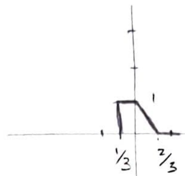
EE210

HOMEWORK 03:

Answers:

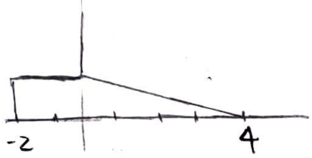
1) a) $x(3t)$

$x\left(\frac{t}{1/3}\right)$



b) $x\left(\frac{1}{2}t\right)$

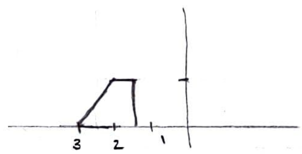
$x\left(\frac{t}{\frac{1}{1/2}}\right)$



c) $x(-2t-4)$

$= x(-(2t+4))$

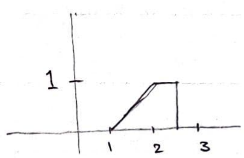
$= x\left(-\left(\frac{t-2}{1/2}\right)\right)$



d) $x(-2t+4)$

$x(-(2t-4))$

$x\left(-\left(\frac{t-2}{1/2}\right)\right)$



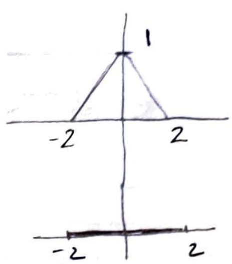
2) a) $F\{x(2t)\} = \int x(2t)\cdot e^{-j2\pi f t}\, dt$

$\tau = 2t \qquad t = \frac{1}{2}\tau$

$d\tau = 2\,dt$

$= \int_{\tau=-\infty}^{\infty} x(\tau)\, e^{-j2\pi f(\frac{1}{2})\tau} (\tfrac{1}{2})\, d\tau$

$= \frac{1}{2} X(\frac{f}{2})$



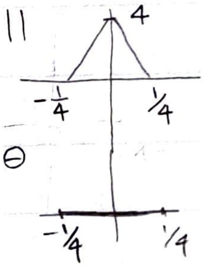
b) $F\{x(\frac{1}{4}t)\} = \int_{-\infty}^{\infty} x(\frac{1}{4}t)\, e^{-j2\pi f t}\, dt$

$\tau = \frac{1}{4}t \qquad t = 4\tau$

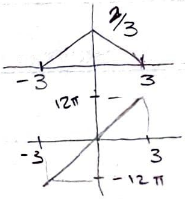
$4\, d\tau = dt$

$= \int_{\tau=-\infty}^{\infty} X(\tau)\cdot e^{-j2\pi f 4t} (4)\, d\tau$

$= 4X(4f) = 4X(\frac{f}{1/4})$



c) $F\{X(-3t-6)\} = \int_{-\infty}^{\infty} X(-3t-6)\cdot e^{-j2\pi f t}\, dt$



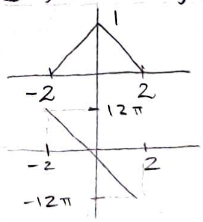
$\tau = -3t-6 \qquad t = -\frac{1}{3}\tau - 2$

$dt = -\frac{1}{3}\, d\tau$

$= \int_{\tau=-\infty}^{\infty} X(\tau)\, e^{-j2\pi f(-\frac{1}{3}\tau - 2)} = \int_{\tau=-\infty}^{\infty} X(\tau) e^{j2\pi \frac{f}{3}\tau} \cdot e^{j2\pi f\cdot 2} \cdot \frac{1}{3}\, d\tau$

$= \frac{1}{3} X(\frac{1}{3}f)\cdot e^{j4\pi f}$; $\theta = 4\pi f$

d) $F\{x(-2t+6)\} = F\{x(\frac{(t-3)}{-1/2})\}$



$= F\{x(\frac{t}{-1/2}) * \delta(t-3)\}$

$= \frac{1}{2} X(\frac{f}{-1/2})\, e^{-j2\pi f(3)}$

$\theta = -6\pi f$

Alvin Maningding

3.

a. $\int_{-\infty}^{\infty} e^{-t}\delta(t-1)\,dt = \int_{-\infty}^{\infty} e^{-1}\delta(t-1)\,dt$

$= e^{-1}\int_{-\infty}^{\infty} \delta(t-1)\,dt$

$= \boxed{e^{-1}}$

b. $\int_{0}^{\infty} e^{-t}\delta(t-1)\,dt = \int_{0}^{\infty} e^{-1}\delta(t-1)\,dt$

$= e^{-1}\int_{0}^{\infty} \delta(t-1)\,dt$

$= \boxed{e^{-1}}$

c. $\int_{0}^{\infty} e^{-t}\delta(t+1)\,d\tau = \int_{0}^{\infty} e^{-(-1)}\delta(t+1)\,d\tau$

$= e^{1}\int_{0}^{\infty} \delta(t+1)\,d\tau$ $\rightarrow 0$

$= 0$ (impulse at $t=-1$ is outside $0 \leq t \leq \infty$)

d. $\int_{-\infty}^{\infty} (t^3+t^2+t+1)\delta(t)\,dt = \int_{-\infty}^{\infty} (0+0+0+1)\delta(t)\,d\tau$

$= \int_{-\infty}^{\infty} (1)\delta(t)\,dt$

$= \boxed{1}$

e. $\int_{-\infty}^{\infty} \cos^2(2\pi t + 0.1\pi)\,\delta(t+1)\,dt$

$$= \int_{-\infty}^{\infty} \cos^2(2\pi(-1) + 0.1\pi)\,\delta(t+1)\,dt$$

$$= 0.9045 \int_{-\infty}^{\infty} \delta(t+1)\,dt$$

$$= \boxed{0.9045}$$

f. $\int_{-\infty}^{\infty} e^{-t}\,\delta(-t-1)\,dt$

Let $\begin{cases} \tau = -t-1 \\ t = -\tau - 1 \end{cases}$, $dt = -d\tau$, and new bounds of integration,

$\tau = (-(-\infty), -(+\infty))$ $= (+\infty, -\infty)$. : $-\int_{-\infty}^{\infty} e^{-(-\tau-1)}\,\delta(\tau)\,(-d\tau)$

$$= \int_{-\infty}^{\infty} \overset{e^1}{\cancel{e^{-(0-1)}}}\,\delta(\tau)\,d\tau$$

$$= \boxed{e^1}$$

g. $\int_{-\infty}^{\infty} t^2\,\delta\left(\frac{-1}{2}t + \frac{1}{2}\right)dt.$

Let $\begin{cases} \tau = -t/2 + 1/2 \\ t = -2(\tau - \frac{1}{2}) = -2\tau + 1 \end{cases}$, $dt = -2d\tau$, and

new bounds of integration $\tau = (+\infty, -\infty)$:

$$-\int_{-\infty}^{\infty} \left(-\frac{t}{2} + \frac{1}{2}\right)^2 \delta(\tau)\,(-2)\,d\tau = 2\int_{-\infty}^{\infty} \left(0 + \frac{1}{2}\right)^2 \delta(\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} 2(1/4)\,\delta(\tau)\,d\tau = \frac{1}{2}\int_{-\infty}^{\infty} \delta(\tau)\,d\tau = \boxed{\frac{1}{2}}$$

h. $\int_{-\infty}^{\infty} e^{t}\delta(3t-1)\,dt.$

Let $\begin{cases} \tau = 3t-1 \\ t = \frac{1}{3}\tau + \frac{1}{3} \end{cases}$, $dt = \frac{1}{3}d\tau$, and bounds of

integration $\tau = (3(-\infty), 3(\infty)) \simeq (-\infty, \infty)$

$$\int_{-\infty}^{\infty} \exp\left(\tfrac{1}{3}\tau + \tfrac{1}{3}\right)\delta(\tau)\left(\tfrac{1}{3}\right)d\tau$$

$$= \frac{1}{3}\int_{-\infty}^{\infty} \exp\left(\tfrac{1}{3}(0) + \tfrac{1}{3}\right)\delta(\tau)\,d\tau$$

$$= \frac{1}{3}e^{1/3}\int_{-\infty}^{\infty}\delta(\tau)\,d\tau)$$

$$= \boxed{\frac{1}{3}e^{1/3}}$$

Answers:

a) $4\sqcap(\frac{f}{10})e^{-j2\pi(0.05)f}$

b) $4\text{sinc}(0.1t) \star \delta(t-0.0$

e) $12\sqcap e^{-j10.1\pi f}$

$-\sqcap(10.2+j6.3)$

f) $11.3\,\text{sinc}(10.1t) \star \delta(t-5.05)$

4) a) Slope: $\frac{-0.5\pi}{5} = -0.1\pi$

$\boxed{X(f) = 4\sqcap(\frac{f}{10})\cdot e^{-j2\pi(0.05)f}}$

b) $x(t) = X(f)\cdot e^{-j2\pi f t_0}$

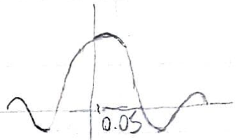
$x(t) = \int 4\cdot e^{-j2\pi f(0.05)}\cdot e^{-j2\pi f t}\, X(f)\, df$

$= 2\cdot A\cdot f_0\, \text{sinc}(2\cdot f_0\cdot t) = (2\cdot\frac{4}{0.1}\cdot 0.05)\,\text{sinc}(2\cdot 0.05t)$

$= 4\,\text{sinc}(0.1(t-0.05))$

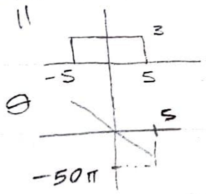
$= 4\,\text{sinc}(0.1t) \star \delta(t-0.05)$

c)



d) $X_2(f) = 3\cdot e^{-j10\pi f}$

||



Slope $= -10\pi$

e) $Y(f) = X(f)\cdot X_2(f) = 4\sqcap(\frac{f}{10})\cdot e^{-j0.1\pi f}\cdot 3\cdot e^{-j10\pi f}$

Polar: $Y(f) = 12\sqcap(0.1f)\,e^{-j2\pi(5.05)f}$

Rectangular: $x = 12\cos(10.1\pi) = 10.2\sqcap$

$y = 12\sin(10.1\pi) = j6.3\sqcap$

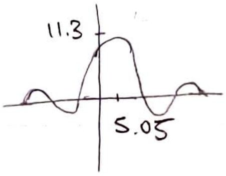
$\sqcap(10.2+j6.3)$

f) $y(t) = \int Y(f)\cdot e^{-j2\pi f t}\, df$

$= 12\int e^{-j2\pi f(5.05)}\cdot e^{-j2\pi f t}\, Y(f)\, df$

$y(t) = 2\cdot A\cdot f_0\, \text{sinc}(2\cdot f_0\cdot t) = 2\cdot 1.2\cdot 5.05\, \text{Sinc}(10.1(t-5.05))$

g)



$12 = 2\cdot A\cdot 5.05$; $A = 1.2$

$11.3\,\text{Sinc}(10.1t) \star \delta(t-5.05)$

EE 210: Homework #3 Solutions

Alvin Maningding

September 27, 2020

5 Convolution

Convolution is defined

$$
\begin{aligned}
y(t) = x(t) * h(t) &= \int_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau)\,\mathrm{d}\tau \\
&= \int_{\tau=-\infty}^{\infty} h(\tau)x(t-\tau)\,\mathrm{d}\tau
\end{aligned}
$$

5.1 $-2\sqcap\left(\frac{t+3}{4}\right) * 2\sqcap\left(\frac{t}{2}\right)$

Let $x(t) = x(\tau)$, as shown in Fig. 2. We choose $h(t)$ to flip and shift, as shown in Fig. 1. To obtain $h(t-\tau)$, flip $h(t) = h(\tau)$ to obtain $h(-\tau)$, which is simply $h(t-\tau)$ with $t = 0$, then substitute t back in.

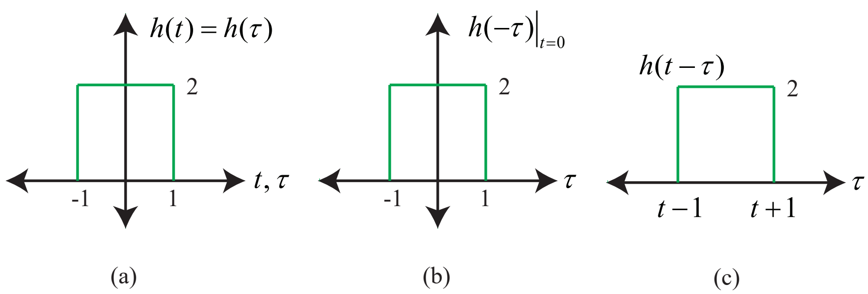


Figure 1: $h(t)$, for problems 5(a) and 5(b), is flipped and shifted to obtain $h(t-\tau)$.

The first region of integration begins when the right edge of $h(t-\tau)$ meets the left edge of $x(\tau)$ at $\tau = t+1 = -5$, or $\boxed{t = -6}$. Applying the convolution integral over the blue-shaded region in Fig. 2(a),

$$
y(t) = \int_{-5}^{t+1} (2)(-2)\,\mathrm{d}\tau = -4t - 24 \quad (-6 \le t < -4)
$$

The next region of integration begins at $\boxed{t = -4}$ and ends when the right edge of $h(t-\tau)$ meets the right edge of $x(\tau)$ at $\tau = t+1 = -1$, or $\boxed{t = -2}$. Applying the convolution integral over the blue-shaded region in Fig. 2(b),

$$
y(t) = \int_{t-1}^{t+1} (2)(-2)\,\mathrm{d}\tau = -8 \quad (-4 \le t < -2)
$$

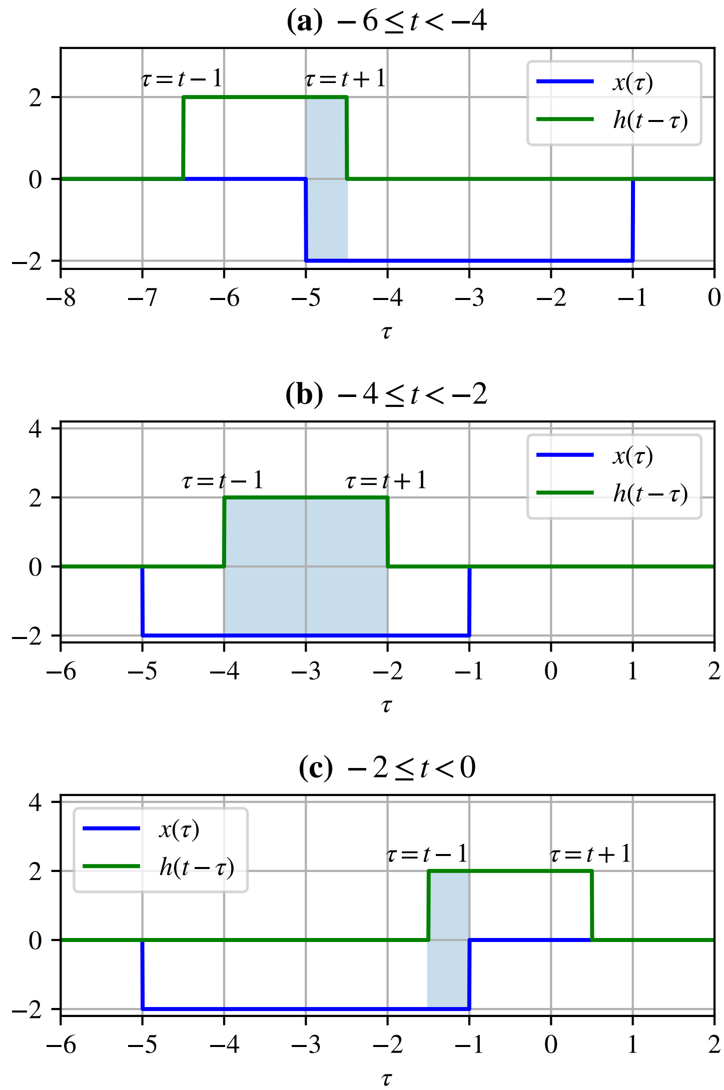


Figure 2

The final region of integration begins at $\boxed{t = -2}$ and ends when the left edge of $h(t-\tau)$ meets the right edge of $x(\tau)$ at $\tau = t - 1 = -1$, or $\boxed{t = 0}$. Applying the convolution integral over the blue-shaded region in Fig. 2(c),

$$y(t) = \int_{t-1}^{1} (2)(-2)\,\mathrm{d}\tau = 4t \quad (-2 \leq t < 0)$$

We have

$$\boxed{y(t) = \begin{cases} -4t - 24 & (-6 \le t < -4) \\ -8 & (-4 \le t < -2) \\ 4t & (-2 \le t < 0) \\ 0 & \text{elsewhere} \end{cases}}$$

The plot is shown in Fig. 3.

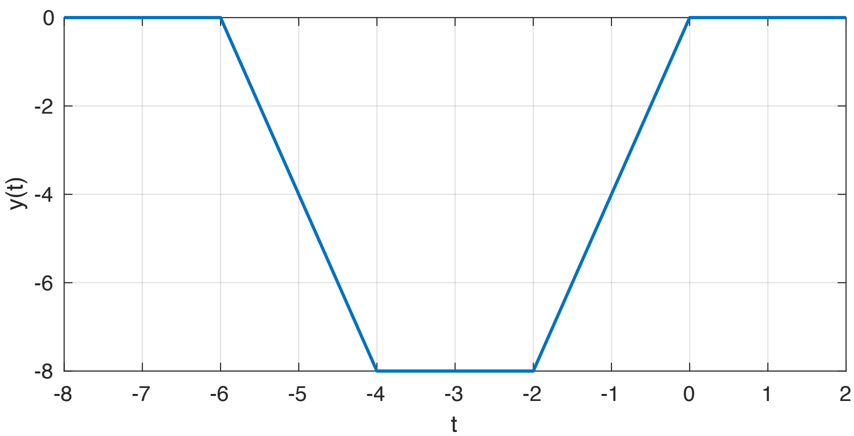


Figure 3

5.2 $\left[2\sqcap\left(\frac{t+3}{3}\right)-3\sqcap\left(\frac{t+3}{4}\right)\right]*2\sqcap\left(\frac{t}{2}\right)$

As before, we choose $h(t)$ to flip and shift. $h(t)$ is identical to the previous problem – refer to Fig. 1.

This problem has many regions of integration, so I will not refer to the "edges" of $x(\tau)$ or $h(t-\tau)$ as before. Note that the shaded areas in Figs. 4, 5, and 6 (see the following pages) may contain more than one region of integration if the shaded area contains a discontinuity in either function. In other words, a discontinuity in $x(\tau)$ or $h(t-\tau)$ requires another integral to be set up.

The regions of integration are given in the labels on each plot. The convolution result is calculated below. Take your time and make sure you are comfortable with obtaining the bounds of integration from the given plots.

$$
y(t)=\begin{cases}
\int_{-5}^{t+1}(2)(-3)\,\mathrm{d}\tau & (-6\le t<-5.5)\\
\int_{-5}^{-4.5}(2)(-3)\,\mathrm{d}\tau+\int_{-4.5}^{t+1}(2)(-1)\,\mathrm{d}\tau & (-5.5\le t<-4)\\
\int_{-t-1}^{-4.5}(2)(-3)\,\mathrm{d}\tau+\int_{-4.5}^{t+1}(2)(-1)\,\mathrm{d}\tau & (-4\le t<-3.5)\\
\int_{t-1}^{t+1}(2)(-1)\mathrm{d}\tau & (-3.5\le t<-2.5)\\
\int_{t-1}^{-1.5}(2)(-1)\mathrm{d}\tau+\int_{-1.5}^{t+1}(2)(-3)\,\mathrm{d}\tau & (-2.5\le t<-2)\\
\int_{t-1}^{-1.5}(2)(-1)\mathrm{d}\tau+\int_{-1.5}^{-1}(2)(-3)\,\mathrm{d}\tau & (-2\le t<-0.5)\\
\int_{t-1}^{-1}(2)(-3)\mathrm{d}\tau & (-0.5\le t<0)\\
0 & \text{elsewhere}
\end{cases}
$$

$$
=\boxed{\begin{cases}
-6t-36 & (-6\le t<-5.5)\\
-2t-14 & (-5.5\le t<-4)\\
4t+10 & (-4\le t<-3.5)\\
-4 & (-3.5\le t<-2.5)\\
-4t-14 & (-2.5\le t<-2)\\
2t-2 & (-2\le t<-0.5)\\
6t & (-0.5\le t<0)\\
0 & \text{elsewhere}
\end{cases}}
$$

$y(t)$ is plotted in Fig. 7 (see last page).

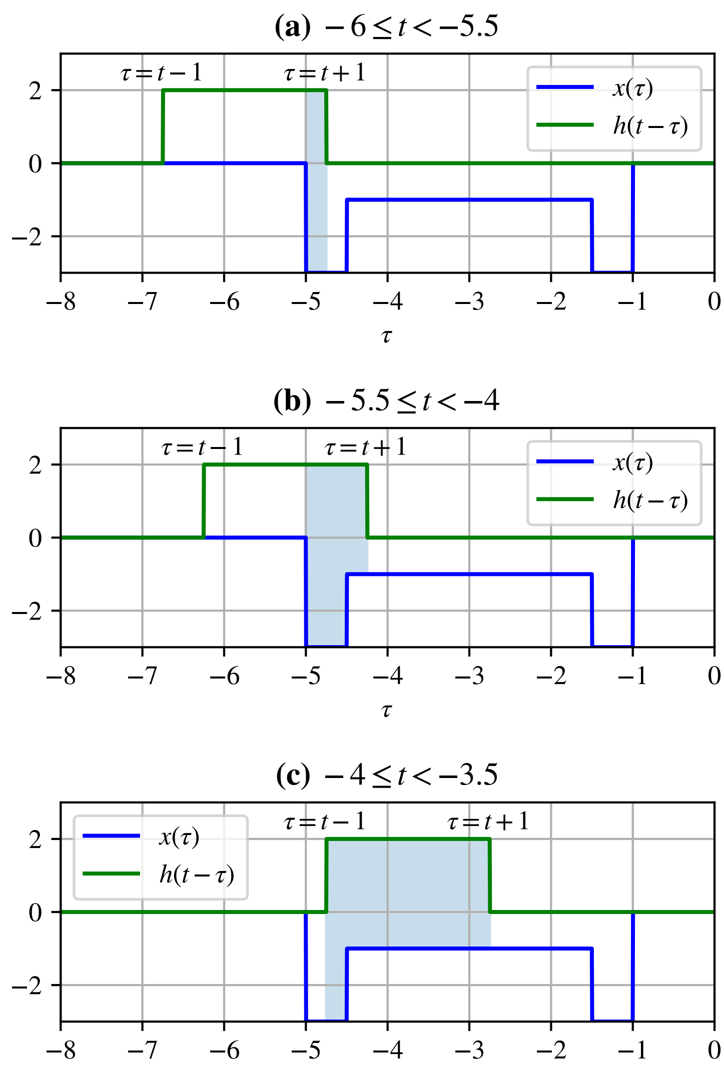

(a) $-6 \le t < -5.5$
$\tau = t - 1$
$\tau = t + 1$
$x(\tau)$
$h(t-\tau)$
(b) $-5.5 \le t < -4$
$\tau = t - 1$
$\tau = t + 1$
$x(\tau)$
$h(t-\tau)$
(c) $-4 \le t < -3.5$
$\tau = t - 1$
$\tau = t + 1$
$x(\tau)$
$h(t-\tau)$
τ

Figure 4

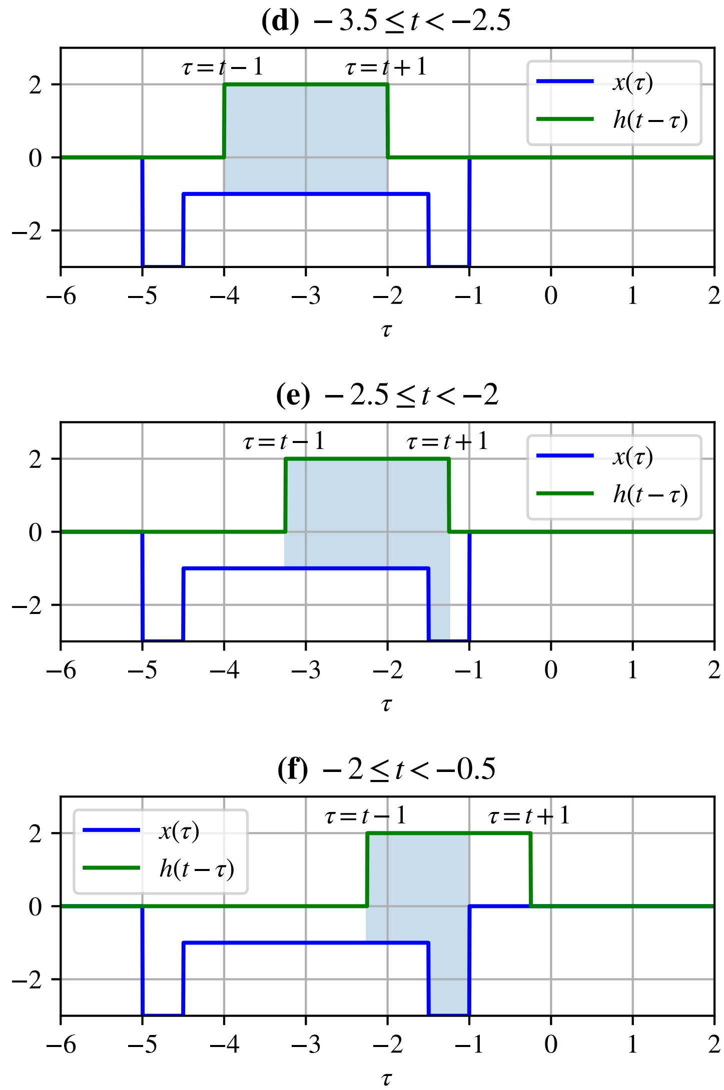


Figure 5

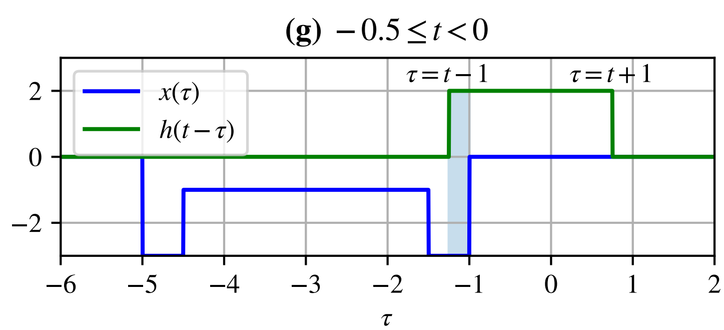


Figure 6

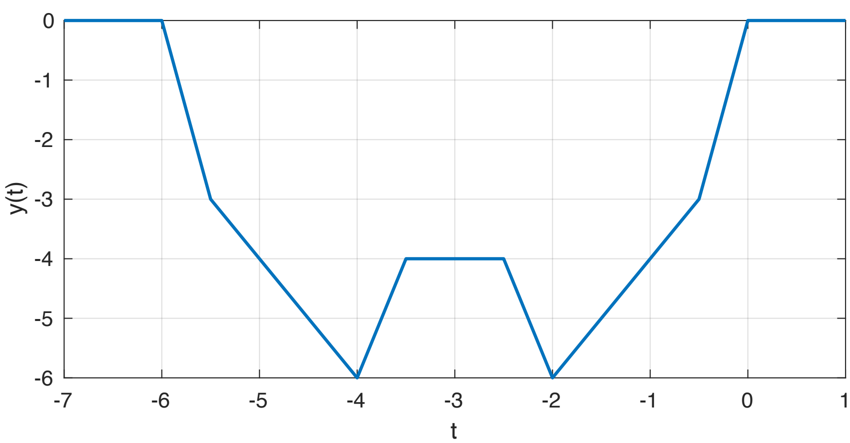


Figure 7: $y(t)$ plotted.