EE 538B CMOS RF IC DESIGN

Midterm Examination No. 1: April 24, 2002

Hi = 100
Lo = 70
$\mu = 86.5$
$\sigma = 8.4$

Time Allowed: 110 Minutes

Student Name: Solutions

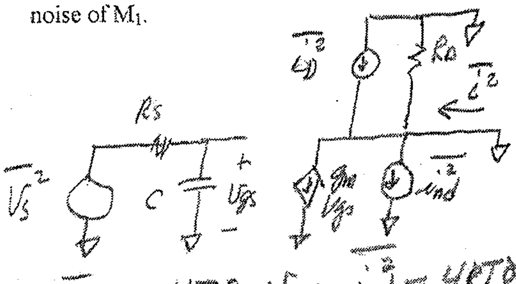
UW Student ID #: 123-45-6789

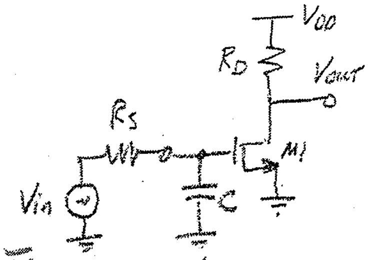
You are allowed one sheet of notes. Write legibly. Show all work. State assumptions.

Problem #	Points Possible	Points
1	**20**	20
2	**10**	10
3	**25**	25
4	**25**	25
5	**20**	20

100

1. (20 points) Find the noise factor of the circuit shown with respect to the source resistance R_S. Neglect channel length modulation effects and all parasitic capacitors. Consider only drain current noise of M_1.





$$\overline{V_S^2} = 4kTR_S\Delta f \; ; \; \overline{i_{nd}^2} = 4kT\gamma g_{do}\Delta f \; ; \; \overline{i_D^2} = \frac{4kT}{R_D}\Delta f$$

The three noise sources are uncorrelated:

(i) Consider $\overline{i_D^2}$: $\overline{i_1^2} = \overline{i_D^2} = \frac{4kT}{R_D}\Delta f$

(ii) Consider $\overline{i_{nd}^2}$: $\overline{i_2^2} = \overline{i_{nd}^2} = 4kT\gamma g_{do}\Delta f$

(iii) Consider $\overline{V_S^2}$:

$$V_{gs} = \frac{1/sC}{R_S + 1/sC}\overline{V_S} = \frac{1}{sR_SC+1}\overline{V_S}$$

$$\Rightarrow \quad \overline{V_{gs}^2} = \frac{1}{(\omega R_S C)^2+1}\overline{V_S^2}$$

$$\overline{i_3^2} = g_m^2 \overline{V_{gs}^2} = \frac{g_m^2}{(\omega R_S C)^2+1}\overline{V_S^2}$$

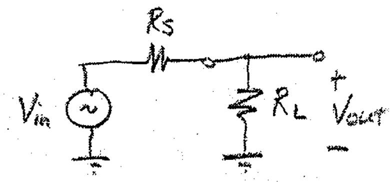
Finally, $F = \dfrac{\overline{i_1^2} + \overline{i_2^2} + \overline{i_3^2}}{\overline{i_3^2}}$

$$= 1 + \frac{\frac{4kT\Delta f}{R_D} + 4kT\gamma g_{do}\Delta f}{\frac{g_m^2}{(\omega R_S C)^2+1} \, 4kTR_S\Delta f}$$

Note that F is increased due to the presence of C as expected.)

$$\therefore F = 1 + \frac{[1+(\omega R_S C)^2]}{g_m^2 R_D R_S} + \frac{[1+(\omega R_S C)^2]\gamma g_{do}}{g_m^2 R_S}$$

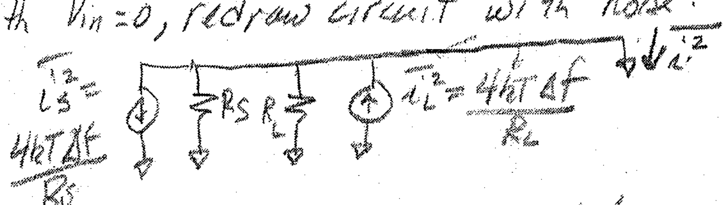
2. (10 points) With respect to a source resistance R_S:
(a) What value of R_L maximizes power transfer?



We obtain maximum power transfer when the load impedance is the complex conjugate of the source impedance; i.e., $Z_L = Z_S^*$
In this case, $R_L = R_S$.

(b) What value of R_L minimizes the noise factor F?

With $V_{in}=0$, redraw circuit with noise:

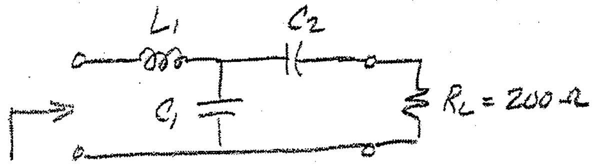


The noise sources are uncorrelated.
Hence, $\overline{i^2} = \overline{i_S^2} + \overline{i_L^2}$

$$\therefore F = \frac{\overline{i^2}}{\overline{i_S^2}} = 1 + \frac{\frac{4kT\Delta f}{R_L}}{\frac{4kT\Delta f}{R_S}} = 1 + \frac{R_S}{R_L}$$

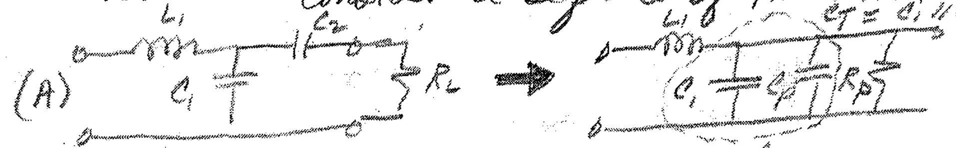
Clearly $F = F_{min} = 1$ when $R_L = \infty$.
This simple examples illustrates the key point that optimum gain match and optimum noise match are different.

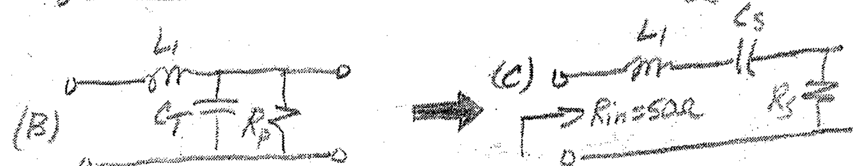
3. (25 points) With Q=10, the matching circuit below is used to convert $R_L = 200\Omega$ to $R_{IN} = 50\Omega$ at f_O = 5.6GHz. Determine the required component values.



(This is a bandpass matching network.)

$R_{in} = 50\,\Omega$ Consider a sequence of transformations:





The final circuit is pure series RLC circuit with:

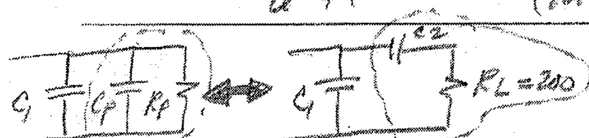
$Q = 10$; $R_S = R_{in} = 50\Omega$; $f_0 = 5.6\,GHz$

For (C): $Q = 10 = \frac{\omega_0 L_1}{R_S} \Rightarrow L_1 = \frac{Q R_S}{\omega_0} = \boxed{14.2\,nH}$ ← L_1

$Q = 10 = \frac{1}{\omega_0 R_S C_S} \Rightarrow C_S = \frac{1}{Q R_S \omega_0} = \boxed{56.8\,fF}$

For (B): $R_p = R_S(Q^2+1) = 50\,(101) = \boxed{5050\,\Omega}$

$C_T = \frac{C_S\, Q^2}{Q^2+1} = \frac{(56.8\,fF)(100)}{(101)} = \boxed{56.3\,fF}$



$Q_2 = \sqrt{\frac{R_p}{R_L} - 1} = \boxed{4.92}$

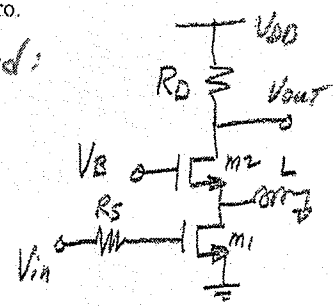
$Q_2 = \frac{1}{\omega_0 R_L C_2} \Rightarrow C_2 = \frac{1}{Q_2 \omega_0 R_L} = \boxed{28.9\,fF}$ ← C_2

$Q_2 = \omega_0 R_p C_p \Rightarrow C_p = \frac{Q_2}{\omega_0 R_p} = \boxed{27.7\,fF}$

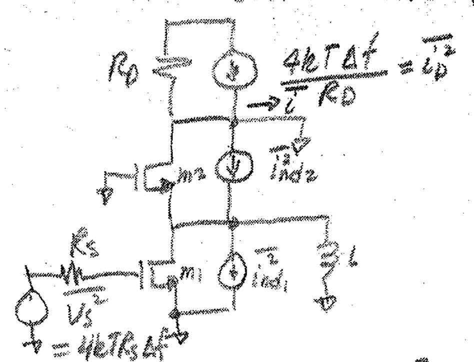
Finally, $C_1 = C_T - C_p = \boxed{28.6\,fF}$ ← C_1

This means $r_{ds1} = r_{ds2} = \infty$.

4. (25 points) Find an expression for the noise factor of the circuit below with respect to the source resistance R_S. Neglect channel length modulation and all parasitic capacitors; also, neglect induced gate current noise. Assume that the DC bias voltage across L is zero.



The noise sources are uncorrelated:

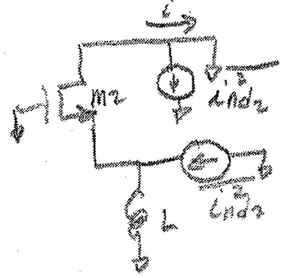


$$\overline{i^2_{n,R_S}} = \underbrace{4kTR_S\Delta f}_{\overline{V_S^2}} \cdot g_{m1}^2 \cdot \left[\frac{g_{m2}^2(\omega L)^2}{1+g_{m2}^2(\omega L)^2}\right]$$

Note: current divider between L and M2.

$$\overline{i^2_{n,m1}} = 4kT\gamma_1 g_{d01}\Delta f \cdot \left[\frac{g_{m2}^2(\omega L)^2}{1+g_{m2}^2(\omega L)^2}\right]$$

$$\overline{i^2_{n,R_D}} = \frac{4kT\Delta f}{R_D}$$



Note: When we use this equivalent circuit, the components in $\overline{i}$ are obviously correlated. Hence,

$$\overline{i^2_{n,m2}} = 4kT\gamma_2 g_{d02}\Delta f \left|\frac{sL}{\frac{1}{g_{m2}}+sL} - 1\right|^2$$

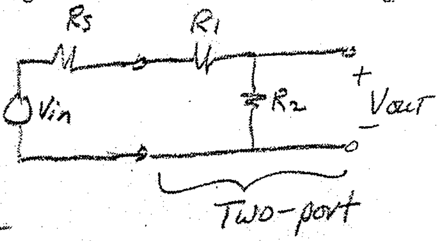
$$= 4kT\gamma_2 g_{d02}\Delta f \left[\frac{1}{1+g_{m2}^2(\omega L)^2}\right]$$

Finally,

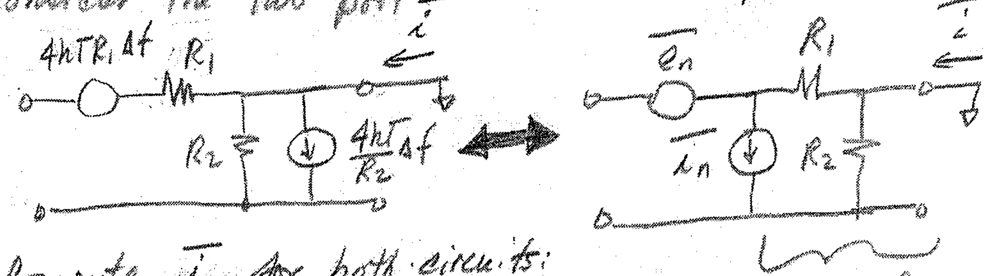
$$F = 1 + \frac{\gamma g_{d01}}{g_{m1}^2 R_S} + \frac{1}{g_{m1}^2 R_S R_D} + \frac{1+\gamma_2 g_{d02} R_D}{g_{m1}^2 g_{m2}^2 R_S R_D (\omega L)^2} \quad \text{F}$$

Note: L decreases F; in HW C increased F;

5. (20 points) Determine expressions for the equivalent two-port input noise voltage and current power sources for the two-port network shown assuming it is driven from a source resistor R_S.



Consider the two port:



Compute $\overline{i}$ for both circuits:

R_1 and R_2 assumed noise free in this case.

(a) Short Input:

$$\overline{i^2} = \frac{4kTR_1\Delta f}{R_1^2} + \frac{4kT\Delta f}{R_2} = 4kT\Delta f\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

(a) Short input:

$$\overline{i^2} = \frac{\overline{e_n^2}}{R_1^2}$$

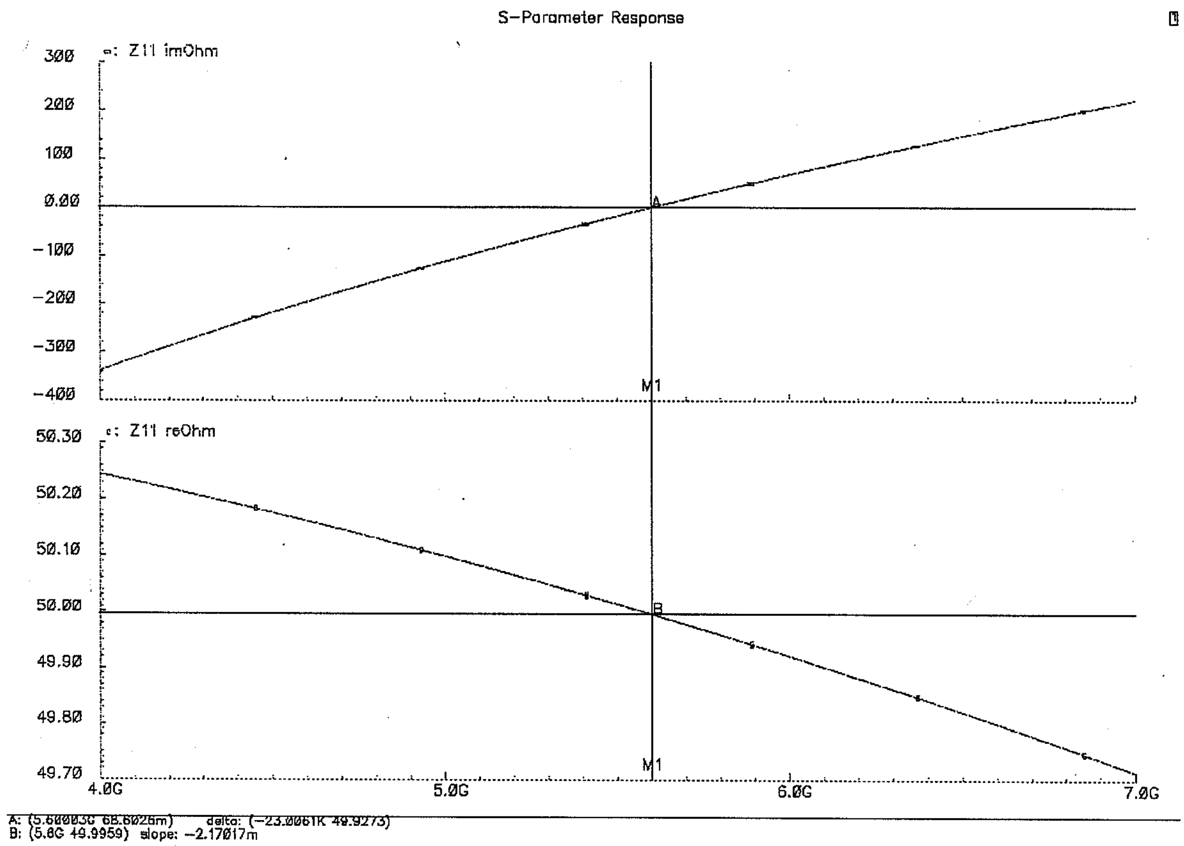
Hence, $\overline{e_n^2} = R_1^2\,\overline{i^2} = \boxed{4kT\Delta f\, R_1^2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$ ← $\overline{e_n^2}$

(b) Open input:

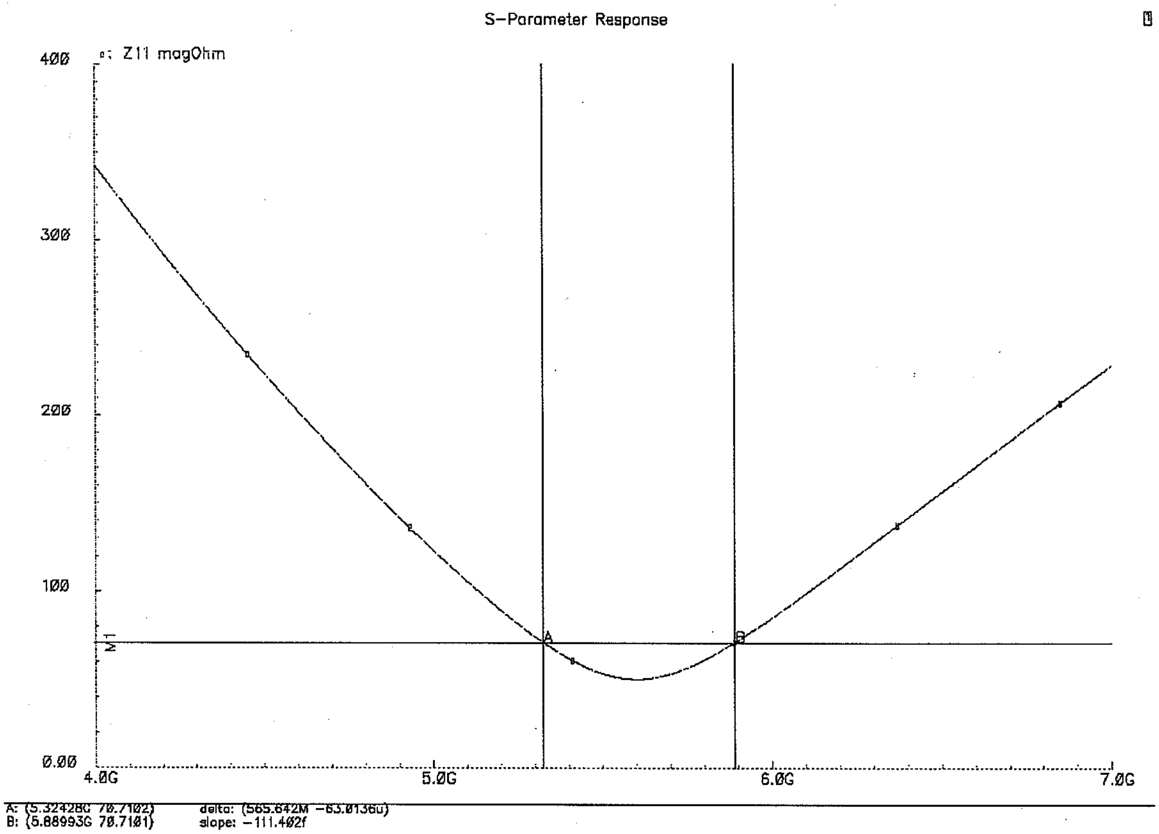
$$\overline{i^2} = \frac{4kT\Delta f}{R_2}$$

$$\overline{i^2} = \overline{i_n^2}$$

Hence, $\overline{i_n^2} = \boxed{\frac{4kT\Delta f}{R_2}}$ ← $\overline{i_n^2}$



Prob-3 <u>Zin@5.6GHz</u> = 50 ohm



Prob-3 BW$\cong 560MHz$