Fall 2020

EE 210

HW#: 10

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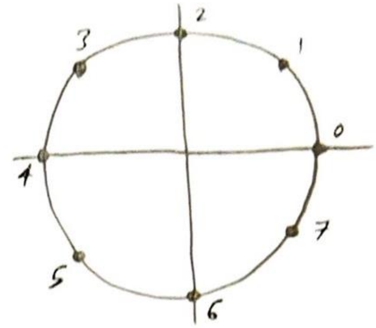
Date: **11/21/2020**

Assigned question #s: **9**

HW 10

(11.1) a) $x[n]$ for $0 \leq n \leq 7$

$N = 8$



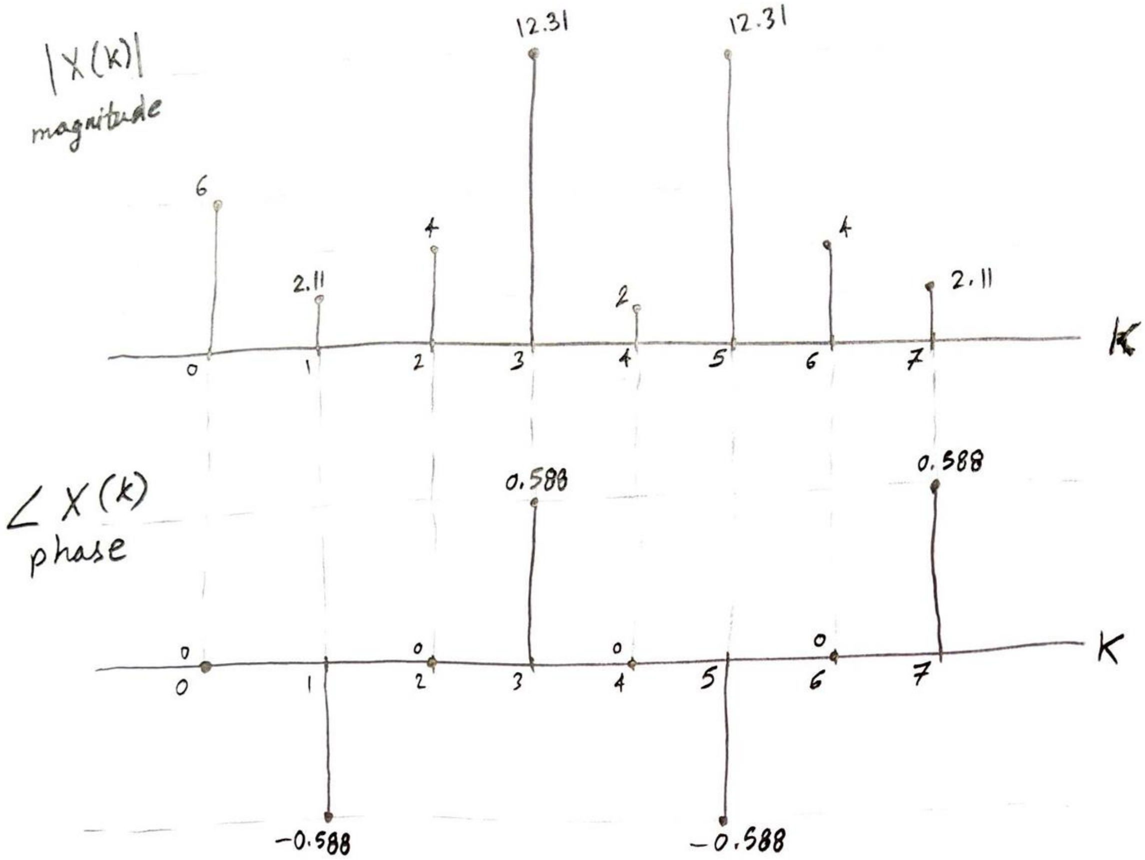
$$x[n] = [5, -2, 2, 1, -1, 3, -2, 0]$$

DFT: $$X(k) = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k n}{N}}$$

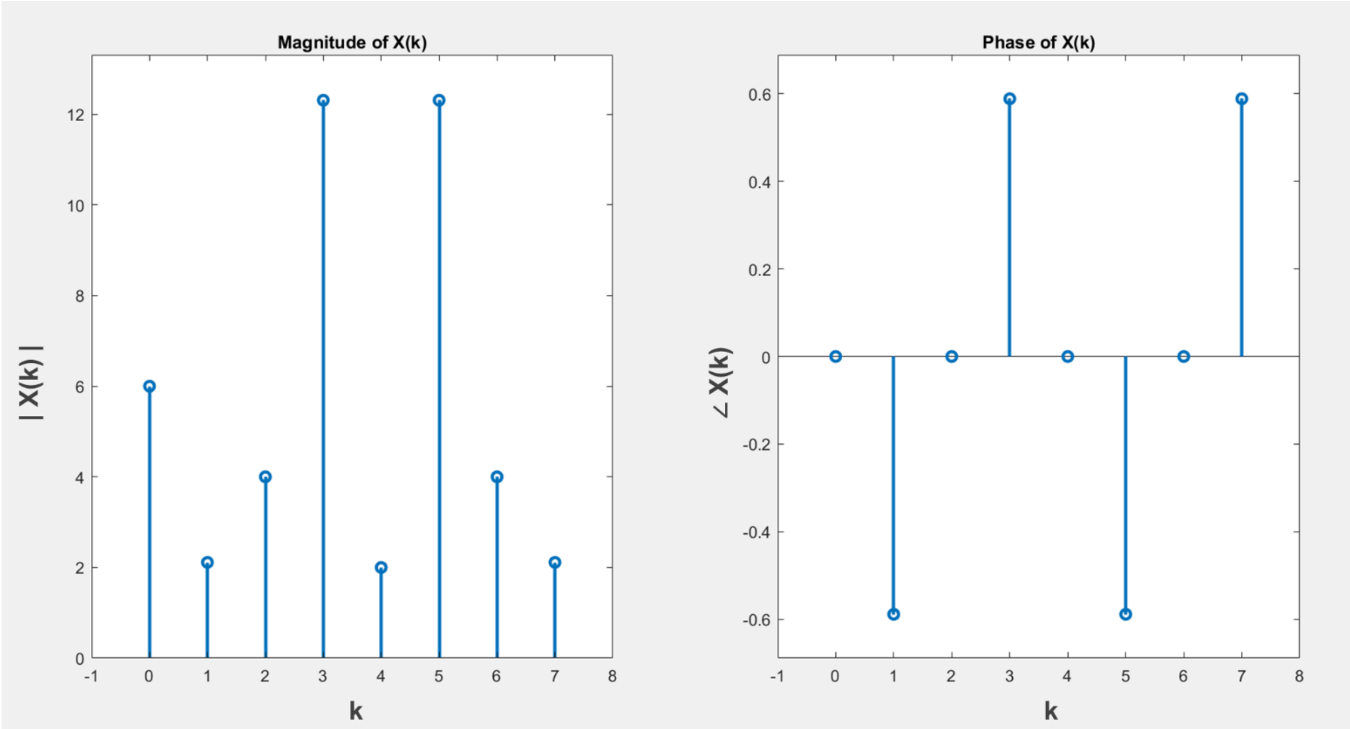
$$X[k] = W \cdot x[n]$$

$$X[k] = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}j & -j & -\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}j & -1 & -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j & j & \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}j & j & \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}j & -1 & \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j & -j & -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j & -j & \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j & -1 & \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}j & j & -\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}j \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & \frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j & j & -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}j & -1 & -\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}j & -j & \frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}j \end{bmatrix} \end{array} \begin{bmatrix} 5 \\ -2 \\ 2 \\ 1 \\ -1 \\ 3 \\ -2 \\ 0 \end{bmatrix}$$

$$\therefore \boxed{X[K] = \begin{bmatrix} 6 \\ 1.757 - j1.1716 \\ 4 \\ 10.243 + j6.828 \\ 2 \\ 10.243 - j6.828 \\ 4 \\ 1.757 + j1.1716 \end{bmatrix} = \begin{bmatrix} 6 \\ 2.11\, e^{-j0.588} \\ 4 \\ 12.31\, e^{j0.588} \\ 2 \\ 12.31\, e^{-j0.588} \\ 4 \\ 2.11\, e^{j0.588} \end{bmatrix}}$$



b) Periodicity of the spectra is $\boxed{N = 8}$

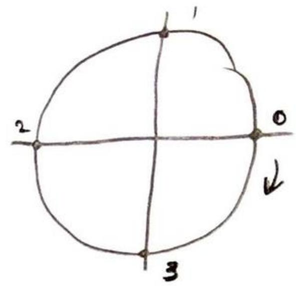


Matlab code for **Q 11.1**:

```
clc; clear all; close all;
x = [5 -2 2 1 -1 3 -2 0];
n = 0:1:length(x)-1;
%FFT N=8
X = fft(x);
%X' = ctranspose(X) which is the conjugate transpose

figure(1)
subplot(1,2,1); stem(n,abs(transpose(X)),'Linewidth',2); title('Magnitude of X(k)');
axis([-1 length(x) 0 max(abs(X))+1])
subplot(1,2,2); stem(n,angle(transpose(X)),'Linewidth',2); title('Phase of X(k)');
axis([-1 length(x) min(angle(X))-0.1 max(angle(X))+0.1])

xx = ifft(X)
```

11.2 $x[n] = e^{-0.5n}(u[n] - u[n-4])$ $\quad n = 0 \to 3 \;,\; N = 4$

$x[n] = [1 \,,\, 0.6065 \,,\, 0.3679 \,,\, 0.2231]$

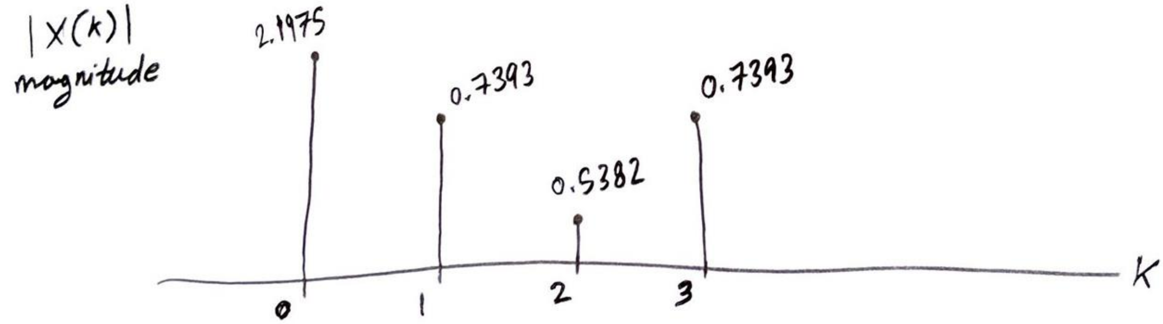


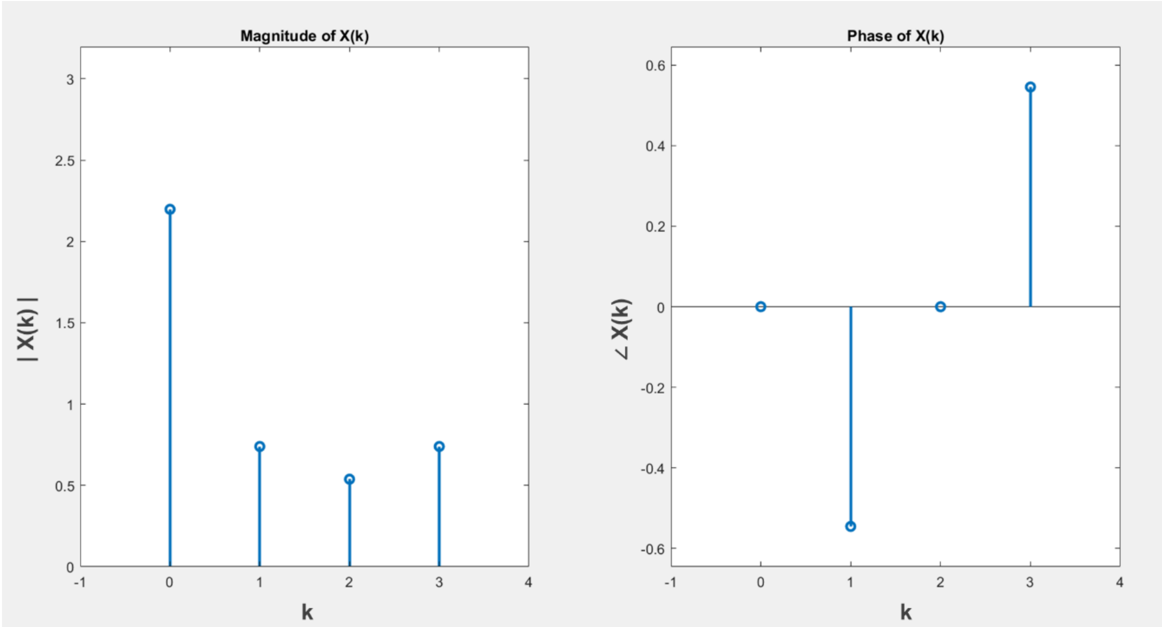
a) DFT: $X(k) = \sum_{n=0}^{3} x[n]\, e^{-j\frac{2\pi kn}{3}}$

$X(k) = W \cdot x[n]$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 0.6065 \\ 0.3679 \\ 0.2231 \end{bmatrix}$$

$$\boxed{X(k) = \begin{bmatrix} 2.1975 \\ 0.6321 - j\,0.3834 \\ 0.5382 \\ 0.6321 + j\,0.3834 \end{bmatrix} = \begin{bmatrix} 2.1975 \\ 0.7393\, e^{-j\,0.5452} \\ 0.5382 \\ 0.7393\, e^{j\,0.5452} \end{bmatrix}}$$

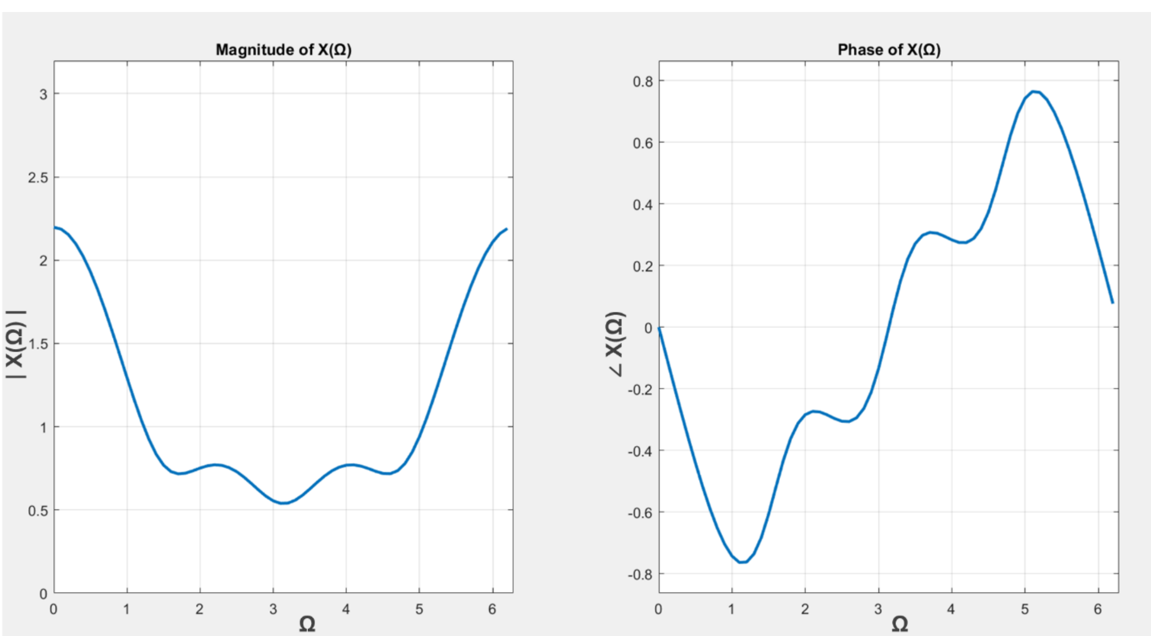


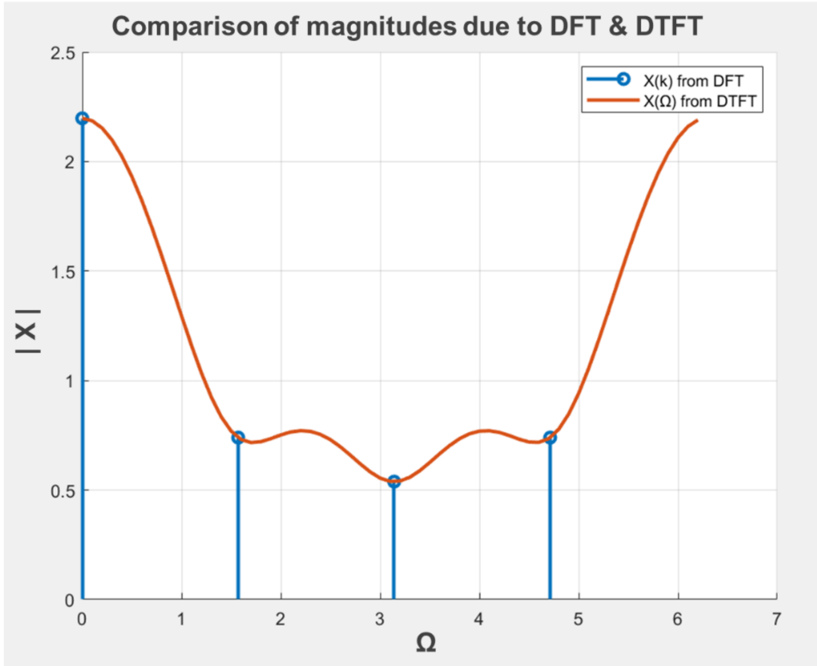


b) DTFT: $X(\Omega) = \sum_{n=0}^{3} x[n]\, e^{-j\Omega n}$

$$\boxed{X(\Omega) = 1 + 0.6065\, e^{-j\Omega} + 0.3679\, e^{-j2\Omega} + 0.2231\, e^{-j3\Omega}}$$

where $\Omega = 0 \rightarrow 2\pi$





Matlab code for **Q 11.2**:

```
clc; clear all; close all;

%DFT
N = 4; n = 0:1:N-1;
x = exp(-0.5*n);

X = fft(x); X = transpose(X);
figure(1)
subplot(1,2,1); stem(n,abs(X),'Linewidth',2); title('Magnitude of X(k)');
axis([-1 length(x) 0 max(abs(X))+1])
subplot(1,2,2); stem(n,angle(X),'Linewidth',2); title('Phase of X(k)');
axis([-1 length(x) min(angle(X))-0.1 max(angle(X))+0.1])
xx = ifft(X)
%----------------------------------------------------
%DTFT
cnt=1;
for OM = 0:0.1:2*pi;
    tmp =0;
    for indx = 0:3;
       tmp = tmp + x(indx+1)*exp(-j*OM*indx);
    end
    Xdtft(cnt) = tmp;
    OM_V(cnt) = OM;
    cnt = cnt+1;
end
figure(2)
subplot(1,2,1); plot(OM_V,abs(Xdtft),'Linewidth',2);
title('Magnitude of X('+string(char(937))+')');
axis([0 (2*pi) 0 max(abs(Xdtft))+1]); grid on
subplot(1,2,2); plot(OM_V,angle(Xdtft),'Linewidth',2);
title('Phase of X('+string(char(937))+')');
axis([0 (2*pi) min(angle(Xdtft))-0.1 max(angle(Xdtft))+0.1]); grid on
%----------------------------------------------------
%Comparison
figure; hold on; grid on;
n_new = (n*2*pi/N);
stem(n_new,abs(X),'Linewidth',2); plot(OM_V,abs(Xdtft),'Linewidth',2);
legend('X(k) from DFT','X('+string(char(937))+') from DTFT')
```

(11.3) DTFT: $H(\Omega) = 1 - 0.2\, e^{-j\Omega} + 0.35\, e^{-j2\Omega}$

$$\boxed{H(\Omega) = \sum_{n=-\infty}^{\infty} h[n]\, e^{-j\Omega}}$$

$\therefore h[n] = 1\,\delta[n] - 0.2\,\delta[n-1] + 0.35\,\delta[n-2]$

$= [1, -0.2, 0.35]$

$N = 8$, DFT:

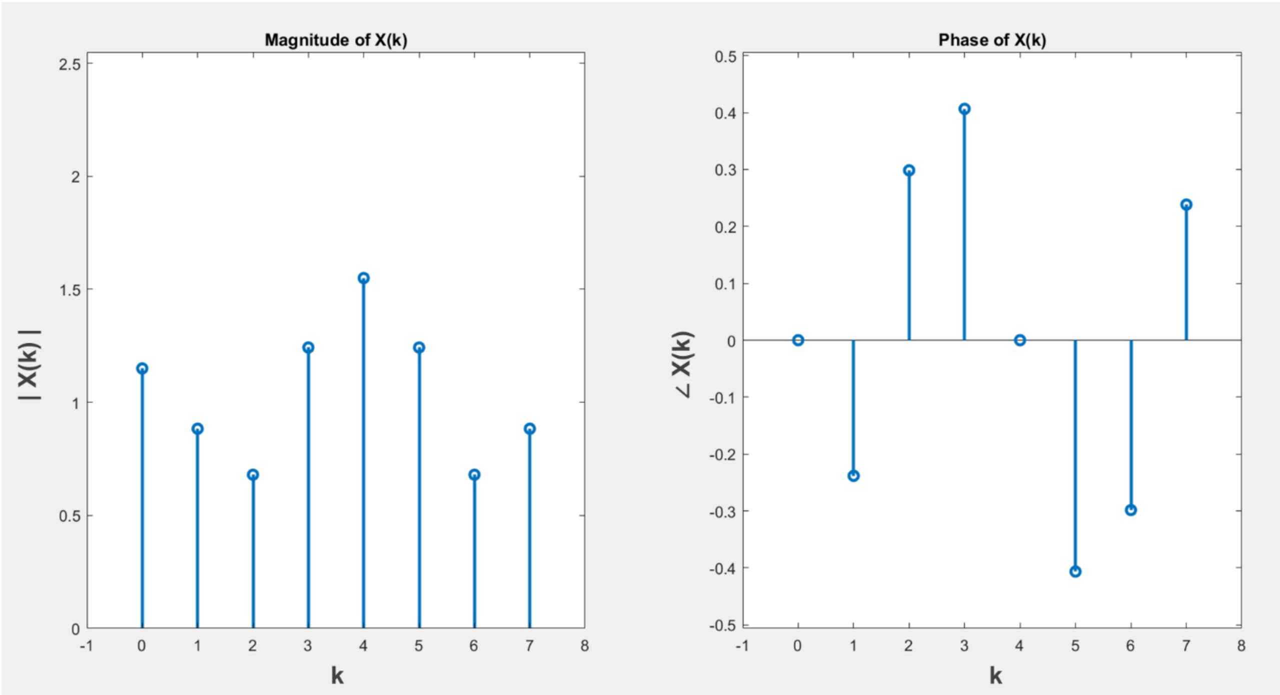
Padding $h[n]$ with Zeros at the end of the sequence for 8-point DFT

$\therefore h[n] = [1, -0.2, 0.35, 0, 0, 0, 0, 0]$

$\therefore H(k) = \sum_{n=0}^{7} h[n]\, e^{-j\frac{2\pi kn}{N}}$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{\sqrt2}{2} - j\frac{\sqrt2}{2} & -j & -\frac{\sqrt2}{2} - j\frac{\sqrt2}{2} & -1 & -\frac{\sqrt2}{2} + j\frac{\sqrt2}{2} & j & \frac{\sqrt2}{2} + j\frac{\sqrt2}{2} \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -\frac{\sqrt2}{2} - j\frac{\sqrt2}{2} & j & \frac{\sqrt2}{2} - j\frac{\sqrt2}{2} & -1 & \frac{\sqrt2}{2} + j\frac{\sqrt2}{2} & -j & -\frac{\sqrt2}{2} - j\frac{\sqrt2}{2} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\frac{\sqrt2}{2} + j\frac{\sqrt2}{2} & -j & \frac{\sqrt2}{2} + j\frac{\sqrt2}{2} & -1 & \frac{\sqrt2}{2} - j\frac{\sqrt2}{2} & j & -\frac{\sqrt2}{2} - j\frac{\sqrt2}{2} \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & \frac{\sqrt2}{2} + j\frac{\sqrt2}{2} & j & -\frac{\sqrt2}{2} - j\frac{\sqrt2}{2} & -1 & -\frac{\sqrt2}{2} - j\frac{\sqrt2}{2} & -j & \frac{\sqrt2}{2} - j\frac{\sqrt2}{2} \end{bmatrix} \begin{bmatrix} 1 \\ -0.2 \\ 0.35 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{\therefore H(k) = \begin{bmatrix} 1.15 \\ 0.8586 - j0.2086 \\ 0.65 + j0.2 \\ 1.1414 + j0.4914 \\ 1.55 \\ 1.1414 - j0.4914 \\ 0.65 - j0.2 \\ 0.8586 + j0.2086 \end{bmatrix} = \begin{bmatrix} 1.15 \\ 0.8836\, e^{-j0.2383} \\ 0.6801\, e^{j0.2985} \\ 1.2427\, e^{j0.4065} \\ 1.55 \\ 1.2427\, e^{-j0.4065} \\ 0.6801\, e^{-j0.2985} \\ 0.8836\, e^{j0.2383} \end{bmatrix}}$$

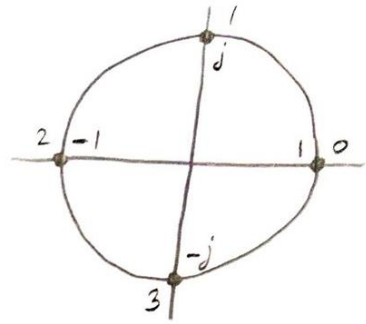


<u>Matlab code for **Q 11.3**:</u>

```
clc; clear all; close all;

%DFT
N = 8;
n = 0:1:N-1;
x = [1 -0.2 0.35];

%DFT N=8
X = fft(x,N); X=transpose(X)

figure(1)
subplot(1,2,1); stem(n,abs(X),'Linewidth',2); title('Magnitude of X(k)');
axis([-1 N 0 max(abs(X))+1])
subplot(1,2,2); stem(n,angle(X),'Linewidth',2); title('Phase of X(k)');
axis([-1 N min(angle(X))-0.1 max(angle(X))+0.1])

xx = ifft(X)
```

11.4 $x[n] = \sin\left(\frac{n\pi}{2}\right)$ for $n = 0, 1, 2, 3$

$\therefore x[n] = [0, 1, 0, -1]$



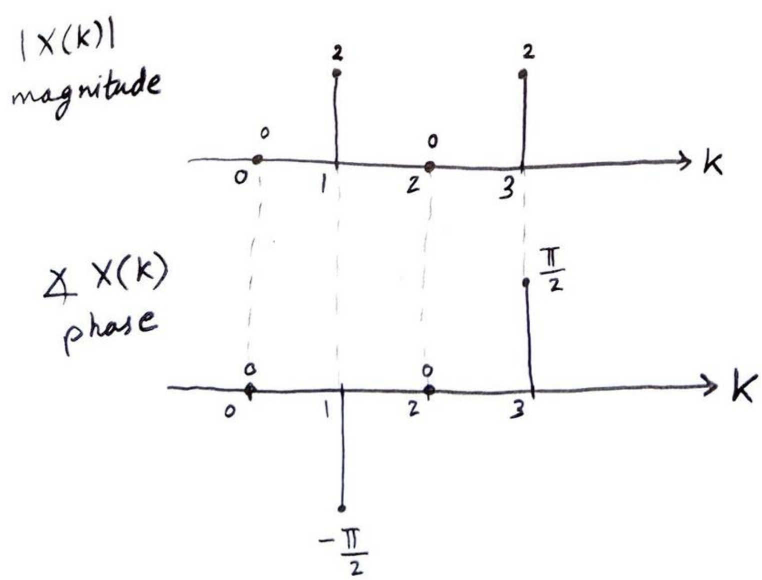
$N = 4$

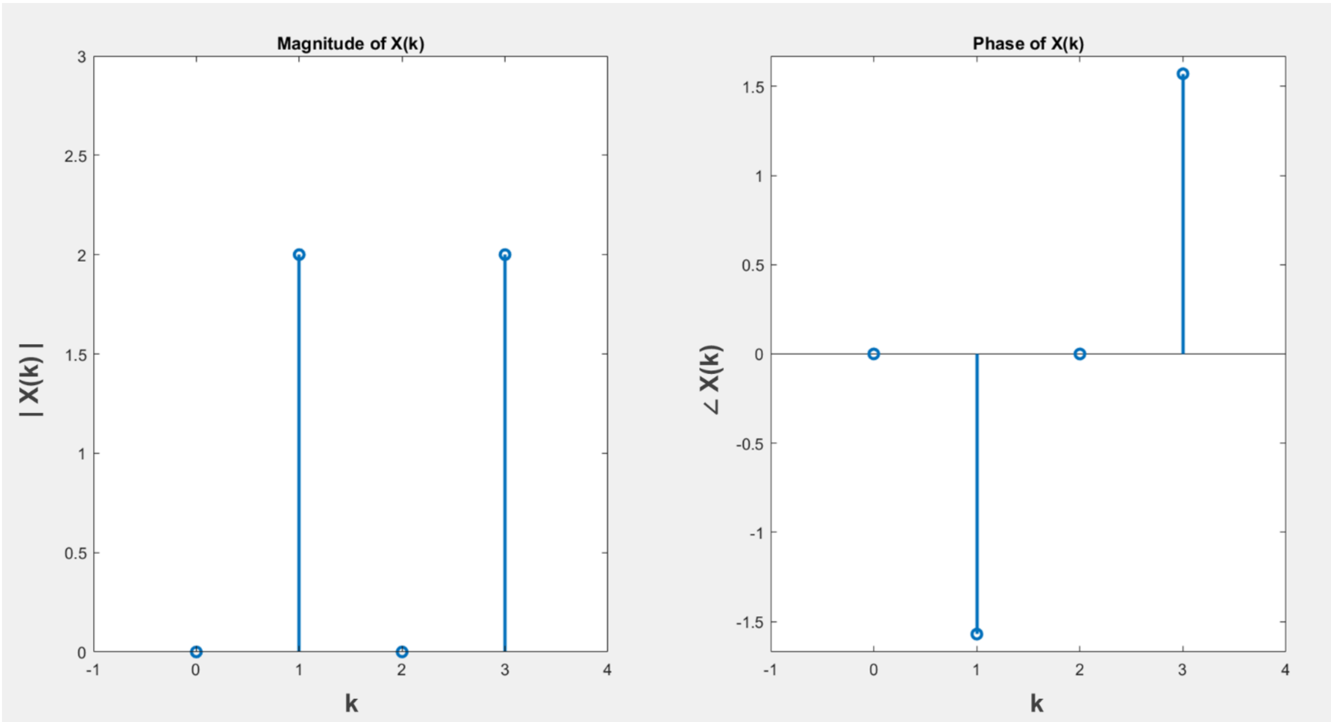
DFT : $X(k) = \sum_{n=0}^{3} x[n]\, e^{-j\frac{2\pi k n}{N}}$

$X(k) = W \cdot x[n]$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\boxed{X(k) = \begin{bmatrix} 0 \\ -2j \\ 0 \\ +2j \end{bmatrix} = \begin{bmatrix} 0 \\ 2e^{-j\frac{\pi}{2}} \\ 0 \\ 2e^{j\frac{\pi}{2}} \end{bmatrix}}$$





Matlab code for **Q 11.4**:

```
clc; clear all; close all;

%DFT
N = 4;
n = 0:1:N-1;
x = sin(n*pi/2);

%DFT N=4
X = fft(x); X = transpose(X)

figure(1)
subplot(1,2,1); stem(n,abs(X),'Linewidth',2); title('Magnitude of X(k)');
axis([-1 N 0 max(abs(X))+1])
subplot(1,2,2); stem(n,angle(X),'Linewidth',2); title('Phase of X(k)');
axis([-1 N min(angle(X))-0.1 max(angle(X))+0.1])

xx = ifft(X)
```

11.5 $h[n] = (-0.95)^n \quad , \quad 0 \leq n \leq 3$

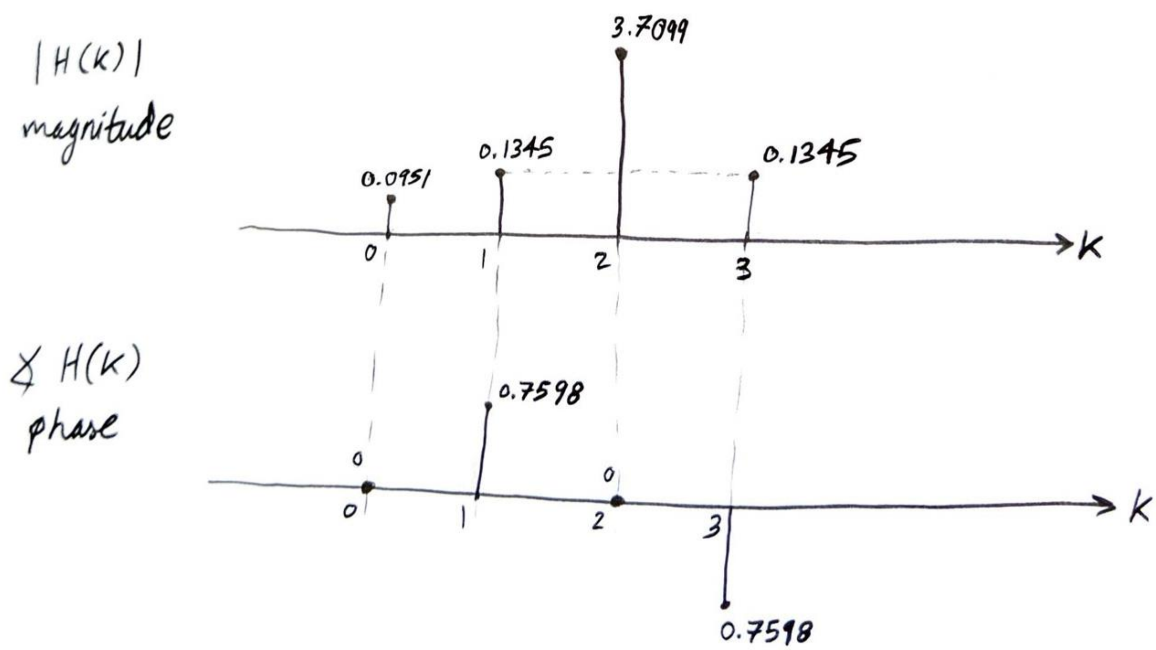
$$h[n] = [\,1\,,\ -0.95,\ 0.9025,\ -0.8574\,]$$

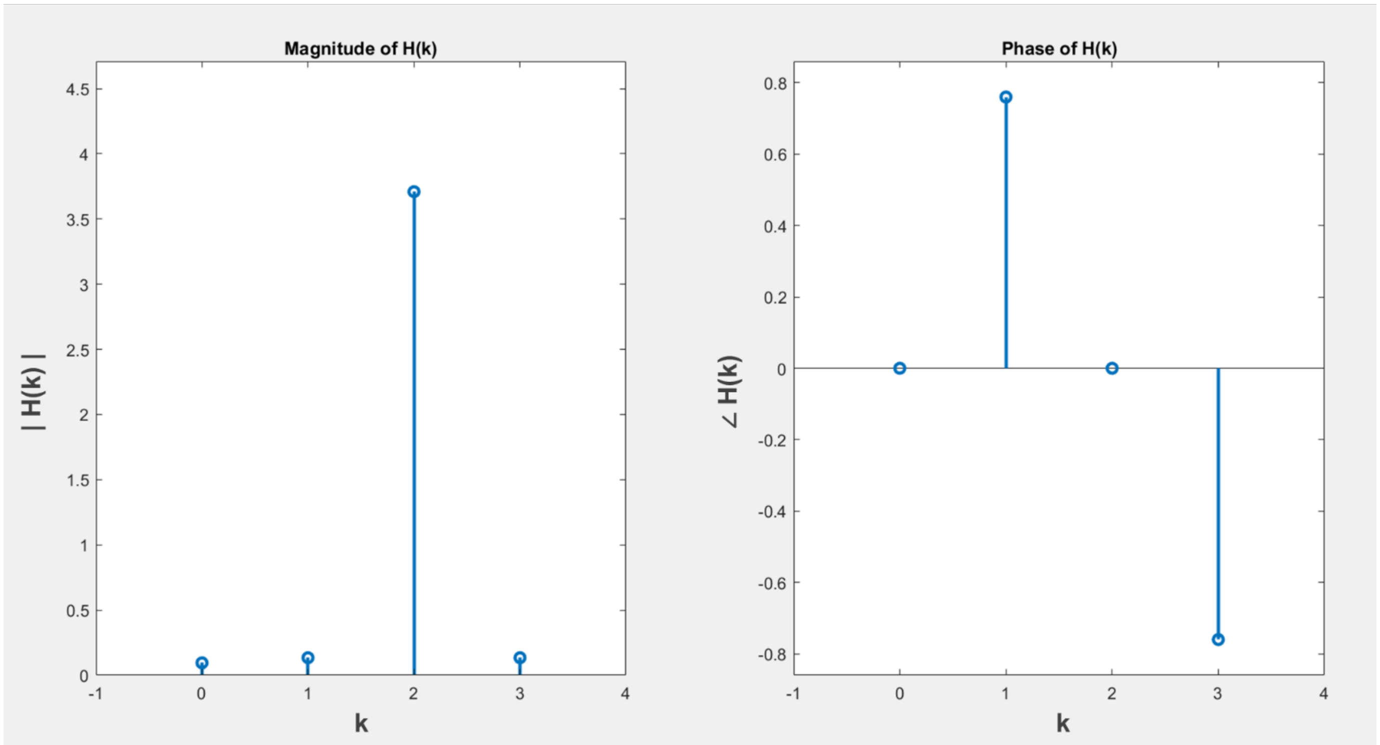
DFT: $$H(k) = \sum_{n=0}^{3} h[n]\ e^{-j\frac{2\pi k n}{N}}$$

$$H(k) = W \cdot h[n]$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -0.95 \\ 0.9025 \\ -0.8574 \end{bmatrix}$$

$$H(k) = \begin{bmatrix} 0.0951 \\ 0.0975 + 0.0926j \\ 3.7099 \\ 0.0975 - 0.0926j \end{bmatrix} = \begin{bmatrix} 0.0951 \\ 0.1345\ e^{j\,0.7598} \\ 3.7099 \\ 0.1345\ e^{-j\,0.7598} \end{bmatrix}$$





<u>Matlab code for **Q 11.5**:</u>

```
clc; clear all; close all;

%DFT
N = 4;
n = 0:1:N-1;
h = (-0.95).^n;

%DFT N=4
H = fft(h); H = transpose(H)

figure(1)
subplot(1,2,1); stem(n,abs(H),'Linewidth',2); title('Magnitude of H(k)');
axis([-1 N 0 max(abs(H))+1])
subplot(1,2,2); stem(n,angle(H),'Linewidth',2); title('Phase of H(k)');
axis([-1 N min(angle(H))-0.1 max(angle(H))+0.1])

hh = ifft(H)
```

Fall 2020

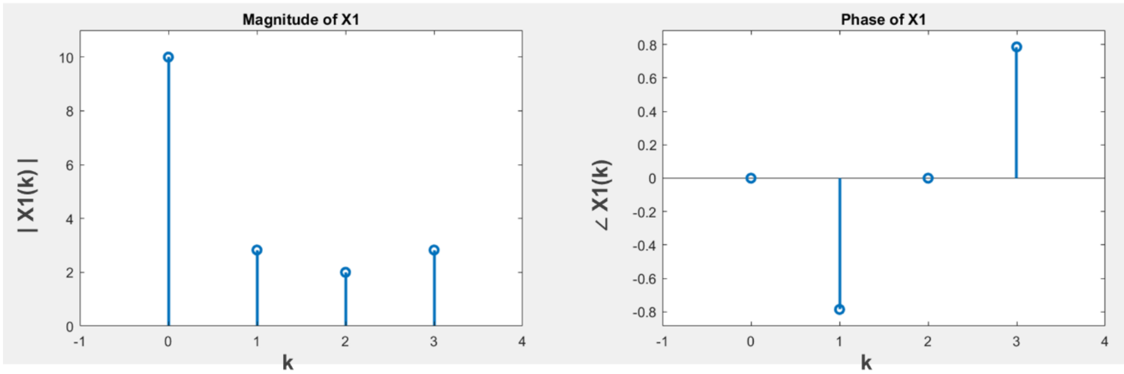
DFT

$$X(k) = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k n}{N}}$$

11.6 a) $X_1(k) = \sum_{n=0}^{3} x_1[n]\, e^{-j\frac{2\pi k n}{4}}$

$= W \cdot x_1[n]$

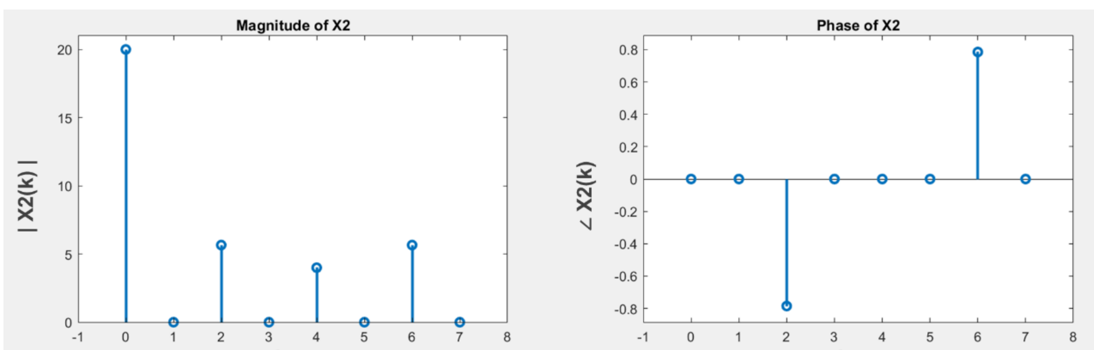
4x4 matrix



b) $X_2(k) = \sum_{n=0}^{7} x_2[n]\, e^{-j\frac{2\pi k n}{8}}$

$= W \cdot x_2[n]$

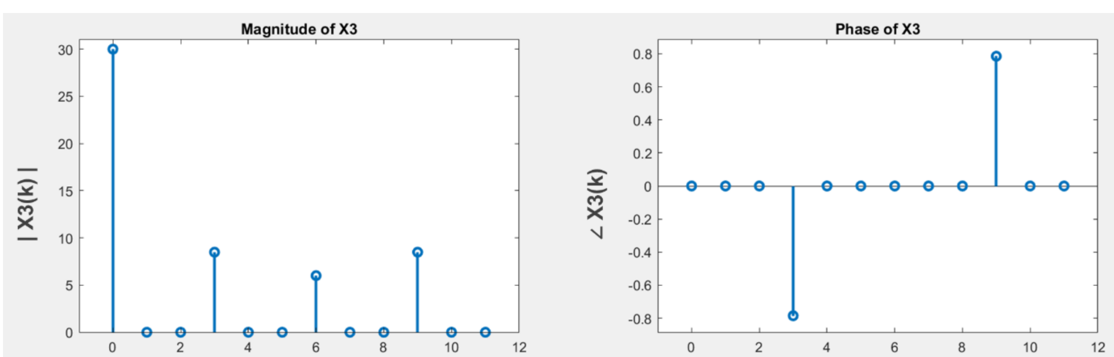
8 x 8 matrix



c) $X_3(k) = \sum_{n=0}^{11} x_3[n]\, e^{-j\frac{2\pi k n}{12}}$

$= W \cdot x_3[n]$

12 x 12 matrix



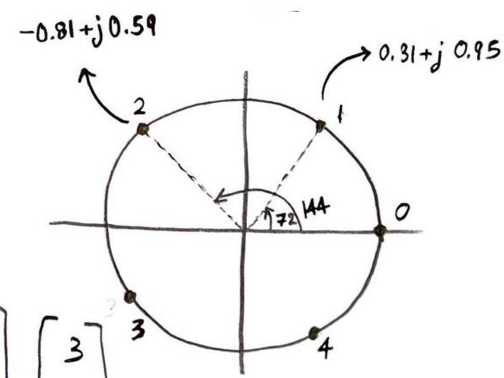
<u>Matlab code for **Q 11.6**:</u>

```
clc; clear all; close all;

x1 = [4 3 2 1];      N1 = length(x1);
x2 = [x1 x1];        N2 = length(x2);
x3 = [x1 x1 x1];     N3 = length(x3);

n1 = 0:1:((N1)-1);
n2 = 0:1:((N2)-1);
n3 = 0:1:((N3)-1);

%DFT
X1 = fft(x1); X1 = transpose(X1);
X2 = fft(x2); X2 = transpose(X2);
X3 = fft(x3); X3 = transpose(X3);

figure(1)
subplot(1,2,1); stem(n1,abs(X1),'Linewidth',2); title('Magnitude of X1');
axis([-1 N1 0 max(abs(X1))+1])
subplot(1,2,2); stem(n1,angle(X1),'Linewidth',2); title('Phase of X1');
axis([-1 N1 min(angle(X1))-0.1 max(angle(X1))+0.1])

figure(2)
subplot(1,2,1); stem(n2,abs(X2),'Linewidth',2); title('Magnitude of X2');
axis([-1 N2 0 max(abs(X2))+1])
subplot(1,2,2); stem(n2,angle(X2),'Linewidth',2); title('Phase of X2');
axis([-1 N2 min(angle(X2))-0.1 max(angle(X2))+0.1])

figure(3)
subplot(1,2,1); stem(n3,abs(X3),'Linewidth',2); title('Magnitude of X3');
axis([-1 N3 0 max(abs(X3))+1])
subplot(1,2,2); stem(n3,angle(X3),'Linewidth',2); title('Phase of X3');
axis([-1 N3 min(angle(X3))-0.1 max(angle(X3))+0.1])

%figure; hold on; stem(abs(X1)); stem(abs(X2)); stem(abs(X3));
%figure; hold on; stem(angle(X1)); angle(abs(X2)); angle(abs(X3));

xx1 = ifft(X1);
xx2 = ifft(X2);
xx3 = ifft(X3);
```

11.7 $\quad x[n] = [3, -1, 0, 2, 1]$

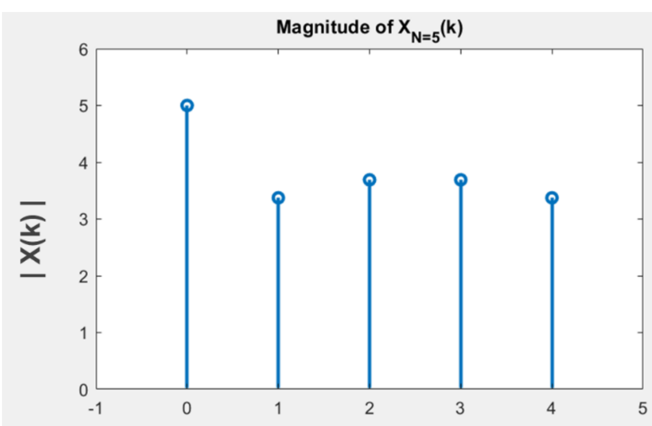
a) $N=5$, DFT

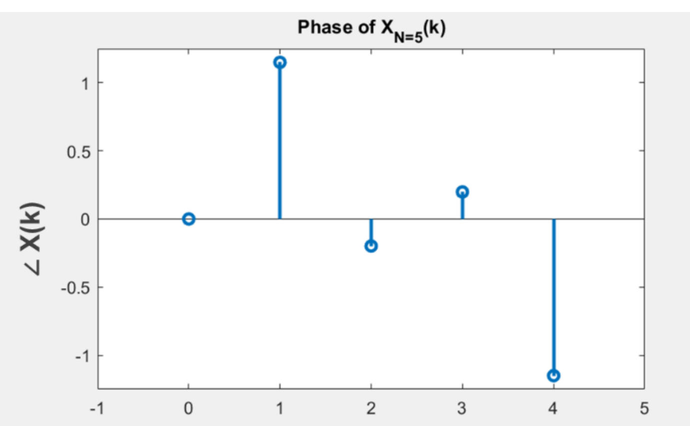
$$X(k) = \sum_{n=0}^{4} x[n]\, e^{-j\frac{2\pi k n}{5}} = W \cdot x[n]$$



$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0.31-j0.95 & -0.81-j0.59 & -0.81+j0.59 & 0.31+j0.95 \\ 1 & -0.81-j0.59 & 0.31+j0.59 & 0.31-j0.95 & -0.81+j0.59 \\ 1 & -0.81+j0.59 & 0.31-j0.95 & 0.31+j0.95 & -0.81-j0.59 \\ 1 & 0.31+j0.95 & -0.81+j0.59 & -0.81-j0.59 & 0.31-j0.95 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\boxed{X(k) = \begin{bmatrix} 5 \\ 1.38+j3.08 \\ 3.62-j0.73 \\ 3.62+j0.73 \\ 1.38-j3.08 \end{bmatrix} = \begin{bmatrix} 5 \\ 3.374\, e^{j1.1488} \\ 3.69\, e^{-j0.1982} \\ 3.69\, e^{j0.1982} \\ 3.374\, e^{-j1.1488} \end{bmatrix}}$$





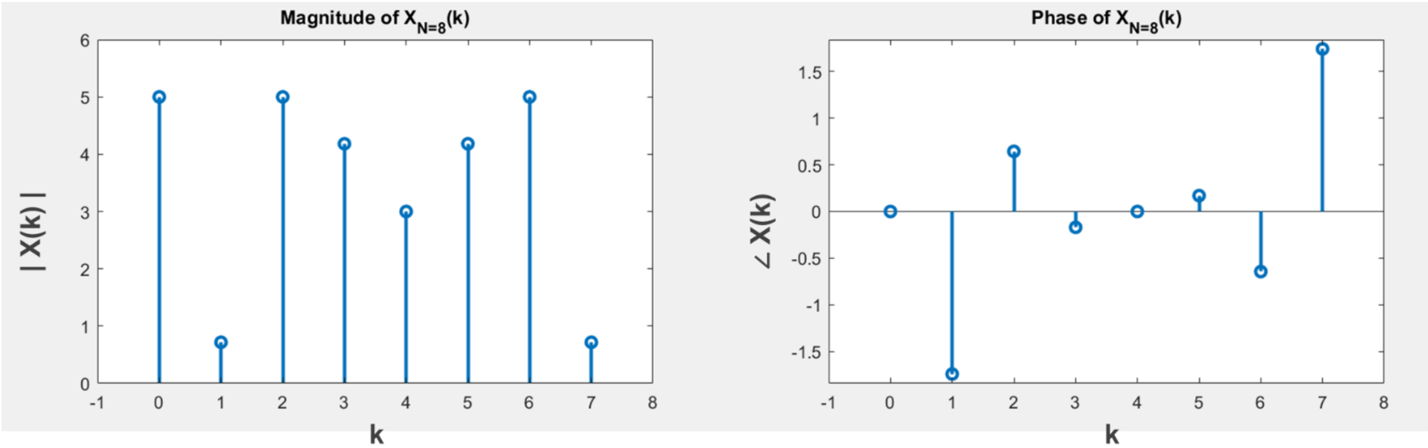
b) $N=8$, FFT

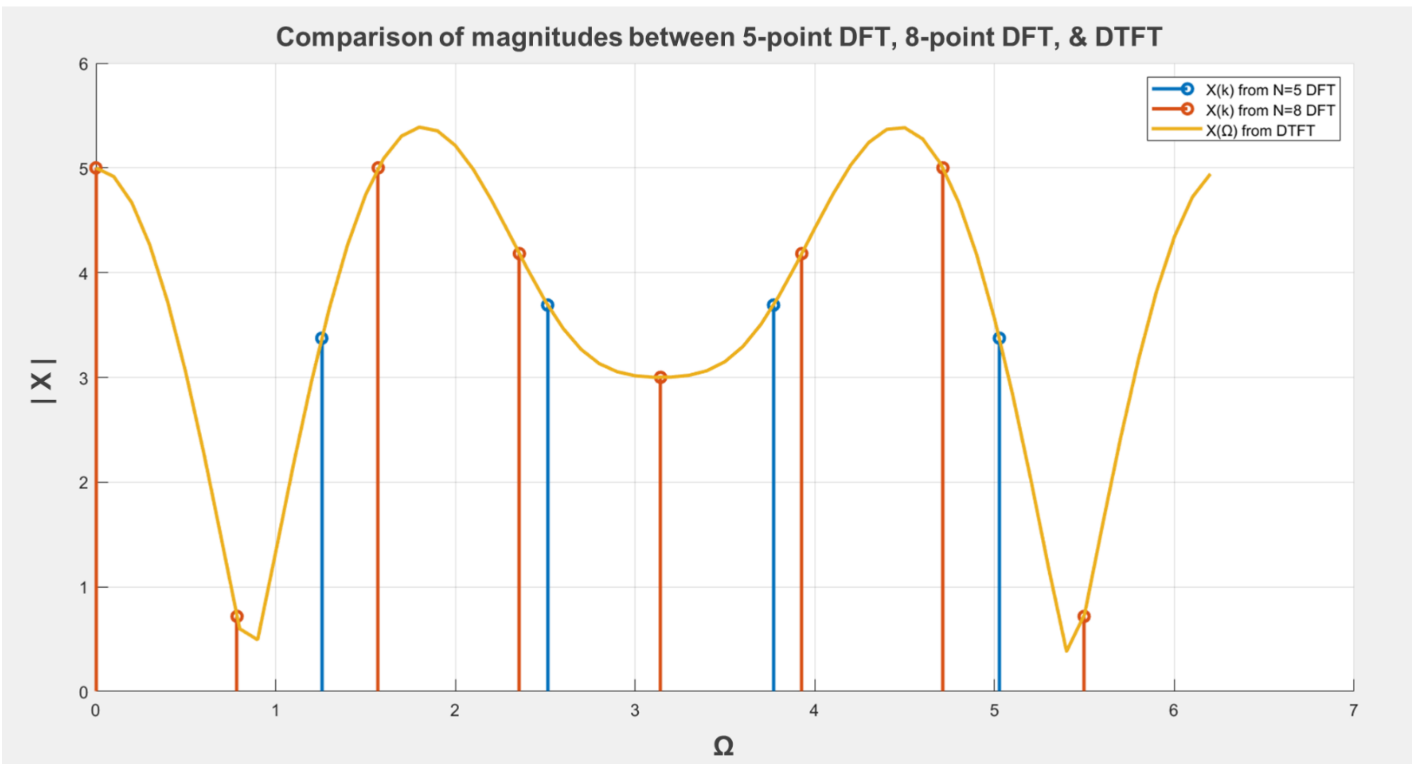
$$x[n] = [3, -1, 0, 2, 1, 0, 0, 0]$$

$$X(k) = \sum_{n=0}^{7} x[n]\, e^{-j\frac{2\pi k n}{8}} = W \cdot x[n]$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & -j & \frac{-\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & -1 & \frac{-\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} & j & \frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & \frac{-\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & j & \frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & -1 & \frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} & -j & \frac{-\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & \frac{-\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} & -j & \frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} & -1 & \frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & j & \frac{-\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & \frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} & j & \frac{-\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & -1 & \frac{-\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & -j & \frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 0 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{X(k) = \begin{bmatrix} 5 \\ -0.12-j\,0.707 \\ 4+j3 \\ 4.12-j\,0.707 \\ 3 \\ 4.12+j\,0.707 \\ 4-j3 \\ -0.12+j\,0.707 \end{bmatrix} = \begin{bmatrix} 5 \\ 0.717\, e^{-j1.74} \\ 5\, e^{j0.64} \\ 4.182\, e^{-j0.17} \\ 3 \\ 4.182\, e^{j0.17} \\ 5\, e^{-j0.64} \\ 0.717\, e^{j1.74} \end{bmatrix}}$$


Magnitude of $X_{N=8}(k)$
| X(k) |
k
Phase of $X_{N=8}(k)$
∠ X(k)
k


Comparison of magnitudes between 5-point DFT, 8-point DFT, & DTFT
X(k) from N=5 DFT
X(k) from N=8 DFT
X(Ω) from DTFT
| X |
Ω

Matlab code for **Q 11.7**:

```
clc; clear all; close all;

%DFT
N1 = 5;
n1 = 0:1:N1-1;
x1 = [3 -1 0 2 1];

X1 = fft(x1); X1 = transpose(X1)
figure(1)
subplot(1,2,1); stem(n1,abs(X1),'Linewidth',2); title('Magnitude of X_N_=_5(k)');
axis([-1 N1 0 max(abs(X1))+1])
subplot(1,2,2); stem(n1,angle(X1),'Linewidth',2); title('Phase of X_N_=_5(k)');
axis([-1 N1 min(angle(X1))-0.1 max(angle(X1))+0.1])

xx1 = ifft(X1)

%FFT
N2 = 8;
n2 = 0:1:N2-1;
x2 = [3 -1 0 2 1 0 0 0];

X2 = fft(x2); X2 = transpose(X2)
figure(2)
subplot(1,2,1); stem(n2,abs(X2),'Linewidth',2); title('Magnitude of X_N_=_8(k)');
axis([-1 N2 0 max(abs(X2))+1])
subplot(1,2,2); stem(n2,angle(X2),'Linewidth',2); title('Phase of X_N_=_8(k)');
axis([-1 N2 min(angle(X2))-0.1 max(angle(X2))+0.1])

xx2 = ifft(X2)

%DTFT
cnt=1;
for OM = 0:0.1:2*pi;
    tmp =0;
    for indx = 0:length(x1)-1;
       tmp = tmp + x1(indx+1)*exp(-j*OM*indx);
    end
    Xdtft(cnt) = tmp;
    OM_V(cnt) = OM;
    cnt = cnt+1;
end

figure(3)
subplot(1,2,1); plot(OM_V,abs(Xdtft),'Linewidth',2); title('Magnitude of X('+string(char(937))+')');
axis([0 (2*pi) 0 max(abs(Xdtft))+1]); grid on
subplot(1,2,2); plot(OM_V,angle(Xdtft),'Linewidth',2); title('Phase of X('+string(char(937))+')');
axis([0 (2*pi) min(angle(Xdtft))-0.1 max(angle(Xdtft))+0.1]); grid on

%----------------------------------------------------
%Comparison
figure; hold on; grid on;
n1_new = (n1*2*pi/N1);
stem(n1_new,abs(X1),'Linewidth',2);
n2_new = (n2*2*pi/N2);
stem(n2_new,abs(X2),'Linewidth',2);
plot(OM_V,abs(Xdtft),'Linewidth',2);
legend('X(k) from N=5 DFT','X(k) from N=8 DFT','X('+string(char(937))+') from DTFT')
```

11.12 $f_s = 16\ kHz$, $N = 512$ $(n = 0 \rightarrow 511)$

a) DFT resolution $= \frac{16\ kHz}{512} = \boxed{31.25\ Hz}$

b) Equivalent frequencies:

(i) $k = 0 \longrightarrow \boxed{f_0 = 0\ Hz}$

(ii) $k = 127 \longrightarrow \boxed{f_{127} = 127 \times \frac{16k}{512} = 3.969\ kHz}$

(iii) $k = 255 \longrightarrow \boxed{f_{255} = 255 \times \frac{16K}{512} = 7.969\ kHz}$

(iv) $k = 511 \longrightarrow \boxed{f_{511} = 511 \times \frac{16k}{512} = 15.968\ kHz}$

11.13 $f_{analog} = 6\ kHz$, $f_s = 40\ kHz$

a) $N = 32$ DFT $\rightarrow$ resolution $= \frac{40\ kHz}{32} = 1.25\ kHz$

$$\boxed{\therefore K_{peak} = \frac{6\ KHz}{1.25\ kHz} = 4.8 \simeq 5}$$

b) $N = 64$ DFT $\rightarrow$ resolution $= \frac{40\ kHz}{64} = 0.625\ kHz$

$$\boxed{\therefore K_{peak} = \frac{6\ kHz}{0.625\ KHz} = 9.6 \simeq 10}$$

c) $N = 128$ DFT $\rightarrow$ resolution $= \frac{40\ kHz}{128} = 0.3125\ kHz$

$$\boxed{\therefore K_{peak} = \frac{6\ kHz}{0.3125\ kHz} = 19.2 \simeq 19}$$