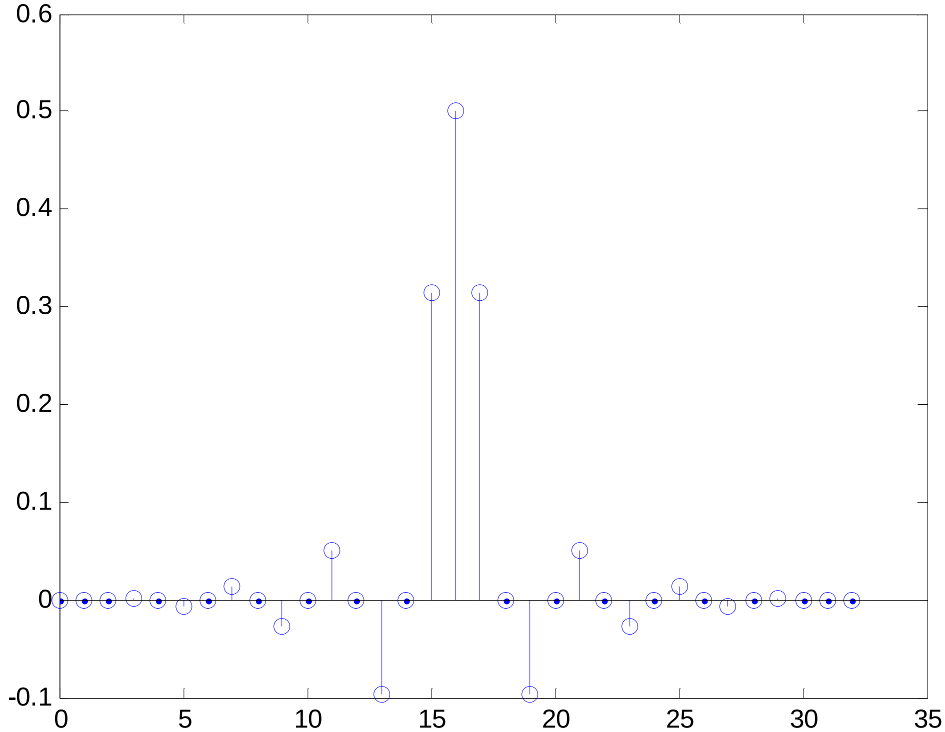
$$= 0.5 + 0.5\cos\left(\frac{2\pi n}{32}\right)$$

The impulse response is given by

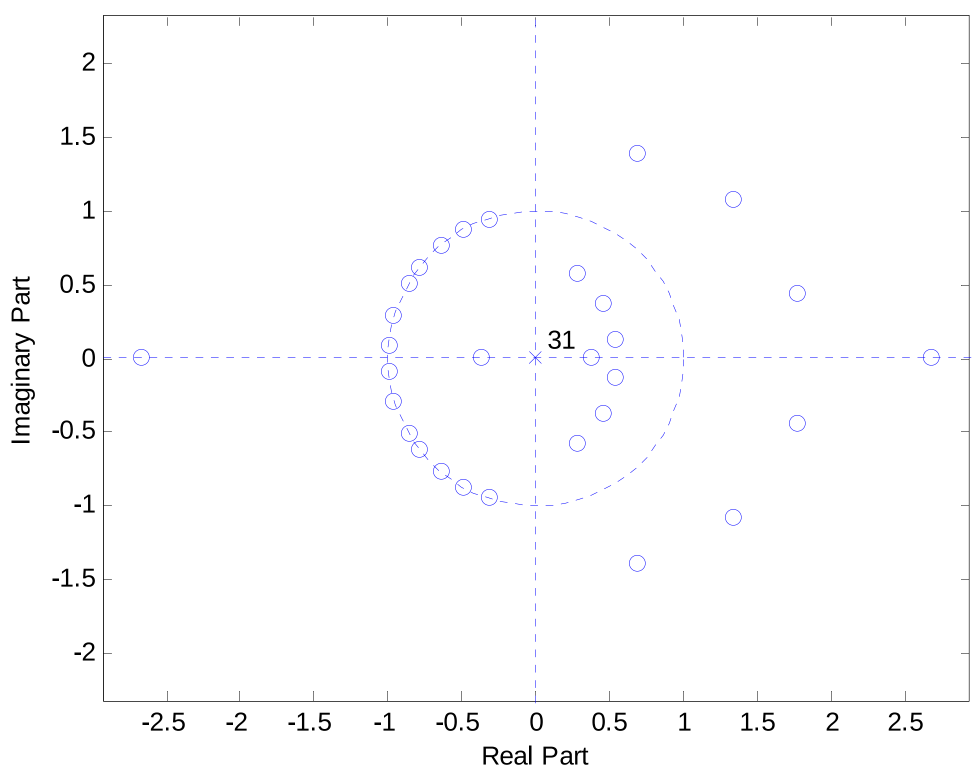
$$h[n] = h_1[n] \cdot w[n] \qquad \text{for } |n| \le 16$$

- The renumbering in the last column of the Table in the next page records the shifting of the impulse response values for causality.
- To obtain a causal realization of the filter, the $h[n]$ values must be associated with these new values of $n$.
- The impulse response values can also be used to find the transfer function $\mathrm{H}(z)$, from which the poles and zeros may be computed

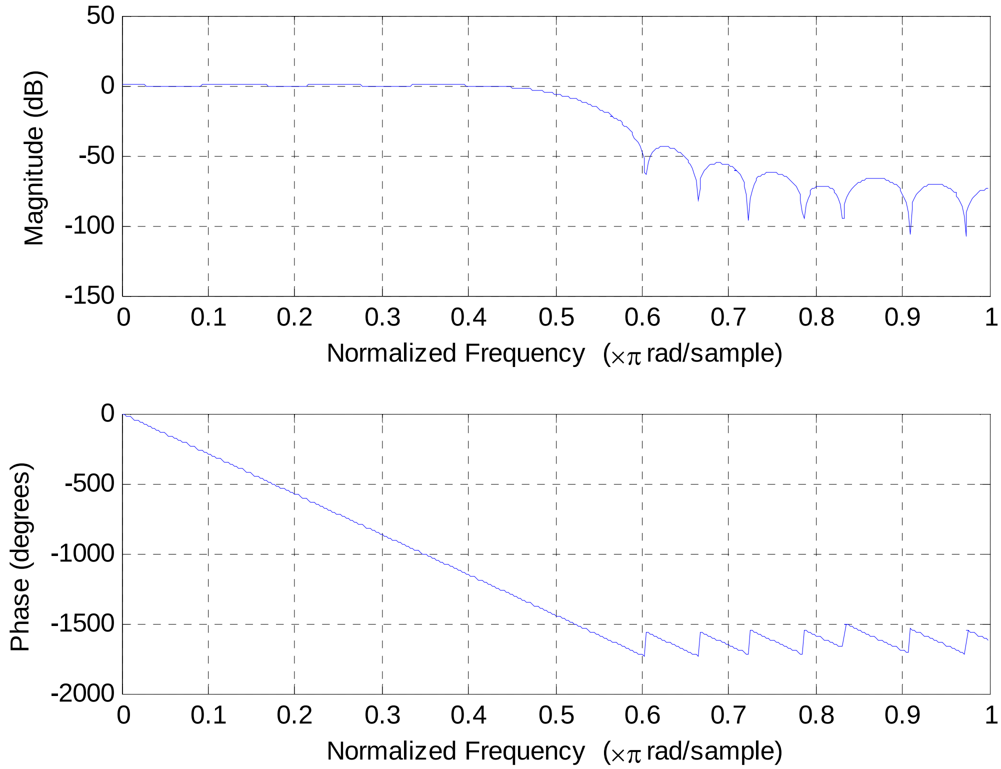
| $n$ | $h_1[n]$ | $w[n]$ | $h[n]$ | *new n causal* |
|---|---|---|---|---|
| -16 | 0.0000 | 0.0000 | 0.0000 | 0 |
| -15 | -0.0212 | 0.0096 | -0.0002 | 1 |
| -14 | 0.0000 | 0.0381 | 0.0000 | 2 |
| -13 | 0.0245 | 0.0843 | 0.0021 | 3 |
| -12 | 0.0000 | 0.1464 | 0.0000 | 4 |
| -11 | -0.0289 | 0.2222 | -0.0064 | 5 |
| -10 | 0.0000 | 0.3087 | 0.0000 | 6 |
| -9 | 0.0354 | 0.4025 | 0.0142 | 7 |
| -8 | 0.0000 | 0.5000 | 0.0000 | 8 |
| -7 | -0.0455 | 0.5975 | -0.0272 | 9 |
| -6 | 0.0000 | 0.6913 | 0.0000 | 10 |
| -5 | 0.0637 | 0.7778 | 0.0495 | 11 |
| -4 | 0.0000 | 0.8536 | 0.0000 | 12 |
| -3 | -0.1061 | 0.9157 | -0.0972 | 13 |
| -2 | 0.0000 | 0.9619 | 0.0000 | 14 |
| -1 | 0.3183 | 0.9904 | 0.3153 | 15 |
| 0 | 0.5000 | 1.0000 | 0.5000 | 16 |
| 1 | 0.3183 | 0.9904 | 0.3153 | 17 |
| 2 | 0.0000 | 0.9619 | 0.0000 | 18 |
| 3 | -0.1061 | 0.9157 | -0.0972 | 19 |
| 4 | 0.0000 | 0.8536 | 0.0000 | 20 |
| 5 | 0.0637 | 0.7778 | 0.0495 | 21 |
| 6 | 0.0000 | 0.6913 | 0.0000 | 22 |
| 7 | -0.0455 | 0.5975 | -0.0272 | 23 |
| 8 | 0.0000 | 0.5000 | 0.0000 | 24 |
| 9 | 0.0354 | 0.4025 | 0.0142 | 25 |
| 10 | 0.0000 | 0.3087 | 0.0000 | 26 |
| 11 | -0.0289 | 0.2222 | -0.0064 | 27 |
| 12 | 0.0000 | 0.1464 | 0.0000 | 28 |
| 13 | 0.0245 | 0.0843 | 0.0021 | 29 |
| 14 | 0.0000 | 0.0381 | 0.0000 | 30 |
| 15 | -0.0212 | 0.0096 | -0.0002 | 31 |
| 16 | 0.0000 | 0.0000 | 0.0000 | 32 |



0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
5
10
15
20
25
30
35

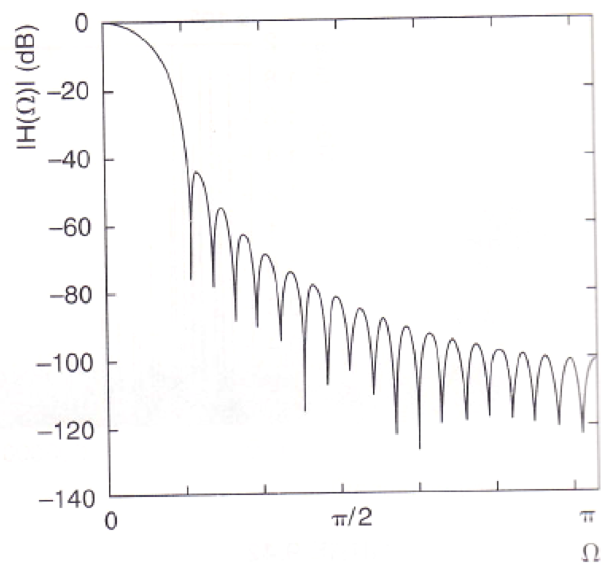
Imaginary Part
Real Part
31
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5



50
0
-50
-100
-150
Magnitude (dB)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Normalized Frequency (×π rad/sample)
0
-500
-1000
-1500
-2000
Phase (degrees)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Normalized Frequency (×π rad/sample)

## Band pass and high pass FIR filters

**FIGURE 9.43**
One-sided magnitude response of low pass filter.



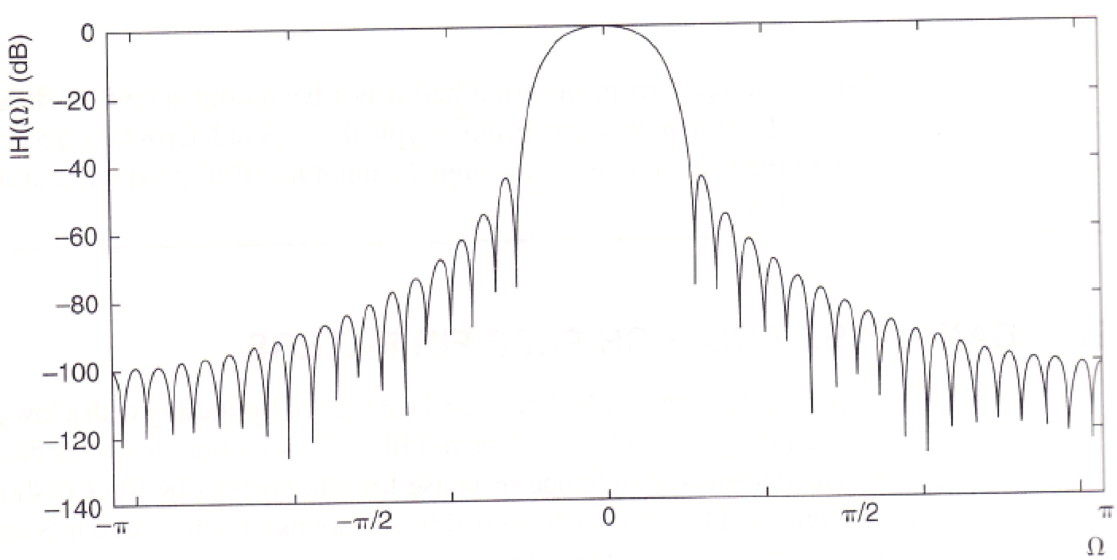




**FIGURE 9.44**
Two-sided magnitude response of low pass filter.

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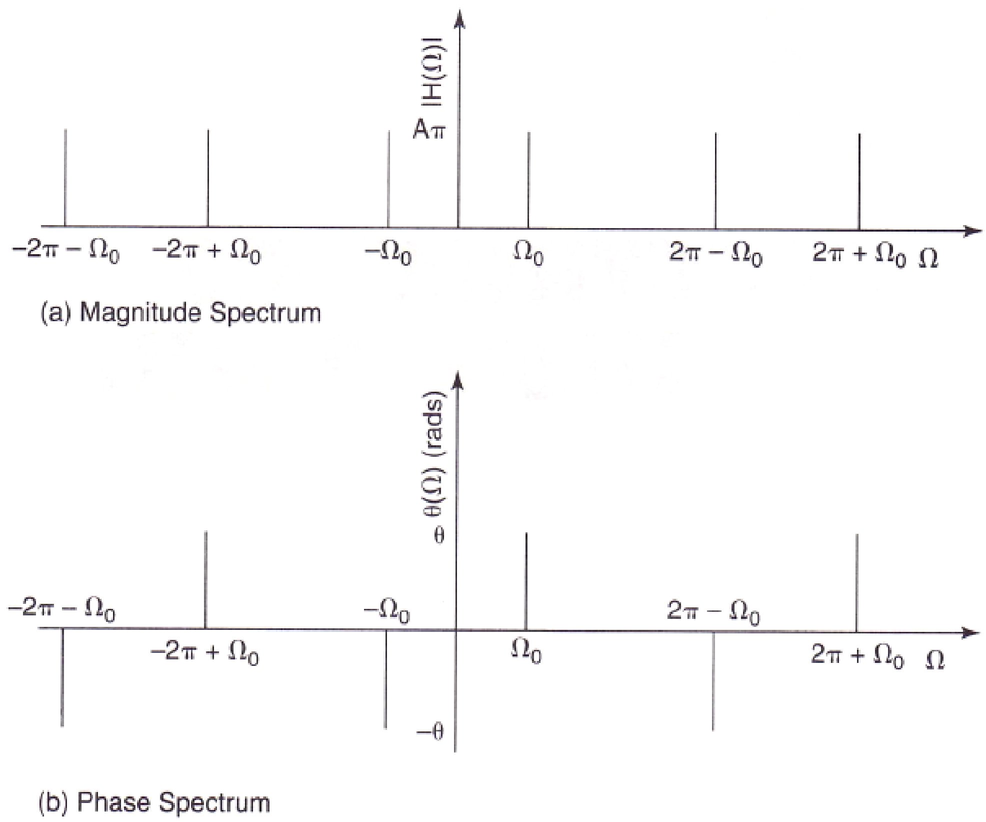


**FIGURE I.1**
Magnitude and phase spectra of cosine signal.

The impulse response of the low pass filter must be multiplied by the cosine function

$$q[n] = \cos(n\Omega_0)$$

to produce

$$h[n] = h_1[n]w[n]\cos(n\Omega_0).$$

For **high pass filter**

$$h_{high}[n] = \cos(n\pi)h_{low}[n] = (-1)^n h_{low}[n].$$

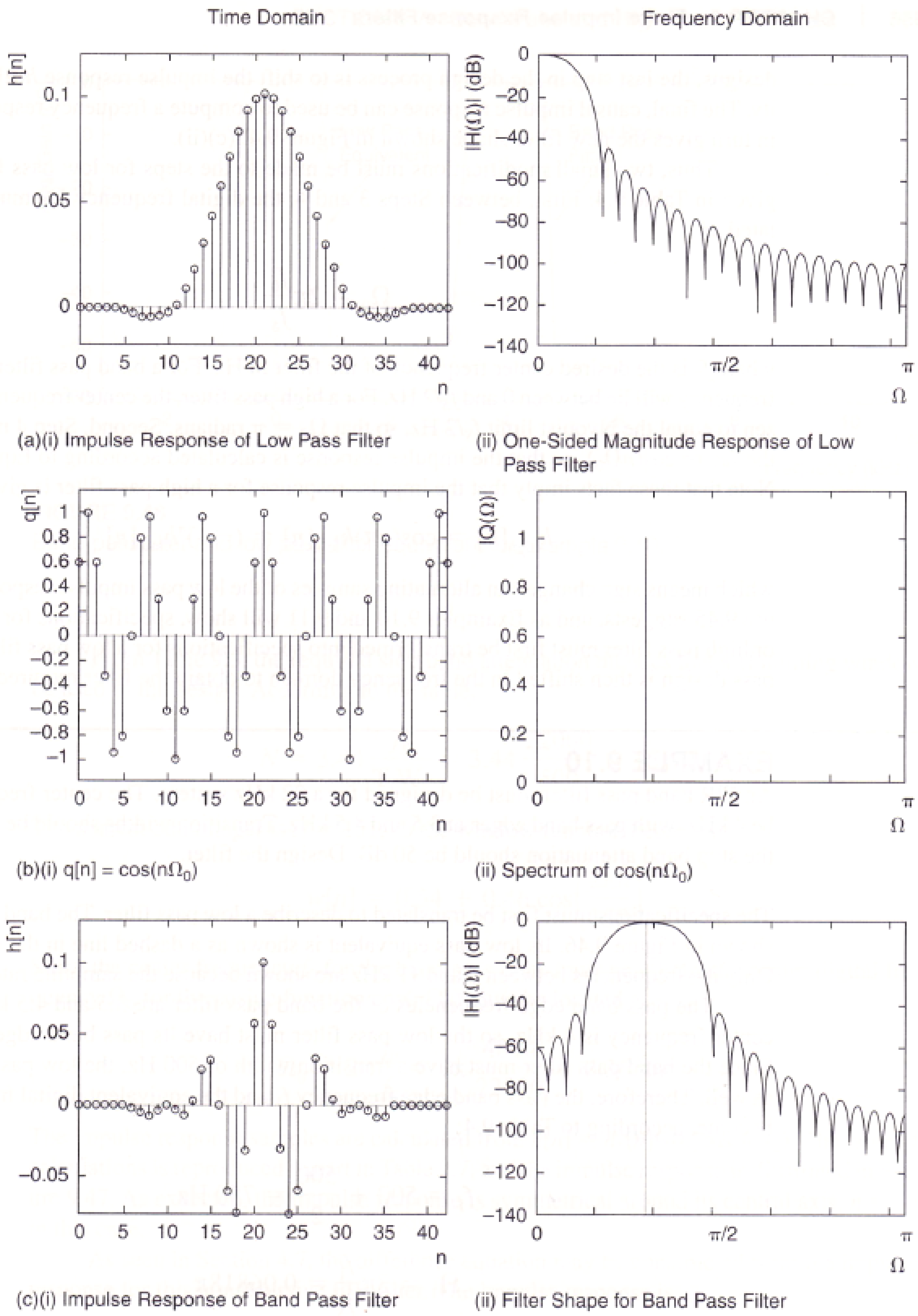


**FIGURE 9.45**
Construction of a band pass filter.

## EXAMPLE 9.10

FIR **band pass filler** must be designed for a 22 kHz (sampling rate) system. The center frequency must be 4 kHz, with pass band edges at 3.5 and 4.5 kHz. Transition widths should be 500 Hz. and the stop band attenuation should be 50 dB. Design the filler.

The specifications must first be translated to describe a low pass filter. The band pass filter is shown in Figure 9.46. Its low pass equivalent is shown as a dashed line in the same figure. Only the frequencies between 0 and 11 kHz are shown because the sampling rate is 22 kHz. The pass band edge frequencies of the band pass filter are 3.5 and 4.5 kHz and the center frequency is 4 kHz, so the low pass filter must have its pass band edge at 500 Hz. Since the band pass filter must have a transition width of 500 Hz, the low pass filter must as well. Therefore, the pass band edges frequency $f_1$ , and the equivalent digital frequency $\Omega_1$ , become, according to Table 9.4:

$$f_1 = 500 + \frac{500}{2} = \; 750\ Hz$$

$$\Omega_1 = 2\pi \frac{f_1}{fs} = 0.06818\pi$$

The impulse response for the ideal low pass filter with this pass band edge is

SECTION 9.7 Band Pass and High Pass FIR Filters | 357

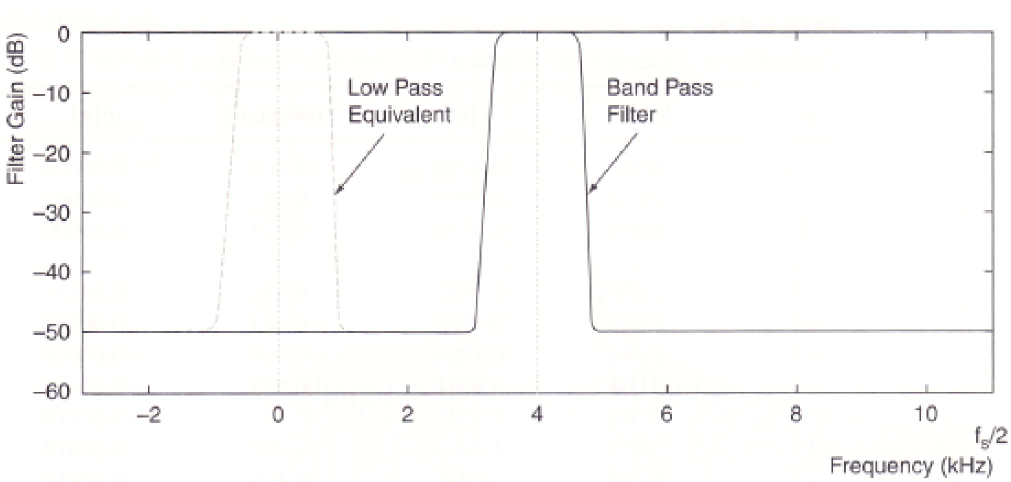


**FIGURE 9.46**
Band pass filter and low pass filter equivalent for Example 9.10.

$$h_1[n] = \frac{\sin(n\Omega_1)}{n\pi} = \frac{\sin(0.0681\pi)}{n\pi}$$

From Table 9.3, the required stop band attenuation dictates that a **Hamming** window be used in the design. According to the table,

$$N = 3.44\frac{fs}{T.W.} = 3.44\left[\frac{22000}{500}\right] = 151.4 \rightarrow \underline{N = 151}$$

The nearest odd integer is $N = 151$, so the window function becomes

$$w[n] = 0.54 + 0.46\cos\left(\frac{2\pi n}{150}\right)$$

Since the center frequency $f_0$ for the band pass filter must be 4 kHz. The center digital frequency $\Omega_0$ for the cosine function $\cos(n\Omega_0)$ must be located at

$$\Omega_0 = 2\pi\frac{f_0}{fs} = 2\pi\frac{4000}{22000} = 0.3636\pi$$

The impulse response samples are calculated from

$$h[n] = h_1[n]w[n]\cos(n\Omega_0)\,.$$

| $n$ | $h_1[n]$ | $w[n]$ | $\cos(n\Omega_0)$ | $h[n]$ | new n |
|---|---|---|---|---|---|
| -75 | -0.001 | 0.0800 | -0.661 | 0.00008 | 0 |
| -74 | -0.001 | 0.0804 | -0.957 | 0.00005 | I |
| -73 | 0.000 | 0.0816 | -0.134 | 0.00000 | 2 |
| -5 | 0.056 | 0.9899 | 0.841 | 0.04651 | 70 |
| -4 | 0.060 | 0.9936 | -0.143 | -0.00853 | 71 |
| -3 | 0.064 | 0.9964 | -0.960 | -0.06079 | 72 |
| -2 | 0.066 | 0.9984 | -0.655 | -0.04321 | 73 |
| -1 | 0.068 | 0.9996 | 0.416 | 0.02810 | 74 |
| 0 | 0.068 | 1.0000 | 1.000 | 0.06818 | 75 |
| 1 | 0.068 | 0.9996 | 0.416 | 0.02810 | 76 |
| 2 | 0.066 | 0.9984 | -0.655 | -0.04321 | 77 |
| 3 | 0.064 | 0.9964 | -0.960 | -0.06079 | 78 |
| 4 | 0.060 | 0.9936 | -0.143 | -0.00853 | 79 |
| 5 | 0.056 | 0.9899 | 0.841 | 0.04651 | 80 |
| 73 | 0.000 | 0.0816 | -0.134 | 0.00000 | 148 |
| 74 | -0.001 | 0.0804 | -0.957 | 0.00005 | 149 |
| 75 | -0.00! | 0.0800 | -0.661 | 0.00008 | 150 |

**FIGURE 9.47**
Band pass filter impulse response for Example 9.10.

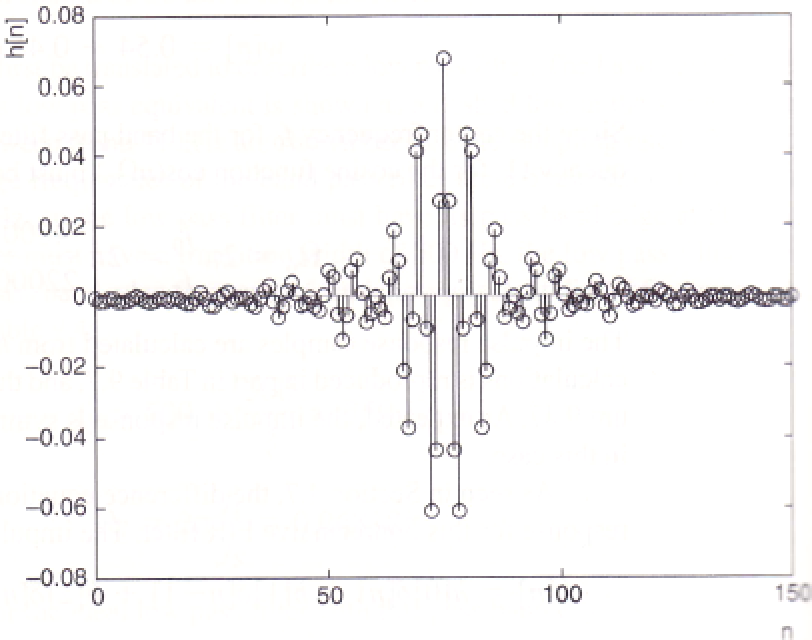


**FIGURE 9.48**
Filter shape of band pass filter response for Example 9.10.