EE223 Analog Integrated Circuits
Fall 2018

Lecture 4: MOS Small Signal Model

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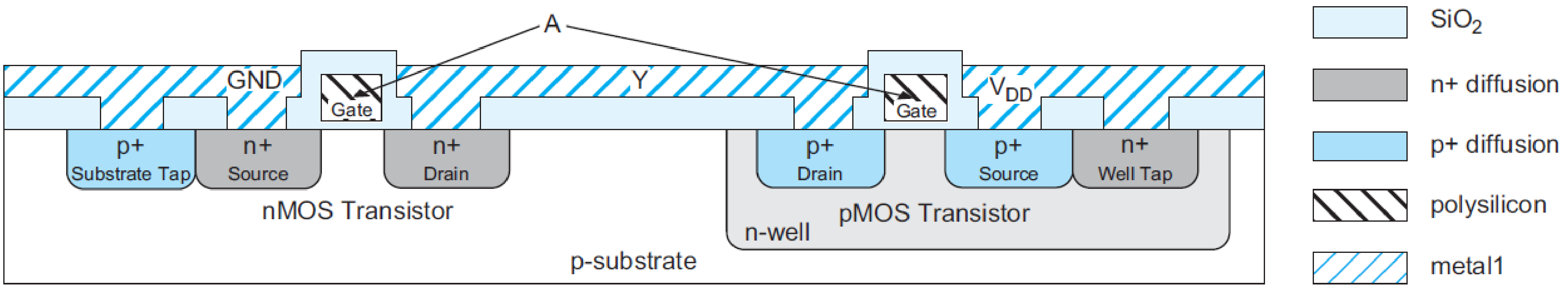
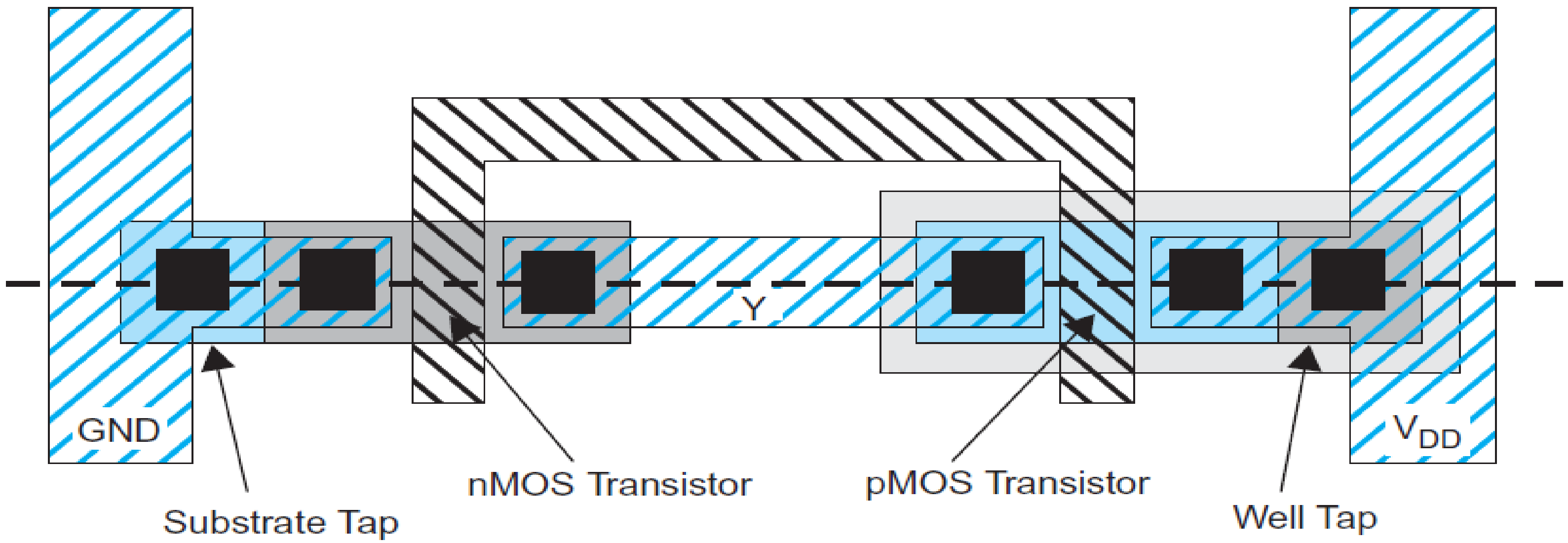
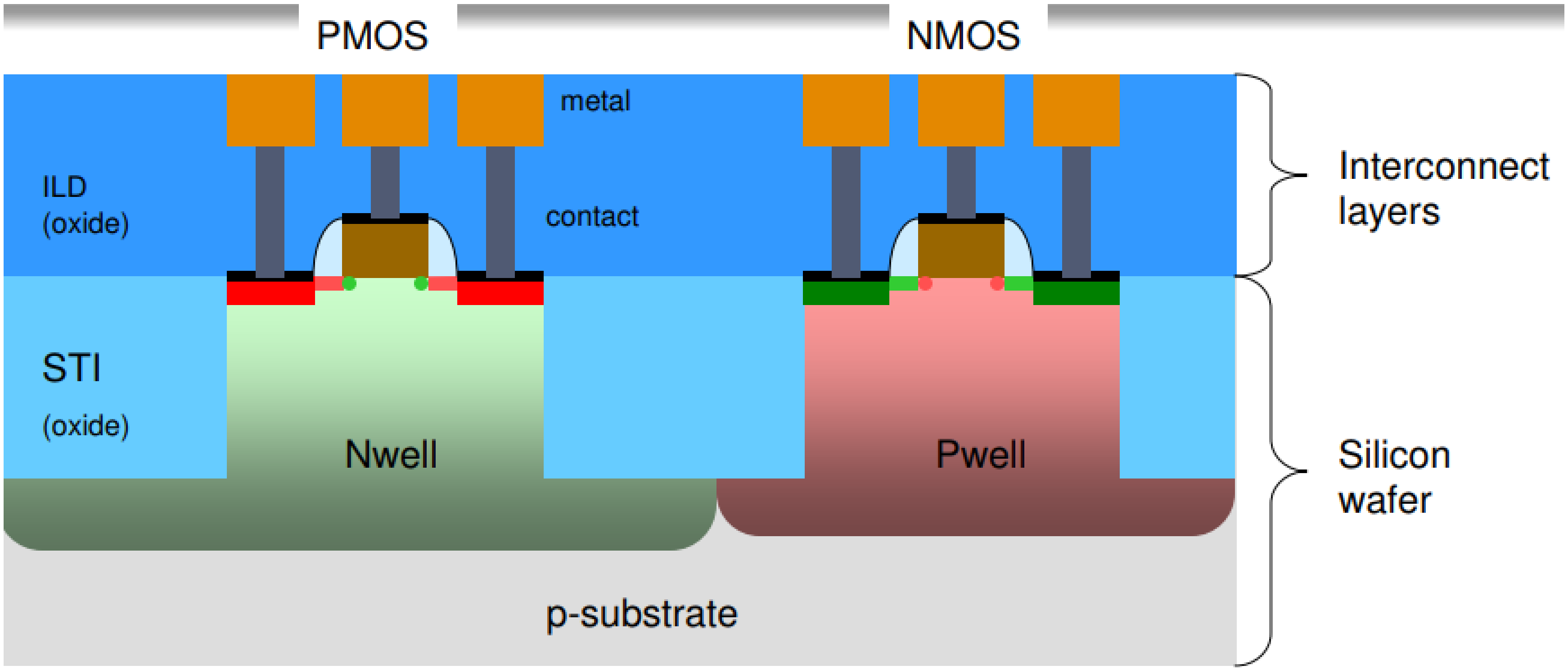
ENG-259

Cross-sectional and Top Views

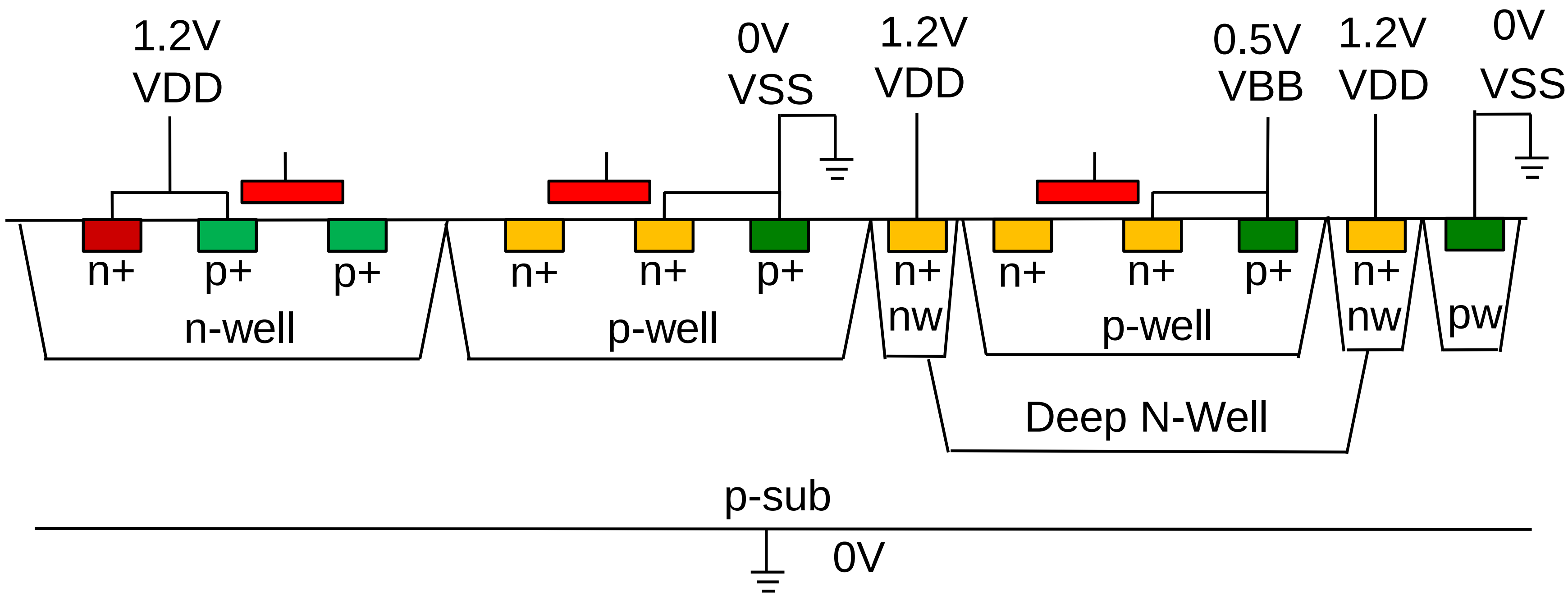




CMOS Cross-section

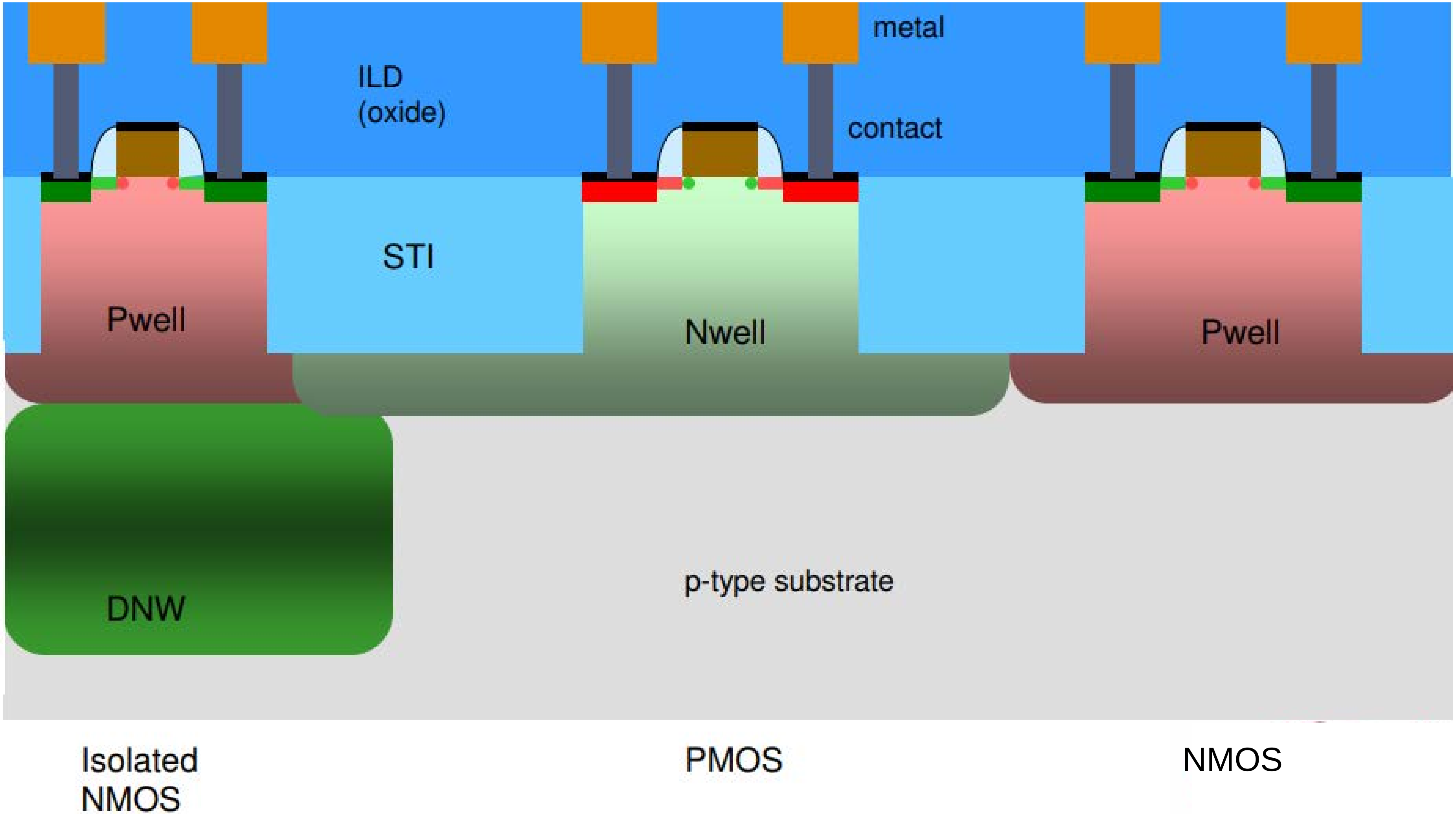


Cross-section with Deep N-Well (DNW)



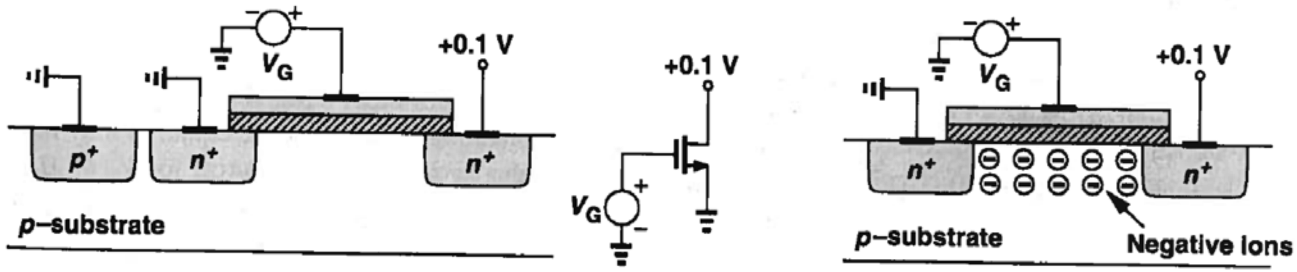
All pn junctions must be reverse-biased at all times.

Cross-section with Deep N-Well (DNW)



Threshold Voltage, V_T

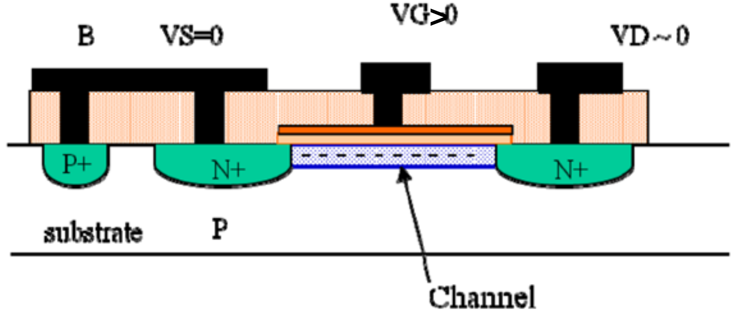
[Razavi]



- Applying a positive voltage to the gate repels holes in the p-substrate under the gate, leaving negative ions (depletion region) to mirror the gate charge

- Before a "channel" forms, the device acts as 2 series caps from the oxide cap and the depletion cap
- If V_G is increased to a sufficient value the area below the gate is "inverted" and electrons flow from source to drain

Threshold Voltage, V_T



- The threshold voltage, V_T, is the voltage at which an "inversion layer" is formed
  - For an NMOS this is when the concentration of electrons equals the concentration of holes in the p^- substrate

$$V_{TH} = \Phi_{MS} + 2\Phi_F + \frac{Q_{dep}}{C_{ox}}$$

Φ_{MS} is the difference between the work functions of the polysilicon gate and the silicon substrate

Φ_F is the Fermi potential, $\Phi_F = \frac{kT}{q}\ln\left(\frac{N_{sub}}{n_i}\right)$

Q_{dep} is the depletion region charge, $Q_{dep} = \sqrt{4q\varepsilon_{si}|\Phi_F|N_{sub}}$

C_{ox} is the gate cap/area, $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$

NMOS with Vgs > V_T, Vds < Vgs-V_T

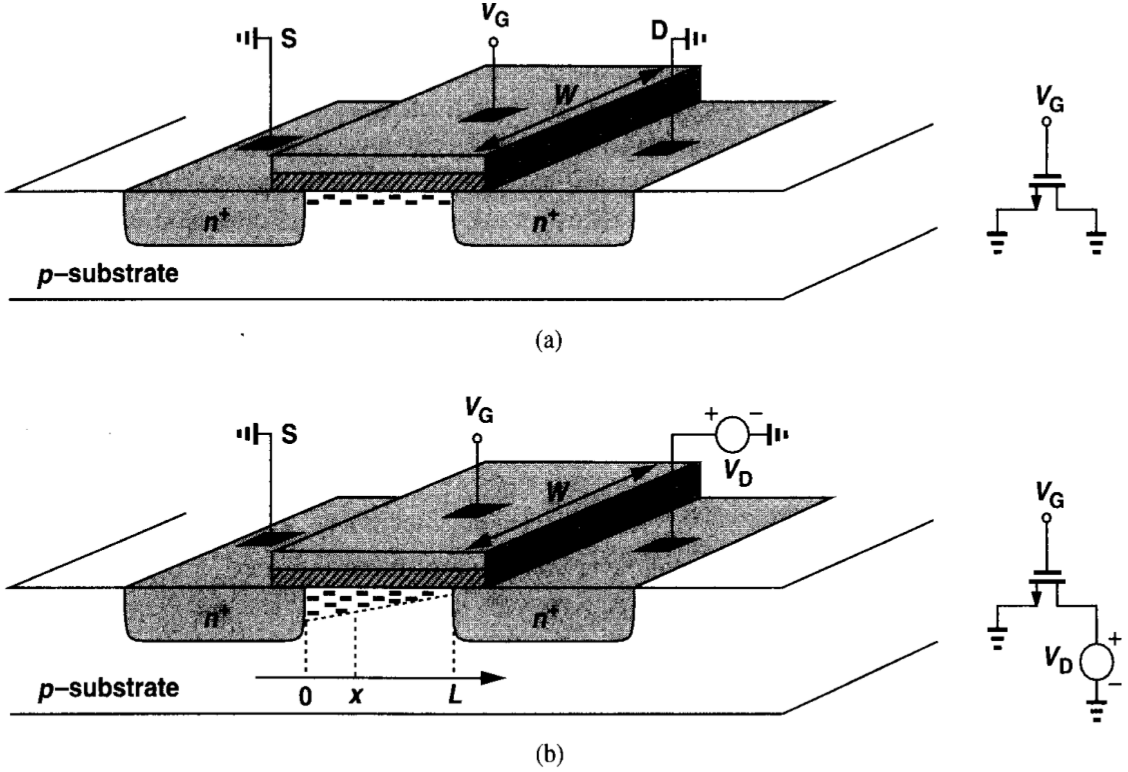
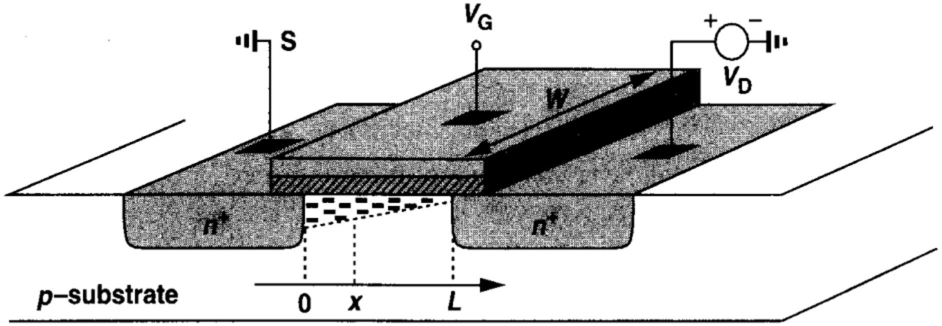


Figure 2.10 Channel charge with (a) equal source and drain voltages, (b) unequal source and drain voltages.

NMOS with Vgs > V_T, Vds < Vgs-V_T



$$I = Q_d \cdot v.$$

$$Q_d = WC_{ox}(V_{GS} - V_{TH})$$

$$Q_d(x) = WC_{ox}[V_{GS} - V(x) - V_{TH}]$$

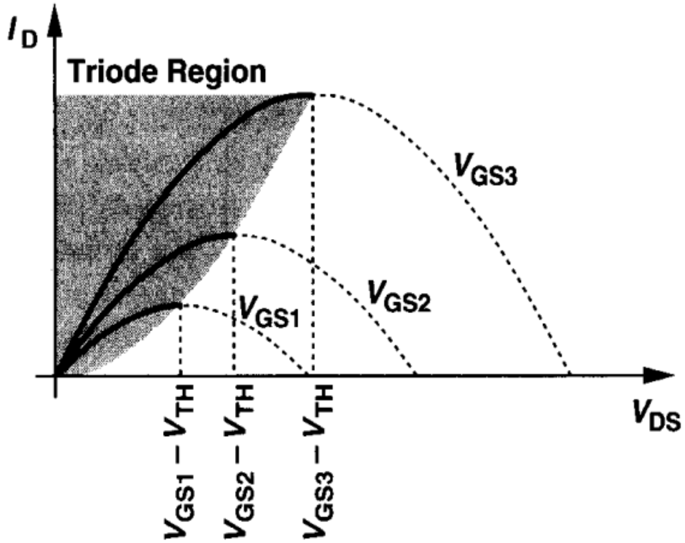
$$I_D = -WC_{ox}[V_{GS} - V(x) - V_{TH}]v$$

$$I_D = WC_{ox}[V_{GS} - V(x) - V_{TH}]\mu_n \frac{dV(x)}{dx}$$

$$\int_{x=0}^{L} I_D dx = \int_{V=0}^{V_{DS}} WC_{ox}\mu_n[V_{GS} - V(x) - V_{TH}]dV.$$

$$\boxed{I_D = \mu_n C_{ox}\frac{W}{L}\left[(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2\right]}$$

NMOS in Triode Region



$$I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS}-V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

If V_{DS} is small,

$$I_D = \mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{TH})V_{DS}$$

Figure 2.11 Drain current versus drain-source voltage in the triode region.

NMOS in Triode Region

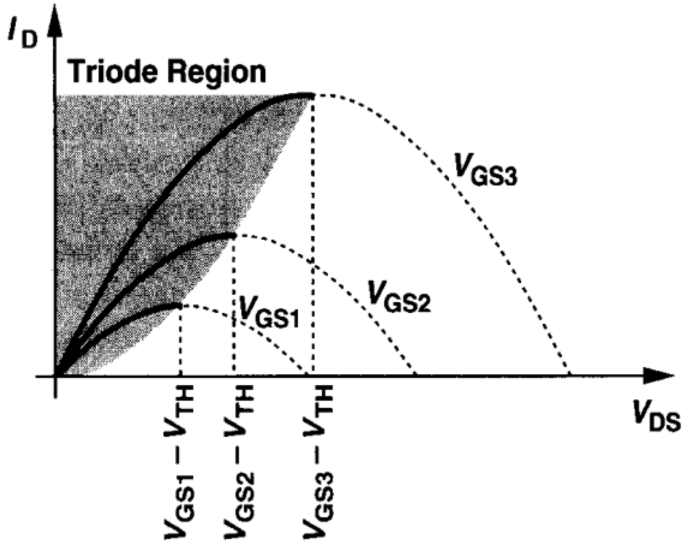


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Figure 2.11 Drain current versus drain-source voltage in the triode region.

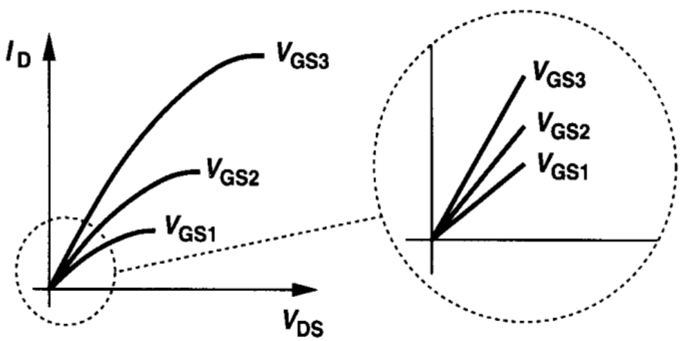
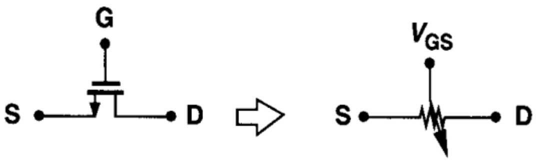
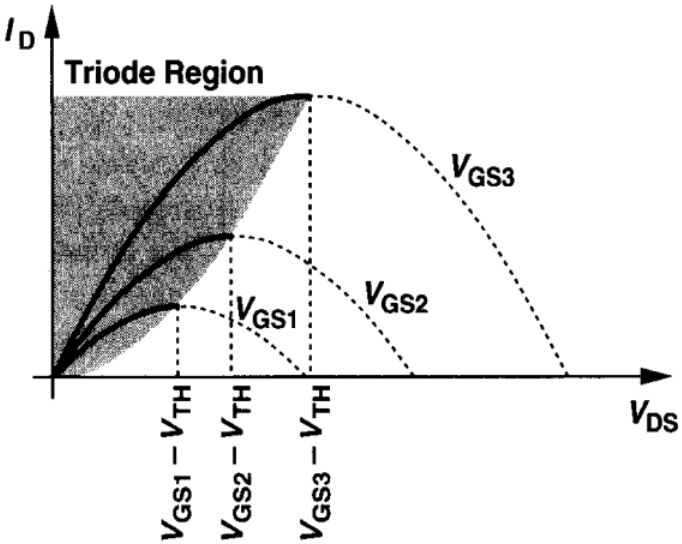


Figure 2.12 Linear operation in deep triode region.



$$R_{on} = \frac{1}{\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})}$$

Peak Current in Triode Region



$$I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$\frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH} - V_{DS})$$

$$I_D \ peaks \ at \ V_{DS} = (V_{GS} - V_{TH})$$

Figure 2.11 Drain current versus drain-source voltage in the triode region.

What is the peak current ?

$$I_{D,max} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})^2$$

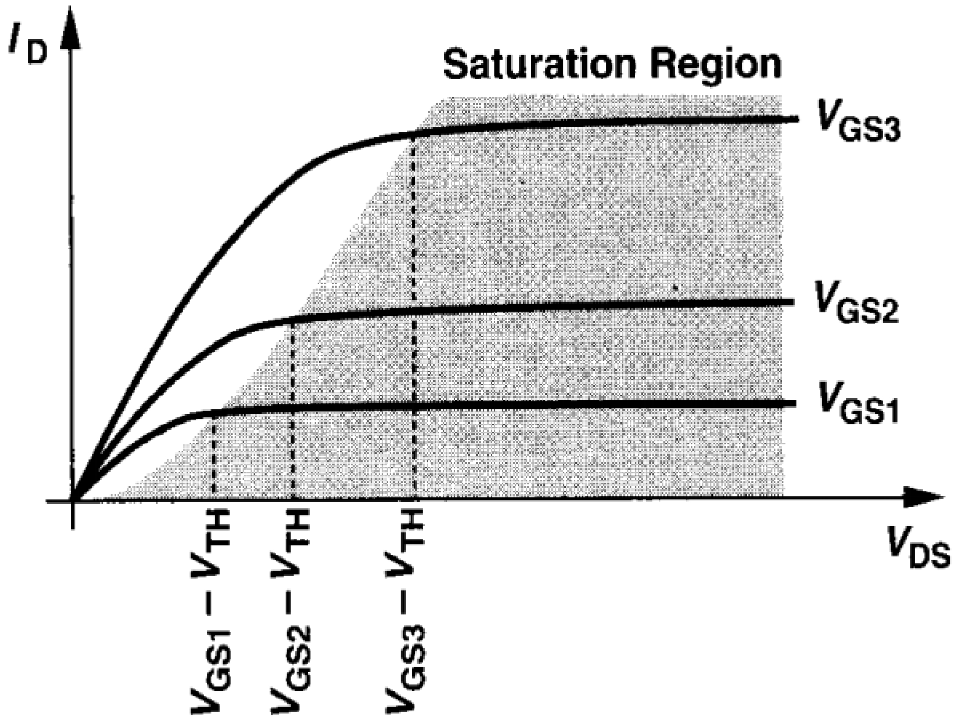
NMOS in Saturation Region

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$$

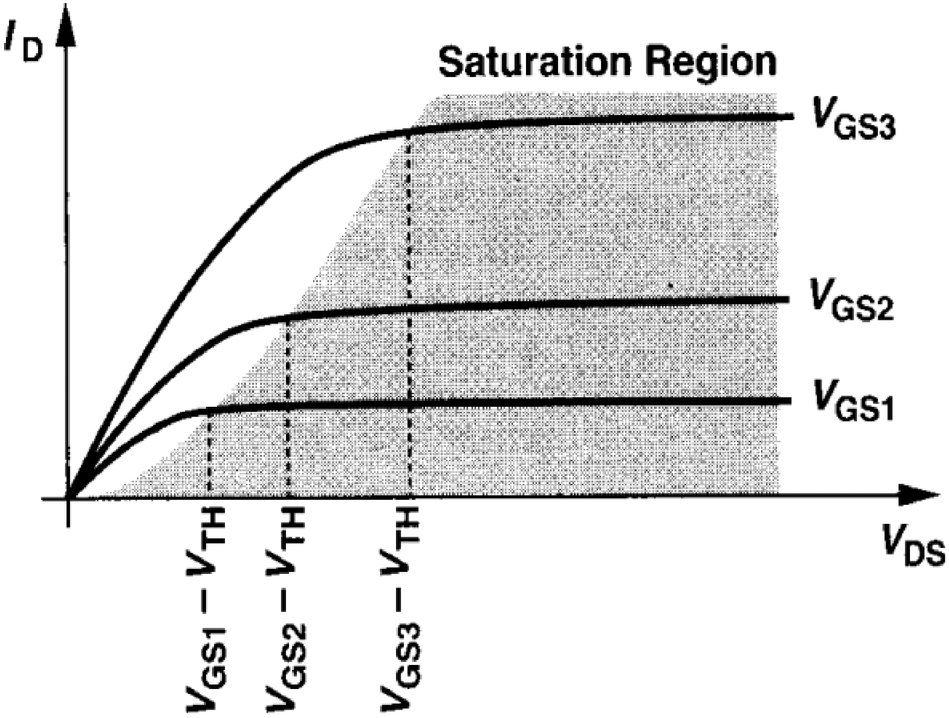
⇐ **Golden Equation**

$$k'_n = \mu_n C_{ox}$$

$$V_{Dsat} = V_{GS} - V_{TH}$$



NMOS Current Source



$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$$

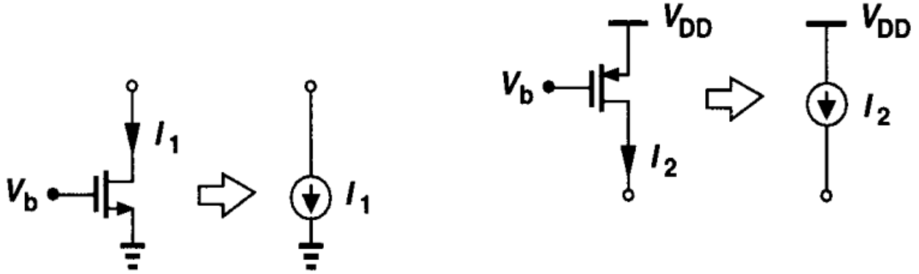
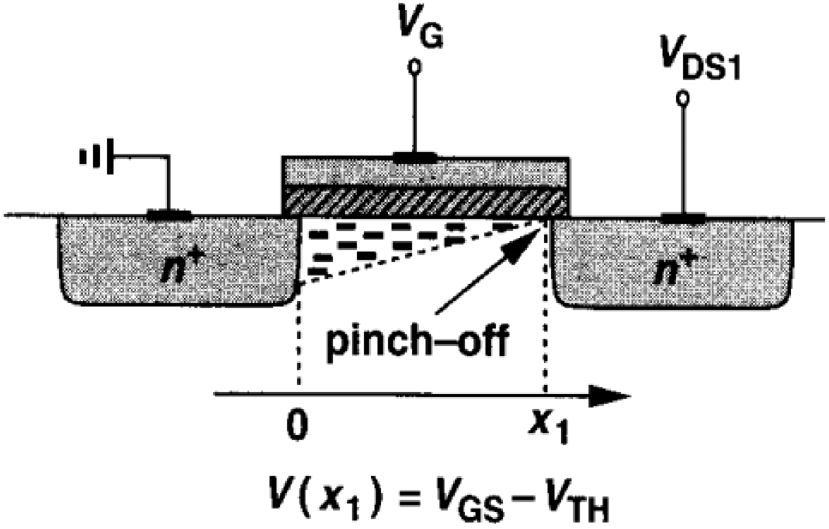
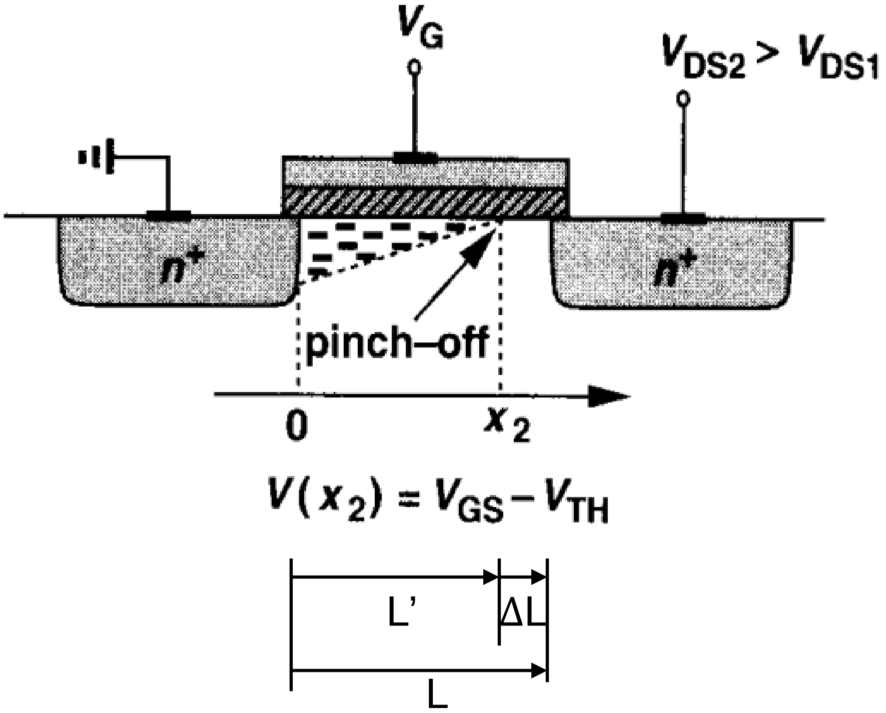


Figure 2.17 Saturated MOSFETs operating as current sources.

Channel Pinch-off





$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L'}(V_{GS} - V_{TH})^2,$$ where *L' = L - ΔL*

Channel Length Modulation

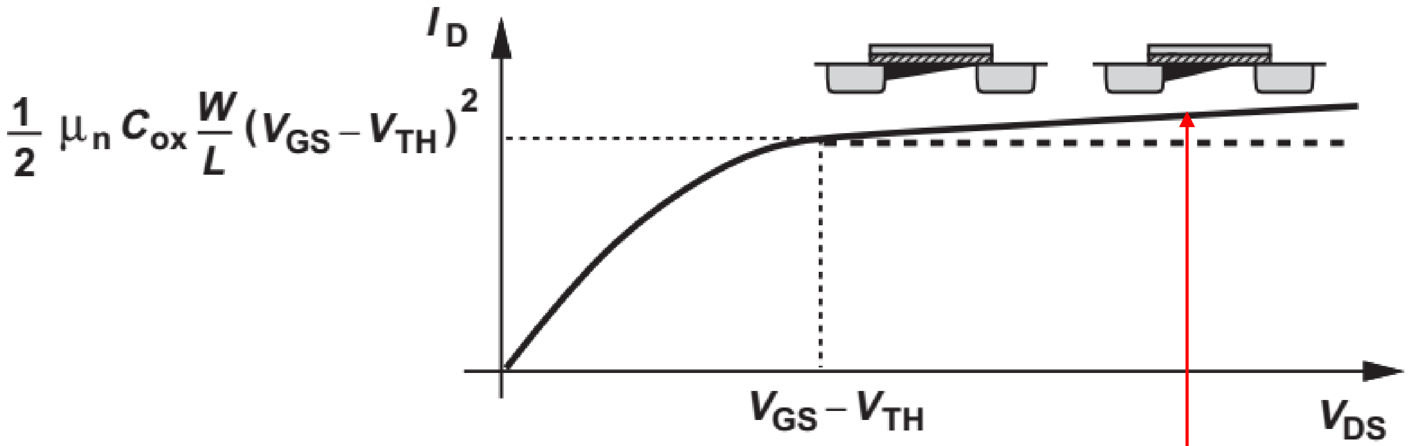


Figure 6.25 Variation of I_D in saturation region.

Slope represents $1/r_o$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$$

$$r_O = \frac{\Delta V_{DS}}{\Delta I_D}$$

Channel Length Modulation Coefficient（λ）

$$r_O = \frac{\Delta V_{DS}}{\Delta I_D}$$

Longer channel length has smaller **λ**

$$\lambda \propto \frac{1}{L}$$

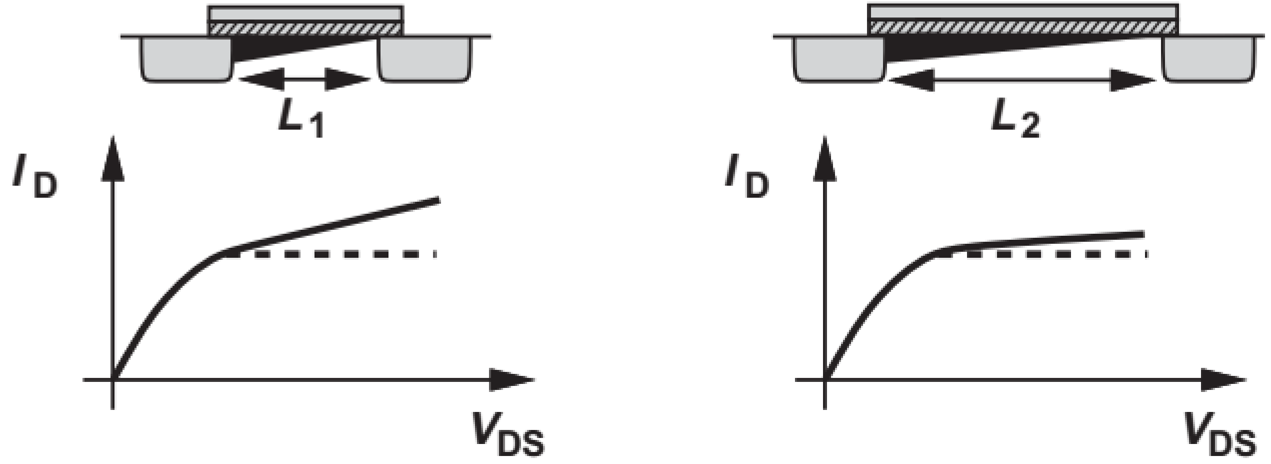
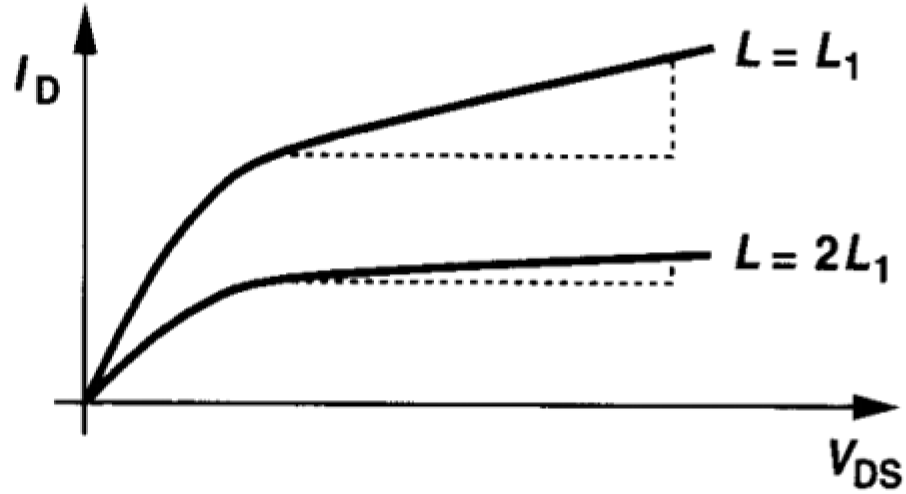


Figure 6.26 Channel-length modulation.

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$$

Effect of Channel Length Modulation

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$$



Effect of doubling channel length.

Good current source requires Longer channel length.

Effect of Channel Length Modulation

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$$

$$\frac{\Delta I_D}{\Delta V_{DS}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2\lambda \approx \lambda I_D$$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{1}{\lambda I_D} \qquad \lambda \propto \frac{1}{L} \qquad A_v = g_m\, r_o$$

$$L\uparrow \quad \lambda\downarrow \quad r_o\uparrow \quad A_v\uparrow$$

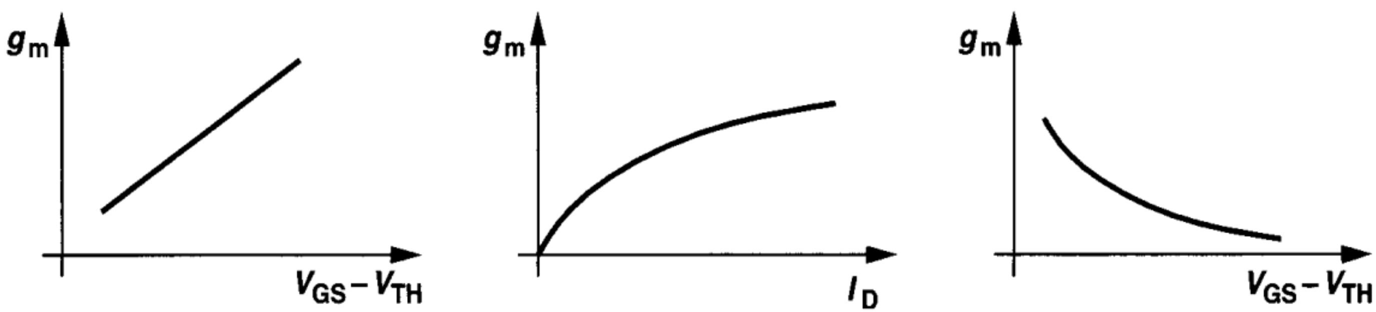
For higher amplifier gain, use Longer channel length.

MOS Transconductance in Saturation

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$$

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{VDS,\text{const.}} = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})$$

$$= \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \frac{2I_D}{V_{GS} - V_{TH}}$$



W/L Constant   W/L Constant I_D Constant

MOS Transconductance in Triode

Saturation

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$$

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{VDS,\text{const.}}$$

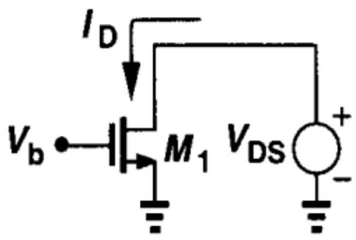
$$= \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})$$

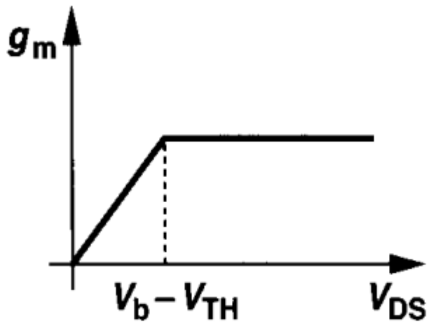
Triode

$$I_D = \mu_n C_{ox}\frac{W}{L}\left[(V_{GS} - V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

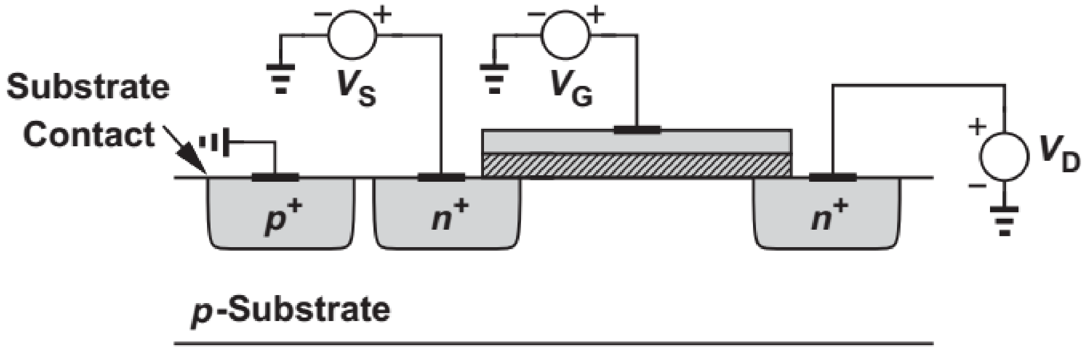
$$g_m = \frac{\partial}{\partial V_{GS}}\left\{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2\right]\right\}$$

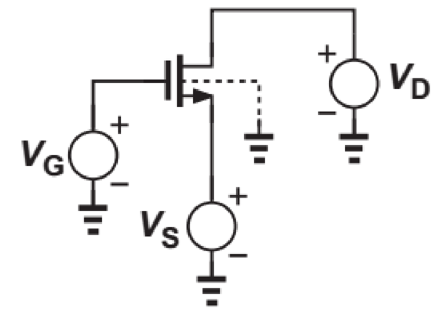
$$= \mu_n C_{ox}\frac{W}{L}V_{DS}.$$





Body Effect





$$V_T = V_{T0} + \gamma\left(\sqrt{|2\Phi_F + V_{SB}|} - \sqrt{|2\Phi_F|}\right)$$

Body effect coefficient, $\gamma = \dfrac{\sqrt{2q\varepsilon_{si}N_{sub}}}{C_{ox}}$

γ typically ranges from 0.3 to 0.4V$^{1/2}$

Body (or Bulk) Transconductance, g_{mb}

The small-signal drain current changes with V_{BS} modulation due to changes in V_T

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L'}(V_{GS} - V_{TH})^2$$

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{|2\Phi_F + V_{SB}|} - \sqrt{|2\Phi_F|}\right)$$

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})\left(-\frac{\partial V_{TH}}{\partial V_{BS}}\right)$$

$$\frac{\partial V_{TH}}{\partial V_{BS}} = -\frac{\partial V_{TH}}{\partial V_{SB}} = -\frac{\gamma}{2}(2\Phi_F + V_{SB})^{-1/2}$$

$$g_{mb} = g_m\frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}} = \eta g_m$$