## Ch 11 DFT and FFT Processing

- The **DFT** (Discrete Fourier Transform) identifies the frequency contents in a signal from a set of the signal's samples.

- Most efficient implementation of DFT is **FFT** (Fast Fourier Transform)

The definition of DTFT was given

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$
$$= \left| X(\Omega) \right| e^{j\theta(\Omega)}$$

Problems of implementing DTFT on a computer: An infinite number of samples are required.

Solution: DFT – by making the number of samples finite.

The DFT equation: This equation transforms a time domain signal $x[n]$ into a frequency domain signal $X[k]$.

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi \frac{k}{N} n} \qquad \text{for } k = 0 \ldots N\text{-}1$$

$$= \left| X[k] \right| e^{j\theta[k]}$$

The number, $N$, of time samples $x[n]$ used as input to the DFT is the same as the number, $N$, of frequency samples $X[k]$ produced as output.

- DFT magnitude spectrum against *k*.
- DFT phase spectrum against *k*.
- $X[k]$ is periodic.

$$X[k+N] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi\frac{(k+N)}{N}n}$$

$$= \sum_{n=0}^{N-1} x[n] e^{-j2\pi\frac{k}{N}n} e^{-j2\pi\frac{N}{N}n}$$

$$= \sum_{n=0}^{N-1} x[n] e^{-j2\pi\frac{k}{N}n} \underbrace{\cancel{e^{-j2\pi\frac{N}{N}n}}}_{=1}$$

$$= \sum_{n=0}^{N-1} x[n] e^{-j2\pi\frac{k}{N}n} = X[k]$$

The **IDFT** (Inverse Discrete Fourier Transform) is also periodic with period, *N*

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j2\pi\frac{k}{N}n} \qquad \text{for } k = 0 \ldots N\text{-}1$$

When the DFT and DTFT operate on the same set of time samples, the DFT matches the DTFT exactly, but **at a finite number of points only.**

Given the following $x[n]$

$$x[n]=\delta[n]+\delta[n-1]+\delta[n-2]$$

a. Find the Fourier transform of $x[n]$, that is $FT\{x[n]\}=X\left(e^{j\Omega}\right)$, and plot the $\left|X\left(e^{j\Omega}\right)\right|$
b. Give the magnitude of the four-point DFT of the first four samples of $x[n]$ and plot
c. Give the magnitude of the eight-point DFT of the first eight samples of $x[n]$ and plot
d. Is the DFT the sampled version of $X\left(e^{j\Omega}\right)$
e. Is it possible to reconstruct the $X\left(e^{j\Omega}\right)$ from the eight-point DFT of part (c)

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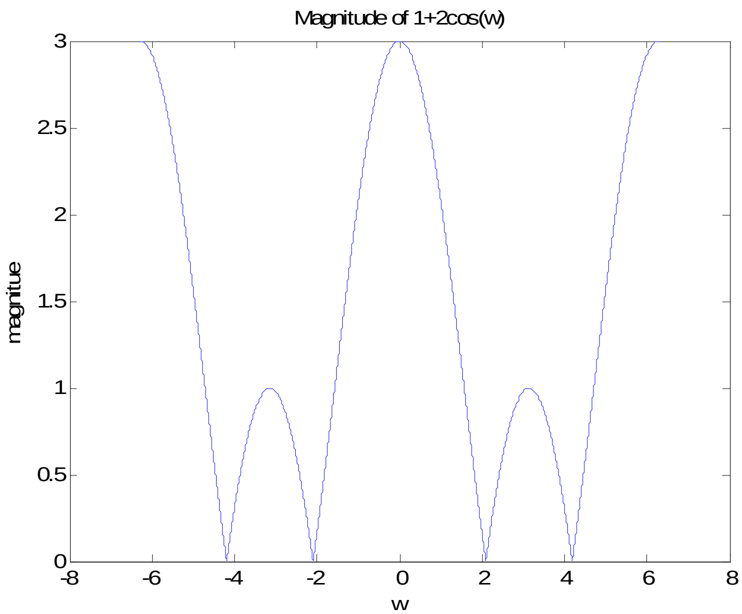
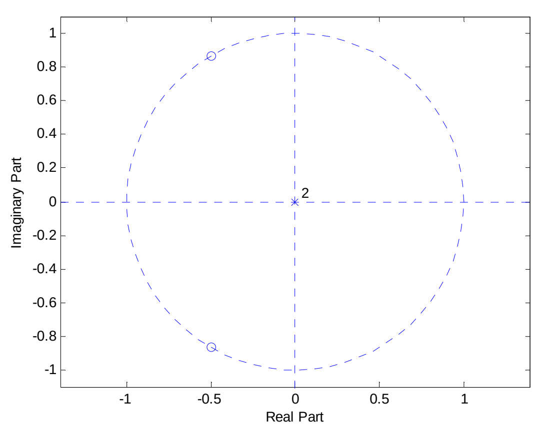
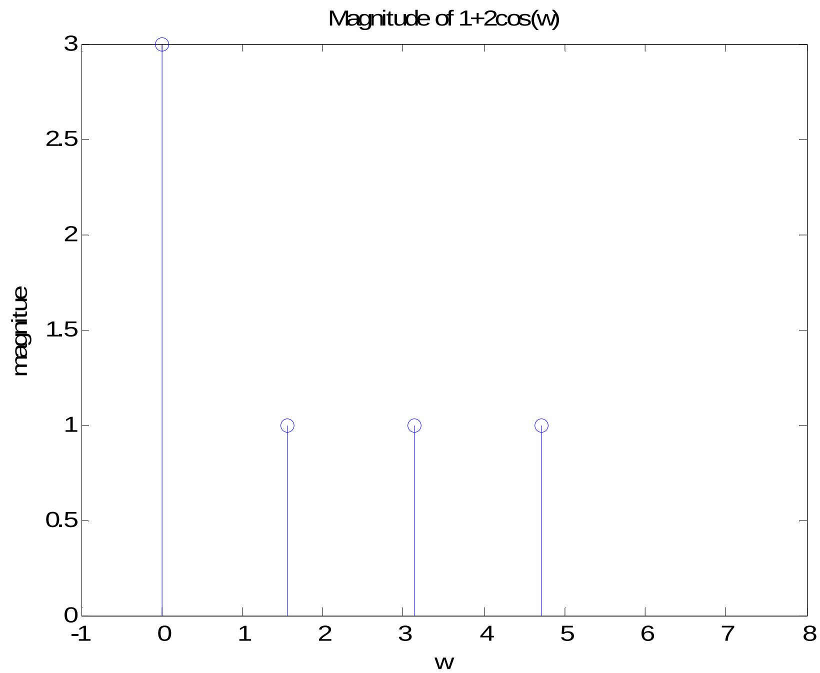
$$X\left(e^{j\Omega}\right)=1+e^{-j\Omega}+e^{-2j\Omega}=e^{-j\Omega}\left(e^{-j\Omega}+1+e^{j\Omega}\right)$$
$$=e^{-j\Omega}\left[1+2\cos(\Omega)\right]$$

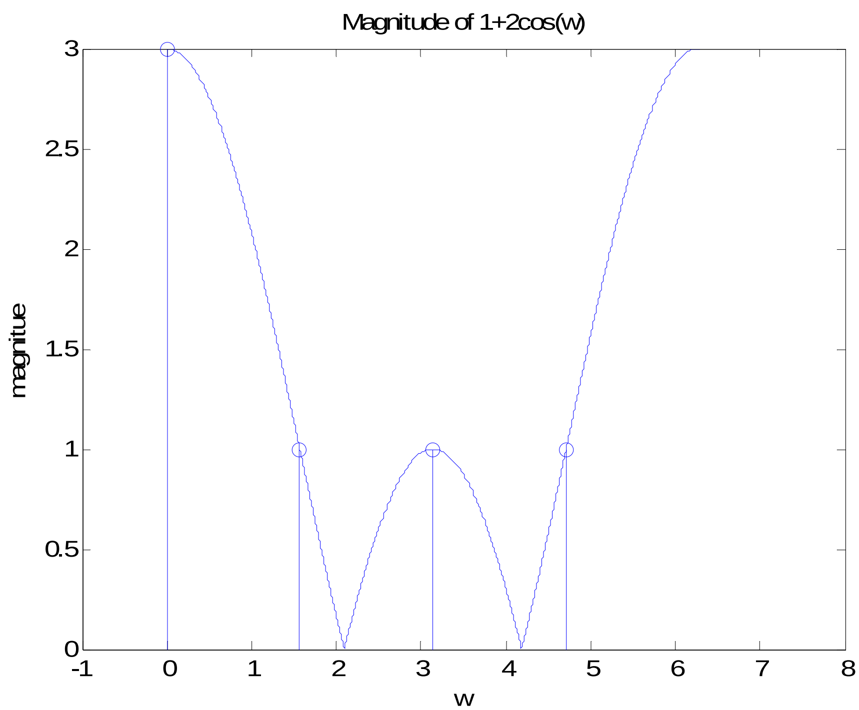
$$\left|X\left(e^{j\Omega}\right)\right|=\left|e^{-j\Omega}\left[1+2\cos(\Omega)\right]\right|=\left|e^{-jw}\right|\left|1+2\cos(\Omega)\right|=\left|1+2\cos(\Omega)\right|$$



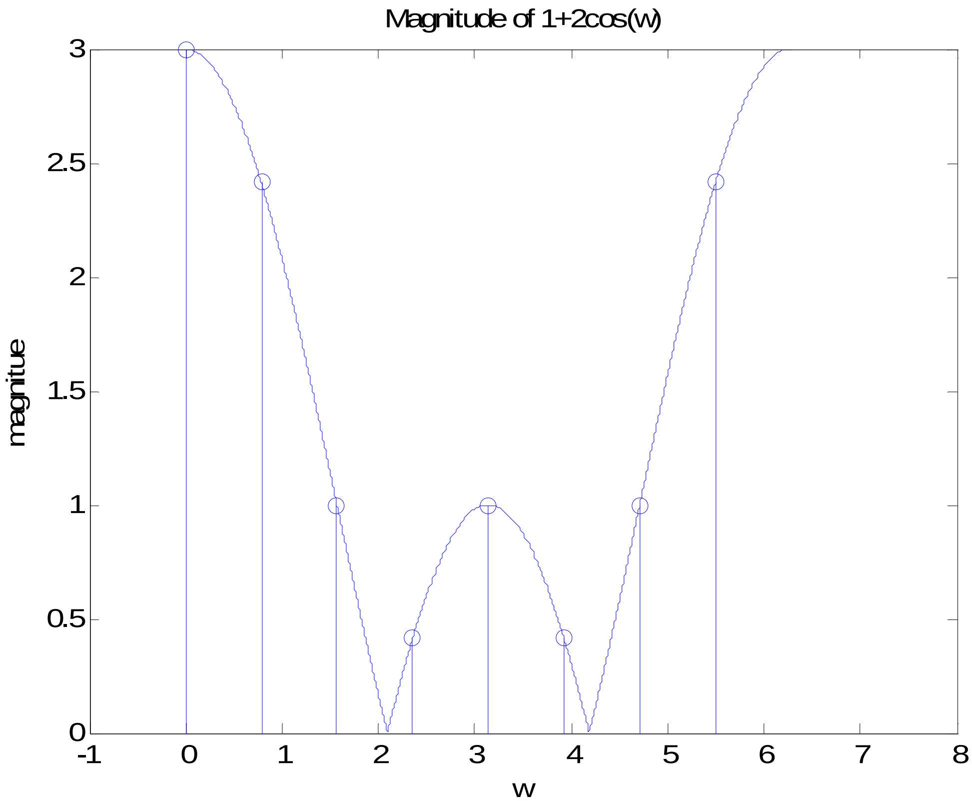




-0.5000 + 0.8660i
-0.5000 - 0.8660i




3.0000    1.0000    1.0000    1.0000

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Columns 1 through 7

  3.0000    2.4142    1.0000    0.4142    1.0000    0.4142    1.0000

Column 8

  2.4142
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d. Is the DFT the sampled version of $X\left(e^{j\omega}\right)$: YES

e. Is it possible to reconstruct the $X\left(e^{j\omega}\right)$ from the eight-point DFT of part (c)

YES from complex values
NO from only magnitudes unless other info are given.

**Ex 11.1)**

Find the DFT magnitude and phase spectra for samples of the signal and verify that the inverse DFT reproduces these samples

$$x[n] = 5\delta[n] + 2\delta[n-1] - 2\delta[n-2] + 4\delta[n-3]$$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi\frac{k}{N}n} = x[0] + x[1]e^{-j2\pi\frac{k}{4}1} + x[2]e^{-j2\pi\frac{k}{4}2} + x[3]e^{-j2\pi\frac{k}{4}3}$$

$$= 5 \quad + \quad 2e^{-j\pi\frac{k}{2}} \quad - \quad 2e^{-j\pi k} \quad + \quad 4e^{-j\pi\frac{k3}{2}}$$

$$X[0] = 5 + 2 - 2 + 4 = 9$$
$$X[1] = 5 + 2(-j) - 2(-1) + 4(j) = 7 + 2j$$
$$X[2] = 5 + 2(-1) - 2(1) + 4(-1) = -3$$
$$X[3] = 5 + 2(j) - 2(-1) + 4(-j) = 7 - 2j$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi\frac{k}{N}n} \qquad \text{for } k = 0 \ldots N\text{-}1$$

$$= \frac{1}{N}\left[ X[0] + X[1]e^{j\pi\frac{n}{2}} + X[2]e^{j\pi} + X[3]e^{j\pi\frac{3n}{2}} \right]$$

$$x[0] = \frac{1}{4}\left[9 + (7+2j) - 3 + (7-2j)\right]$$
$$= \frac{1}{4}\left[9 + 7 + 7 - 3 + (+2j - 2j)\right]$$
$$= \frac{20}{4}$$
$$= 5$$