Done by: J Kim

Decimation in Frequency (DIF) processing

$$
\begin{aligned}
X[k] &= \sum_{n=0}^{N-1} x[n]\cdot e^{-\frac{j2\pi kn}{N}} \\
&= \sum_{n=0}^{\frac{N}{2}-1} x[n]\cdot e^{-\frac{j2\pi kn}{N}} + \sum_{n=\frac{N}{2}}^{N-1} x[n]\cdot e^{-\frac{j2\pi kn}{N}} \\
&= \sum_{n=0}^{\frac{N}{2}-1} x[n]\cdot e^{-\frac{j2\pi kn}{N}} + \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right]\cdot e^{-\frac{j2\pi k\left(n+\frac{N}{2}\right)}{N}} \\
&= \sum_{n=0}^{\frac{N}{2}-1} x[n]\cdot e^{-\frac{j2\pi kn}{N}} + \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right]\cdot e^{-\frac{j2\pi kn}{N}}\cdot e^{-\frac{j\cancel{2}\pi k\cancel{N}/\cancel{2}}{\cancel{N}}} \leftarrow \left\{e^{-j\pi k} = \left(e^{-j\pi}\right)^{k} = (-1)^{k}\right\} \\
&= \sum_{n=0}^{\frac{N}{2}-1} x[n]\cdot e^{-\frac{j2\pi kn}{N}} + (-1)^{k}\cdot \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right]\cdot e^{-\frac{j2\pi kn}{N}} \\
&= \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + (-1)^{k}\cdot x\left[n+\frac{N}{2}\right]\right)\cdot e^{-\frac{j2\pi kn}{N}}
\end{aligned}
$$

$$
X[k] = \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + (-1)^{k}\cdot x\left[n+\frac{N}{2}\right]\right)\cdot e^{-\frac{j2\pi kn}{N}} \qquad \leftarrow [k = 0,\ 1,\ 2,\ 3]
$$

$$
\left[\begin{aligned}
&\left[\begin{aligned}
X[2k] &= \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + (-1)^{2k}\cdot x\left[n+\frac{N}{2}\right]\right)\cdot e^{-\frac{j2\pi(2k)n}{N}} \\
&= \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + x\left[n+\frac{N}{2}\right]\right)\cdot e^{-\frac{j2\pi kn}{N/2}}
\end{aligned}\right. \\
&[k = 0,\ 1] \\
&\left[\begin{aligned}
X[2k+1] &= \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] - x\left[n+\frac{N}{2}\right]\right)\cdot e^{-\frac{j2\pi(2k+1)n}{N}} \\
&= \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] - x\left[n+\frac{N}{2}\right]\right)\cdot e^{-\frac{j2\pi kn}{N/2}}\cdot e^{-\frac{j2\pi n}{N}}
\end{aligned}\right.
\end{aligned}\right.
$$

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Assume that a given sample $\underline{x[n]=[1\ 2\ 3\ 4]}$

$@k=0$

$$X[(2k)=0]=\sum_{n=0}^{\frac{N}{2}-1}\left(x[n]+x\left[n+\frac{N}{2}\right]\right)\cdot e^{-\frac{j2\pi kn}{N/2}}$$

$$=\underbrace{(x[0]+x[2])\cdot e^{-\frac{j2\pi\cdot 0\cdot 0}{4/2}}}_{n=0\ \&\ k=0}+\underbrace{(x[1]+x[3])\cdot e^{-\frac{j2\pi\cdot 0\cdot 1}{4/2}}}_{n=1\ \&\ k=0}$$

$$=\overbrace{(x[0]+x[2])\cdot 1}^{Y_1}+\overbrace{(x[1]+x[3])\cdot 1}^{Y_2}$$

$$=Y_1+Y_2$$

$$X[(2k+1)=1]=\sum_{n=0}^{\frac{N}{2}-1}\left(x[n]-x\left[n+\frac{N}{2}\right]\right)\cdot e^{-\frac{j2\pi kn}{N/2}}\cdot e^{-\frac{j2\pi n}{N}}$$

$$=\underbrace{(x[0]-x[2])\cdot e^{-\frac{j2\pi\cdot 0\cdot 0}{4/2}}\cdot e^{-\frac{j2\pi\cdot 0}{4}}}_{n=0\ \&\ k=0}+\underbrace{(x[1]-x[3])\cdot e^{-\frac{j2\pi\cdot 0\cdot 1}{4/2}}\cdot e^{-\frac{j2\pi\cdot 1}{4}}}_{n=1\ \&\ k=0}$$

$$=(x[0]-x[2])\cdot(1)\cdot(1)+(x[1]-x[3])\cdot(1)\cdot(-j)$$

$$=(x[0]-x[2])\cdot 1-j(x[1]-x[3])$$

$$=\overbrace{(x[0]-x[2])}^{Y_3}-j\cdot\overbrace{(x[1]-x[3])}^{Y_4}$$

$$=Y_3-j\cdot Y_4$$

$@k=1$

$$X[(2k)=2]=\sum_{n=0}^{\frac{N}{2}-1}\left(x[n]+x\left[n+\frac{N}{2}\right]\right)\cdot e^{-\frac{j2\pi kn}{N/2}}$$

$$=\underbrace{(x[0]+x[2])\cdot e^{-\frac{j2\pi\cdot 1\cdot 0}{4/2}}}_{n=0\ \&\ k=1}+\underbrace{(x[1]+x[3])\cdot e^{-\frac{j2\pi\cdot 1\cdot 1}{4/2}}}_{n=1\ \&\ k=1}$$

$$=(x[0]+x[2])\cdot 1+(x[1]+x[3])\cdot(-1)$$

$$=\underbrace{(x[0]+x[2])}_{Y_1}-\underbrace{(x[1]+x[3])}_{Y_2}$$

$$=Y_1-Y_2$$

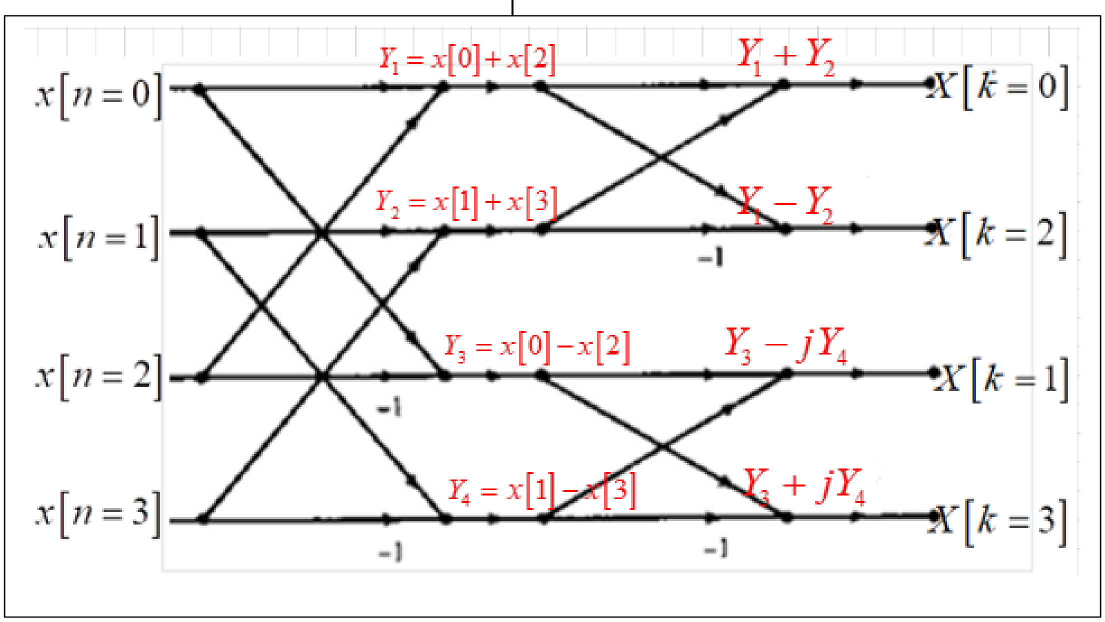
$$X[(2k+1)=3]=\sum_{n=0}^{\frac{N}{2}-1}\left(x[n]-x\left[n+\frac{N}{2}\right]\right)\cdot e^{-\frac{j2\pi kn}{N/2}}\cdot e^{-\frac{j2\pi n}{N}}$$

$$=\underbrace{(x[0]-x[2])\cdot e^{-\frac{j2\pi\cdot 1\cdot 0}{4/2}}}_{n=0\ \&\ k=1}+\underbrace{(x[1]-x[3])\cdot e^{-\frac{j2\pi\cdot 1\cdot 1}{4/2}}\cdot e^{-\frac{j2\pi 1}{4}}}_{n=1\ \&\ k=1}$$

$$=(x[0]-x[2])\cdot 1+(x[1]-x[3])\cdot(-1)\cdot(-j)$$

$$=\underbrace{(x[0]-x[2])}_{Y_1}+j\underbrace{(x[1]-x[3])}_{Y_2}$$

$$=Y_3+j\cdot Y_4$$



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```
clc; clear;
%This prog. is trying to show the Decimation In Frequency(DIF)
%It is working good
x = 1:4;
N = length(x);

for k = 0:N/2-1
    tmp_1 = 0;
    tmp_2 = 0;
    for n = 0:N/2-1
        tmp_1 = tmp_1 + (x(n+1) + x(n+N/2+1))*exp(-j*2*pi*k*n/(N/2));
        tmp_2 = tmp_2 + (x(n+1) - x(n+N/2+1))*exp(-j*2*pi*k*n/(N/2))*exp(-j*2*pi*n/N)
    end
    X(2*k+1) =   tmp_1;
    X(2*k+1+1) = tmp_2;
end

[X.' fft(x,4).']% This verifies the output is correct.
```

```
clc; clear;

x = 1:4;

Y1 = x(1) + x(3);
Y2 = x(2) + x(4);
Y3 = x(1) - x(3);
Y4 = x(2) - x(4);

for k=0:N-1
   if k==0
       X(k+1) = Y1 + Y2;
   elseif k==1
       X(k+1) = Y3 - j*Y4;
   elseif k==2
       X(k+1) = Y1 - Y2;
   else
      X(k+1) = Y3 + j*Y4;
   end

end

[X.' fft(x,4).']% This verifies the output is correct.
```

```
clc; clear;

x = 1:4;
N = length(x);

for k = 0:N-1
    tmp_1 = 0;
%     tmp_2 = 0;
    for n = 0:N/2-1
        tmp_1 = tmp_1 + (x(n+1) + (-1)^(k)*x(n+N/2+1))*exp(-j*2*pi*k*n/(N))
%         tmp_2 = tmp_2 + (x(n+1) - x(n+N/2+1))*exp(-j*2*pi*k*n/(N/2))*exp(-j*pi*n/N)
    end
    X(k+1) =   tmp_1;
%     X(2*k+1+1) = tmp_2;
end

[X.' fft(x,4).']
```