Pattern Recognition
Assignment 1

Department of Computer Science and Engineering
Indian Institute of Technology, Madras

Singular Value Decomposition, Eigen Value Decomposition and Polynomial Regression

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Group 17

Contents

1 Singular Value Decomposition

1.1 SVD on Gray Scale Image

Intuition

Images can be compressed and stored by significant Eigen vectors. But, there can be degradation in the quality. We experimented SVD on Gray scale image by converting the given color image to gray scale form.

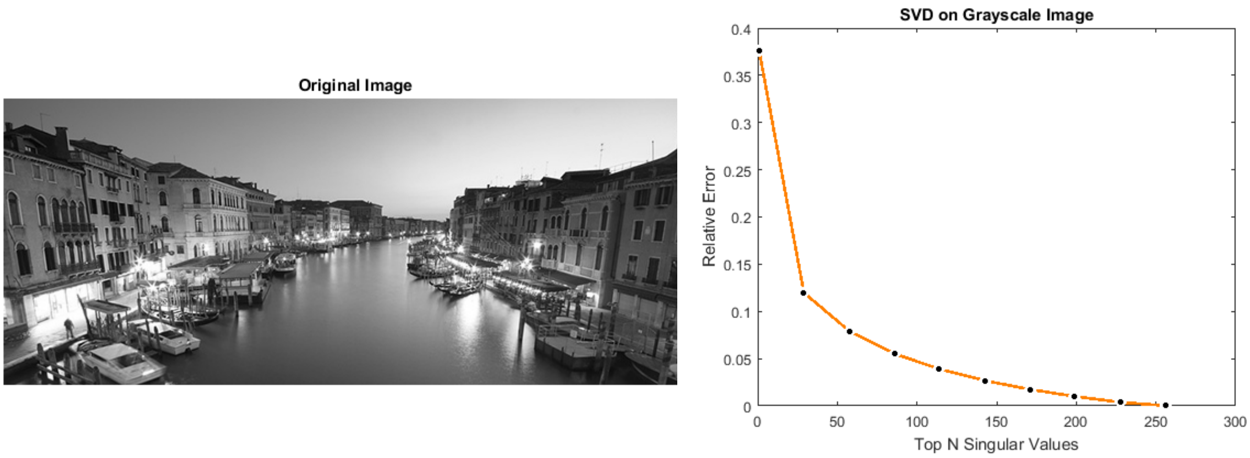
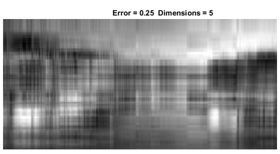


Figure 1: Original image and its corresponding error plot

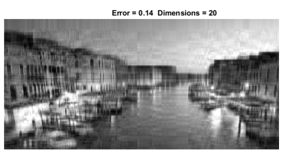
Inference

As we increase the number of Eigen values, Frobenius error decreases. Reconstructed images are shown below.

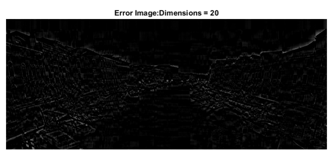


(a) Reconstruction with Top 5 Dimension

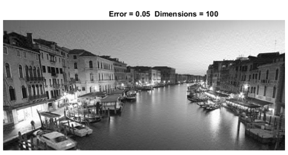
(b) Error Image with Top 5 Dimension



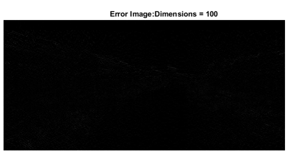
(c) Reconstruction with Top 20 Dimension



(d) Error Image with Top 20 Dimension

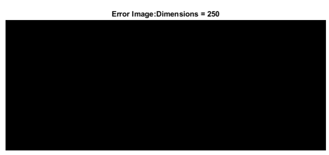


(e) Reconstruction with Top 100 Dimension



(f) Error Image with Top 100 Dimension

(g) Reconstruction with Top 250 Dimension



(h) Error Image with Top 250 Dimension

Figure 2: Reconstruction of Gray scale Image with SVD

1.2 SVD on Color bands

Intuition

As compression and reconstruction of Gray scale Image was successful, we applied SVD on each color bands of the rectangular image.

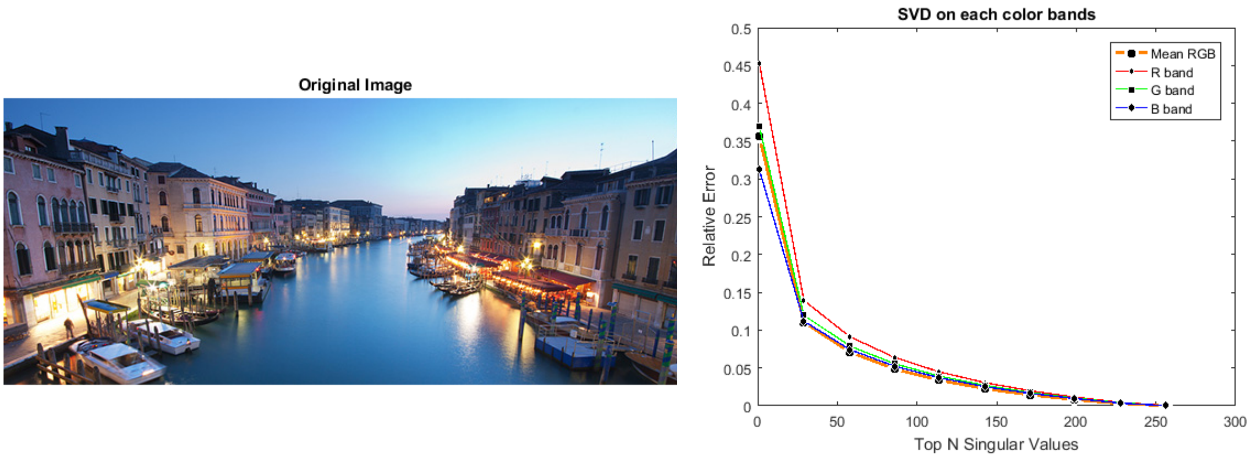
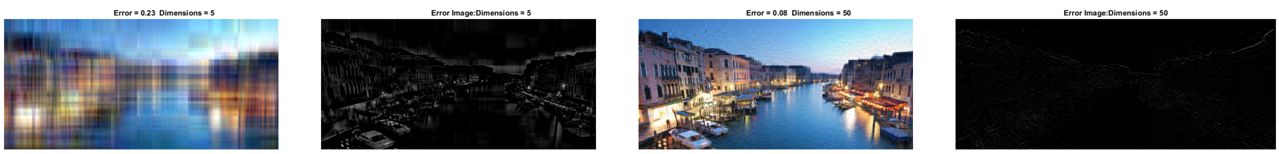


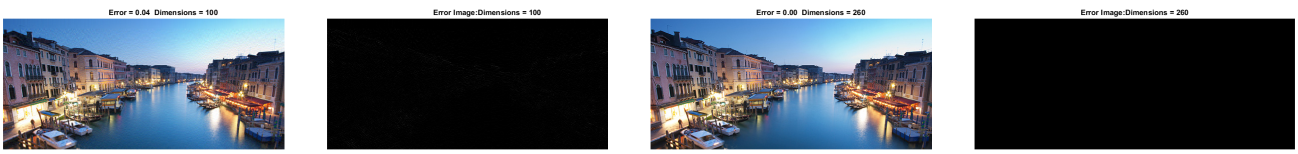
Figure 3: Original image and error plot of 3 color bands

Inference

For the given Image, decrease in Frobenius error varies for each color band. Reconstructed images are shown below.



(a) Reconstruction with Top 5 Dimension
(b) Error Image with Top 5 Dimension
(c) Reconstruction with Top 50 Dimension
(d) Error Image with Top 50 Dimension



(e) Reconstruction with Top 100 Dimension
(f) Error Image with Top 100 Dimension
(g) Reconstruction with Top 260 Dimension
(h) Error Image with Top 260 Dimension

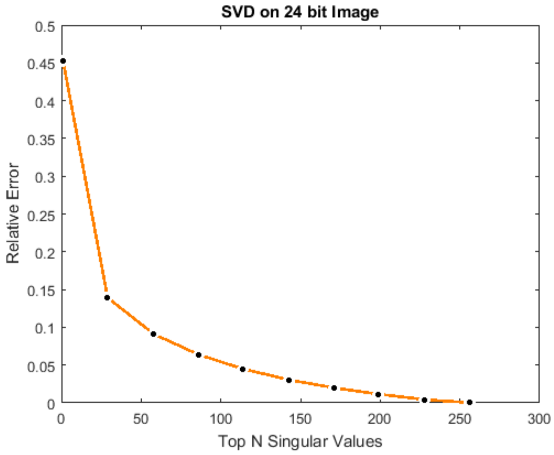
Figure 4: Reconstruction of Rectangular Image with SVD on each color bands

1.3 SVD on Concatenated color bands

Intuition

We experiment SVD by concatenating Red, Green and Blue color bands into a 24 bit binary value and reconstructed the image by splitting it into 8 bit binary value for each color band.

(a) Original Image

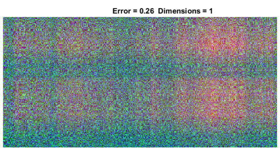


(b) Error Plot

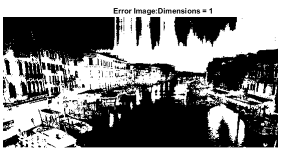
Figure 5: Original image and error plot

Inference

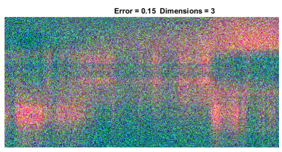
For the given Image, there is a decrease in Frobenius error as we increase the number of Eigen values. Reconstructed images are shown below.



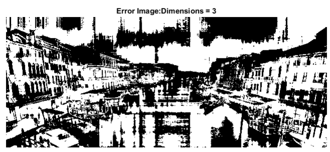
(a) Reconstruction with Top 1 Dimension



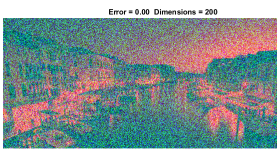
(b) Error Image with Top 1 Dimension



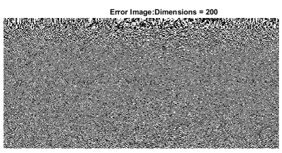
(c) Reconstruction with Top 3 Dimension



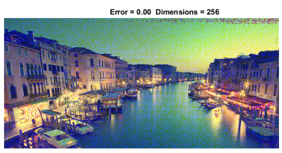
(d) Error Image with Top 3 Dimension



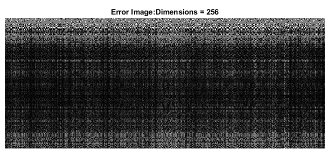
(e) Reconstruction with Top 200 Dimension



(f) Error Image with Top 200 Dimension



(g) Reconstruction with Top 256 Dimension



(h) Error Image with Top 256 Dimension

Figure 6: Reconstruction of Rectangular Image with SVD on concatenated color band

Inference

Reconstructed Images are so noisy. Although we take all the Eigen values, color information is spoiled in the reconstructed image. The The silver lining is that we could get the context of the image.

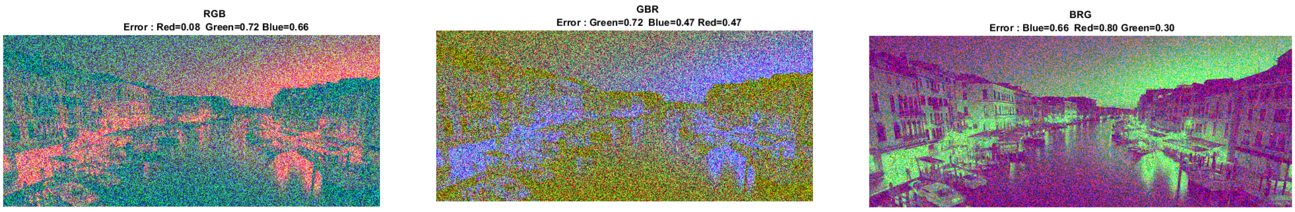
1.4 Rearrangement of color bands in Concatenation

Intuition

We experiment SVD by concatenating Red, Green and Blue color bands into a 24 bit binary value and reconstructed the image by splitting it into 8 bit binary value for each color bands. But, we concatenate the bands in various ways. Reconstructed images are shown below.

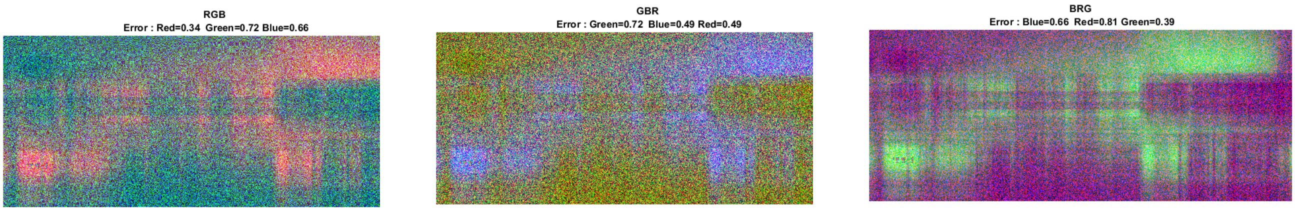
(a) RGB (b) GBR (c) BRG

Figure 7: Top 256 Dimensions



(a) RGB (b) GBR (c) BRG

Figure 8: Top 240 Dimensions



(a) RGB (b) GBR (c) BRG

Figure 9: Top 3 Dimensions

Inference

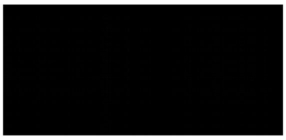
Reconstructed images are so noisy. Although we take all the Eigen values, color information is spoiled in the reconstructed image. The silver lining is that we could get the context of the image.

1.5 SVD with Random Singular Values

Intuition

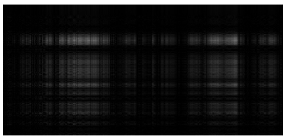
Instead of taking top N singular values, we reconstructed images by choosing random singular values. Reconstructed gray scale and colored images are shown below.

1.5.1 Gray Scale Image



(a) Reconstruction with 1 random dimension

(b) Error image for 1 random dimension



(c) Reconstruction with 10 random dimensions

(d) Error image for 10 random dimensions

(e) Reconstruction with 100 random dimensions

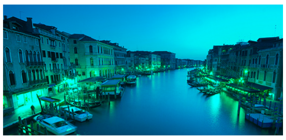
(f) Error image for 100 random dimensions

(g) Reconstruction with 200 random dimensions

(h) Error image for 200 random dimensions

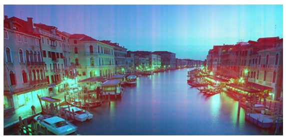
Figure 10: Reconstruction of Gray Scale Image with SVD using Random Singular Values

1.5.2 Colored Image



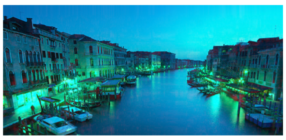
(a) Reconstruction with 1 random dimension

(b) Error image for 1 random dimension



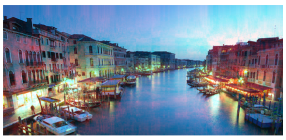
(c) Reconstruction with 10 random dimensions

(d) Error image for 10 random dimensions



(e) Reconstruction with 100 random dimensions

(f) Error image for 100 random dimensions



(g) Reconstruction with 200 random dimensions

(h) Error image for 200 random dimensions

Figure 11: Reconstruction of Colored Image with SVD using Random Singular Values

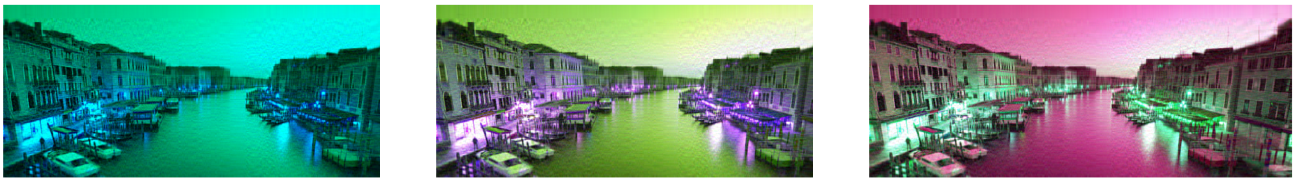
Inference

We observed that the error of reconstructed image does not follow any order when we increase the number of dimensions randomly as the strength of the Eigen values varies randomly.

1.6 Rearrangement of color bands

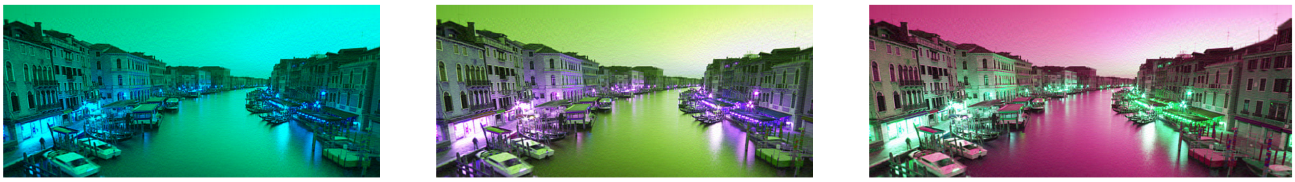
Intuition

To perform SVD on each color band and permute the reconstructed color bands while reconstructing the images. Reconstructed images are shown below.



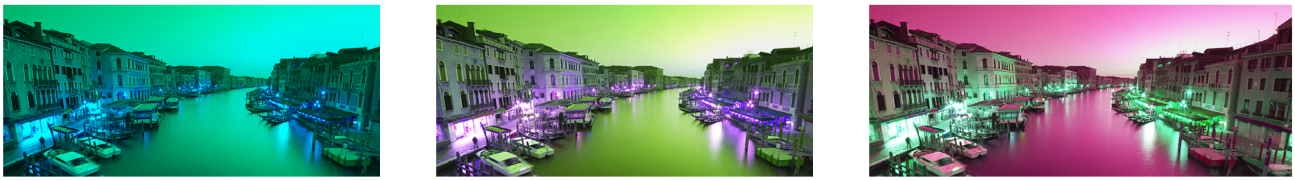
(a) RGB (b) GBR (c) BRG

Figure 12: Top 50 Dimensions



(a) RGB (b) GBR (c) BRG

Figure 13: Top 100 Dimensions



(a) RGB (b) GBR (c) BRG

Figure 14: Top 260 Dimensions

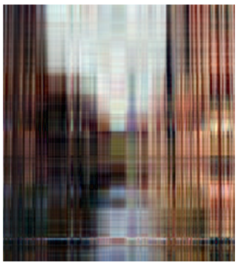
Inference

In color bands rearrangement, we observed that reconstruction is better than that of concatenated color bands. But, color information is not preserved in the prior method.
For the given image, RGB permutation emphasize shaded green, GBR permutation emphasize pale green and BRG permutation emphasize redness.

1.7 SVD on Square Image

Intuition

To perform SVD on square images. Reconstructed images are shown below.



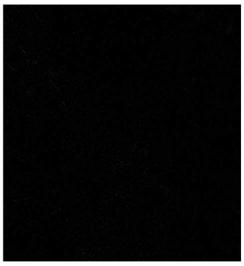
(a) Reconstruction with Top 5 Dimension

(b) Error Image with Top 5 Dimension

(c) Reconstruction with Top 50 Dimension

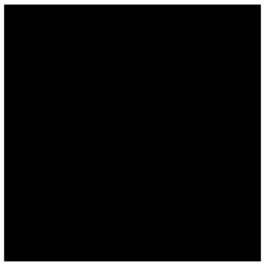
(d) Error Image with Top 50 Dimension

(e) Reconstruction with Top 100 Dimension



(f) Error Image with Top 100 Dimension

(g) Reconstruction with Top 256 Dimension



(h) Error Image with Top 256 Dimension

Figure 15: Reconstruction of square image using SVD

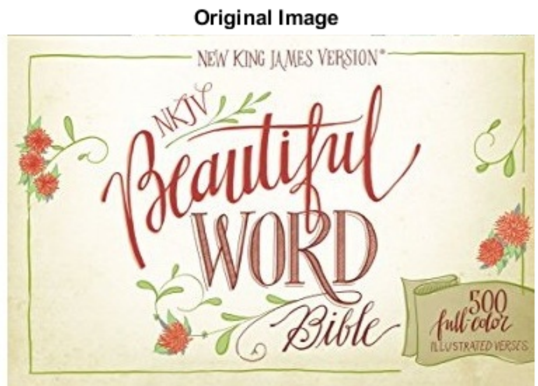
Inference

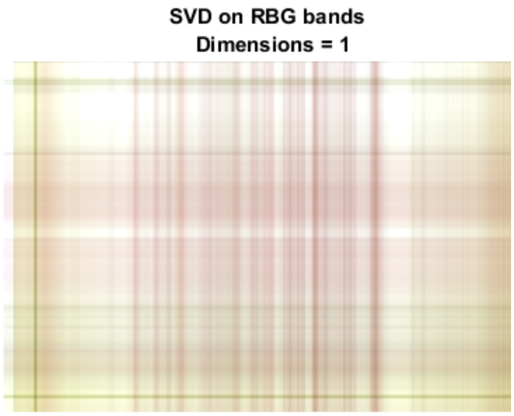
We observed that the top N singular values in SVD contribute better on image reconstruction than top N Eigen values in EVD. Analysis on the same is discussed in the later section.

1.8 Context Attention with SVD

Intuition

From the inference of the last experiment, we found that context is retained in the reconstructed image. What if context present only in a single color band? We can represent the context with Singular values of significant color band. We captured an image with information in the Red band. We reduced dimensions of Green and Blue bands and retained dimensions of Red band.





(a) Reconstruction with one Dimension in each color bands

(b) Reconstruction with one Dimension in Green and Blue bands and all Dimensions in Red band

Figure 16: Context Attention with SVD

Inference

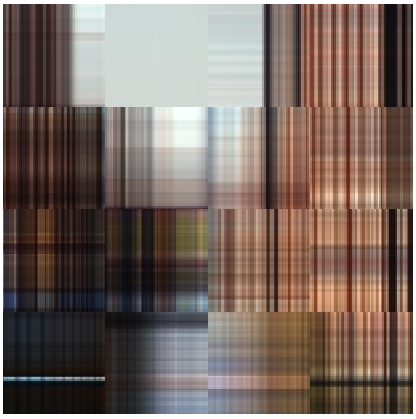
We observed that we can reconstruct image and context with the significant color band.

1.9 Region Attention with SVD

Intuition

In the last experiment, we observed that context can be retained when information is in the significant color band. What if the information is distributed across bands and localized in the spatial domain? In that case, we divided image into regions and applied SVD on each region. Region with context can be reconstructed with all Eigen values while remaining regions can be compressed.

(a) Original Image



(b) SVD on various regions of Image

(c) Information Retained Image

Figure 17: Context Attention with SVD

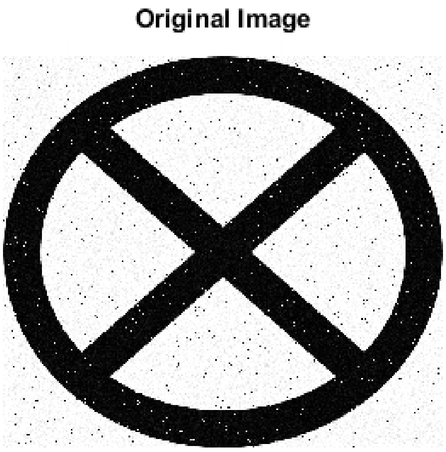
Inference

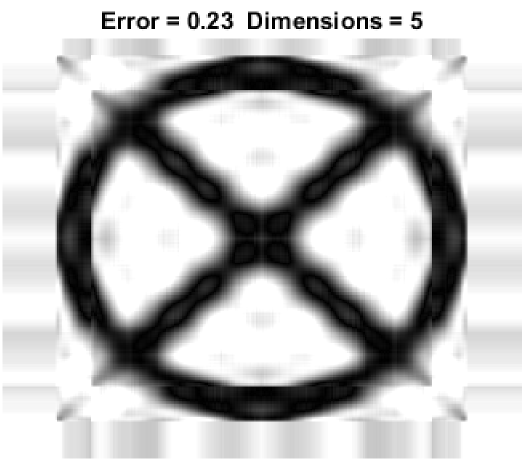
We observed that we can reconstruct image and context with locality.

1.10 Smoothing Noisy Image with SVD

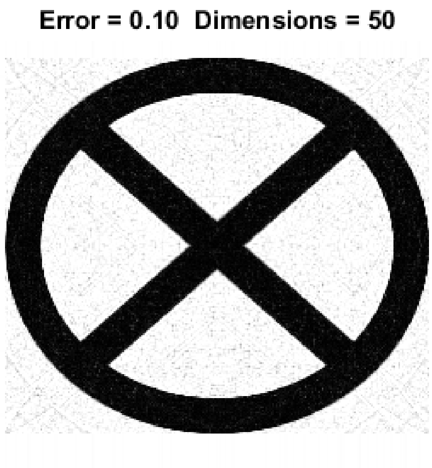
Intuition

As SVD discards dimensions with lower variance, noisy images can be smoothed out by discarding few Eigen dimensions.





(a) Reconstruction of Image with 5 Dimensions



(b) Reconstruction of Image with 50 Dimensions

Figure 18: Smoothing Noisy Image with SVD

Inference

We observed that reconstruction with very few dimensions can spoil the image. At the same time, reconstruction with many dimensions can retain noise in the image. So, we have to maintain a trade off on blur and noisy image reconstruction with number of dimensions to be considered.

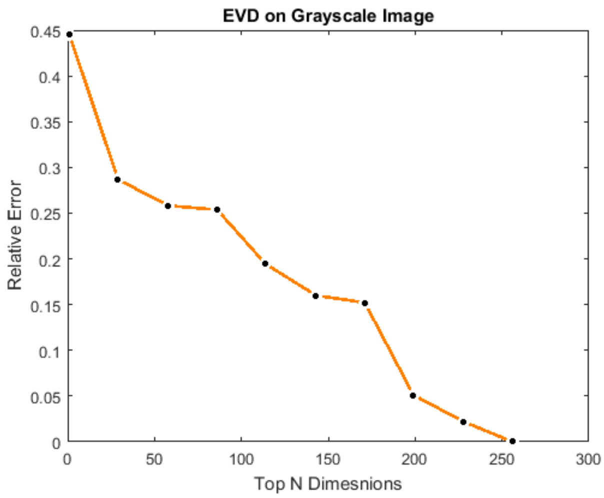
2 Eigen Value Decomposition

2.1 EVD on Gray scale Image

Intuition

To experiment EVD on Gray scale image by converting the given color image to gray scale form. Original image and EVD reconstruction error are shown below.

(a) Original Image

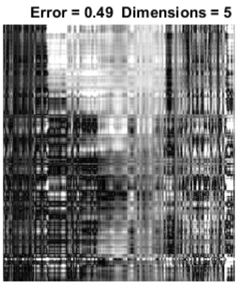


(b) Error Plot

Figure 19: Original image and error plot

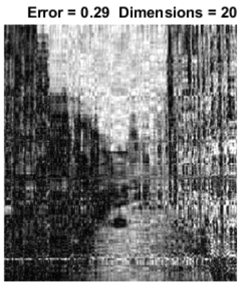
Inference

Frobenius error on reconstructed image is not smoothly decreased as we increase top Eigen values. Reconstructed images are shown below.

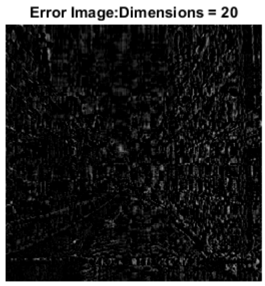


(a) Reconstruction with Top 5 Dimension

(b) Error Image with Top 5 Dimension



(c) Reconstruction with Top 20 Dimension

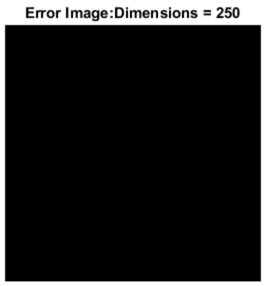


(d) Error Image with Top 20 Dimension

(e) Reconstruction with Top 100 Dimension

(f) Error Image with Top 100 Dimension

(g) Reconstruction with Top 250 Dimension



(h) Error Image with Top 250 Dimension

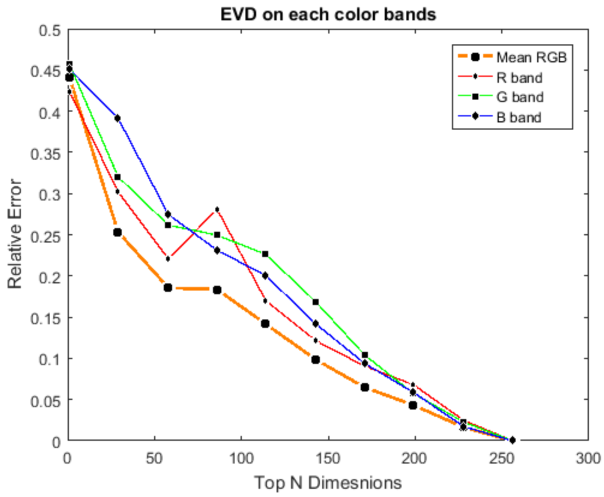
Figure 20: Reconstruction of Gray scale Image with EVD

2.2 EVD on Color bands

Intuition

We experiment EVD on various color bands of same image. Original image and EVD reconstruction error are shown below.

(a) Original Image

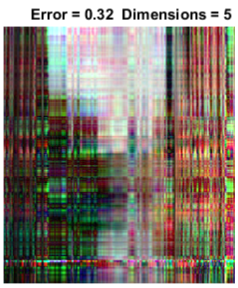


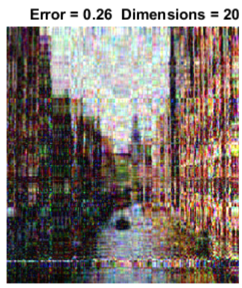
(b) Error Plot

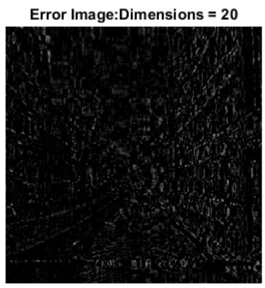
Figure 21: Original image and error plot

Inference

We observed that Frobenius error on each band is independent of every other band while reconstructing the image. Also, we observed that reconstruction error of all bands are lower than that of any color band individually. Reconstructed images are shown below.





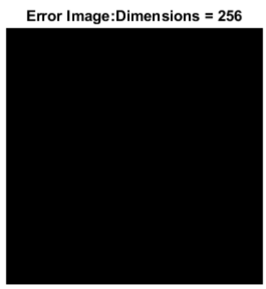


(a) Reconstruction with Top 5 Dimension

(b) Error Image with Top 5 Dimension

(c) Reconstruction with Top 20 Dimension

(d) Error Image with Top 20 Dimension



(e) Reconstruction with Top 100 Dimension

(f) Error Image with Top 100 Dimension

(g) Reconstruction with Top 256 Dimension

(h) Error Image with Top 256 Dimension

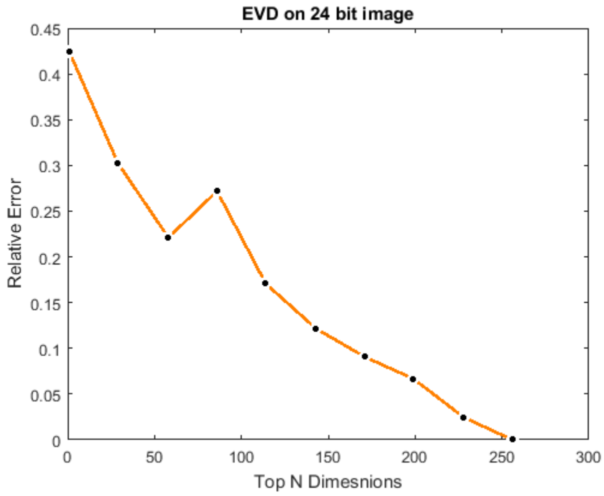
Figure 22: Reconstruction of Image with EVD on color bands

2.3 EVD on Concatenated color bands

Intuition

We experiment EVD by concatenating Red, Green and Blue color bands into a 24 bit binary value and reconstructed the image by splitting it into 8 bit binary value for each color band. Original image and EVD reconstruction error are shown below.

(a) Original Image

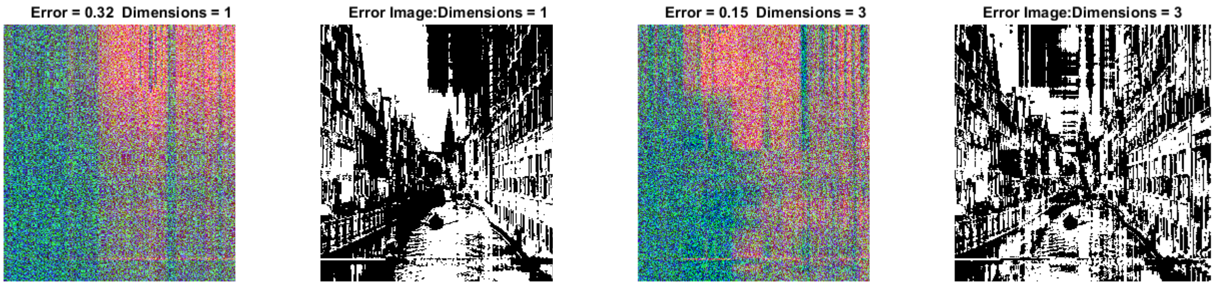


(b) Error Plot

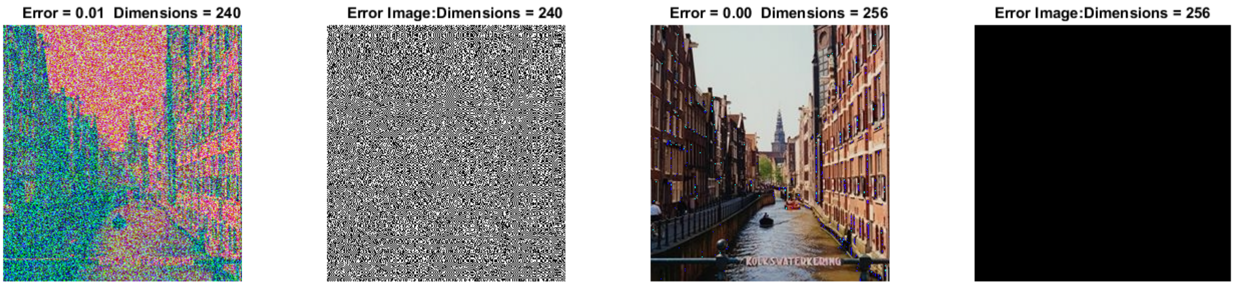
Figure 23: Original image and error plot

Inference

For the given Image, there is a decrease in Frobenius error as we increase the number of Eigen values. Reconstructed images are shown below.



(a) Reconstruction with Top 1 Dimension (b) Error Image with Top 1 Dimension (c) Reconstruction with Top 3 Dimension (d) Error Image with Top 3 Dimension



(e) Reconstruction with Top 240 Dimension (f) Error Image with Top 240 Dimension (g) Reconstruction with Top 256 Dimension (h) Error Image with Top 256 Dimension

Figure 24: Reconstruction of Image with EVD on concatenated color bands

Inference

Even though we take most of the Eigen values, reconstructed images are so noisy. But, when we take all the Eigen values, image is reconstructed successfully.

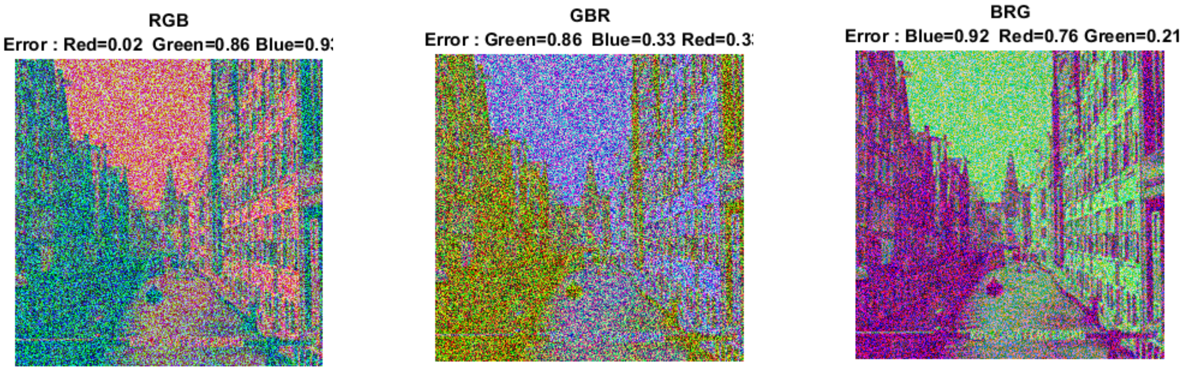
2.4 EVD on Rearranged color bands

Intuition

We experiment EVD by rearranged Red, Green and Blue color bands and reconstructed the image. Reconstructed images are shown below.

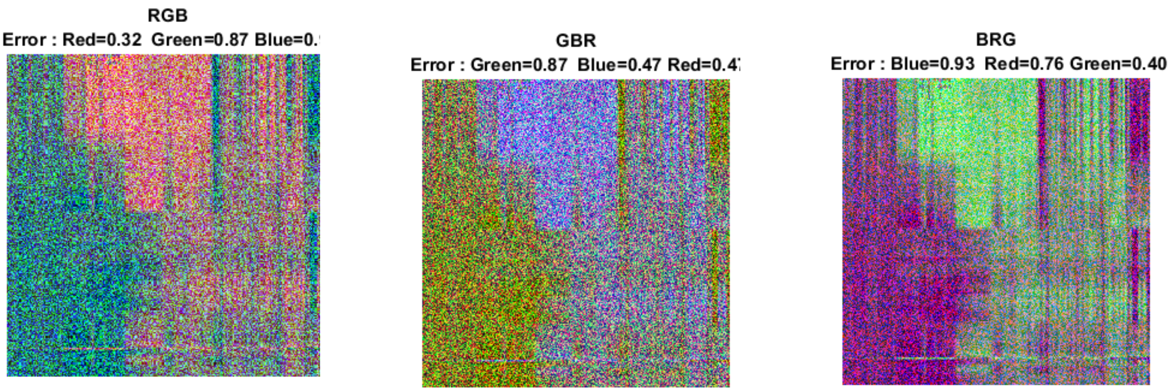
(a) RGB (b) GBR (c) BRG

Figure 25: Top 256 Dimensions



(a) RGB (b) GBR (c) BRG

Figure 26: Top 240 Dimensions



(a) RGB (b) GBR (c) BRG

Figure 27: Top 3 Dimensions

Inference

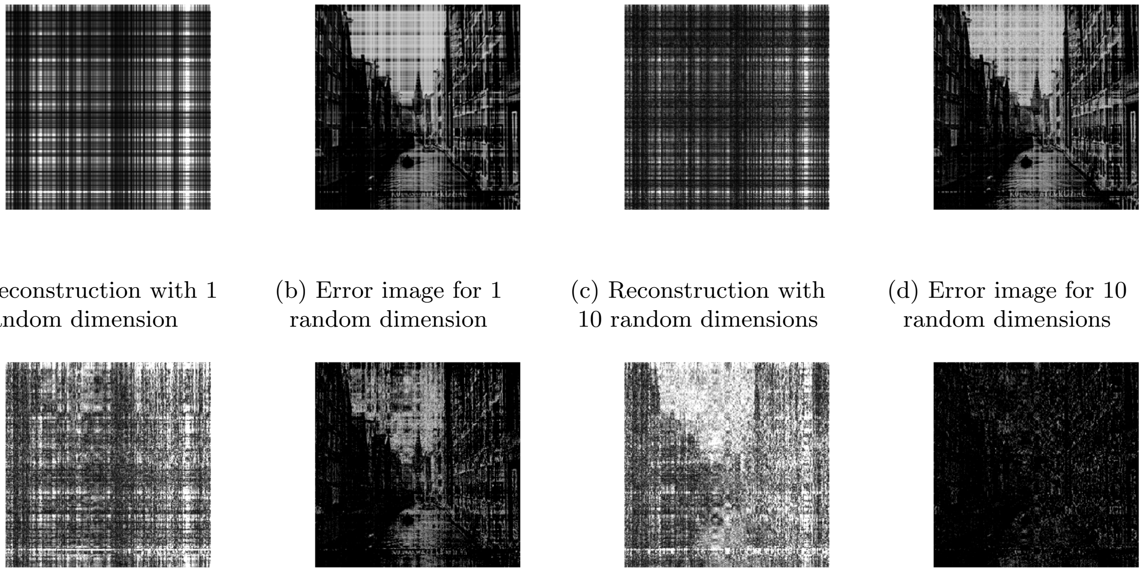
Even though we take most of the Eigen values, reconstructed images are so noisy. But, while we take all the Eigen values, image is reconstructed successfully. Also we observed that rearranging color bands leads to a drastic change in reconstructed images.

2.5 EVD with Random Eigen Values

Intuition

Rather than taking top N Eigen values, we reconstructed images by choosing random Eigen values. Reconstructed gray scale, colored images are shown below.

2.5.1 Gray Scale Image



(a) Reconstruction with 1 random dimension

(b) Error image for 1 random dimension

(c) Reconstruction with 10 random dimensions

(d) Error image for 10 random dimensions

(e) Reconstruction with 100 random dimensions

(f) Error image for 100 random dimensions

(g) Reconstruction with 200 random dimensions

(h) Error image for 200 random dimensions

Figure 28: Reconstruction of Gray Scale Image with EVD using random Eigen values

2.5.2 Color Image

(a) Reconstruction with 1 random dimension

(b) Error image for 1 random dimension

(c) Reconstruction with 10 random dimensions

(d) Error image for 10 random dimensions

(e) Reconstruction with 100 random dimensions

(f) Error image for 100 random dimensions

(g) Reconstruction with 200 random dimensions

(h) Error image for 200 random dimensions

Figure 29: Reconstruction of Colored Image with EVD using random Eigen values

Inference

We observed that the error of reconstructed image does not follow any order when we increase the number of dimensions randomly as the strength of the Eigen values varies randomly.

3 Analysis on SVD and EVD

Here we present inferences of performance of EVD and SVD.

3.1 Performance of SVD on Rectangular Image

We have shown the performance of SVD for Gray scale and colored images. For colored images, we have shown performance on various color bands and concatenated color bands.

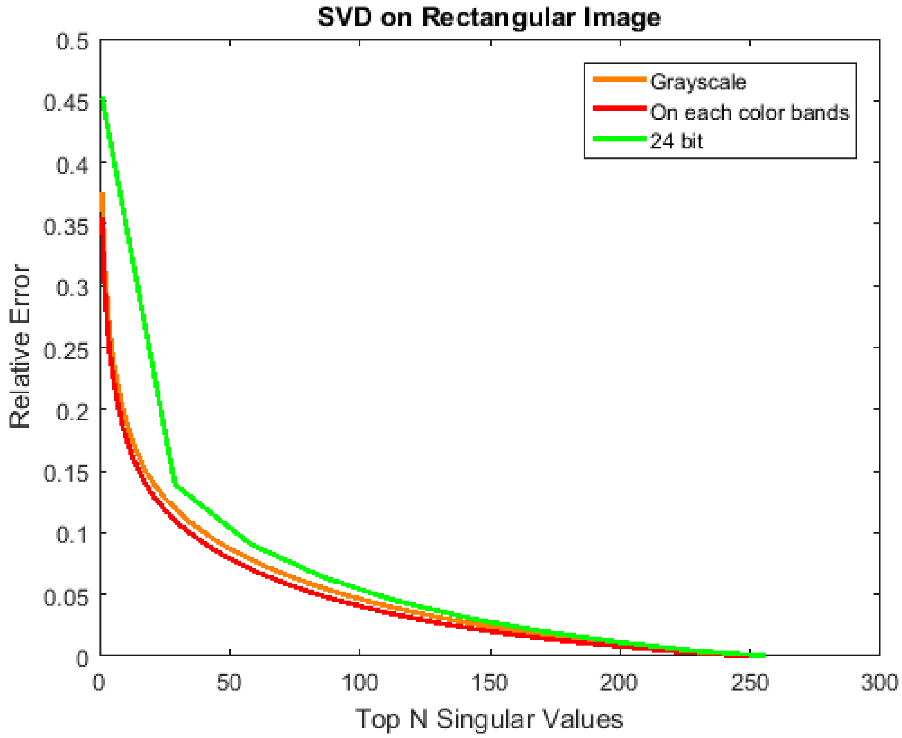


Figure 30: Performance of SVD on rectangular image

Top N Singular Values	Gray scale	Color Bands (RGB)	Concatenated Color Bands
1	0.376	0.356	0.453
29	0.120	0.110	0.139
58	0.079	0.071	0.091
86	0.055	0.049	0.064
114	0.039	0.034	0.045
143	0.027	0.022	0.030
171	0.017	0.014	0.020
199	0.010	0.008	0.011
228	0.004	0.003	0.004
256	0.000	0.000	0.000

Table 1: Error for Various SVD methods

Also, we can see that performance of SVD is almost same in all the cases.

3.2 Performance of EVD on Square Image

We shown the performance of EVD for Gray scale and colored images. For colored images, we shown performance on various color bands and concatenated color bands.

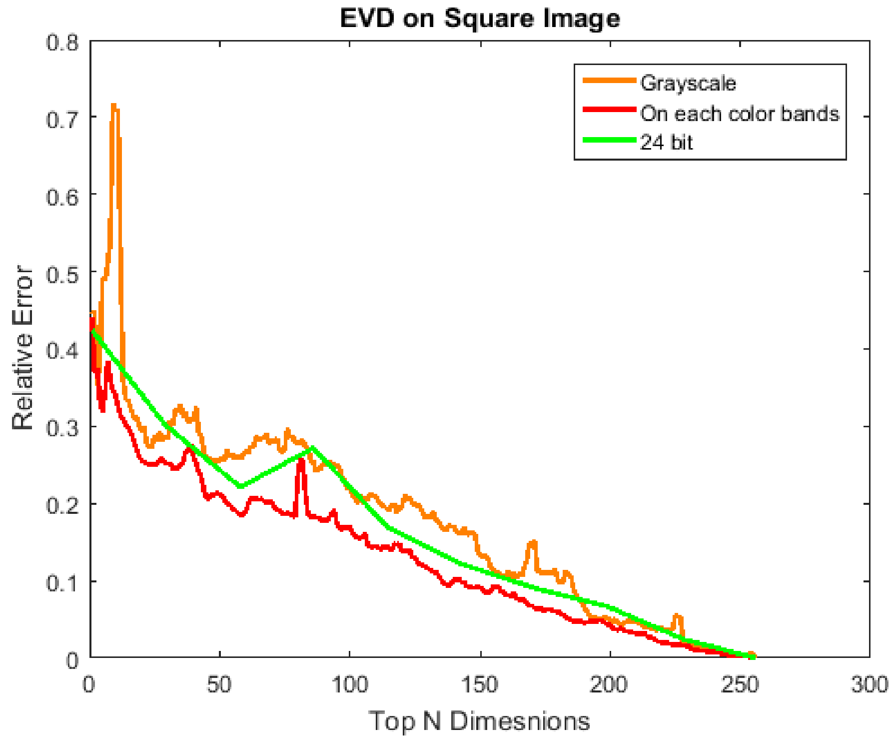


Figure 31: Performance of EVD on square image

Top N Eigen Values	Gray scale	Color Bands (RGB)	Concatenated Color Bands
1	0.445	0.441	0.424
29	0.287	0.253	0.302
58	0.258	0.185	0.221
86	0.254	0.184	0.272
114	0.194	0.142	0.171
143	0.160	0.098	0.122
171	0.152	0.065	0.091
199	0.051	0.043	0.067
228	0.022	0.016	0.025
256	0.000	0.000	0.000

Table 2: Error for Various EVD methods

From the error plot and table, we can infer that performance of EVD fluctuates along the increase in Eigen dimensions. At the same time error is being reduced as we increase the number of Eigen values.

3.3 Comparison between EVD and SVD

We applied SVD on both square and rectangular images while EVD allow us to perform reconstruction only on square image. Why? Applying EVD on rectangular image is same as SVD. We shown the EVD and SVD performance graph for squared image.

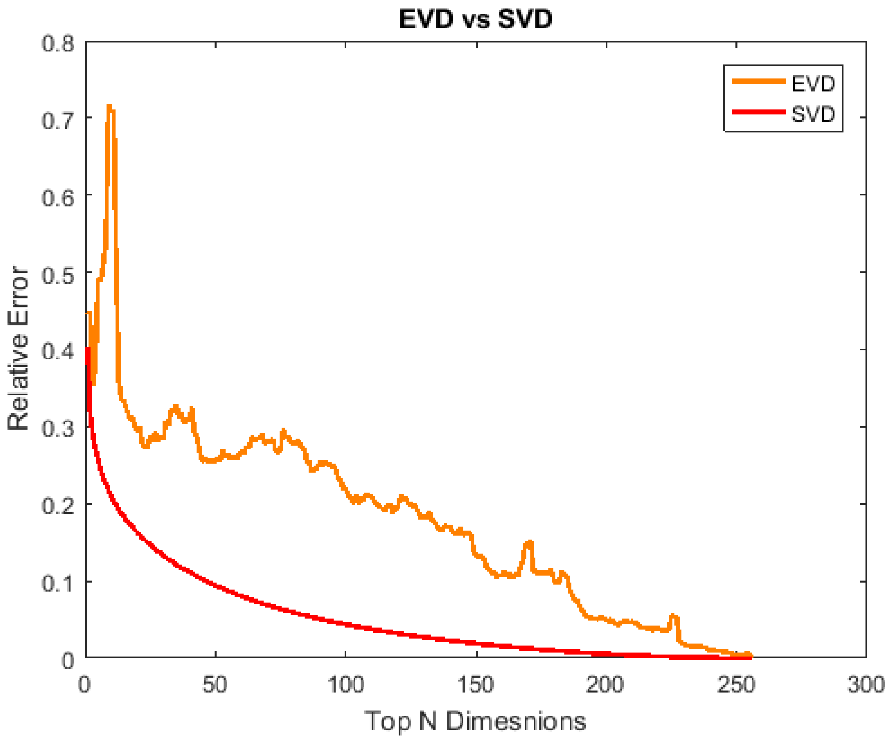


Figure 32: Accuracy of EVD vs SVD

We can see that performance of EVD fluctuates along the increase in Eigen dimensions while performance of SVD follows a exponential curve. But, we can't generalize as it can be the case for given data.

3.4 Miscellaneous Inferences

- Applying EVD on rectangular image is same as SVD.
- Eigen vectors are not unique. To reconstruct image properly, we have to verify the Eigen vectors by $AV = U\Sigma$.
- Using SVD or EVD, we can store and transmit compressed images by picking up the proper Eigen vectors and Eigen values.
- We observed that the SVD and EVD are lossy compression in general. But, when we take all the Eigen values, it can be lossless compression.

4 Polynomial Regression

4.1 Univariate Regression

Intuition

From the visualization of data, we infer that we may need polynomial of order 5 or 6 to best fit the curve.

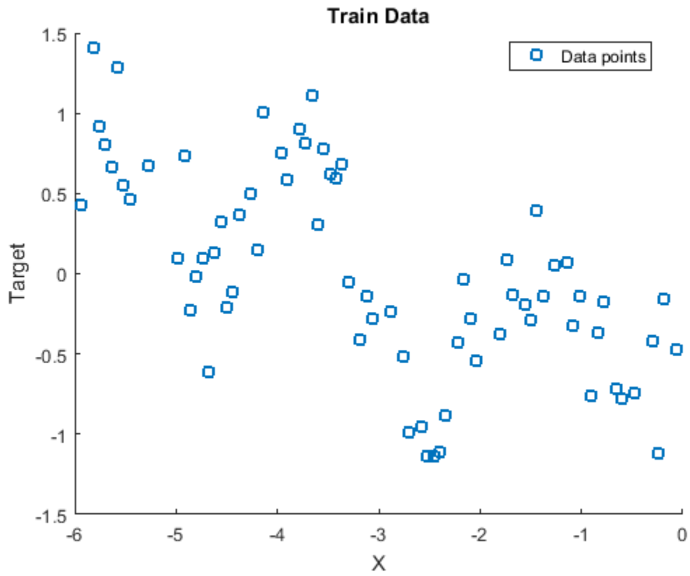
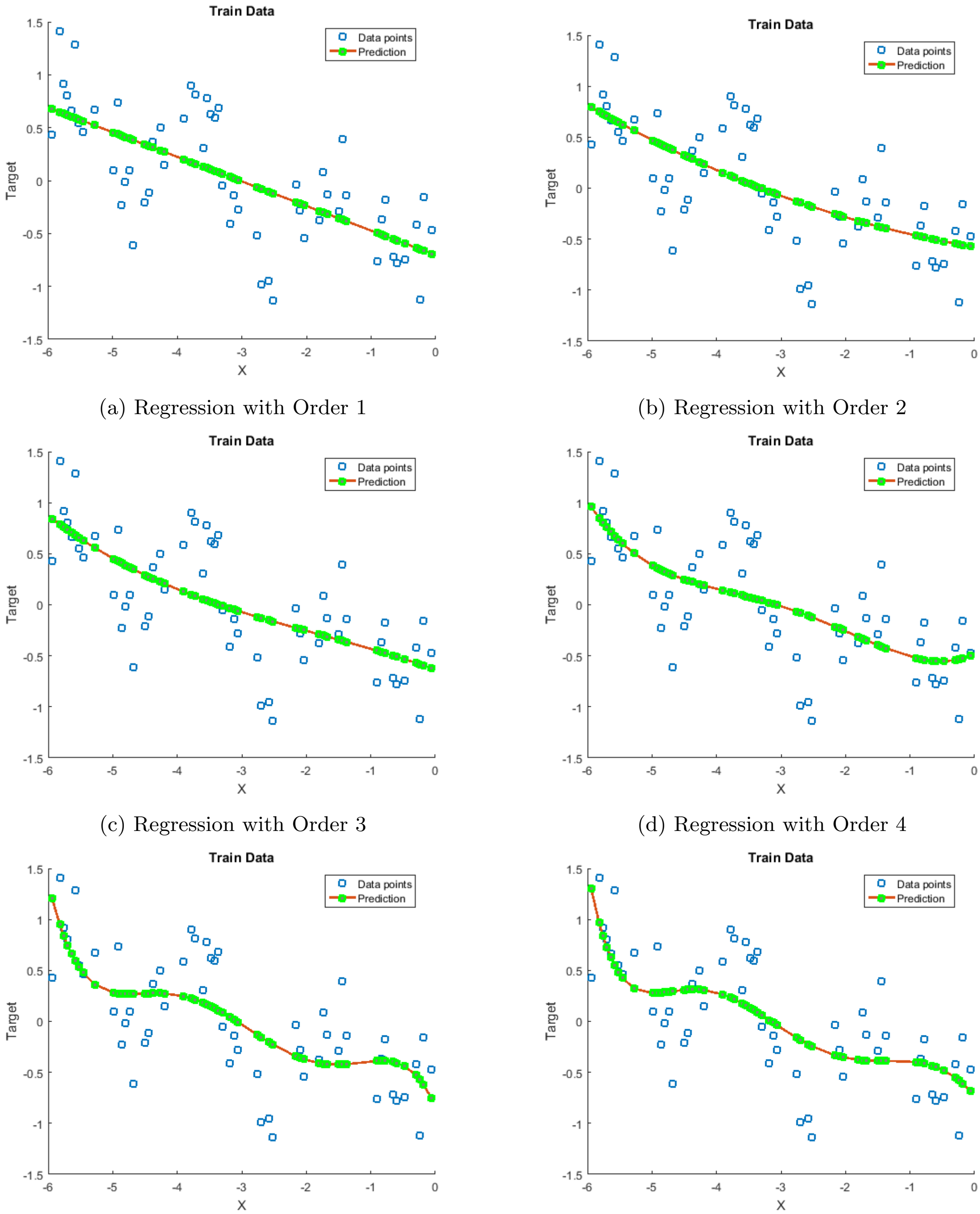


Figure 33: Visualization of Univariate data



(a) Regression with Order 1 (b) Regression with Order 2

(c) Regression with Order 3 (d) Regression with Order 4

(e) Regression with Order 5 (f) Regression with Order 6

Figure 34: Univariate Regression

Intuition and Inference

Curve fit for order 5 and order 6 seems same. Increasing the complexity of the model may overfit the train data. We confirm the intuition by plotting the error.

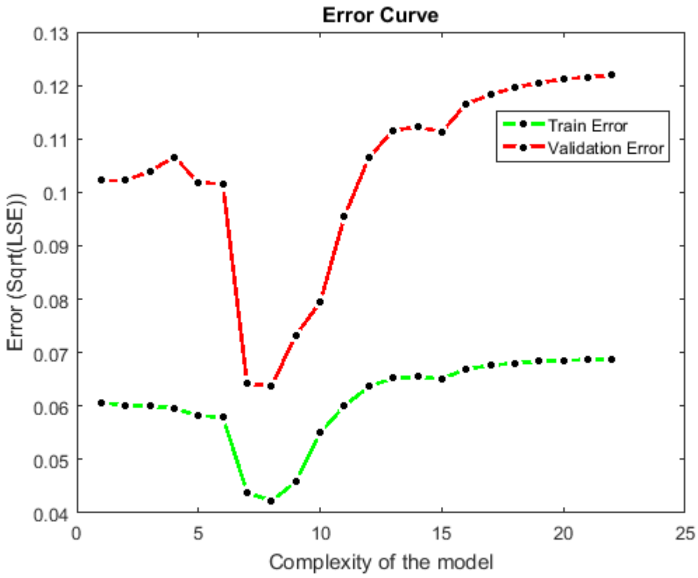


Figure 35: Complexity and Error on Train and Validation univariate data

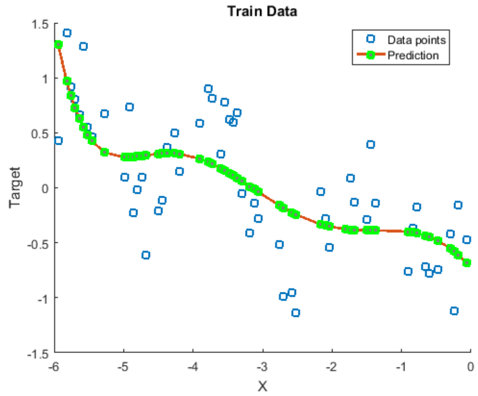
Order of Regression	Train Error (Sqrt(LSE))	Validation Error (Sqrt(LSE))
1	0.10	0.10
2	0.09	0.11
3	0.09	0.11
5	0.08	0.11
6	0.08	0.11
7	0.05	0.08
8	0.06	0.08
10	0.08	0.09
15	0.10	0.10
20	0.11	0.10

Table 3: Comparison between Train and Validation Error across various regression orders for Univariate data

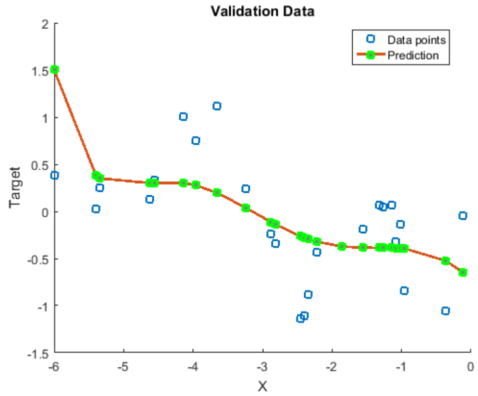
Intuition and Inference

From the error plot and table, we observed that our intuition is wrong as model performs best at order 7. Regression on train, validation and test data is shown below.

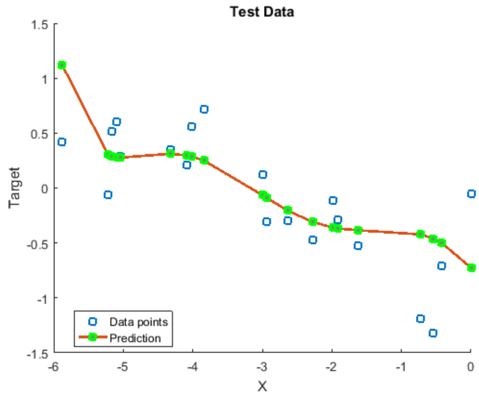
Univariate Regression with order 7



(a) Regression on Train data



(b) Regression on Validation data



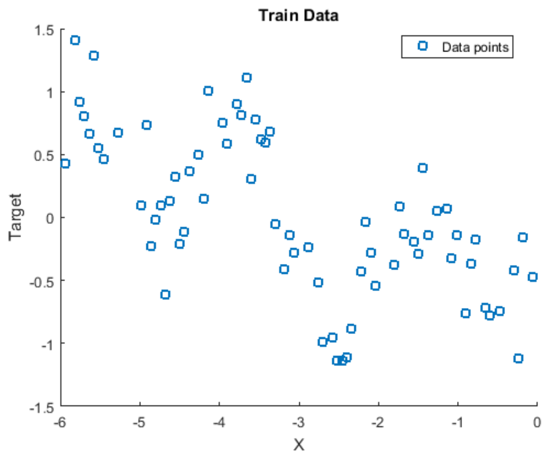
(c) Regression on Test data

Figure 36: Univariate Regression with order 7

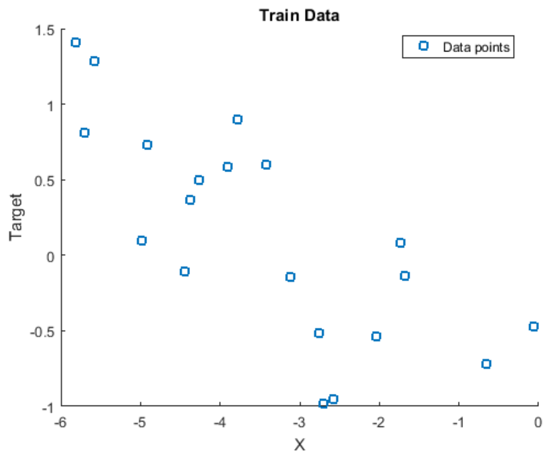
4.1.1 Univariate Regression with Regularization

Intuition and Inference

As the number of data points is large and close to each other, over fitting may not possible. So, we reduced the number of train data points to incorporate over fitting.



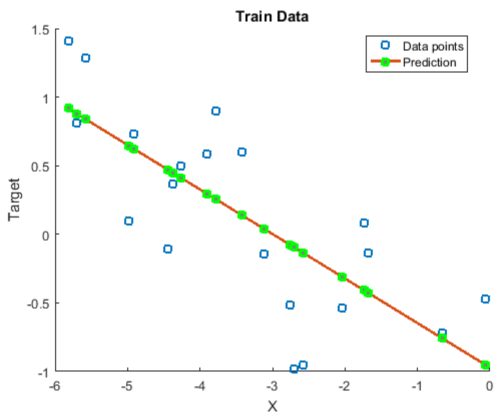
(a) Given Univariate Train data



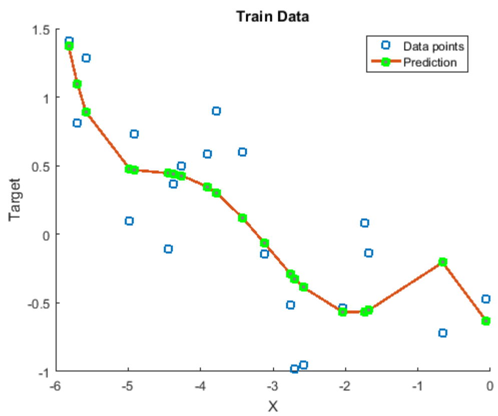
(b) Reduced Univariate Train data

Figure 37: Reduction on Univariate Train data

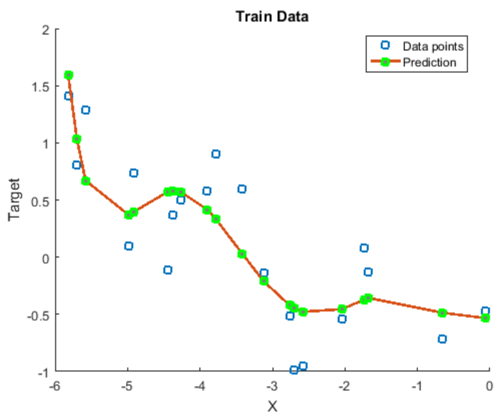
Univariate Regression on Reduced Train data



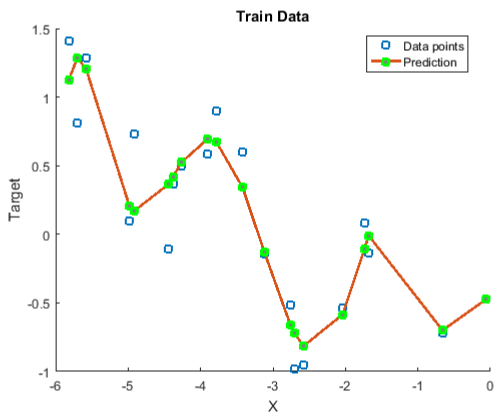
(a) Regression with Order 1



(b) Regression with Order 5



(c) Regression with Order 6



(d) Regression with Order 7

Figure 38: Univariate Regression on Reduced Train data

Intuition and Inference

We found that the polynomial regression of order 7 seems to over fit the train data. We confirm the same by plotting the error.

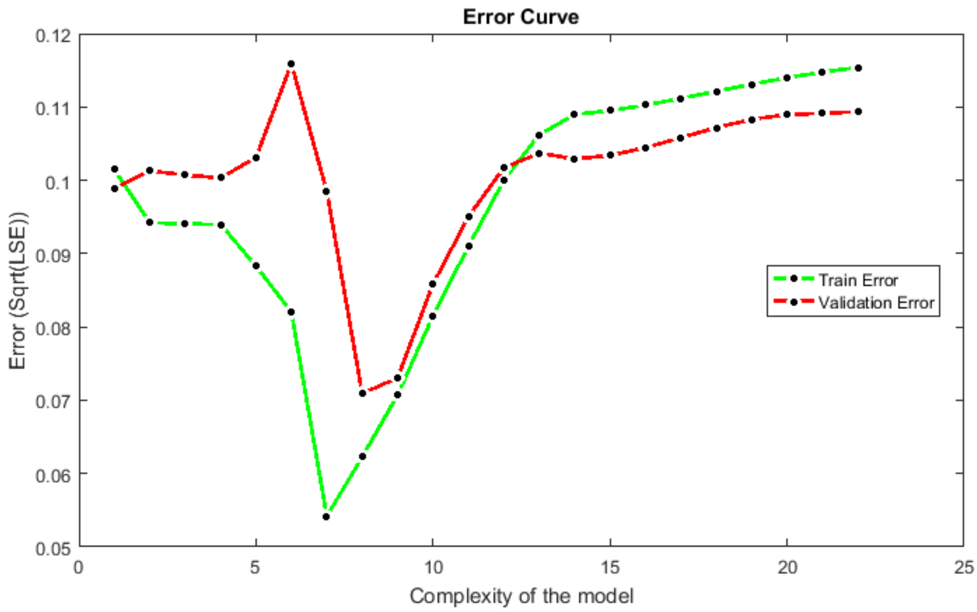


Figure 39: Complexity and Error on reduced Train and Validation univariate data

Intuition and Inference

We infer that the train data is over fitted at polynomial order 7. To overcome over fitting of train data, L2 regularization is applied. Error plot is shown below.

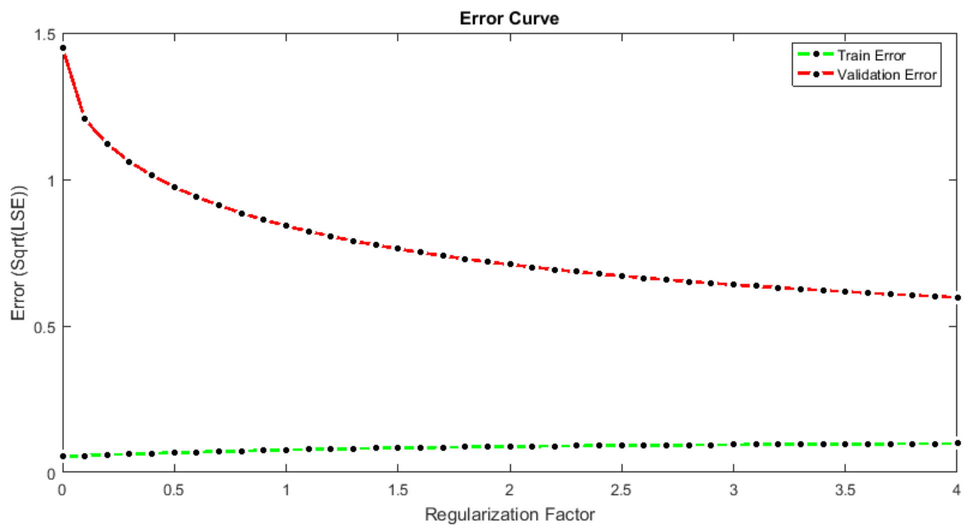
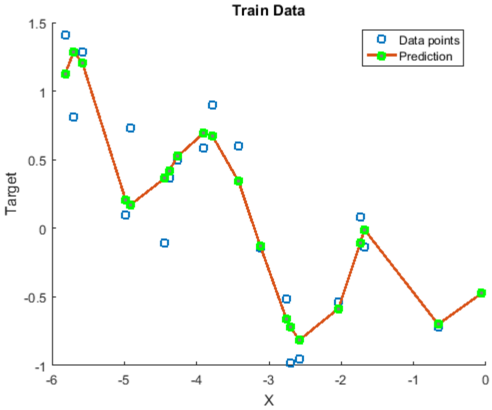


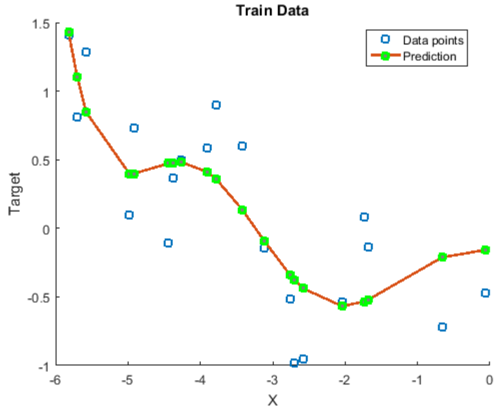
Figure 40: Regularization factor and Error on reduced Train and Validation univariate data with order 7

Inference

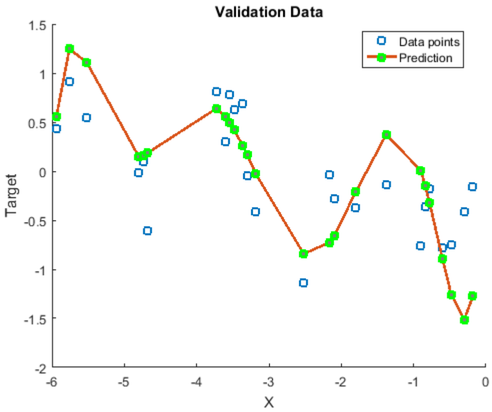
As we configure regularization factor λ, error is gradually decreasing in validation data and increasing in train data. Also, in our case, error became constant beyond particular λ. We are choosing $\lambda = 1.5$.



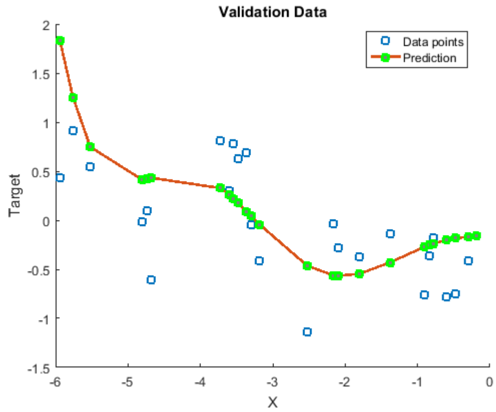
(a) Regression on Train data with Order 7



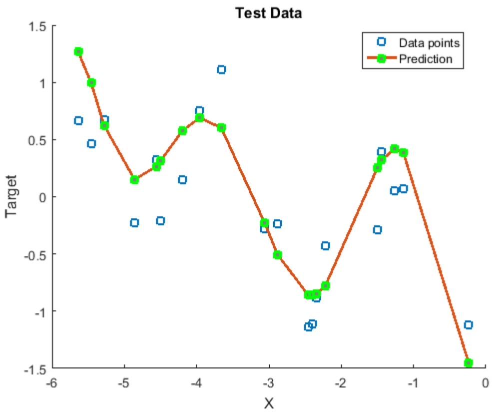
(b) Regression on Train data with Order 7 and $\lambda = 1.5$



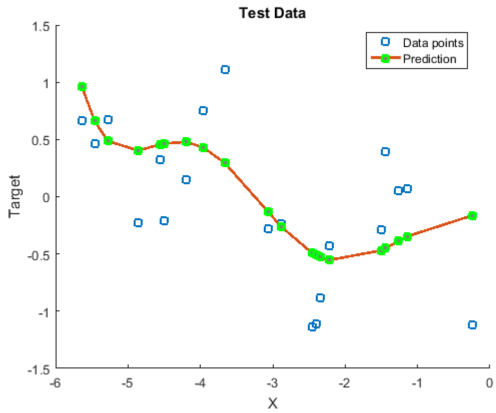
(c) Regression on Validation data with Order 7



(d) Regression on Validation data with Order 7 and $\lambda = 1.5$



(e) Regression on Test data with Order 7



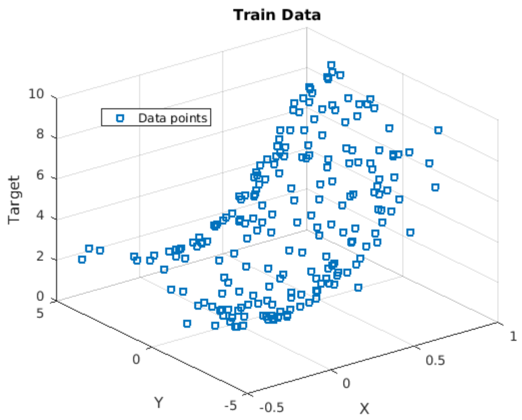
(f) Regression on Test data with Order 7 and $\lambda = 1.5$

Figure 41: Comparision on Univariate Regression with over fitting and regularization

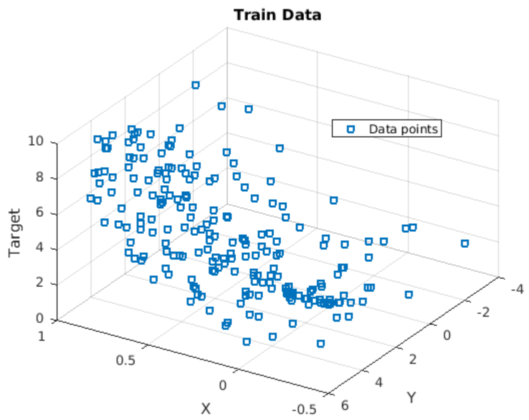
Inference

Applying L2 regularization on Univariate data smoothens curve. Also, it prevents the regression from over fitting.

4.2 Bivariate Regression



(a) Visualization of Univariate data in View 1

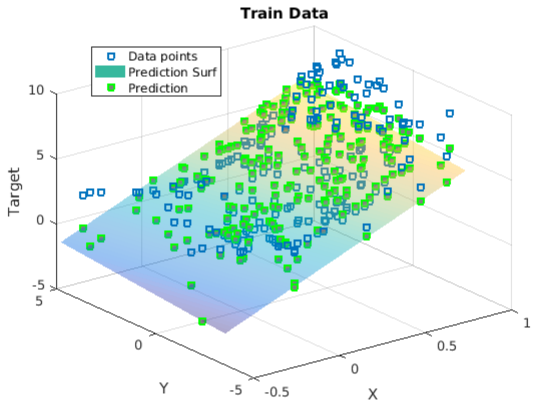


(b) Visualization of Univariate data in View 2

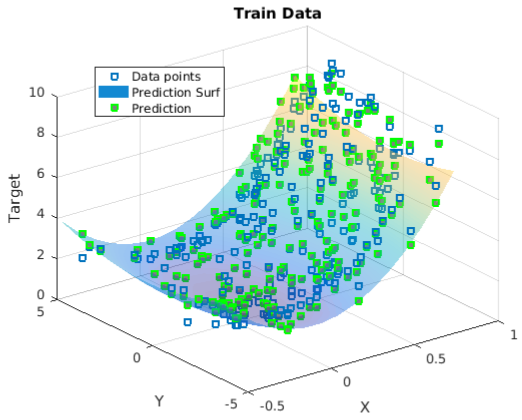
Figure 42: Visualization of Univariate data

Intuition

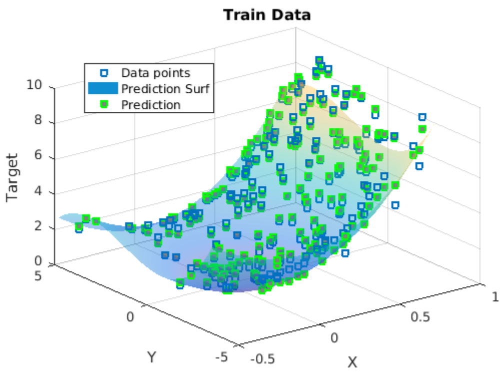
From the visualization of data, we infer that we may need polynomial of order 4 or 5 to best fit the curve.



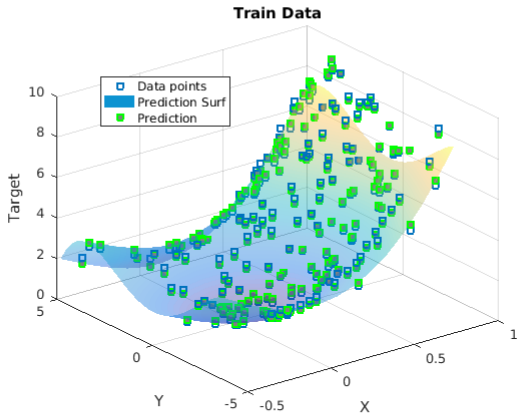
(a) Regression with Order 1



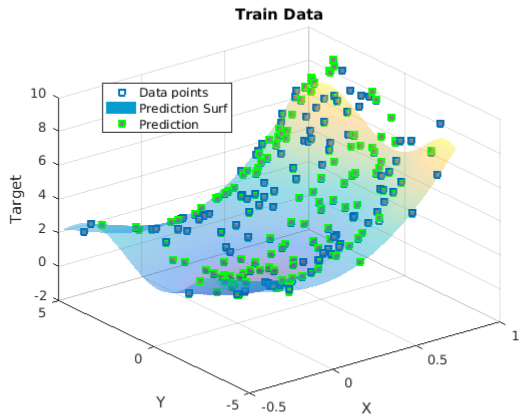
(b) Regression with Order 2



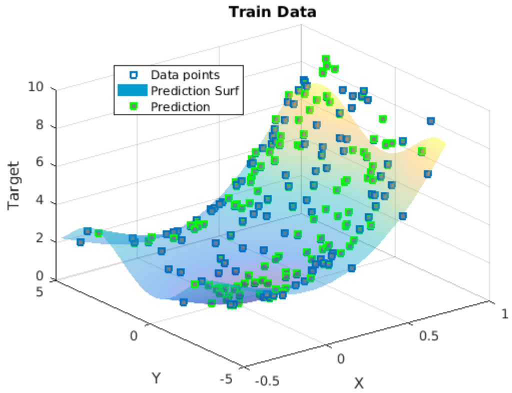
(c) Regression with Order 3



(d) Regression with Order 4



(e) Regression with Order 5



(f) Regression with Order 6

Figure 43: Bivariate Regression

Intuition and Inference

Observation is that increasing the complexity of the model may overfit the train data. We confirm the intuition by plotting the error.

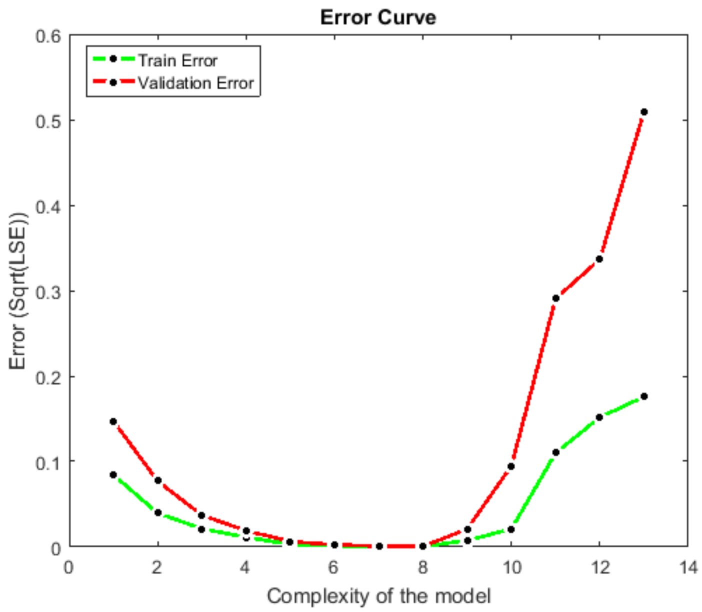


Figure 44: Complexity and Error on Train and Validation bivariate data

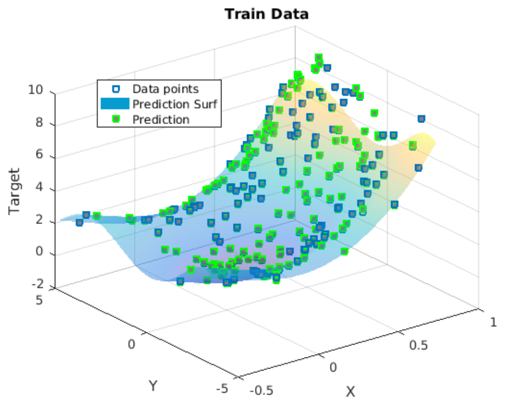
Order of Regression	Train Error (Sqrt(LSE))	Validation Error (Sqrt(LSE))
1	0.182	0.158
2	0.070	0.075
3	0.043	0.047
5	0.005	0.026
6	0.001	0.004
7	0.001	0.004
8	0.001	0.005
10	0.039	0.391
12	0.257	2.807

Table 4: Comparison between Train and Validation Error across various regression orders for Bivariate data

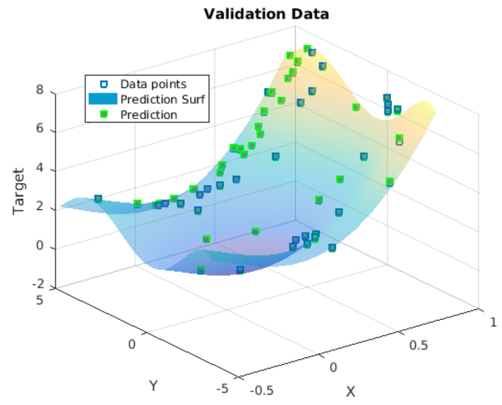
Inference

From the error plot and table, we observed that regression model performs better with order 6. Regression on train, validation and test data is shown below.

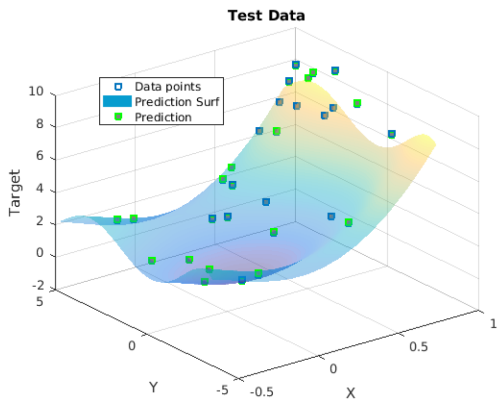
Bivariate Regression with order 6



(a) Regression on Train data



(b) Regression on Validation data



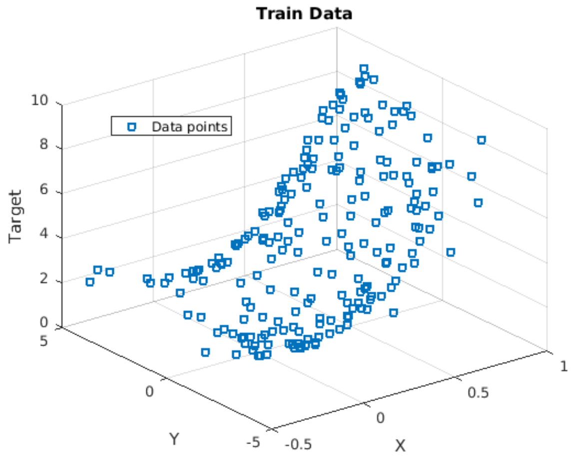
(c) Regression on Test data

Figure 45: Bivariate Regression with order 6

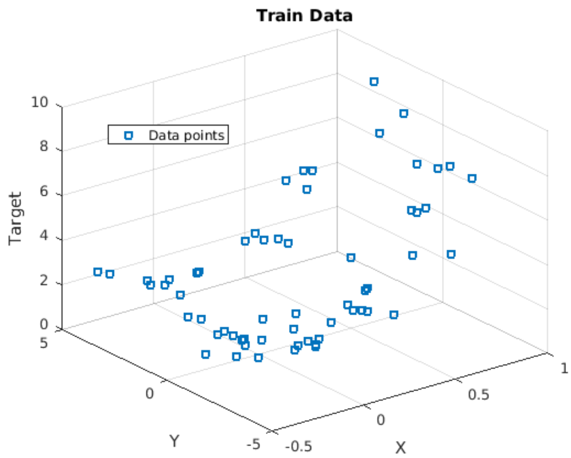
4.2.1 Bivariate Regression with Regularization

Intuition and Inference

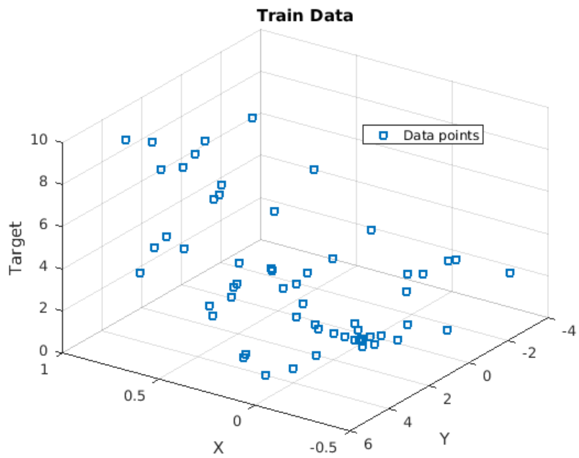
As the number of data points is large and close to each other, over fitting may not possible. So, we reduced the number of train data points to incorporate over fitting.



(a) Given Bivariate Train data



(b) Reduced Bivariate Train data in View 1



(c) Reduced Univariate Train data in View 2

Figure 46: Reduction on Bivariate Train data

Intuition

We can find the over fit of train data by plotting the error.

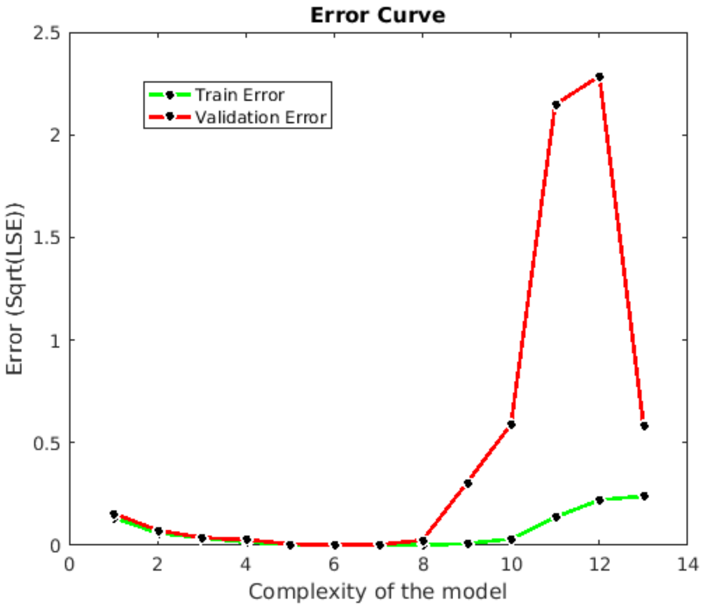


Figure 47: Complexity and Error on reduced Train and Validation bivariate data

Intuition and Inference

We infer that the train data is over fitted at polynomial order 9. To overcome over fitting of train data, L2 regularization is applied. Error plot is shown below.

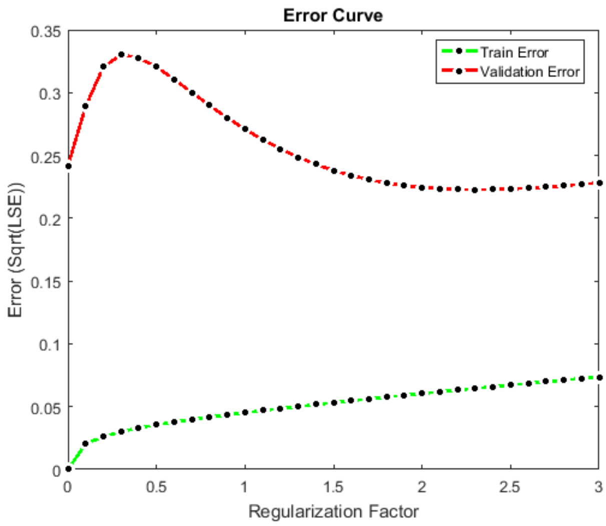
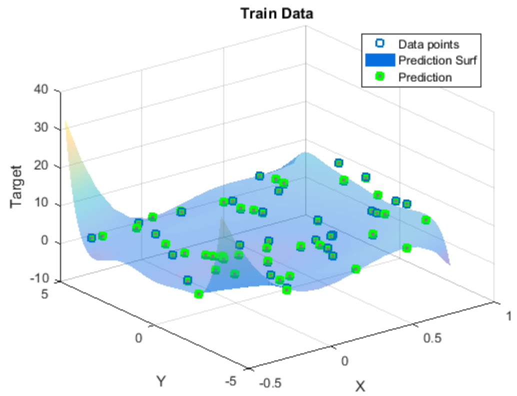


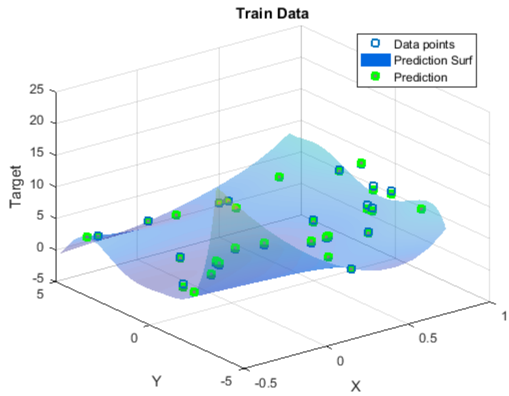
Figure 48: Regularization factor and Error on reduced Train and Validation univariate data with order 9

Inference

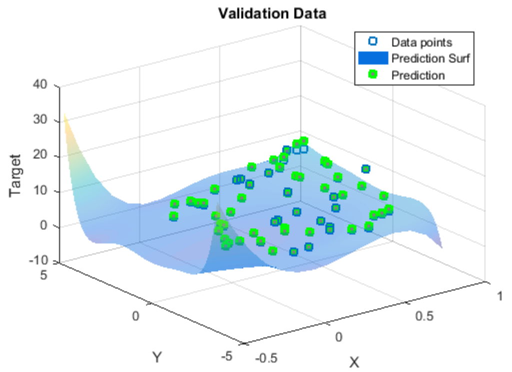
As we increase regularization factor λ, there is an increase in error for validation data. But, after that there is a gradual decrease in error. We infer that error becomes minimal when $\lambda = 2$.



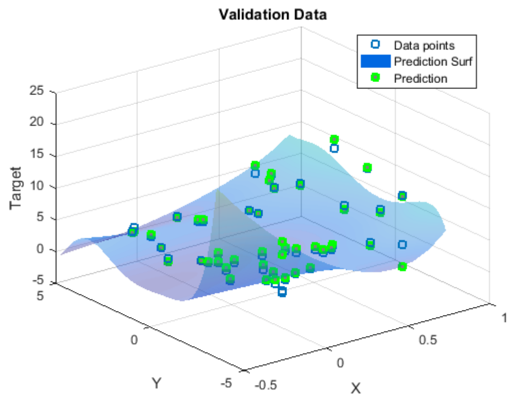
(a) Regression on Train data with Order 9



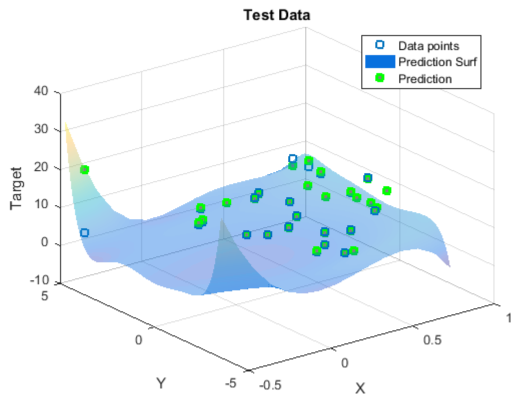
(b) Regression on Train data with Order 9 and $\lambda = 2$



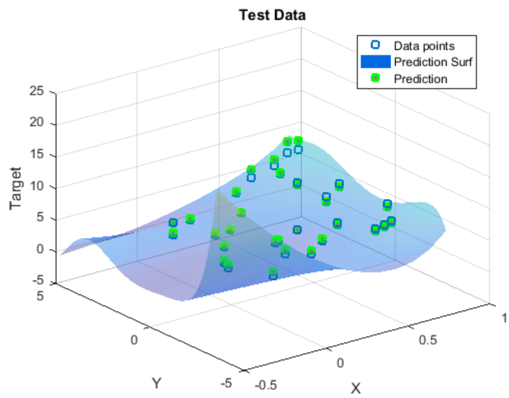
(c) Regression on Validation data with Order 9



(d) Regression on Validation data with Order 7 and $\lambda = 2$



(e) Regression on Test data with Order 9



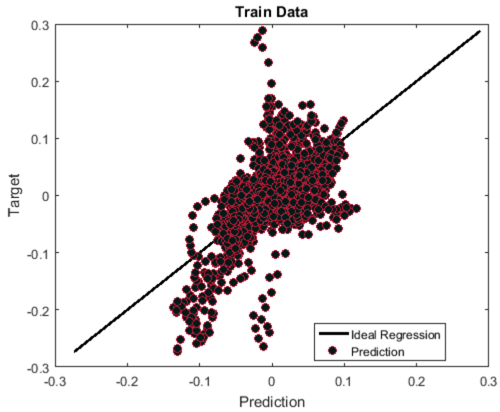
(f) Regression on Test data with Order 9 and $\lambda = 2$

Figure 49: Comparision on Bivariate Regression with over fitting and regularization

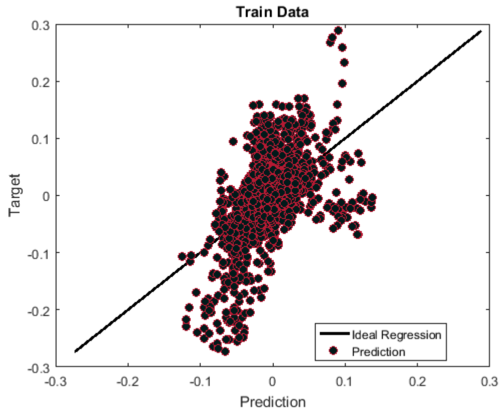
Inference

Applying L2 regularization on Bivariate data smoothens the surf. Also, it prevents the regression model from over fitting.

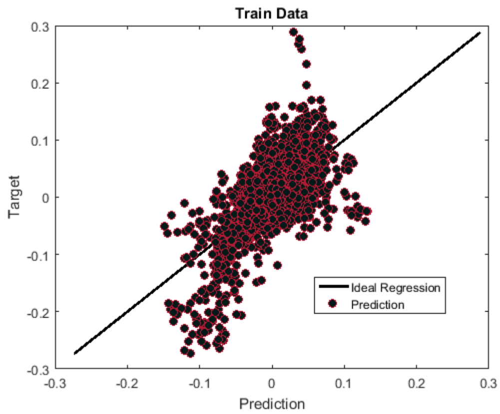
4.3 Multivariate Regression



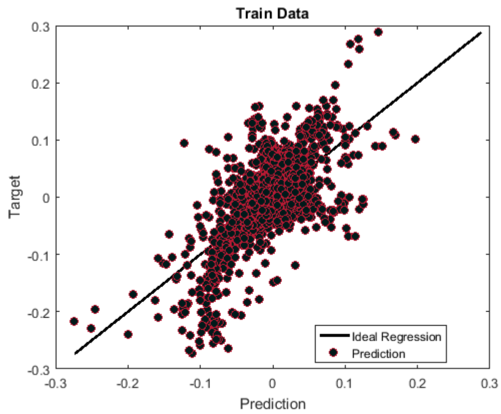
(a) Regression with Order 1



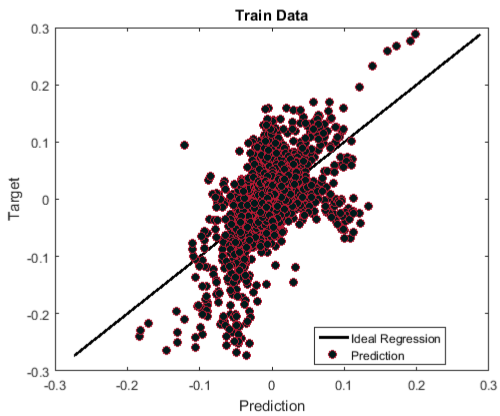
(b) Regression with Order 2



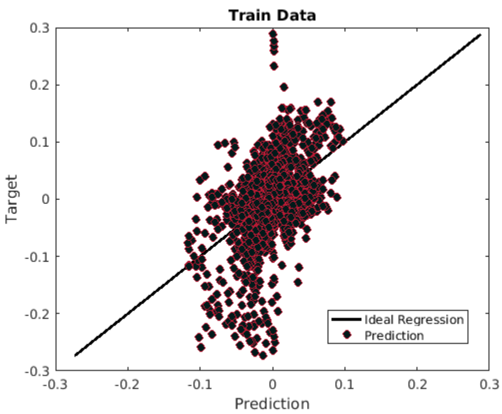
(c) Regression with Order 3



(d) Regression with Order 4



(e) Regression with Order 5



(f) Regression with Order 6

Figure 50: Multivariate Regression

Inference

We infer that regression with order 5 performs better than that of order 6. We can confirm this by plotting error.

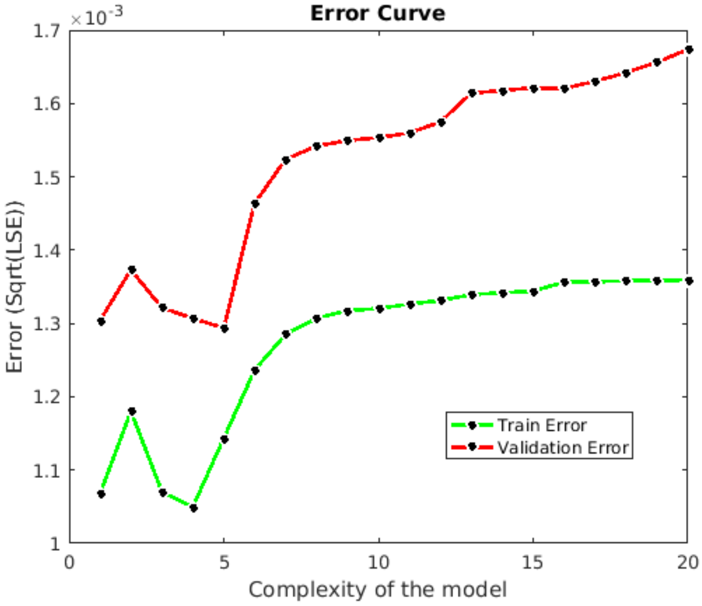


Figure 51: Complexity and Error on Train and Validation Multivariate data

Order of Regression	Train Error (Sqrt(LSE))[1]	Validation Error (Sqrt(LSE))[2]
1	1.067	1.303
2	1.180	1.373
3	1.049	1.306
4	1.143	1.297
5	1.143	1.297
6	1.236	1.468
7	1.285	1.523
8	1.307	1.542
10	1.320	1.553
12	1.331	1.574

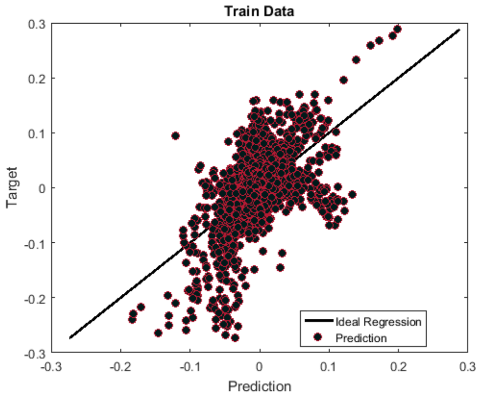
Table 5: Comparison between Train and Validation Error across various regression orders for Multivariate data

Inference

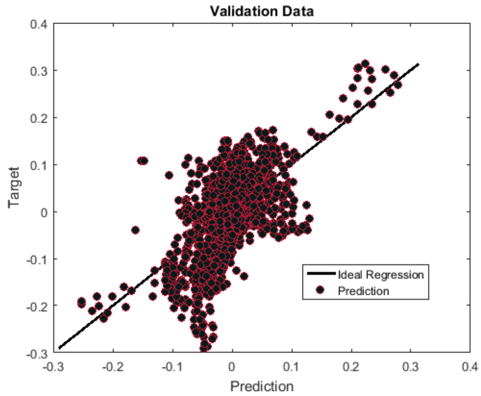
We found that the error is minimal in order 4 and 5 for train and validation data respectively. Further beyond order 6, error is higher in both the cases. Regression on train, validation and test data is shown below.

[1] Values shown in $\exp^{-3}$
[2] Values shown in $\exp^{-3}$

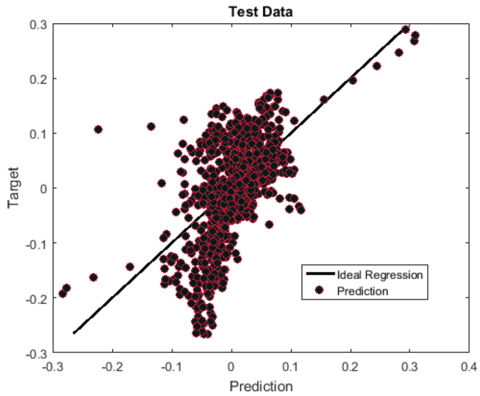
Bivariate Regression with order 5



(a) Regression on Train data



(b) Regression on Validation data



(c) Regression on Test data

Figure 52: Multivariate Regression with order 5

Intuition and Inference

We infer that train data is over fitted at polynomial order 7. To overcome over fitting of train data, L2 regularization is applied. Error plot is shown below.

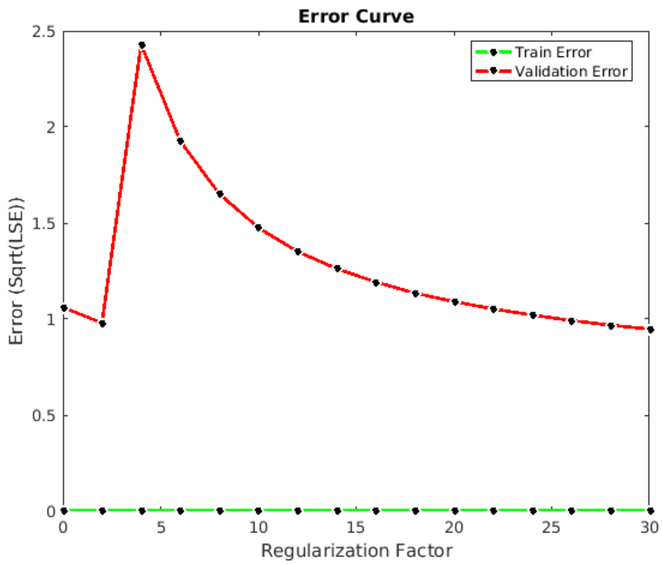
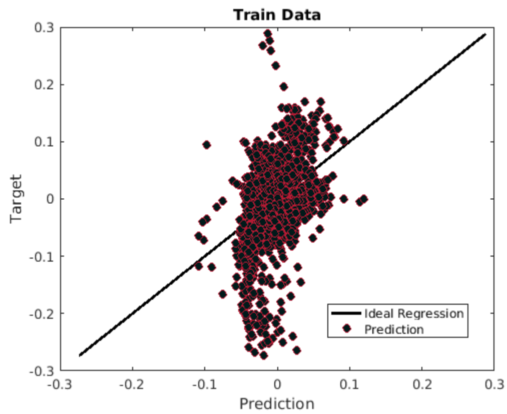


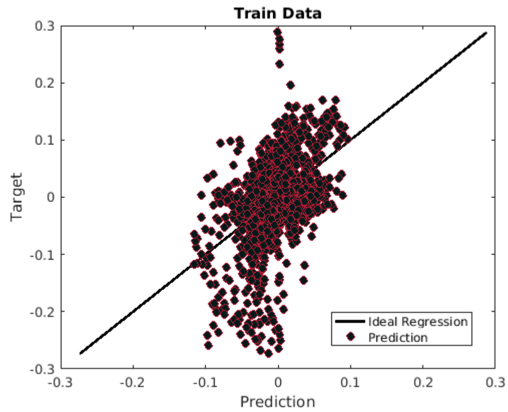
Figure 53: Regularization factor and Error on reduced Train and Validation Multivariate data with order 7

Inference

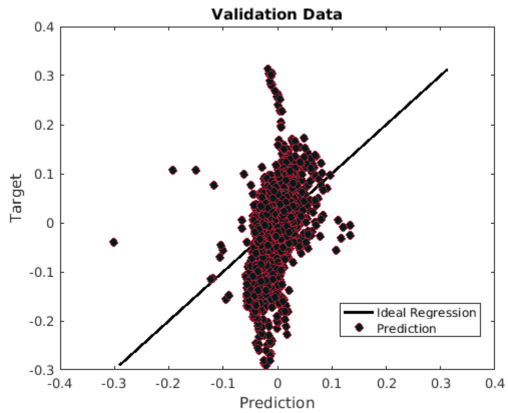
As we increase regularization factor λ, there is an increase in error for validation data. But, after that there is a gradual decrease in error. We infer that error becomes minimal when $\lambda = 30$.



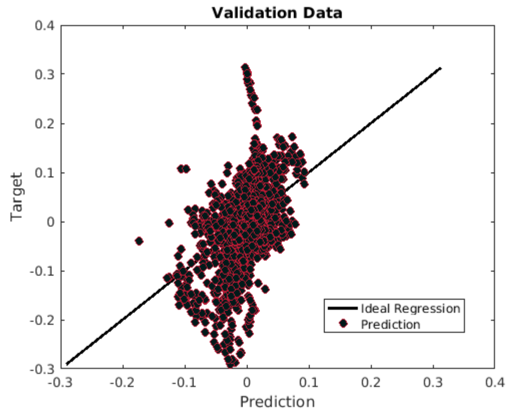
(a) Regression on Train data with Order 7



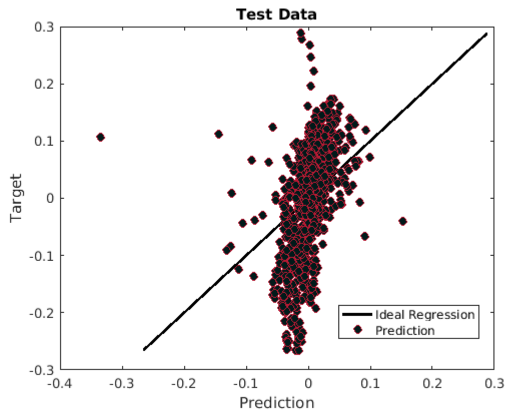
(b) Regression on Train data with Order 7 and λ = 30



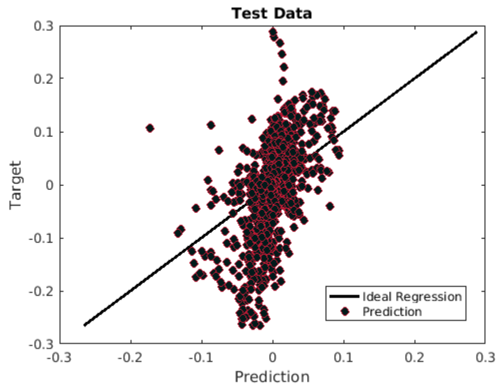
(c) Regression on Validation data with Order 7



(d) Regression on Validation data with Order 7 and λ = 30



(e) Regression on Test data with Order 7



(f) Regression on Test data with Order 7 and λ = 30

Figure 54: Comparision on Multivariate Regression with over fitting and regularization

Inference

Applying L2 regularization on Multivariate data performs better and prevents the regression model from over fitting.

5 References

- Bishop, Christopher M., “Pattern Recognition and Machine Learning”, Information Science and Statistics, 2006.
- Gilbert Strang, “Linear Algebra and Its Applications”, Thomson Learning, Inc, 2006.
- R.O.Duda, P.E.Hart and D.G.Stork, “Pattern Classification”, John Wiley, 2001.
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