EE 210

HW#: 07

Last Name: **Aldacher**

First Name: **Muhammad**

ID: **011510317**

Date: **10/17/2020**

Assigned question #s: **3**

HW07

3.1. (a) $x[n] = \left(\frac{1}{2}\right)^n u[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n]\, z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$-$ $\quad X(z) = 1 + \left(\frac{1}{2}\right)^1 z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \left(\frac{1}{2}\right)^3 z^{-3} + \cdots$ multiply $\left(\frac{1}{2}\right)z^{-1}$ on both sides

$$\left(\frac{1}{2}z^{-1}\right) X(z) = \left(\frac{1}{2}\right)z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \cdots$$

$=$

$$\left(1 - \frac{1}{2}z^{-1}\right) X(z) = 1 \qquad \left|\frac{1}{2}z^{-1}\right| < 1$$

$$\boxed{\therefore X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{z}{z-\frac{1}{2}}} \qquad \boxed{|z| > \left|\frac{1}{2}\right|} \text{ ROC}$$

(b) $x[n] = -\left(\frac{1}{2}\right)^n u[-n-1] \rightarrow -n-1 \geq 0$, $n \leq -1$

Removing the "$-$" sign in $x[n]$ & will attach in the end

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[-n-1]\, z^{-n}$$

$$= \sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^n z^{-n} = \sum_{m=1}^{\infty} \left(\frac{1}{2}\right)^{-m} z^{m}$$

$X(z) = \left(\frac{1}{2}\right)^{-1} z + \left(\frac{1}{2}\right)^{-2} z^2 + \cdots$ multiply $\left(\frac{1}{2}\right)^{-1} z$ on both sides

$$\left(\left(\frac{1}{2}\right)^{-1} z\right) X(z) = \left(\frac{1}{2}\right)^{-2} z^2 + \left(\frac{1}{2}\right)^{-3} z^3 + \cdots$$

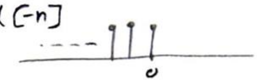
subtract

$$\left(1 - \left(\frac{1}{2}\right)^{-1} z\right) X(z) = \left(\frac{1}{2}\right)^{-1} z \qquad \left|\left(\frac{1}{2}\right)^{-1} z\right| < 1$$

$$\boxed{\therefore X(z) = \frac{-\left(\frac{1}{2}\right)^{-1} z}{1 - \left(\frac{1}{2}\right)^{-1} z} = \frac{z}{z-\frac{1}{2}}} \qquad \boxed{|z| < \left|\frac{1}{2}\right|} \text{ ROC}$$

(c) $x[n] = \left(\frac{1}{2}\right)^n u[-n] \longrightarrow \begin{matrix} -n \geq 0 \\ n \leq 0 \end{matrix}$

$u[-n]$



$$X(z) = \sum_{n=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^n u[-n]\, z^{-n} = \sum_{n=-\infty}^{0} \left(\tfrac{1}{2}\right)^n z^{-n}$$

Let $m = -n$

$n = 0 \rightarrow m = 0$

$n = -\infty \rightarrow m = \infty$

$$= \sum_{m=0}^{\infty} \left(\tfrac{1}{2}\right)^{-m} z^{m}$$

$$X(z) = 1 + \left(\tfrac{1}{2}\right)^{-1} z^{1} + \left(\tfrac{1}{2}\right)^{-2} z^{2} + \text{-----}$$

$$\left(\left(\tfrac{1}{2}\right)^{-1} z^{1}\right) X(z) = \left(\tfrac{1}{2}\right)^{-1} z^{1} + \left(\tfrac{1}{2}\right)^{-2} z^{2} + \text{------}$$

subtract

$$\left(1 - \left(\tfrac{1}{2}\right)^{-1} z^{1}\right) X(z) = 1 \qquad \text{where } \left|\left(\tfrac{1}{2}\right)^{-1} z^{1}\right| < 1$$

$$\boxed{X(z) = \frac{1}{1 - \left(\frac{1}{2}\right)^{-1} z} = \frac{1/2}{1/2 - z}} \qquad \boxed{|z| < \left|\tfrac{1}{2}\right|} \text{ ROC}$$

Another way for (c)

$$x[n] = \left(\tfrac{1}{2}\right)^n u[-n]$$

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^{n+1-1} u[-n+1-1]\, z^{-n}$$

$-(n-1)-1$

let $n-1 = m$

$n = m+1$

n	m
$-\infty \rightarrow$	$-\infty$
$\infty \rightarrow$	∞

$$= \sum_{m=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^{m+1} u[-m-1]\, z^{-m-1}$$

$$= \left(\tfrac{1}{2}\right) z^{-1} \underbrace{\sum_{-\infty}^{\infty} \left(\tfrac{1}{2}\right)^{m} u[-m-1]\, z^{-m}}$$

$$= \frac{1}{2} z^{-1} \frac{-z}{z - \left(\frac{1}{2}\right)}$$

$$\therefore \boxed{X(z) = \frac{1/2}{1/2 - z}} \qquad \text{ROC } \boxed{|z| < \left|\tfrac{1}{2}\right|}$$

(d) $x[n] = \delta(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} \delta(n)\, z^{-n} = z^{(0)} = 1 \qquad \boxed{X(z)=1} \qquad \boxed{-\infty < z < \infty}\ \text{ROC}$$

all z

(e) $x[n] = \delta(n-1)$

$$X(z) = \sum_{n=-\infty}^{\infty} \delta(n-1)\, z^{-n} = z^{-1} \qquad \boxed{X(z) = z^{-1}}$$

$$\text{ROC:} \quad \sum_{n=-\infty}^{\infty} |x[n] z^{-n}| < \infty \quad \rightsquigarrow \quad |z^{-1}| < \infty \quad \rightarrow \quad \boxed{|z| > 0}\ \text{ROC}$$

all z except 0

(f) $x[n] = \delta(n+1)$

$$X(z) = \sum_{n=-\infty}^{\infty} \delta(n+1)\, z^{-n} = z^{-(-1)} \qquad \boxed{X(z) = z}$$

$$\text{ROC:} \quad |z| < \infty \quad \rightsquigarrow \quad \boxed{\begin{array}{c} |z| < \infty \\ -\infty < z < \infty \end{array}}\ \text{ROC}$$

(g) $x[n] = \left(\frac{1}{2}\right)^n u(n) - \underbrace{\left(\frac{1}{2}\right)^n u(n-10)}$

$$X_B(z) = \sum_{n=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^n u(n-10)\, z^{-n}$$

$$= \sum_{n=10}^{\infty} \left(\tfrac{1}{2}\right)^n z^{-n}$$

$$X_B(z) = \left(\tfrac{1}{2}\right)^{10} z^{-10} + \left(\tfrac{1}{2}\right)^{11} z^{-11} + \cdots$$

$$\left(\tfrac{1}{2}\right) z^{-1} X_B(z) = \left(\tfrac{1}{2}\right)^{11} z^{-11} + \left(\tfrac{1}{2}\right)^{12} z^{-12} + \cdots$$

Subtract

$$\left(1 - \tfrac{1}{2} z^{-1}\right) X_B(z) = \left(\tfrac{1}{2}\right)^{10} z^{-10} \qquad \left|\tfrac{1}{2} z^{-1}\right| < 1 \qquad \boxed{|z| > \left|\tfrac{1}{2}\right|}$$

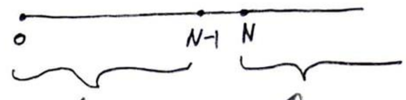
$$X_B(z) = \frac{\left(\frac{1}{2}\right)^{10} z^{-10}}{1 - \frac{1}{2} z^{-1}} = \frac{\left(\frac{1}{2}\right)^{10} z^{-9}}{z - \frac{1}{2}}$$

$$\boxed{\therefore X(z) = \frac{z}{z - \left(\frac{1}{2}\right)} - \frac{z}{z - \left(\frac{1}{2}\right)} \left(\tfrac{1}{2}\right)^{10} z^{-10}}$$

ROC $\boxed{|z| > \left|\tfrac{1}{2}\right|}$

3.2

$$x[n] = \begin{cases} n & 0 \leq n \leq N-1 \\ N & n \geq N \end{cases}$$



$$x[n] = n\big(u[n] - u[n-N]\big) + N\,u[n-N]$$

$$= \underbrace{n\,u[n]}_{x_A} - \underbrace{n\,u[n-N]}_{x_B} + \underbrace{N\,u[n-N]}_{x_C} = x_A - x_B + x_C$$

$$\rightarrow x_A[n] = n\,(1)^{n-1}u[n] \longrightarrow X_A(z) = \frac{z}{(z-1)^2} \qquad |z| > |1|$$

$$\rightarrow x_B[n] = n\,(1)^{n-N}u[n-N]$$

$$= (n-N+N)\,(1)^{n-N}u[n-N]$$

$$= N\,(1)^{n-N}u[n-N] + (n-N)\,(1)^{n-N-1+1}u[n-N]$$

$$= \big(N\,(1)^{n}u[n] + n\,(1)^{n-1}u[n]\big) * \delta[n-N]$$

$$X_B(z) = N\,\frac{z}{z-1}\,z^{-N} + \frac{z}{(z-1)^2}\,z^{-N} \qquad |z| > |1|$$

$$\rightarrow x_C[n] = N\,u[n-N]$$

$$= N\,(1)^{n-N}u[n-N]$$

$$= \big(N\,(1)^{n}u[n]\big) * \delta(n-N) \longrightarrow X_C(z) = \frac{z}{z-1}\,z^{-N} \qquad |z| > |1|$$

$$\therefore X(z) = X_A(z) - X_B(z) + X_C(z)$$

$$= \frac{z}{(z-1)^2} - N\,\frac{z^{-N+1}}{z-1} - \frac{z^{-N+1}}{(z-1)^2} + \frac{z^{-N+1}}{z-1}$$

$$\boxed{\therefore X(z) = \frac{z^{N}-1}{z^{N-1}(z-1)^2} - \frac{N-1}{z^{N-1}(z-1)} \qquad \boxed{|z| > |1|}\ \text{ROC}}$$

3.5

$$X(z) = (1+2z)(1+3z^{-1})(1-z^{-1})$$

$$= (1+2z)(1-z^{-1}+3z^{-1}-3z^{-2})$$

$$= (1+2z)(1+2z^{-1}-3z^{-2})$$

$$= \cancel{1}+2z^{-1}-3z^{-2}+2z+\cancel{4}-6z^{-1}$$

$$= 2z+5-4z^{-1}-3z^{-2}$$

$$\therefore \boxed{x[n] = 2\delta[n+1]+5\delta[n]-4\delta[n-1]-3\delta[n-2]}$$