EE230-02 RFIC II
Fall 2018

Lecture 3: RF Basics Review2

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Non-idealities in RF Circuits

- Definition: Signal Power
- Noise
- Linearity

Voltage Gain and Power Gain

Voltage Gain $$A_V|_{\mathrm{dB}} = 20\log\frac{V_{out}}{V_{in}}$$

Power Gain $$A_P|_{\mathrm{dB}} = 10\log\frac{P_{out}}{P_{in}}$$

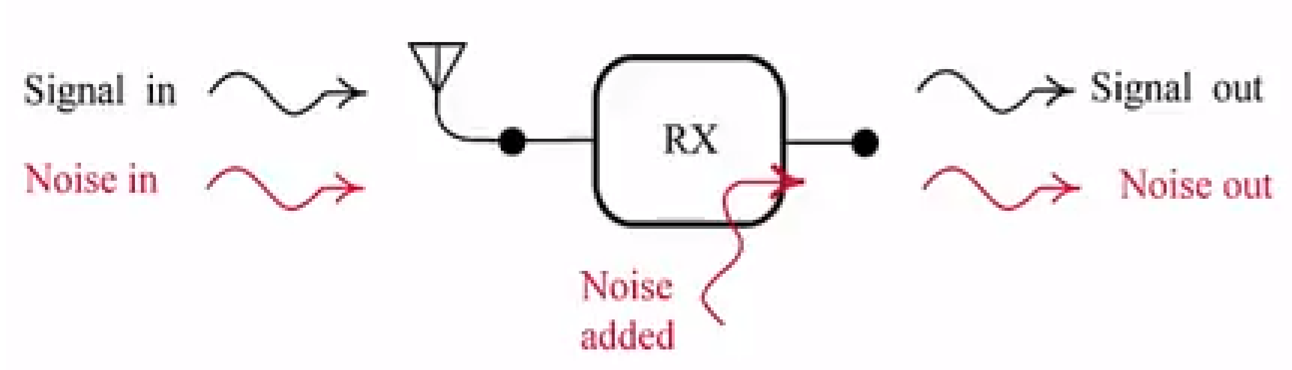
If an amplifier having an input resistance of Ri and driving a load resistance of Ro,

$$\boldsymbol{A_{P|dB} = 10log\frac{\frac{V_{out}^2}{R_o}}{\frac{V_{in}^2}{R_i}} = 10log\frac{V_{out}^2}{V_{in}^2} + 10log\frac{R_i}{R_o}}$$

$$\boldsymbol{= 20log\frac{V_{out}}{V_{in}} + 10log\frac{R_i}{R_o} = A_{V|dB} + 10log\frac{R_i}{R_o}}$$

Voltage Gain ≠ Power Gain if Rin ≠ Rout.

Noise Factor and Noise Figure (dB)



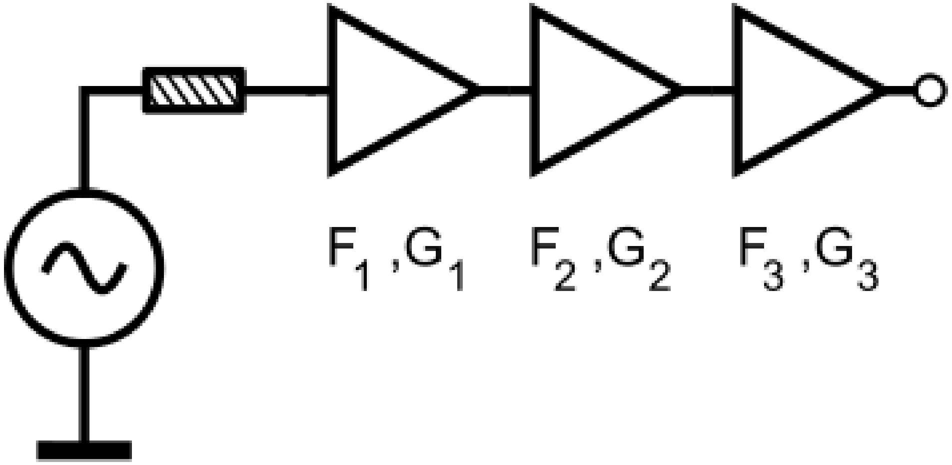
Signal Out = G x Signal In

Noise Out = G x Noise In **+ Noise added**

$$NF = \frac{(SNR)_{in}}{(SNR)_{out}} \geq 1$$ A receiver degrades the SNR!

Friis Formula

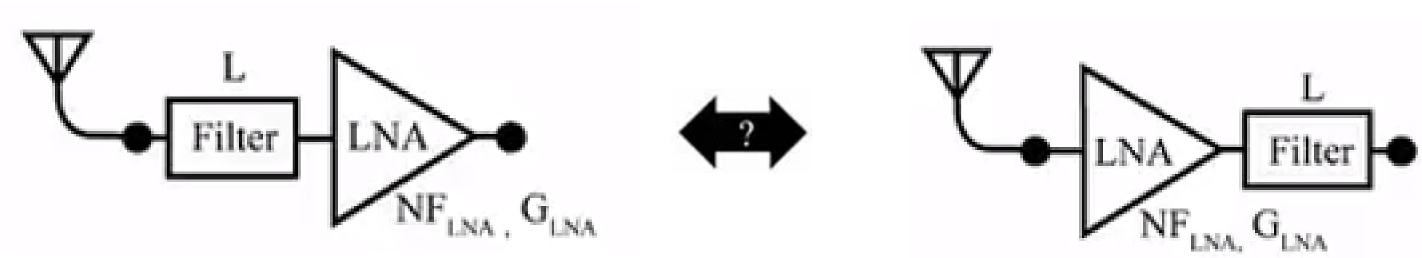
Friis's formula is used to calculate the total noise factor of a cascade of stages, each with its own noise factor and gain where F_i and G_i are the noise factor and available power gain. Note that both magnitudes are expressed as ratios, not in decibels.



$$F_{total} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \ldots + \frac{F_n - 1}{G_1 G_2 \ldots G_{n-1}}$$

Noise Figure of Cascaded Stages

- Which configuration is better in terms of noise?

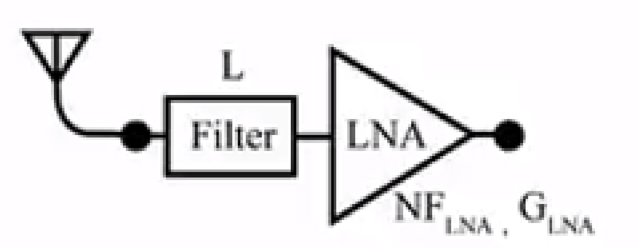


- Filters are passive and usually lossy, i.e. L (=1/G)
- NF of a lossy stage is equal to its loss, i.e. NF = L

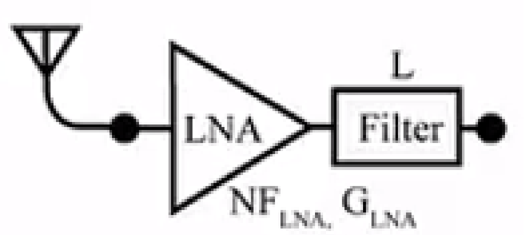
Noise Figure of Cascaded Stages

Example:
$L = 1$ dB = 1.25
G_{LNA} = 10 dB = 10
NF_{LNA} = 3 dB = 2

$$F_{total} = F_1 + \frac{F_2 - 1}{G_1}$$



$$NF_{tot} = L + (NF_{LNA} - 1) \cdot L$$
$$= 1.25 + 1\text{x}1.25 = 2.5 = 4\text{dB}$$



$$NF_{tot} = NF_{LNA} + (L - 1) / G_{LNA}$$
$$= 2 + 0.25/10 = 2.025 = 3dB$$

1. Place those components with lowest NF and highest gain at earlier stages
2. Avoid lossy components at the input

Linear vs. Nonlinear

Linear $$y(t) = \alpha x(t)$$

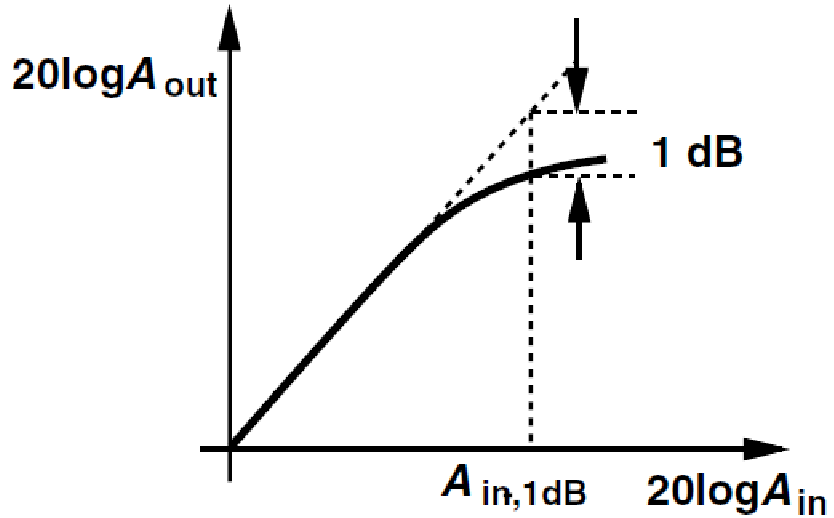
Nonlinear $$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$

$$x(t) = A\cos\omega t$$

$$\begin{aligned} y(t) &= \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3 A^3\cos^3\omega t \\ &= \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}(1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3\cos\omega t + \cos 3\omega t) \\ &= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t \end{aligned}$$

Gain Compression

$$y(t) = \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t$$



$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A^2_{in,1dB}\right| = 20\log|\alpha_1| - 1\text{ dB}$$

$$A_{in,1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

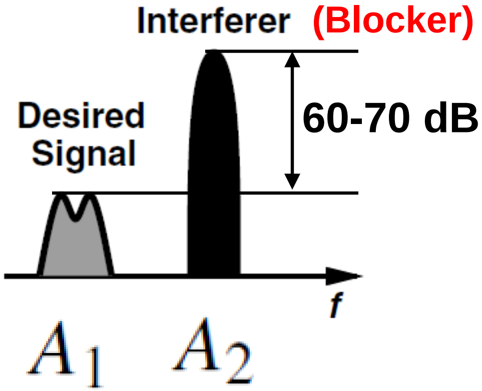
Gain Desensitization

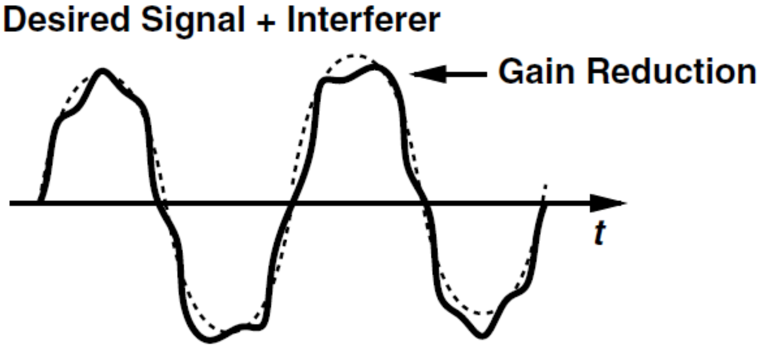
$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$x(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t \qquad A_1 \ll A_2$$

$$y(t) = \left(\alpha_1 + \frac{3}{4}\alpha_3 A_1^2 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots$$

$$\approx \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots$$



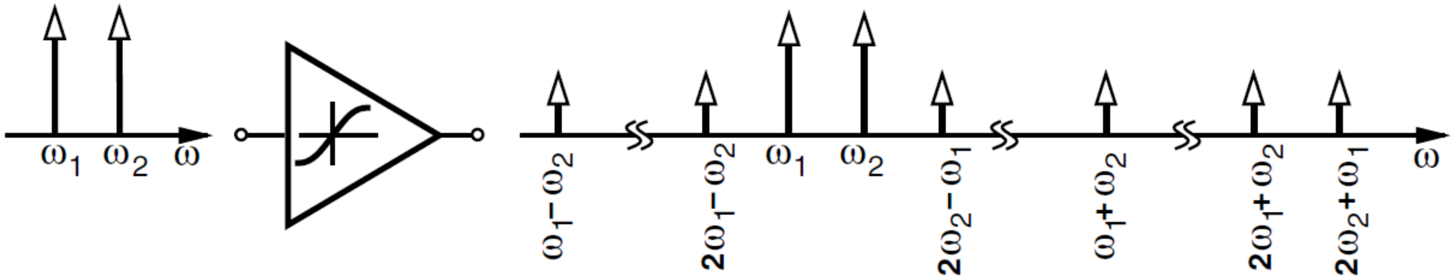


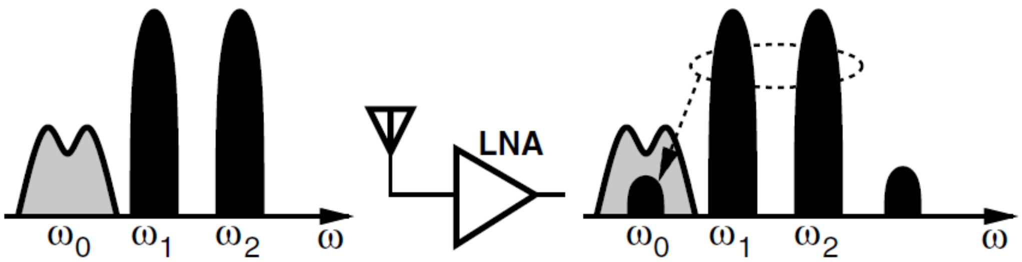
Intermodulation

$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

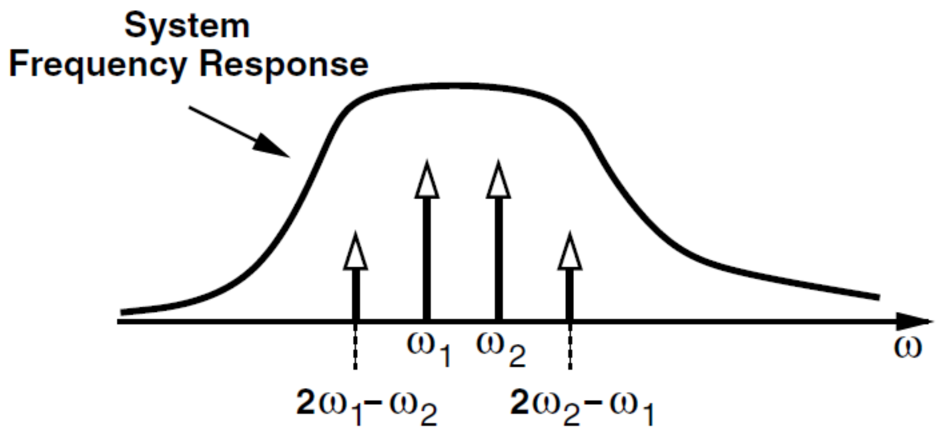
$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

$$y(t) = \alpha_1 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + \alpha_2 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 + \alpha_3 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3.$$

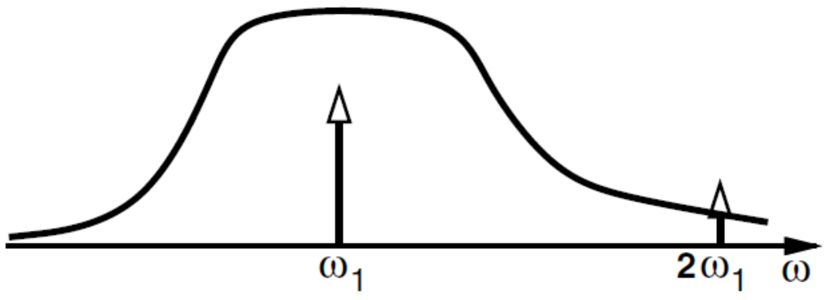




Two-tone vs. Single-tone test



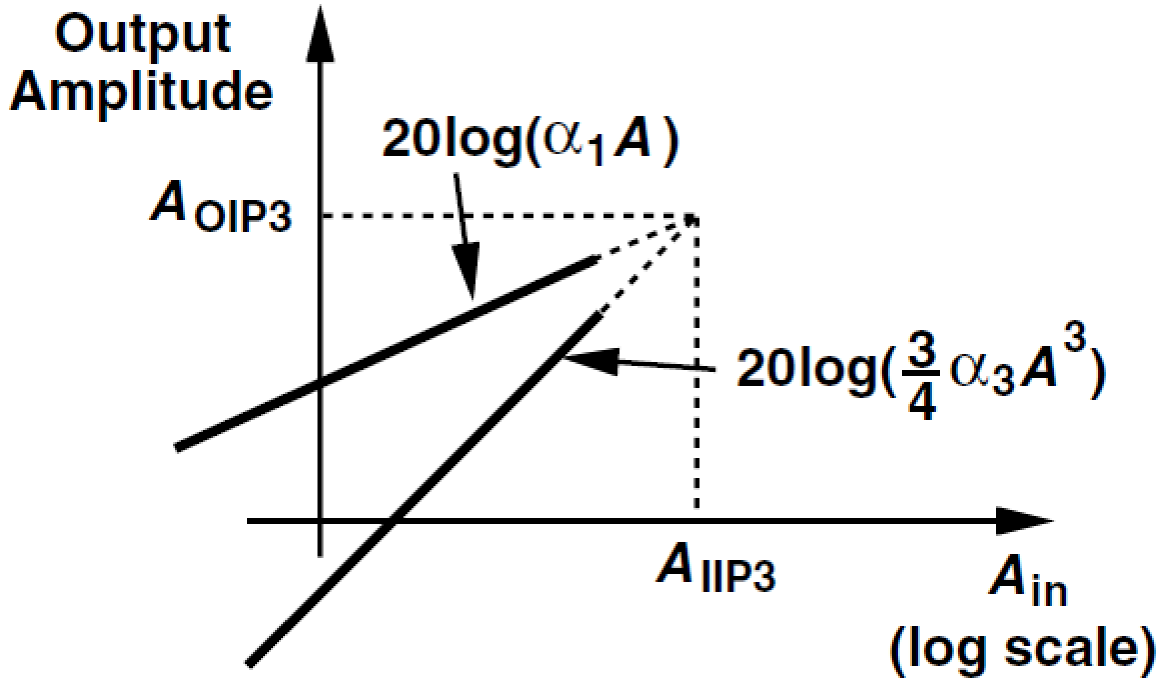
(a)



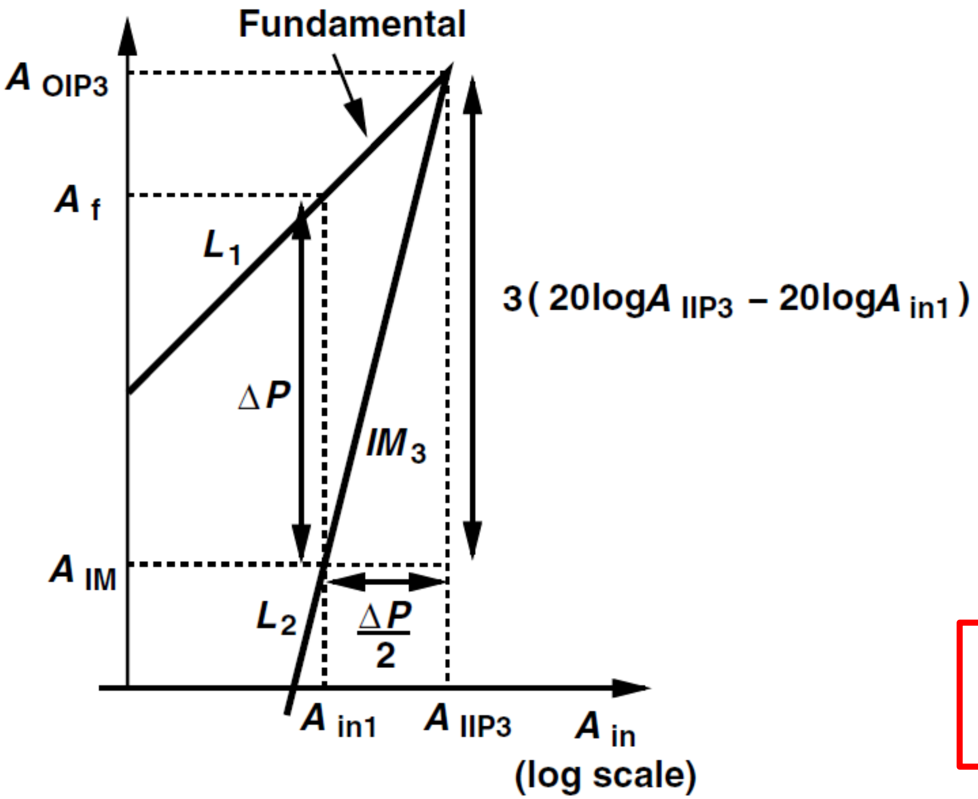
(b)

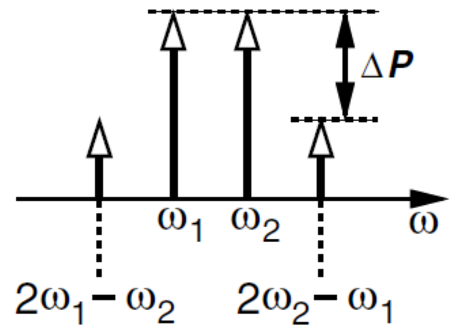
IP3 (Third Intercept Point)

$$
\begin{aligned}
y(t) &= \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3 A^3\cos^3\omega t \\
&= \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}(1+\cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3\cos\omega t + \cos 3\omega t) \\
&= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4}\right)\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t
\end{aligned}
$$



IP3

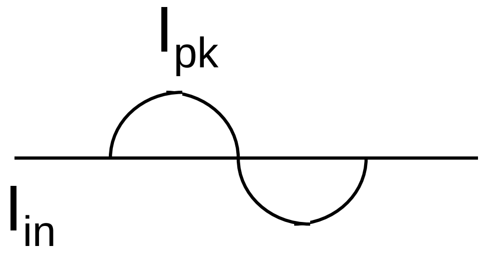


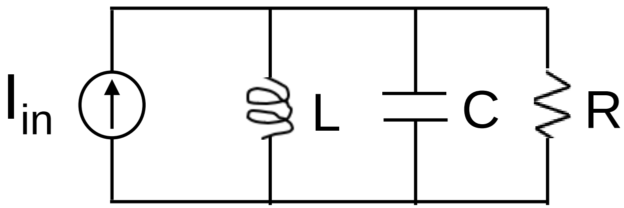


$$IIP_3\big|_{dBm} = \frac{\Delta P\big|_{dB}}{2} + P_{in}\big|_{dBm}$$

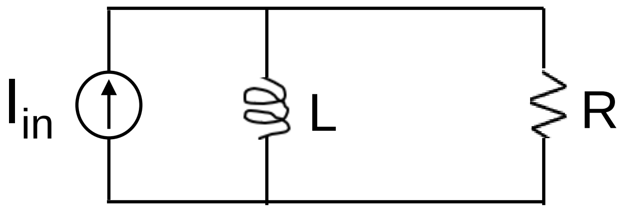
For a given input level (well below *P*1*dB*), the IIP3 can be calculated by halving the difference between the output fundamental and IM levels and adding the result to the input level,
where all values are expressed as logarithmic quantities.

Parallel RLC



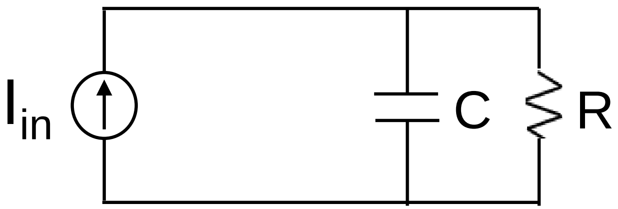


Low Frequency

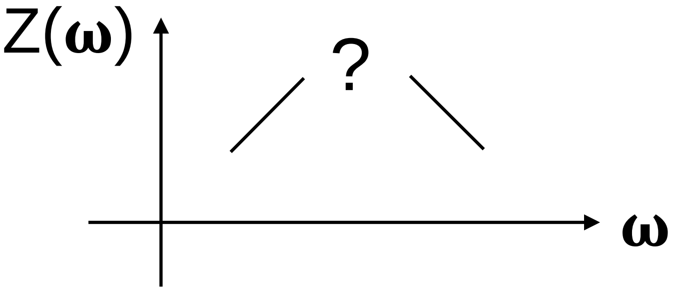


Inductive

High Frequency



Capacitive



Admittance

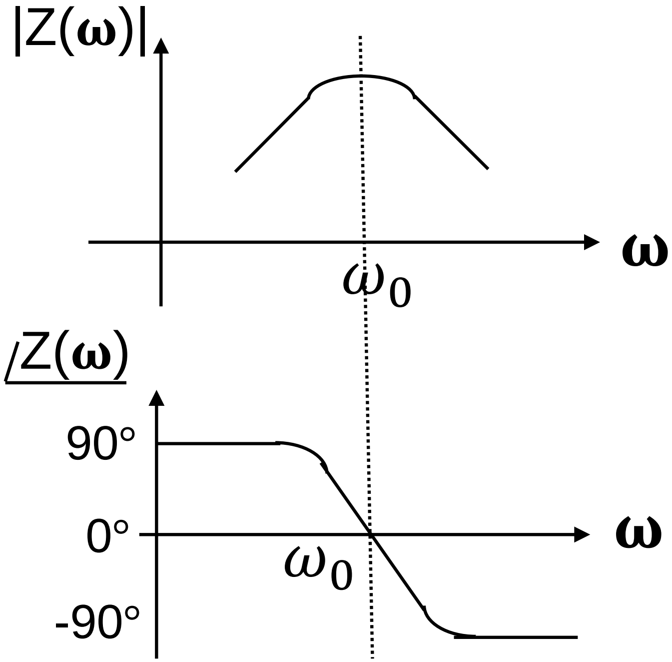
$$Y(\omega) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$

$$At\ resonance, \qquad \omega_0 C = \frac{1}{\omega_0 L}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Y(\omega_0) = \frac{1}{R} \longrightarrow Purely\ resistive$$

Quality Factor

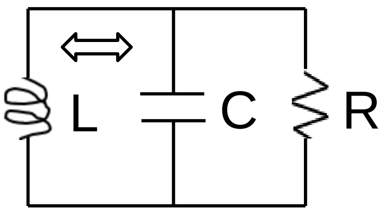


Convenient on-chip value
1pF || 1nH ≈ 5 GHz

Quality Factor

$$Q \equiv \omega_0 \cdot \frac{Energy\ Stored}{Avg\ Power\ Dissipated}$$

At $\omega = \omega_0$, $V_{out} = I_{in} \cdot R$



Stored energy moves back and forth Between L and C

Quality Factor

$$V_{outpk} = I_{pk} \cdot R \quad \Longrightarrow \quad E_{tot.} = \frac{1}{2} C (I_{pk} \cdot R)^2$$

$$P_{ave} = (I_{rms})^2 \cdot R = \left(\frac{I_{pk}}{\sqrt{2}}\right)^2 \cdot R = \frac{1}{2} I_{pk}^2 \cdot R$$

$$Q \equiv \omega_0 \cdot \frac{E_{tot}}{P_{avg}} = \frac{1}{\sqrt{LC}} \cdot \frac{\frac{1}{2} C (I_{pk} \cdot R) 2}{\frac{1}{2} I_{pk}^2 \cdot R} = \frac{CR}{\sqrt{LC}} = \frac{R}{\sqrt{L/C}}$$

Check: If R $\rightarrow \infty$, Q $\rightarrow \infty$

$\sqrt{\frac{L}{C}}$ = Characteristic Impedance of network

At Resonance

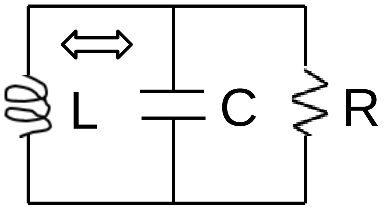
$$|Z_L| = \omega_0 L = \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}}$$

$$|Z_C| = \frac{1}{\omega_0 C} = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}}$$

$Q\ for\ Parallel\ RLC$

$$Q = \frac{R}{|Z_L|} = \frac{R}{\omega_0 L}$$

$$Q = \frac{R}{|Z_C|} = \omega_0 RC$$

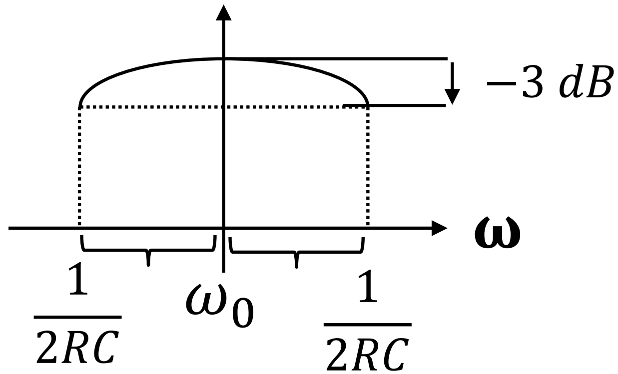


$Beware\ of$
$branch\ current\ at\ resonance$

$$|I_L| = |I_C| = \frac{|V_{out}|}{\omega_0 L} = \frac{|I_{in}|R}{\omega_0 L} = Q \cdot |I_{in}|$$

- Very large current can flow through L&C at resonance!
- Careful layout is necessary

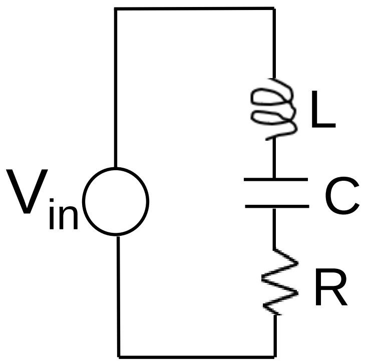
BW & Q relationship



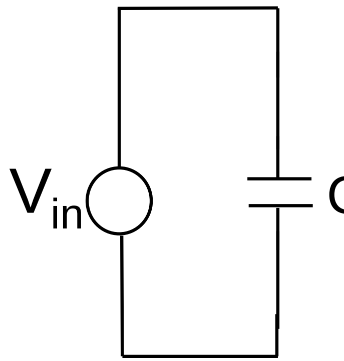
$$Total\ BW = \frac{1}{RC}$$

$$\frac{\omega_0}{BW} = \frac{RC}{\sqrt{LC}} = \frac{R}{\sqrt{L/C}} = Q$$

Series RLC

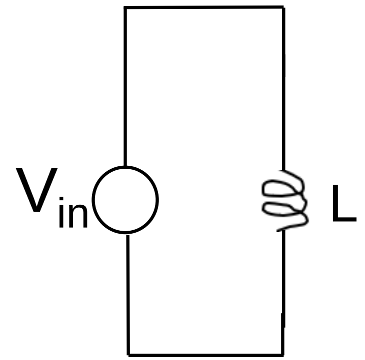


Low Frequency



Capacitive

High Frequency



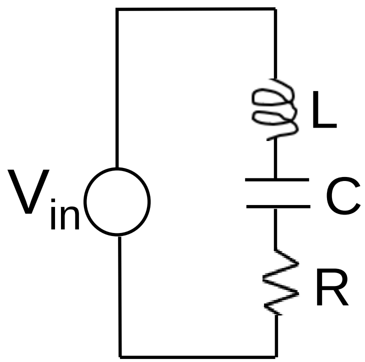
Inductive

$$\mathrm{Z}(\omega) = R + j\omega L + \frac{1}{j\omega C} = R + j(\omega L - \frac{1}{\omega C})$$

$$At\ resonance,\ \omega_0 L = \frac{1}{\omega_0 C} \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Z(\omega_0) = R \longrightarrow Purely\ resistive$$

At Resonance



$$Beware\ of$$
$$branch\ voltage\ at\ resonance$$

$$|V_L| = |V_C| = Q \cdot |V_{in}|$$

- A very large voltage can be developed on L&C at resonance!
- Useful for passive voltage amplification in LNA
- Series LC is not just a short!