MT2002: Statistical Modeling Structure of a PyMC Model

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Parts of a PyMC Model

A PyMC model consists of three core components:

- Priors:
 - Capture prior knowledge or belief about model parameters.
 - Example: $\theta \sim \text{Beta}(1,1)$ (uniform prior, no preference for heads or tails).
 - Choosing Priors:
 - Use domain knowledge if available.
 - If uncertain, use non-informative or weakly informative priors.

2 Likelihood:

- Models how data is generated given the parameters.
- Example: $y \sim \text{Binomial}(n, \theta)$ for coin tosses.
- Choosing Likelihood:
 - Use the data-generating process (e.g., Binomial for binary data).

Inference Engine (Backend):

- Uses algorithms (e.g., MCMC) to sample from the posterior distribution.
- Handles the computation behind the scenes.
- PyMC automatically selects the method or allows you to customize it.

Example: Coin Toss Inference with PyMC

Task: Estimate the probability of heads (θ) from 7 heads out of 10 tosses.

Model: Notations book p-32

- **Prior:** $\theta \sim \text{Beta}(1,1)$ (uniform distribution).
- **Likelihood:** $y \sim \text{Binomial}(n = 10, p = \theta)$ (observed data: 7 heads).
- **Inference:** Sample from posterior $P(\theta \mid \text{data})$ using MCMC.

Python Code:

```
Observed data: 7 heads in 10 tosses data = [1, 1, 1, 1, 1, 1, 1, 0, 0, 0]
with pm.Model() as coin_toss_model: 1. Define Prior
theta = pm.Beta("theta", alpha=1, beta=1)
2. Define Likelihood
y = pm.Binomial("y", n=1, p=theta, observed=data)
3. Perform Inference
trace = pm.sample()
Visualize posterior distribution
pm.plot_posterior(trace, var_names=["theta"])
```

Summary of PyMC Model

What We Explored:

- Structure of a PyMC Model:
 - Priors: Initial beliefs about parameters.
 - Likelihood: Relates observed data to parameters.
 - Inference: Sampling to compute the posterior distribution.
- What the Model Does:
 - Combines priors and data via Bayes' theorem.
 - Provides probabilistic estimates of unknown parameters.
- What the Model Takes as Input:
 - Observed data (e.g., coin toss results).
 - Prior distributions for unknown parameters.
- What the Model Produces as Output:
 - Posterior distribution of parameters.
 - Visualizations (e.g., posterior plots).
- How It Produces the Output:
 - Uses an inference engine (e.g., MCMC sampling) to sample the posterior.
 - Requires sufficient data to infer parameter values effectively.

Let's create a basic pymc model!

Ready to explore PyMC in-depth?