EE210 [redacted]

Homework #2

1) a) i) $\sin(3x)\cdot\cos(5x)$

$$\left(\frac{e^{j3x}-e^{-j3x}}{2j}\right)\left(\frac{e^{j5x}+e^{-j5x}}{2}\right)$$

$$=\left(\frac{1}{2j}\right)\left(\frac{1}{2}\right)\left(e^{8x}+e^{-j2x}-e^{j2x}-e^{-j8x}\right)$$

$$=\frac{1}{2}\left(\frac{e^{8x}-e^{-j8x}}{2j}\right)-\frac{1}{2}\left(\frac{e^{2x}-e^{-j2x}}{2j}\right)$$

$$\boxed{=\frac{1}{2}\sin(8x)-\frac{1}{2}\sin(2x)}$$

ii) $\sin^2(3x)\cdot\cos(2x)=(\sin(3x)(\sin(3x)))\cdot\cos(2x)$

$$=\left(\frac{e^{j3x}-e^{-j3x}}{2j}\right)\left(\frac{e^{j3x}-e^{-j3x}}{2j}\right)\ldots$$

$$=-\frac{1}{4}\left(e^{j6x}\cancel{-e^{-j0x}}^{\,1}\cancel{-e^{j0x}}^{\,1}+e^{-j6x}\right)\cdot\frac{1}{2}\left(e^{j2x}+e^{-j2x}\right)$$

$$=\left(-\frac{1}{4}\right)\left(e^{j6x}+e^{-j6x}-2\right)\cdot\frac{1}{2}\left(e^{j2x}+e^{-j2x}\right)$$

$$=\left(-\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(e^{j8x}+e^{j4x}+e^{-j4x}+e^{-j8x}-2e^{j2x}-2e^{-j2x}\right)$$

$$\boxed{=-\frac{1}{4}\cos(8x)-\frac{1}{4}\cos(4x)+\frac{1}{2}\cos(2x)}$$

iii) $\cos(3x)\cos(3x)\cos(3x)$

$$=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(e^{j3x}+e^{-j3x}\right)\left(e^{j3x}+e^{-j3x}\right)\cdot\ldots$$

$$=\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(e^{j6x}+1+1+e^{-j6x}\right)\frac{1}{2}\left(e^{j3x}+e^{-j3x}\right)$$

$$=\frac{1}{8}\left(e^{j9x}+e^{j3x}+e^{-j3x}+e^{-j9x}+2e^{j3x}+2e^{-j3x}\right)$$

$$\boxed{=\frac{1}{4}\cos(9x)+\frac{1}{4}\cos(3x)+\frac{1}{2}\cos(3x)}$$

$$=\frac{1}{4}\cos(9x)+\frac{3}{4}\cos(3x)$$

1)b) i) $\frac{1}{2}\sin(8x) - \frac{1}{2}\sin(2x)$; $x = 13\pi t$

$\frac{1}{2}\sin(104\pi t) - \frac{1}{2}\sin(26\pi t)$

$\boxed{f = 13 \ \& \ 52\ Hz}$

ii) $-\frac{1}{4}\cos(8x) - \frac{1}{4}\cos(4x) + \frac{1}{2}\cos 2x$

$= -\frac{1}{4}\cos(104\pi t) - \frac{1}{4}\cos(52\pi t) + \frac{1}{2}\cos(26\pi t)$

$\boxed{f = 13, 26, 52\ Hz}$

iii) $\frac{1}{4}\cos(9x) + \frac{3}{4}\cos(3x)$

$= \frac{1}{4}\cos(117\pi t) + \frac{3}{4}\cos(39\pi t)$

$\boxed{f = 19.5 \ \& \ 58.5\ Hz}$

c) i) $\frac{1}{2}\sin(104\pi t) - \frac{1}{2}\sin(26\pi t) \rightarrow \frac{1}{2}\left(\frac{1}{2j}\right)\{\delta(f-52) - \delta(f+52)\} - \left(\frac{1}{2}\right)\left(\frac{1}{2j}\right)\{\delta(f-13) \ldots$

$\ldots - \delta(f+13)\}$

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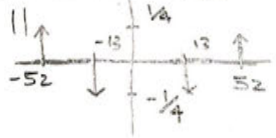

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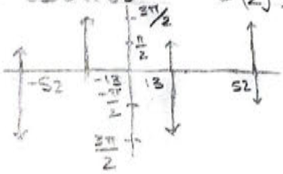

ii) $-\frac{1}{4}\left(\frac{1}{2}\right)\{\delta(f-52) + \delta(f+52)\} - \left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\{\delta(f-26) + \delta(f+26)\} + \frac{1}{2}\left(\frac{1}{2}\right)\{\delta(f-13) + \delta(f+13)\}$

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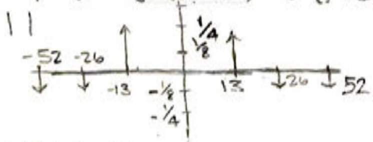

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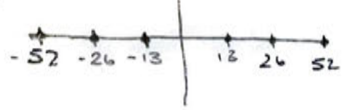
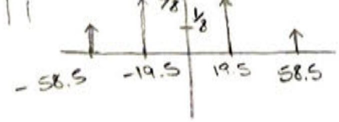


iii) $\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\{\delta(f-58.5) + \delta(f+58.5)\} + \left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\{\delta(f-19.5) + \delta(f+19.5)\}$

$||$



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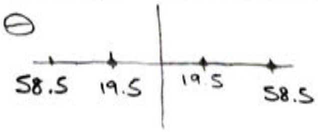
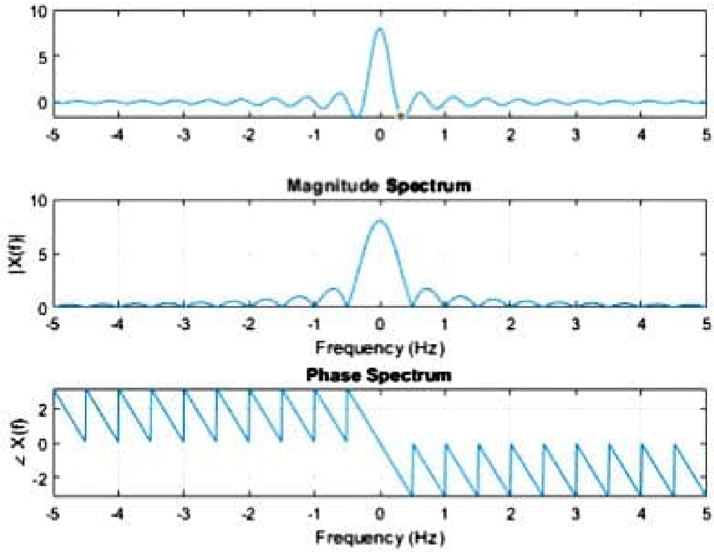
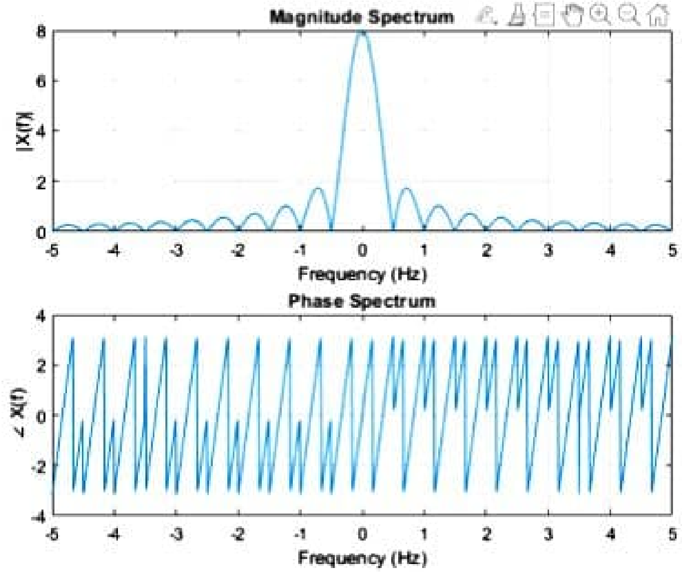


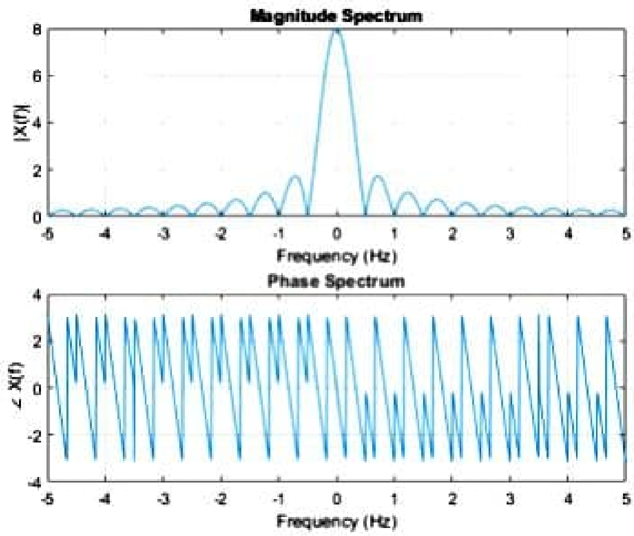
2) a)



b)



c)



d)

The magnitudes of a) and b) are equal from the 0th to the 5th harmonics.
There are differences in phase between a) and b), where the phase is now offset by approximately 180deg and reflected about the y-axis.

3) a) $157° = \tan^{-1}\left(\frac{x}{-3}\right)$

$x = -3\tan(157°)$

$x = 1.273$

$\boxed{-3 + j1.273}$

b) $r = \sqrt{-3^2 + 1.273^2}$

$= 3.26$

$\boxed{3.26e^{j\frac{157\pi}{180}}}$

4) a) $1+j = \sqrt{1^2+1^2}\, e^{j\,atan(\frac{1}{1})} = \boxed{\sqrt{2}\, e^{j\frac{\pi}{4}}}$

$1-j = \sqrt{1^2+-1^2}\, e^{j\,atan(-1)} = \boxed{\sqrt{2}\, e^{j\frac{-\pi}{4}}}$

$-1+j = \sqrt{2}\, e^{j\,atan(-1)} = \boxed{\sqrt{2}\, e^{j\frac{3\pi}{4}}}$

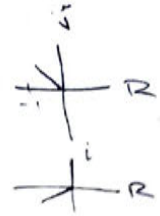
$-1-j = \sqrt{2}\, e^{j\,atan(\frac{-1}{-1})} = \boxed{\sqrt{2}\, e^{j\frac{5\pi}{4}}}$

$2+0j = \sqrt{2^2+0^2}\, e^{j\,atan(0)} = 2e^{j0} = \boxed{2}$

$-2+0j = \sqrt{-2^2+0^2}\, e^{j\,atan(0)} = \boxed{2e^{-j\pi}}$

$0+1j = \sqrt{0^2+1^2}\, e^{j\,atan(\frac{1}{0})} = \boxed{1e^{j\frac{\pi}{2}}}$

$0-1j = \sqrt{0^2+-1^2}\, e^{j\,atan(\frac{-1}{0})} = \boxed{1e^{j\frac{3\pi}{2}}}$



b) $\frac{1+j}{1-j}\cdot\frac{1+j}{1+j} = \left(\frac{1+j+j-1}{1+0j+1}\right) = \frac{2j}{2} = j \Rightarrow \sqrt{0^2+1^2}\, e^{j\,atan\frac{1}{0}} = \boxed{1e^{j\frac{\pi}{2}}}$

$\frac{-1+j}{-1-j}\cdot\frac{-1+j}{-1+j} = \frac{1-j-j-1}{1-j+j+1} = \frac{-2j}{2} = -j \Rightarrow 1e^{j\,atan(-\frac{1}{0})} = \boxed{1e^{j\frac{3\pi}{2}}}$

$\frac{2}{-j}\cdot\frac{j}{j} = \frac{2j}{1} \Rightarrow 2e^{j\,atan(\frac{2}{0})} = \boxed{2e^{j\frac{\pi}{2}}}$

$\frac{j}{-2} = 0-\frac{1}{2}j \Rightarrow \sqrt{0^2+(\frac{1}{2})^2}\, e^{j\,atan(\frac{-\frac{1}{2}}{0})} = \boxed{\frac{1}{2}\, e^{j\frac{3\pi}{2}}}$

5) a) $f = \frac{1}{0.1} = 100$

$$\boxed{\frac{2}{\sqrt{2}} \cos(2\pi(100)t)}$$

$$\boxed{\frac{2}{\sqrt{2}} \sin\left(2\pi(100)\left(t - \frac{\pi}{2}\right)\right)}$$

b) $\frac{2}{\sqrt{2}} \cos(200\pi t)$

$$= \frac{\cancel{2}}{\sqrt{2}} \, \frac{e^{j200\pi t} + e^{-j200\pi t}}{\cancel{2}} = \boxed{\frac{e^{j200\pi t} + e^{-j200\pi t}}{\sqrt{2}}}$$

$\frac{2}{\sqrt{2}} \sin\left(200\pi\left(t - \frac{\pi}{2}\right)\right)$

$$= \frac{\cancel{2}}{\sqrt{2}} \, \frac{e^{j200\pi\left(t-\frac{\pi}{2}\right)} - e^{-j200\pi\left(t-\frac{\pi}{2}\right)}}{\cancel{2}j}$$

$$\boxed{= \frac{e^{j200\pi\left(t-\frac{\pi}{2}\right)} - e^{-j200\pi\left(t-\frac{\pi}{2}\right)}}{j\sqrt{2}}}$$

1) a) i) $\sin(3x)\cdot\cos(5x) = \left(\frac{e^{j3x} - e^{-j3x}}{2j}\right)\left(\frac{e^{j5x} + e^{-j5x}}{2}\right)$

$= \frac{1}{4j}\left[e^{j8x} + e^{-j2x} - e^{j2x} - e^{-j8x}\right]$

$= \frac{1}{4j}\left[e^{j8x} - e^{-j8x} - \left(e^{j2x} - e^{-j2x}\right)\right]$

$= \frac{1}{2}\left[\frac{e^{j8x} - e^{-j8x}}{2j} - \left(\frac{e^{j2x} - e^{-j2x}}{2j}\right)\right]$

$= \boxed{\frac{1}{2}\sin(8x) - \frac{1}{2}\sin(2x)}$

ii) $\sin^2(3x)\cdot\cos(2x) = \left(\frac{e^{j3x} - e^{-j3x}}{2j}\right)\left(\frac{e^{j3x} - e^{-j3x}}{2j}\right)\left(\frac{e^{j2x} + e^{-j2x}}{2}\right)$

$= \frac{-1}{8}\left[\left(e^{j6x} - 1 - 1 + e^{-j6x}\right)\left(e^{j2x} + e^{-j2x}\right)\right]$

$= \frac{-1}{8}\left[e^{j8x} + e^{j4x} - 2e^{j2x} - 2e^{-j2x} + e^{-j4x} + e^{-j8x}\right]$

$= \frac{-1}{8}\left[e^{j8x} + e^{-j8x} + e^{j4x} + e^{-j4x} - 2\left(e^{j2x} + e^{-j2x}\right)\right]$

$= \frac{-1}{4}\left[\frac{e^{j8x} + e^{-j8x}}{2} + \frac{e^{j4x} + e^{-j4x}}{2} - \left(e^{j2x} + e^{-j2x}\right)\right]$

$= \boxed{\frac{-1}{4}\cos 8x - \frac{1}{4}\cos 4x + \frac{1}{2}\cos 2x}$

iii) $\cos(3x)\cdot\cos^2(3x) = \left(\frac{e^{j3x}+e^{-j3x}}{2}\right)\left(\frac{e^{j3x}+e^{-j3x}}{2}\right)\left(\frac{e^{j3x}+e^{-j3x}}{2}\right)$

$= \frac{1}{8}\left[(e^{j6x}+1+1+e^{-j6x})\cdot(e^{j3x}+e^{-j3x})\right]$

$= \frac{1}{8}\left[e^{j9x}+e^{j3x}+2e^{j3x}+2e^{-j3x}+e^{-j3x}+e^{-j9x}\right]$

$= \frac{1}{4}\left[\frac{e^{j9x}+e^{-j9x}}{2}+\frac{e^{j3x}+e^{-j3x}}{2}+(e^{j3x}+e^{-j3x})\right]$

$= \frac{1}{4}\cos(9x)+\frac{1}{4}\cos 3x+\frac{1}{2}\cos 3x$

$= \boxed{\frac{1}{4}\cos(9x)+\frac{3}{4}\cos 3x}$

b) $x = 13\pi t$

i) $\frac{1}{2}\sin(8\cdot 13\cdot\pi\cdot t)-\frac{1}{2}\sin(2\cdot 13\cdot\pi\cdot t) = \frac{1}{2}\sin(2\cdot\pi\cdot 52\cdot t) - \frac{1}{2}\sin(2\cdot\pi\cdot 13\cdot t)$

$$\boxed{\begin{aligned} f_1 &= 52\text{Hz} \\ f_2 &= 13\text{Hz} \end{aligned}}$$

ii) $-\frac{1}{4}\cos(8\cdot 13\cdot\pi\cdot t)-\frac{1}{4}\cos(4\cdot 13\cdot\pi\cdot t)+\frac{1}{2}\cos(2\cdot 13\cdot\pi\cdot t)$

$= -\frac{1}{4}\cos(2\cdot\pi\cdot 52\cdot t)-\frac{1}{4}\cos(2\cdot\pi\cdot 26t)+\frac{1}{2}\cos(2\cdot\pi\cdot 13\cdot t)$

$$\boxed{\begin{aligned} f_1 &= 52\text{ Hz} \\ f_2 &= 26\text{ Hz} \\ f_3 &= 13\text{ Hz} \end{aligned}}$$

1) b) iii) $\frac{1}{4}\cos(9\cdot 13\cdot \pi t) + \frac{3}{4}\cos(3\cdot 13\cdot \pi\cdot t) = \frac{1}{4}\cos(2\cdot\pi\cdot 58.5\cdot t)$

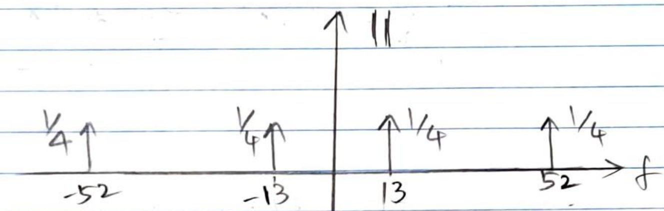
$+\frac{3}{4}\cos(2\cdot\pi\cdot 19.5 t)$

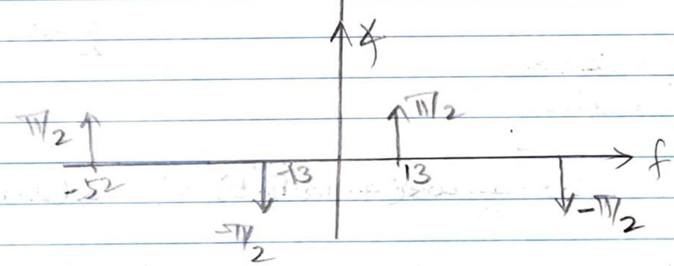
$$\boxed{\begin{aligned} f_1 &= 58.5\text{ Hz} \\ f_2 &= 19.5\text{ Hz} \end{aligned}}$$

c) i) $\frac{1}{2}\sin(2\cdot\pi\cdot 52\cdot t) - \frac{1}{2}\sin(2\pi\cdot 13\cdot t)$

$$X(f) = \frac{1}{2}\left[\frac{1}{2j}\left[\delta(f-52) - \delta(f+52)\right]\right]$$

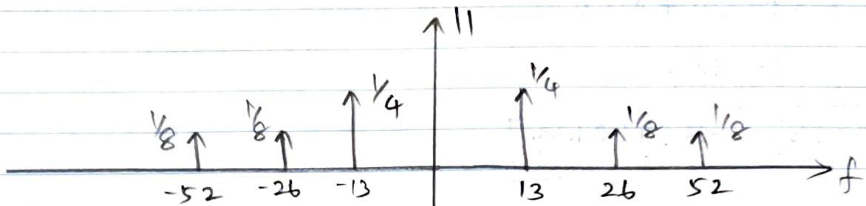
$$-\frac{1}{2}\left[\frac{1}{2j}\left[\delta(f-13) - \delta(f+13)\right]\right]$$

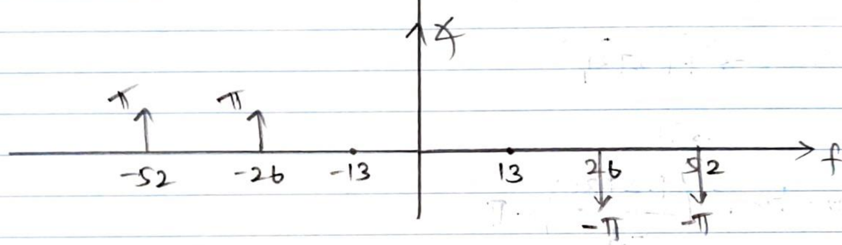




ii) $-\frac{1}{4}\cos(2\cdot\pi\cdot 52t) - \frac{1}{4}\cos(2\cdot\pi\cdot 26t) + \frac{1}{2}\cos(2\cdot\pi\cdot 13\cdot t)$

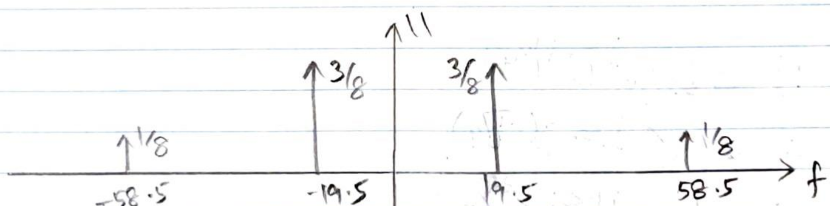
$$X(f) = -\frac{1}{8}\left[\delta(f-52)+\delta(f+52)\right] - \frac{1}{8}\left[\delta(f-26)+\delta(f+26)\right]$$
$$+\frac{1}{4}\left[\delta(f-13)+\delta(f+13)\right]$$

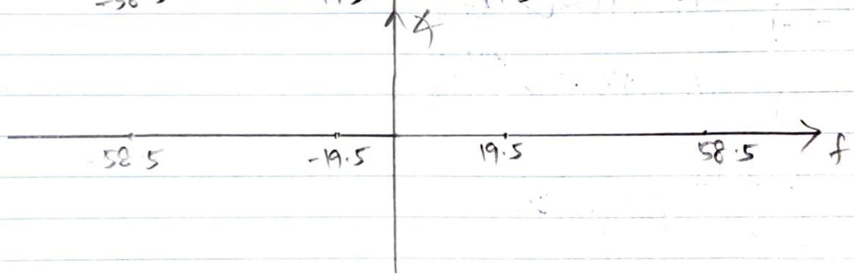




iii) $\frac{1}{4}\cos(2\pi\cdot 58.5t) + \frac{3}{4}\cos(2\pi\cdot 19.5t)$

$$X(f) = \frac{1}{8}\left[\delta(f-58.5)+\delta(f+58.5)\right] + \frac{3}{8}\left[\delta(f-19.5)+\delta(f+19.5)\right]$$





2 a) Plot $F\left\{2\cdot\Pi\left(\frac{t}{4}\right)\right\}$ in magnitude and phase. (it means frequency response of rectangular function with amplitude 2, center at 0, and width of 4) , ranges of the frequencies are $\left[-5Hz \le f \le 5Hz\right]$

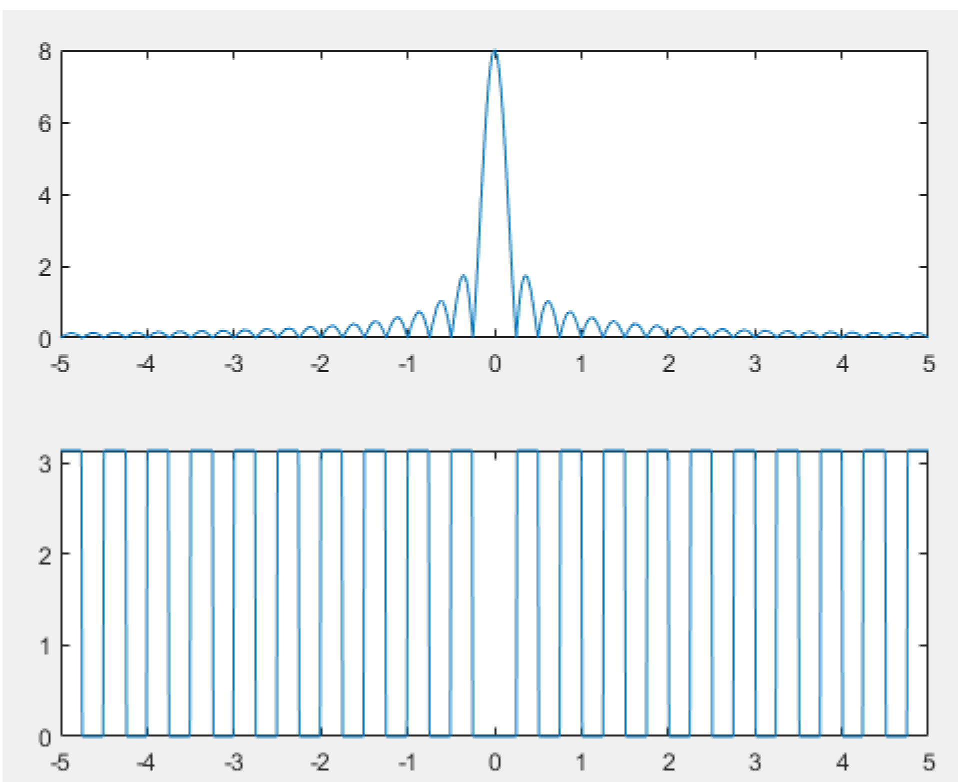
X(f) = 2.A.t_0 *sinc(2.t_0.f) where A=amplitude, t_0= width/2

Code:

```
clc; clear;

f = -5:0.0001:5;
X = 8*sinc(4*f);
X_mag = abs(X);
X_ph = angle(X);
subplot(2,1,1), plot(f,X_mag);
subplot(2,1,2), plot(f,X_ph);
```

Plot:



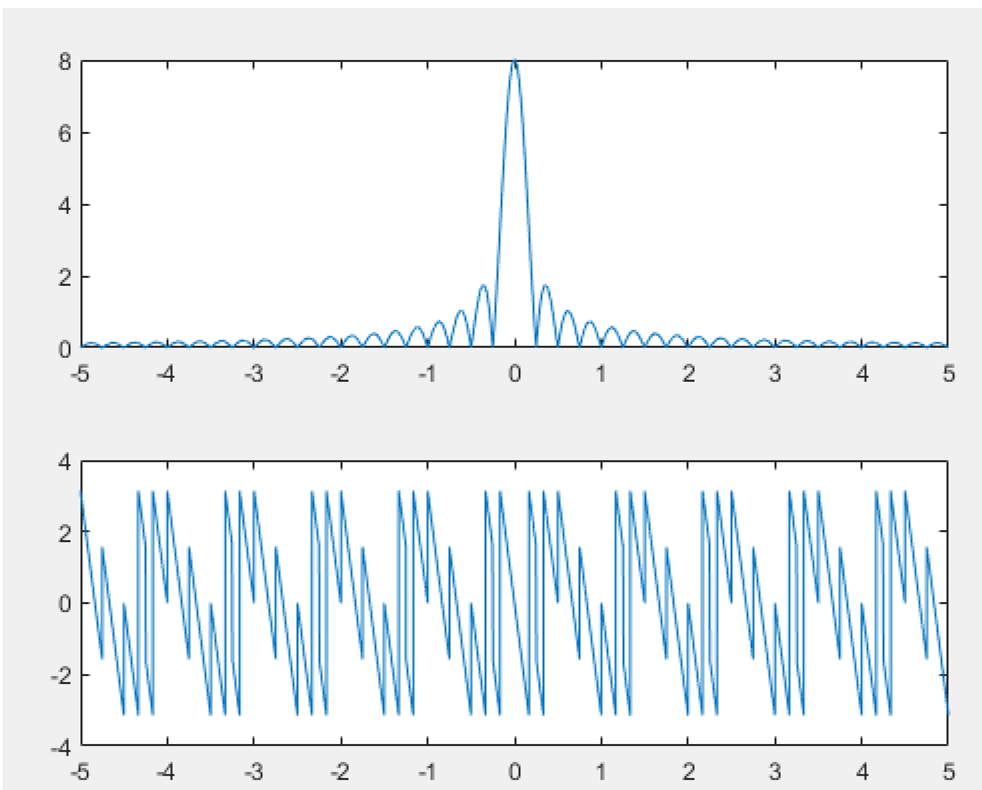
b) Plot $F\left\{2\cdot\Pi\left(\frac{t-3}{4}\right)\right\}$ in magnitude and phase.

Code:

```
clc; clear;

f = -5:0.0001:5;
X = 8*sinc(4*f).*exp(j*2*pi*f*-3);
X_mag = abs(X);
X_ph = angle(X);
subplot(2,1,1), plot(f,X_mag);
subplot(2,1,2), plot(f,X_ph);
```

Plot:



c) Plot $F\left\{2\cdot\Pi\left(\frac{t+3}{4}\right)\right\}$ in magnitude and phase.

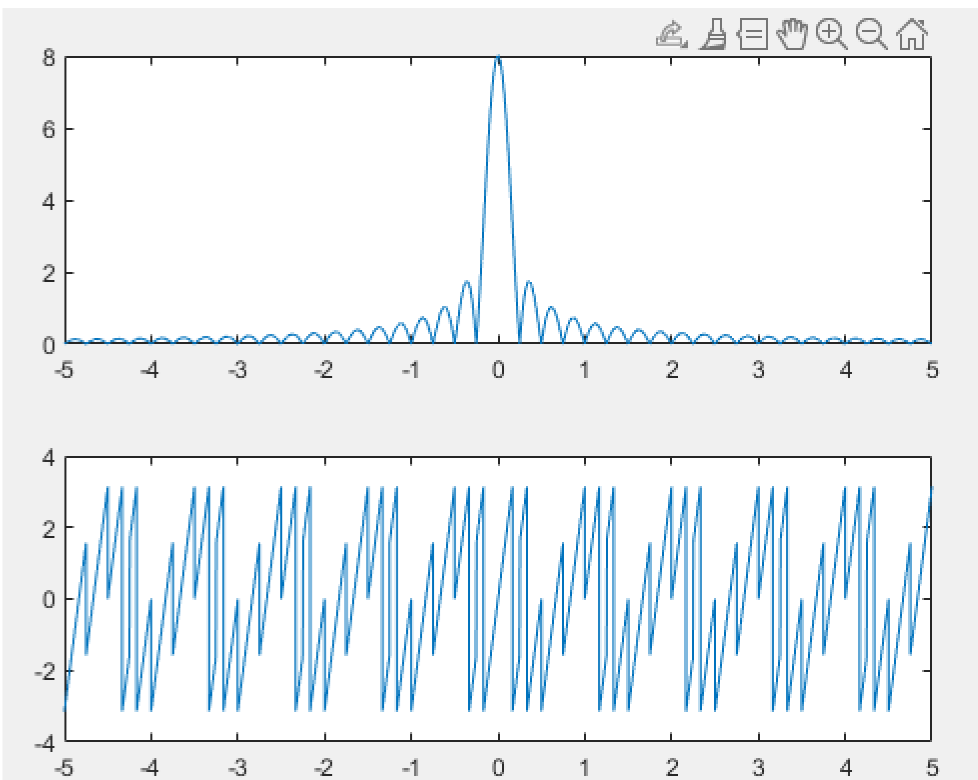
Code:

prb2a.m Untitled3

```
clc; clear;

f = -5:0.0001:5;
X = 8*sinc(4*f).*exp(j*2*pi*f*3);
X_mag = abs(X);
X_ph = angle(X);
subplot(2,1,1), plot(f,X_mag);
subplot(2,1,2), plot(f,X_ph);
```

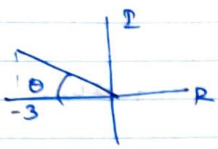
Plot:



d) What is the difference between a) & b) in the frequency domain? Compare magnitude vs. magnitude & phase vs. phase of two signals in the frequency domain.

Magnitude of signals of both a) and b) *remains the same* while the phase of the signal b) is distorted with a negative slope due to introduction of time shift in time domain.

3) $\theta = 23^\circ$



a) $a = -3$

$b = \tan 157^\circ \times -3$

$= 1.2734$

$$\boxed{a + bj = -3 + j1.2734}$$

b) Polar form (in radian)

$|| = \sqrt{-3^2 + 1.2734^2}$

$= 3.2591$

$\theta = \tan^{-1}\left(\frac{1.2734}{-3}\right) - \pi$

$$\Rightarrow \boxed{3.2591\, e^{j\,\mathrm{atan}\left(\frac{1.2734}{-3}\right) - \pi}}$$

4) a) $1+j \Rightarrow \sqrt{2}\, e^{j\tan^{-1}\left(\frac{1}{1}\right)} = \sqrt{2}\, e^{j\pi/4}$

$1-j \Rightarrow \sqrt{2}\, e^{-j\pi/4}$

$-1+j \Rightarrow \sqrt{2}\, e^{j3\pi/4}$

$-1-j \Rightarrow \sqrt{2}\, e^{-j3\pi/4}$

$2 \Rightarrow 2$

$-2 \Rightarrow 2e^{j\pi}$

$j \Rightarrow \sqrt{1}\, e^{j(\tan^{-1}\infty)} = 1.e^{j\pi/2}$

$-j \Rightarrow 1.e^{-j\pi/2}$

b) $\frac{1+j}{1-j} = \frac{(1+j)^2}{1^2-j^2} = \frac{1+j^2+2j}{1+1} = \frac{2j}{2} = j \Rightarrow e^{j\pi/2}$

$\frac{-1+j}{-1-j} = \frac{(-1+j)^2}{(-1)^2-(+j)^2} = \frac{1+j^2-2j}{2} = \frac{-2j}{2} = -j \Rightarrow e^{-j\pi/2}$

$\frac{2}{-j} = -2\times -j = 2j \Rightarrow 2e^{j\pi/2}$

$\frac{j}{-2} = -\frac{1}{2}j \Rightarrow \frac{1}{2}e^{j(-\pi/2)} = \frac{1}{2}e^{-j\pi/2}$

5) a) Amplitude = 2

$RMS = \frac{2}{\sqrt{2}} = \sqrt{2}$

$T = 0.01 s$

$f = \frac{1}{0.01} = 100 Hz$

In terms of sine using RMS:-

$$\boxed{\sqrt{2}\sin\left(2\pi \cdot 100 \cdot t - \frac{3\pi}{2}\right)}$$

In terms of cosine using RMS:-

$$\boxed{\sqrt{2}\cos\left(2\pi \cdot 100 \cdot t\right)}$$

b) $$\sqrt{2}\sin\left(2\pi \cdot 100 \cdot t - \frac{3\pi}{2}\right) = \sqrt{2}\left(\frac{e^{j\left(2\pi 100 t - \frac{3\pi}{2}\right)} - e^{-j\left(2\pi \cdot 100 \cdot t - 3\pi/2\right)}}{2j}\right)$$

$$= \boxed{\frac{1}{\sqrt{2}j}\left(e^{j\left(2\pi 100 \cdot t - 3\pi/2\right)} - e^{-j\left(2 \cdot \pi \cdot 100 \cdot t - 3\pi/2\right)}\right)}$$

$$\sqrt{2}\cos\left(2\pi 100 \cdot t\right) = \frac{\sqrt{2}}{2}\left(e^{j\left(2\pi \cdot 100 t\right)} + e^{-j\left(2\pi \cdot 100 \cdot t\right)}\right)$$

$$= \boxed{\frac{1}{\sqrt{2}}\left(e^{j\left(2\pi 100 t\right)} + e^{-j\left(2\pi \cdot 100 \cdot t\right)}\right)}$$