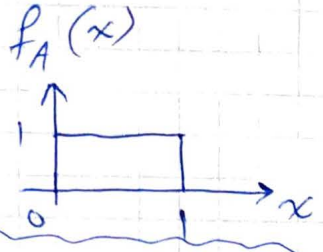
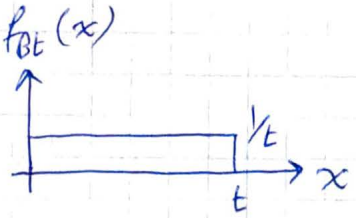
③ Random Process $X(t) = A + Bt$ — A & B are Uniform [0, 1]

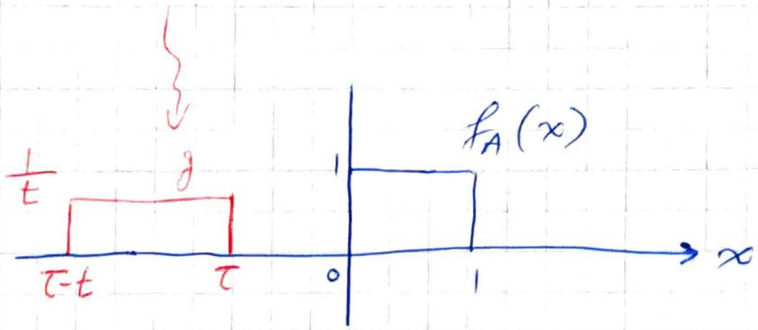


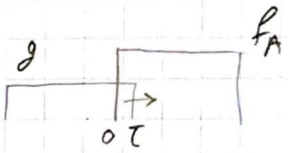


τ is my x

f_X → Convolution of f_A & f_{Bt}

$$f_X = f_A * f_{Bt} = \int_{-\infty}^{\infty} f_A(\tau)\, g(x-\tau)\, d\tau$$



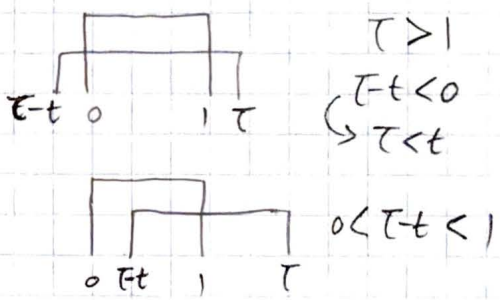


ⓐ $\tau < 0 \rightsquigarrow f_X = 0$

ⓐ $0 < \tau < 1 \rightsquigarrow f_X = \int_0^{\tau} (1)\left(\frac{1}{t}\right) d\tau$

$= \frac{\tau}{t}$

ⓐ $1 < \tau < t \rightsquigarrow f_X = \int_0^{1} (1)\left(\frac{1}{t}\right) d\tau = \frac{1}{t}$



ⓐ $t < \tau < t+1 \rightsquigarrow f_X = \int_{\tau-t}^{1} (1)\left(\frac{1}{t}\right) d\tau = \frac{1-\tau+t}{t}$

τ is my x

So, pdf of f(x) is