inverse row
LU decomposition (factorizing matrix)
Gaussian Elimination
Gauss-Jordan
Linear
non linear } equation difference.

Linear Algebra:

Date: ______

Linear Equations:

2D: $ax + by = c$ — constant a, b are both nonzero

3D: $ax + by + cz = d$ — constants a, b, c are all non zero.

$a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0$ is called homogeneous linear equation.

Note: A linear equation doesnot involve any products or square roots of variables occurs only to the first power and donot appear, as arguments of trignometric, logrithmic, or exponential functions.

Example:

Linear Equation	Non-Linear Equations
i) $x_1 - 2x_2 - 3x_3 + x_4 = 0$	i) $\sin x + y = 0$
ii) $x + 3y = 7$	ii) $3x + 2y - xy = 5$

(Q) Differentiate the following equations as a linear/non-linear.

i) $x_1 - 2x_2 - 3x_3 + x_4 = 0$ Linear
ii) $3x.x + 2y - y = 5$ Non-linear
iv) $\frac{1}{2}x - y + 3z = -1$ Linear
iii) $\cos x + 1 = y$ Non-Linear
v) $\sqrt{2}x + y = 7$ Linear.
vi) $2\sqrt{x} + y = 7$ Non-Linear.
vii) $\ln x + 1 = y$ Non-Linear.

Consistent: if it has atleast one solution.
Inconsistent: if it has no solutions.

SB: $(1,0), (0,1)$

R^3: $(1\ 0\ 0)\ (2\ 2\ 0)\ (3\ 3\ 3)$

R^2: $(2,1)\ (3,0)$

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Leading Diagonal $\begin{bmatrix} 1 & 6 & 10 \\ 30 & 9 & 0 \\ 40 & 0 & 8 \end{bmatrix}$

Secondary Diagonal $\begin{bmatrix} 4 & 3 & 12 \\ 10 & 1 & 5 \\ 2 & 4 & 8 \end{bmatrix}$

Null/Zero Matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Date:

• Upper Triangular Matrix $a_{ij} = 0,\ i > j$

$$A = \begin{bmatrix} 5 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -1 \end{bmatrix}$$

• Lower Triangular Matrix $a_{ij} = 0,\ i < j$

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 3 & -1 & 0 \\ 2 & 4 & 6 \end{bmatrix}$$

Tranpose of Matrix & it property

(i) $(A^t)^t = A$

(ii) $(A \pm B)^t = A^t \pm B^t$

(iii) $(AB)^t = B^t A^t$

$(AB)^t \neq A^t B^t$

Transpose: Rows become column

$$A \begin{bmatrix} 5 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -1 \end{bmatrix} \quad A^t = \begin{bmatrix} 5 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 5 & -1 \end{bmatrix}$$

Same Transpose

$$A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \quad A^t = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

Symetric Matrix. $A^t = A$ (A is sqr Matrix)

$$B = \begin{bmatrix} 1 & 5 & 6 \\ 5 & 2 & 7 \\ 6 & 7 & 3 \end{bmatrix} \quad B^t = \begin{bmatrix} 1 & 5 & 6 \\ 5 & 2 & 7 \\ 6 & 7 & 3 \end{bmatrix}$$

Skew Symmetric Matrix, $A^t = -A$.

$$F = \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -4 \\ 3 & 4 & 0 \end{bmatrix} \quad F^t = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix} \Rightarrow - \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -4 \\ 3 & 4 & 0 \end{bmatrix} \Rightarrow -F$$

Date:

(Q)

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 0 \\ 1 & 4 \end{bmatrix}_{3\times 2} \qquad B = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 2 & 3 \end{bmatrix}_{2\times 3}$$

$$AB = \begin{bmatrix} 5\times1 + 3\times-1 & 5\times3 + 3\times2 & 5\times4 + 3\times3 \\ & & \\ & & \end{bmatrix}$$

Determinant

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 2 \\ 3 & 1 \end{vmatrix} = \overset{5-6}{\cancel{}} = -1$$

$$|B| = \begin{vmatrix} 5 & 2 & 1 \\ 3 & 4 & 7 \\ 2 & -1 & 2 \end{vmatrix}$$

Expanding w.r.t 1^{st} row

$$5\begin{vmatrix} 4 & 7 \\ -1 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 7 \\ 2 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$$

$$5(8+7) - 2(6-14) + 1(-3-8)$$

$$75 + 16 - 11$$

$$80$$

Minor & Cofactor

$$A = \begin{bmatrix} 5 & 2 & 1 \\ 3 & 4 & 7 \\ 2 & -1 & 2 \end{bmatrix}$$

For 7: $M_{23} = \begin{vmatrix} 5 & 2 \\ 2 & -1 \end{vmatrix} = -5-4 = -9$

$$C_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 2 \\ 2 & -1 \end{vmatrix} = 9$$

For 5:

$M_{11} = \begin{vmatrix} 4 & 7 \\ -1 & 2 \end{vmatrix} = 8+7 = 15$, $C_{11} = (-1)^{1+1}\begin{vmatrix} 4 & 7 \\ -1 & 2 \end{vmatrix} = 15$

then A is multiplicative inverse of B & vice versa.

If $\overset{A\times B}{AB}$ is identity Matrix.

Date: ____________

Multiplicative Inverse of a Matrix

$\overset{\to 2\times 2}{A} \quad \overset{\to 2\times 2}{B}$

$A' = \frac{1}{|A|} \cdot$ Adjoint A → interchange value of main diagonal & change sign of secondary diagonal.

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \Rightarrow |A| = 5-6 = -1$ ← if 0 then can't find determinant.

To check:

$AB = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

$\begin{bmatrix} -5+6 & 2-2 \\ -15+15 & 6-5 \end{bmatrix}$

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\text{Adj } A = \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{-1} \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$

$|A| = 0$, Singular Matrix

$|A| \neq 0$ Non Singular Matrix

↳ We find inverse.

Echelon Form (Row Echelon Form)

Properties:

Determinant $= |A|$

i) All non zero rows are above of zero rows.

ii) Each leading entry of a row is in a column to the right of the leading entry of row above it.

(iii) All entries in a column below a leading entry are zeros.

Reduced Echelon Form:

(iv) The leading entry in each non-zero row is 1.

(v) Each leading entry i.e 1 is the only non zero entry in its column.

Posiblility of Solution:

- Solution exist & Unique
- No solution
- Solution exist & infinitly to many.

Homogeneous → $2x+2y=0$

↳ Alway solution exist.

Single eq & 2 unknowns
2 eq & 3 unknowns] Solution will be infinate to many Not unique.

Non-Linear:

$\sin x + y = 0$

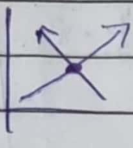
$x + y + xy = 0$

Date:

Non-Homogeneous

Q) $2x - y = 3$

$x + y = 2$



intersection at a point
One Solution.

$2(2-y) - y = 3$

$4 - 2y - y - 3 = 0$

$-3y = -1$

$\boxed{y = \frac{1}{3}}$

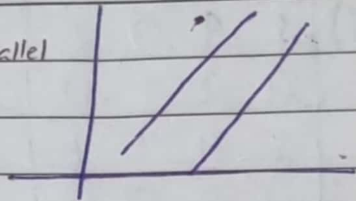
$x = 2 - \frac{1}{3}$

$\boxed{x = \frac{5}{3}}$

Q) $-x - y = 3$

$x + y = 3$

Parallel



$-(3-y) - y = 3$

$-3 + \cancel{y} - \cancel{y} = 3$

$m_1 = m_2$

$-x - y = 3$

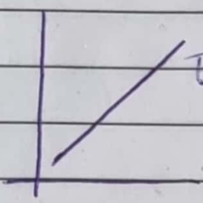
$x + y = 2$] solution of the system of eq doesn't exist.

$0 \neq 5$

Q) $2x + 2y = -2$

$-x - y = 1$

$0 = 0$



→ overlap
↳ Consident.

infinate many solution

$y = -1 - x$

$x = 0$

$y = -1 - 0$

$(0, -1)$

$x = 1$

$y = -1 - 1$

$(1, -2)$

Cooficient Matrix

$A = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Augmented Matrix

$A = \begin{bmatrix} 2 & 2 & 2 \\ -1 & -1 & 1 \end{bmatrix}$

Leeding Entry: First non-zero entry

$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -1 & 3 & 4 \end{bmatrix}$

Echelon Form

(i) $\begin{bmatrix} 1 & 5 & 2 & -9 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 5 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 5 & 2 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Not Echelon

(i) $\begin{bmatrix} 1 & 3 & 4 \\ 0 & 0 & 0 \\ 2 & 4 & 9 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 & -9 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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Row Operation:
- •) interchang rows in matrix
- •) Add/Sub each row in matrix
- •) Multiply row with constant in matrix. ↳ non-zero

Date: ____________

(i) $\begin{bmatrix} 1 & 9 & 0 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 4 & 4 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \underset{\text{Operation}}{\sim R_3 - R_1} \begin{bmatrix} 1 & 9 & 0 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & -5 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \sim -\frac{1}{5} \cdot R_3 \begin{bmatrix} 1 & 9 & 0 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & -4/5 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\sim 2R_3 - R_2 \begin{bmatrix} 1 & 9 & 0 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & \frac{-13}{5} & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} 0 & 0 & 1 & -9 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 \xleftrightarrow{\text{interchange}} R_2 \begin{bmatrix} 0 & 1 & 3 & 7 \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 3 & 4 \\ 0 & 0 & 0 \\ 2 & 4 & 9 \end{bmatrix} \quad R_2 \leftrightarrow R_3 \begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & 9 \\ 0 & 0 & 0 \end{bmatrix} \sim 2R_1 - R_2 \begin{bmatrix} 2 & 6 & 8 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

(iv) $\begin{bmatrix} 3 & -4 & 0 & 9 \\ 2 & 4 & -1 & 0 \\ 10 & 0 & -2 & -4 \end{bmatrix} \sim \frac{1}{3} R_1 \begin{bmatrix} 1 & -\frac{4}{3} & 0 & 3 \\ 2 & 4 & -1 & 0 \\ 10 & 0 & -2 & -4 \end{bmatrix} \begin{matrix} \sim R_2 - 2R_1 \\ \sim R_3 - 10R_1 \end{matrix} \begin{bmatrix} 1 & -4/3 & 0 & 3 \\ 0 & 4/3 & -1 & -6 \\ 0 & \frac{40}{3} & -2 & -34 \end{bmatrix}$

$\sim R_3 - 10R_2 \begin{bmatrix} 1 & -4/3 & 0 & 3 \\ 0 & 4/3 & -1 & -6 \\ 0 & 0 & 8 & 26 \end{bmatrix}$

$\begin{matrix} \sim \frac{3}{4} R_2 \\ \sim \frac{1}{8} R_3 \end{matrix} \begin{bmatrix} 1 & -4/3 & 0 & 3 \\ 0 & 1 & -\frac{3}{4} & -\frac{9}{2} \\ 0 & 0 & 1 & 13/4 \end{bmatrix}$

Echolen Form.

$\begin{matrix} \sim R_1 + \frac{4}{3} R_2 \\ \sim R_2 + \frac{3}{4} R_3 \end{matrix} \begin{bmatrix} 1 & 0 & -1 & -3 \\ 0 & 1 & 0 & \frac{-47}{16} \\ 0 & 0 & 1 & \frac{13}{4} \end{bmatrix}$

$\sim R_1 + R_3 \begin{bmatrix} 1 & 0 & 0 & 1/4 \\ 0 & 1 & 0 & -47/16 \\ 0 & 0 & 1 & 13/4 \end{bmatrix}$

Reduce Echolen F

Guasian Elimination Method → Echelon Method

Date:

(1) $x_1 + x_2 + 2x_3 = 8$ Non Homogeneous

$-x_1 - 2x_2 + 3x_3 = 1$

$3x_1 - 7x_2 + 4x_3 = 10$

Augmented Matrix

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix} \begin{matrix} \sim R_2 + R_1 \\ \sim R_3 - 3R_1 \end{matrix} \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix} \sim R_3 - 10R_2 \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{bmatrix}$$

E.F

$$\sim -\frac{1}{52} R_3 \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_1 + x_2 + 2x_3 = 8$ — (i)

$-x_2 + 5x_3 = 9$ — (ii)

$\boxed{x_3 = 2}$ — (iii)

Put iii in ii

$-x_2 + 5(2) = 9$

$\boxed{x_2 = 1}$

$x_1 + 1 + 2(2) = 8$

$\boxed{x_1 = 3}$

$(x_1, x_2, x_3) = (3, 1, 2).$

Guass-Jorden Method → Reduced Echlon method

$$\sim -1 \times R_1 \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{matrix} \sim R_1 - R_2 \\ \sim R_2 + 5R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 7 & 17 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\sim R_1 - 7R_3 \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_1 = 3$

$x_2 = 1$

$x_3 = 2$ $(x_1, x_2, x_3) = (3, 1, 2).$

Guasian Elimination

Date:

(1) $x - y + 2z - w = -1$

$2x + y - 2z - 2w = -2$

$-x + 2y - 4z + w = 1$

$3x - 3w = -3$

$$A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 2 & 1 & -2 & -2 \\ -1 & 2 & -4 & 1 \\ 3 & 0 & 0 & -3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ -2 \\ 1 \\ -3 \end{bmatrix}$$

Augmented Matix

(1)
$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & -4 & 1 \\ 3 & 0 & 0 & 0 & -3 \end{bmatrix}$$

(2)
$$\begin{matrix} \sim R_2 - 2R_1 \\ \sim R_3 + R_1 \\ \sim R_4 - 3R_1 \end{matrix} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix}$$

(3)
$$\begin{matrix} \sim 3R_3 - R_2 \\ \sim R_4 - R_2 \end{matrix} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x - y + 2z - w = -1$ — (i)

$3y - 6z = 0$ — (ii)

(ii) $\Rightarrow \boxed{y = 2z}$

using in (i)

$x - 2z + 2z - w = -1$

$x - w = -1$

$\boxed{x = w - 1}$

$z = 0$

$w = 1$

infinitly many answers.

Guass-Jordan: (Reduced Echlon)

$$\sim \tfrac{1}{3} R_2 \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim R_1 + R_2 \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Reduced Echlon Form

$x - w = -1$

$y - 2z = 0$

Let $w = \gamma$

$z = s$

$\boxed{x = w - 1}$

$\boxed{y = 2z}$

$\boxed{x = \gamma - 1}$

$\boxed{y = 2s}$

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Homogenous

Date: ______

Guassian Elimination.

Q) $3x_1 + 5x_2 - 4x_3 = 0$
$3x_1 + 2x_2 - 4x_3 = 0$
$6x_1 + x_2 - 8x_3 = 0$

First Solution $(0,0,0)$

Augmented Matrix

$$\begin{bmatrix} 3 & 5 & -4 & 0 \\ 3 & 2 & -4 & 0 \\ 6 & 1 & -8 & 0 \end{bmatrix} \begin{matrix} \sim R_2 - R_1 \\ \sim R_3 - 2R_1 \end{matrix} \begin{bmatrix} 3 & 5 & -4 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{bmatrix}$$

$3x_1 + 5x_2 - 4x_3 = 0$ — (i)
$-3x_2 = 0$ — (ii)

$$\sim R_3 - 3R_2 \begin{bmatrix} 3 & 5 & -4 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \text{Echlon Form}$$

(ii) $\Rightarrow \boxed{x_2 = 0}$

using eqn (i)
$3x_1 + 5(0) - 4x_3 = 0$
$3x_1 - 4x_3 = 0$
$\boxed{x_1 = \frac{4}{3}x_3}$

$x_2 = 0$
$x_3 = 3$

$x_1 = \frac{4}{3}(3)$
$x_1 = 4$

$((4,0,3))$

infinetly many solutions for x_1 & x_3

Q) $x_1 - 5x_2 + 9x_3 = 0$
$-x_1 + 4x_2 - 3x_3 = 0$
$2x_1 - 8x_2 + 9x_3 = 0$

Augmented Matrix

$$\begin{bmatrix} 1 & -5 & 9 & 0 \\ -1 & 4 & -3 & 0 \\ 2 & -8 & 9 & 0 \end{bmatrix} \begin{matrix} \sim R_2 + R_1 \\ \sim R_3 - 2R_1 \end{matrix} \begin{bmatrix} 1 & -5 & 9 & 0 \\ 0 & -1 & 6 & 0 \\ 0 & 2 & -9 & 0 \end{bmatrix}$$

$x_1 - 5x_2 + 9x_3 = 0$ — (i)
$-x_2 + 6x_3 = 0$ — (ii)
$3x_3 = 0$ — (iii)

$$\sim R_3 + 2R_2 \begin{bmatrix} 1 & -5 & 9 & 0 \\ 0 & -1 & 6 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

(iii) $\Rightarrow x_3 = 0$

using (ii)
$-x_2 + 6(0) = 0$
$x_2 = 0$

Put x_2 & x_3 in eqn (i)
$x_1 - 5(0) + 9(0) = 0$
$x_1 = 0$

$(0,0,0)$ only trival solution exist.

Date: ____________

Q) Solve linear equa using Gaussian Elimination Method:

Q.3) $x_2 - 4x_3 = 8$

$2x_1 - 3x_2 + 2x_3 = 1$

$5x_1 - 8x_2 + 7x_3 = 1$

Augmented Matrix

$$\begin{bmatrix} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 5 & -8 & 7 & 1 \end{bmatrix}$$

$R_2 + 3R_1$, $R_3 + 8R_1$

$$\begin{bmatrix} 0 & 1 & -4 & 8 \\ 2 & 0 & -10 & 25 \\ 5 & 0 & -25 & 65 \end{bmatrix}$$

$\frac{1}{5}R_3$

$$\begin{bmatrix} 0 & 1 & -4 & 8 \\ 2 & 0 & -10 & 25 \\ 1 & 0 & -5 & 13 \end{bmatrix}$$

$2R_3 - R_2$

$$\begin{bmatrix} 0 & 1 & -4 & 8 \\ 2 & 0 & -10 & 25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$x_2 - 4x_3 = 8$

$x_1 - 10x_3 = 25$

Q) $3x + 4y - z = -6$, $-2y + 10z = -8$, $4y - 2z = -2$

$$\begin{bmatrix} 3 & 4 & -1 & -6 \\ 0 & -2 & 10 & -8 \\ 0 & 4 & -2 & -2 \end{bmatrix}$$

$R_3 - R_1$

$$\begin{bmatrix} 3 & 4 & -1 & -6 \\ 0 & -2 & 10 & -8 \\ -3 & 0 & -1 & 4 \end{bmatrix}$$

$-\frac{1}{3}R_3$

$$\begin{bmatrix} 3 & 4 & -1 & -6 \\ 0 & -2 & 10 & -8 \\ 1 & 0 & \frac{1}{3} & \frac{-4}{3} \end{bmatrix}$$

$\frac{1}{2}R_3 + R_2$

$$\begin{bmatrix} 3 & 4 & -1 & -6 \\ 0 & -2 & 10 & -8 \\ 0 & 0 & 9 & -9 \end{bmatrix}$$

$3x + 4y - z = -6 \Rightarrow x = -1$

$-2y + 10z = -8 \Rightarrow -2y + 10(-1) = -8$

$9z = -9$ $\quad \boxed{y = -1}$

$\boxed{z = -1}$

$\frac{1}{2}R_1 + R_2$

$$\begin{bmatrix} \frac{3}{2} & 0 & \frac{19}{2} & -11 \\ 0 & -2 & 10 & -8 \\ 0 & 0 & 9 & -9 \end{bmatrix}$$

$\frac{2}{3}R_1$, $-\frac{1}{2}R_2$

$$\begin{bmatrix} 1 & 0 & \frac{19}{3} & \frac{-22}{3} \\ 0 & 1 & -5 & 4 \\ 0 & 0 & 9 & -9 \end{bmatrix}$$

$x + \frac{19}{3}z = \frac{-22}{3}$ $\quad x = -1$

$y - 5z = 4 \Rightarrow y = -1$

$z = -1$

Invertible Matrix. $\longrightarrow$ Singular Matrix = determinant is zero

Inverse of a Matrix:

Date: ______

If A is an $n \times n$ matrix, A matrix C of order.

$AA^{-1} = A^{-1}A = I$ identity Matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$ $\quad A^{-1} = \dfrac{1}{|A|} \text{Adj } A$

$|A| = 2 \times -2 - 1 \times 3$

$= 7$

$\text{Adj } A = \begin{bmatrix} -2 & \\ & 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 5 & -3 \\ 2 & -1 & 0 \\ -3 & 0 & 4 \end{bmatrix}$ $\quad \text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$

Inverse Matrix using row operations: (REF)

$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The augmented matrix is:

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$

$\sim R_2 - 2R_1, \sim R_3 - 3R_1$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 1 & -2 & -3 & 0 & 1 \end{array}\right]$

$\sim R_3 - R_2$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 \end{array}\right]$

$\sim R_1 + R_3, \sim R_2 - R_3, R_3 \times -1$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right]$

So,

$A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & 1 & -1 \end{bmatrix}$

Ans

To prove $AA^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ = will give indentity matrix. which is

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Inverse of Matrix using row operation
Rank of Matrix
Guassian Elimination
Guass-Jordan
Factorize — LU Decomposition.

Date:

$$A=\begin{bmatrix}5&2&3\\0&-1&4\\0&0&3\end{bmatrix} \qquad A=\begin{bmatrix}5&2&3\\0&-1&4\\0&0&0\end{bmatrix}$$

Rank = 3 $\qquad$ Rank = 2

To find rank we need to convert in echlon Form.

Q) $A=\begin{bmatrix}1&0&3\\2&2&1\\-1&2&3\end{bmatrix}$ $\quad R_2-2R_1,\ R_3+1R_1$ $\begin{bmatrix}1&0&3\\0&2&-5\\0&2&6\end{bmatrix}$ $\quad R_3-R_2$ $\begin{bmatrix}1&0&3\\0&2&-5\\0&0&-17\end{bmatrix}$ Rank = 3

Q) $A=\begin{bmatrix}1&2&1\\-2&-3&1\\3&5&0\end{bmatrix}$ $\quad R_2+2R_1,\ R_3-3R_1$ $\begin{bmatrix}1&2&1\\0&1&3\\0&-1&-3\end{bmatrix}$ $\quad R_3-R_2$ $\begin{bmatrix}1&2&1\\0&1&3\\0&0&0\end{bmatrix}$ Rank = 2

LU Decomposition: 2 row operation possible, interchanging rows is not possible.

We can make changes in L
inverse of upper
such that
if we multiple so in L we do division
if we add we do subtract
if we sub we do Addition.

Factorize: $12 = 3\times 4 \qquad 12 = 6\times 2$

$$U=\begin{bmatrix}x&x&x\\0&x&x\\0&0&x\end{bmatrix},\quad L=\begin{bmatrix}1&0&0\\x&1&0\\x&x&1\end{bmatrix}$$

$$x^2-4 = x^2-2^2$$
$$= (x-2)(x+2)$$

Q) $A=\begin{bmatrix}6&-2&0\\9&-1&1\\3&7&5\end{bmatrix} \xrightarrow{\sim \frac{1}{6}R_1} \begin{bmatrix}1&-\frac{1}{3}&0\\9&-1&1\\3&7&5\end{bmatrix},\ L=\begin{bmatrix}6&0&0\\x&1&0\\x&x&1\end{bmatrix} \xrightarrow{\sim R_2-9R_1,\ \sim R_3-3R_1} \begin{bmatrix}1&-\frac{1}{3}&0\\0&2&1\\0&8&5\end{bmatrix},\ L=\begin{bmatrix}6&0&0\\9&1&0\\3&x&1\end{bmatrix}$

$A = LU$

$$A=\begin{bmatrix}6&0&0\\9&1&0\\3&4&1\end{bmatrix}\begin{bmatrix}1&-\frac{1}{3}&0\\0&2&1\\0&0&1\end{bmatrix} \Rightarrow \begin{bmatrix}6&-2&0\\9&-1&1\\3&7&5\end{bmatrix}$$

$\sim R_3-4R_2$

$$U=\begin{bmatrix}1&-\frac{1}{3}&0\\0&2&1\\0&0&1\end{bmatrix},\quad L=\begin{bmatrix}6&0&0\\9&1&0\\3&4&1\end{bmatrix}$$

$6 \quad 12 \quad -12$

$\cancel{0 \quad -3 \quad 2}$

$0 \quad -3 \quad \frac{5}{2}$

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$$\begin{bmatrix} 2 & 4 & -4 \\ 1 & -4 & 3 \\ -6 & -9 & 5 \end{bmatrix} \qquad U = \begin{bmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ x & x & 1 \end{bmatrix}$$

$\frac{1}{2} R_1$

$$\begin{bmatrix} 1 & 2 & -2 \\ 1 & -4 & 3 \\ -6 & -9 & 5 \end{bmatrix}, L = \begin{bmatrix} 2 & 0 & 0 \\ x & 1 & 0 \\ x & x & 1 \end{bmatrix}$$

$R_2 - R_1$, $R_3 + 6R_1$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -6 & 5 \\ 0 & 3 & -7 \end{bmatrix}, L = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ -6 & x & 1 \end{bmatrix}$$

$R_3 + \frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -6 & 5 \\ 0 & 0 & -\frac{9}{2} \end{bmatrix}, L = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ -6 & -\frac{1}{2} & 1 \end{bmatrix}$$

$A = LU$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ -6 & -\frac{1}{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & -2 \\ 0 & -6 & 5 \\ 0 & 0 & \frac{-9}{2} \end{bmatrix} = \begin{bmatrix} 2 & 4 & -4 \\ 1 & -4 & 3 \\ -6 & -9 & 5 \end{bmatrix}$$ Proved!

$12 - \frac{5}{2} - \frac{9}{2} = 5$

$\cancel{-8+12}$

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Solving Linear System by LU Decomposition.

(1) $2x_1 + 6x_2 + 2x_3 = 2$
$-3x_1 - 8x_2 = 2$
$4x_1 + 9x_2 + 2x_3 = 3$

$$A = \begin{bmatrix} 2 & 6 & 2 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$AX = B$

$LUX = B$

$LY = B$, when $UX = Y$ & $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$\sim \frac{1}{2} R_1$

$$\begin{bmatrix} 1 & 3 & 1 \\ -3 & -8 & 0 \\ 4 & 9 & 2 \end{bmatrix}, L = \begin{bmatrix} 2 & 0 & 0 \\ \times & 1 & 0 \\ \times & \times & 1 \end{bmatrix}$$

$\sim R_2 + 3R_1,\ R_3 - 4R_1$

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & -2 \end{bmatrix}, L = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & & 1 \end{bmatrix}$$

$\sim R_3 + 3R_2$

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix}, L = \begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 1 \end{bmatrix}$$

To check $A = LU$ which gives same matrix

$LY = B$

$$\begin{bmatrix} 2 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore UX = Y$

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 14 \end{bmatrix}$$

$x_1 + 3x_2 + x_3 = 1$ — (i), $x_2 + 3x_3 = 5$ — (ii), $7x_3 = 14$ —

$\boxed{x_3 = 2}$ $\boxed{x_2 = -1}$ $\boxed{x_1 = 2}$

$2y_1 = 2$, $-3y_1 + y_2 = 2$, $4y_1 - 3y_2 + y_3 = 3$

$\boxed{y_1 = 1}$ $-3(1) + y_2 = 2$ $4(1) - 3(5) + y_3 = 3$

$\boxed{y_2 = 5}$ $y_3 = 14$

$2x_1 + 6x_2 + 2x_3 = 2$

$2(2) + 6(-1) + 2(2)$

$4 - 6 + 4$

$\boxed{2}$ Proved!

$2\vec{a}$ Scaling Up

$\frac{1}{2}\vec{a}$ Scaling down

$-1\times\vec{a}$ inverse direction.

Euclidean Vector Spaces:

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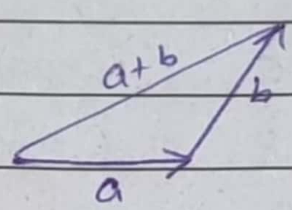
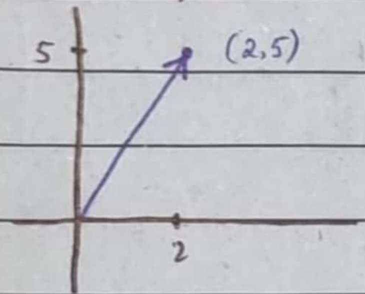
5 km Distance Scalar

5 km Distance due to east Vector.

$\vec{a} = 5$ $\longrightarrow$ $\vec{b}$ - $\longrightarrow$



$\vec{a} = (2,5) \to$ 2D vector / 2-tuple.
" Space



$\vec{a} = (2,5)$, $\vec{b} = (-1,4)$

$\vec{a} + \vec{b} = (1,9)$

$\vec{a} - \vec{b} = (3,1)$

Linear Combination

$2\vec{a} + 3\vec{b}$

Nom: $\sqrt{x^2 + y^2}$

$||a|| = \sqrt{(2)^2 + (5)^2}$

$||a|| = \sqrt{29}$

Eigen Value Can: Scale up
Scale down
Change direction of the vector
Zero.

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Q1(i)

$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}, X = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$AX = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 6 \end{bmatrix} \Rightarrow 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$AX = 3X$

$AX = \lambda X$

$AX - \lambda X = 0$

$AX - \lambda I X = 0$

$(A - \lambda I) X = 0$

$\det(A - \lambda I) = 0$

↳ Formula for finding Eigen Value of Eigen Vector (X)

Q) Find characteristic equation & characteristic polynomial.

Continue →

$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

Eigen Vector:

for $\lambda = -1$

$(A - \lambda I) X = 0$

$\left\{ \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - (-1) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} X = 0$

$\begin{bmatrix} 4 & 0 \\ 8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Augmented Matrix $\sim R_2 - 2R_1$

$\begin{bmatrix} 4 & 0 & 0 \\ 8 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$4x_1 = 0$

$x_1 = 0$

$X = \begin{bmatrix} 0 \\ 5 \end{bmatrix} = 5 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ which is Eigen Vector for $\lambda = -1$

Q1(i)

$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$

$\det(A - \lambda I) = 0$

$A - \lambda I = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\det(A - \lambda I) = \det \begin{bmatrix} 3-\lambda & 0 \\ 8 & -1-\lambda \end{bmatrix} = 0$

Characteristic Eqn

$(3-\lambda)(-1-\lambda) - 0 = 0$

$-3 - 3\lambda + \lambda + \lambda^2 = 0$

$\lambda^2 - 2\lambda - 3 = 0$ Characteristic Polynomial.

$\lambda^2 - 3\lambda + \lambda - 3 = 0$

$\lambda(\lambda - 3) + 1(\lambda - 3) = 0$

$(\lambda - 3)(\lambda + 1) = 0$

$\boxed{\lambda = -1}$ $\boxed{\lambda = 3}$ Eigen Value.

← Continue

$(\lambda + 1)(\lambda - 3) = 0$

$\lambda = 3$ $\lambda = -1$

$\begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$

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Q1(ii)

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Q) $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$\det(A-\lambda I)=0$ Characteristic Equa.

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}=0$$

expanding wrt 1^{st} row

$4-\lambda((1-\lambda)(1-\lambda)-0)-0+$
$1(0+2(1-\lambda))=0$

$(4-\lambda)(1-\lambda)^2+2(1-\lambda)=0$

$(1-\lambda)[(4-\lambda)(1-\lambda)+2]=0$

$(1-\lambda)(4-4\lambda-\lambda+\lambda^2+2)=0$

$(1-\lambda)(\lambda^2-5\lambda+6)=0$ Characteristic Polynomial

$1-\lambda=0$

$\boxed{\lambda=1}$

$\lambda^2-5\lambda+6=0$

$(\lambda-2)(\lambda-3)=0$

Eigen Values $\boxed{\lambda=2}$, $\boxed{\lambda=3}$

for $\lambda=1$:

$(A-\lambda I)X=0$

$$\left\{\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}-1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\}X=0$$

$$\begin{bmatrix} 4-1 & 0 & 1 \\ -2 & 1-1 & 0 \\ -2 & 0 & 1-1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}X$$

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Augmented Matrix: $\sim R_3-R_2$ $\sim 3R_2+2R_1$

$$\begin{bmatrix} 3 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 3 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 3 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$3x_1+x_3=0$ — (i)

$2x_3=0 \Rightarrow \boxed{x_3=0}$

$3x_1+0=0 \Rightarrow \boxed{x_1=0}$

Let $x_2=S$.

$$X=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ S \\ 0 \end{bmatrix}=S\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

In Pie $\lambda = 2$ & 3

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For: $\lambda = 2$:

$$\left\{\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\} x = 0$$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\sim R_2 + R_1,\ R_3 + R_1$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$2x_1 + x_3 = 0$

$-x_2 + x_3 = 0$

$\therefore \boxed{x_3 = S}$

$+x_2 = +S$

$\boxed{x_2 = S}$

$2x_1 + S = 0$

$\boxed{x_1 = -\frac{S}{2}}$

$$X = \begin{bmatrix} -\frac{S}{2} \\ S \\ S \end{bmatrix} = S\begin{bmatrix} -\frac{1}{2} \\ 1 \\ 1 \end{bmatrix}$$

$$\frac{S}{2}\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

For: $\lambda = 3$

$$\left\{\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\sim R_2 + 2R_1,\ \sim R_3 + 2R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 + x_3 = 0$

$-2x_2 + 2x_3 = 0$

$\therefore x_3 = S$

$-2x_2 + 2(S) = 0$

$-\cancel{2}x_2 = -\cancel{2}S$

$\boxed{x_2 = S}$

$\boxed{x_1 = -S}$

$$X = \begin{bmatrix} -S \\ S \\ S \end{bmatrix} \Rightarrow S\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Q2(ii)

Date: ______

(1) Find eigenvalues and bases (eigen vectors) for the eigenspaces of:

$$\begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix}$$

$A - \lambda I = 0$

$$\begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & 1 \\ -6 & -2 & 0 \\ 19 & 5 & -4 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

Characteristic Equation $= \begin{vmatrix} -2-\lambda & 0 & 1 \\ -6 & -2-\lambda & 0 \\ 19 & 5 & -4-\lambda \end{vmatrix} = 0$

$$-2-\lambda \begin{bmatrix} -2-\lambda & 0 \\ 5 & -4-\lambda \end{bmatrix} - 0 + 1 \begin{bmatrix} -6 & -2-\lambda \\ 19 & 5 \end{bmatrix}$$

$$-2-\lambda((-2-\lambda)(-4-\lambda)-0) + (-30)-(-2-\lambda)19$$

$$\cancel{-2-\lambda(8+2\lambda+4\lambda+\lambda^2)+24+6\lambda}$$

$$\cancel{(-4-\lambda)[(-2-\lambda(-2-\lambda))+(-6)] = 0}$$

$$\cancel{(-4-\lambda)[4+2\lambda+2\lambda+\lambda^2-6] = 0}$$

$$\cancel{-4-\lambda = 0} \quad ; \quad \cancel{\lambda^2+4\lambda-2 = 0}$$

$$\cancel{\lambda = -4}$$

$$-2-\lambda((-2-\lambda)(-4-\lambda)) + [-570(-2-\lambda)]$$

$$[(-2-\lambda)^2(-4-\lambda)] + [-570(-2-\lambda)]$$

$$(-2-\lambda)[(-2-\lambda)(-4-\lambda) - 570] = 0$$

$$-2-\lambda = 0 \quad ; \quad 8+2\lambda+4\lambda+\lambda^2-570 = 0$$

$$\lambda^2+6\lambda-262 = 0$$

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$$(-2-\lambda)^2(-4-\lambda)+[-30-19(-2-\lambda)]$$

$$-16-8\lambda-8\lambda-4\lambda^2-4\lambda-2\lambda^2-2\lambda^2-\lambda^3+8+19\lambda=0$$

$\boxed{\lambda^3+8\lambda^2+\lambda+8=0}$ Charateritic Polynomial.

$$\lambda^2(\lambda+8)+1(\lambda+8)=0$$

$\lambda+8=0$ $\quad\quad$ $\lambda^2+1=0$

$\boxed{\lambda=-8}$ $\quad\quad$ $\lambda=\pm\sqrt{-1}$ $\quad$ $\boxed{\text{Eigen Value}}$

No Sol

For $\lambda=-8$:

$$(A-\lambda I)x=0$$

$$\begin{bmatrix}-2&0&1\\-6&-2&0\\19&5&-4\end{bmatrix}-(-8)\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}x=0$$

$$\begin{bmatrix}6&0&1\\-6&6&0\\19&5&4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$\sim R_2+R_1$, $\sim R_3-2R_1$

$$\begin{bmatrix}6&0&1\\0&6&1\\7&5&2\end{bmatrix}$$

$\frac{1}{7}R_3$

$$\begin{bmatrix}6&0&1\\0&6&1\\1&\frac{5}{7}&\frac{2}{7}\end{bmatrix}$$

$\sim 6R_3-R_1$

$$\begin{bmatrix}6&0&1\\0&6&1\\0&\frac{30}{7}&\frac{5}{7}\end{bmatrix}$$

$\sim\frac{7}{30}R_3$

$$\begin{bmatrix}6&0&1\\0&6&1\\0&1&\frac{1}{6}\end{bmatrix}$$

$\sim 6R_3-R_2$

$$\begin{bmatrix}6&0&1\\0&6&1\\0&0&0\end{bmatrix}$$

$6x_1+x_3=0$

$6x_2+x_3=0$

$\therefore \boxed{x_3=S}$

$6x_1+S=0$ $\quad\quad$ $6x_2+S=0$

$\boxed{x_1=\frac{-S}{6}}$ $\quad\quad$ $\boxed{x_2=\frac{-S}{6}}$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-\frac{S}{6}\\-\frac{S}{6}\\S\end{bmatrix}\Rightarrow S\begin{bmatrix}-\frac{1}{6}\\-\frac{1}{6}\\1\end{bmatrix}$$

$$\frac{S}{6}\begin{bmatrix}-1\\-1\\6\end{bmatrix}$$

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Diagonalization:

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$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

For egenvalues

let $(A-\lambda I) = 0$

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = 0$$

$\lambda = 1,\ \lambda = 2,\ \lambda = 3$

For Eigen Vector:

$(A - \lambda I)X = 0$

for $\lambda = 1$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad P_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

for $\lambda = 2$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \gamma\begin{bmatrix} -\frac{1}{2} \\ 1 \\ 1 \end{bmatrix} = \frac{\gamma}{2}\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \qquad P_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \qquad P_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$P^{-1} = ?$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & -1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\sim R_1 + 2R_2,\ \sim R_3 + 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 1 \end{array}\right]$$

$\sim R_1 - R_3,\ \sim R_2 - R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 2 & 0 & 1 \end{array}\right]$$

$\sim -1 \times R_2,\ \sim -1 \times R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 & -1 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix}$$

Now, $D = P^{-1}AP$

$$D = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & -2 & -3 \\ 1 & 4 & 3 \\ 0 & 4 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

5.4 (Q3)

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Differential Equations: (Diagonalization Method)

① Solve the system.

(a) $y_1' = y_1 + 4y_2$

$y_2' = 2y_1 + 3y_2$

(b) Find the solution that satisfies the initial condition $y_1(0) = 0,\ y_2(0) = 0$.

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

For EigenValues:

$$\begin{bmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(3-\lambda) - 8 = 0$$

$$\boxed{\lambda = -1} \; ; \; \boxed{\lambda = 5}$$

For EigenVector:

$$(A - \lambda I)X = 0$$

For $\lambda = -1$

$$\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Augmented Matrix:

$$\begin{bmatrix} 2 & 4 & 0 \\ 2 & 4 & 0 \end{bmatrix}$$

$\sim R_2 - R_1$

$$\begin{bmatrix} 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$2x_1 + 4x_2 = 0$$

$$x_1 + 2x_2 = 0$$

$$x_1 = -2x_2$$

Let $x_2 = \gamma$

$$x_1 = -2\gamma$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2\gamma \\ \gamma \end{bmatrix} = \gamma\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\boxed{P_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}}$$

For $\lambda = 5$

$$\begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Augmented Matrix

$$\begin{bmatrix} -4 & 4 & 0 \\ 2 & -2 & 0 \end{bmatrix}$$

$\sim 2R_2 + R_1$

$$\begin{bmatrix} -4 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$-4x_1 + 4x_2 = 0$$

$$x_1 = x_2$$

Let $x_2 = t$

$$x_1 = t$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix} \Rightarrow t\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \boxed{P_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$\cancel{2x^2 - 4x + 2}$

$\cancel{2x}$

making the substitution

$Y = PU$ then $Y' = PU'$

$U' = DU = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$

$U_1' = -U_1$ | $U_2' = 5U_2$

$U_1 = C_1 e^{-x}$ | $U_2 = C_2 e^{5x}$.

Date:

$P = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}$ ✓

$P^{-1} = ?$

$D = P^{-1} A P$

$D = \begin{bmatrix} -1 & 0 \\ 0 & 5 \end{bmatrix}$

Now $Y = PU$

$Y = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$Y = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_1 e^{-x} \\ C_2 e^{5x} \end{bmatrix}$

$Y_1 = -2C_1 e^{-x} + C_2 e^{5x}$

$Y_2 = C_1 e^{-x} + C_2 e^{5x}$

$D = P^{-1} A P$

$D = \begin{bmatrix} & \\ & \end{bmatrix}$

(b) Now using the initial condition:

$Y_1(0) = 0$ means, $y = 0$ when $x = 0$

$Y_1 = -2C_1 e^{-x} + C_2 e^{5x}$

$0 = -2C_1 e^{0} + C_2 e^{5(0)}$

$0 = -2C_1 + C_2$

$-2C_1 + C_2 = 0$ — (i)

And

$Y_2(0) = 0$

$Y_2 = C_1 e^{-x} + C_2 e^{5x}$

$0 = C_1 e^{0} + C_2 e^{5(0)}$

$0 = C_1 + C_2$

$C_1 + C_2 = 0$ — (ii)

Solving eqs (i) & (ii)

$C_1 = 0$ and $C_2 = 0$

So, $Y_1 = -2(0)e^{-x} + 0e^{5x}$

$\boxed{Y_1 = 0}$

and $Y_2 = 0e^{-x} + 0e^{5x}$

$\boxed{Y_2 = 0}$

Date:

Q) (a) Solve the system: $y_1' = y_1 + 3y_2$
$y_2' = 4y_1 + 5y_2$

(b) Find the solution that satisfies the conditions: $y_1(0) = 2$, $y_2(0) = 1$

$A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ For Eigen Vectors:

let $(A - \lambda I) = 0$

$\begin{bmatrix} 1-\lambda & 3 \\ 4 & 5-\lambda \end{bmatrix}$

$(1-\lambda)(5-\lambda) - 12$

$\lambda^2 - 6\lambda - 7$

$\boxed{\lambda = 7}$; $\boxed{\lambda = -1}$

For Eigen Vector:

$(A - \lambda I)X = 0$

For $\lambda = -1$

$\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} + 1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\sim R_2 - 2R_1$

$\begin{bmatrix} 2 & 3 & 0 \\ 4 & 6 & 0 \end{bmatrix}$ $\begin{bmatrix} 2 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$2x_1 + 3x_2 = 0$

$2x_1 = -3x_2$

$x_1 = \frac{-3}{2}x_2$

let $x_2 = \gamma$ then,

$x_1 = \frac{-3}{2}\gamma$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3/2\,\gamma \\ \gamma \end{bmatrix}$

$\frac{1}{2}\gamma \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ $P_1 = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$\rightarrow$ C

$C \rightarrow$

For $\lambda = 7$

$\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} - 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\sim 6R_2 + 4R_1$

$\begin{bmatrix} 6 & 3 & 0 \\ 4 & -2 & 0 \end{bmatrix}$ $\begin{bmatrix} -6 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$-6x_1 + 3x_2 = 0$ let $x_2 = \gamma$

$-6x_1 = -3x_2$ $x_1 = \frac{1}{2}\gamma$

$x_1 = \frac{1}{2}x_2$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/2\,\gamma \\ \gamma \end{bmatrix}$

$\frac{1}{2}\gamma \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$P = \begin{bmatrix} -3 & 1 \\ 2 & 2 \end{bmatrix}$ $P_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Now,

$D = P^{-1}AP$ $\begin{bmatrix} 7 & 0 \\ 0 & -1 \end{bmatrix}$

Making the substitution: $y = Pu$ then $y' = Pu'$

$u' = Du \begin{bmatrix} 7 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

$u_1' = 7u_1$; $u_2' = -u_2$

Integrating ; Integrating

$u_1 = C_1 e^{7x}$; $u_2 = C_2 e^{-x}$

Now $y = Pu$

$y = \begin{bmatrix} -3 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ $y = \begin{bmatrix} -3 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} C_1 e^{7x} \\ C_2 e^{-x} \end{bmatrix}$

$y_1 = -3C_1 e^{7x} + C_2 e^{-x}$

$y_2 = 2C_1 e^{7x} + 2C_2 e^{-x}$

Date:

(b) x $y = 2$ when $x = 0$

$y_1(0) = 2$

$2 = -3C_1 e^{7(0)} + C_2 e^{-0}$

$2 = -3C_1 + C_2$

$-3C_1 + C_2 = 2$ — (i)

$y_2(0) = 1$

$1 = 2C_1 e^{7(0)} + 2C_2 e^{-0}$

$1 = 2C_1 + 2C_2$

$2C_1 + 2C_2 = 1$ — (ii)

Mul (i) by 2 then Sub (ii)

$$\begin{array}{r} 2C_1 + 2C_2 = 1 \\ \mp 6C_1 \pm 2C_2 = \mp 4 \\ \hline 8C_1 = -3 \end{array}$$

$\boxed{C_1 = -3/8}$

(ii) $-3/8 + 2C_2 = 1$

$\boxed{C_2 = 7/8}$

$y_1 = -3\left(\frac{-3}{8}\right)e^{7x} + \left(7/8\right)e^{-x}$

$y_1 = \frac{9}{8}e^{7x} + \frac{7}{8}e^{-x}$

$y_2 = 2\left(-3/8\right)e^{7x} + 2\left(7/8\right)e^{-x}$

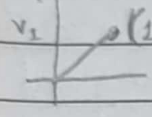
$y_2 = -3/4\, e^{7x} + 7/8\, e^{-x}$

Date: ______

Set V $\quad V_1 \quad V_2 \quad V_3$

$V = \{(1,2), (3,4), (2,5) \cdots\}$

2D Vector, (2 tuples), R^2



Properties:

(i) $V_1 + V_2 = (4,6) \in V$ Closure Property

(ii) $V_1 + V_2 = V_2 + V_1$ Commutative

(iii) $V_1 + (V_2 + V_3) = (V_1 + V_2) + V_3$ Associative

(iv) $kV_1 = (kV_1, kV_2)$ $k=5$ $V_1 = (1,2)$

$kV_1 = (5,10) \in V$

(v) $k(V_1 + V_2) = kV_1 + kV_2$ Distributive

(vi) $(kl)V_1 = k(lV_1)$ $k=2, l=3, V_1=(1,2)$

$lV_1 = (3,6)k$ $\quad kl = 6 \quad kl(V) = 6(1,2)$

$l = (6,12)$ $\quad = (6,12)$

(vii) $V_1 + \underbrace{0}_{(0,0)} = V_1$

(viii) $V_1 = (1,2)$, $V_1' = (-1,-2)$

(ix) $1V_1 = V_1$, $1V_2 = V_2$

(x) $(k+l)V_1 = kV_1 + lV_1$

Vector having all these properties is called vector spaces.

Set of Real Num:

$R = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$

As real num obey all properties of vector then this can also be called a vector space.

$N = \{1, 2, 3, 4, 5, \cdots\}$

Natural No. is not a vector space cuz it doesn't obey some of the 10 properties.

Additive Identity X: $1+0 = 1$ X, $1+(-1) = 0$ X.

The subset of set V having values (vectors, real No) and if that subset obeys the 10 then that subset is called sub-spaces.

Linear Combination:

$u = (1,0)$, $v = (0,1)$, $w = (2,3)$

$(2,3) = 2(1,0) + 3(0,1)$

$(2,3) = (2,3)$

$w = \underbrace{2u + 3v}_{\text{Standard Vector.}}$

W is the linear combination of u & v.

MIGHTY PAPER PRODUCT

Q5(ii)

Date: ______

Q) Show that $W=(2,2,2)$ is a linear Combination of $u=(0,-2,2)$, $v=(1,3,-1)$

$W = k_1 u + k_2 v$

$(2,2,2) = k_1(0,-2,2) + k_2(1,3,-1)$

$(2,2,2) = (0k_1, -2k_1, 2k_1) + (k_2, 3k_2, -k_2)$

$(0k_1 + k_2, -2k_1 + 3k_2, 2k_1 - k_2)$

$0k_1 + k_2 = 2$ — (i) $\boxed{k_2 = 2}$

$-2k_1 + 3k_2 = 2$ — (ii)

$2k_2 - k_2 = 2$ — (iii)

$2k_1 - 2 = 2 \Rightarrow \boxed{k_1 = 2}$

Since $W = 2u + 2v$

then W is a lccom of u & v.

Spanning set is a set of 2D vectors that generate the vector space V.

$S = \{(1,0),(0,1)\}$

$R^2 = V = \{(1,2),(-1,2),(3,4)\}$

$(1,2) = 1(1,0) + 2(0,1)$

$(-1,2) = -1(1,0) + 2(0,1)$

$(3,4) = 3(1,0) + 4(0,1)$

The elements of Vector Space V is a linear combination of Spanning space S.

Q5(i)

Q) Determine whether $V_1 = (1,1,2)$, $V_2 = (1,0,1)$, $V_3 = (2,1,3)$ spanning Vector Space R

$(b_1, b_2, b_3) = k_1V_1 + k_2V_2 + k_3V_3$

$= k_1(1,1,2) + k_2(1,0,1), k_3(2,1,3)$

$k_1 + k_2 + 2k_3 = b_1$

$k_1 + 0k_2 + k_3 = b_2$

$2k_1 + k_2 + 3k_3 = 3b_3$

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{vmatrix}$$

Find determinent

$\sim R_2 - R_1, \sim R_3 - 2R_1$

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{vmatrix}$$

Expanding wrt row 1

$$1\begin{vmatrix} -1 & -1 \\ -1 & -1 \end{vmatrix} - 0 + 0$$

$1 - 1 = 0$

Since $\det(A) = 0$

then V_1, V_2 & V_3 doesn't span R^3

Date:

Linear Independent/Dependent

$V = \{V_1, V_2, V_3, \cdots V_r\}$

$k_1V_1, k_2V_2, k_3V_3 \cdots k_rV_r = 0$

$k_1 = k_2 = k_3 = \cdots = k_r = 0$

When scalar multiplication & addition = 0 then that case is linear Independence but if the k set of values ≠ 0 & their sum = 0 then that case is called linear dependent.

Q7(i)

Q) Determine whether $V_1 = (1,-2,3), V_2 = (5,6,-1), V_3 = (3,2,1)$ are linearly Independent or dependent in R^3.

$$|A| = \begin{vmatrix} 1 & 5 & 3 \\ -2 & 6 & 2 \\ 3 & -1 & 1 \end{vmatrix}$$

$\sim R_2 + 2R_1, \sim R_3 - 3R_1$

$$|A| = \begin{vmatrix} 1 & 5 & 3 \\ 0 & 16 & 8 \\ 0 & -16 & -8 \end{vmatrix}$$

$$|A| = 1 \begin{vmatrix} 16 & 8 \\ -16 & -8 \end{vmatrix}$$

$= -128 - (-128) = 0$

Since $\det(A) = 0$

therefor V_1, V_2 & V_3 are ~~lo~~ dependent.

Q7(ii)

Q) $V_1 = (-3,0,4), V_2 = (5,-1,2), V_3 = (1,1,3)$

$$A = \begin{bmatrix} -3 & 5 & 1 \\ 0 & -1 & 1 \\ 4 & 2 & 3 \end{bmatrix} \Rightarrow 3R_3 + 4R_1 \begin{bmatrix} -3 & 5 & 1 \\ 0 & -1 & 1 \\ 0 & 26 & 14 \end{bmatrix}$$

$$-5 \begin{vmatrix} 0 & 1 \\ 0 & 14 \end{vmatrix} + 1 \begin{vmatrix} 0 & -1 \\ 0 & 26 \end{vmatrix}$$

$= -120$

$$|A| = -3 \begin{vmatrix} -1 & 1 \\ 26 & 14 \end{vmatrix} - \cancel{0}$$

$= -3(26) - (3)(-14)$

$= -78 - (239)$

$= -117$

∴ Linearly Independent.

Date:

① $V_1 = (1,-2,3)$, $V_2 = (5,6,-1)$, $V_3 = (3,2,1)$

$k_1V_1 + k_2V_2 + k_3V_3 = 0$

$k_1(1,-2,3) + k_2(5,6,-1) + k_3(3,2,1)$

$k_1 + 5k_2 + 3k_3 = 0$

$-2k_1 + 6k_2 + 2k_3 = 0$

$3k_1 - 2k_2 + k_3 = 0$

Augmented Matrix: $\sim R_2 + 2R_1, \sim R_3 - 3R_1$ $\sim R_3 + R_2$

$$\begin{bmatrix} 1 & 5 & 3 & 0 \\ -2 & 6 & 2 & 0 \\ 3 & -1 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 3 & 0 \\ 0 & 16 & 8 & 0 \\ 0 & -16 & -8 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 5 & 3 & 0 \\ 0 & 16 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$k_1 + 5k_2 + 3k_3 = 0$ — (i)

$16k_2 + 8k_3 = 0$ — (ii)

$16k_2 = -8k_3$

$k_2 = -\frac{1}{2}k_3$

Assume $k_3 = t$

$\boxed{k_2 = -\frac{1}{2}t}$

(i) $k_1 + 5(-\frac{1}{2}t) + 3(t)$

$\boxed{k_1 = -\frac{1}{2}t}$

$\therefore$ Linearly Dependent.

Bases: Date:

$S = \{V_1, V_2, V_3\} \Rightarrow$ subset of vector spaces.

Set is bases if: (i) L. Independent (ii) S spans V. Space V.

Just find the det of vector & if it $\neq 0$ then it is a base.

Q) Find a basis for the solution space of the homogenious linear system & find the dimensions of that space.

$$x_1 + x_2 - x_3 = 0$$
$$-2x_1 - x_2 + 2x_3 = 0$$
$$-x_1 + x_3 = 0$$

Augmented Matrix

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ -2 & -1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{\sim R_2 + 2R_1,\ R_3 + R_1} \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{\sim R_3 - R_2} \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 + x_2 - x_3 = 0$ — (i)

$x_2 = 0$ — (ii)

$x_1 + (0) - x_3 = 0$

let $x_3 = t$

$x_1 = t$

$x_2 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ t \end{bmatrix} \Rightarrow t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

Base: $(1, 0, 1)$

Dimension: 1

Date:

Q) $x_1 + x_2 + 2x_3 = 8$
$-x_1 - 2x_2 + 3x_3 = 1$
$3x_1 - 7x_2 + 4x_3 = 10$

$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 1 \\ 10 \end{bmatrix}$

Augmented Matrix

$$A = \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & \end{matrix} \\ \begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix} \end{array}$$

After Multiplication

$A = \begin{bmatrix} 3 & 1 & 4 \\ -3 & -2 & 6 \\ 9 & -7 & 8 \end{bmatrix}$

$x_1 C_1 + x_2 C_2 + x_3 C_3$
$3(1,-1,3) + 1(1,-2,-7) + 2(2,3,4)$
$(3,-3,9) + (1,-2,-7) + (4,6,8)$
$(8,1,10)$

$x_1 = 3,\ x_2 = 1,\ x_3 = 2$

↳ multiply C_1, C_2, C_3 with x_1, x_2, x_3.

$$3\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 1 \\ -2 \\ -7 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$B = \begin{bmatrix} 8 \\ 1 \\ 10 \end{bmatrix}$

↳ Column Space (linear Combination of Matrix)

Solution space of a Homogenous system of Equation called a Null Space.

Q) Show that the vectors $V_1 = (1,2,1)$, $V_2 = (2,9,0)$ & $V_3 = (3,3,4)$ form basis of R^3.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{bmatrix} \quad \sim R_2 - 2R_1, \sim R_3 - R_1 \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & -3 \\ 0 & -2 & 1 \end{bmatrix}$$

$$1\begin{vmatrix} 5 & -3 \\ -2 & 1 \end{vmatrix} - 0 + 0$$

$5 - 6 = -1$

∴ The det is non-zero which proves that V_1, V_2 & V_3 form basis.

Date: ________

Q5(i) Show that vector $w = (9,2,7)$ is a linear combination of $u = (1,2,-1)$ and $v = (6,4,2)$ for $k_1 = -3$ and $k_2 = 2$.

$W = k_1 u + k_2 V$

$$(9,2,7) = -3(1,2,-1) + 2(6,4,2)$$
$$= (-3,-6,3) + (12,8,4)$$
$$(-3+12, -6+8, 3+4)$$
$$(9,2,7) \quad \therefore \text{Proved!}$$

(Q.6)(ii) Determine whether $u = (2,2,2)$, $v = (0,0,3)$ & $w = (0,1,1)$ span vector R^3.

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix} \quad \sim R_3 - R_1, \sim R_2 - R_1 \quad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 3 & 1 \end{bmatrix}$$

$$2\begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix} - 0 + 0$$

$$2(0 \times 1) - 2(3 \times 1)$$
$$0 - 6$$

$\therefore$ Non-Zero Det So u, v & w span vector R^3.

Q9

Date: ____________

Q) Find bases for the row & column spaces of:

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

$$\begin{array}{cccccc} C_1 & & C_3 & & C_5 & \end{array}$$

$\sim R_2 - 2R_1 , R_3 - 2R_1 , \sim R_4 + R_1$

$$\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 0 & 0 & 1 & 3 & -2 & -6 \\ 0 & 0 & 1 & 3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\sim R_3 - R_2$

$$\begin{array}{c|c} & \begin{array}{cccccc} C_1 & & C_3 & & C_5 & \end{array} \\ \begin{array}{c} r_1 \rightarrow \\ r_2 \rightarrow \\ r_3 \rightarrow \\ R = \end{array} & \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 0 & 0 & 1 & 3 & -2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Bases for Row Spaces:

$r_1 = [1, -3, 4, -2, 5, 4]$

$r_2 = [- - - - - - - - -]$

$r_3 = [- - - - - - - - -]$

Bases for Column Spaces (R)

$$C_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, C_3 = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, C_5 = \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

Q10(i)

Q) Find Bases for null space for A.

$\sim R_2 - 5R_1 , R_3 - 7R_1$

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & -19 & 0 \\ 0 & 1 & -19 & 0 \end{bmatrix}$$

$\sim R_3 - R_2$

$$\begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & -19 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - x_2 + 3x_3 = 0$ — (i)

$x_2 - 19x_3 = 0$ — (ii)

(ii) $x_2 = 19x_3 \quad \therefore x_3 = t$

$\boxed{x_2 = 19t}$

using x_2, x_3 in (i)

$x_1 - 19t + 3t = 0$

$x_1 - 16t = 0$

$\boxed{x_1 = 16t}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16t \\ 19t \\ t \end{bmatrix} = t \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix}$$

Bases for null spaces

$$\begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix}$$

Date: ____________

Q1) Find Components of vector $\overrightarrow{P_1P_2}$

(a) $P_1(3,5)$ $P_2(2,8)$ (b) $P_1(5,-2,1)$, $P_2(2,4,2)$ (c) $P_1(0,0,0)$, $P_2(-1,6,1)$

$P_2 - P_1$

$\overrightarrow{P_1P_2} = (-1, 3)$ $\overrightarrow{P_1P_2} = (-3, 6, 1)$ $\overrightarrow{P_1P_2} = (-1, 6, 1)$

Q.2) Let $u = (4,-1)$, $v = (0,5)$ and $w = (-3,-3)$ - Find the components of:

(a) $u + w$ (b) $v - 3u$ (c) $3v - 2(u+2w) \Rightarrow (0,15) - 2(-2,-7)$

$(1,-4)$ $(0,5) - (12,-3)$ $(0,15) - (-4,-18)$

$(-12, 8)$ $(4, 29)$

Q.4) If $u = (-2,-1,4,5)$, $v = (3,1,-5,7)$ & $w = (-6,2,1,1)$ evaluate the expression.

(a) $\|3u - 5v + w\|$

$(-6,-3,12,15) - (15,5,-25,35) + (-6,2,1,1) \Rightarrow (27,-6,38,19)$

$\sqrt{(27)^2 + (-6)^2 + (38)^2 + (19)^2} \Rightarrow \sqrt{2570}$

(b) $\|3u\| - 5\|v\| + \|w\|$

$|(-6,-3,12,15)| - 5|(3,1,-5,7)| + |(-6,2,1,1)|$

$\sqrt{6^2+3^2+12^2+15^2} - 5\sqrt{3^2+1^2+5^2+7^2} + \sqrt{6^2+2^2+1^2+1^2}$

$3\sqrt{46} - 10\sqrt{21} + \sqrt{42}$

Q.5) Let $u = (2,-2,3)$, $v = (1,-3,4)$ and $w = (3,6,-4)$

(a) $\|u + v\| = (3,-5,7)$ (b) $\|u\| + \|v\|$

$\sqrt{9+25+49}$ $\sqrt{4+4+9} + \sqrt{1+9+16}$

$\sqrt{83}$ $\sqrt{17} + \sqrt{26}$

Q6) Find the Euclidean distance between u and v.

(a) $u = (3,3,3)$ & $v = (1,0,4)$ (b) $u = (0,-2,-1,1)$ & $v = (-3,2,4,4)$

$\sqrt{(1-3)^2 + 3^2 + (4-3)^2}$ $\sqrt{(-3)^2 + (2+2)^2 + (4+1)^2 + (4-1)^2}$

$\sqrt{14}$ $\sqrt{59}$