

$$-2^{n-1} \rightarrow +2^{n-1} - 1$$

DATE ___ / ___ / 20___

Conventional Method \rightarrow 1's Complement method

$$2^3 = 8$$

1's Complement

$$000 + 0$$

$$001 + 1$$

$$010 + 2$$

$$011 + 3$$

$$\boxed{100 - 0}$$

$$101 - 1$$

$$110 - 2$$

$$\boxed{111 - 3}$$

$$000 + 0$$

$$001 + 1$$

$$010 + 2$$

$$011 + 3$$

$$100 - 3$$

$$101 - 2$$

$$110 - 1$$

$$\boxed{111 - 0}$$

bitwise Operation

$$\downarrow \downarrow \downarrow$$

$$111$$

$$001 \rightarrow 110$$

$$-17$$

$$00010001$$

$$\textcircled{1} 10010001$$

$$\textcircled{2} 11101110$$

$$\textcircled{3} 11101111$$

2's Complement

$$\downarrow \downarrow \downarrow$$

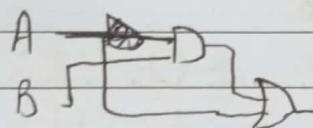
$$111$$

Simplification

Using Boolean Algebra:

Rule / Properties / Law.

$$A + AB$$



$$A + AB$$

$$A \cdot 1 + A \cdot B \quad \because A \cdot 1 = A$$

$$A(1+B) \quad \because DL$$

$$A(B+1) \quad \because CL$$

$$A(1) \quad \because A+1=1$$

$$\boxed{A}$$

$$\bar{A}B(\bar{A}+B)$$

$$\bar{A}B \cdot \bar{A} + \bar{A}B \cdot B \quad DL$$

$$(\bar{A}+\bar{B}) \cdot \bar{A} + (\bar{A}+\bar{B}) \cdot B \quad \text{De Morgan}$$

$$\bar{A}\bar{A} + \bar{A}\bar{B} + \bar{A}B + B\bar{B} \quad DL$$

$$\bar{A} + \bar{A}\bar{B} + \bar{A}B + 0 \quad \bar{A} \cdot \bar{A} = \bar{A} \text{ & } \bar{A} \cdot A = 0$$

$$A(1+\bar{B}+B)$$

$$\boxed{A(1)}$$

$$A + \bar{A}B$$

$$A + (\bar{A}+\bar{B}) \quad \because A+\bar{A}=1$$

$$1 + \bar{B}$$

$$\boxed{1}$$

$$\boxed{1}$$

$$\textcircled{1}) A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C$$

$$C(\bar{A}B + \bar{A}B + \bar{A}\bar{B})$$

$$C[(\bar{B}(A+\bar{A}) + \bar{A}B)]$$

$$C(\bar{B} + \bar{A}B)$$

$$C\bar{B} + \bar{A}BC$$

$$\textcircled{1}) (A+B)(AD+A\bar{D}) + AB + B$$

$$AA\bar{D} + AA\bar{A}\bar{D} + ABD + AB\bar{D} + AB + B$$

$$AA(D+\bar{D}) + AB(D+\bar{D}) + AB + B$$

$$AA + AB + AB + B$$

$$AA + AB + B$$

$$A + AB + B$$

$$A(1+B) + B$$

$$\boxed{A+B}$$

$$\bar{B}C(A+\bar{A}) + \bar{A}BC$$

$$\bar{B}C + \bar{A}BC$$

$$AB(\bar{C}D + \bar{E}F)$$

$$\bar{A} + \bar{B} + (\bar{C}D + \bar{E}F)$$

$$\bar{A} + \bar{B} + \bar{\bar{C}}D \cdot \bar{\bar{E}}F$$

$$\bar{A} + \bar{B} + (\bar{\bar{C}} + \bar{D} \cdot \bar{\bar{E}} + \bar{F})$$

$$\bar{A} + \bar{B} + [(C+\bar{D}) \cdot (E+\bar{F})]$$

$$\bar{A}B \cdot (\bar{C}D + \bar{E}F) \cdot (\bar{A}B + \bar{C}D)$$

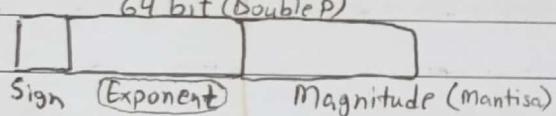
$$\bar{A}B + \bar{C}D + \bar{E}F + ABCD$$

$$AB + \bar{C} + \bar{D} + E + \bar{F} + ABCD$$

Number Standard.

32 bit (Single P)

64 bit (Double P)



① +ve FPN	$\leftarrow + \rightarrow$	
② -ve FPN	$1.34 \times 10^4 \leftarrow +13.4$	-13.4
③ +ve Power FPN	$1.34 \times 10^3 \leftarrow +0.00134$	-0.00134
④ -ve Power FPN	$1.34 \times 10^0 \leftarrow +1.34$	-1.34

2-9

① Normalization ($a \cdot b \dots * 10^x$)

non-binary #

①) 19 \leftarrow

- ① Convert binary
- ② normalization

100011. \leftarrow

1	8	23	
0	00000100	001000...	
1.0011 $\times 2^{+4}$	Sign	Exponent	Mantissa

Formula for unsigned No. $[2^n - 1]$ $n = \text{no of bits in register}$ Range: $0 - 2^n - 1$ Signed No. $(2^{n-1} - 1)$. Sign flag is high then the number is +ve.+0 to +3 $\underline{\underline{1111,1111}} = -1$

-0 to -3

OR & AND are commutative, associative and distributive

NAND is not associative

Ex-OR is commutative, associative but not distributive.

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$$\textcircled{1} (P' + Q + R)(P' + Q' + R)(Q' + R')$$

$$(P'P' + P'Q' + P'R + P'Q + QQ' + QR + PR + RQ' + RR)(Q' + R')$$

$$(P' + P'(Q + Q) + P'R + Q + R(Q + Q') + R)(Q' + R')$$

$$P' + P'R + R + (Q' + R')$$

$$P'(1 + R) + R + (Q' + R')$$

$$[P' + R](Q' + R')$$

$$P'Q' + P'R' + RQ' + RR'$$

$$[P'Q' + P'R' + RQ']$$

$$\textcircled{1} AB + AB'C + AB'C'$$

$$AB + AB'C$$

$$A(B + B'C)$$

$$A(B + C)$$

We need to solve this way.

$$A(B + B'C + B'C)$$

$$A(B + C + B'C)$$

$$A(B + C + C)$$

$$[A(B + C)]$$

$$\textcircled{2} a + a'b + a'b'c + a'b'c'd + a'b'c'd'e$$

$$a + a'(b + b'c + b'c'd + b'c'd'e) \quad \therefore a + a'b = a + b$$

$$a + b + b'c + b'c'd + b'c'd'e$$

$$a + b + c + b'c'd + b'c'd'e$$

$$a + b + c + b'c'(d + d'e)$$

$$a + b + c + b'c'(d + e)$$

$$\textcircled{2} x'y + yz' + yz + xy'z$$

$$x'y + y(z' + z) + xy'z$$

$$y + x'y + xy'z$$

$$y(1 + x') + xy'z$$

$$y + xy'z$$

$$\textcircled{3} w(w' + x' + z')(w' + xz)$$

$$0 + wx' + wz'(w' + xz)$$

$$w(x' + z')(w' + xz)$$

$$(x' + z')(wxz)$$

$$wxz' + wxzz'$$

$$\boxed{0}$$

$$\textcircled{4} (P + \bar{Q} + \bar{R})(P + \bar{Q} + R)(P + Q + \bar{R})$$

$$(PP + P\bar{Q} + P\bar{R} + \bar{Q}\bar{R} + \bar{Q}\bar{Q} + \bar{Q}R + \bar{R}\bar{P} + \bar{R}\bar{Q} + \bar{R}R)(P + Q + \bar{R})$$

$$(P + P(1) + P(1) + \bar{Q} + \bar{Q}(1) + 0)(P + Q + \bar{R})$$

$$(P + \bar{Q})(P + Q + \bar{R})$$

$$PP + P\bar{Q} + P\bar{R} + \bar{Q}\bar{P} + \bar{Q}\bar{Q} + \bar{Q}\bar{R}$$

$$P + P(1) + P\bar{R} + \bar{Q}\bar{R}$$

$$P(1 + \bar{R}) + \bar{Q}\bar{R}$$

$$\boxed{P + \bar{Q}\bar{R}}$$



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Two's Complement

Sign Magnitude

$$\underline{0 \ 1 \ 0 \ 0} = 4$$

0 = +ve

1 = -ve

1's Complement

$$0 \ 1 \ 1 \ 0 \ 1$$

Reverse

$$1 \ 0 \ 0 \ 1 \ 0$$

2's Complement

$$1 \ 0 \ 0 \ 1 \ 0$$

$$+ \underline{\quad \quad \quad 1}$$

$$1 \ 0 \ 0 \ 1 \ 1$$

Binary Addition/Subtraction Using 1's Complement:

$$3 = + 0 \ 0 \ 1 \ 1$$

$$2 = \underline{- 0 \ 0 \ 1 \ 0}$$

$$2 - 3 = - 1$$

$$3 - 2$$

1st Complement

$$1 \ 1 \ 0 \ 1$$

$$0 \ 0 \ 1 \ 1$$

$$1 \ 1 \ 0 \ 1$$

$$\underline{0 \ 0 \ 0 \ 0 \ 0}$$

$$\rightarrow 1$$

$$\underline{0 \ 0 \ 0 \ 1}$$

$$- 3 - 4$$

$$1 \ 1 \ 0 \ 0$$

$$1 \ 0 \ 1 \ 1$$

$$1 \ 0 \ 1 \ 1 \ 1$$

$$0 \ 1 \ 1 \ 1$$

$$\underline{1 \ 0 \ 0 \ 0}$$

$$\boxed{-0 \ 1 \ 1 \ 1}$$

$$0 \ 0 \ 1 \ 0$$

$$+ 1 \ 1 \ 0 \ 0 \rightarrow 1\text{'s Complm}$$

$$1 \ 1 \ 1 \ 0$$

1's Complement

$$- 0 \ 0 \ 0 \ 1$$



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Addition/SubtractionUsing 2's Complement:

$$3 - 2 = 1$$

$$\begin{array}{r} +0011 \\ -0010 \end{array} \xrightarrow{1's\ Comp} \begin{array}{r} 1101 \\ +1 \\ \hline 1110 \end{array}$$

over
flow
 \downarrow
we ignore
this bit

$$2 - 3 = -1$$

$$\begin{array}{r} 0010 \\ -0011 \end{array} \rightarrow \begin{array}{r} 1100 \\ +1 \\ \hline 1101 \end{array}$$

$$\begin{array}{r} 0010 \\ +1101 \\ \hline (-1)1111 \end{array} \rightarrow \begin{array}{r} 0000 \\ \boxed{0001} +1 \\ -0001 (-1) \end{array}$$

$$-3 - 2 = -5$$

$$\begin{array}{r} 1101 \\ 1110 \\ \hline 11011 \\ 0100 \\ \hline 1 \\ 0101 \end{array}$$

Binary Representation of Fraction values.

$$(93.75)_{10} = (?)_2$$

$$\begin{array}{l} 93 = 1011101 \\ .75 = .11 \end{array}$$

$$(1011101.11)_2$$

64 32 16 8 4 2 1 $\frac{1}{2} \frac{1}{4}$

$$64 + 16 + 8 + 4 + 1 = 93$$

$$0.75 = \frac{1}{2} + \frac{1}{4}$$

$$(93.75)_{10}$$

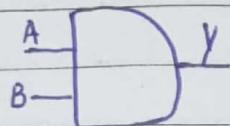
$$(93.75)_{10} = (1011101.11)_2$$

$$\begin{array}{l} 0.75 \times 2 = 1 \\ 0.50 \times 2 = 1 \end{array} \downarrow$$

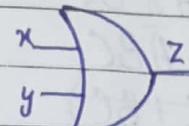


Logic Gates & Boolean Algebra:

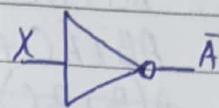
Three basic gates AND (\cdot) OR (+) NOT ($\bar{\cdot}$), ('), (\sim)
 $Y = A \cdot B$ $Z = x + y$ $X = \bar{A}$



Switches in Series

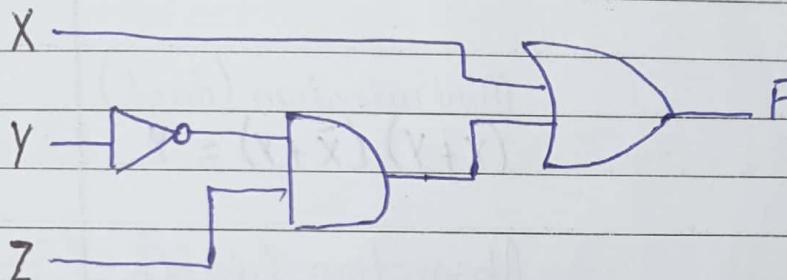


Switches in Parallel



Normally closed switch

$$F = X + \bar{Y}Z$$



Order of evaluation

- 1) Parentheses
- 2) NOT
- 3) AND
- 4) OR

Double Negation

$$\text{NOT } \bar{A} = A$$

Laws of Boolean Algebra:

$$A + 1 = 1$$

$$A \cdot 1 = A \quad \text{identity}$$

$$\bar{\bar{X}} = X \quad \text{involution}$$

$$A + 0 = A$$

$$A \cdot 0 = 0 \quad \text{Annulment}$$

$$A + A = A$$

$$A \cdot A = A \quad \text{Idempotent}$$

$$AB + \bar{A}C + BC = \bar{A}B + \bar{A}C \quad (\text{Consensus Theorem})$$

$$A + \bar{A} = 1$$

$$A \cdot \bar{A} = 0 \quad \text{Complement}$$

$$A + B = B + A$$

$$A \cdot B = B \cdot A \quad \text{Commutative}$$

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

$$\overline{A \cdot B} = \bar{A} + \bar{B} \quad \text{de Morgan's Theorem}$$

$$(A+B)+C = A+(B+C) \quad (AB)C = A.(B.C) \quad \text{Associative}$$

$$A(B+C) = AB + AC \quad A+BC = (A+B)(A+C) \quad \text{Distributive}$$

$$A \cdot A + A \cdot C + A \cdot B + B \cdot C$$

$$A + A \cdot C + A \cdot B + B \cdot C$$

$$A(1+C) + AB + BC \quad \therefore 1+C=1$$

$$A \cdot 1 + AB + BC$$

$$A(1+B) + BC \quad \therefore 1+B=1$$

$$A \cdot 1 + BC$$

$$\boxed{A + BC}$$

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Consensus Theorem: $AB + \bar{A}C + BC = AB + \bar{A}C$

$$AB + \bar{A}C + 1 \cdot BC$$

$$AB + \bar{A}C + (A + \bar{A}) \cdot BC$$

$$AB + \bar{A}C + ABC + \bar{A}BC$$

$$A(B + BC) + \bar{A}(C + BC)$$

$$AB(1 + C) + \bar{A}C(1 + B)$$

$$AB \cdot 1 + \bar{A}C \cdot 1$$

$$\boxed{AB + \bar{A}C}$$

Useful Theorems:

Minimization

$$XY + \bar{X}Y = Y$$

Minimization (dual)

$$(X+Y)(\bar{X}+Y) = Y$$

Absorption

$$X + XY = X$$

Absorption (dual)

$$X \cdot (X+Y) = X$$

Simplification

$$X + \bar{X}Y = X + Y$$

Simplification (dual)

$$X \cdot (\bar{X} + Y) = X \cdot Y$$

DeMorgan's

$$\overline{X+Y} = \bar{X} \cdot \bar{Y}$$

DeMorgan's (dual)

$$\overline{X \cdot Y} = \bar{X} + \bar{Y}$$



Practice Questions:

(i) $(x+y)(x+\bar{y})(\bar{x}+z)$

(ii) $XYZ + X\bar{Y}Z + XY\bar{Z}$

$(xx + x\bar{y} + x\bar{y} + y\bar{y})(\bar{x}+z)$

$XY(Z + \bar{Z}) + X\bar{Y}Z$

$(x+x(\bar{y}+y)+0)(\bar{x}+z)$

$XY \cdot 1 + X\bar{Y}Z$

$(x+x \cdot 1)(\bar{x}+z)$

$XY + X\bar{Y}Z$

$x(\bar{x}+z)$

X(X+Y) Distributive Law

$\bar{x}\bar{x} + XZ$

$0 + XZ$

\boxed{XZ}

$X(Y+\bar{Y})(Y+Z)$

$X \cdot 1 \cdot (Y+Z)$

$\boxed{X(Y+Z)}$

Example 6

a) $X = ABC + \bar{A}B + AB\bar{C}$

b) $X = \bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$X = AB(C + \bar{C}) + \bar{A}B$

$X = \bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$= AB \cdot 1 + \bar{A}B$

$= \bar{A}\bar{C}(B + \bar{B}) + A\bar{B}\bar{C}$

$= B(A + \bar{A})$

$= \bar{A}\bar{C} \cdot 1 + A\bar{B}\bar{C}$

$= B \cdot 1$

$= \bar{C}(\bar{A} + A\bar{B})$

$\boxed{X = B}$

$= \bar{C} \cdot 1 \cdot (\bar{A} + \bar{B})$

c) $AB + \bar{A}C + BC = AB + \bar{A}C$

$\boxed{X = (\bar{A} + \bar{B})\bar{C}}$

L.H.S

$AB + \bar{A}C + (A + \bar{A})BC$

L.H.S = $(A\bar{A} + AC + \bar{A}B + BC)(B + C)$

$AB + \bar{A}C + ABC + \bar{A}BC$

$ABC + \bar{A}BB + BBC + ACC + \bar{A}BC + BCC$

$AB(1+C) + \bar{A}C(1+B)$

$ABC + \bar{A}B + BC + AC + \bar{A}BC + BC$

$AB \cdot 1 + \bar{A}C \cdot 1$

$BC(A + \bar{A}) + \bar{A}B + AC + BC$

$\boxed{AB + \bar{A}C}$

$\bar{A}B + AC + BC$

Proved!

$A\bar{A} + \bar{A}B + C(A + B)$

$\bar{A}(A + B) + C(A + B)$

$\boxed{(\bar{A} + C)(A + B)} \text{ Proved!}$



GO GETTERS

Binary Multiplication: 01010011 (multiplicand) 11000101 (multiplier)

Step 1: The sign bit of the multiplicand is 0 and the sign bit for the multiplier is 1 - The sign bit for the product will be 1 (negative)

Step 2: Take 2's Complement of multiplier to put it in true form.

$$11000101 \rightarrow 00111011$$

Note: Only the magnitude bits are used in these steps.

$$\begin{array}{r}
 1010011 \text{ Multiplicand} \\
 \times 0111011 \text{ Multiplier} \\
 \hline
 1010011 \text{ 1'st Partial Product} \\
 + 1010011 \text{ 2'nd Partial Product} \\
 \hline
 11111001 \text{ Sum of 1'st \& 2'nd} \\
 0000000 \text{ 3'rd Partial Product} \\
 \hline
 111111001 \text{ Sum} \\
 1010011 \text{ 4'th Partial P} \\
 \hline
 111110010001 \text{ Sum} \\
 1010011 \text{ 5'th Partial Product} \\
 \hline
 1001100100001 \text{ Sum} \\
 0000000 \text{ 7'th} \\
 \hline
 1001100100001 \\
 011001101110 \text{ 1's Complement} \\
 \hline
 \end{array}$$

Sign Bit 1 011001101111 Answer.

Division:

Step 1: Subtract the divisor from the dividend using 2's complement addition to get the first partial remainder and add 1 to the quotient - If this partial remainder is positive, go to Step 2 - If the partial remainder is zero or negative, the division is complete.

Step 2: Subtract the divisor from the partial remainder and add 1 to the quotient - If the result is positive, repeat for the next partial remainder - If the result is zero or negative, the division is complete -

Divide 01100100 by 00011001

Step 1: Subtract the divisor from the dividend using 2's complement addition
(remember the final carries are discarded)

$$\begin{array}{r}
 01100100 \quad \text{Dividend} \\
 + \underline{11100111} \quad \text{2's complement of divisor} \\
 \hline
 01001011 \quad \text{Positive 1st partial remainder}
 \end{array}$$

Add 1 to quotient $00000000 + 00000001 = 00000001$

Step 2: Subtract the divisor from the 1st partial remainder using 2's complement

$$\begin{array}{r}
 01001011 \quad 1^{\text{st}} \text{ Partial remainder} \\
 - \underline{11100111} \quad \text{2's complement of divisor} \\
 \hline
 00110010 \quad \text{Positive 2nd partial remainder}
 \end{array}$$

Add 1 to quotient $00000001 + 00000001 = 00000010$

Step 3 00110010 2nd Partial

$$\begin{array}{r}
 \underline{11100111} \quad \text{2's comp divisor} \\
 - 00011001 \quad 3^{\text{rd}} \text{ Partia Rem}
 \end{array}$$

Add 1) 00000011

Step 4 00011001 3rd Partial

$$\begin{array}{r}
 \underline{11100111} \quad \text{2' Compl divisor} \\
 - 00000000 \quad \text{Zero remainder}
 \end{array}$$

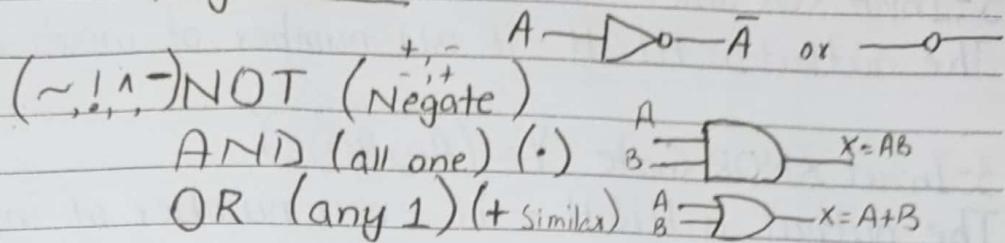
Add 1 to quotient $00000011 + 00000001 = \boxed{00000100}$
quotient.



GO GETTERS

3 Design:

- 1) Truth Table
- 2) Logic Diagram
- 3) Expression

Logic Gates

$$X = A \cdot B$$

number of input = number of column
for rows 2

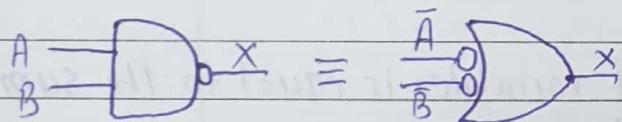
$$\sim(AB) \neq \sim AB$$

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$$\text{NAND (Not of AND)} \quad X = \overline{AB} \quad A \overline{\cdot} B \rightarrow \overline{AB}$$

$$\text{NOR (Not of OR)} \quad X = \overline{A+B} \quad A \overline{+} B \rightarrow \overline{A+B}$$

NAND = Negative-OR



NOR = Negative-AND

$$\overline{AB} = \overline{A} + \overline{B}$$

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

Exclusive-OR (XOR) $A \oplus B$

Same = 0

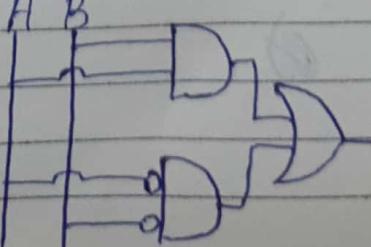
Different = 1

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive-NOR (XNOR)

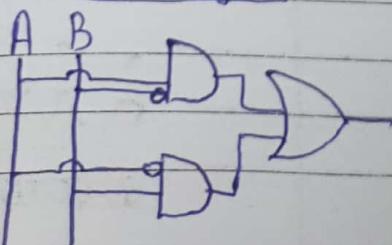
Same = 1 Different = 0

Equivilant



GO GETTERS

Equivilant Diagram:



3-Input XOR Gate: $Y = A \oplus B \oplus C$

The output is HIGH - if odd number of inputs are 1

3-Input XNOR Gate $Y = (A \oplus B \oplus C)'$

The output is HIGH - if even number of inputs are 1

De Rule: 11

$$A + \bar{A}B = A + B$$

$$(A + AB) + \bar{A}B$$

$$(AA + AB) + \bar{A}B$$

$$A(A + AB) + \bar{A}B + A\bar{A}$$
 adding $A\bar{A} = 0$

$$A(A + AB) + \bar{A}(A + B)$$

$$(A + B)(A + \bar{A})$$

$$(A + B) \cdot 1$$

$$\boxed{A + B}$$

Rule: 12

$$(A + B)(A + C) = A + BC$$

$$AA + AC + AB + BC$$

$$A + AC + AB + BC$$

$$A(1+C) + AB + BC$$

$$A(1+B) + BC$$

$$\boxed{A + BC}$$

De Morgan's First theorem

The complement of a product of variables is equal to the sum of the complements of the variables.

Stated another way:

The complement of two or more ANDed variable is equivalent to the OR of the complement of the individual variables.

$$\textcircled{1} \quad \overline{XY} = \bar{X} + \bar{Y}$$

$$\overline{xy} = \overline{x} \odot \overline{y} = \bar{x} + \bar{y}$$

$$\textcircled{2} \quad \overline{X + Y} = \bar{X} \cdot \bar{Y}$$

$$\overline{\overline{A + BC} + D(\bar{E} + \bar{F})}$$

$$\overline{x} \odot \overline{y} = \overline{x} \cdot \overline{y} = \bar{x} + \bar{y}$$

$$\textcircled{3} \quad \overline{(A + B + C)D}$$

$$\overline{(A + B + C) + \bar{D}}$$

$$(A \cdot \bar{B} \cdot \bar{C}) + \bar{D}$$

$$\overline{\overline{A + BC} \cdot \overline{D(E + F)}} \\ A + BC \cdot [\bar{D} + (\bar{E} + \bar{F})]$$



1110
0001
0010

-8+4+2

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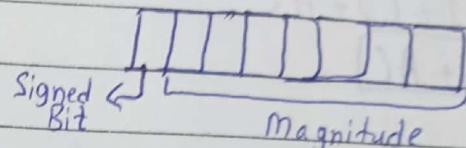
The Largest decimal number = $2^n - 1$

Example five bits ($n=5$) ($n=6$)

$$2^5 - 1 = 31$$

$$2^6 - 1 = 63$$

(Q) Decimal value of 1's Complement



a) 00010111

b) 11101000

$$16+4+2+1
(+23)$$

$$\begin{array}{r} -128 \ 64 \ 32 \ 16 \ 8 \ 4 \ 2 \ 1 \\ -128 + 64 + 32 + 8 \\ -24 \end{array}$$

00011001 (+25)

10011001 (-25)

Add 1

$$-24 + 1 = \boxed{-23}$$

(Q) 2's Complement

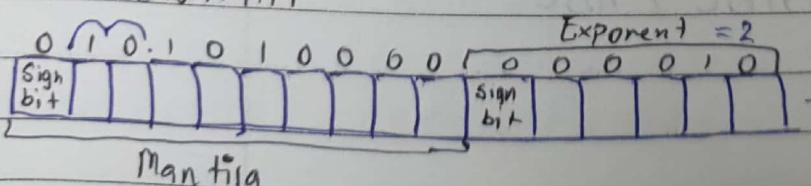
a) 01010110

$$64+16+4+2
(+86)$$

b) 10101010

$$\begin{array}{r} -128 \ 64 \ 32 \ 16 \ 8 \ 4 \ 2 \ 1 \\ -128 + 32 + 8 + 2 \\ \boxed{-86} \end{array}$$

Normalization: FPN



$$10.101 \cdot 2^2$$

2.625

! 01000000000111110 Exponent = -2
0.001
(0.125)_0



$$\begin{aligned} Q) AB + A(B+C) + B(B+C) \\ AB + AB + AC + BB + BC \\ AB + B + AC + BC \\ B(1+A) + AC + BC \\ B(1+C) + AC \\ \boxed{B+AC} \end{aligned}$$

$$\begin{aligned} Q) [A\bar{B}(C+BD) + \bar{A}\bar{B}]C \\ A\bar{B}CC + A\bar{B}BDC + \bar{A}\bar{B}C \\ A\bar{B}C + 0 + \bar{A}\bar{B}C \\ \bar{B}C(A+\bar{A}) \\ \boxed{\bar{B}C} \end{aligned}$$

Standard Forms of Boolean Expressions:

- 2 Standard forms: ① Sum of Product (SOP)
 ② Product of Sum (POS).

SOP: When two or more product terms are summed by Boolean addition, the resulting expression is a sum-of-product (SOP).
 Example: $AB + ABC$, $ABC + CDE + \bar{B}CD$, $\bar{A}B + \bar{A}BC + A\bar{C}$.

Standard SOP Form: is one which all variables in the domain appear in each product term in the expression.

Example: $A\bar{B}CD + \bar{A}BC\bar{D} + ABC\bar{D}$

SOP expression: $\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$

A	B	C	Output
0	0	0	0
✓0	0	1	1
0	1	0	0
0	1	1	0
✓1	0	0	1
1	0	1	0
1	1	0	0
✓1	1	1	1

SOP: 1

POS: 0

00 01 11 10
01 10 11 12
11 10 12 13
10 12 13 14

5

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The Karnaugh Map:

A Karnaugh map provides systematic method for simplifying Boolean expressions and, if properly used, will produce the simplest SOP or POS expression possible, known as the minimum expression

3-Variable		C	AB	0	1
000	000	0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	
001	010	1	$\bar{A}B\bar{C}$	$\bar{A}BC$	
010	011	2	$A\bar{B}\bar{C}$	$A\bar{B}C$	
011	110	3	$AB\bar{C}$	ABC	
110	100	6	$AB\bar{C}$	ABC	
111	101	7	$A\bar{B}\bar{C}$	$A\bar{B}C$	
100	10	4	$A\bar{B}\bar{C}$	$A\bar{B}C$	
101	11	5	$AB\bar{C}$	ABC	
110	10	8	$AB\bar{C}$	ABC	
111	11	9	$A\bar{B}\bar{C}$	$A\bar{B}C$	
000, 010, 110					

is adjacent to

000, 010, 110

4-Variable		C	D	AB	00	01	11	10
0000	0000	0	0					
0001	0100	1	0					
0010	0110	2	0					
0011	1100	3	0					
0100	0101	4	1					
0101	1101	5	1					
0110	1110	6	1					
0111	1111	7	1					
1100	1101	8	1					
1101	1110	9	1					
1110	1111	10	1					
1111	1111	11	1					
1111	1111	12	1					
1111	1111	13	1					
1111	1111	14	1					
1111	1111	15	1					
1111	1111	16	1					

Mapping SOP expression on K-map : $\bar{A} + A\bar{B} + AB\bar{C}$

$$\bar{A}: \bar{A}(\bar{B}+B) = \bar{A}\bar{B} + \bar{A}B \Rightarrow (\bar{A}\bar{B} + \bar{A}B)(\bar{C}+C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC$$

$$A\bar{B}: A\bar{B}(\bar{C}+C) = A\bar{B}\bar{C} + A\bar{B}C$$

The process that results in an expression containing the fewest possible terms with the fewest possible variables is called minimization.

1	1	1	1
1	1	1	1

- Variables that occurs both uncomplemented & complemented within the group are eliminated - These are called contradictory variables.
- Determine the product terms for the k-map in the figure and write resulting minimum SOP expression.

Product term: $\bar{A}\bar{B}\bar{C} + \bar{A}BC + ABC\bar{C} + ABC$

For minimization:

$$AB\bar{C} + AB\bar{C} = AB$$

$$\bar{A}BC + ABC = BC$$

$$\bar{A}\bar{B}C$$

$$\text{Minimum SOP} = AB + BC + \bar{A}\bar{B}\bar{C}$$

1	
	1
1	1
1	1

$$\bar{C}\bar{D} + \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{C}$$

$$(C+D) \cdot (\bar{A}+\bar{B}\bar{D}) \cdot (\bar{A}+\bar{B}+C)$$

$$A+B\bar{C}$$

$$A \cdot (\bar{B}+C)$$

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POS : 3-Variable

C	\bar{C}
A+B+C	A+B+ \bar{C}
A+ \bar{B} +C	A+ \bar{B} + \bar{C}
$\bar{A}+\bar{B}+C$	$\bar{A}+\bar{B}+\bar{C}$
$\bar{A}+B+C$	$\bar{A}+B+\bar{C}$

4-Variable

$C+D$	$C+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
A+B			
A+ \bar{B}			
$\bar{A}+\bar{B}$			
$\bar{A}+B$			

$$\textcircled{Q} (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$$

$$(A+B+\ell)(A+B+\bar{\ell})(A+\bar{B}+\ell)(A+\bar{B}+\bar{\ell})$$

$$(A+B)(A+\bar{B}) = A$$

$$(A+\bar{B}+C)(\bar{A}+\bar{B}+C) = \bar{B}+C$$

0	0
0	0
0	

Conversion (POS to SOP) : For POS expression all the cells that do not contain 0's, contain 1's for which the SOP expression is derived. Likewise for an SOP expression all the cells that do not contain 1's contain 0's from which the POS expression is derived.

$$\text{POS to SPOS: } F(A,B,C) = (A+B) \cdot (A+C) \cdot (A+\bar{C})$$

$$(A+B+C\bar{C}) \cdot (A+C+B\bar{B}) \cdot (B+\bar{C}+A\bar{A})$$

$$(A+B+C) \cdot (A+B+\bar{C}) \cdot (A+B+C) \cdot (A+\bar{B}+C) \cdot (B+\bar{C}+A) \cdot (B+\bar{C}+\bar{A})$$

$$(A+B+C) \cdot (A+B+\bar{C}) \cdot (A+\bar{B}+C) \cdot (\bar{A}+B+C).$$



Don't Care Condition in K'Map:

Sometimes a situation arises in which some input variables combinations are not allowed. For example in BCD code.

Decimal	Binary	SOP Term	7-segment display
0	0000	$\bar{A}\bar{B}\bar{C}\bar{D}$	a, f, e, d, c, b
1	0001	$\bar{A}\bar{B}\bar{C}D$	b, c
2	0010	$\bar{A}\bar{B}C\bar{D}$	a, b, g, e, d
3	00100	$\bar{A}\bar{B}C\bar{D}$	a, b, g, c, d
4	0100	$\bar{A}B\bar{C}\bar{D}$	f, g, b, c
5	0101	$\bar{A}B\bar{C}D$	a, f, g, c, d
6	0110	$\bar{A}BC\bar{D}$	a, f, e, g, c, d
7	0111	$\bar{A}BCD$	a, b, c
8	1000	$A\bar{B}\bar{C}\bar{D}$	a, b, f, e, g, c, d
9	1001	$A\bar{B}\bar{C}D$	a, f, b, g, c
10	1010	$A\bar{B}\bar{C}D$ X	
11	1011	X	
12	1100	X	
13	1101	X	
14	1110	X	
15	1111	X	

POS (Product of Sum)

$$(+) \cdot (+) \cdot (+)$$

0 - Var

1 - Var'

Σ = Sum.

		POS	SOP		
0 → Var'	1 → Var	Maxterm	A	B	minterm,
		$A + B$	0	0	$\bar{A} \cdot \bar{B}$
		$A + \bar{B}$	0	1	$\bar{A} \cdot B$
		$\bar{A} + B$	1	0	$A \cdot \bar{B}$
		$\bar{A} + \bar{B}$	1	1	$A \cdot B$

 $\Pi(M_0, M_1, M_2)$ Product of Maxterm
M

$$\begin{array}{c} A \\ \backslash \\ B \end{array} \quad \begin{array}{c} A+B \\ \bar{A}\bar{B} \\ 00 \end{array} \quad \begin{array}{c} \bar{A}B/A\bar{B} \\ 01 \end{array}$$

0	1
2	3

For k-map POS
we consider 0

$$\begin{array}{c} A \\ \backslash \\ B \end{array} \quad \begin{array}{c} AB \\ \bar{A}\bar{B} \\ 10 \end{array} \quad \begin{array}{c} AB/\bar{A}+\bar{B} \\ 11 \end{array}$$

For SOP we
consider 1

$$\begin{array}{c} B \\ \backslash \\ \bar{B} \end{array} \quad \text{AND POS}$$

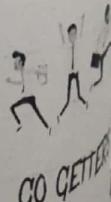
A	0	0
\bar{A}	0	1

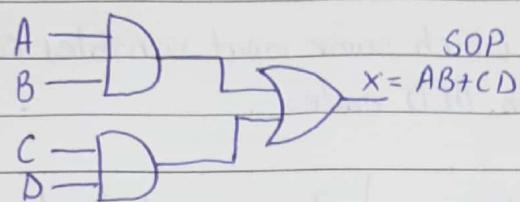
SOP: $F(A, B) = m_3$

POS: $F(A, B) = \Pi(M_0, M_1, M_2)$

F(A, B, C) = (A + B + C) \cdot (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B + \bar{C}) \cdot (\bar{A} + \bar{B} + C)

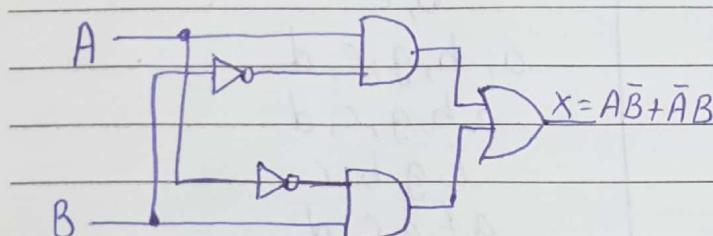
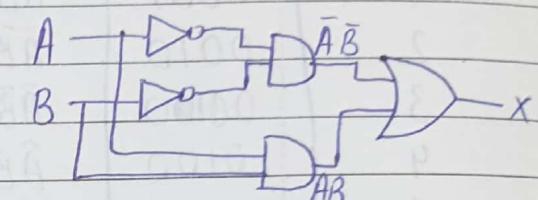
		BC		A					
		00	01	11	10	00	01	11	10
		0	1	3	2	0	1	0	1
0	4	5	7	6		1	1	0	1
1						1	0	1	0



Basic Combinational Logic Circuits:1- AND - OR2. AND-OR-Invert

$$\overline{AB} + \overline{CD}$$

POS: $(\bar{A} + \bar{B})(\bar{C} + \bar{D})$

3. Exclusive-OR4) Exclusive-NOR

$$x = A\bar{B} + \bar{A}B = (\bar{A}\bar{B})(\bar{A}B)$$

$$= (\bar{A} + B)(A + \bar{B}) = \bar{A}\bar{B} + AB$$

Q) Reduce the combinational logic circuit

$$x = (\bar{A}\bar{B}\bar{C})C + \bar{A}\bar{B}\bar{C} + D$$

$$= (\bar{A} + \bar{B} + \bar{C})C + \bar{A} + \bar{B} + \bar{C} + D$$

$$= AC + BC + C + A + B + C + D$$

$$= C(A + B + 1) + A + B + D$$

$$x = A + B + C + D$$

Universal Property of NAND Gate:

$$\textcircled{1} \quad A \rightarrow \text{NAND} \rightarrow \bar{A} = A \rightarrow \text{NOT}$$

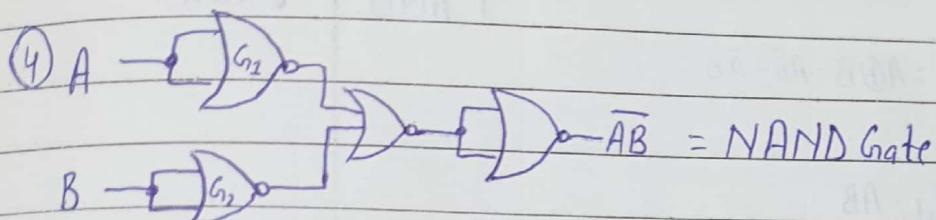
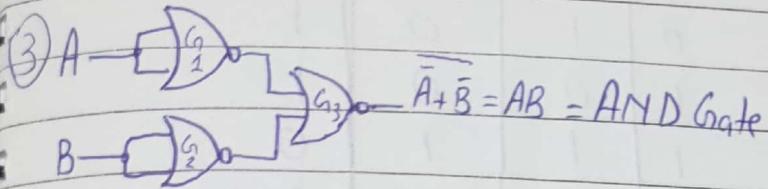
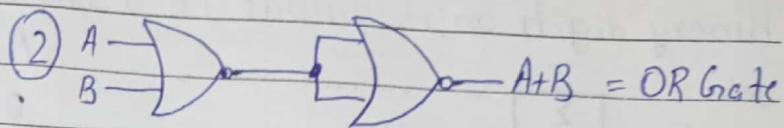
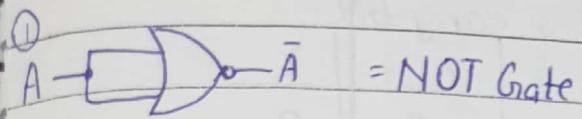
$$\textcircled{2} \quad \begin{array}{c} A \\ \text{NAND} \\ B \end{array} = AB = \text{AND Gate}$$

$$\textcircled{3} \quad \begin{array}{c} A \\ \text{NAND} \\ B \end{array} = \overline{AB} = A + B = \text{OR Gate}$$

$$\textcircled{4} \quad \begin{array}{c} A \\ \text{NAND} \\ B \end{array} \rightarrow \text{NAND} \rightarrow \overline{AB} \rightarrow \overline{\overline{AB}} = A + B = \text{NOR Gate}$$



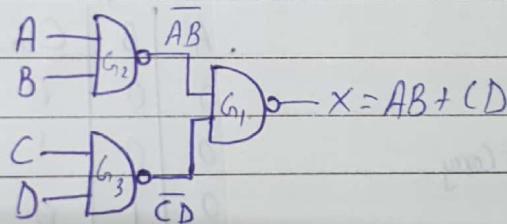
Universal Property of NOR Gate



NAND Logic (De Morgan)

$$\overline{AB} = \bar{A} + \bar{B}$$

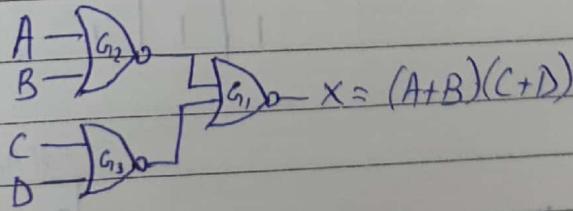
$$\begin{aligned} X &= (\overline{AB})(\overline{CD}) \\ &= (\bar{A} + \bar{B})(\bar{C} + \bar{D}) \\ &= (\bar{A}\bar{B}) + (\bar{C}\bar{D}) \\ &= \bar{A} \cdot \bar{B} + \bar{C} \cdot \bar{D} \\ &= AB + CD \end{aligned}$$



NOR Logic

$$\overline{A+B} = \bar{A}\bar{B}$$

$$\begin{aligned} X &= \overline{\overline{A+B} + \overline{C+D}} \\ &= (\overline{\overline{A+B}})(\overline{\overline{C+D}}) \\ &= (A+B)(C+D) \end{aligned}$$

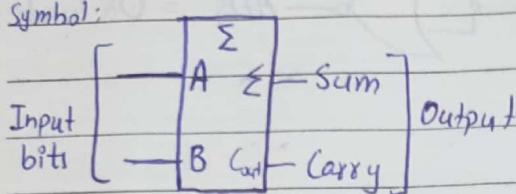


GO GETTERS

The Half Adder:

The half adder accept two binary digits on its inputs and produce two binary digits on its output i.e. a sum bit & a carry bit.

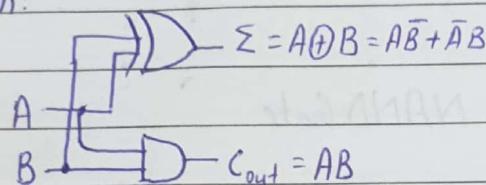
Logic Symbol:



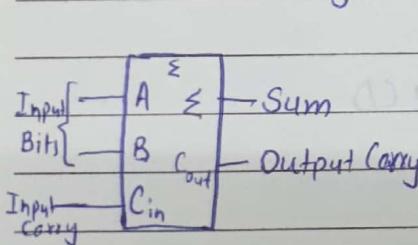
A	B	Cout	Σ
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

AND XOR

Logic Diagram:

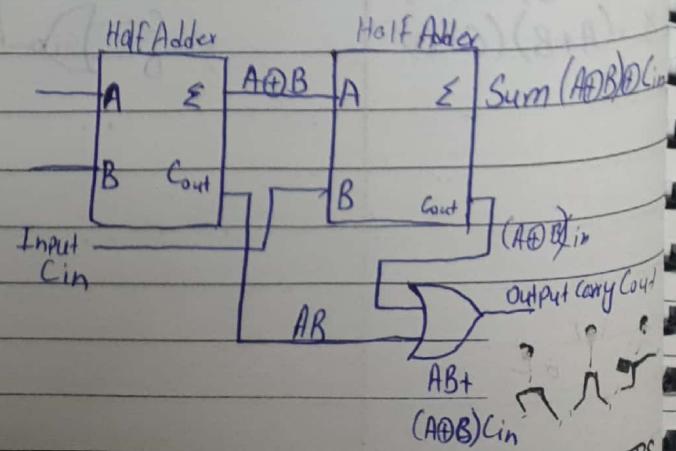
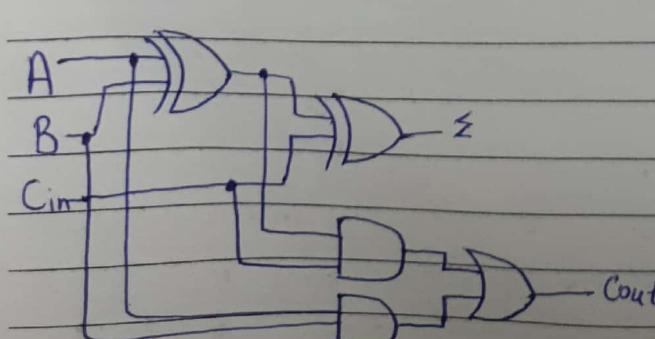


The Full Adder: accepts two inputs bits and an input carry and generates a sum output and a output carry.



A	B	Cin	Cout	Σ
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Logic Diagram



GO GETTERS

Parallel Binary Adder:

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Two or more full adder are connected to form a parallel binary adder

$$\begin{array}{r} \boxed{1} \quad \boxed{1} \\ + \quad \underline{0} \quad \underline{1} \end{array} \text{ carry bit from right column}$$

In this case carry $\leftarrow 100$

bit from second column became a sum bit

$$\begin{array}{r} \boxed{1} \quad \boxed{1} \\ + \quad \underline{0} \quad \underline{1} \\ \hline 1 \quad 0 \quad 0 \quad 0 \end{array}$$

$$\sum_1 \sum_3 \sum_2 \sum_4$$

~~Note~~

Note: Output carry bit

became MSB in sum(Σ_4)

$$\begin{array}{r} A_2 \quad A_1 \\ + B_2 \quad B_1 \\ \hline \Sigma_3 \quad \Sigma_2 \quad \Sigma_1 \end{array}$$

$$A_4 \quad A_3 \quad A_2 \quad A_1$$

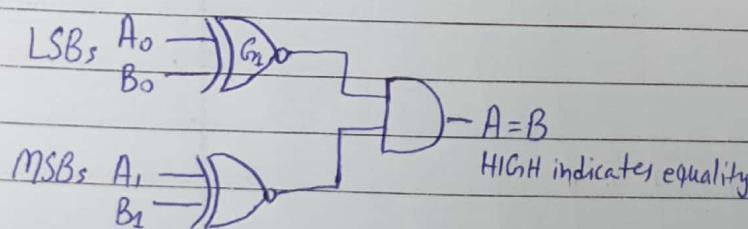
$$B_4 \quad B_3 \quad B_2 \quad B_1$$

$$C_4 \quad \Sigma_4 \quad \Sigma_3 \quad \Sigma_2 \quad \Sigma_1$$

General format addition
of two 2-bit numbers

Comparators: The basic function of a comparators is to compare the magnitudes of two binary numbers to determine the relationship of those two numbers. In its simplest form, a comparator circuit determines whether two numbers are equal.

Equality: In order to compare binary numbers containing two bits each, an additional exclusive-NOR gate is necessary.



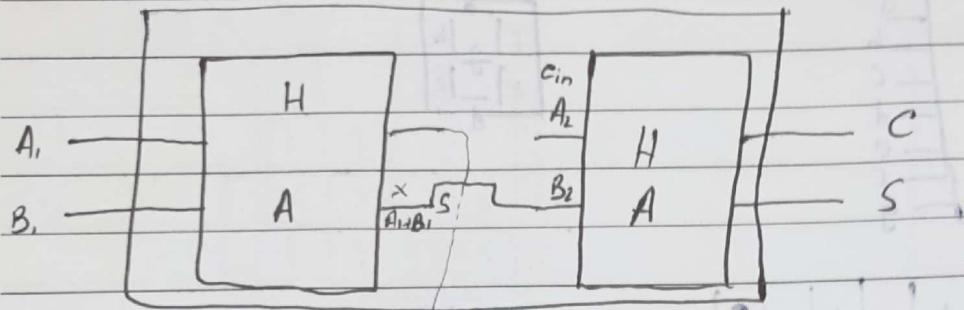
Inequality: To determine an inequality of binary numbers A and B, you first examine the highest order bit in each number - The following conditions are possible:

- 1) If A₃ = 1 and B₃ = 0, number A is greater than number B
- 2) If A₃ = 0 and B₃ = 1, number A is less than number B
- 3) If A₃ = B₃ you must examine the next lower bit position for an inequality



GO GETTERS

FA



$$(A+B)+C \\ X+C$$

A	B	C_{in}	C_{out}	S
0	0	0	0	m_0
0	0	1	0	m_1
0	1	0	0	m_2
0	1	1	1	m_3
1	0	0	0	m_4
1	0	1	1	m_5
1	1	0	1	m_6
1	1	1	1	m_7

HA		
1	0	1
1	1	1
$S=0$	$S=1$	$S=0$
$C=1$	$C=0$	$C=1$
carry	Sum	
1	0	1

$A_{in}=1$	$C_{in}=1$	FA	C_{out}	C_{in}
1	1	1	1	A
1	0	1	1	B
$S=1$	$S=0$	$S=0$		

$$C_{out}=1 \quad C_{out}=1 \quad C_{out}=1 \\ S=1, \quad S=0, \quad S=0$$

$$\text{Carry} \quad \text{Sum} \\ 1 \quad 1 \quad 0 \quad 0$$

Ripple Effect
delay occurs.

A	BC
0	111
1	111
0	111
1	111

(20B)

$$\text{Carry} \quad \text{Sum} \\ 1 \quad 1 \quad 0 \quad 0$$

(10B)

(10B)



Encryption / Decryption.

$$\begin{matrix} \text{input} \\ \uparrow \\ n = 2^m \end{matrix}$$

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Encoders

An encoder is a combinational logic circuit that essentially accept an active high level on one of its inputs representing a digit, such as decimal or octal digit and converts it to a coded output such as BCD or binary.

SOP: $\Sigma \rightarrow (1)$

POS: $\Pi \rightarrow (0)$

$(x) \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \end{bmatrix}$ Don't Care

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BCD - Excess Code:

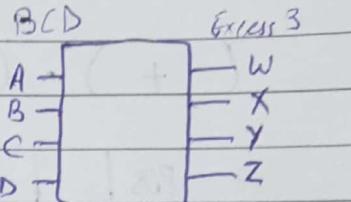
Equation for

$$W = \Sigma(m_5, m_6, m_7, m_8, m_9), X(m_{10}, m_{11}, m_{12}, m_{13}, m_{14}, m_{15}) + \dots$$

$$X = \Sigma(m_1, m_2, m_3, m_4, m_5), X(\dots)$$

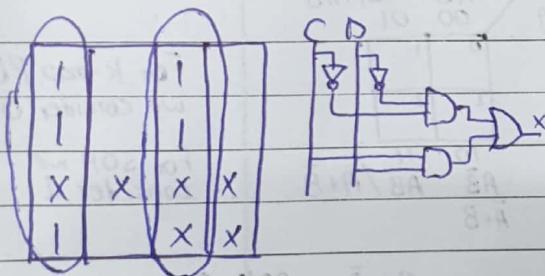
$$Y = \Sigma(m_0, m_3, m_4, m_7, m_8), X(\dots)$$

$$Z = \Sigma(m_0, m_2, m_4, m_6, m_8), X(\dots)$$

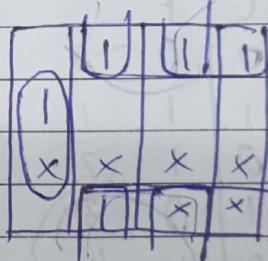


$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

$D \rightarrow \square \rightarrow Z$



$$Z = \bar{D}$$



$$Y = \bar{C}\bar{D} + CD$$

~~AB~~

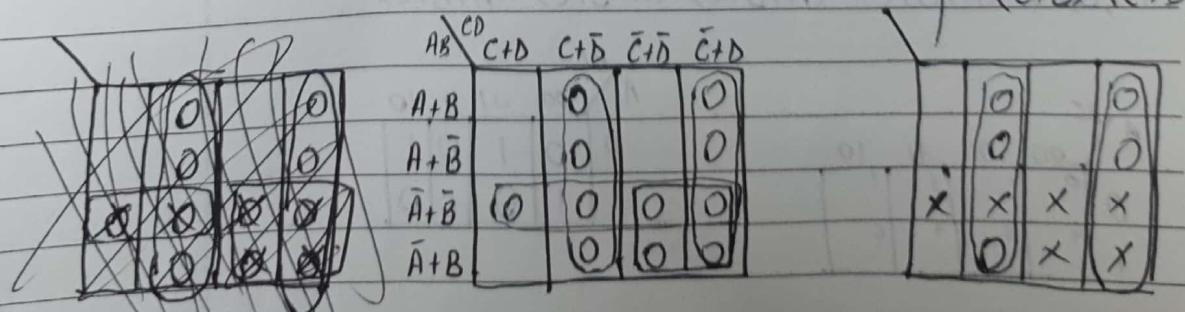
$$X = \bar{B}C + \bar{B}\bar{C}D + B\bar{C}\bar{D}$$

$$X = \bar{B}C + \bar{B}D + B\bar{C}\bar{D}$$

$$Y = \Sigma(m_0, m_3, m_4, m_7, m_8)$$

$$X = \Pi(m_1, m_2, m_5, m_9, m_{10}, m_{11}, m_{12}, m_{13}, m_{14}, m_{15})$$

$$\Rightarrow (C+\bar{D}) \cdot (\bar{C}+D)$$



$$(C+\bar{D}) \cdot (\bar{C}+D) \cdot (\bar{A}+\bar{B}) \cdot (\bar{A}+C)$$

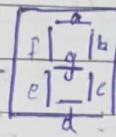
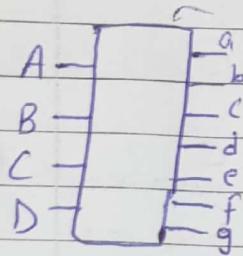


→ Don't Care

BCD to 7 Segment Decoder:

① Active High
② Active Low
 $\hookrightarrow \text{ON} = 1$
 $\text{OFF} = 0$

No matter
For SOP 1
For POS 0
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Active High:

A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0
3	0	0	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1
5	0	1	0	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1
7	0	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1
9	1	0	0	1	1	1	1	0	1	1

③ For eqs we consider column
so in this case we need to
find 7 equation

a(SOP)

AB	CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	1	1	1	1	1
$\bar{A}B$		1	1	1	1
$A\bar{B}$			1	1	1
AB				X	X
				X	X
				X	X

$(A + C + BD + \bar{B}\bar{D})$

a(POS)

$C+D$	$\bar{C}+\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
$A+B$	0	0	0
$A+\bar{B}$	0	0	0
$\bar{A}+\bar{B}$	0	X	X
$\bar{A}+B$	X	X	X

$(\bar{B} + C + D) \cdot (A + B + C + \bar{D})$

b(SOP)

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
X	X	X	X	X	X	X
X	X	X	X	X	X	X
X	X	X	X	X	X	X
X	X	X	X	X	X	X
X	X	X	X	X	X	X

$(\bar{A}B + CD + \bar{C}\bar{D} + \bar{A}\bar{B})$

$(B + C + D') \cdot (\bar{B}' + C' + D)$

c(SOP)

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
X	X	X	X	X	X	X
X	X	X	X	X	X	X
X	X	X	X	X	X	X
X	X	X	X	X	X	X
X	X	X	X	X	X	X

$(\bar{C} + D + B)$

g(SOP)

1	1			1
X	X	X	X	
1	1	X	X	

$$(A + C\bar{D} + B\bar{C} + \bar{B}C)$$

e(SOP)

1				1
X	X	X	X	
1		X	X	

$$\bar{B}\bar{D} + C\bar{D}$$

d(SOP)

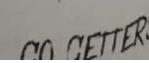
1			1
1			1
X	X	X	X
1	1	X	X

$$A + \bar{B}\bar{D} + B\bar{C}D + C\bar{D} + \bar{B}C$$

f(SOP)

1			1
1			1
X	X	X	X
1		X	X

~~$$A + B\bar{D} + \bar{C}\bar{D} + \bar{C}B + \bar{D}B$$~~



I/P \rightarrow Encoder \rightarrow Decoder \rightarrow O/P
 Machine Understood Human Understood



2×1
 4×2
 8×3

DATE 1/1/20

Octal to Binary (Encoder)

E = Enable Pin

If input[i]
zero enable[i]
1.

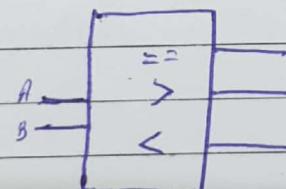
D ₀	D ₁	D ₂	D ₃	B ₄	D ₅	D ₆	D ₇	A ₂	A ₁	A ₀	E
1	0	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	1	0	
								0	1	0	
								0	1	1	
								1	0	0	
								1	0	1	
								1	1	0	
								1	1	1	

$$a == b$$

$$a \neq b \quad \begin{cases} a > b \\ a < b \end{cases}$$

1bit

A	B	$=$	$<$	$>$
0	0	1	0	0
0	1	0	1	0
1	0	0	0	1
1	1	1	0	0



2 bit

A	B	$A=B$	$A \neq B$
0	0	1	00
0	0	0	11
0	0	0	102
0	0	0	113
1	0	0	000
1	0	1	011
1	0	1	02
1	0	1	113
2	1	0	000
2	1	0	011
2	1	0	102
2	1	0	113
3	1	1	000
3	1	1	011
3	1	1	102
3	1	1	113

Quiz

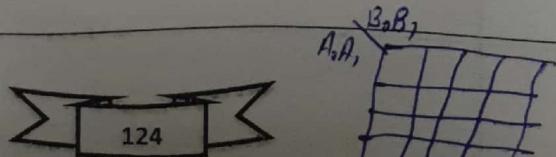
$$\text{Equal} = \bar{A}\bar{B} + AB$$

$$LT = \bar{A}B$$

$$GT = A\bar{B}$$

2 Bit

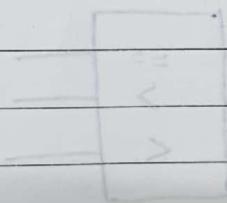
$$\text{Equal} = \bar{A}_0\bar{A}_1\bar{B}_0\bar{B}_1 + \bar{A}_0A_1\bar{B}_0B_1 + A_0\bar{A}_1B_0\bar{B}_1 + A_0A_1B_0B_1$$



GO GETTERS

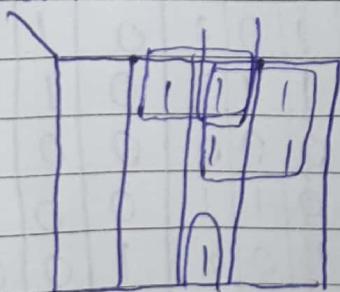
Multiplexer: Many input but only one output.

S_1, S_0	D_0	D_1	D_2	D_3	Output (y)
0 0					\bar{S}, \bar{S}_0, D_0
0 1					\bar{S}, \bar{S}_0, D_1
1 0					S, \bar{S}_0, D_2
1 1					S, S_0, D_3

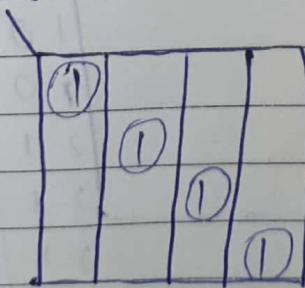


Comparator:

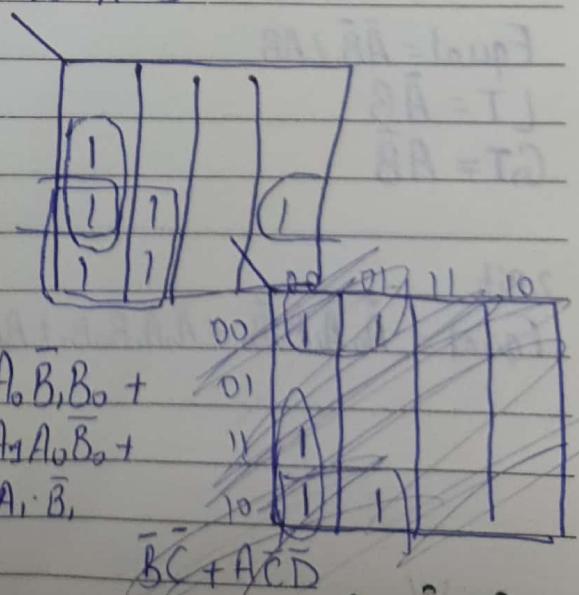
For $A < B$



for $A = B$



for $A > B$



$$\bar{A}_1 \bar{A}_0 B_0 + \bar{A}_1 B_1 + \bar{A}_0 B_1 B_0$$

$$(A_1 \oplus B_1)(A_0 \oplus B_0)$$

$$A_1 \bar{B}_1 B_0 + \\ A_1 A_0 \bar{B}_0 + \\ A_1 \cdot \bar{B}_1$$



CO-SITTERS

data line → no. of select line
 $2^m = n$
 ↓ No. of input.

$m = \log_2 n$
 $n = 4$
 $m = \log_2 2^2 = 2$

DATE 1/120

Multiplexers:

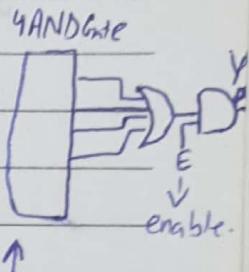
A MUX selects one data line from two or more input lines and routes data from the selected lines to output. The particular data line that is selected is determined by the select inputs.

Advantages:

-) Reduce no. of wires -) Reduce circuit complexity & cost.
-) Implementing of various circuit using MUX.

Types:

2:1 MUX, 4:1 MUX, 8:1 MUX, 16:1 MUX and 32:1 MUX.

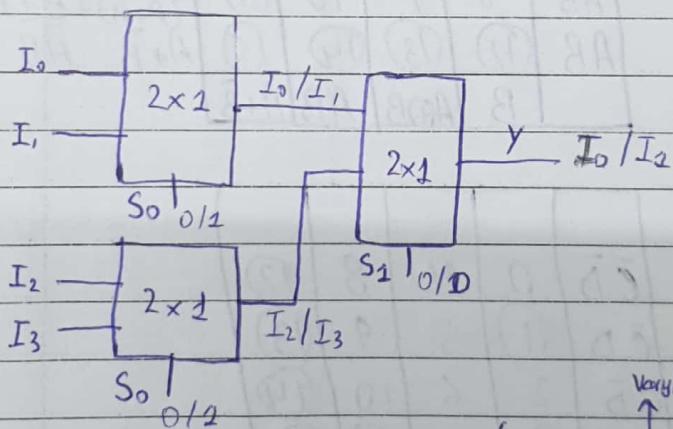


Implement 4x1 MUX using 2x1 MUX .

$$n = 4$$

$$\frac{4}{2} = \frac{2}{1}$$

+ ③ $\rightarrow 2 \times 1 \text{ MUX}$

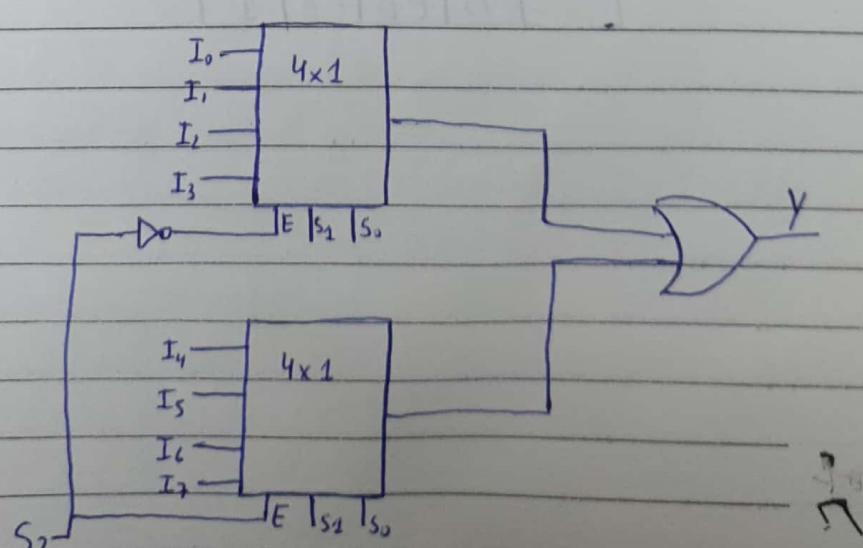


4x1 MUX

S ₁	S ₀	Y
0	0	I ₀
0	1	I ₁
1	0	I ₂
1	1	I ₃

Implementing 8x1 MUX Using 4x1 MUX (we use enable to implement 8x1 MUX)

S ₂	S ₁	S ₀	Y
0	0	0	I ₀
0	0	1	I ₁
0	1	0	I ₂
0	1	1	I ₃
1	0	0	I ₄
1	0	1	I ₅
1	1	0	I ₆
1	1	1	I ₇



8:1 MUX $\rightarrow 2^3 = 8$ 3 select lines

$$\textcircled{Q} F(A, B, CD) = \Sigma(1, 3, 4, 11, 12, 13, 14, 15) \text{ using } 8 \times 1 \text{ MUX}$$

No. of lines \neq No. of variables input

\therefore Remaining 1 variable will be input variable & other 3 will be selection variable

	I ₀	I ₁	I ₂	I ₃	I ₄	I ₅	I ₆	I ₇
\bar{D}	0	2	4	6	8	10	12	14
D	1	3	5	7	9	11	13	15
	0	0	0	1	1			

A, B, C \rightarrow select lines

D \rightarrow will be mapped
as input

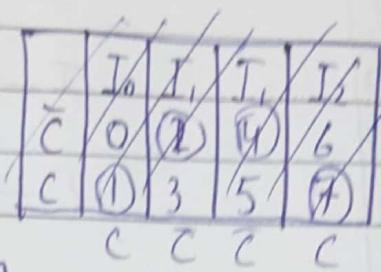
Same on 4x1 MUX

	I ₀	I ₁	I ₂	I ₃	
$\bar{A}\bar{B}$	0	1	2	3	$\bar{A}B + AB = B(A + \bar{A}) = B$
$\bar{A}B$	4	5	6	7	$\bar{A}\bar{B} + A\bar{B} + AB$
$A\bar{B}$	8	9	10	11	$\bar{A}B + A$
AB	12	13	14	15	$(A + \bar{A}) + A + \bar{B} = A + \bar{B}$
	B	$A \oplus B$	AB	$A + \bar{B}$	$\bar{A}B = A + \bar{B}$

$\bar{C}\bar{D}$	0	4	8	12
$\bar{C}D$	1	5	9	13
$C\bar{D}$	2	6	10	14
CD	3	7	11	15
	D	$\bar{C}\bar{D}$	CD	1

1-Bit Full Adder using Multiplexer.

A	B	Cin	S	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1



For Sum

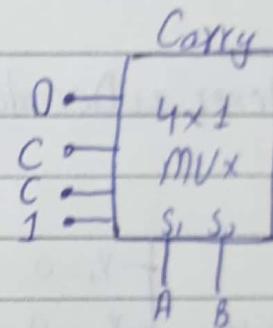
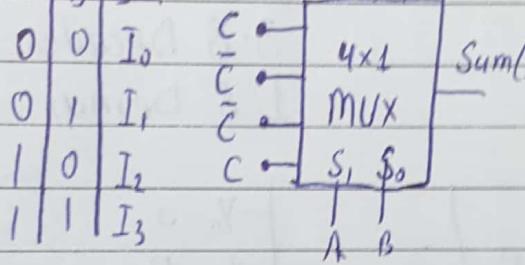
A	B	Cin	Sum
0	0	0	00
0	1	0	01
1	0	0	11
1	1	1	10

For carry

A	B	Cin	Carry
0	0	0	00
0	1	0	01
1	0	0	11
1	1	1	10

S₁, S₀

A	B	S
0	0	I ₀
0	1	I ₁
1	0	I ₂
1	1	I ₃



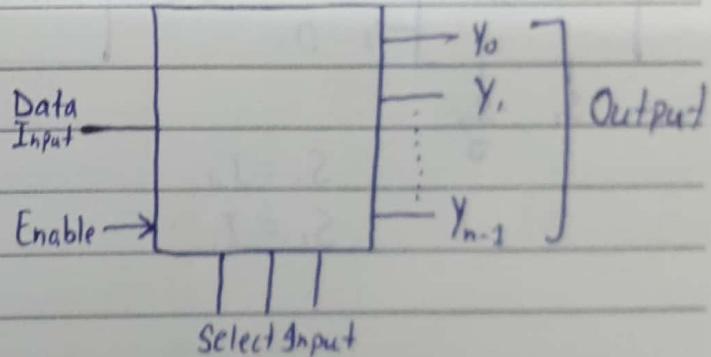
De-Multiplexer:

- One input & many output
- Reverse operation of multiplexer
- One to many ckt or data distributor.

n → O/P lines

m → Select lines

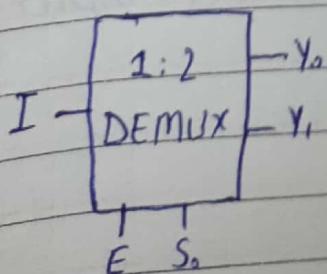
$$n = 2^m, m = \log_2 n$$



1:2 DEMUX

$$m = \log_2 2$$

$$m = 1$$

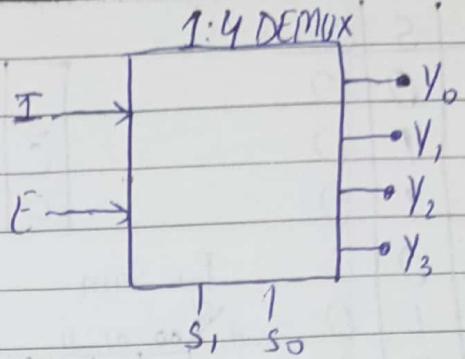


E	S ₀	Y ₀	Y ₁
0	0	0	0
0	1	0	0
1	0	1	0
1	1	0	1

$$Y_0 = E \bar{S}_0 I$$

$$Y_1 = E S_0 I$$

E	S ₁	S ₀	Y ₀	Y ₁	Y ₂	Y ₃
0	X	X	0	0	0	0
1	0	0	I	0	0	0
1	0	1	0	I	0	0
1	1	0	0	0	I	0
1	1	1	0	0	0	I



$$\begin{aligned} n &= 4 \\ m &= \log_2 n \\ m &= \log_2 2^2 \\ m &= 2 \end{aligned}$$

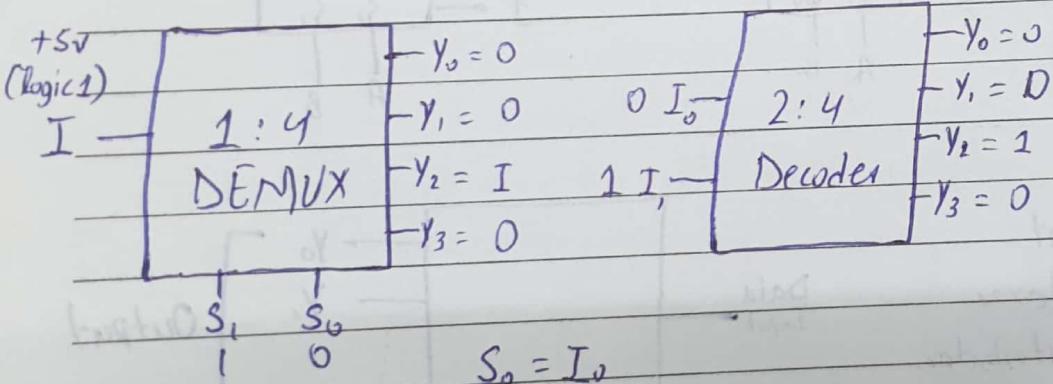
$$\begin{aligned} Y_0 &= E\bar{S}_1\bar{S}_0I & Y_2 &= ES_1\bar{S}_0I \\ Y_1 &= E\bar{S}_1S_0I & Y_3 &= ES_1S_0I \end{aligned}$$

4 × (4 input AND Gate)

Demultiplexers as Decoder:

3:8 Decoder

1:8 Demux



$$\begin{aligned} S_0 &= I_0 \\ S_1 &= I_1 \end{aligned}$$

I	I ₀	I ₁	Y ₀	Y ₁	Y ₂	Y ₃
0	0	1	1	0	0	0
0	1	0	0	1	0	0
1	0	0	0	0	1	0
1	1	1	0	0	0	1

A DEMUX performs the opposite function from MUX - It switches data from one input line or two or more data lines depending on select input.