HW07

Q.1) a) $$Z[x[n]] = X(z) = \sum_{n=-\infty}^{\infty} x[n]\cdot z^{-n}.$$

if $x[n] = a^n u(n)$, $\quad X(z) = \dfrac{z}{z-a}$; $|z| > |a|$.

$\therefore x[n] = \left(\frac{1}{2}\right)^n u(n)$, $\quad \boxed{X(z) = \dfrac{z}{z-1/2} \; ; \; |z| > \dfrac{1}{2}.}$

b) $x[n] = -\left(\frac{1}{2}\right)^n u[-n-1].$

if $x[n] = -a^n u[-n-1] = \dfrac{z}{z-a}$; $|z| < |a|.$

$$\therefore X(z) = -\sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[-n-1]\cdot z^{-n} = -\sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^n \cdot z^{-n}$$

$$= -\sum_{n=1}^{\infty} (2z)^n = -\left(\frac{2z}{1-2z}\right). = \frac{z}{z-1/2} \; ; \; |z| < \frac{1}{2}$$

$$\boxed{Z\left\{-\left(\frac{1}{2}\right)^n u[-n-1]\right\} = \frac{z}{z-1/2} \; ; \; \text{Roc} \Rightarrow |z| < \frac{1}{2}}$$

c) $x[n] = \left(\frac{1}{2}\right)^n u[-n]$

$$\therefore X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[-n].z^{-n} = \sum_{n=-\infty}^{0} (2z)^n = \frac{1}{1\cancel{z}-2z} \; ; \; |z| < \frac{1}{2}$$

$$\boxed{Z\left\{\left(\frac{1}{2}\right)^n u[-n]\right\} = \frac{1}{1-2z} \; ; \; \text{Roc} \Rightarrow |z| < \frac{1}{2}}$$

d) $x[n] = \delta[n]$

$\therefore X(z) = \displaystyle\sum_{n=-\infty}^{\infty} \delta[n-0]\cdot z^{-n} = z^0 = 1.$; for all values of z

$$\boxed{Z\{\delta[n]\} = 1 \; ; \; \text{Roc} \Rightarrow \text{all values of } z}$$

e) $x[n] = \delta[n-1]$.

$$\therefore X(z) = \sum_{n=-\infty}^{\infty} \delta[n-1] \cdot z^{-n} = z^{-1} \quad ; \quad |z| > 0$$

$$\boxed{Z\{\delta[n-1]\} = z^{-1} \quad ; \quad \text{Roc} \Rightarrow |z| > 0}$$

g) $x[n] = \left(\frac{1}{2}\right)^n \left(u[n] - u[n-10]\right)$.

$$\therefore X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n] - \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n-10].$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n] - \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-10} u[n-10] \left(\frac{1}{2}\right)^{10}$$

We know that, $Z\{a^n u[n]\} = \frac{z}{z-a} \quad ; \quad |z| > |a|$

$$\therefore X(z) = \frac{z}{z - 1/2} - \left(\frac{1}{2}\right)^{10} \times \frac{z}{z - 1/2} \times z^{-10} \quad ; \quad |z| > 0$$

$$= \frac{z}{z - 1/2} \left\{1 - \left(\frac{1}{2z}\right)^{10}\right\} = \frac{2z}{2z-1} \left\{\frac{2z^{10} - 1}{2z^{10}}\right\}$$

$$\therefore \boxed{X(z) = \frac{1 - (2z)^{-10}}{1 - (2z)^{-1}} \quad ; \quad |z| > 0}$$

a) $X(z) = \frac{z}{z - 1/2} \quad ; \quad |z| > 1/2$

b) $X(z) = \frac{z}{z - 1/2} \quad ; \quad |z| < 1/2$

c) $X(z) = \frac{1}{1 - 2z} \quad ; \quad |z| < 1/2$

d) $X(z) = 1 \quad ;$ for all values of z

e) $X(z) = z^{-1} \quad ; \quad |z| > 0$

g) $X(z) = \frac{1 - (2z)^{-10}}{1 - (2z)^{-1}} \quad ; \quad |z| > 0$

Q.2) $x[n] = \begin{cases} n , & 0 \le n \le N-1 \\ N , & N \le n \end{cases}$

$\therefore \; x[n] = n\,u[n] - (n-N)\,u[n-N]$

We know that, $Z\{a^n u[n]\} = \dfrac{z}{z-a} \; ; \; |z| > |a|$ &

$$Z\{n\,x[n]\} = -z \frac{d}{dz} X(z).$$

$$Z\{x[n-n_0]\} = X(z) \cdot z^{-n_0}$$

$$\therefore \; Z\{n\,u[n]\} = -z \frac{d}{dz}\left(\frac{1}{1-z^{-1}}\right) \; ; \; |z|>1.$$

$$= -z \times \frac{1}{(1-z^{-1})^2} \times (0 + 1 \cdot z^{-2}) = \frac{z^{-1}}{(1-z^{-1})^2} \; ; |z|>1$$

$$\therefore \; Z\{(n-N)u[n-N]\} = -z^{-N} \times z \frac{d}{dz}\left(\frac{1}{1-z^{-1}}\right)$$

$$= -z^{-N+1} \times \frac{1}{(1-z^{-1})^2} \times 1 \cdot z^{-2} \; ; \; |z|>1$$

$$= \frac{-z^{-N-1}}{(1-z^{-1})^2} \; ; \; |z|>1$$

$$\therefore \; X(z) = \frac{z^{-1}}{(1-z^{-1})^2} - \frac{z^{-N-1}}{(1-z^{-1})^2} = \frac{z^{-1} - z^{-N-1}}{(1-z^{-1})^2}$$

$$\boxed{\therefore \; X(z) = \frac{z^{-1}(1-z^{-N})}{(1-z^{-1})^2} \; ; \; |z|>1}$$

Q.3)

$$X(z) = (1+2z)(1+3z^{-1})(1-z^{-1})$$

$$= (1+3z^{-1}+2z+6)(1-z^{-1})$$

$$= (7+2z+3z^{-1})(1-z^{-1})$$

$$= 7+2z+3z^{-1}-7z^{-1}-2-3z^{-2}$$

$$\therefore X(z) = 2z+5-4z^{-1}-3z^{-2}$$

we know that, $Z\{k\delta[n]\} = k$

$$Z\{\delta[n-1]\} = z^{-1}$$

$$Z\{\delta[n+1]\} = z.$$

$$\boxed{\therefore x[n] = 2\delta[n+1] + 5\delta[n] - 4\delta[n-1] - 3\delta[n-2].}$$