EE 538B CMOS RF IC DESIGN

Midterm Examination No. 2: May 15, 2002

Time Allowed: 110 Minutes

Student Name: Solutions

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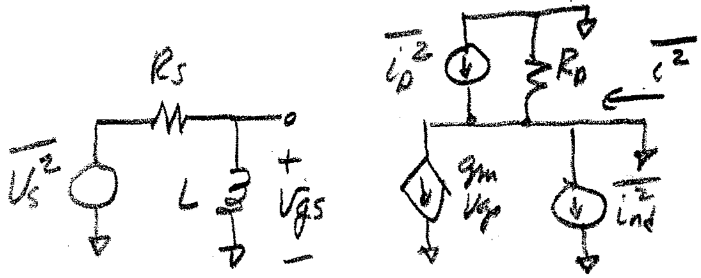
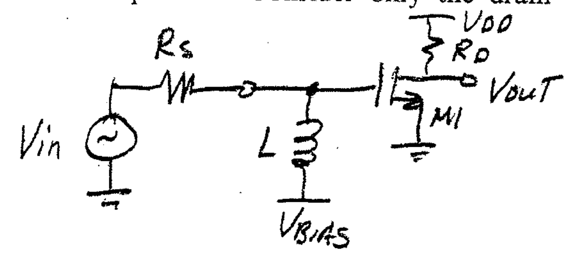
You are allowed two sheets of notes. Write legibly. Show all work. State assumptions.

Problem #	Points Possible	Points
1	**20**	20
2	**30**	30
3	**20**	20
4	**30**	30

100

1. (20 points) Find the noise factor of the circuit shown with respect to the source resistance R_S. Neglect channel length modulation effects and all parasitic capacitors. Consider only the drain current noise component of M_1.





$$\overline{V_s^2} = 4kTR_s\Delta f;\quad \overline{i_{nd}^2} = 4kT\gamma g_{do}\Delta f;\quad \overline{i_D^2} = \frac{4kT}{R_D}\Delta f$$

The three noise sources are uncorrelated:

(i) Consider $\overline{i_D^2}$: $\overline{i_1^2} = \overline{i_D^2} = \frac{4kT}{R_D}\Delta f$

(ii) Consider $\overline{i_{nD}^2}$: $\overline{i_2^2} = \overline{i_{nd}^2} = 4kT\gamma g_{do}\Delta f$

(iii) Consider $\overline{V_s^2}$:

$$V_{gs} = \frac{SL}{SL+R_s}\overline{V_s} = \frac{SL/R_s}{SL/R_s+1}$$

$$\Rightarrow \quad V_{gs}^2 = \frac{(\omega L/R_s)^2}{(\omega L/R_s)^2+1}\overline{V_s^2}$$

(Here we neglect phase shift since we're dealing with uncorrelated noise)

$$\therefore \overline{i_3^2} = g_m^2 V_{gs}^2 = \frac{g_m^2(\omega L/R_s)^2}{(\omega L/R_s)^2+1}\overline{V_s^2}$$

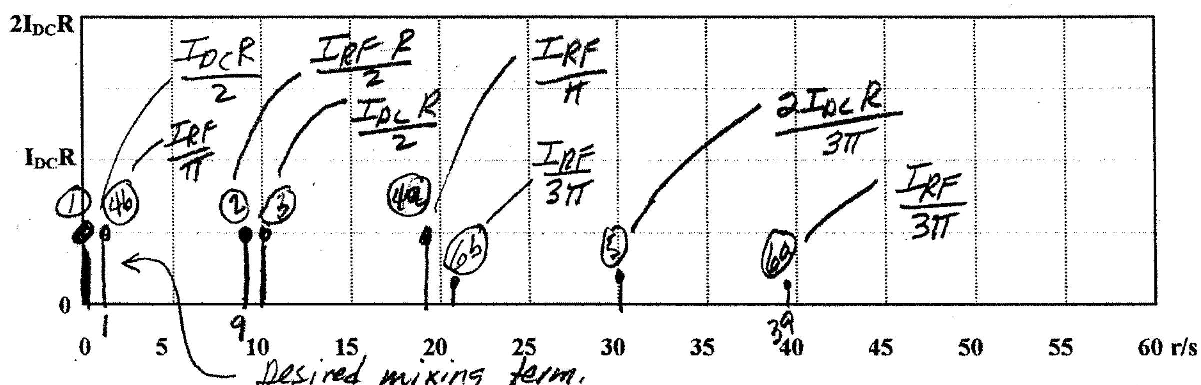
$$F = \frac{\overline{i_1^2}+\overline{i_2^2}+\overline{i_3^2}}{\overline{i_3^2}} = 1 + \frac{\frac{4kT}{R_D}\Delta f}{\frac{g_m^2(\omega L/R_s)^2\, 4kTR_s\Delta f}{(\omega L/R_s)^2+1}} + \frac{4kT\gamma g_{do}\Delta f}{\frac{g_m^2(\omega L/R_s)^2\, 4kTR_s\Delta f}{(\omega L/R_s)^2+1}}$$

$$= 1 + \frac{[1+(\omega L/R_s)^2]}{g_m^2 R_s R_D(\omega L/R_s)^2} + \frac{\gamma[1+(\omega L/R_s)^2]}{\alpha g_m R_s(\omega L/R_s)^2} \leftarrow$$

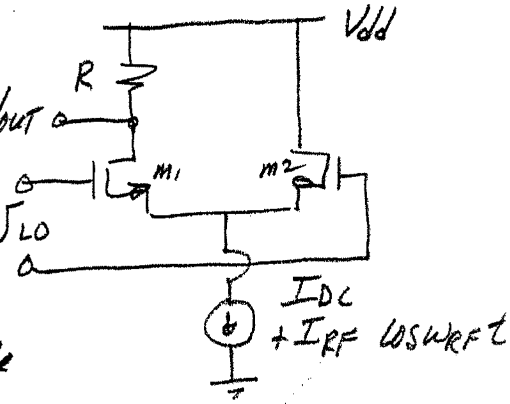
Where $\alpha = g_m/g_{do}$

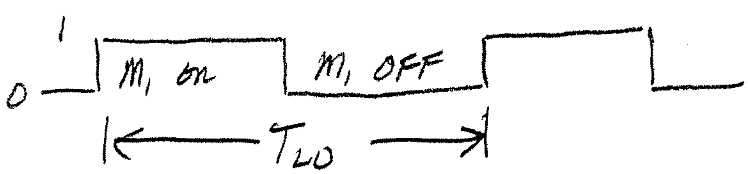
2. (30 points) Assume that M_1 and M_2 switch completely and instantaneously at the zero crossings of the RF signal that has a 50% duty cycle. The LO signal is $V_{LO}\cos\omega_{LO}t$ and the tail current is $(I_{DC} + I_{RF}\cos\omega_{RF}t)$. For simplicity, let $\omega_{LO}=10$ and $\omega_{RF}=9$.

   Determine and plot on the graph below the peak voltage frequency components in V_{out}. Carefully label both the amplitude and frequency of each component. (Neglect switching frequency components higher than the third harmonic)



Let's assume that V_{LO} is large enough to fully switch M_1-M_2. Since the circuit is unbalanced, either all or none of the tail current is passed through R to create V_{IF}. Hence, the appropriate switch waveform is at ω_{LO} is:





The Fourier series of this switching waveform is:

$$V(t) = \underbrace{\frac{1}{2}}_{\substack{\text{DC}\\ \text{Component}\\ \text{Of switching}\\ \text{waveform}}} + \sum_{n=1}^{\infty} a_n \cos n\omega_{LO} t$$

Where $a_n = \dfrac{\sin n\pi/2}{n\pi/2}$ $\quad\begin{cases} a_1 = \dfrac{2}{\pi} \\ a_2 = 0 \\ a_3 = \dfrac{-2}{3\pi} \\ a_4 = 0 \\ \vdots \end{cases}$

2. (cont—blank work page)

$$V_{IF} = \left[\frac{1}{2} + \frac{2}{\pi}\cos\omega_{LO}t - \frac{2}{3\pi}\cos 3\omega_{LO}t\right]\cdot\left[I_{DC} + I_{RF}\cos\omega_{RF}t\right]R$$

$$= \Big[\overset{①}{\frac{I_{DC}}{2}} + \overset{②}{\frac{I_{RF}}{2}}\cos\omega_{RF}t + \overset{③}{\frac{2I_{DC}}{\pi}\cos\omega_{LO}t}$$

$$+ \overset{④}{\frac{2I_{RF}}{\pi}\cos\omega_{LO}t\cos\omega_{RF}t} - \overset{⑤}{\frac{2I_{DC}}{3\pi}\cos 3\omega_{LO}t}$$

$$- \overset{⑥}{\frac{2I_{RF}}{3\pi}\cos 3\omega_{LO}t\cos\omega_{RF}t}\Big]R$$

Interpretation of terms: (Plotted on graph on previous page.)

① $\dfrac{I_{DC}R}{2}$ ➡ DC term at output

② $\dfrac{I_{RF}R}{2}\cos\omega_{RF}t$ ➡ RF feedthrough term at output — not present in balanced version of circuit

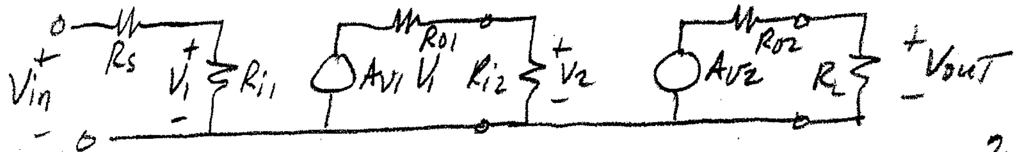
③ $\dfrac{2I_{DC}R}{\pi}\cos\omega_{LO}t$ ➡ LO feedthrough term at output

④ $= \dfrac{I_{RF}}{\pi}\Big[\underbrace{\overset{4a}{\cos(\omega_{LO}+\omega_{RF})t}}_{\text{upconverted term}} + \underbrace{\overset{4b}{\cos(\omega_{LO}-\omega_{RF})t}}_{\text{Downconverted term}}\Big]$

⑤ $\dfrac{2I_{DC}R}{3\pi}\cos 3\omega_{LO}t$ ➡ third harmonic of LO at output

⑥ $= \dfrac{I_{RF}}{3\pi}\Big[\overset{6a}{\cos(3\omega_{LO}+\omega_{RF})t} + \overset{6b}{\cos(3\omega_{LO}-\omega_{RF})t}\Big]$

3. (20 points) An amplifier with NF_1= 6dB, power gain A_{P1}= 4dB, and $IIP_{3,1}$= 0dBm is cascaded with a second amplifier with NF_2= 10dB, power gain A_{P2}= 10dB, and $IIP_{3,1}$= -6dBm. Assuming a matched condition at all nodes, what are the overall NF, A_P and IIP_3 values?



$$P_{OUT} = V_{in}^2\left(\frac{R_{i1}}{R_s+R_{i1}}\right)^2 \cdot A_{v1}^2 \cdot \left(\frac{R_{i2}}{R_{o1}+R_{i2}}\right)^2 A_{v2}^2 \cdot \left(\frac{R_L}{R_L+R_{o2}}\right)^2 \Big/ R_L$$

$$P_{in} = V_{in}^2\left(\frac{R_{i1}}{R_s+R_{i1}}\right)^2 \Big/ R_{i1}$$

- $$A_p = \frac{P_{OUT}}{P_{in}} = A_{v1}^2\left(\frac{R_{i2}}{R_{o1}+R_{i2}}\right)^2 \cdot A_{v2}^2\left(\frac{R_L}{R_L+R_{o2}}\right)^2 \cdot \frac{R_{i1}}{R_L}$$

Also, $A_{P1} = \dfrac{P_{OUT,1}}{P_{in}} = A_{v1}^2\left(\dfrac{R_{i2}}{R_{o1}+R_{i2}}\right)^2 \cdot \dfrac{R_{i1}}{R_{i2}} = \dfrac{A_{v1}^2}{4}$ with matched condition

$\therefore A_p = \dfrac{P_{OUT}}{P_{in}} = \dfrac{A_{v1}^2}{4} \cdot \dfrac{A_{v2}^2}{4}$ (with matched conditions)

$$A_p = A_{p1} \cdot A_{p2} = \boxed{14dB} \longleftarrow A_p$$

- $NF_2 = 10dB \Rightarrow F_2 = 10^1 = 10$ $\qquad A_{P1} = 4dB \Rightarrow 10^{.4} = 2.51$

$NF_1 = 6dB \Rightarrow F_1 = 10^{.6} = 3.98$

Friis Equation $\Rightarrow NF = 1 + (F_1 - 1) + \dfrac{(F_2-1)}{A_{P1}}$

$$= 1 + (3.98-1) + \frac{(10-1)}{2.51} = 7.566 = 8.79\ dB \longleftarrow NF$$

$$\frac{1}{IIP_3^2} = \frac{1}{IIP_{3,Amp1}^2} + \frac{A_{v1}^2}{IIP_{3,Amp2}^2} \qquad \frac{A_{v1}^2}{4} = 4dB = 2.51 \quad \therefore A_{v1} = 3.17$$

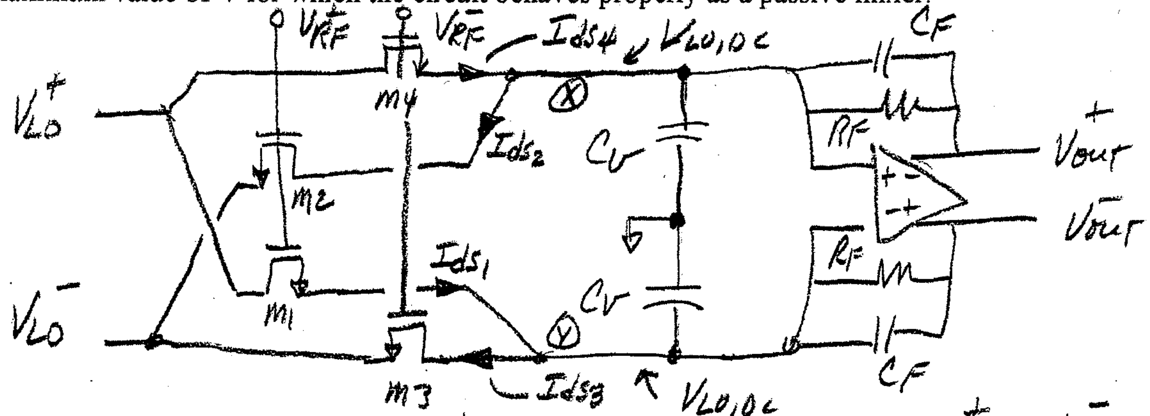
$IIP_{3,Amp1} = 0\ dBm = 10\log\dfrac{V_{rms}^2/50}{10^{-3}} \Rightarrow V_{rms1}^2 = 50\times10^{-3}\ V^2$

$IIP_{3,Amp2} = -6dBm \Rightarrow V_{rms2}^2 = 12.5\times10^{-3}\ V^2$

$\therefore \dfrac{1}{IIP_3^2} = \dfrac{1}{50\times10^{-3}} + \dfrac{10}{12.5\times10^{-3}} \Rightarrow IIP_3 = 34.9\times10^{-3}\ V \Rightarrow 10\log\dfrac{V_{rms}^2/50}{10^{-3}}$

$$= -16.1\ dBm \longleftarrow$$

4. (20 points) A passive mixer circuit is shown below. Assume $V_{RF,DC}$=3.85V, $V_{LO,DC}$=1.15V, V_{DD}=5.0V, and V_T=1.0V. Also assume that V_{RF}=$V\cos\omega_{RF}t$ and V_{LO}=$V\cos\omega_{RF}t$. Determine the maximum value of V for which the circuit behaves properly as a passive mixer.



Note $V_{RF} = V_{RF}^+ - V_{RF}^-$ and $V_{LO} = V_{LO}^+ - V_{LO}^-$

$\Rightarrow V_{RF}^+ = V_{RF,DC} + \frac{V}{2}\cos\omega_{RF}t;\; V_{RF}^- = V_{RF,DC} - \frac{V}{2}\cos\omega_{RF}t$

and $V_{LO}^+ = V_{LO,DC} + \frac{V}{2}\cos\omega_{LO}t;\; V_{LO}^- = V_{LO,DC} - \frac{V}{2}\cos\omega_{LO}t$

Transistors $M_1 - M_4$ must remain ON and in non-saturation:

① $(V_{GS}-V_T) \geq$ for M_4 $\Rightarrow V_{RF,DC} - \frac{V}{2}\cos\omega_{RF}t - V_{LO,DC} - V_T \geq 0$

$\underbrace{\cos\omega_{RF}t = 1}_{\text{WORST-CASE must be satisfied}} \Rightarrow V_{RF,DC} - \frac{V}{2} - V_{LO,DC} - V_T \geq 0$

$3.85V - \frac{V}{2} - 1.15V - 1.0V \geq 0$

$\therefore V \leq 3.4V$

② $V_{DS} < (V_{GS} - V_T)$ for M_4 in non-saturation:

$\Rightarrow (V_{LO}^+ - V_{LO,DC}) < (V_{RF}^- - V_{LO,DC} - V_T)$

$\Rightarrow V_{LO}^+ < (V_{RF}^- - V_T)$

$(V_{LO,DC} + \frac{V}{2}\cos\omega_{LO}t) < (V_{RF,DC} - \frac{V}{2}\cos\omega_{RF}t - V_T)$

WORST-CASE CONDITION $\left\{ (1.15V + \frac{V}{2}) < (3.85V - \frac{V}{2} - 1.0) \right.$

$V < 1.7V$

Condition ② dominates $\Rightarrow \boxed{V < 1.7V}$