

**HW04**

1. Two signals,  $x_1(t)$  and  $x_2(t)$ , are defined as follows,

$$x_1(t) = \begin{cases} 1 & -1 \leq t \leq 0 \\ 2 & 0 \leq t \leq 1 \\ 0 & \text{elsewhere} \end{cases}, \quad x_2(t) = \begin{cases} 2 & -2 \leq t \leq 1 \\ 1 & 1 \leq t \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

and are shown in Figure 1. Perform the convolution  $y(t) = x_1(t) * x_2(t)$ . Plot the result  $y(t)$  in the time domain and, using MATLAB, in the frequency domain ( $\mathcal{F}\{y(t)\}$ ) (both magnitude & phase plots).

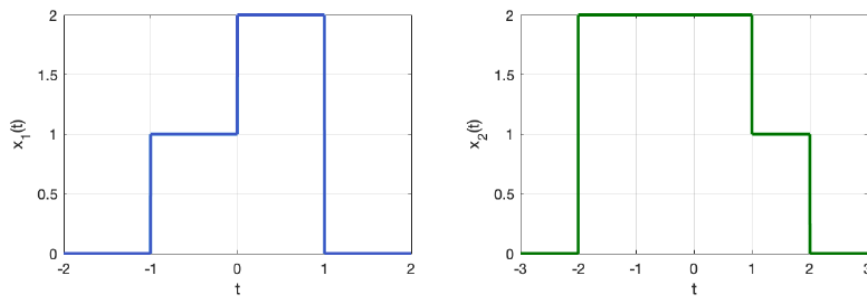


Figure 1:  $x_1(t)$  and  $x_2(t)$  for Problem 1.

2. Two signals,  $x_3(t)$  and  $x_4(t)$ , are defined as follows,

$$x_3(t) = \begin{cases} t^2 & -2 \leq t \leq 0 \\ 0 & \text{elsewhere} \end{cases}, \quad x_4(t) = \begin{cases} -3t+15 & 4 \leq t \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

and are shown in Figure 2. Perform the convolution  $x_3(t) * x_4(t)$  and plot the result in the time domain only.

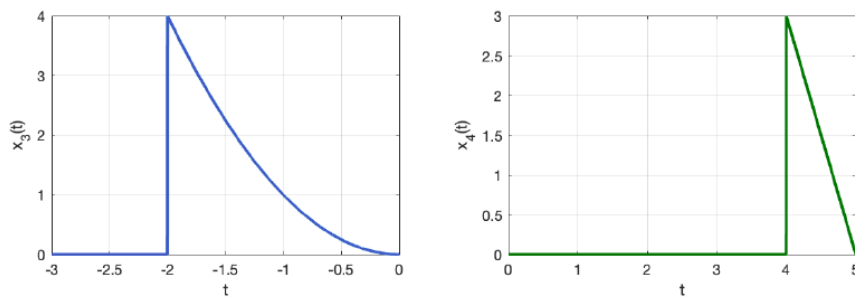
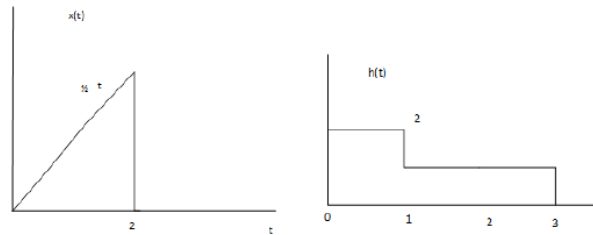


Figure 2:  $x_3(t)$  and  $x_4(t)$  for Problem 2.

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3. If a function is given as

$$y(t) = 3\Delta\left(\frac{t-5}{4}\right)$$

Note: As you may know that  $\left[3\Delta\left(\frac{t-5}{4}\right)\right]$  means just like the rect function. Amplitude: 3, centered at: 5, width: 4

Find a function  $h(t)$  and plot it so the following equation holds

a)  $y(t) = 1 \cdot \Pi\left(\frac{t-1}{2}\right) * h(t)$

b)  $y(t) = 2 \cdot \Pi\left(\frac{t-7}{2}\right) * h(t)$

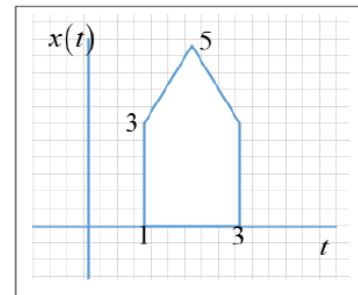
c)  $y(t) = -1 \cdot \Pi\left(\frac{t+5}{2}\right) * h(t)$

4. Find convolution of two functions

$$y(t) = 1 \cdot \Pi\left(\frac{t-1}{2}\right) * \left[2 \cdot \Pi\left(\frac{t-2}{2}\right) + 2 \cdot \Pi(t-5) + 2 \cdot \delta(t-7)\right]$$

Yes there is no mistake; you see a Dirac delta function on the right functions

- Find  $y(t)$
- Plot  $y(t)$
- Find frequency response of  $y(t)$
- Plot magnitude and phase of part (c)



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- 5 In class, we proved that convolution in the time domain corresponds to multiplication in the frequency domain. That is,

$$\mathcal{F}\{x(t) * h(t)\} = \mathcal{F}\left\{\int_{-\infty}^{\infty} x(\tau) \cdot h(t - \tau) d\tau\right\} = X(f)H(f)$$

where  $\mathcal{F}\{x(t)\} = X(f)$  denotes the Fourier transform of  $x(t)$ . **Now show** that convolution in the frequency domain corresponds to multiplication in the time domain:

$$\mathcal{F}^{-1}\{X(f) * H(f)\} = x(t)h(t)$$

where  $\mathcal{F}^{-1}\{X(f)\}$  denotes the inverse Fourier transform of  $X(f)$ .

## 6 Convolution

- a.  $x(t)$  is shown in Figure 1. Perform the convolution  $y(t) = x(t) * x(t)$ ; that is,  $x(t)$  convolved with itself. And plot  $Y(f)$  in magnitude and phase where  $[-10 \leq f \leq 10]$

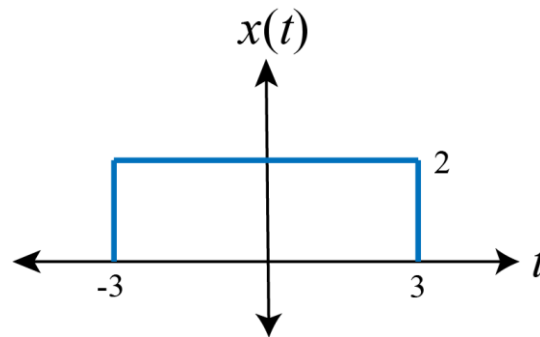


Figure 1:  $x(t)$  for problem 2(a).

- b.  $x_1(t)$  and  $x_2(t)$  are shown in Figure 2. Perform the convolution  $y(t) = x_1(t) * x_2(t)$ . And plot  $Y(f)$  in magnitude and phase where  $[-10 \leq f \leq 10]$

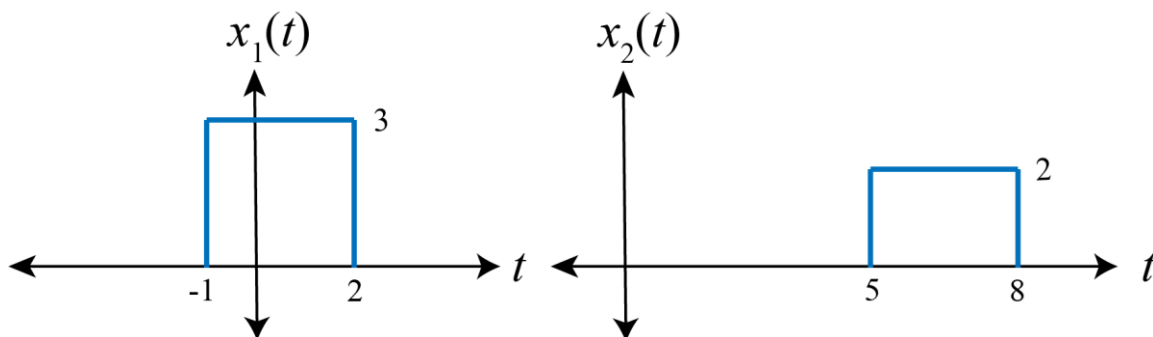
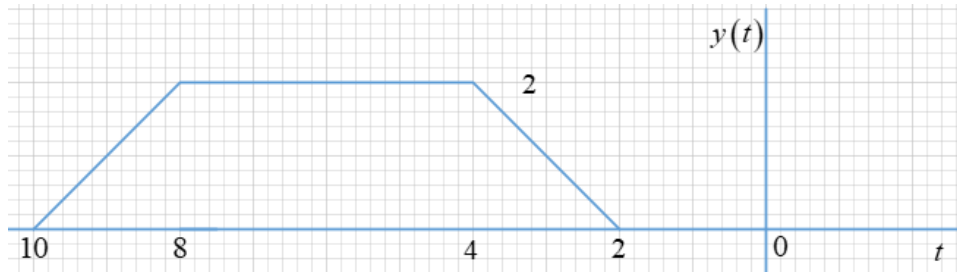


Figure 2:  $x_1(t)$  and  $x_2(t)$  for problem 2(b).

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7 Output of a filter,  $y(t)$ , is shown below.



If input,  $x(t)$ , is given as shown below,

$$y(t) = x(t) * h(t)$$

- Find  $h(t)$  and plot it.
- Find  $Y(f)$  based on  $F\{x(t)\}$  &  $F\{h(t)\}$
- Plot magnitude and phase of  $Y(f)$  using tools.  $[-10 \leq f \leq 10]$



If filter,  $h(t)$ , is given as shown below,

$$y(t) = x(t) * h(t)$$

- Find  $x(t)$  and plot it.
- Find  $Y(f)$  based on  $x(t)$  &  $h(t)$
- Plot magnitude and phase of  $Y(f)$  using tools  $[-10 \leq f \leq 10]$

