Fall 2020

EE 210

HW#: 03

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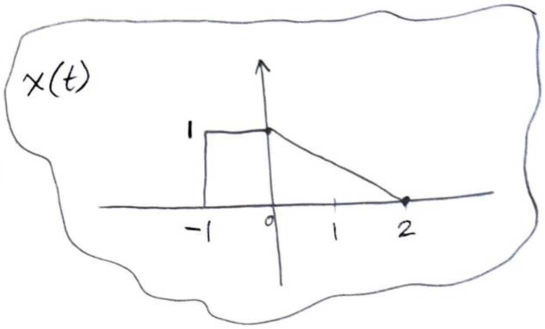
ID: **011510317**

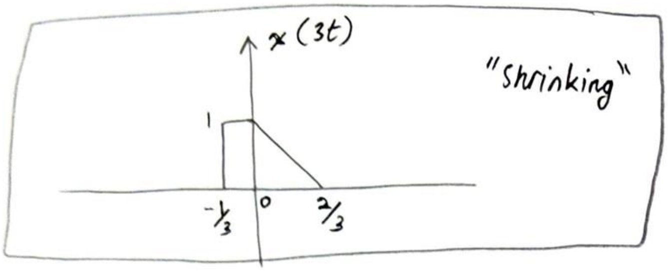
Date: **9/17/2020**

Assigned question #s: **5**

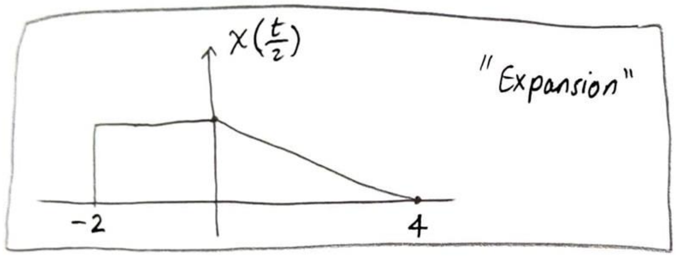
HW03

① a) $x(3t) = x\left(\frac{t}{1/3}\right)$

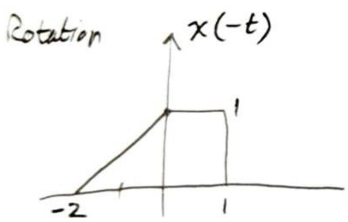


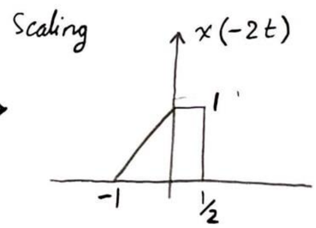


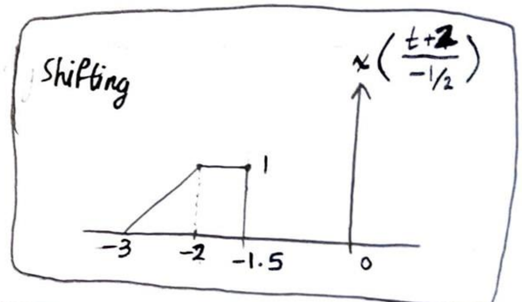
b) $x\left(\frac{t}{2}\right)$



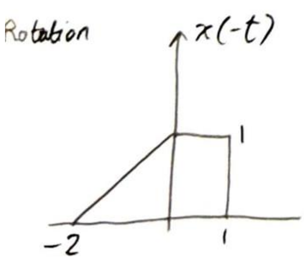
c) $x(-2t-4) = x(-2(t+2)) = x\left(\frac{t+2}{-1/2}\right)$

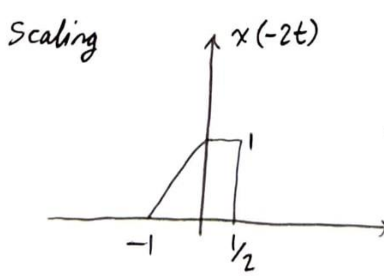


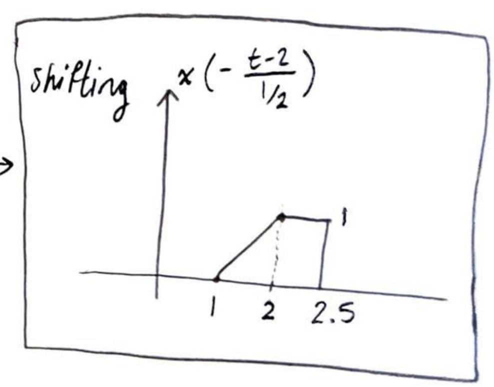




d) $x(-2t+4) = x(-2(t-2)) = x\left(-\frac{t-2}{1/2}\right)$



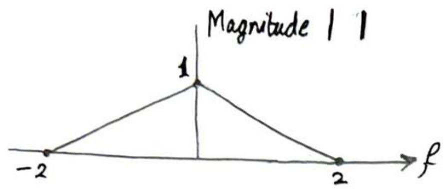


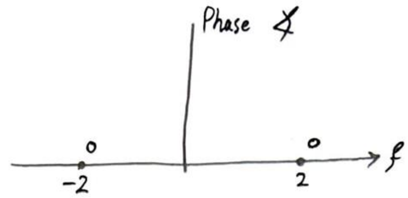


② a) $\mathcal{F}\{x(2t)\} = \mathcal{F}\{x(\frac{t}{1/2})\} = \frac{1}{2}X(\frac{1}{2}f)$

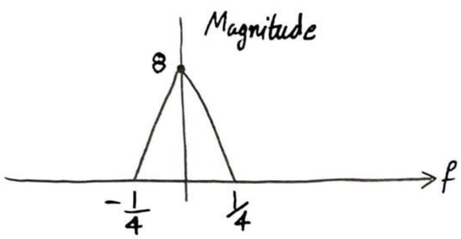
$= \boxed{\Lambda(\frac{f}{4})}$

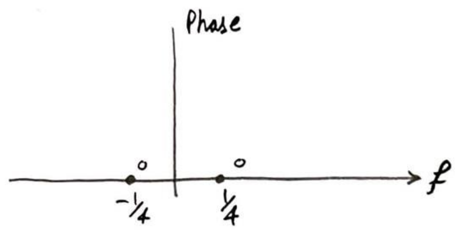
$\boxed{\mathcal{F}\{x(t)\} \rightarrow 2\,\Lambda(\frac{f}{2})}$





b) $\mathcal{F}\{x(\frac{t}{4})\} = 4\,X(4f) = \boxed{8\,\Lambda(2f)}$





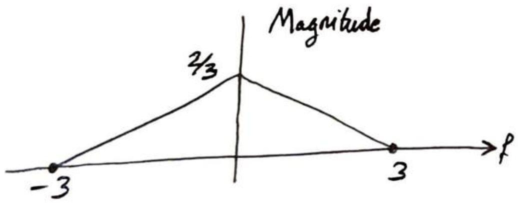
c) $\mathcal{F}\{x(-3t-6)\} = \mathcal{F}\{x(\frac{t+2}{-1/3})\} = \frac{1}{3}X(-\frac{1}{3}f)\,e^{-j2\pi f(-2)}$

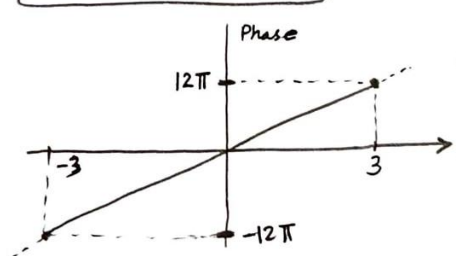
$= \boxed{\frac{2}{3}\Lambda(\frac{-f}{6})\;e^{j4\pi f}}$

$\theta = 4\pi f$

at $f=-3 \rightsquigarrow \theta = -12\pi$

at $f=3 \rightsquigarrow \theta = 12\pi$





θ could also be drawn between -2π and 2π

→ Harder

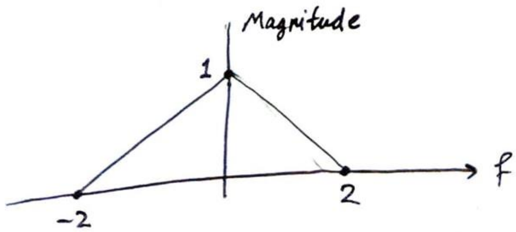
d) $\mathcal{F}\{x(-2t+6)\} = \mathcal{F}\{x(\frac{t-3}{-1/2})\} = \frac{1}{2}X(-\frac{1}{2}f)\,e^{-j2\pi f(3)}$

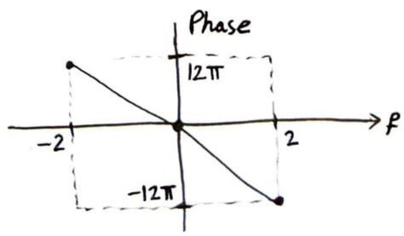
$= \boxed{\Lambda(\frac{-f}{4})\,e^{-j6\pi f}}$

$\theta = -6\pi f$

at $f=-2 \rightsquigarrow \theta = +12\pi$

at $f=2 \rightsquigarrow \theta = -12\pi$





3 a) $\int_{-\infty}^{\infty} e^{-t}\,\delta(t-1)\,dt = e^{-1}\int_{-\infty}^{\infty} \delta(t-1)\,dt = \boxed{e^{-1} = 0.3679}$

b) $\int_{0}^{\infty} e^{-t}\,\delta(t-1)\,dt = e^{-1}\int_{0}^{\infty} \delta(t-1)\,dt = \boxed{e^{-1} = 0.3679}$

c) $\int_{0}^{\infty} e^{-t}\,\delta(t+1)\,dt = \boxed{0}$ "as $t=-1$ is outside the integration limits"

d) $\int_{-\infty}^{\infty} (t^3+t^2+t+1)\,\delta(t)\,dt = (0^3+0^2+0+1)\int_{-\infty}^{\infty}\delta(t)\,dt = \boxed{1}$

e) $\int_{-\infty}^{\infty} \cos^2(2\pi t + 0.1\pi)\,\delta(t+1)\,dt = \cos^2(2\pi(-1)+0.1\pi)\int_{-\infty}^{\infty}\delta(t+1)\,dt$

$= \cos^2(0.1\pi - 2\pi)$

$\approx \boxed{0.9045}$

f) $\int_{-\infty}^{\infty} e^{-t}\,\delta(-t-1)\,dt = \int_{-\infty}^{\infty} e^{-t}\,\delta(-(t+1))\,dt = e^{-(-1)}\int_{-\infty}^{\infty}\delta(t+1)\,dt = \boxed{e^{1} = 2.718}$

g) $\int_{-\infty}^{\infty} t^2\,\delta(-\frac{1}{2}t+\frac{1}{2})\,dt = \int_{-\infty}^{\infty} t^2\,\delta(-\frac{1}{2}(t-1))\,dt$

$= (1)^2\int_{-\infty}^{\infty}\frac{1}{|1/2|}\,\delta(t-1)\,dt = \boxed{2}$

h) $\int_{-\infty}^{\infty} e^{t}\,\delta(3t-1)\,dt = \int_{-\infty}^{\infty} e^{t}\,\delta(3(t-\frac{1}{3}))\,dt$

$= e^{(1/3)}\int_{-\infty}^{\infty}\frac{1}{3}\,\delta(t-\frac{1}{3})\,dt = \boxed{\frac{1}{3}e^{1/3} = 0.4652}$

4 a) $X(f) = |X(f)| \cdot e^{-j2\pi f t_0}$

$$\boxed{X(f) = 4\,\mathrm{rect}\left(\frac{f}{10}\right) \cdot e^{-j2\pi f\left(\frac{1}{20}\right)} = \begin{cases} 4\,e^{-j\frac{\pi}{10}f} & \text{if } -5 \leq f \leq 5 \\ 0 & \text{otherwise} \end{cases}}$$

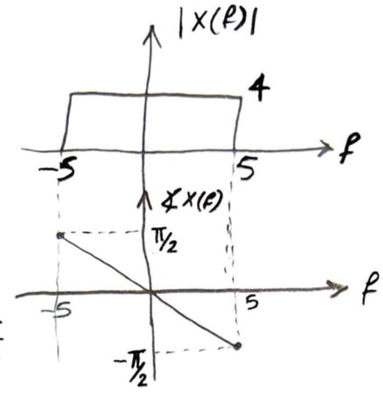
$\theta = -2\pi t_0 f$

at $f=-5$:

$\theta = -2\pi t_0(-5) = \frac{\pi}{2}$

$10\pi\, t_0 = \pi/2$

$\boxed{t_0 = \frac{1}{20}}$



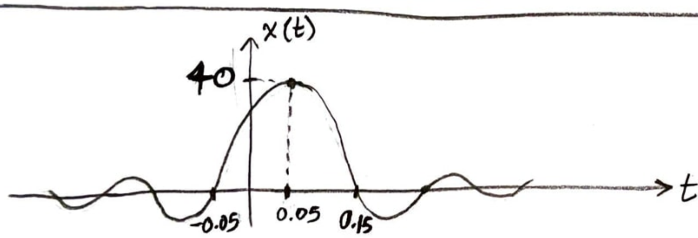
b) $x(t) = \mathcal{F}^{-1}\left\{ |X(f)|\, e^{-j2\pi t_0 f} \right\}$

$= 2 \cdot A \cdot f_0 \cdot \mathrm{Sinc}(2 \cdot f_0 \cdot (t - t_0))$

$= \boxed{40\,\mathrm{Sinc}\left(10\left(t - \frac{1}{20}\right)\right)}$

$A = 4$
$f_0 = 5$
$t_0 = \frac{1}{20}$

c)



Zero crossings at

$10\left(t - \frac{1}{20}\right) = n$

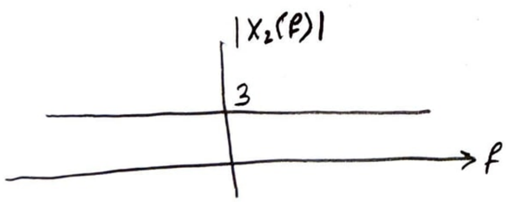
$t - 0.05 = 0.1n$

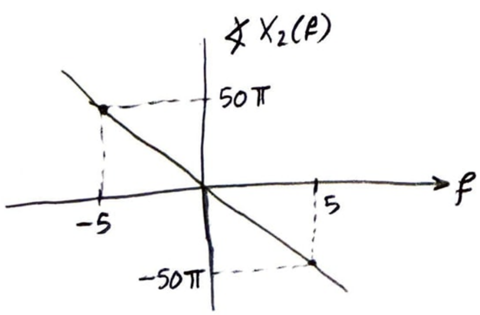
$t = 0.1n + 0.05$

d) $X_2(f) = 3\, e^{-j10\pi f}$ $\rightsquigarrow$ $\theta = -10\pi f$ $\rightarrow$ slope $= -10\pi$

at $f = -5$: $\theta = 50\pi$

at $f = 5$: $\theta = -50\pi$





e) $Y(f) = X(f) \cdot X_2(f)$

$$= 4\, e^{-j\frac{\pi}{10}f} \cdot 3\, e^{-j10\pi f} \quad \text{if } -5 \leq f \leq 5$$

$$\boxed{Y(f) = \begin{cases} 12\, e^{-j\pi(10.1)f} & \text{if } -5 \leq f \leq 5 \\ 0 & \text{otherwise} \end{cases}} \quad \text{OR} \quad \boxed{Y(f) = 12\, \text{rect}\left(\frac{f}{10}\right) e^{-j\pi(10.1)f}}$$

at $-5 \leq f \leq 5$:

$$\boxed{\begin{array}{ll} \text{Polar format:} & Y(f) = 12\, e^{-j\pi(10.1)f} \\ \text{Rectangular:} & Y(f) = 12 \cos(10.1\pi f) - j\, 12 \sin(10.1\pi f) \end{array}}$$

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f) $\cancel{y}y(t) = \mathcal{F}^{-1}\left\{12\, \text{rect}\left(\frac{f}{10}\right) e^{-j\pi(10.1)f}\right\} = 2 A f_0\, \text{Sinc}(2 f_0 (t - t_0))$

$A = 12$
$f_0 = 5$
$t_0 = 10.1$

$$= \boxed{120\, \text{Sinc}\left(10(t - 10.1)\right)}$$

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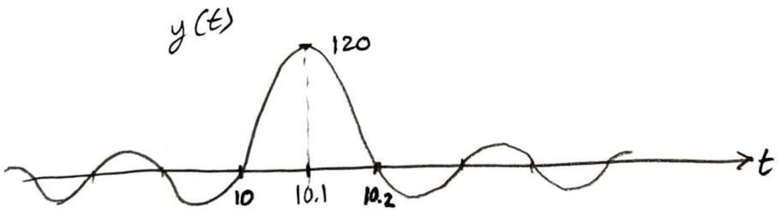
g)



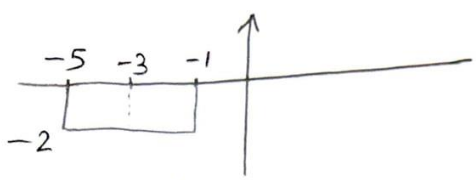
Zero crossings at

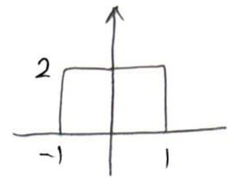
$10(t - 10.1) = n$

$t - 10.1 = 0.1n$

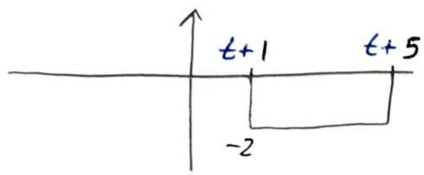
$t = 0.1n + 10.1$

5 a) $x(t) = -2\,\Pi\left(\frac{t+3}{4}\right) \quad * \quad h(t) = 2\,\Pi\left(\frac{t}{2}\right)$





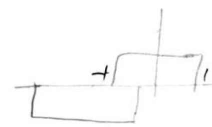
$\to x(t-\tau)$



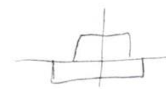
$\boxed{1}$ $t+5 < -1 \to t < -6 \rightsquigarrow y(t) = 0$

$\boxed{2}$ $-1 \leq t+5 \leq 1 \to -6 \leq t \leq -4 \rightsquigarrow y(t) = \int_{-1}^{t+5} (-2)(2)\,d\tau$

$= -4\,[\tau]_{-1}^{t+5} = -4(t+6)$

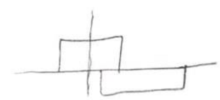


$\boxed{3}$ $\begin{array}{l} t+5 \geq 1 \\ t+1 < -1 \end{array} \to -4 < t \leq -2 \rightsquigarrow y(t) = \int_{-1}^{1} (-2)(2)\,d\tau = -8$



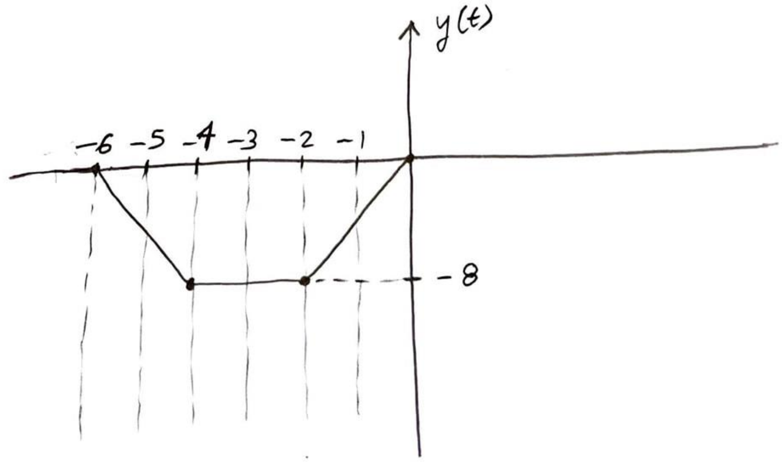
$\boxed{4}$ $-1 < t+1 < 1 \to -2 < t \leq 0 \rightsquigarrow y(t) = \int_{t+1}^{1} (-2)(2)\,d\tau$

$= -4[\tau]_{t+1}^{1} = 4(t)$

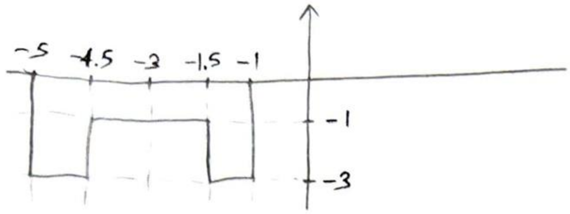


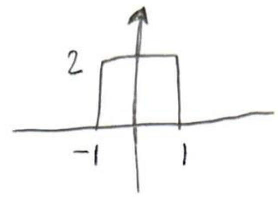
$\boxed{5}$ $t > 0 \to y(t) = 0$

$$\boxed{y(t) = \begin{cases} 0 & t < -6 \\ -4(t+6) & -6 \leq t \leq -4 \\ -8 & -4 < t \leq -2 \\ 4t & -2 < t \leq 0 \\ 0 & t > 0 \end{cases}}$$



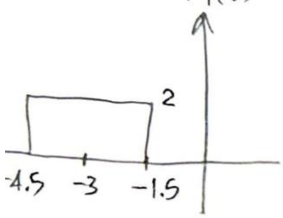
b) $x(t) = 2\Pi\left(\frac{t+3}{3}\right) - 3\Pi\left(\frac{t+3}{4}\right)$ $\quad * \quad$ $h(t) = 2\Pi\left(\frac{t}{2}\right)$



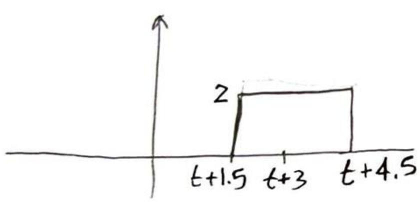


$x(t) = x_1(t) - x_2(t)$

$x_1(t) = 2\Pi\left(\frac{t+3}{3}\right)$



$x_1(t-\tau)$



$\boxed{1}$ $t+4.5 < -1 \rightarrow t < -5.5$

$y(t) = 0$

$\boxed{2}$ $-1 < t+4.5 < 1 \rightarrow -5.5 \leq t \leq -3.5$

$y(t) = \int_{-1}^{t+4.5} (2)(2)\,d\tau$

$= 4(t+5.5)$

$\boxed{3}$ $t > -3.5$, $t+1.5 \leq -1$ $\rightarrow -3.5 < t \leq -2.5$

$y(t) = \int_{-1}^{1} (2)(2)\,d\tau = 8$

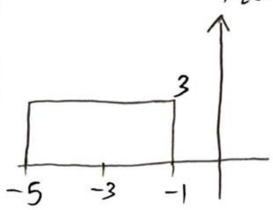
$\boxed{4}$ $-1 < t+1.5 \leq 1 \rightarrow -2.5 < t \leq -0.5$

$y(t) = \int_{t+1.5}^{1} (2)(2)\,d\tau$

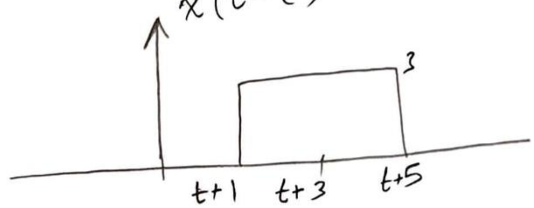
$= -4(t+0.5)$

$\boxed{5}$ $t > 0.5 \rightarrow y(t) = 0$

$x_2(t) = 3\Pi\left(\frac{t+3}{4}\right)$



$x(t-\tau)$



$\boxed{1}$ $t+5 < -1 \rightarrow t < -6$

$y(t) = 0$

$\boxed{2}$ $-1 \leq t+5 \leq 1 \rightarrow -6 \leq t \leq -4$

$y(t) = \int_{-1}^{t+5} (3)(2)\,d\tau$

$= 6(t+6)$

$\boxed{3}$ $t > -4$, $t+1 \leq -1$ $\rightarrow -4 < t \leq -2$

$y(t) = \int_{-1}^{1} (3)(2)\,d\tau = 12$

$\boxed{4}$ $-1 < t+1 \leq 1 \rightarrow -2 < t \leq 0$

$y(t) = \int_{t+1}^{1} (3)(2)\,d\tau = -6t$

$\boxed{5}$ $t > 0 \rightarrow y(t) = 0$

$$y_1(t) = \begin{cases} 0 & t < -5.5 \\ 4t+22 & -5.5 \leq t \leq -3.5 \\ 8 & -3.5 \leq t \leq -2.5 \\ -4t-2 & -2.5 \leq t \leq -0.5 \\ 0 & t > -0.5 \end{cases}$$

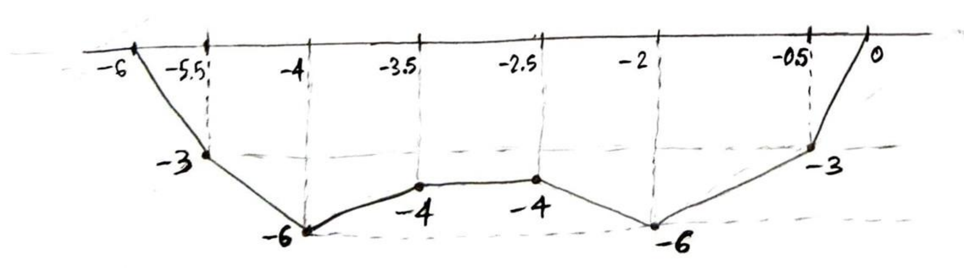
$$y_2(t) = \begin{cases} 0 & t < -6 \\ 6t+36 & -6 \leq t \leq -4 \\ 12 & -4 < t \leq -2 \\ -6t & -2 < t \leq 0 \\ 0 & t > 0 \end{cases}$$

$$y(t) = y_1(t) - y_2(t)$$

y_1	0	0	$4t+22$	$4t+22$	8	$-4t-2$	$-4t-2$	0	0
		−	−	−	−	−	−	−	
y_2	0	$6t+36$	$6t+36$	12	12	12	$-6t$	$-6t$	0
	−6		−5.5	−4	−3.5	−2.5	−2	−0.5	0

$$\boxed{y(t) = \begin{cases} 0 & t < -6 \\ -6t-36 & -6 \leq t \leq -5.5 \\ -2t-14 & -5.5 \leq t \leq -4 \\ 4t+10 & -4 \leq t \leq -3.5 \\ -4 & -3.5 \leq t \leq -2.5 \\ -4t-14 & -2.5 \leq t \leq -2 \\ 2t-2 & -2 \leq t \leq -0.5 \\ 6t & -0.5 \leq t \leq 0 \\ 0 & t > 0 \end{cases}}$$

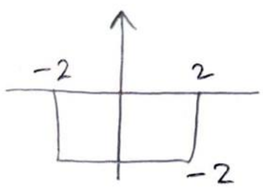
$y(t)$



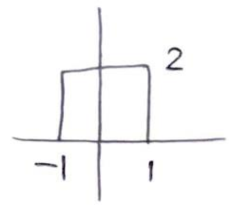
5) 2^{nd} Method

a) $x(t) = -2\Pi\left(\frac{t+3}{4}\right)$ $\qquad$ $h(t) = 2\Pi\left(\frac{t}{2}\right)$

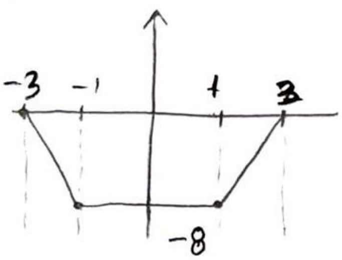
$\Downarrow$ $\qquad$ $\Downarrow$



$* \ \delta(t+3)$



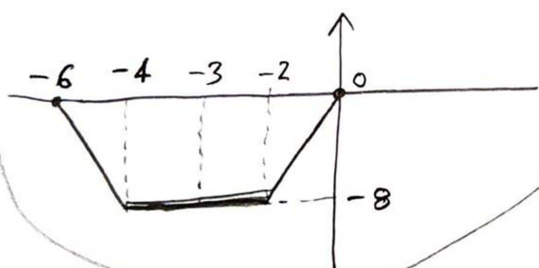
$y(t) =$



$* \ \delta(t+3)$

$\Downarrow$

$y(t)$



b) $x(t) = 2\Pi\left(\frac{t+3}{3}\right) - 3\Pi\left(\frac{t+3}{4}\right)$ $\qquad h(t) = 2\Pi\left(\frac{t}{2}\right)$