This transform represents a very practical way to identify the frequency content of the signal. It operates on a signal with N samples and produces a DFT with N points. Each index k marks a point of the DFT.

2. The DFT $X[k]$ provides a DFT magnitude spectrum $|X[k]|$ and a DFT phase spectrum $\theta[k]$.
3. The DFT is periodic with period N.
4. The DFT samples the DTFT at the digital frequencies described by the equation

$$\Omega = 2\pi \frac{k}{N}$$

5. The DFT indices k refer to analog frequencies in Hz given by

$$f = k\frac{f_S}{N}$$

where N is the number of points in the DFT and f_S is the sampling frequency. The spacing between these frequencies determines the resolution of the DFT. Because of its limited resolution, the DFT cannot report all frequencies in a signal perfectly. The DFT tends to smear sharp peaks in a spectrum, although increasing the number of points in the DFT can reduce the amount of smearing.

6. The spectra of both nonperiodic and periodic signals may be obtained using the DFT. The DFT simply operates on a group of signal samples selected by the DFT's window. The type of window used (rectangular, Hanning, Hamming, Blackman, or Kaiser) affects the details of the spectrum reported.
7. Spectrograms combine multiple DFTs together to show how a signal's frequency content changes with time.
8. The FFT is a computationally efficient way of implementing the DFT. It obtains exactly the same results as the DFT.
9. The spectra for digital images are calculated using the 2D DFT (or 2D FFT).

REVIEW QUESTIONS

11.1 A digital signal $x[n]$ is shown in Figure 11.43.

a. Plot the DFT magnitude and phase spectra for the samples of $x[n]$ for $0 \le n \le 7$.

b. What is the period of the spectra in (a)?

11.2 For the digital signal $x[n] = e^{-0.5n}(u[n] - u[n-4])$, plot:

a. The DFT magnitude spectrum

b. The DTFT magnitude spectrum

11.3 The DTFT for a signal is $H(\Omega) = 1 - 0.2e^{-j\Omega} + 0.35e^{-j2\Omega}$. Find the magnitudes and phases for an 8-point DFT of the same signal.

11.4 A digital signal is described as $x[n] = \sin(n\pi/2)$ for $0 \le n \le 3$. Plot a 4-point DFT $X[k]$ for this signal.

11.5 The impulse response for a filter is $h[n] = (-0.95)^n$ for $0 \le n \le 3$. Plot a 4-point DFT $H[k]$ for this filter.

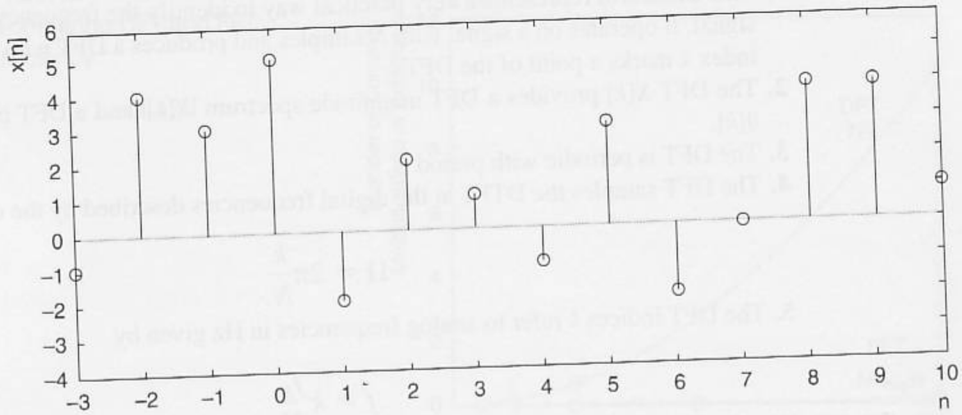

FIGURE 11.43
Signal for Question 11.1.

11.6 Compare the DFT magnitude spectra for the signals:

a. $x_1[n] = [4\ 3\ 2\ 1]$
b. $x_2[n] = [4\ 3\ 2\ 1\ 4\ 3\ 2\ 1]$
c. $x_3[n] = [4\ 3\ 2\ 1\ 4\ 3\ 2\ 1\ 4\ 3\ 2\ 1]$

11.7 Five samples of a digital signal $x[n]$ are $[3\ {-1}\ 0\ 2\ 1]$.

a. Find a 5-point DFT magnitude spectrum for these samples.
b. Zero-pad the signal to 8 points, and find an 8-point DFT magnitude spectrum. Compare the result to the spectrum in (a). To understand the results, it may be necessary to plot the DTFT magnitude spectrum for $x[n]$.

11.8 The DFT magnitudes and phases computed from four samples of a signal are listed in Table 11.7. Find the values of the four samples.

TABLE 11.7
DFT Values for Question 11.8

k	$\lvert X[k]\rvert$	$\theta[k]$
0	10.000	0.0000
1	2.8284	−0.7854
2	2.0000	0.0000
3	2.8284	0.7854

11.9 The 8-point DFT of a signal $x[n]$ defined beginning at $n = 0$ is $X[k]$. The inverse DFT of $X[k]$ is shown in Figure 11.44.

a. Plot the signal $x[n]$.
b. How do 4-point and 8-point DFT magnitude spectra for $x[n]$ compare?

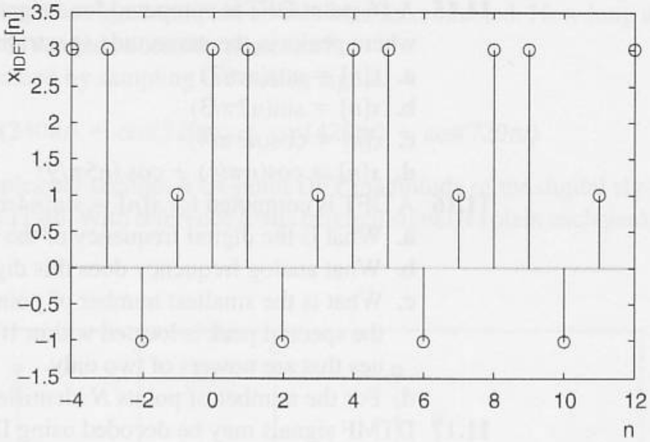

FIGURE 11.44 Inverse DFT for Question 11.9.

11.10 The frequency response for a (noncausal) equiripple filter is given by

$$H_1(\Omega) = 0.2273(\cos\Omega)^3 + 0.5778(\cos\Omega)^2 + 0.3505\cos\Omega + 0.0311$$

Find the noncausal impulse response $h_1[n]$ that corresponds to this frequency response.

11.11 An equiripple filter is described by the optimal filter shape

$$H_1(\Omega) = 0.6269\cos^3\Omega + 0.5134\cos^2\Omega - 0.1084\cos\Omega - 0.0551$$

Obtain a causal impulse response $h[n]$ for the filter.

11.12 An analog signal is sampled at 16 kHz. A 512-point DFT is computed.

a. What is the resolution of the DFT?

b. Find the equivalent frequency in Hz for each of the following points of the DFT:

(i) $k = 0$
(ii) $k = 127$
(iii) $k = 255$
(iv) $k = 511$

11.13 A 6 kHz analog sine wave is sampled at 40 kHz. Determine where the peaks in its DFT magnitude spectrum will occur for a:

a. 32-point DFT
b. 64-point DFT
c. 128-point DFT

11.14 A 6 kHz analog sine wave is sampled at 7.5 kHz. Determine where the peaks in its DFT magnitude spectrum will occur for a:

a. 32-point DFT
b. 64-point DFT
c. 128-point DFT