The American University in Cairo

Computer Science and Engineering Department

Fall 2022

CSCE 363/3611 Digital Signal Processing

Assignment #1

(Due on: October 15, 2022 at mid-night)

(The assignment is individual - Submit on Blackboard as one .zip file)

Problem 1

Consider the discrete-time signal given by:

$$x(n)=\{-1,-2,\underset{\uparrow}{0},1,1,1,0,0\}$$

Sketch and label carefully each of the following signals:

a) $x(n-5)$

b) $3x(n+2)$

c) $x(-n-7)$

d) $x(n)u(n)$

Solution:

a)

The American University in Cairo

Computer Science and Engineering Department

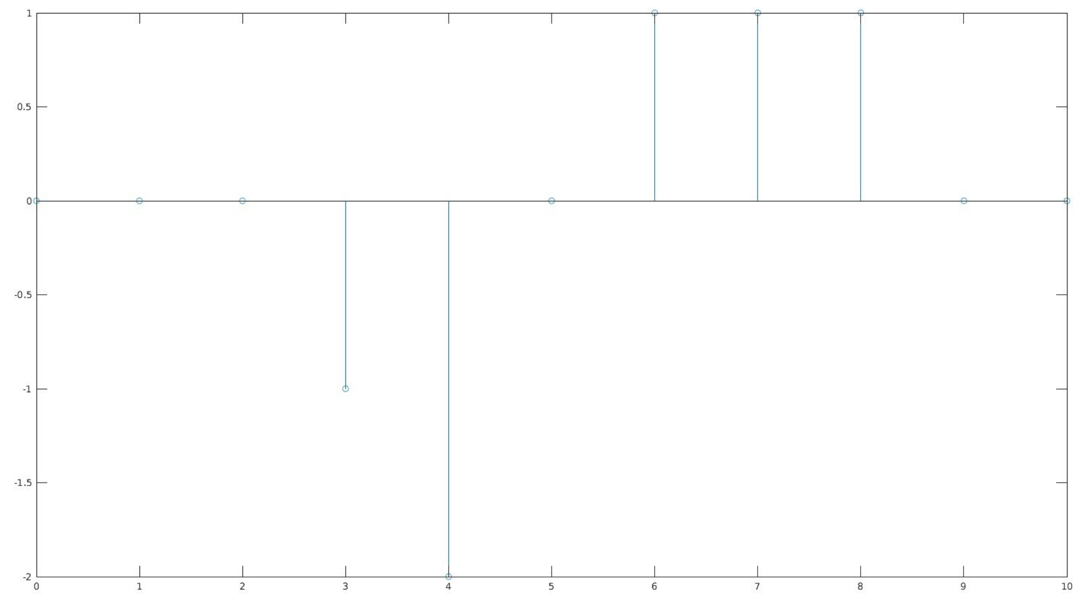
Fall 2022

CSCE 363/3611 Digital Signal Processing

Assignment #1

(Due on: October 15, 2022 at mid-night)

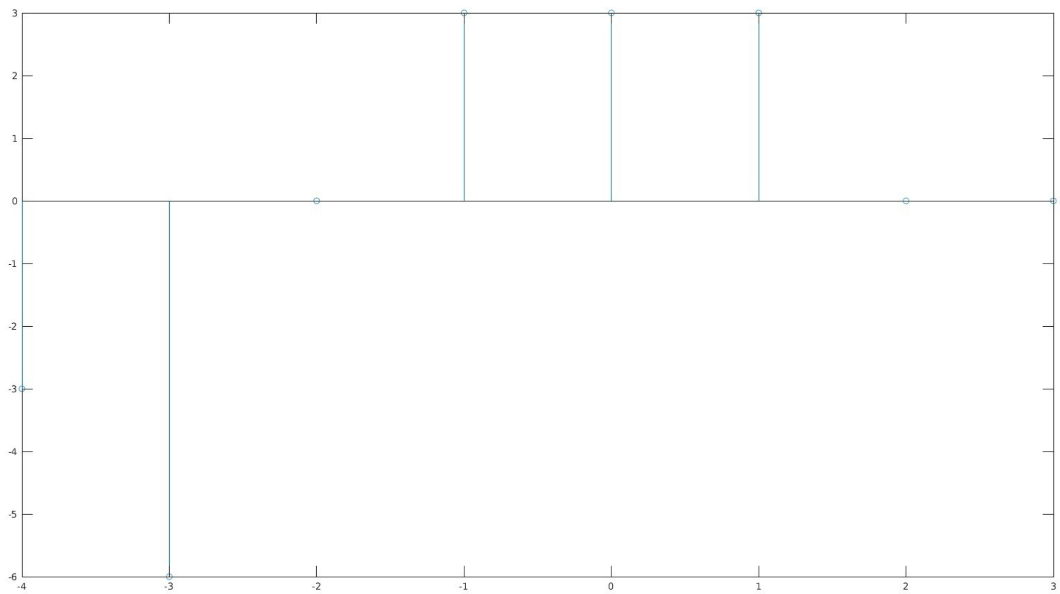
(The assignment is individual - Submit on Blackboard as one .zip file)



n	0	1	2	3	4	5	6	7	8	9	10
$x(n)$	0	0	0	-1	-2	0	1	1	1	0	0

$$x(n-5)=\{0,0,0,-1,-2,0,1,1,1,0,0\}$$

b)



n	-4	-3	-2	-1	0	1	2	3
$x(n)$	-3	-6	0	3	3	3	0	0

$$3x(n+2)=\{-3,-6,0,3,3,3,0,0\}$$

c)

The American University in Cairo

Computer Science and Engineering Department

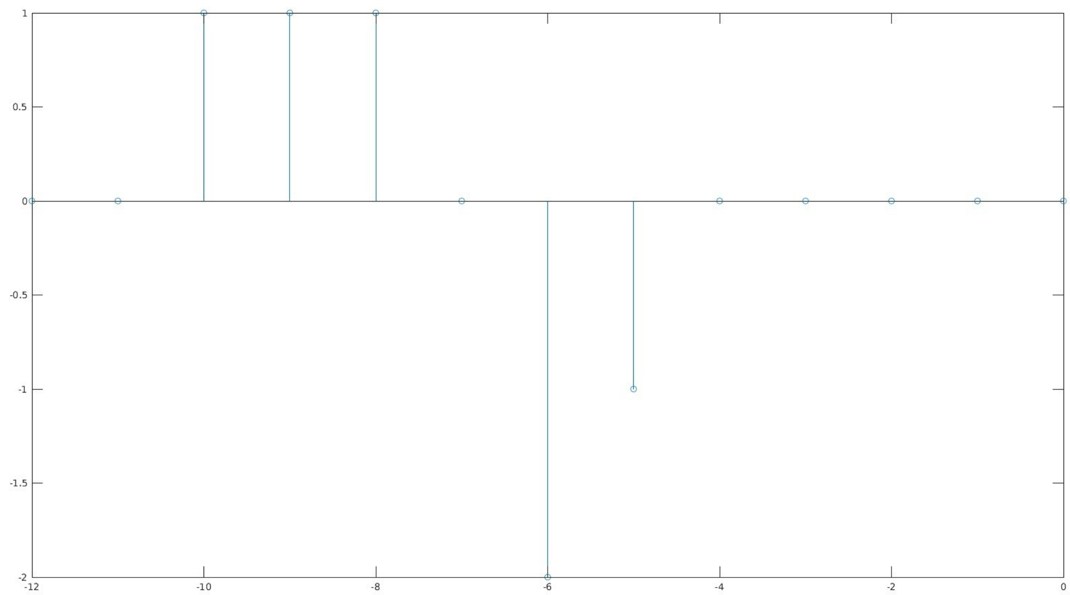
Fall 2022

CSCE 363/3611 Digital Signal Processing

Assignment #1

(Due on: October 15, 2022 at mid-night)

(The assignment is individual - Submit on Blackboard as one .zip file)



n	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
$x(n)$	0	0	1	1	1	0	-2	-1	0	0	0	0	0

$$x(-n-7)=x(-(n+7))=\{0,0,1,1,1,0,-2,-1,0,0,0,0,\underset{\uparrow}{0}\}$$

d)

The American University in Cairo

Computer Science and Engineering Department

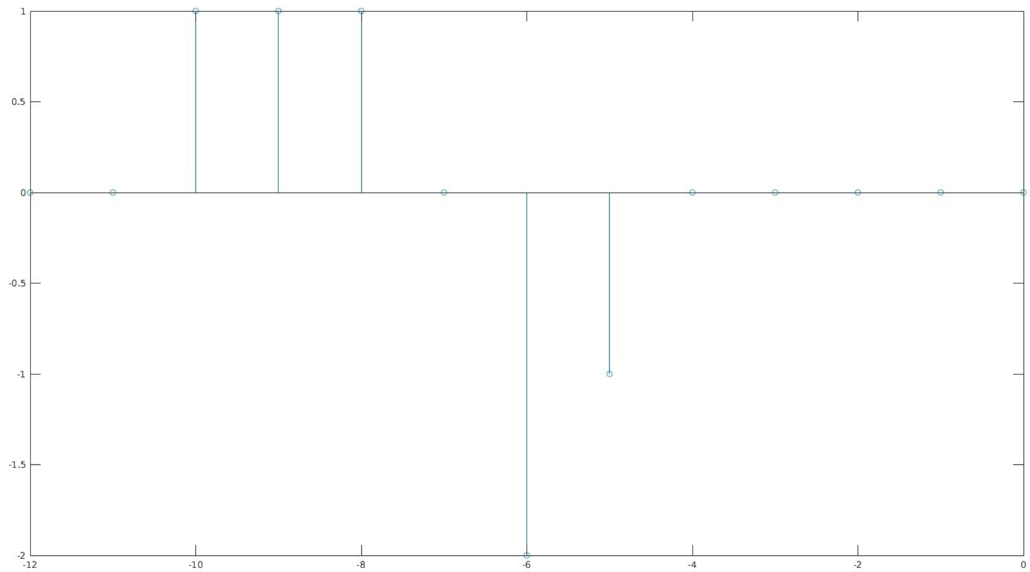
Fall 2022

CSCE 363/3611 Digital Signal Processing

Assignment #1

(Due on: October 15, 2022 at mid-night)

(The assignment is individual - Submit on Blackboard as one .zip file)



n	-2	-1	0	1	2	3	4	5
$x(n)$	0	0	0	1	1	1	0	0

$$x(n)u(n)=\{0,0,0,1,1,1,0,0\}$$

The American University in Cairo

Computer Science and Engineering Department

Fall 2022

CSCE 363/3611 Digital Signal Processing

Assignment #1

(Due on: October 15, 2022 at mid-night)

(The assignment is individual - Submit on Blackboard as one .zip file)

Problem 2

Represent the system given by the following input-output equation as a block-diagram:

$$y(n)=x(n-2)+2x(n-3)+x^2(n-2)+y(n-5)$$

Solution:

The American University in Cairo

Computer Science and Engineering Department

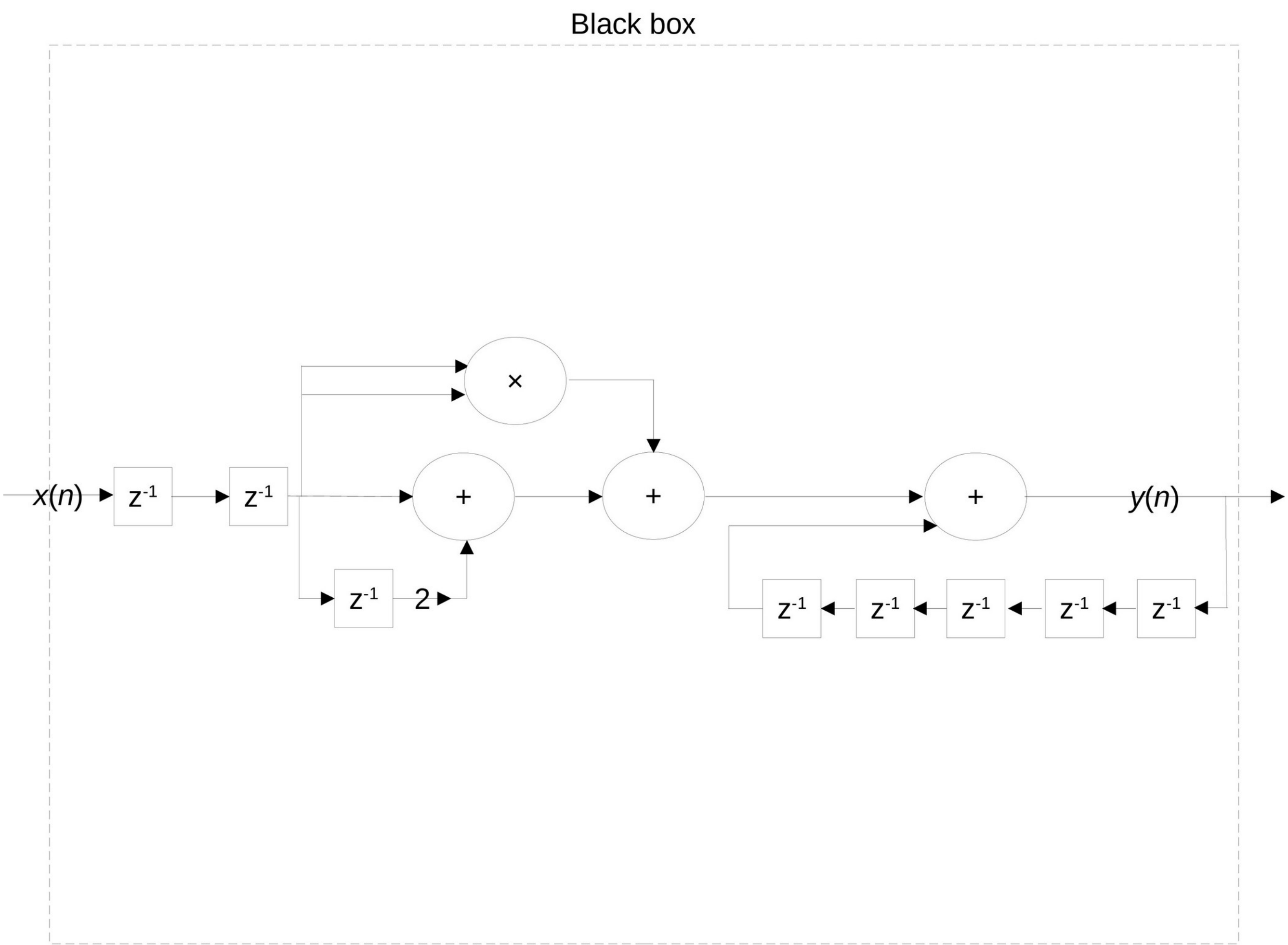
Fall 2022

CSCE 363/3611 Digital Signal Processing

Assignment #1

(Due on: October 15, 2022 at mid-night)

(The assignment is individual - Submit on Blackboard as one .zip file)



The American University in Cairo

Computer Science and Engineering Department

Fall 2022

CSCE 363/3611 Digital Signal Processing

Assignment #1

(Due on: October 15, 2022 at mid-night)

(The assignment is individual - Submit on Blackboard as one .zip file)

Problem 3

For each variant of the following systems, determine whether it is static or dynamic, time variant or time invariant, linear or non-linear, and causal or non-causal. Justify your answer:

a) $y(n)=x(n-7)+2x(n-3)$

b) $y(n)=(n+1)x(n)$

c) $y(n)=\cos(x(n))+x(n-3)$

Solution:

a) The system is dynamic because its output at any instant n depends on past samples of the input. $y(n, k) = x(n - k - 7) + 2x(n - k - 3)$ and $y(n - k) = x(n - k - 7) + 2x(n - k - 3)$. Since $y(n, k) = y(n - k) \rightarrow$ Time invariant. If we input $x_1(n)$ to the system, we get $y_1(n) = x_1(n - 7) + 2x_1(n - 3)$. If we input $x_2(n)$ to the system, we get $y_2(n) = x_2(n - 7) + 2x_2(n - 3)$. If we input $a_1x_1(n) + a_2x_2(n)$ to the system we get $y_3(n) = a_1(x_1(n - 7) + 2x_1(n - 3)) + a_2(x_2(n - 7) + 2x_2(n - 3))$. If we add $a_1y_1(n)$ and $a_2y_2(n)$, we get $a_1y_1(n) + a_2y_2(n) = a_1(x_1(n - 7) + 2x_1(n - 3)) + a_2(x_2(n - 7) + 2x_2(n - 3))$. Since $y_3(n) = a_1y_1(n) + a_2y_2(n)$, then the system is linear. The system is causal because its output at any time point depends only on past inputs.

The American University in Cairo

Computer Science and Engineering Department

Fall 2022

CSCE 363/3611 Digital Signal Processing

Assignment #1

(Due on: October 15, 2022 at mid-night)

(The assignment is individual - Submit on Blackboard as one .zip file)

b) The system is static because its output at any instant n depends at most on the input sample at the same time. $y(n, k) = (n + 1)x(n - k)$ and $y(n - k) = (n - k + 1)x(n - k)$. Since $y(n, k) \neq y(n - k) \rightarrow$ Time invariant. If we input $x_1(n)$ to the system, we get $y_1(n) = (n + 1)x_1(n)$. If we input $x_2(n)$ to the system, we get $y_2(n) = (n + 1)x_2(n)$. If we input $a_1x_1(n) + a_2x_2(n)$ to the system we get $y_3(n) = a_1((n + 1)x_1(n)) + a_2((n + 1)x_2(n))$. If we add $a_1y_1(n)$ and $a_2y_2(n)$, we get $a_1y_1(n) + a_2y_2(n) = a_1((n + 1)x_1(n)) + a_2((n + 1)x_2(n))$. Since $y_3(n) = a_1y_1(n) + a_2y_2(n)$, then the system is linear. The system is non-causal because its output at any time point depends on future inputs.

c) The system is dynamic because its output at any instant n depends on past samples of the input. $y(n, k) = \cos(x(n - k)) + x(n - k - 3)$ and $y(n - k) = \cos(x(n - k)) + x(n - k - 3)$. Since $y(n, k) = y(n - k) \rightarrow$ Time invariant. If we input $x_1(n)$ to the system, we get $y_1(n) = \cos(x_1(n)) + x_1(n - 3)$. If we input $x_2(n)$ to the system, we get $y_2(n) = \cos(x_2(n)) + x_2(n - 3)$. If we input $a_1x_1(n) + a_2x_2(n)$ to the system we get $y_3(n) = \cos(a_1x_1(n)) + a_1x_1(n - 3) + \cos(a_2x_2(n)) + a_2x_2(n - 3)$. If we add $a_1y_1(n)$ and $a_2y_2(n)$, we get $a_1y_1(n) + a_2y_2(n) = a_1\cos(x_1(n)) + a_1x_1(n - 3) + a_2\cos(x_2(n)) + a_2x_2(n - 3)$. Since $y_3(n) \neq a_1y_1(n) + a_2y_2(n)$, then the system is non-linear. The system is causal because its output at any time point depends only on past inputs.

The American University in Cairo

Computer Science and Engineering Department

Fall 2022

CSCE 363/3611 Digital Signal Processing

Assignment #1

(Due on: October 15, 2022 at mid-night)

(The assignment is individual - Submit on Blackboard as one .zip file)

Problem 4

Implement the following functions using MATLAB or Python:

- Downsample: A function that takes as input the file name of an input audio file, the downsampling factor (integer only), and the file name of the output downsampled file. This function should change the sampling rate of the input audio from F_S to (F_S /downsampling factor) without changing the duration of the audio.
- IncreaseSpeed: A function that takes as input the file name of an input audio file, the speeding factor (integer only), and the file name of the output increased speed audio file. This function should change the duration of the input audio from T to (T/speeding factor) without changing the sampling rate.

Deliverables:

- Your code (either MATLAB .m files or Python .py or Jupyter notebook files).
- Apply the Downsample function to the audio file provided "Audio1.wav" using downsampling factor of 2, 3, and 10. Name the output files "Audio1_Down_2.wav", "Audio1_Down_3.wav", and "Audio_Down_10.wav", respectively. The sampling rate of the original audio is 14.4kHz.
- The file "Audio1.wav" represents the following signal:

$$x_a(t) = 0.1\sin(2\pi \times 1000t) + 0.9\sin(2\pi \times 3000t + \frac{\pi}{3})$$

Comment on which of the downsampling factors results in a noticeable change in the audio and the reasons for such a change.

- Apply the Downsample function to the audio file provided “Audio2.wav” using a downsampling factor of 2, 10, and 35. Name the output files “Audio2_Down_2.wav”, “Audio2_Down_10.wav”, and “Audio2_Down_35.wav”, respectively. The sampling rate of the original audio is 44.1kHz.
- Comment on the quality of the audio obtained after downsampling the file “Audio2.wav”.
- Apply the IncreaseSpeed function to the audio file provided “Audio1.wav” using a speeding factor of 2, 3, and 4. Name the output files “Audio1_Inc_2.wav”, “Audio1_Inc_3.wav”, and “Audio1_Inc_4.wav”, respectively.
- Apply the IncreaseSpeed function to the audio file provided “Audio2.wav” using a speeding factor of 2, 3, and 4. Name the output files “Audio2_Inc_2.wav”, “Audio2_Inc_3.wav”, and “Audio2_Inc_4.wav”, respectively.

Solution:

$$x_a(nT) = x(n)$$

Using a downsampling factor of 2, we get

$$x_2(n)=0.1\sin\left(\frac{5\pi}{18}n\right)+0.9\sin\left(\frac{5\pi}{6}n+\frac{\pi}{3}\right)$$

Using a downsampling factor of 4, we get

$$x_4(n)=0.1\sin\left(\frac{5\pi}{12}n\right)+0.9\sin\left(\frac{5\pi}{4}n+\frac{\pi}{3}\right)$$

Mapping ω to be in the range $-\pi \le \omega \le \pi$

$$\frac{5\pi}{4}-2\pi=\frac{-3\pi}{4}$$

$$x_{10}(n)=0.1\sin\left(\frac{5\pi}{12}n\right)+0.9\sin\left(\frac{-3\pi}{4}n+\frac{\pi}{3}\right)$$

Using a downsampling factor of 10, we get

$$x_{10}(n)=0.1\sin\left(\frac{25\pi}{18}n\right)+0.9\sin\left(\frac{25\pi}{6}n+\frac{\pi}{3}\right)$$

Mapping each ω to be in the range $-\pi \le \omega \le \pi$

$$\frac{25\pi}{18}-2\pi=\frac{-11\pi}{18}$$

$$\frac{25\pi}{6}-2\pi=\frac{\pi}{6}$$

The American University in Cairo

Computer Science and Engineering Department

Fall 2022

CSCE 363/3611 Digital Signal Processing

Assignment #1

(Due on: October 15, 2022 at mid-night)

(The assignment is individual - Submit on Blackboard as one .zip file)

$$x_{10}(n)=0.1\sin\left(\frac{-11\pi}{18}n\right)+0.9\sin\left(\frac{\pi}{6}n+\frac{\pi}{3}\right)$$

The Sampling Theorem

If the highest frequency contained in an analog signal $x_a(t)$ is F_{max} and the signal is sampled at a rate $F_S > 2F_{max}$, then $x_a(t)$ can be exactly recovered from its sample values

The sampling rate $F_N = 2F_{max}$ is called the Nyquist rate

The frequencies present in the signal are $F_1 = 1000$ Hz and $F_2 = 3000$ Hz

Thus, $F_{max} = 3000$ Hz

The Nyquist rate is then 3000 × 2 = 6000 Hz

The downsampling factors that result in a noticeable change in the audio are 4 and 10. The reasons for such a change is that dividing the sampling frequency of 14.4 kHz by 4 results in a sampling frequency of 4.8 kHz. 4.8 kHz is less than twice the highest frequency in the audio. Similarly, dividing the sampling frequency of 14.4 kHz by 10 results in a sampling frequency of 1.44 kHz. This frequency is much less than twice the highest frequency in the audio.

The quality of the audio obtained after downsampling the file “Audio2.wav” dramatically decreased.

Important Notes:

- All deliverables should be included in one .zip file.

The American University in Cairo

Computer Science and Engineering Department

Fall 2022

CSCE 363/3611 Digital Signal Processing

Assignment #1

(Due on: October 15, 2022 at mid-night)

(The assignment is individual - Submit on Blackboard as one .zip file)

- For Problems 1, 2, and 3, write your answers in a Word or PDF file and include it in the .zip file.
- For Problem 4, you need to implement the downsampling and increasing speed from scratch. Don’t use ready made functions that you might find available in MATLAB or Python.
- This is an individual assignment.