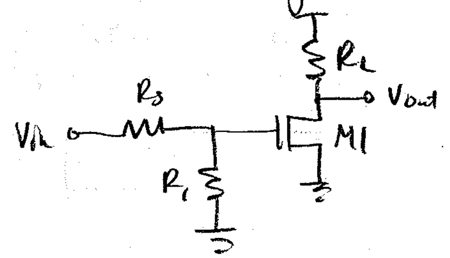
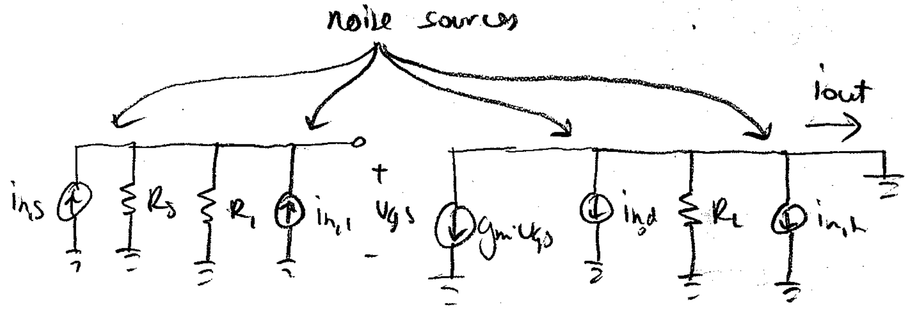
Lee: Problem 12.5 (a)

Draw small signal model (neglect all capacitances)



$\Rightarrow$



→ there are 4 seperate noise sources, calculate iout due to each (while neglecting the others)

① Noise due to source resistor (only $i_{n,S}$ noise source is active, others are open-circuited)

$$V_{gs} = i_{n,S} \cdot R_S \| R_1$$

−assume $R_S = R_1 = R$ (for input match)

$$\therefore V_{gs} = i_{n,S} \cdot \frac{R}{2}$$

$$\therefore i_{out} = -g_m V_{gs} = -g_m \cdot i_{n,S} \cdot \frac{R}{2}$$

→ We want power : $\overline{i_{out}^2} = g_m^2 \cdot \frac{R^2}{4} \cdot \overline{i_{n,S}^2}$

−sub in expression for $\overline{i_{n,S}^2}$: $\overline{i_{n,S}^2} = \frac{4kT\Delta f}{R_S} = \frac{4kT\Delta f}{R}$

$$\therefore \overline{i_{out}^2} = g_m^2 \cdot \frac{R^2}{4} \cdot \frac{4kT\Delta f}{R} = \boxed{\frac{g_m^2 \cdot R \cdot 4kT\Delta f}{4} = \overline{i_{out}^2}}$$

noise at O/P due to R_S

② Now repeat for noise due to R_1 ($i_{n,1}$)

−from inspection, this will experience the same gain to the output as $i_{n,S}$ did.

$$\therefore \boxed{\overline{i_{out}^2} = \frac{g_m^2 \cdot R \cdot 4kT\Delta f}{4}}$$

noise at O/P due to R_1

③ Now consider noise due to drain thermal noise ($i_{n,d}$)

- from inspection, $i_{out} = i_{n,d}$

$$\therefore \overline{i_{out}^2} = \overline{i_{n,d}^2}$$

→ Sub in expression for $\overline{i_{n,d}^2}$: $\overline{i_{n,d}^2} = 4kT\gamma \cdot g_{do} \cdot \Delta f$

$$\therefore \boxed{\overline{i_{out}^2} = 4kT\gamma \cdot g_{do} \cdot \Delta f}$$

↑ noise at O/P due to drain thermal noise of M1.

④ Finally, consider noise from R_L ($i_{n,L}$)

- also appears directly at O/P

$$\therefore \overline{i_{out}^2} = \overline{i_{n,L}^2}$$

→ Sub in $\overline{i_{n,L}^2} = \frac{4kT\Delta f}{R_L}$

$$\therefore \boxed{\overline{i_{out}^2} = \frac{4kT\Delta f}{R_L}}$$

↑ O/P noise due to resistor R_L.

→ Now, to find noise factor: $F = \frac{\text{total noise at O/P}}{\text{noise at O/P due to } R_S}$

$$= \frac{\left[\frac{g_m^2 \cdot R \cdot 4kT\Delta f}{4} + \frac{g_m^2 \cdot R \cdot 4kT\Delta f}{4} + 4kT\gamma \cdot g_{do} \cdot \Delta f + \frac{4kT\Delta f}{R_L}\right]}{\left[\frac{g_m^2 \cdot R \cdot 4kT\Delta f}{4}\right]}$$

$$= 1 + 1 + \frac{\cancel{4kT}\gamma \cdot g_{do} \cancel{\Delta f} \cdot 4}{g_m^2 \cdot R \cdot \cancel{4kT\Delta f}} + \frac{\cancel{4kT\Delta f} \cdot 4}{R_L \cdot g_m^2 \cdot R \cdot \cancel{4kT\Delta f}}$$

$$= 2 + \frac{4 \cdot \gamma \cdot g_{do}}{g_m^2 \cdot R} + \frac{4}{g_m^2 \cdot R \cdot R_L}$$

Sub in $\alpha = \frac{g_m}{g_{do}}$:

$$\boxed{F = 2 + \frac{4\gamma}{\alpha \cdot R \cdot g_m} + \frac{4}{g_m^2 \cdot R \cdot R_L} \; \triangle}$$

Last term due to load was not included in the expression given in class.